

$Z_c(3900)$ from Lattice QCD Simulation

Yoichi IKEDA (RCNP, Osaka Univ.)

HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, D. Kawai, T. Miyamoto, K. Sasaki (YITP, Kyoto Univ.)

T. Doi, T. Hatsuda (RIKEN)

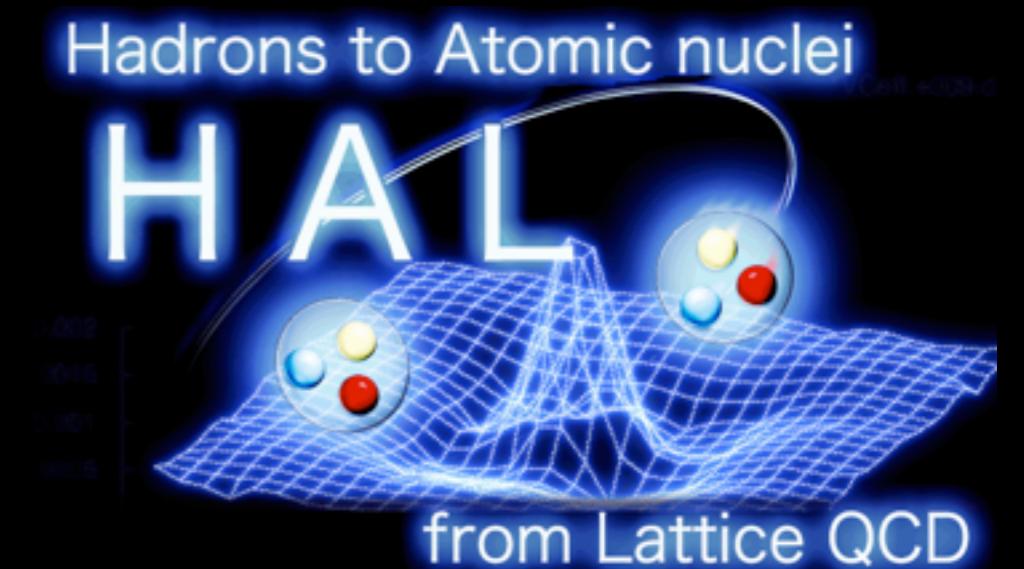
S. Gongyo (Univ. Tours)

T. Inoue (Nihon Univ.)

Y. Ikeda, N. Ishii, K. Murano (RCNP, Osaka Univ.)

T. Iritani (Stony Brook Univ.)

H. Nemura (Univ. Tsukuba)



Charm Hadron Interactions from Lattice QCD Simulation

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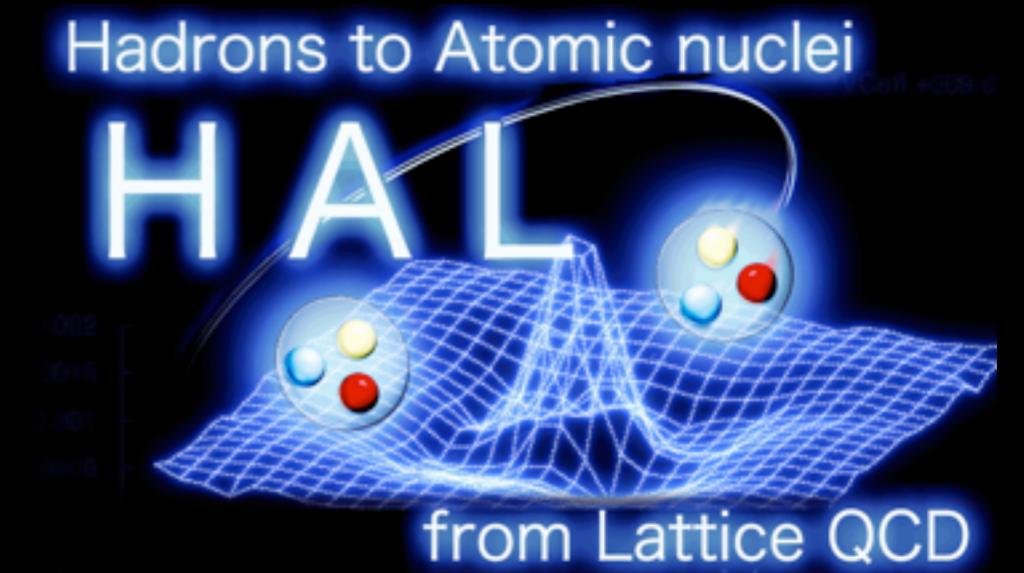
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The 14th International Conference on Meson-Nucleon Physics and the Structure of the Nucleon
@Kyoto, Japan (Jul. 25 -- 30, 2016.)

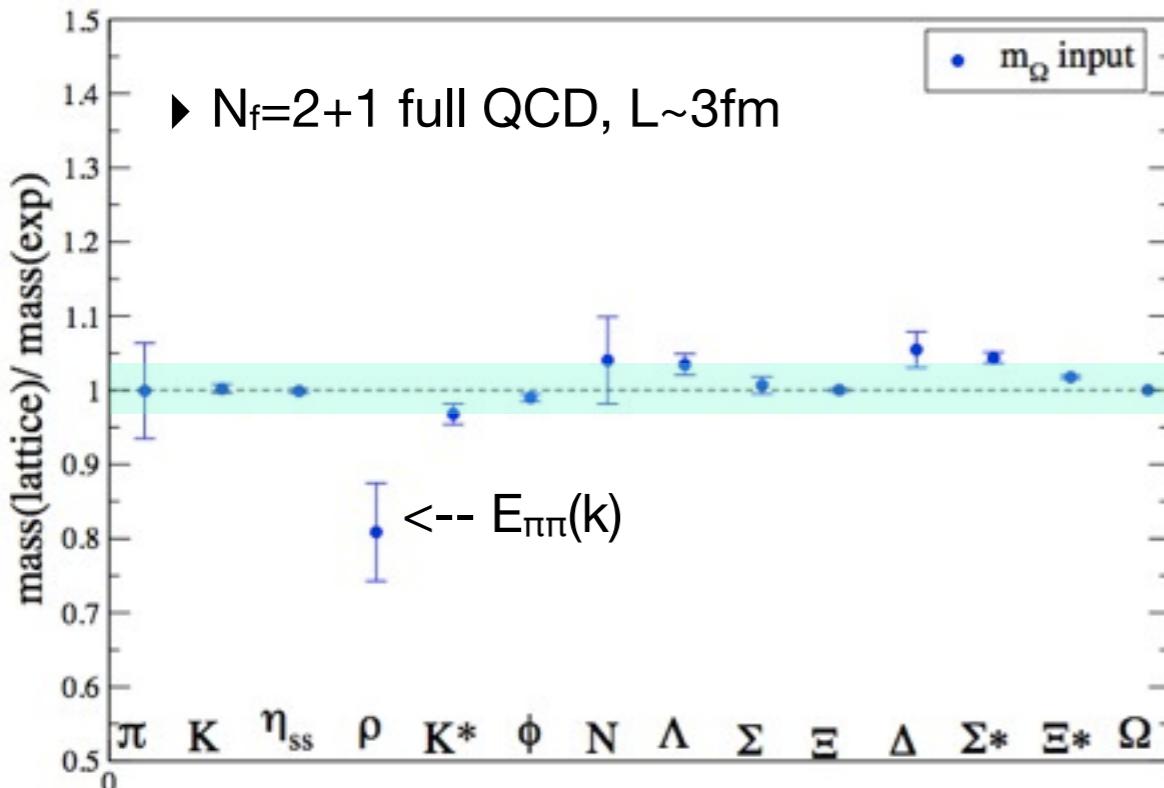
Contents

- ✓ Lattice QCD at physical point -- single hadron spectroscopy --
- ✓ Problems in multi-hadron scatterings in LQCD simulation
 - ▶ large volume problem (lattice has to accommodate hadron interactions)
 - ▶ coupled-channel problem (many resonances are embedded into c.c. continuum)
- ✓ HAL QCD approach to extract hadron interactions
- ✓ Results on charm hadron interactions
 - ▶ $Z_c(3900)$ in s-wave meson-meson scattering
 - ▶ $D^{\bar{b}a}$ & Λ_c interactions
- ✓ Summary

Single hadron spectroscopy from LQCD

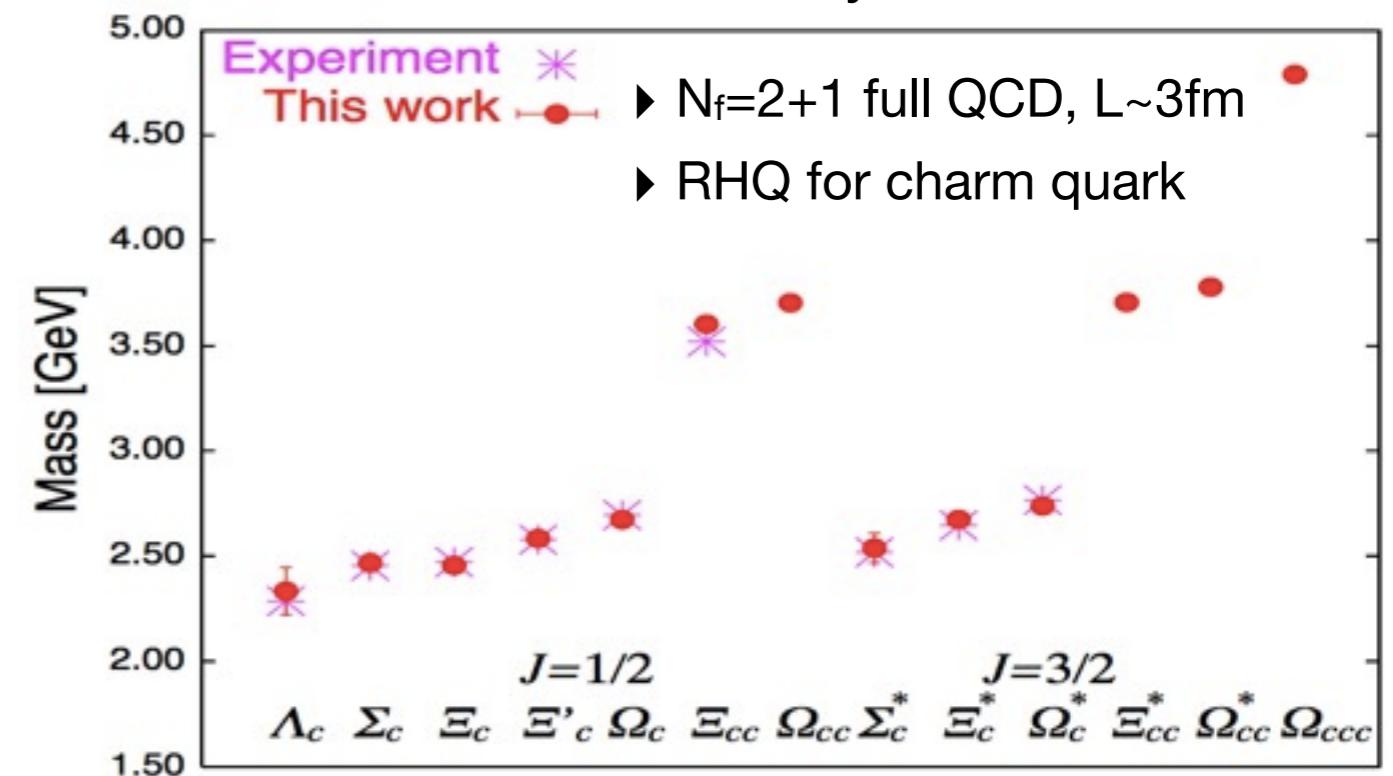
★ Low-lying hadrons on physical point (physical m_q)

light-quark sector

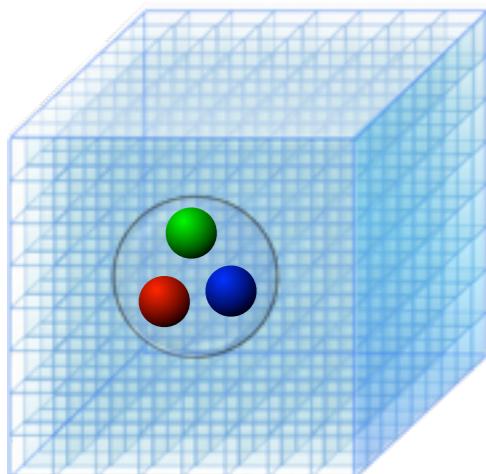


Aoki et al. (PACS-CS), PRD81 (2010).

charm baryons



Namekawa et al. (PACS-CS), PRD84 (2011); PRD87 (2013).



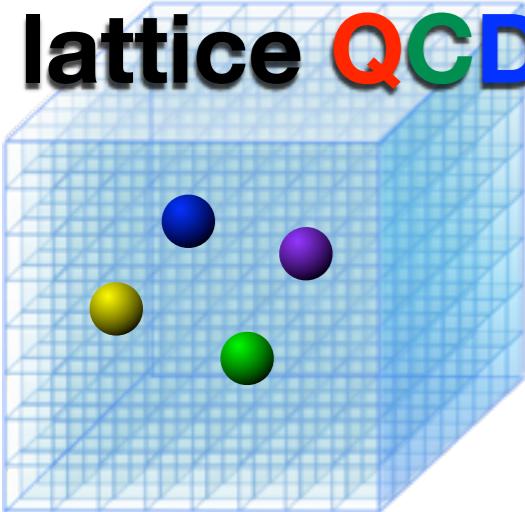
a few % accuracy already achieved for single hadrons

LQCD now can predict undiscovered charm hadrons (Ξ_{cc} , Ξ_{cc}^* , Ω_{ccc} , ...)

→ Next challenge : multi-hadron systems

Problems in multi-hadron scatterings (1)

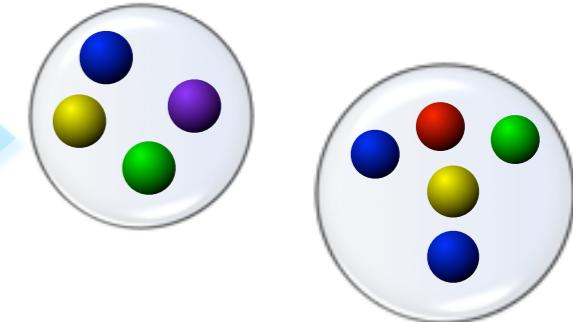
lattice QCD



Scattering on the lattice

- (1) large V
- (2) coupled-channel

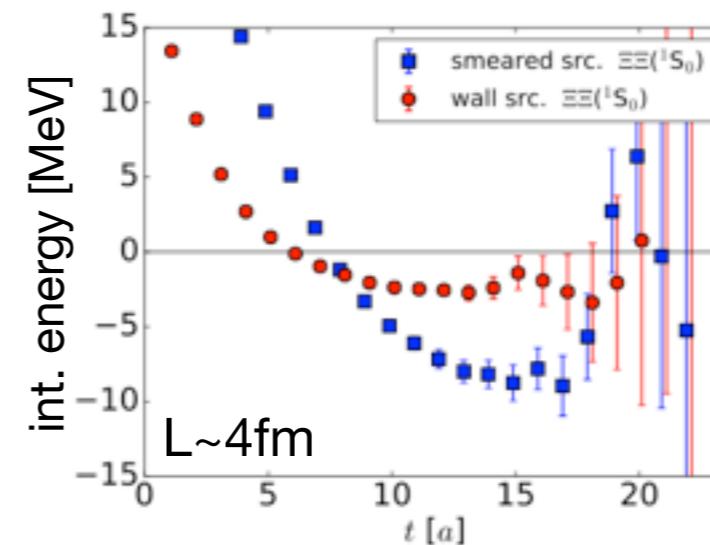
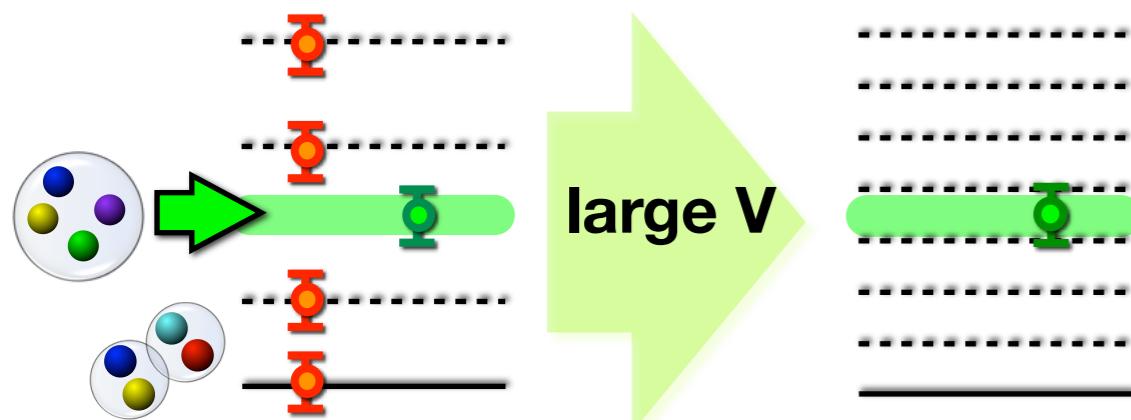
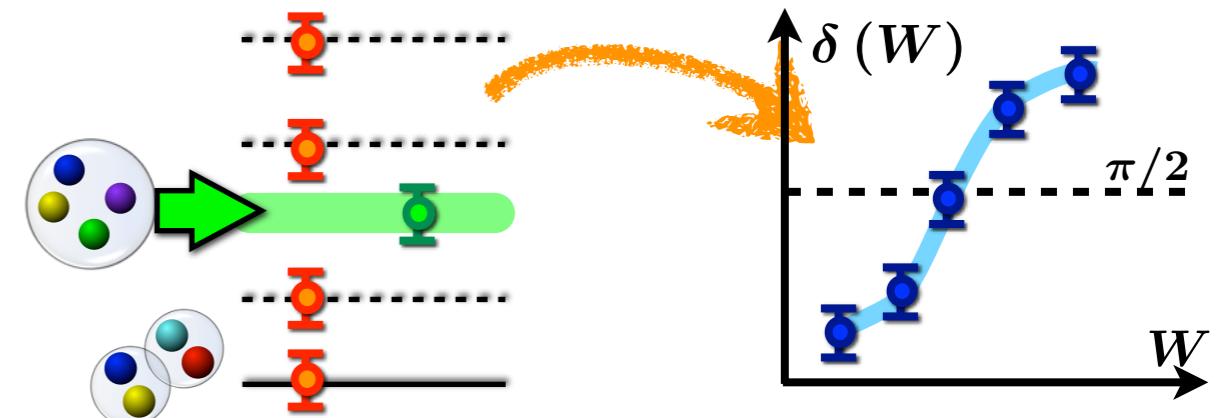
$\checkmark \underline{X, Y, Z, P_c, \dots}$



★ single channel scattering

$$\langle 0 | \Phi(\tau) \Phi^\dagger(0) | 0 \rangle = \sum_n A_n e^{-W_n \tau}$$

- ❖ Lüscher's formula Lüscher, NPB354 (1991).
 - ▶ finite V spectrum \rightarrow phase shift $\delta(W_n)$

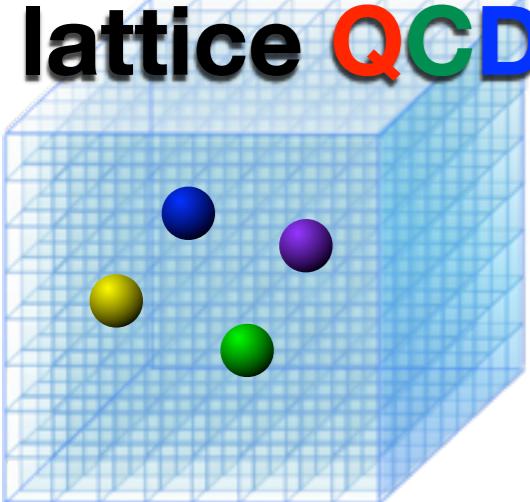


▶ fake plateau easily appears...

Iritani [HAL QCD], arXiv:1607.06371(2016).

Problems in multi-hadron scatterings (1)

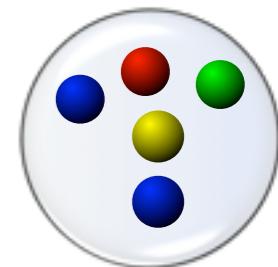
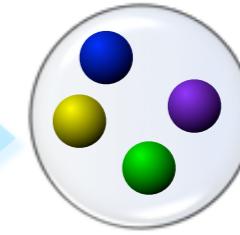
lattice QCD



Scattering on the lattice

- (1) large V
- (2) coupled-channel

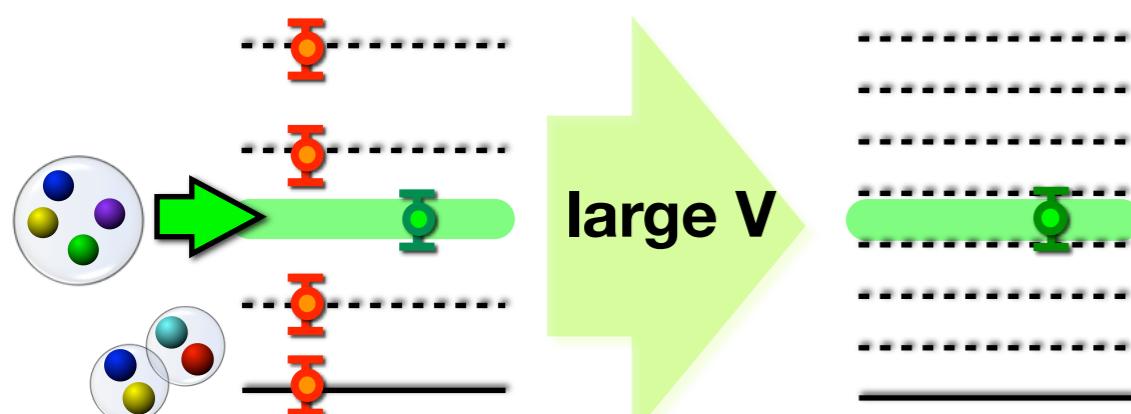
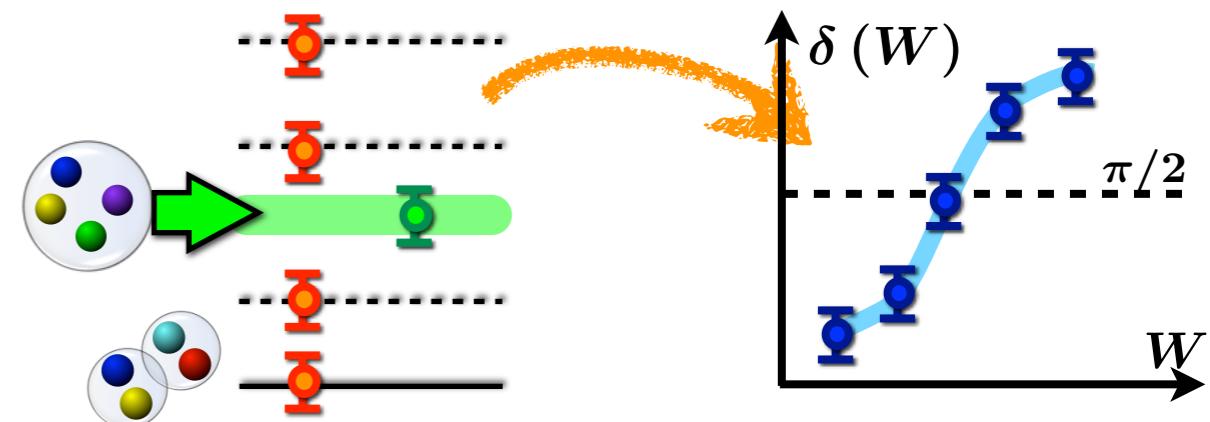
✓ X, Y, Z, P_c,...



★ single channel scattering

$$\langle 0 | \Phi(\tau) \Phi^\dagger(0) | 0 \rangle = \sum_n A_n e^{-W_n \tau}$$

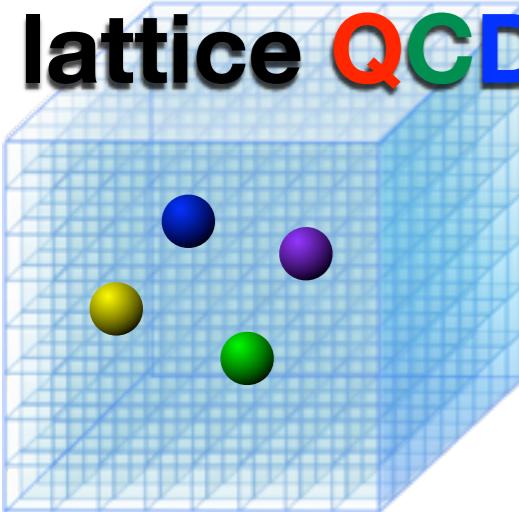
- ❖ Lüscher's formula Lüscher, NPB354 (1991).
 - ▶ finite V spectrum --> phase shift $\delta(W_n)$



- ★ difficult to identify W_n due to contamination
 - energy-independent quantity from LQCD to obtain S-matrix is a key

Problems in multi-hadron scatterings (2)

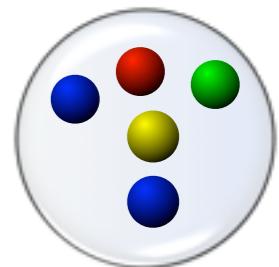
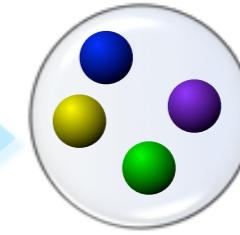
lattice QCD



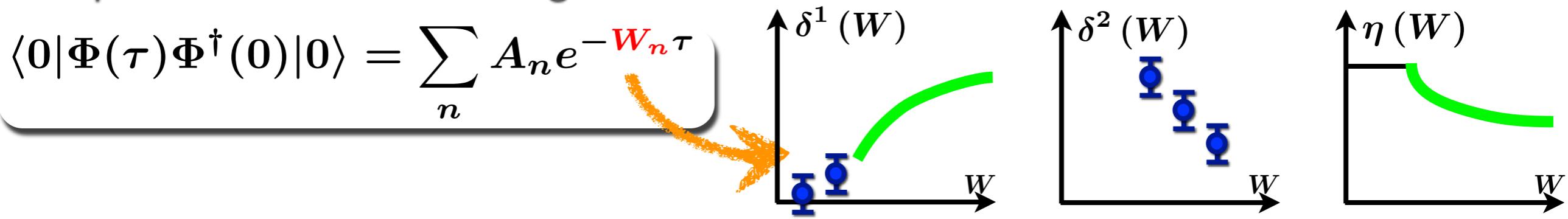
Scattering on the lattice

- (1) large V
- (2) coupled-channel

$\checkmark \underline{X, Y, Z, P_c, \dots}$



★ coupled-channel scattering



→ coupled-channel Lüscher's formula

e.g., 2-channel : $\delta^1(W)$, $\delta^2(W)$, $\eta(W)$ --> find $W(L_1)=W(L_2)=W(L_3)$

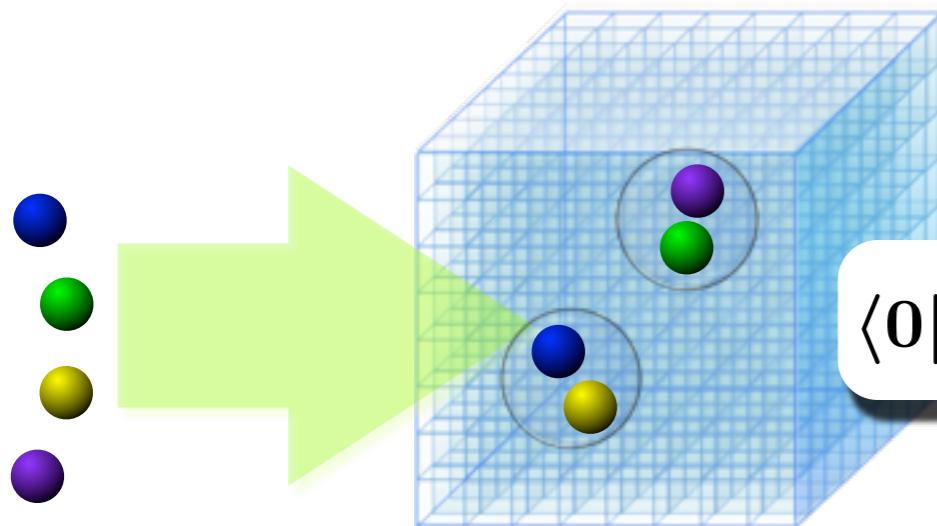
→ **assumptions** about interaction or K-matrices necessary...

★ indicate **more information mandatory** to solve coupled-channel scatterings

→ What can we measure in addition to temporal correlations?

HAL QCD “potential” approach

- ◆ HAL QCD approach: derive **energy-independent** interaction kernel
 - measure **spatial** correlation as well as temporal correlation

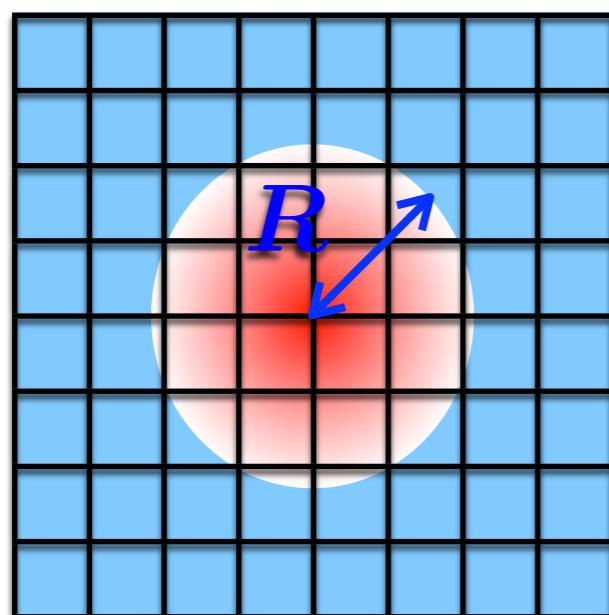


$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) | W_n \rangle = \sqrt{Z_1 Z_2} \psi_n(\vec{r}) e^{-W_n \tau}$$

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al,(HAL QCD), PLB712, 437(2012).

★ Nambu-Bethe-Salpeter wave functions: $\Psi_n(\vec{r})$

→ Equal-time choice of NBS amplitudes



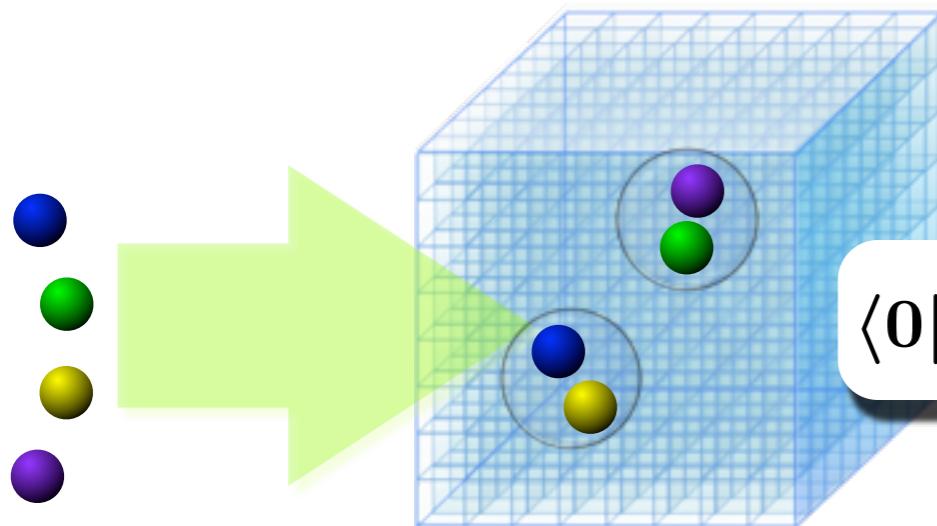
✓ NBS wave functions satisfy **Helmholtz equation outside interactions** (phase shift --> S-matrix)

$$(\nabla^2 + \vec{k}_n^2) \psi_n(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

→ Nonrelativistic approx. is **NOT** required

HAL QCD “potential” approach

- ◆ HAL QCD approach: derive **energy-independent** interaction kernel
 - measure **spatial** correlation as well as temporal correlation

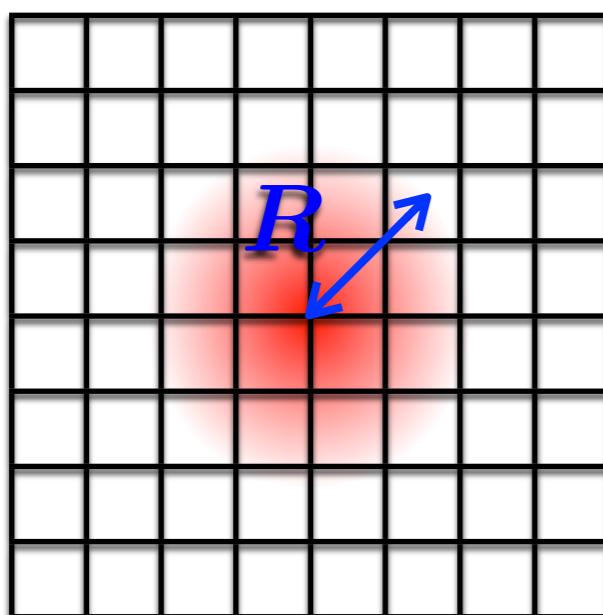


$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) | W_n \rangle = \sqrt{Z_1 Z_2} \psi_n(\vec{r}) e^{-W_n \tau}$$

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
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Ishii et al,(HAL QCD), PLB712, 437(2012).

- ★ Nambu-Bethe-Salpeter wave functions: $\Psi_n(\vec{r})$

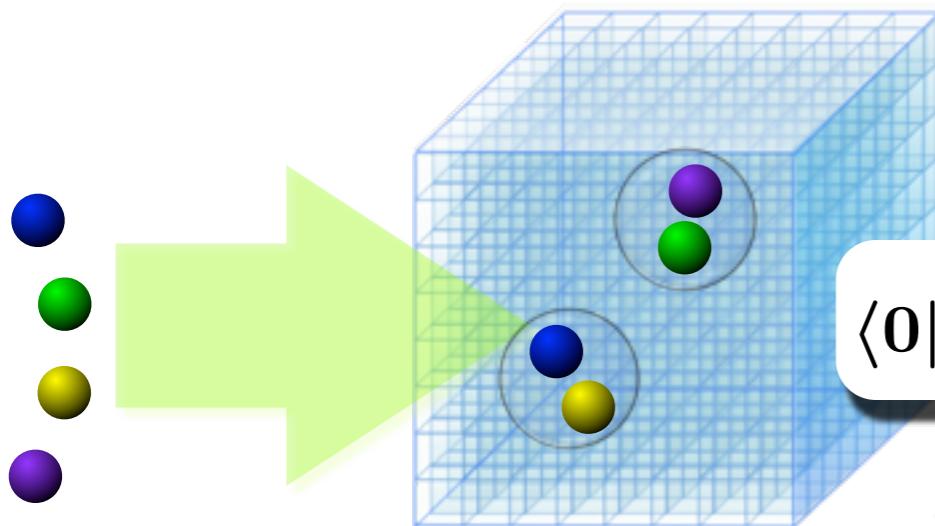
$$(\nabla^2 + \vec{k}_n^2) \psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_n(\vec{r}')$$



- $U(r, r')$ is energy-independent (until new threshold opens)
- $U(r, r')$ contains all 2PI contributions
- $U(r, r')$ is NOT an observables (faithful to S-matrix)

Coupled-channel HAL QCD approach

- ◆ HAL QCD approach: derive **energy-independent** interaction kernel
 - measure **spatial** correlation as well as temporal correlation



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al,(HAL QCD), PLB712, 437(2012).

$$\langle 0 | \phi_1^a(\vec{x} + \vec{r}, \tau) \phi_2^a(\vec{x}, \tau) | W_n \rangle = \sqrt{Z_1^a Z_2^a} \psi_n^a(\vec{r}) e^{-W_n \tau}$$

★ NBS wave functions for each channel: $\Psi_n^a(\vec{r})$

$$(\nabla^2 + (\vec{k}_n^a)^2) \psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r}, \vec{r}') \psi_n^b(\vec{r}')$$

✓ derivative expansion:

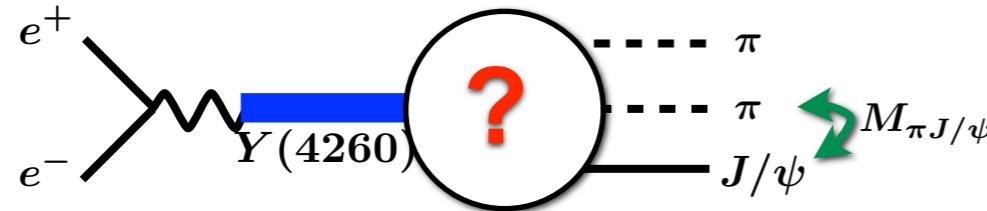
$$U(\vec{r}, \vec{r}') = \{V_{\text{LO}}(\vec{r}) + \mathcal{O}(\nabla)\} \delta(\vec{r} - \vec{r}')$$

This work: extract effective LO potential

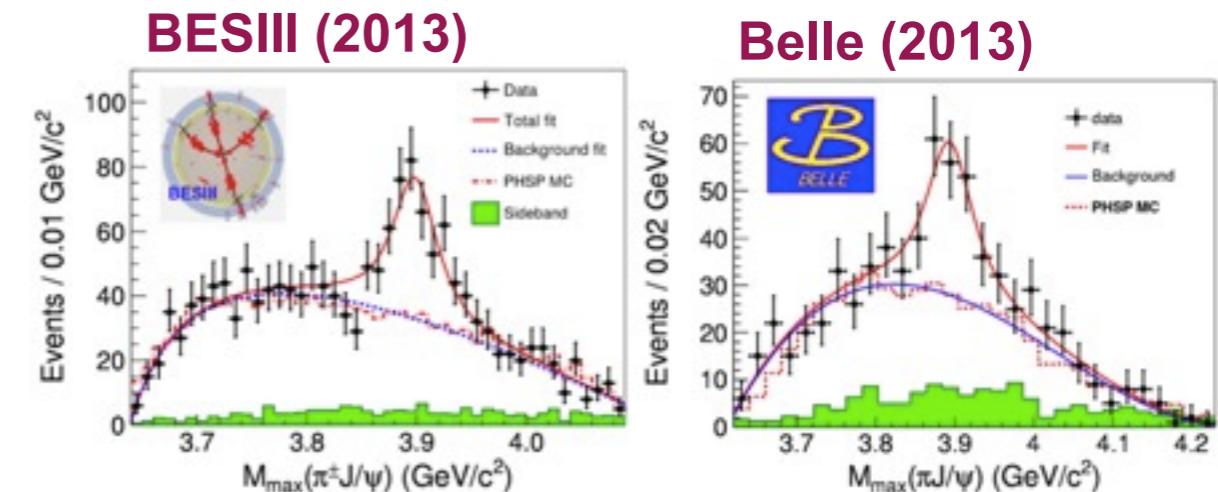
$$(\nabla^2 + (\vec{k}_n^a)^2) \psi_n^a(\vec{r}) = 2\mu^a \sum_b V_{\text{LO}}^{ab}(\vec{r}) \psi_n^b(\vec{r})$$

Targets: charm tetraquark & charm nuclei

✓ Tetraquark candidate $Z_c(3900)$

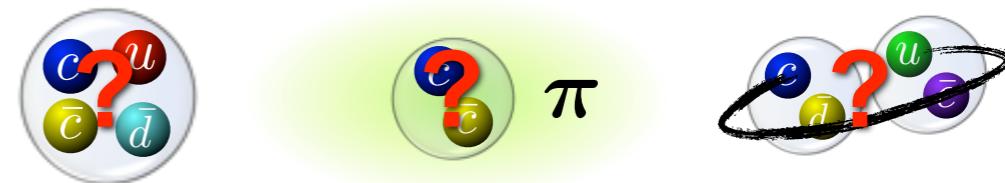


► $Z_c(3900)$ found in $\pi^{+-}J/\psi$ ($cc^{\bar{b}a}ud^{\bar{b}a}$)



structure of $Z_c(3900)$

- tetraquark?, hadro-charmonium?, $D^{\bar{b}a}D^*$ molecule?
- threshold kinematical effect?



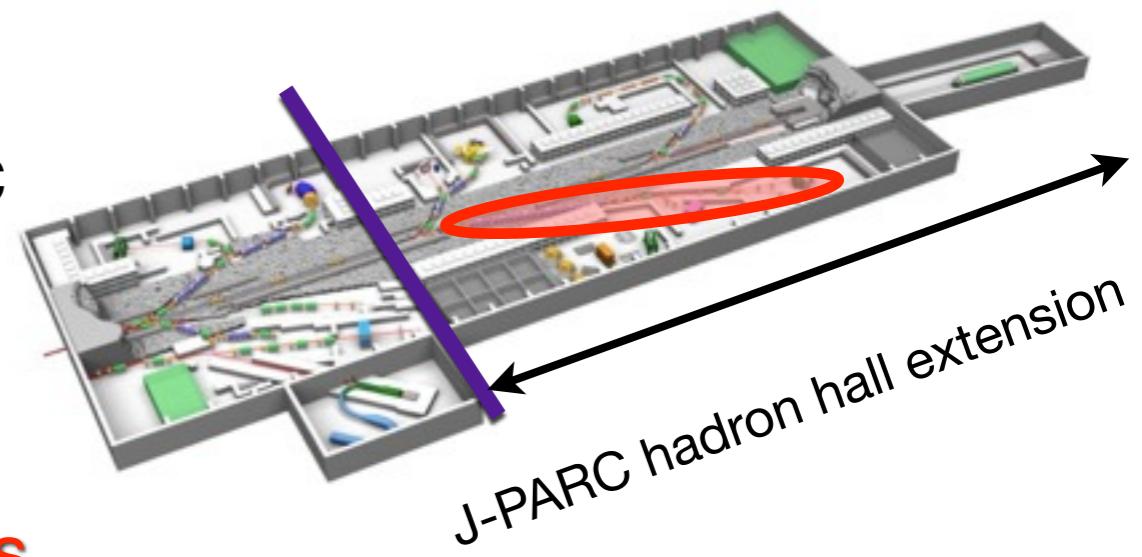
📌 $Z_c(3900)$ in s-wave $\pi J/\psi - \rho \eta_c - D^{\bar{b}a}D^*$ coupled-channel system

✓ $D^{\bar{b}a}$ & Λ_c nuclei

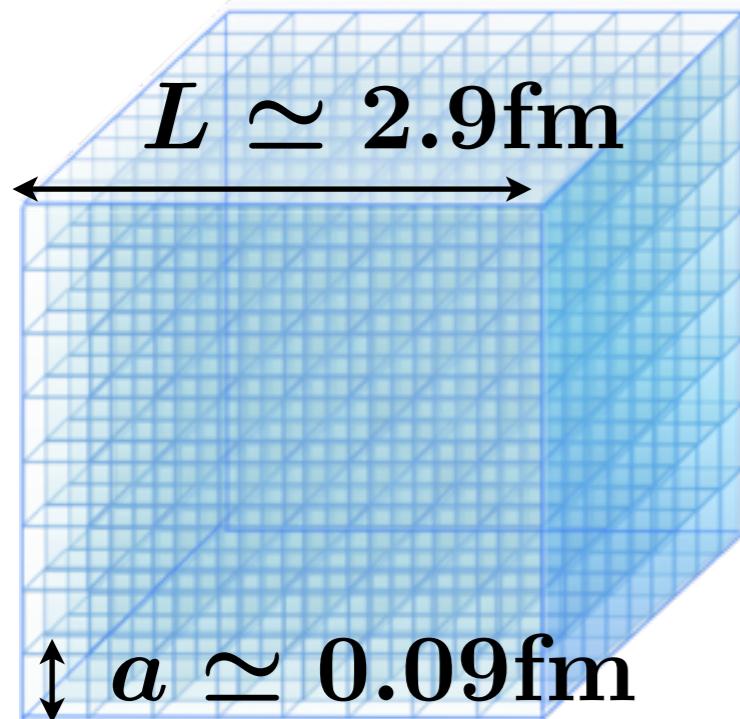
- Charm nuclei will be produced @K10, J-PARC
- What are possible structures?

see e.g., Yasui, Sudoh, Yamaguchi, Hyodo, Hosaka, arXiv:1606.08685 (2016).

📌 s-wave $D^{\bar{b}a}N$ & $\Lambda_c N$ interactions



Lattice QCD setup



★ N_f=2+1 full QCD

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Iwasaki gauge & O(a)-improved Wilson quark actions
- $a=0.0907(13)$ fm $\rightarrow L \sim 2.9$ fm ($32^3 \times 64$)

★ Relativistic Heavy Quark action for charm

S. Aoki et al., PTP109, 383 (2003).

Y. Namekawa et al., PRD84, 074505 (2011).

- remove leading cutoff errors $O((m_c a)^n)$, $O(\Lambda_{QCD} a)$, ...

⇒ We are left with $O((a\Lambda_{QCD})^2)$ syst. error (~ a few %)

- three sets of full QCD gauge configs. used ($m_\pi \sim 410\text{-}700\text{MeV}$)

light hadron mass (MeV)

$m_\pi = 411, 572, 701$

$m_K = 635, 714, 787$

$m_\rho = 896, 1000, 1097$

$m_N = 1215, 1411, 1583$

Charmed meson mass (MeV)

$m_{\eta_c} = 2988, 3005, 3024$

$m_{J/\psi} = 3097, 3118, 3143$

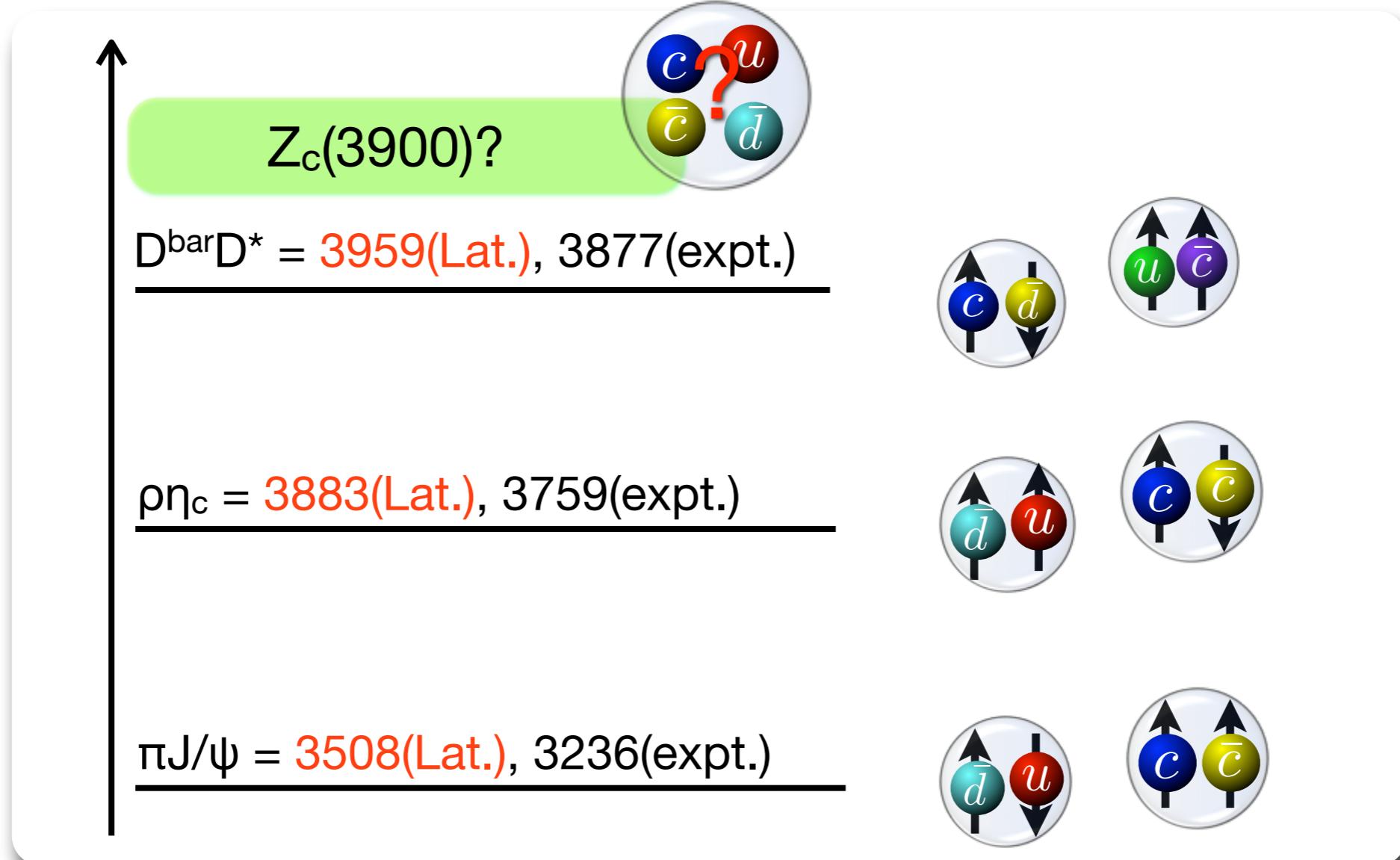
$m_D = 1903, 1947, 2000$

$m_{D^*} = 2056, 2101, 2159$

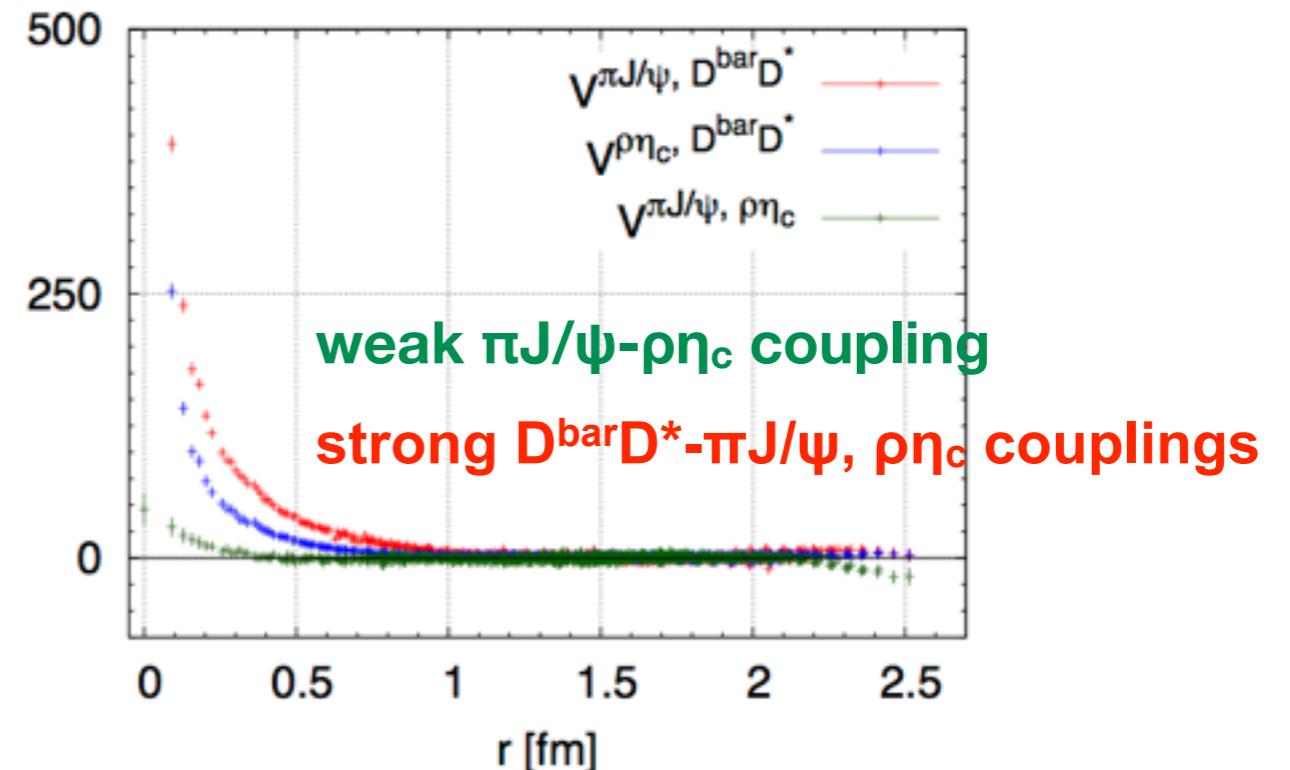
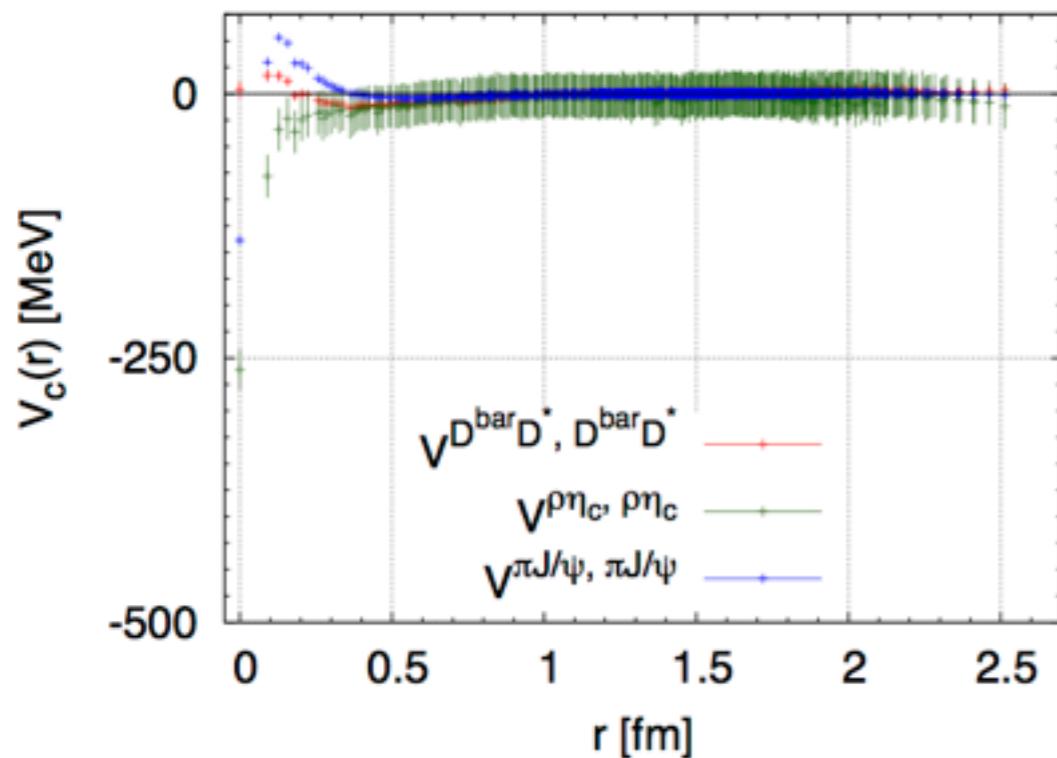
$m_{\Lambda_c} = 2434, 2584, 2710$

Structure of $Z_c(3900)$

-- $\pi J/\psi - \rho \eta_c - D^{\bar{b}ar}D^*$ in $|G(J^{PC})=1^+(1^{+-})$ --



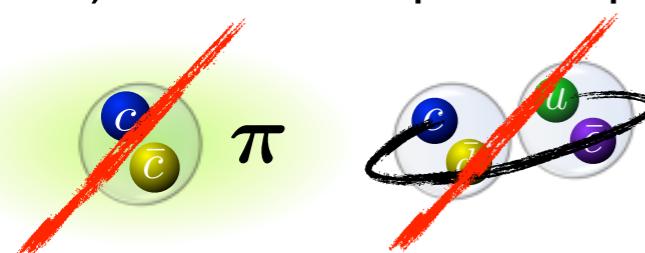
S-wave $\pi J/\psi - \rho \eta_c - D^{\bar{b}ar}D^*$ potential @ $m_\pi=410\text{MeV}$



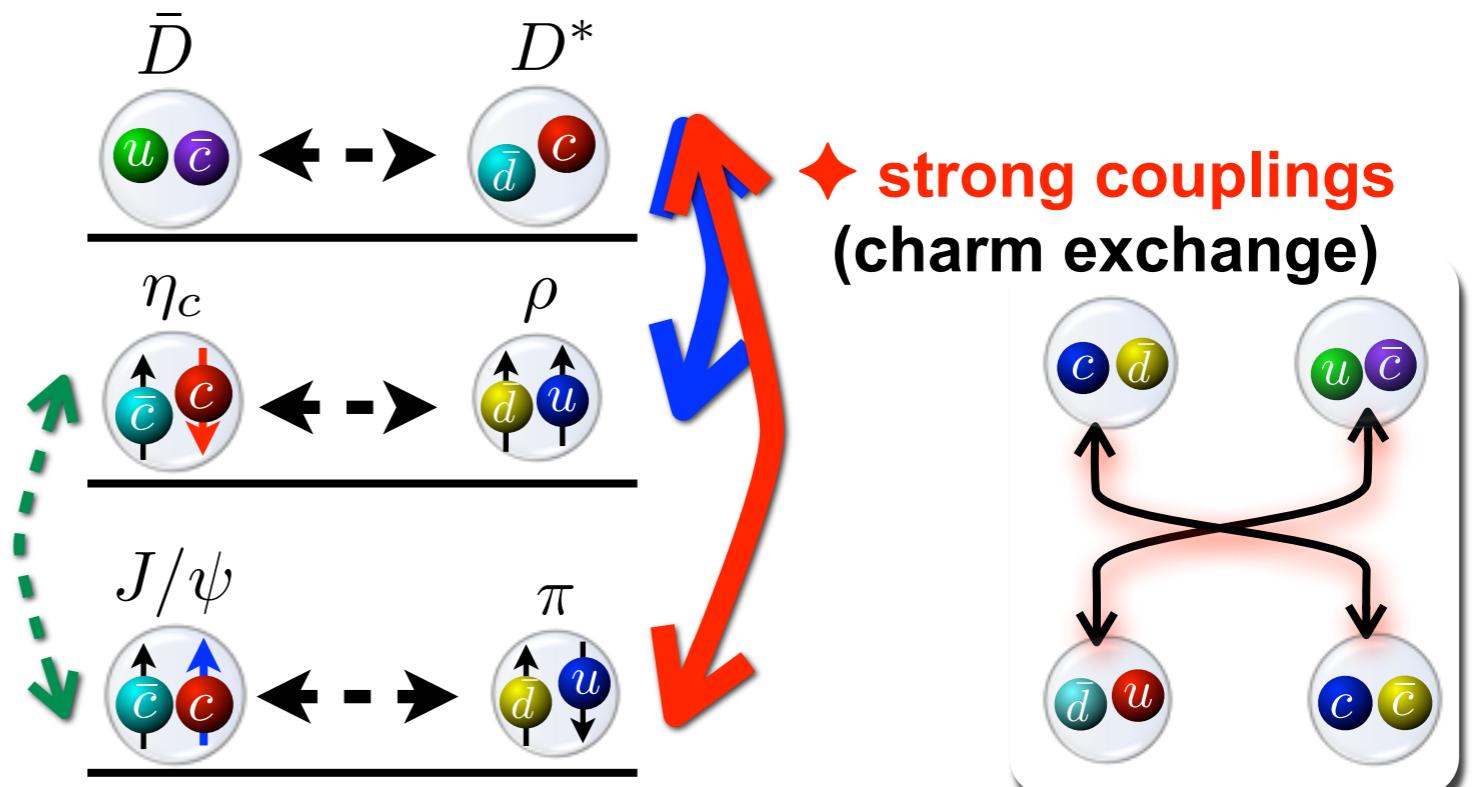
[Y. Ikeda \(HAL QCD\), arXiv.1602.03465\(hep-lat\).](#)

★ weak diagonal potentials:

$Z_c(3900)$ is NOT simple $\pi J/\psi$ & $D^{\bar{b}ar}D^*$



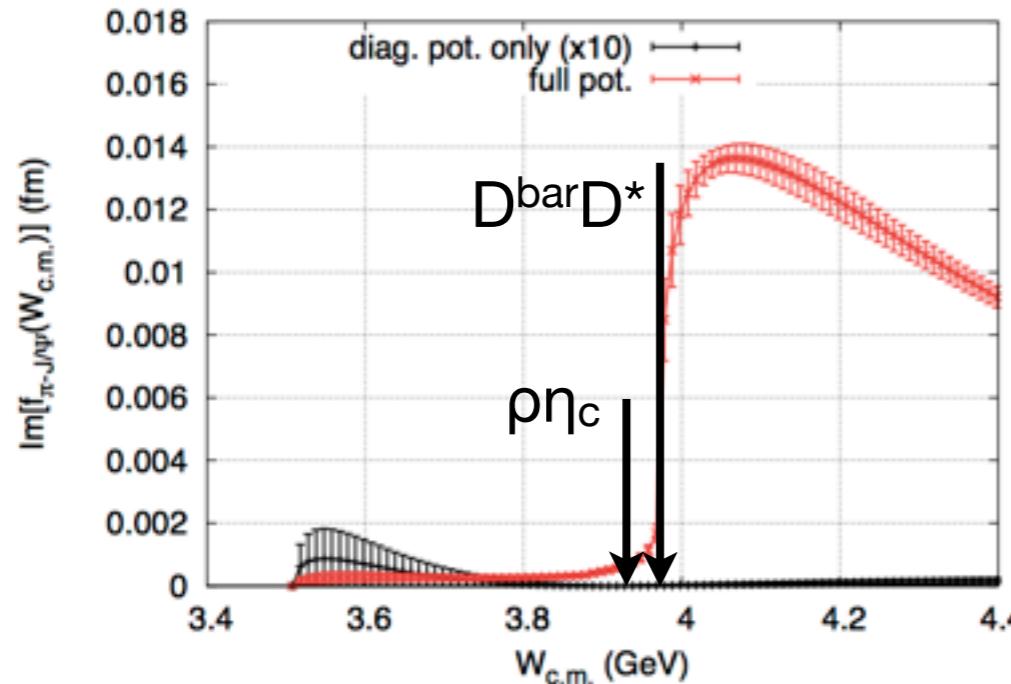
◆ heavy quark spin symmetry



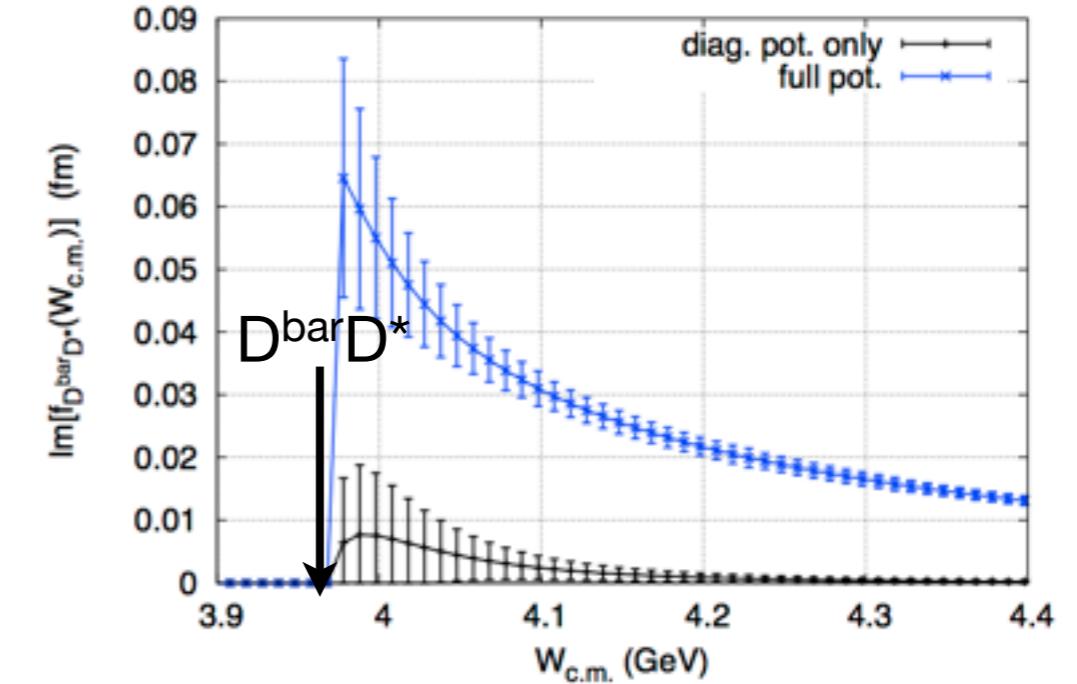
Invariant mass spectra of $\pi J/\psi$ & $D^{\bar{b}ar}D^*$

★ 2-body scattering (ideal setting to understand $Z_c(3900)$ structure)

- $\pi J/\psi$ invariant mass



- $D^{\bar{b}ar}D^*$ invariant mass



$$\frac{\Gamma(Zc(3900) \rightarrow \bar{D}D^*)}{\Gamma(Zc(3900) \rightarrow \pi J/\psi)} = 6.2(1.1)(2.7)$$

[BESIII Coll., PRL112 \(2014\).](#)

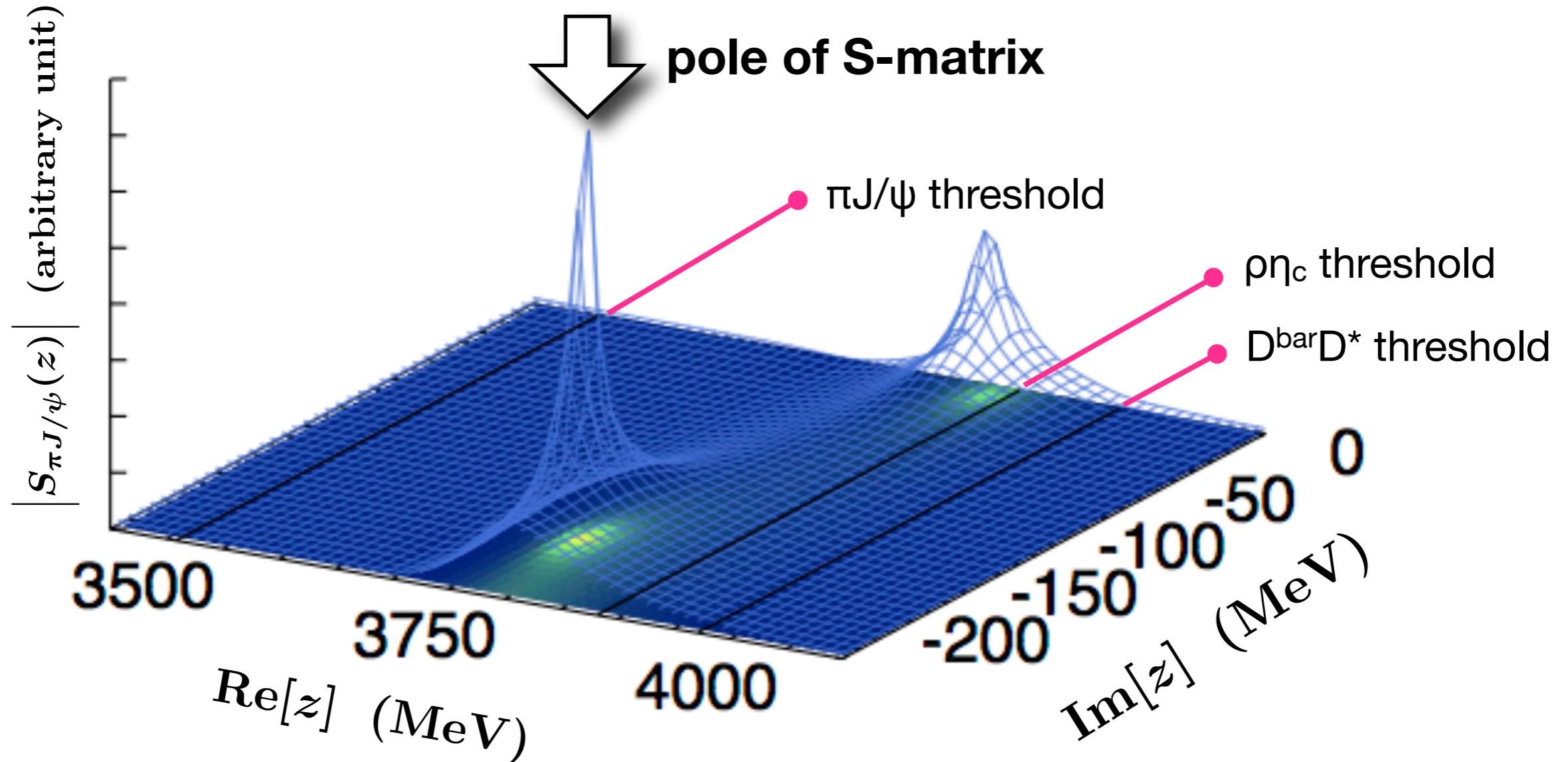
✓ Enhancement near $D^{\bar{b}ar}D^*$ threshold due to strong $\sqrt{\pi J/\psi, D^{\bar{b}ar}D^*}$

- Peak in $\pi J/\psi$ (not Breit-Wigner line shape)
- Threshold enhancement in $D^{\bar{b}ar}D^*$

✓ Is $Z_c(3900)$ a conventional resonance? --> pole positions

Complex pole position ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{\bar{b}ar}D^*$:2nd)

✿ analytic continuation of S-matrix onto complex energy plane



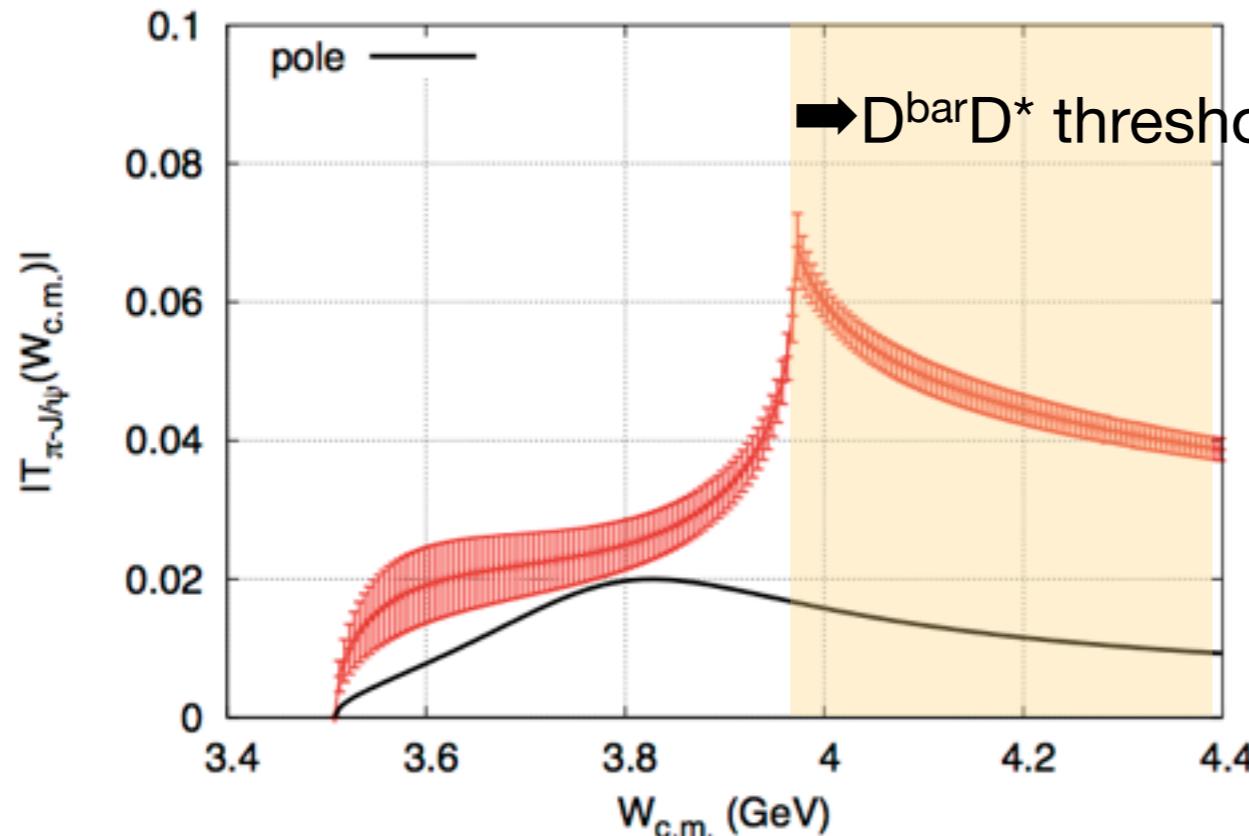
- “Virtual” pole on [2nd, 2nd, 2nd] sheet is found (far from $D^{\bar{b}ar}D^*$ threshold)
- $Z_c(3900)$ is **not a conventional resonance**
- How large does the pole contribute to the amplitude?

T-matrix of $\pi J/\psi$ & $D^{\bar{b}ar}D^*$

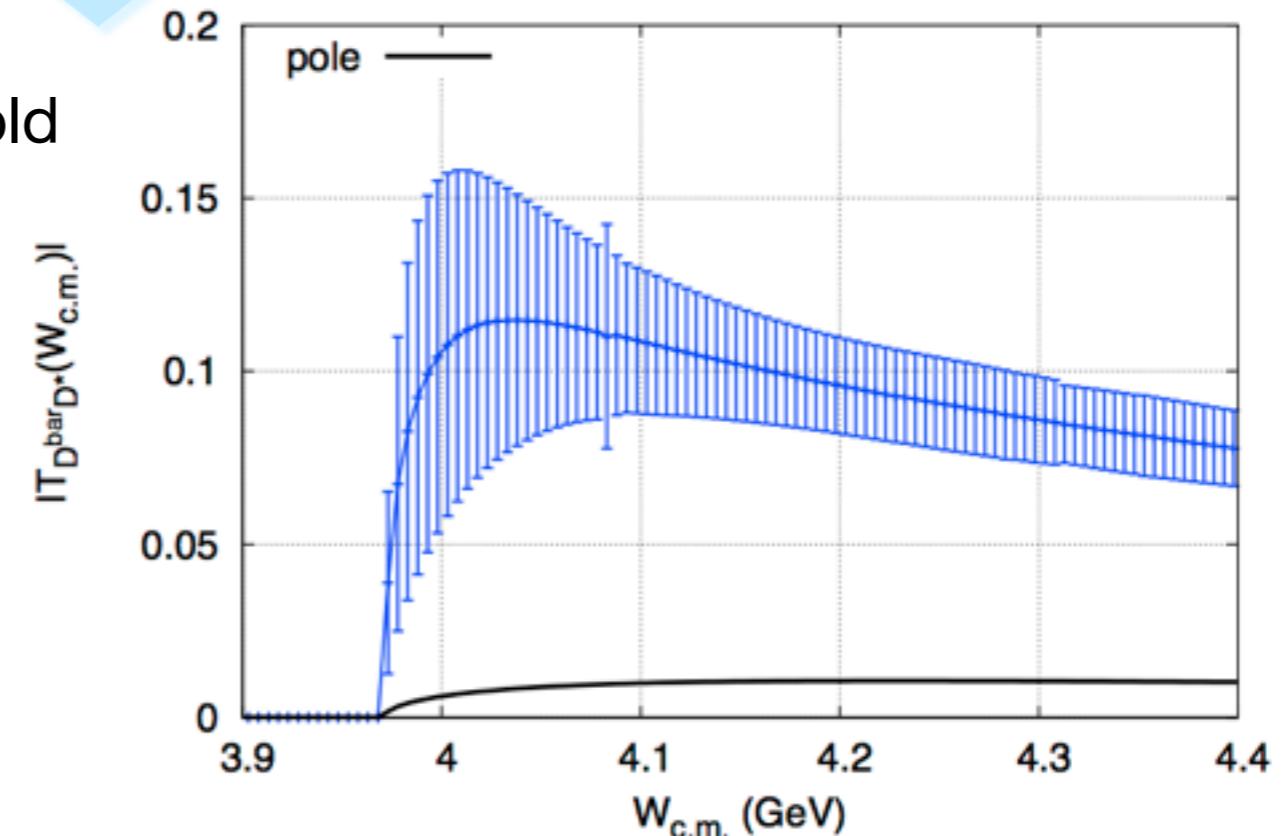
- calculate residues of T-matrices in $\pi J/\psi$ & $D^{\bar{b}ar}D^*$ channels

$$S(k) = 1 + 2iT(k)$$

- $\pi J/\psi - \pi J/\psi$ T-matrix



- $D^{\bar{b}ar}D^* - D^{\bar{b}ar}D^*$ T-matrix



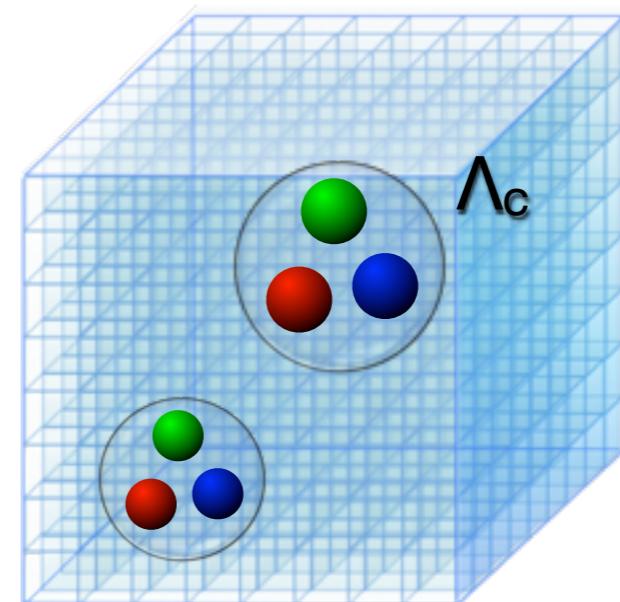
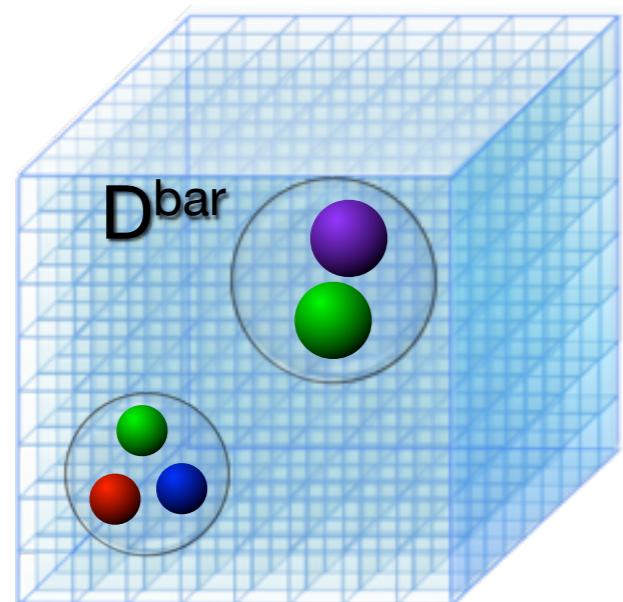
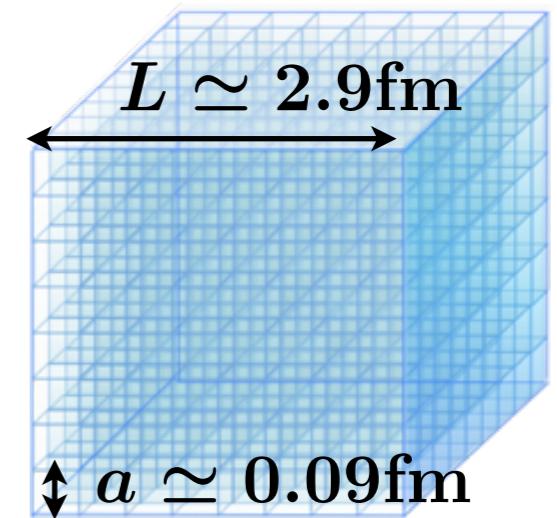
- contribution from virtual pole to T-matrix is small
- $Z_c(3900)$ is cusp at $D^{\bar{b}ar}D^*$ threshold induced by off-diagonal $V^{\pi\psi}, D^{\bar{b}ar}D^*$

S-wave $D^{\bar{b}ar}N$ & $\Lambda_c N$ interactions

-- implications to $D^{\bar{b}ar}$ & Λ_c nuclei --

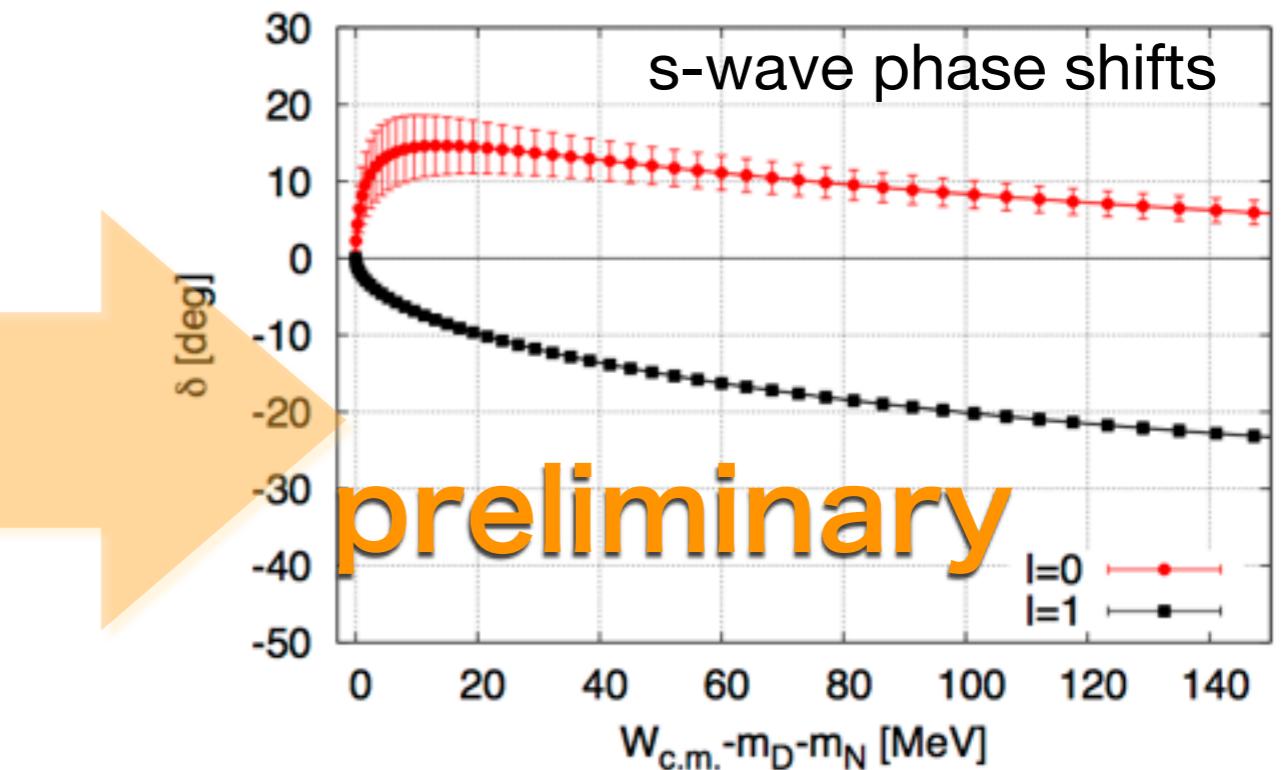
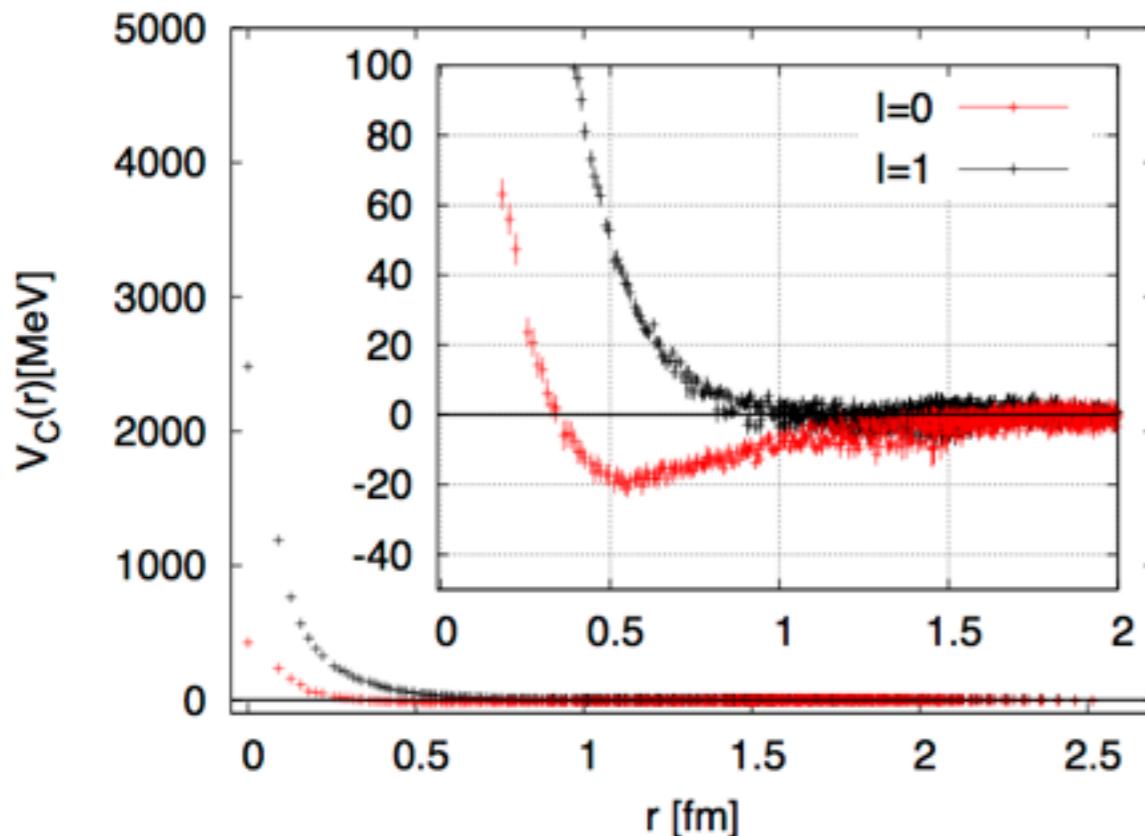
- ▶ $N_f=2+1$ full QCD configs. (PACS-CS collaboration)
- ▶ RHQ for charm

$m_\pi =$	411, 572, 701
$m_N =$	1215, 1411, 1583
$m_D =$	1903, 1947, 2000
$m_{\Lambda_c} =$	2434, 2584, 2710



S-wave $D^{\bar{b}ar}N$ scattering in $|l|=0, 1$ @ $m_{\pi}=410\text{MeV}$

$|l|=0: (\bar{c}^{\bar{b}ar}u)(udd), |l|=1: (\bar{c}^{\bar{b}ar}u)(uud)$



$$a_{\bar{D}N} = \begin{cases} 0.79(42) & (I = 0) \\ -0.20(1) & (I = 1) \end{cases}$$

$D^{\bar{b}ar}N$ potentials

($|l|=0$) short-range repulsion + mid-range attractions, **net attraction**

($|l|=1$) short-range repulsion, **weak repulsion**

[Y. Ikeda \[HAL QCD\], in preparation](#)

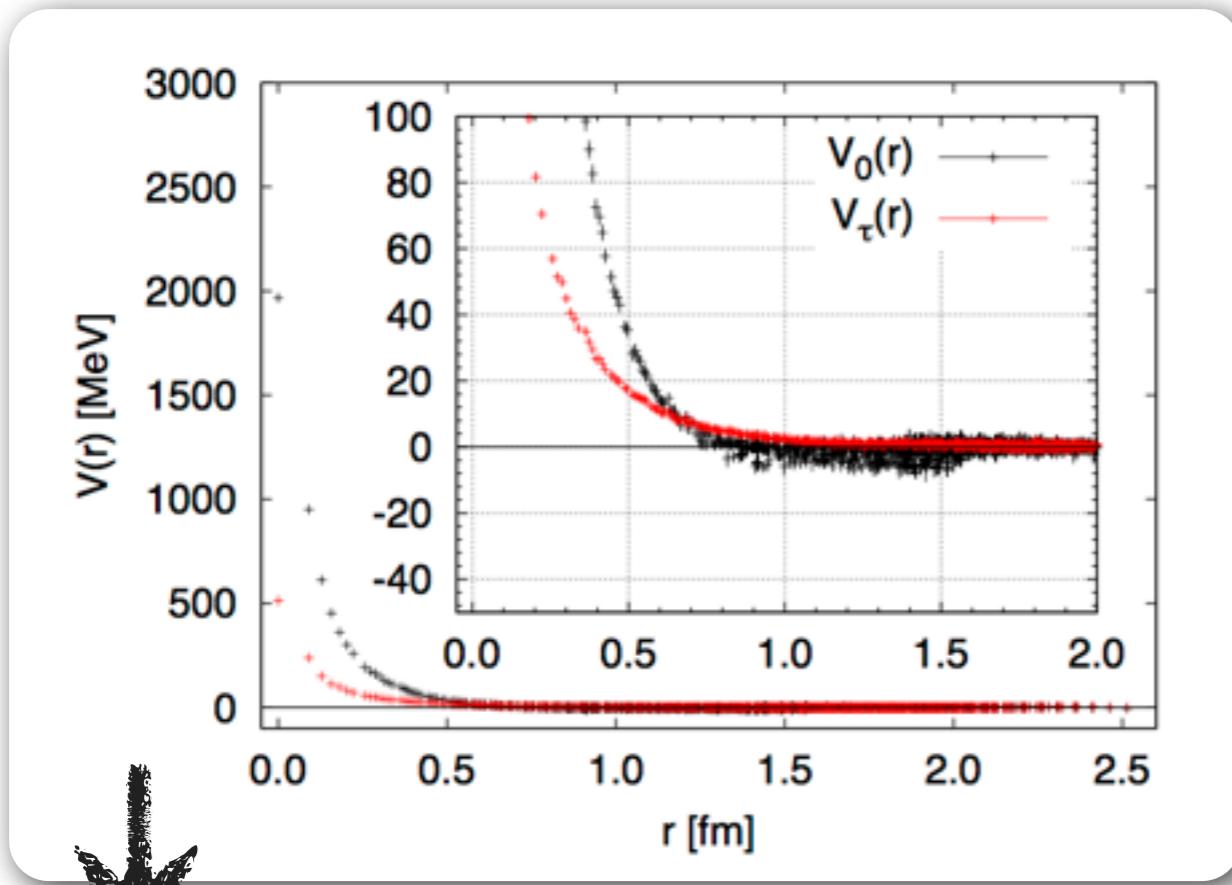
Pauli-blocking at work in meson-baryon system

Machida, Namiki, PTP33 (1965).

Isospin dependence of potentials

✓ decomposition of potentials -- input for nuclear calculations --

$$V_{\bar{D}N}(\vec{r}) = V_0(\vec{r}) + V_\tau(\vec{r})(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_N)$$

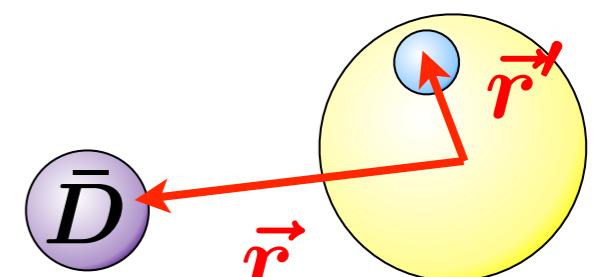


\bar{D}^{bar} -nucleus folding potential

$$V_{\bar{D}A}(\vec{r}) = \sum_{i=1}^A \int d^3r' \rho_{N_i}(\vec{r}') [V_0(\vec{r} - \vec{r}') + V_\tau(\vec{r} - \vec{r}')(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_{N_i})]$$

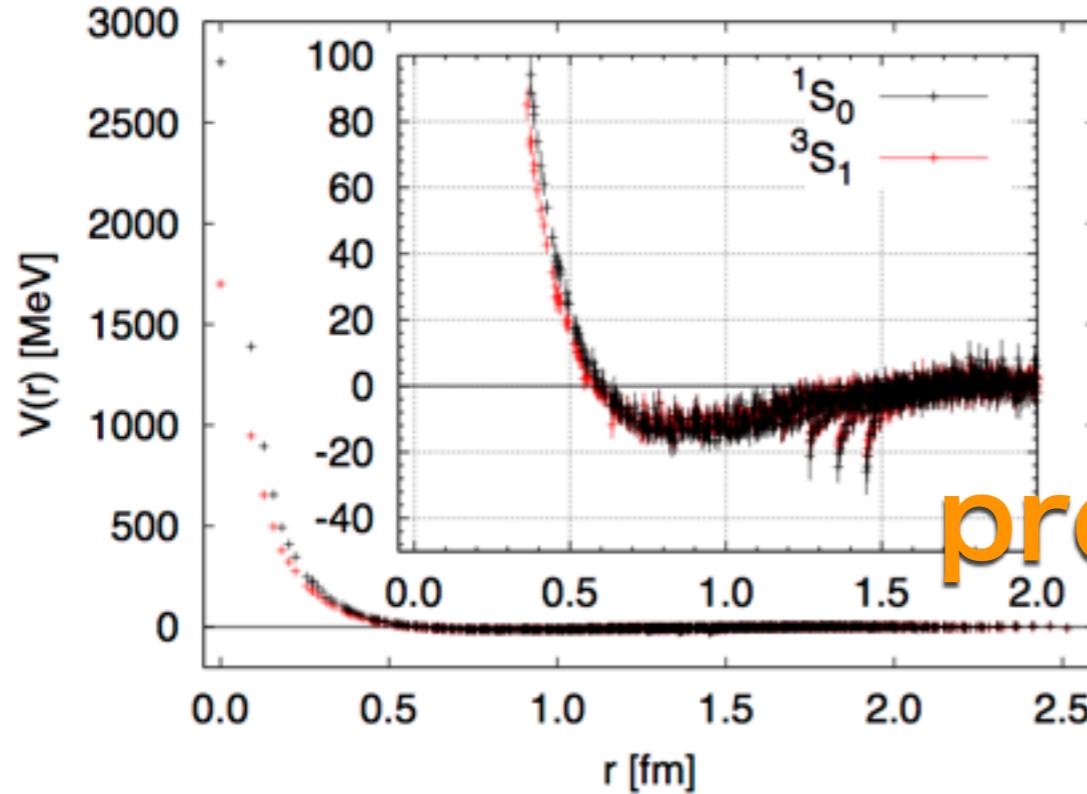
→ \bar{D}^{bar} can be bound in neutron/proton rich nuclei

AMD calc.: M. Isaka, YI, E. Hiyama, in progress



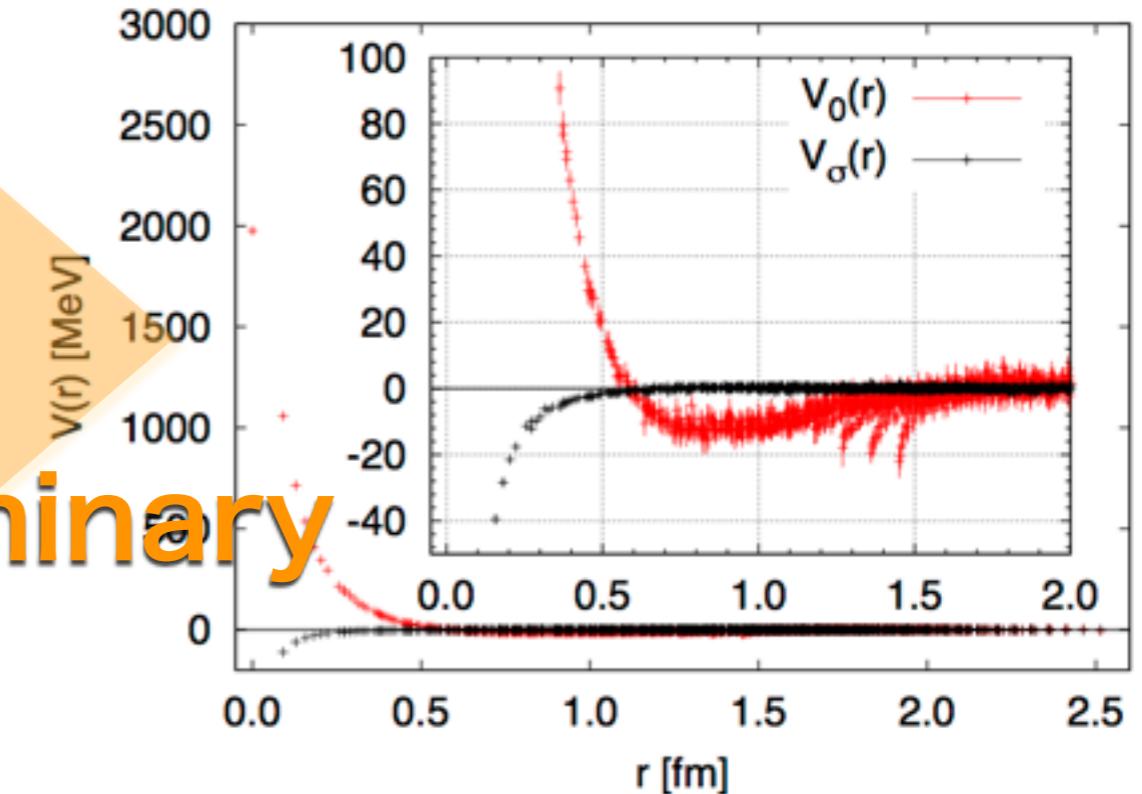
S-wave Λ_c N potentials @ $m_\pi=410\text{MeV}$

1S_0 & 3S_1 channels



$$a_{\Lambda_c N} = \begin{cases} 0.29(16) & (^1S_0) \\ 0.28(13) & (^3S_1) \end{cases}$$

$$V_{\Lambda_c N}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})(\vec{\sigma}_{\Lambda_c} \cdot \vec{\sigma}_N)$$



preliminary

► $V_0(r)$ is **attractive**, $V_\sigma(r)$ is **very small**

Λ_c N potentials in 1S_0 & 3S_1 channels

- (1) weak net attractions in both 1S_0 and 3S_1 channels
- (2) **weak spin dependence --> HQ spin symmetry at work**
- (3) Λ_c can be bound in heavy nuclei

Summary

◆ HAL QCD approach for coupled-channel hadron interactions

- ▶ energy-independent “potentials” from equal-time NBS wave functions
- ▶ a solution of multi-hadron problems (large V , coupled-channel)

❖ Charmed tetraquark candidate $Z_c(3900)$ in $J^P=1^+$ channel

- ▶ $Z_c(3900)$ is threshold cusp induced by strong $V^{D\bar{D}^* - \pi J/\psi}$

❖ $D\bar{D}$ nuclei

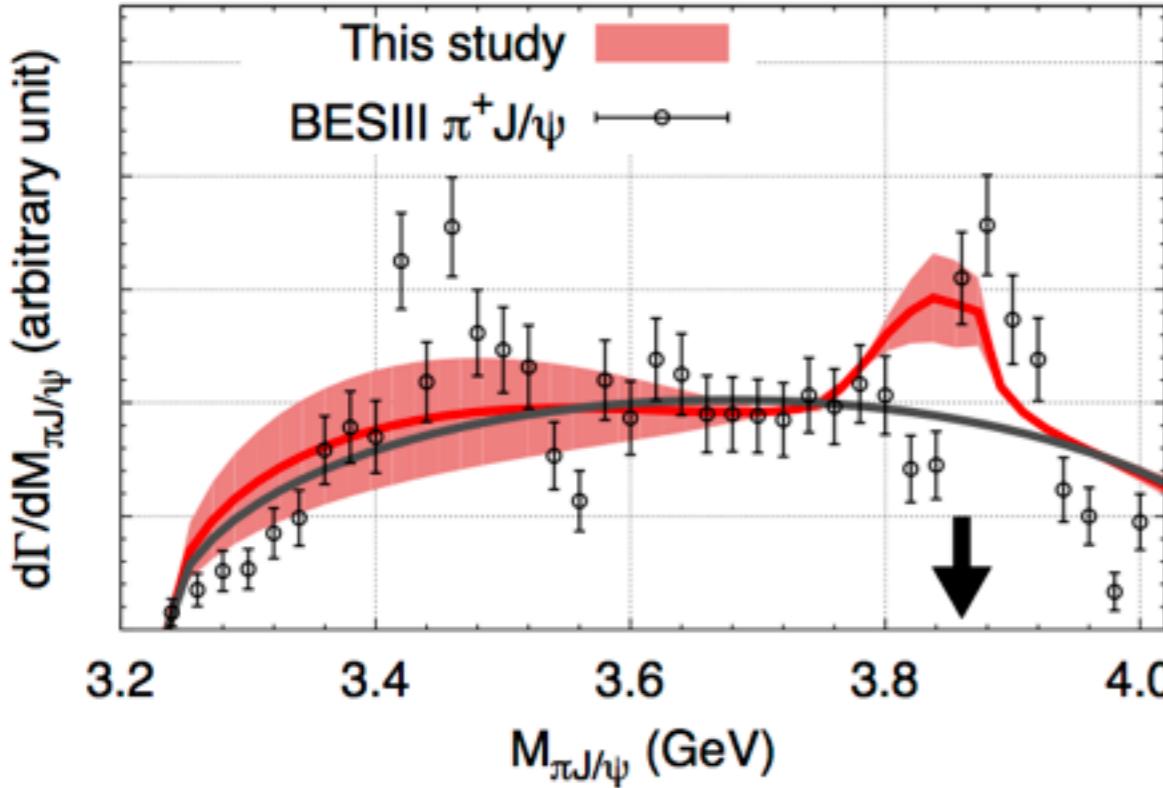
- ▶ Isospin dependent interaction plays important role (neutron/proton rich favored)

❖ Λ_c nuclei

- ▶ Spin independent interaction is weak net attraction (heavy nuclei favored)

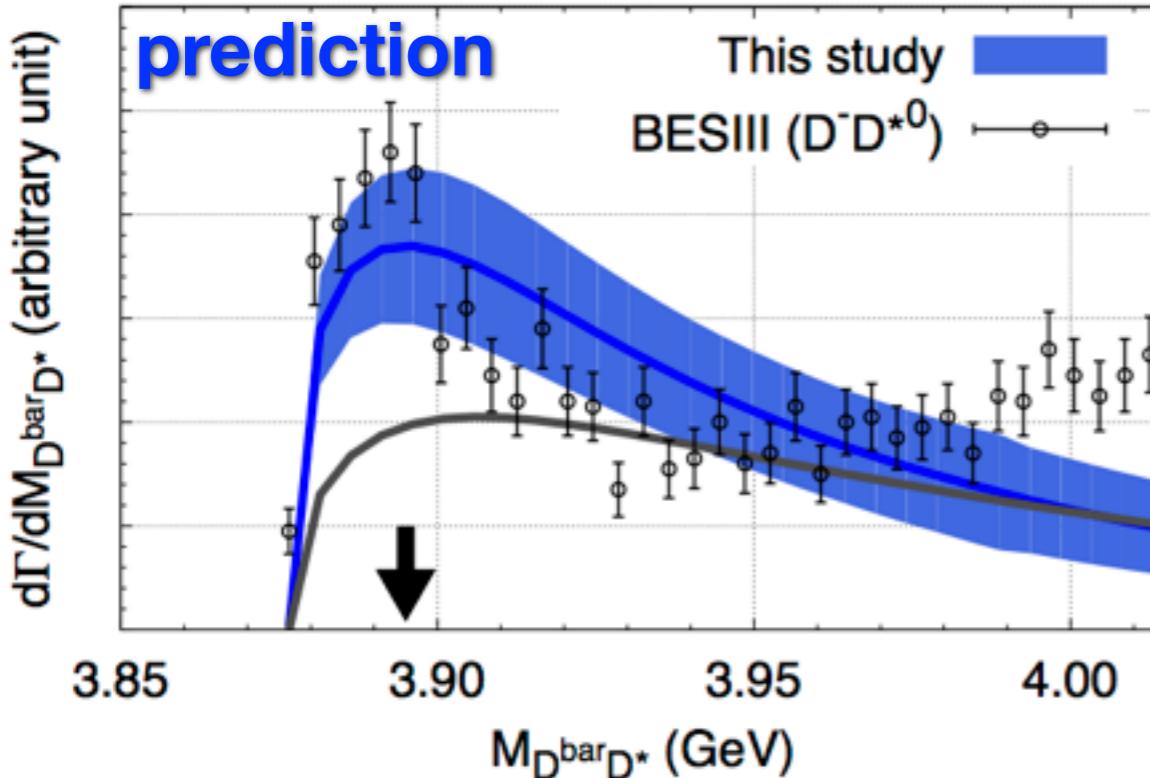
◆ Prospects : physical point simulation is next step

Mass spectra ($\pi J/\psi$ & $D^{\bar{b}ar}D^*$)



parameters: $C_{\pi D^{\bar{b}ar}D^*}/C_{\pi \pi J/\psi} = Re^{i\theta}$
 $\rightarrow R=0.95(18), \theta=-58(44)$ deg. (+overall factor)

- ★ **$Z_c(3900)$ production rate in expt.**
 $21.5 \pm 3.3\%$ (BESIII)
 $29.0 \pm 8.9\%$ (Belle)
- ★ **Calculation w/ LQCD potential**
 $32 \pm 1\%$



- ★ **Good agreement around 3.9 GeV**
- Deviation from expt. data at high energies
 - ▶ explicit $D^{\bar{b}ar}D^*$ channel coupling?
 - ▶ higher partial wave?

