Construction and application of the $\bar{K}N$ local potential based on chiral unitary approach

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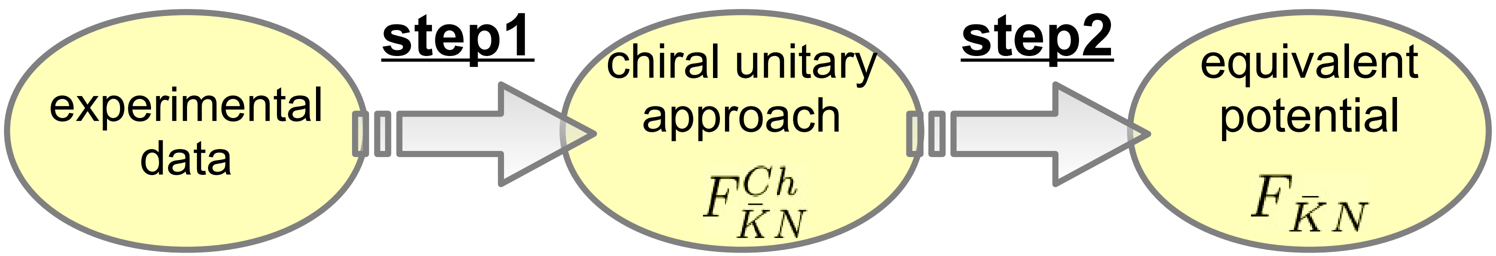

3. This work (new potential)
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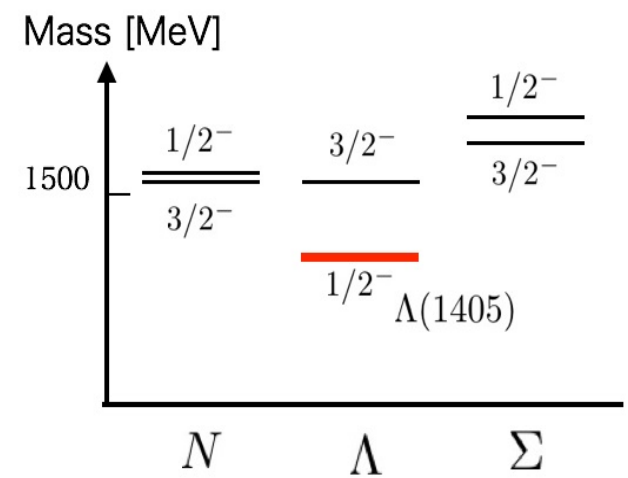
Motivation

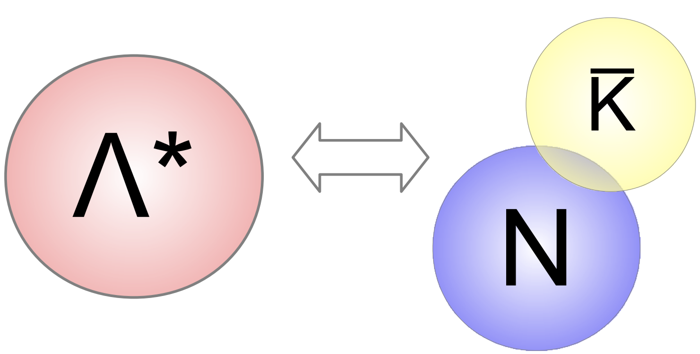
Λ(1405) ↔ quasi bound state of $\bar{K}$N.

$\bar{K}$N interaction is **strongly attractive.**



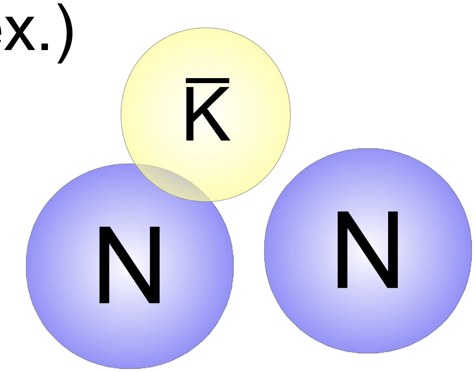
- Y. Akaishi and T. Yamazaki, Phys. Rev. C 65, 044005 (2002)
- T. Hyodo and D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)

molecular state of Λ(1405)



$\bar{K}$ nuclei

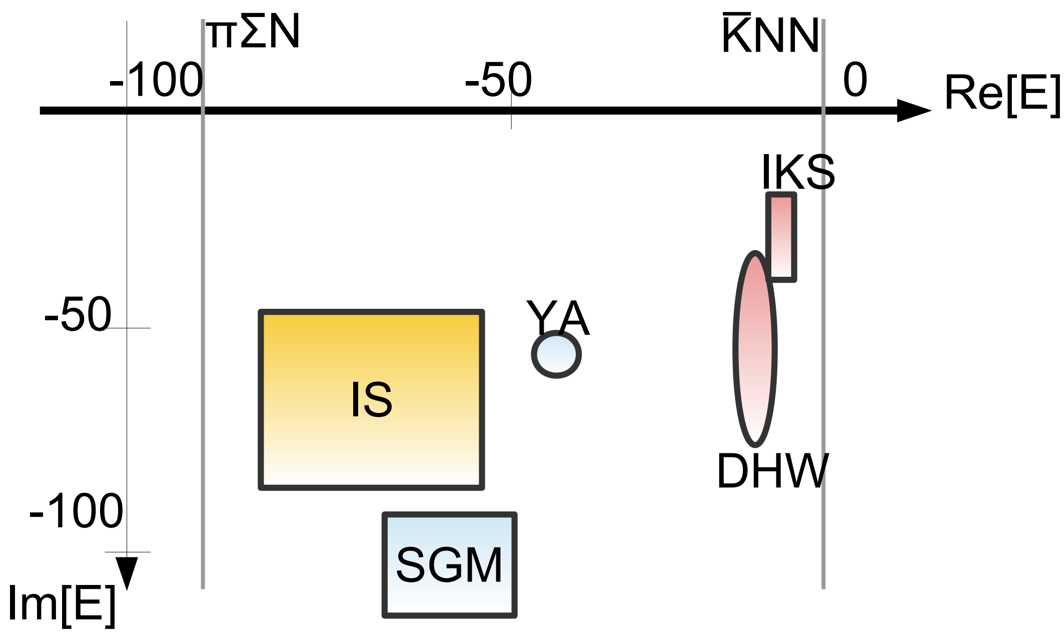
ex.)



ex.) deeply binding?
compact state?

Motivation

Theoretical calculation of $\bar{K}NN$ ($I=1/2, J^p=0^-$)

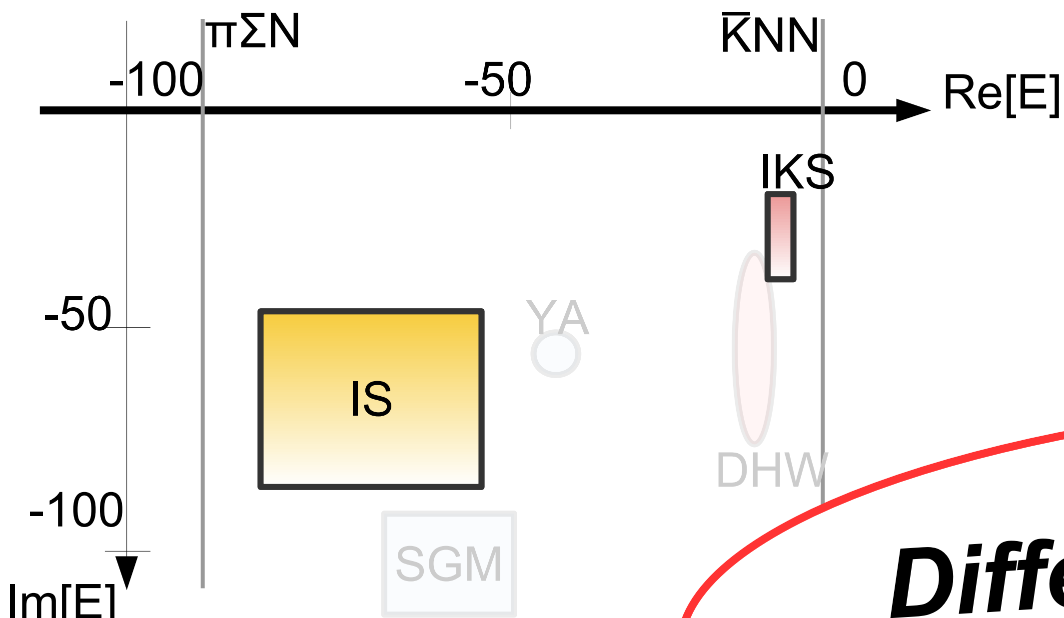


- [SGM] : Shevchenko, Gal, Mares, Phys. Rev. C 76, 044004 (2007)
- [YA] : Yamazaki, Akaishi, Phys. Rev. C 76, 045201 (2007)
- [IS] : Ikeda, Sato, Phys. Rev. C 76, 035203 (2007)
- [DHW] : Dote, Hyodo, Weise, Phys. Rev. C 79, 014003 (2009)
- [IKS] : Ikeda, Kamano, Sato, Prog. Theor.Phys. 124, 3 (2010)

Conclusive result has not been achieved in theoretical calculations

Motivation

Theoretical calculation of $\bar{K}NN$ ($I=1/2, J^p=0^-$)



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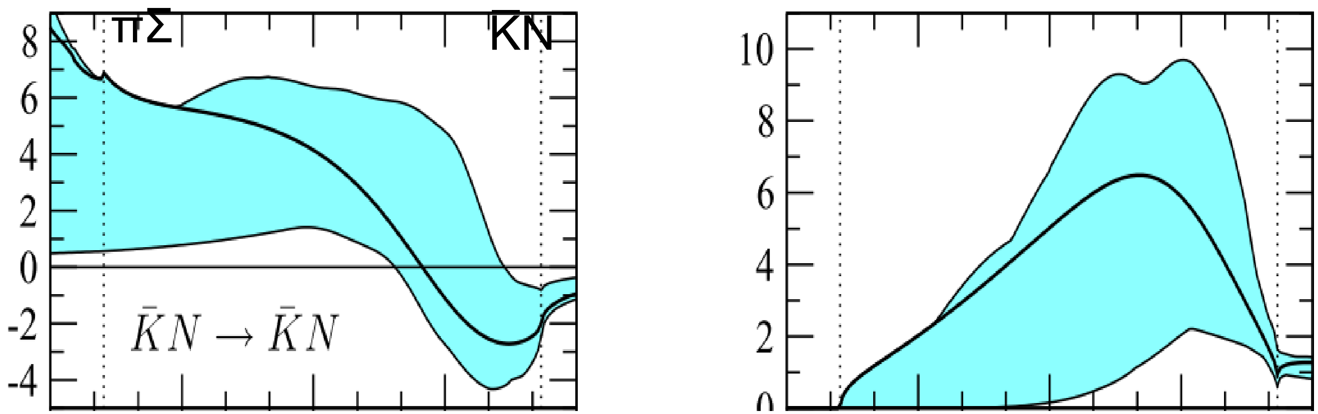
- [YA] : Yamazaki, Akaishi, Phys. Rev. C 76, 045201 (2007)
- [IS] : Ikeda, Sato, Phys. Rev. C 76, 035203 (2007)
- [DHW] : Dote, Hyodo, Weise, Phys. Rev. C 79,

Different results are caused by $\bar{K}N$ interaction

Conclusive result has not been achieved in theoretical calculations

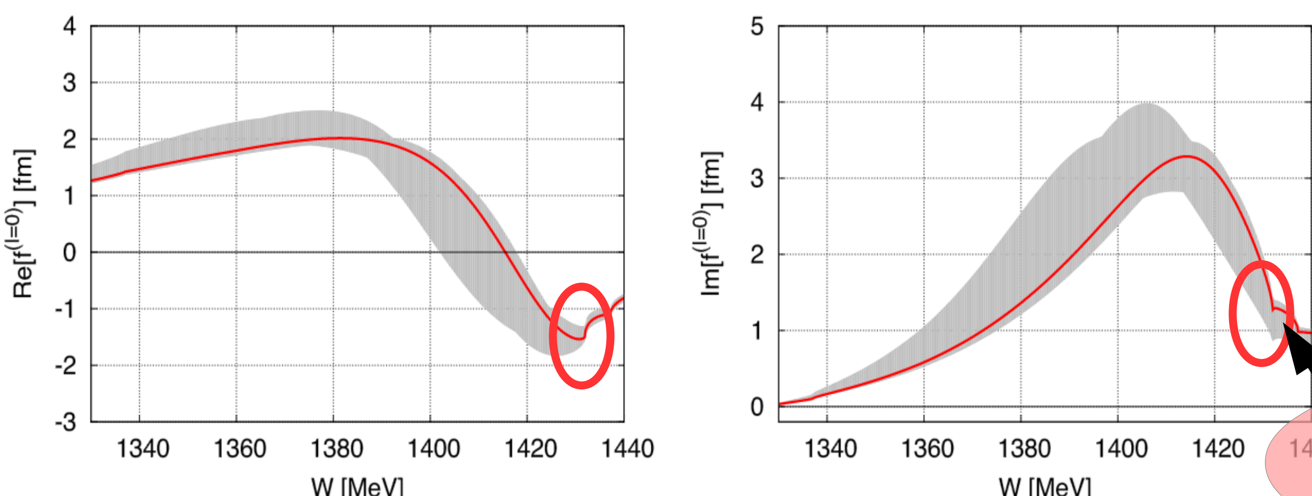
Motivation

$\bar{K}N$ subthreshold amplitude



- Borasoy et al. Phys. Rev. C 74, 055201 (2006)
- R.Nissler, Ph.D thesis (2007)

large uncertainty



- Ikeda, Hyodo, Weise Nucl. Phys. A 881, 98 (2012)
- Kamiya, et al., Nucl. Phys. A 954, 41 (2016)

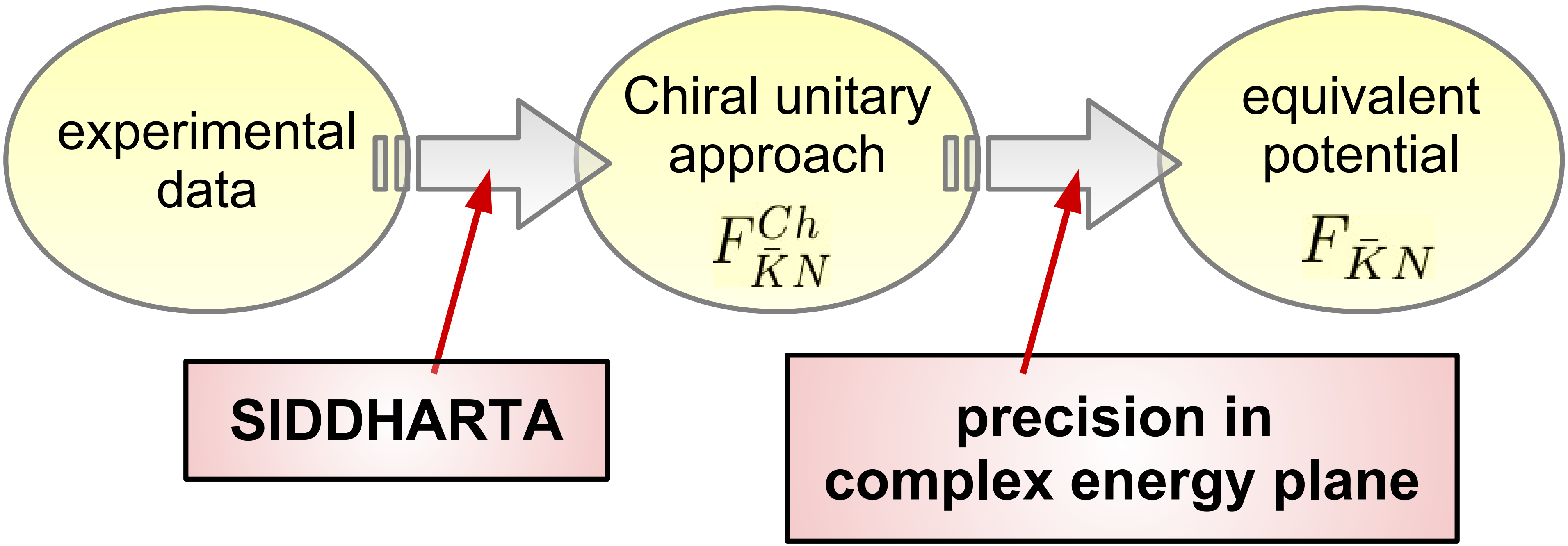
Bazzi et al. Phys. Lett. B 704, 113 (2011)

SIDDHARTA

Uncertainty is significantly reduced by SIDDHARTA

Motivation

Construction of *r*-dep. local potential



high precision
$\bar{K}N$ local potential

reliable prediction ⟹

- spatial structure of Λ(1405)
- few-body calculation

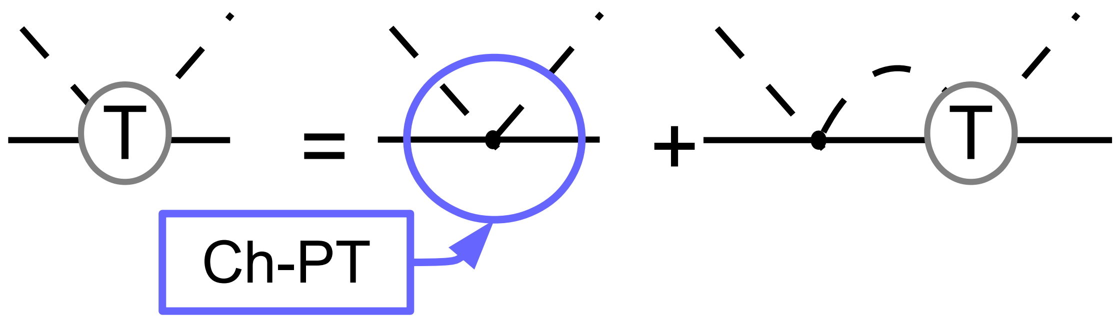
Previous work

$\bar{K}N$ potential from Ch-U

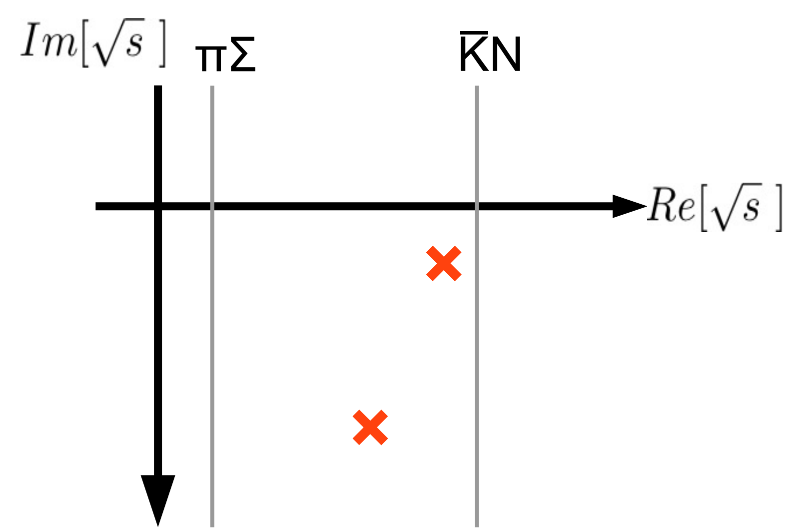
T. Hyodo and W. Weise, Phys. Rev. C 77, 035204 (2008)

- **$\bar{K}N$ amplitude from chiral unitary approach**

• chiral unitary approach



Jido et al. Nucl. Phys. A 725, 181 (2003)

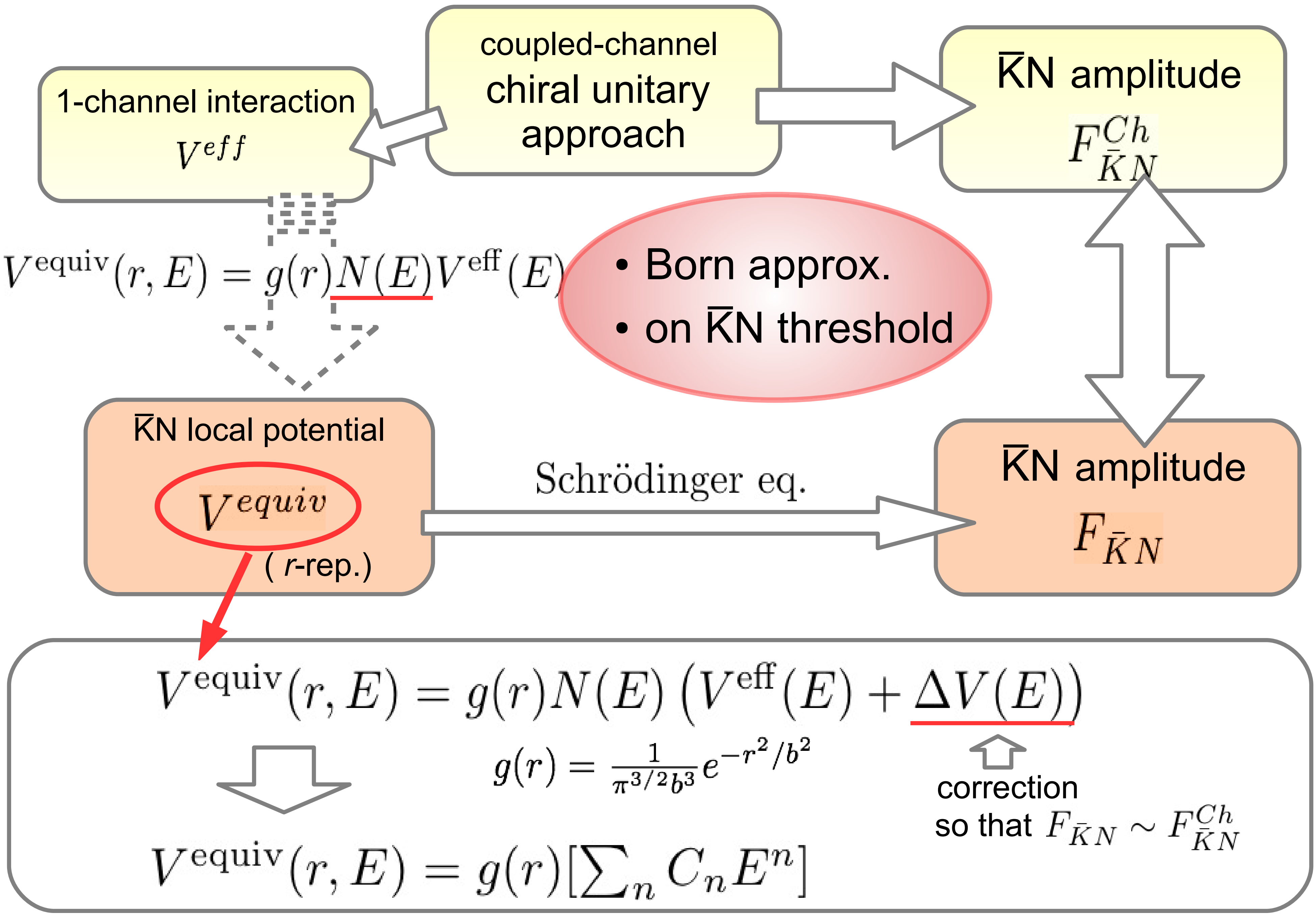


• channel coupling

in S=-1 , I=0 sector

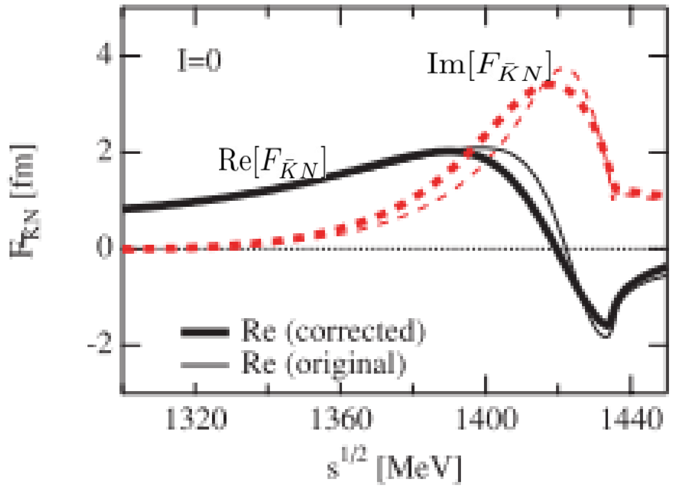
Attractions in $\bar{K}N$ and πΣ leads to **double pole** structure

equivalent local potential



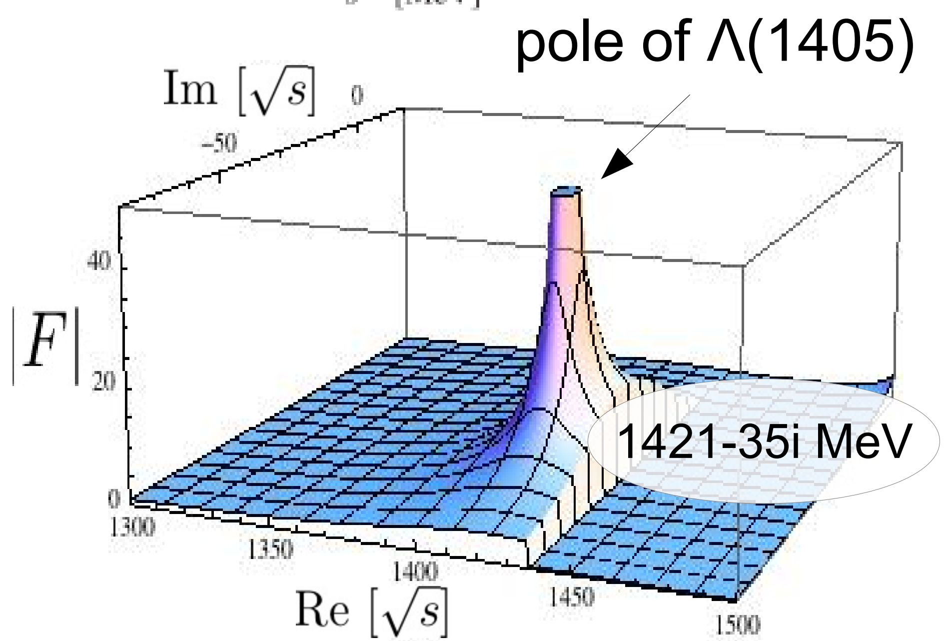
➢ problem

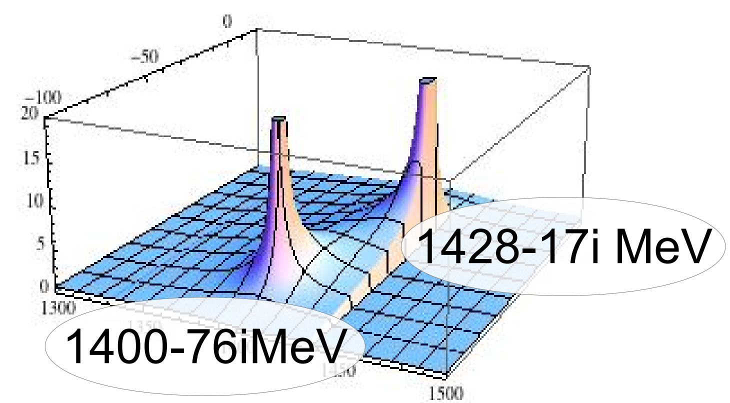
amplitude on $\mathrm{Re}\,[\sqrt{s}]$



$F_{\bar{K}N}$ almost reproduced $F^{Ch}_{\bar{K}N}$

- analytic continuation of $F_{\bar{K}N}$ with V^{equiv} to the complex energy plane



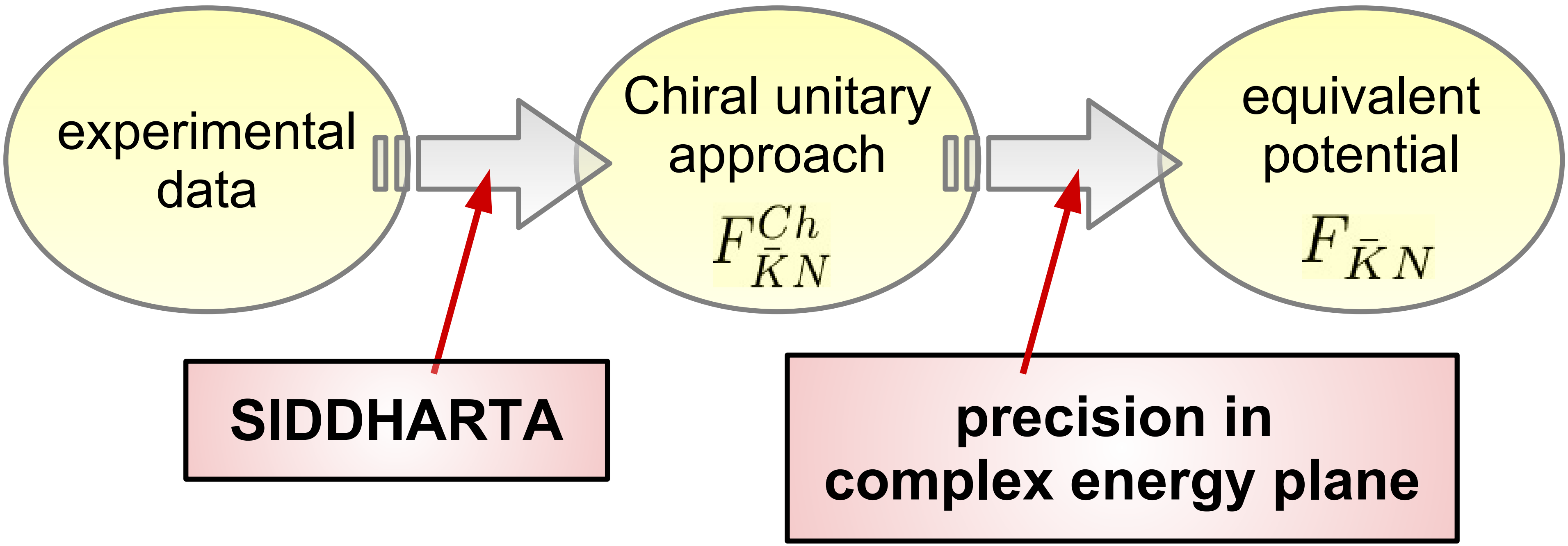


$F^{Ch}_{\bar{K}N}$ from chiral unitary approach

V^{equiv} **does not reproduce the pole structure of** $F^{Ch}_{\bar{K}N}$

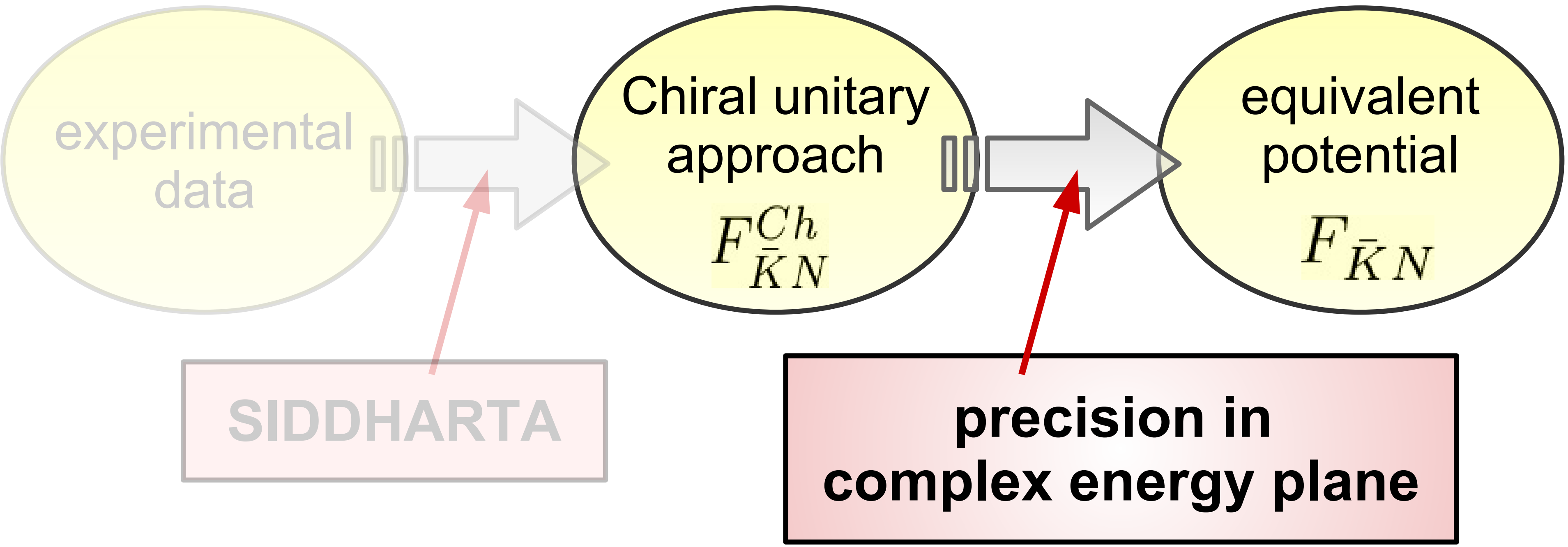
This work

Construction of *r*-dep. local potential



This work

Construction of *r*-dep. local potential



reproduce

- $\bar{K}N$ pole (higher pole)
- $\pi\Sigma$ pole (lower pole)

This work

➢ **Improvement (K̄N pole)**

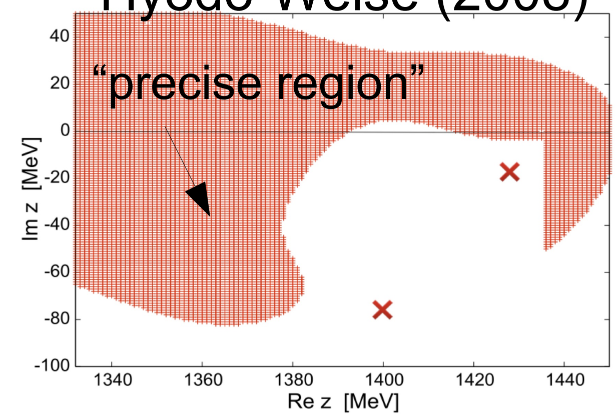
$$\Delta F_{\rm real} = \frac{\int d\sqrt{s}\, |F^{\rm Ch}_{\bar{K}N}(\sqrt{s}) - F_{\bar{K}N}(\sqrt{s})|}{\int d\sqrt{s}\, |F^{\rm Ch}_{\bar{K}N}|} \times 100$$

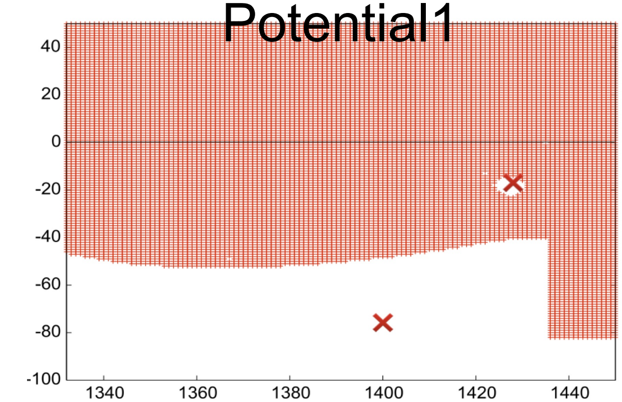
——— deviation of the amplitude on the real axis

——— change ΔV and fitting range

	Hyodo-Weise	Potential1 (This work)	Chiral unitary
ΔV	real	complex	
fit range [MeV]	1300~1410	1332~1450	
ΔFreal [%]	14	0.48	
Pole [MeV]	1421-35i	1427-17i	1428-17i 1400-76i

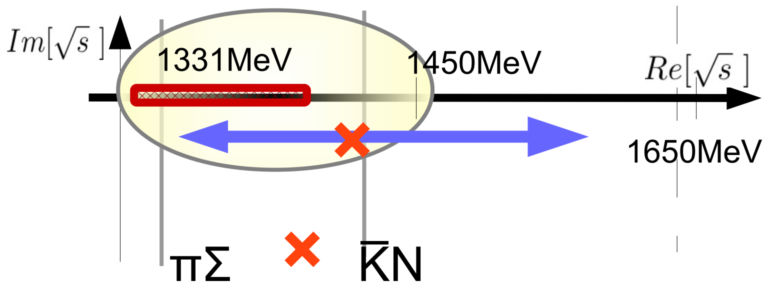
Hyodo-Weise (2008)





ΔFreal and K̄N pole position are improved

Improvement (πΣ pole)

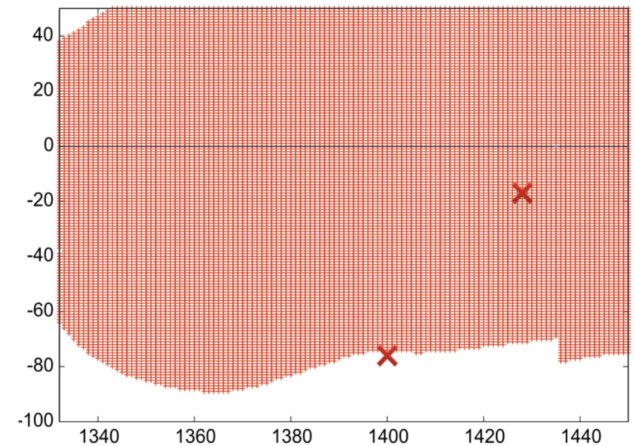


second pole did not appear

⟶ change fit range and polynomial type of V^{equiv}

	Potential1	Potential2	Chiral unitary
polynomial type in E	3rd order	10th order	
fit range [MeV]	1332~1410	1332~1520	
Pole [MeV]	1427-17i	1428-17i 1400-77i	1428-17i 1400-76i

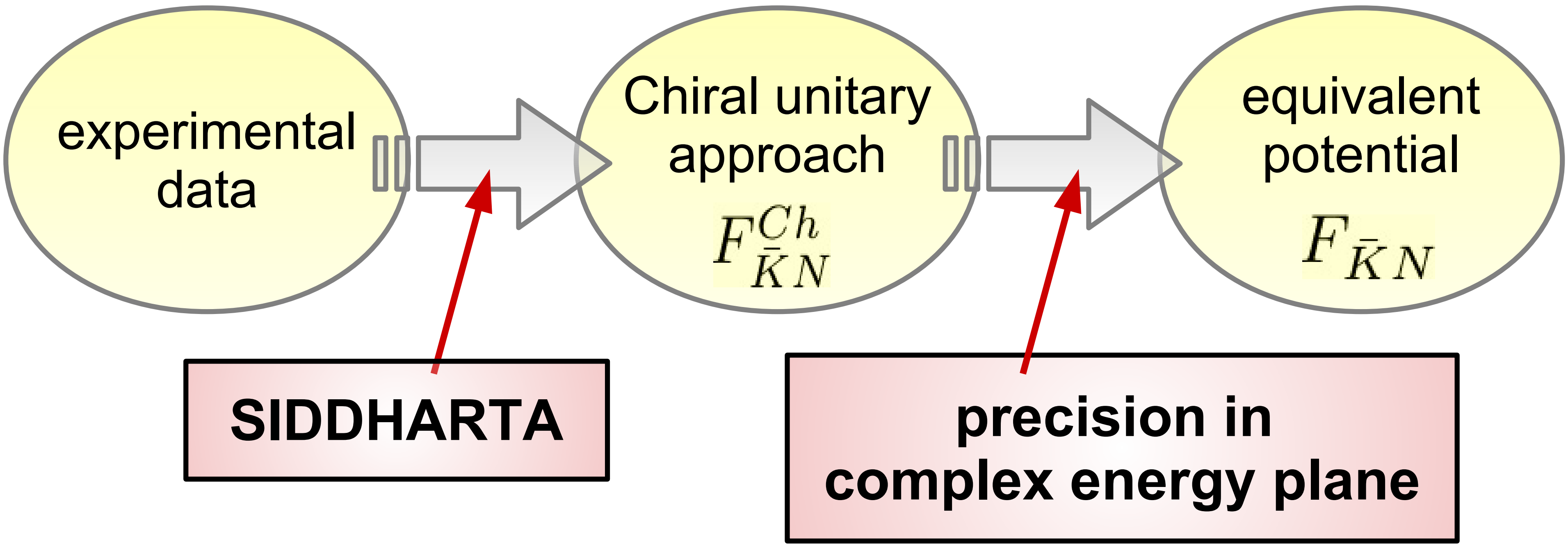
Potential2



πΣ pole appears at correct position

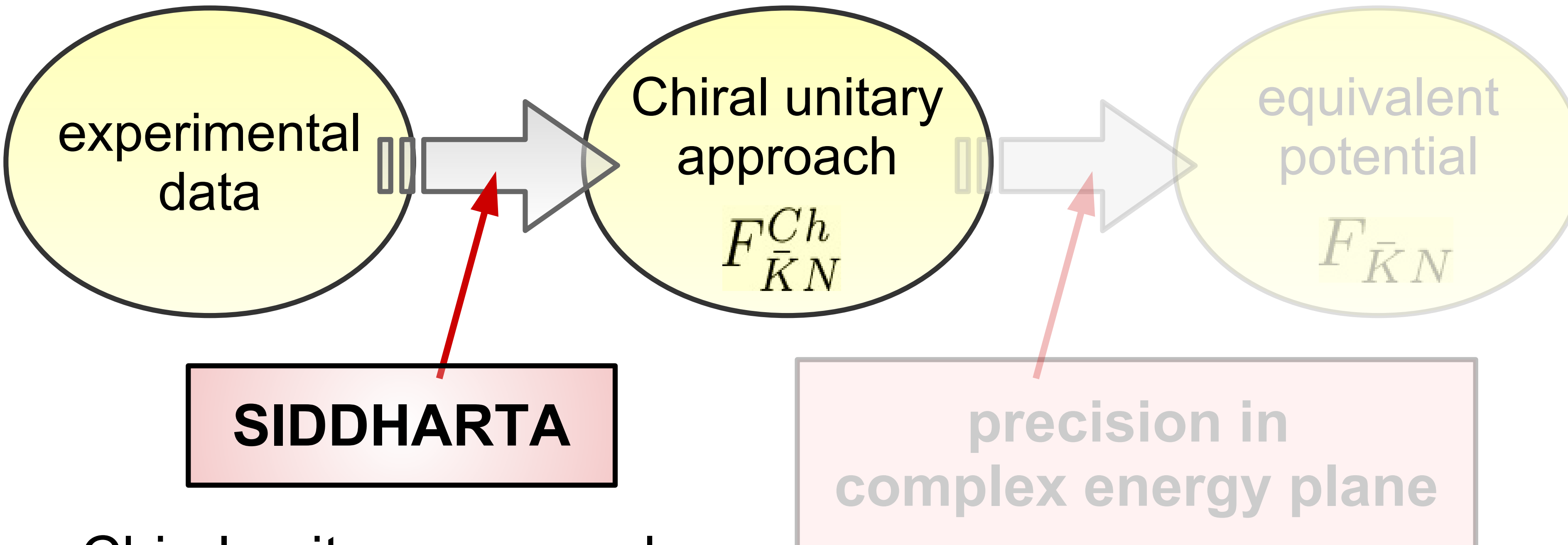
This work

Construction of *r*-dep. local potential



This work

Construction of *r*-dep. local potential



Chiral unitary approach
with NLO term

Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. A881, 98 (2012)

⇒ **quantitatively reliable potential**

➢ Results with SIDDHARTA

I=0

b = 0.38 fm

fit function : 10^{th} order in $\sqrt{s}$

fit range : 1332~1657 MeV

→ $\Delta F_{\rm real} = 0.53\ \%$

pole : 1424-26i MeV

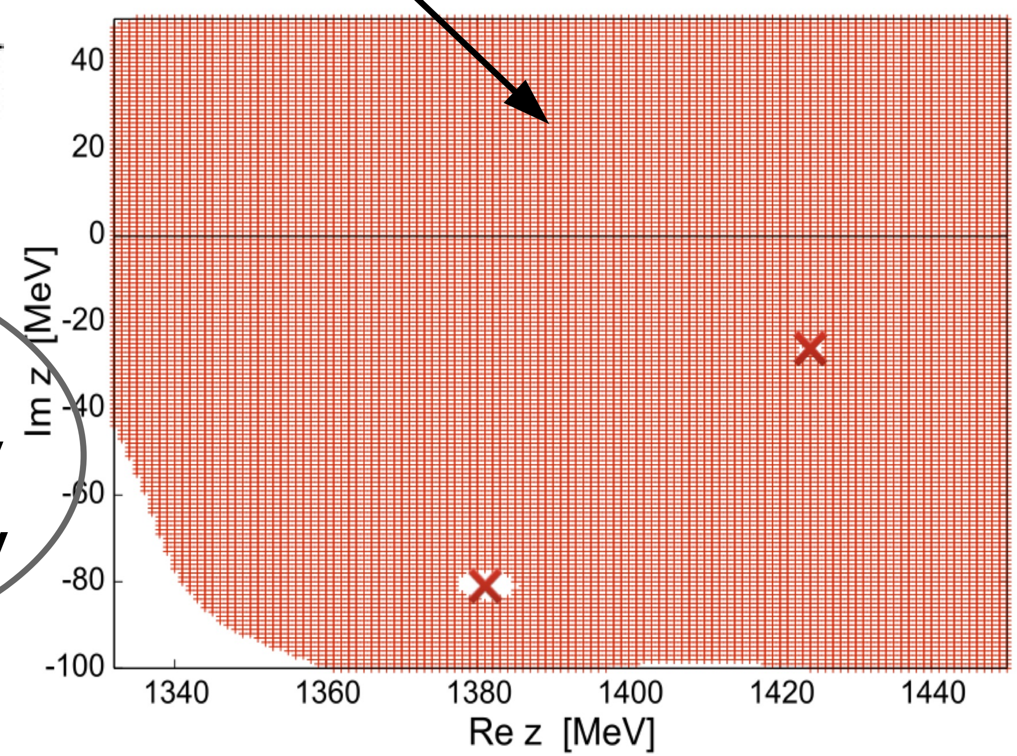
1381-81i MeV

original pole

1424-26i

1381-81i

"precise region"



I=1 (with same framework)

→ $\Delta F_{\rm real} = 1.1\ \%$

Precise potential with SIDDHARTA

Application

➢ Spatial structure of Λ(1405)

E-dep. complex potential

- normalization : $\int d\boldsymbol{r} \left[1 - \frac{\partial V(r,E)}{\partial E}\right] \psi_{\mathrm{G}}^{E\text{-dep.}\,2}(r) = 1$
- expectation value : $\langle r^2 \rangle_{\mathrm{G}}^{E\text{-dep.}} = \int d\boldsymbol{r}\ r^2 \left[1 - \frac{\partial V(r,E)}{\partial E}\right] \psi_{\mathrm{G}}^{E\text{-dep.}\,2}(r)$

mean-squared radius becomes complex

$E = E_{\mathrm{res}},\ b \to 0$ (zero range potential)

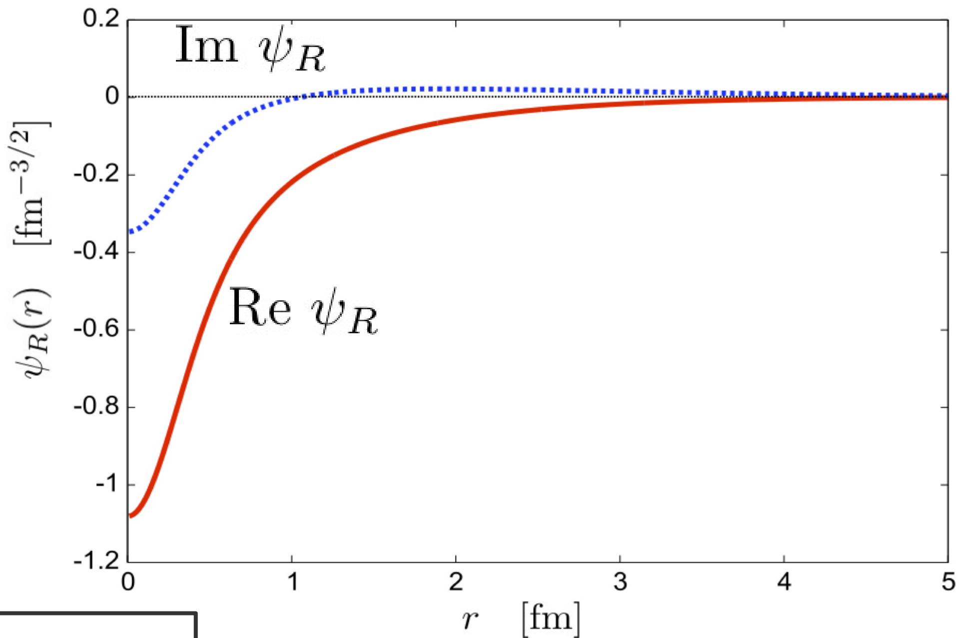
$$\psi(r) \to \frac{A}{|A|}\sqrt{\frac{\mathrm{Im}[k]}{2\pi}}\frac{e^{ikr}}{r}, \quad \langle r^2 \rangle = \int d\boldsymbol{r}\ r^2 |\psi(r)|^2$$

$$\to \frac{\mathrm{Im}k}{2\pi} 4\pi \int_0^\infty dr\ r^2 e^{-2\mathrm{Im}[k]r} = \frac{1}{2(\mathrm{Im}[k])^2}$$

dumping

We use usual <r²> to interpret spatial structure.

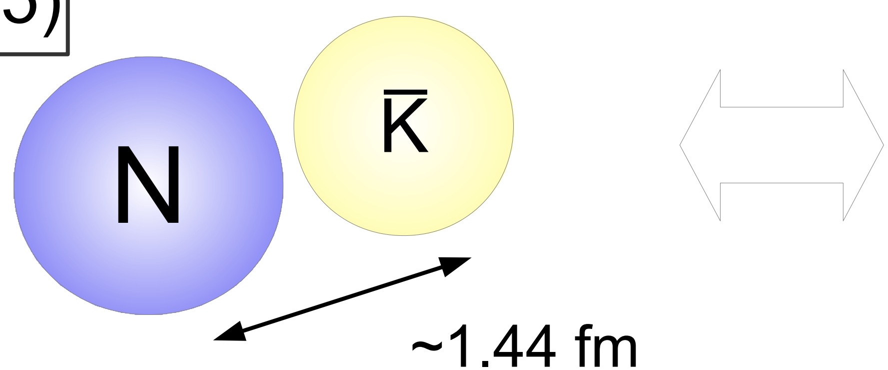
Application



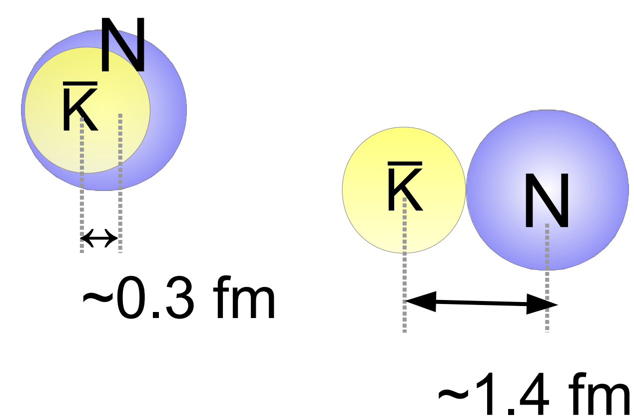
$$\sqrt{< r^2 >} = 1.44 \text{ fm}$$

p : ~0.85fm
K^- : ~0.55fm

Λ(1405)



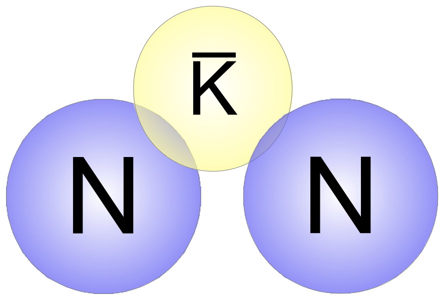
cf.



Λ(1405) is relatively larger (molecular state).

Other applications

- **$\bar{K}$ nuclei (ex. $\bar{K}NN$)**



calculate up to 7-body system

Collaborate with

S. Ohnishi, T. Hoshino, W. Horiuchi, T. Hyodo

- **K^-p correlation in heavy-ion collision**

$$C(\mathbf{q},\mathbf{P}) \sim \int d\mathbf{r} S_{12}(\mathbf{r}) |\Psi_{12}(\mathbf{r},\mathbf{q})|^2$$

Ψ_{12} : relative wave function

S_{12} : hadron source function

A. Ohnishi, K. Morita, K. Miyahara, T. Hyodo, Nucl. Phys. A954 (2016) 294

Coupled-channel potential

dynamical $\pi\Sigma$ channel ($\bar{K}N$-$\pi\Sigma$), Coulomb force (K^-p-$\bar{K}^0n$)

Summary

- We have improved the potential construction procedure by changing ΔV, fit range, and fit function

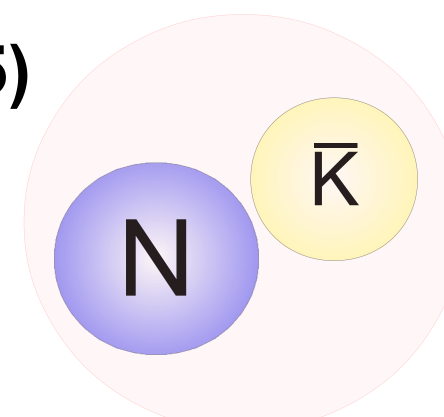
  → $F^{Ch}_{\bar{K}N}$ is reproduced **precisely in complex E plane**

- We have constructed the realistic $\bar{\mathrm{K}}$N local potentials in both I=0 and I=1 channels with **SIDDHARTA** constraint

- We have discussed the **structure of Λ(1405)**

  → $\sqrt{< r^2 >} = 1.44 \text{ fm}$

  molecular state of Λ(1405)

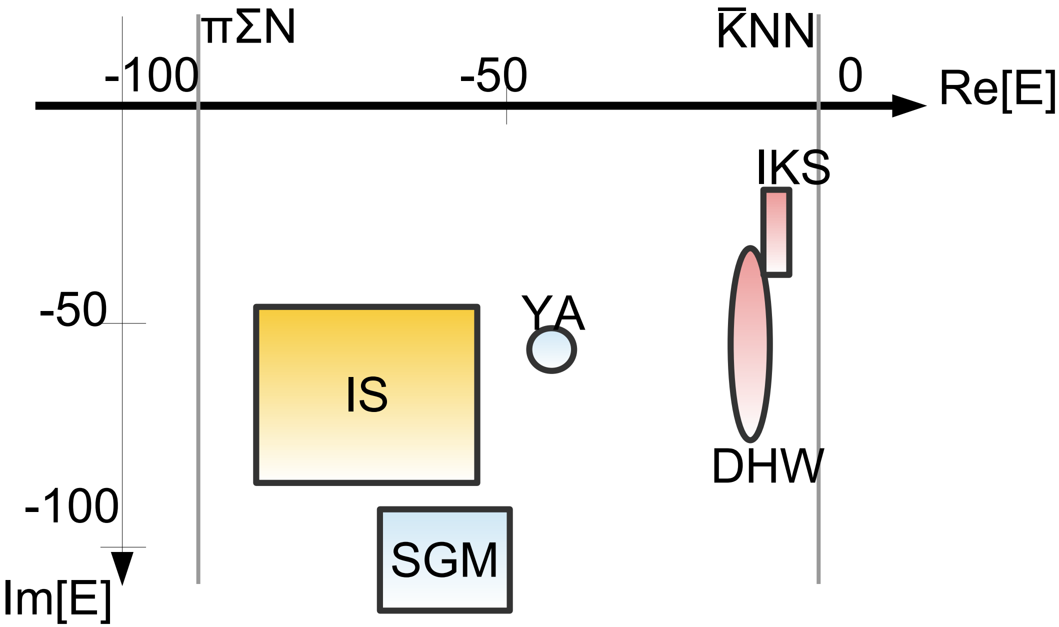


Published in **K. Miyahara and T. Hyodo, Phys. Rev. C93 (2016) 015201**

Backup slides

Motivation

$\bar{K}NN$ ($I=1/2, J^p=0^-$)



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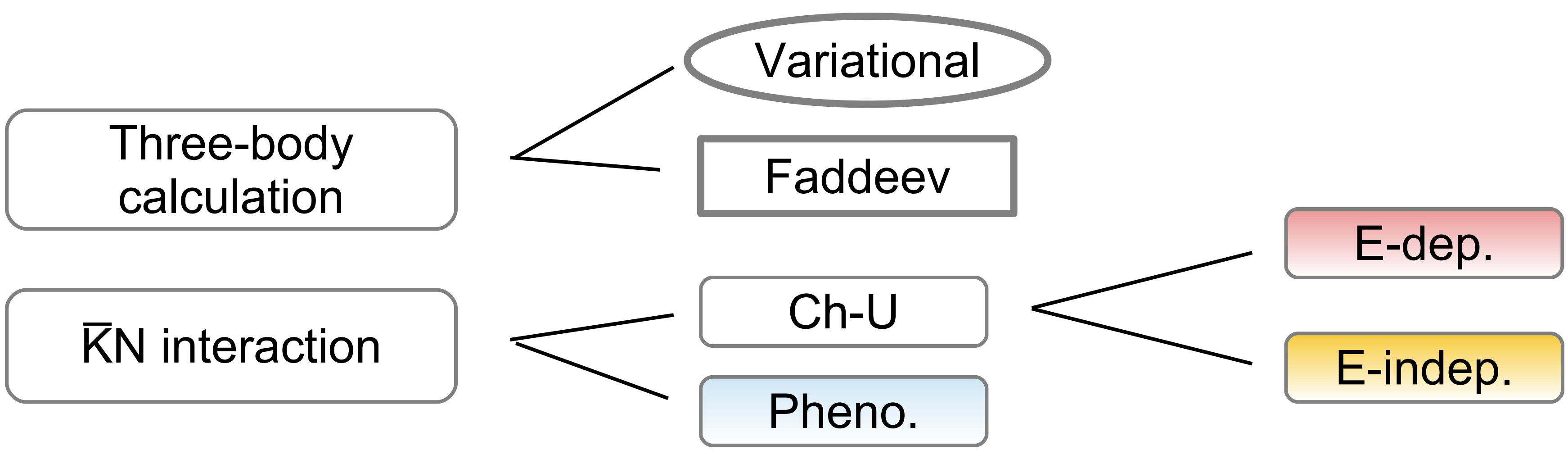
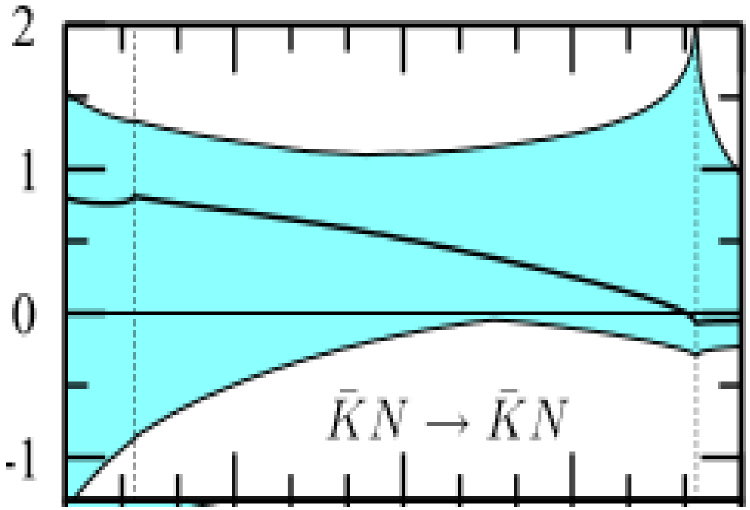
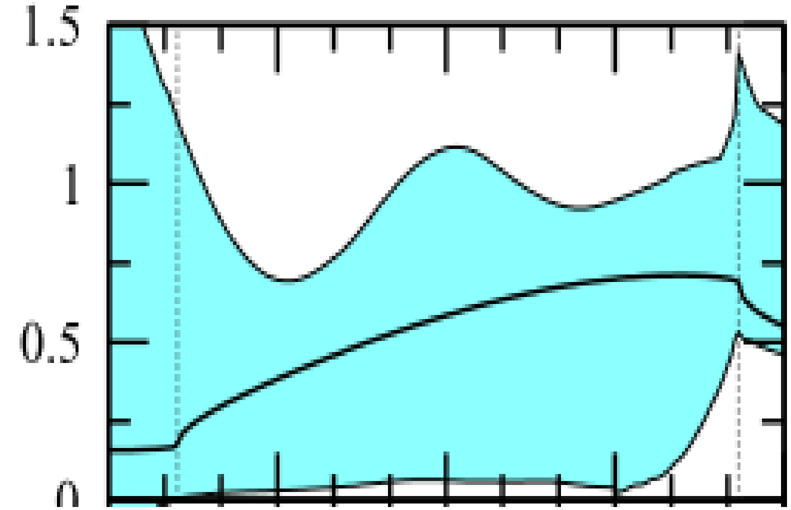


Error of amplitude (without SIDDHARTA)

(I=1)

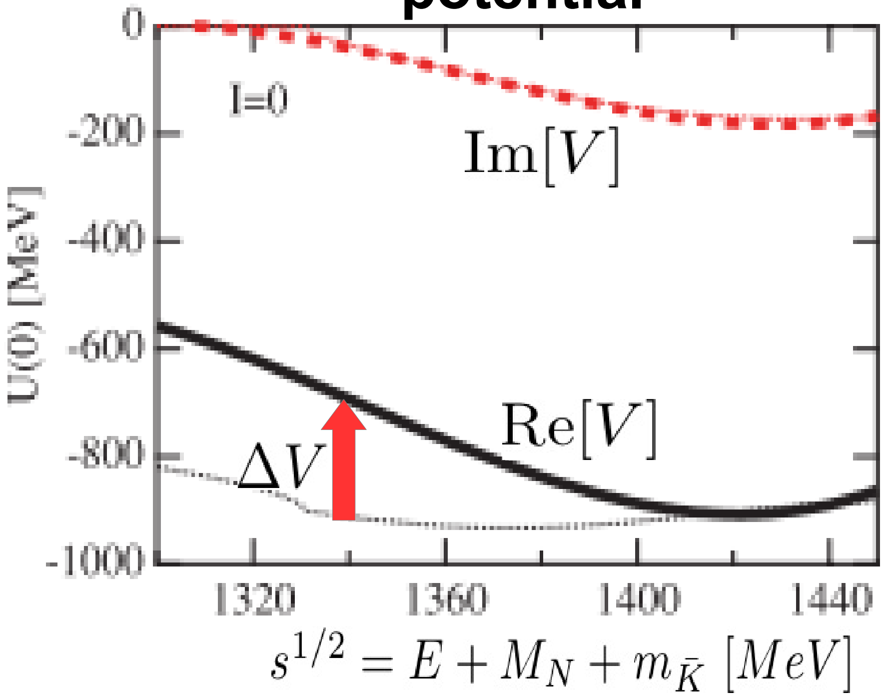




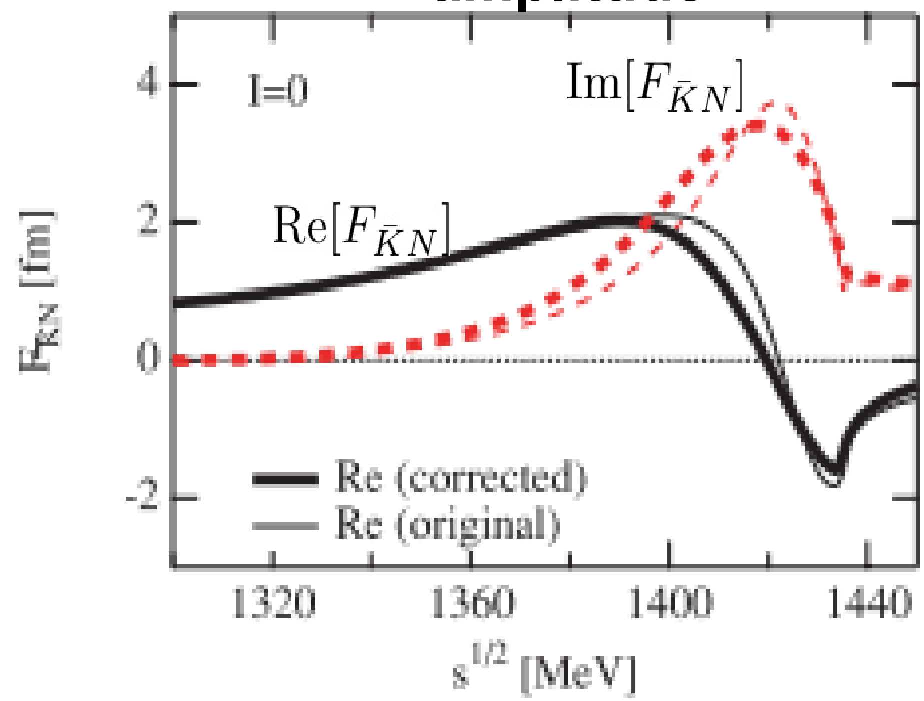
➢ Results

T. Hyodo and W. Weise, Phys. Rev. C 77, 035204 (2008)

potential



amplitude



- E dependence of $V^{equiv}(0, E)$

  real ΔV

  Im[ΔV]=0

- E-dep. of $F_{\bar{K}N}$ and $F^{Ch}_{\bar{K}N}$

$F_{\bar{K}N}$ almost reproduced $F^{Ch}_{\bar{K}N}$

$\bar{K}NN$ system

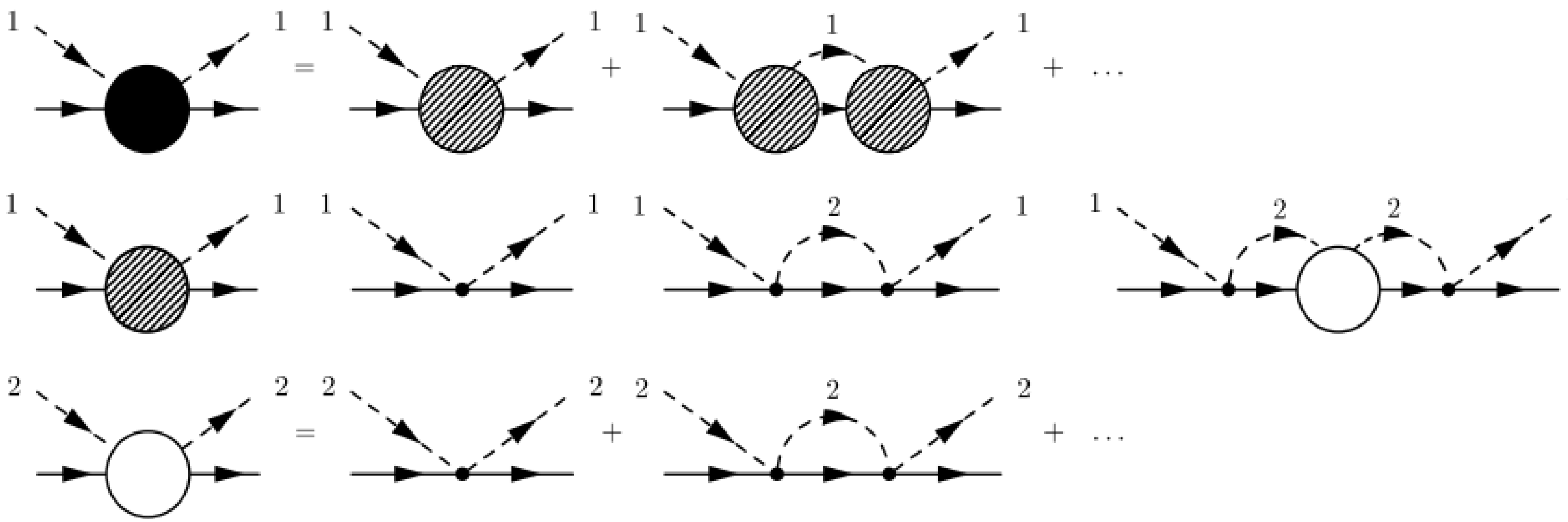
Paper	Shevchenko Gal, Mares, Revai (2007)	Yamazaki Akaishi (2007)	Ikeda Sato (2007)	Dote, Hyodo, Weise (2008,2009)	Ikeda, Kamano, Sato (2010)
$\bar{K}N$ interaction	E-indep. Pheno.	E-indep. Pheno.	E-indep. Chiral	E-dep. Chiral	E-dep. Chiral
Pole of Lambda(1405) (roughly)	1406.5-25i	1406-25i	1420-30i	1428-17i 1400-76i	1417-16i 1335-65i
Method (πΣN dynamics)	Faddeev (explicit)	Variational (effective)	Faddeev (explicit)	Variational (effective)	Faddeev (explicit)
Binding energy	50-70	48	60-95	17-23	9-16
Width	90-110	60	45-80	40-70	34-46

cf. exp. (B,Γ) = (115,67) MeV Angello et al. (2005)
(95,162) MeV Ichikawa et al. (20013)

$\bar{K}N$ single-channel effective interaction

situation : nonrelativistic and S-wave

→ **WT term** is LO



$$T^{eff} = V^{eff} + V^{eff} G_1 T^{eff}$$

$$= T^{11}$$

➢ equivalent local potential

Gaussian : $g(r) = \frac{1}{\pi^{3/2} b^3} e^{-r^2/b^2}$

way to decide "b"

$$V^{\rm equiv}(r,E) = g(r) N(E) V^{\rm eff}(E)$$

$$\Rightarrow \quad F_{\bar{K}N} = F^{\rm Ch}_{\bar{K}N}$$

- Born approx.
- on K̄N threshold

- Previous work — at resonance energy ⇒ $b = 0.47 \ {\rm fm}$
- This work — on K̄N threshold ⇒ $b = 0.46 \ {\rm fm}$

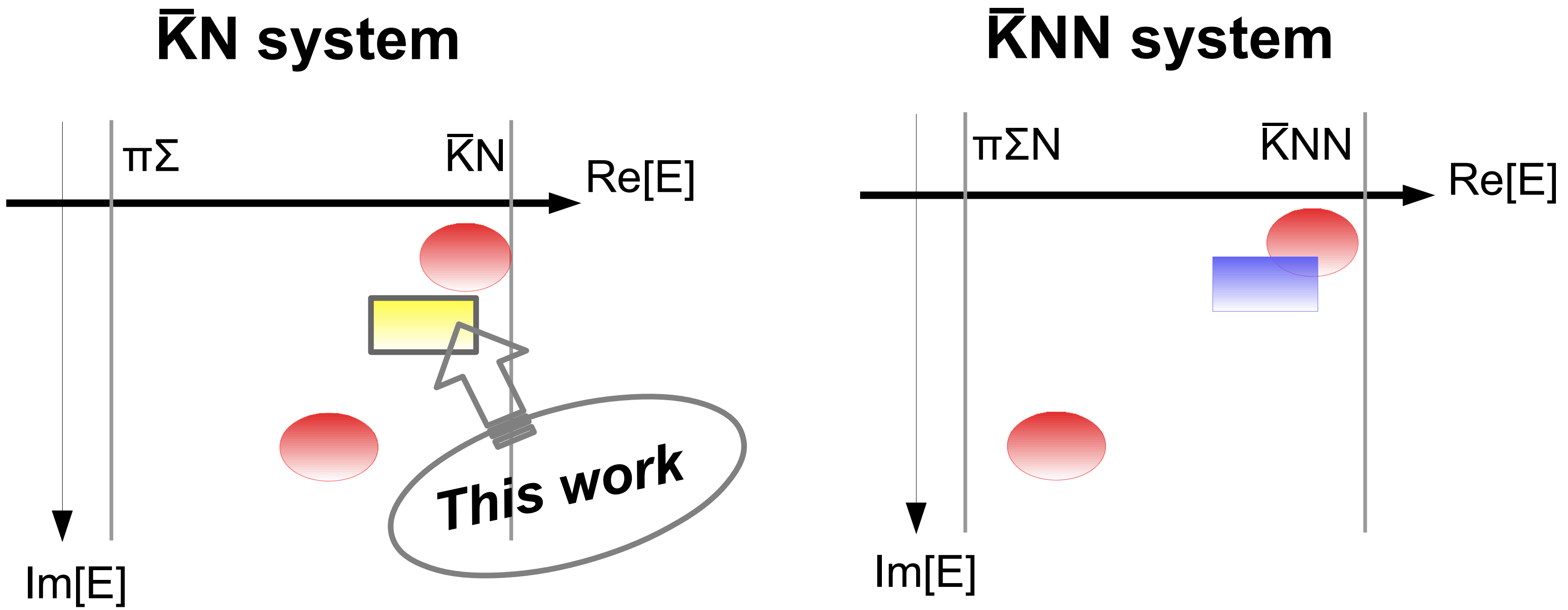
Consistent with original strategy

Determination of "b" is improved

➢ Consequence in few-body system

cf. Dote, Hyodo, Weise, Phys. Rev. C 79, 014003 (2009) :

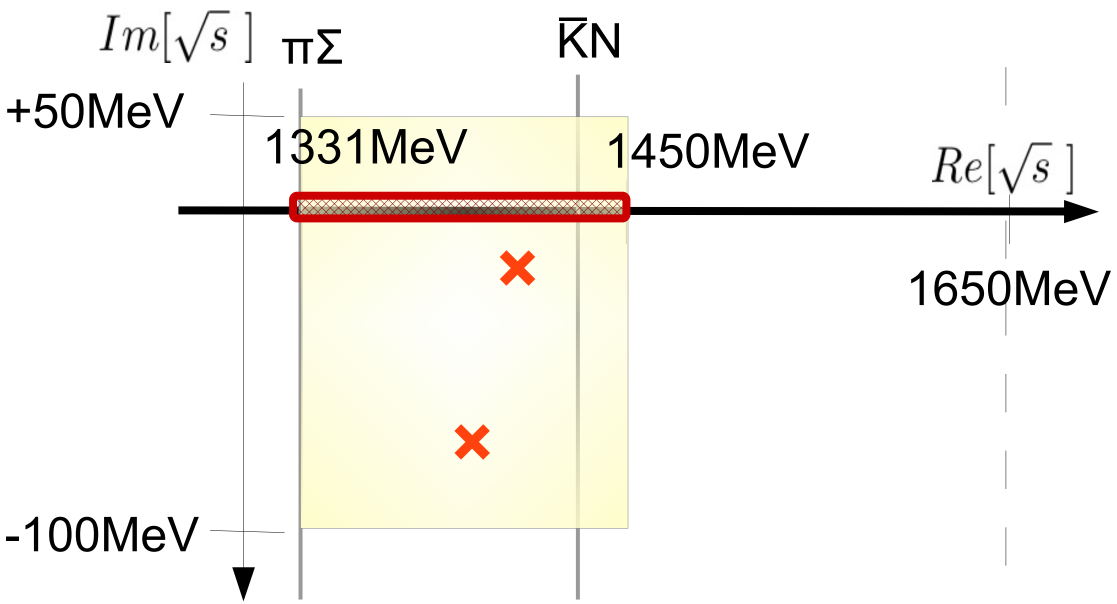
Ikeda, Kamano, Sato, Prog. Theor. Phys. 124, 3 (2010) :



Information in the complex energy plane is important for physical state

➢ definition

to discuss the "good" V^{equiv}

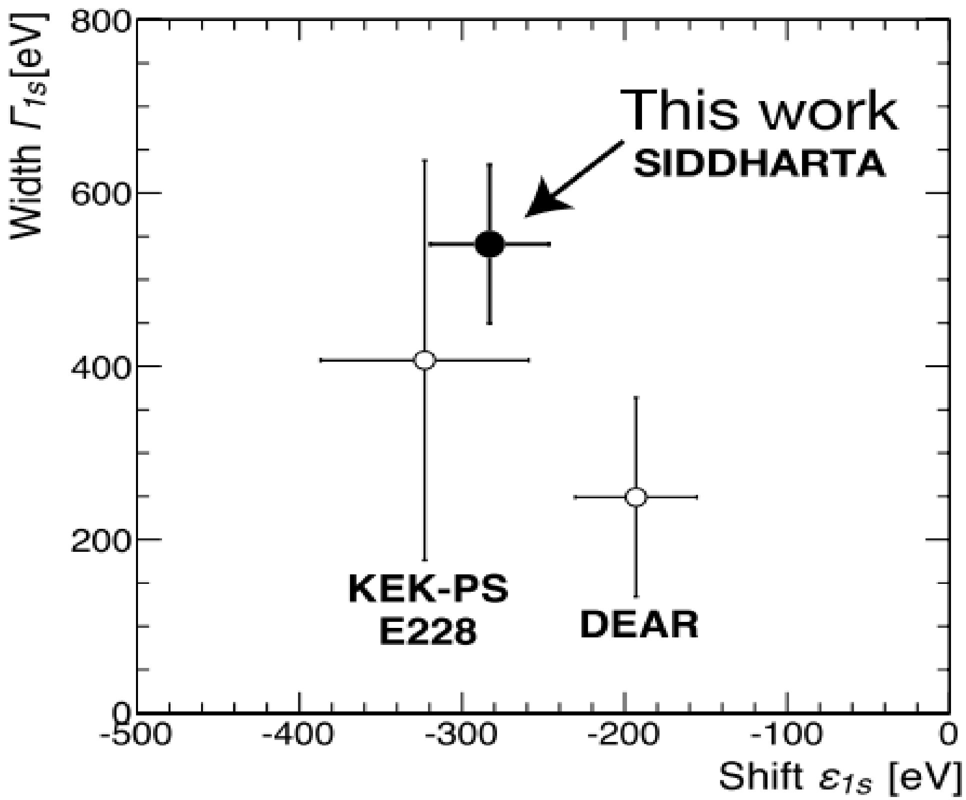


$$\Delta F_{real} = \frac{1}{N}\sum_i \left| \frac{F^{Ch}_{\bar{K}N} - F_{\bar{K}N}}{F^{Ch}_{\bar{K}N}} \right| \times 100$$

: average deviation of the amplitude on the real axis

P_{comp} : the percentage of the area in the complex plane which satisfies $\left| \frac{F^{Ch}_{\bar{K}N} - F_{\bar{K}N}}{F^{Ch}_{\bar{K}N}} \right| \times 100 \; < \; 20\%$

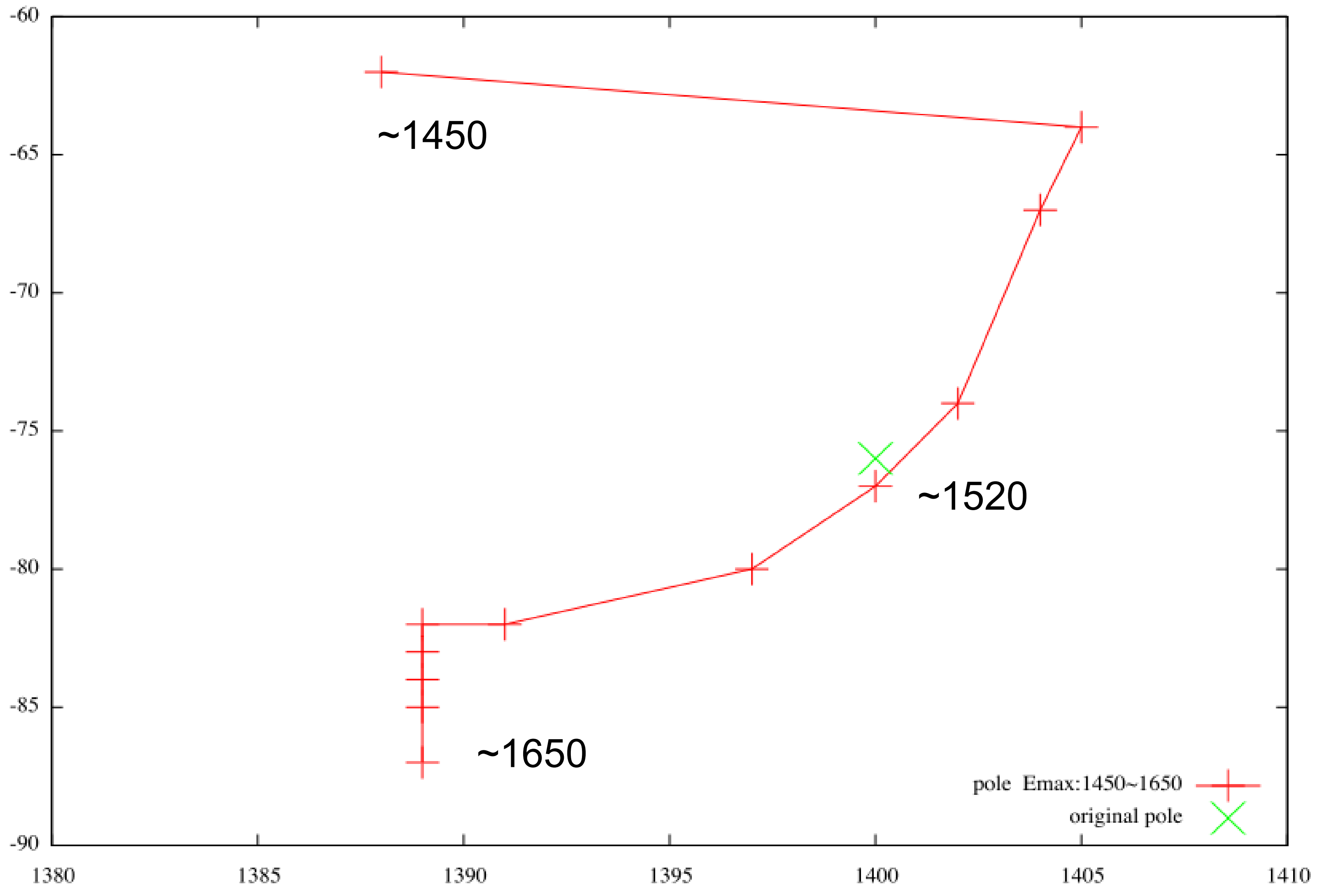
➢ SIDDHARTA



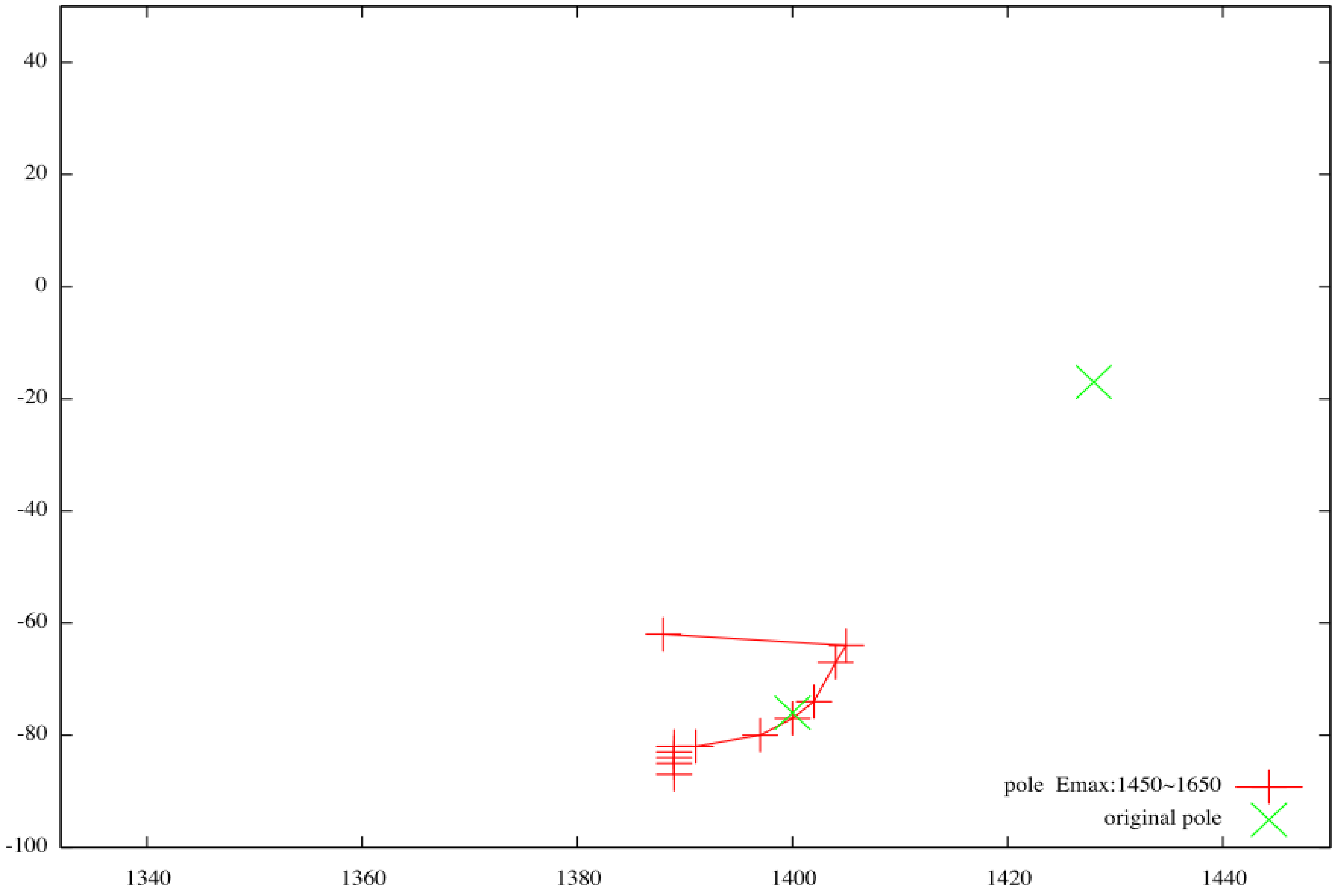
From this energy shift, scattering length can be calculated
→ scattering length is important to decide amplitude below threshold

➢ **Improvement (2/2)**

πΣ pole position against fit range

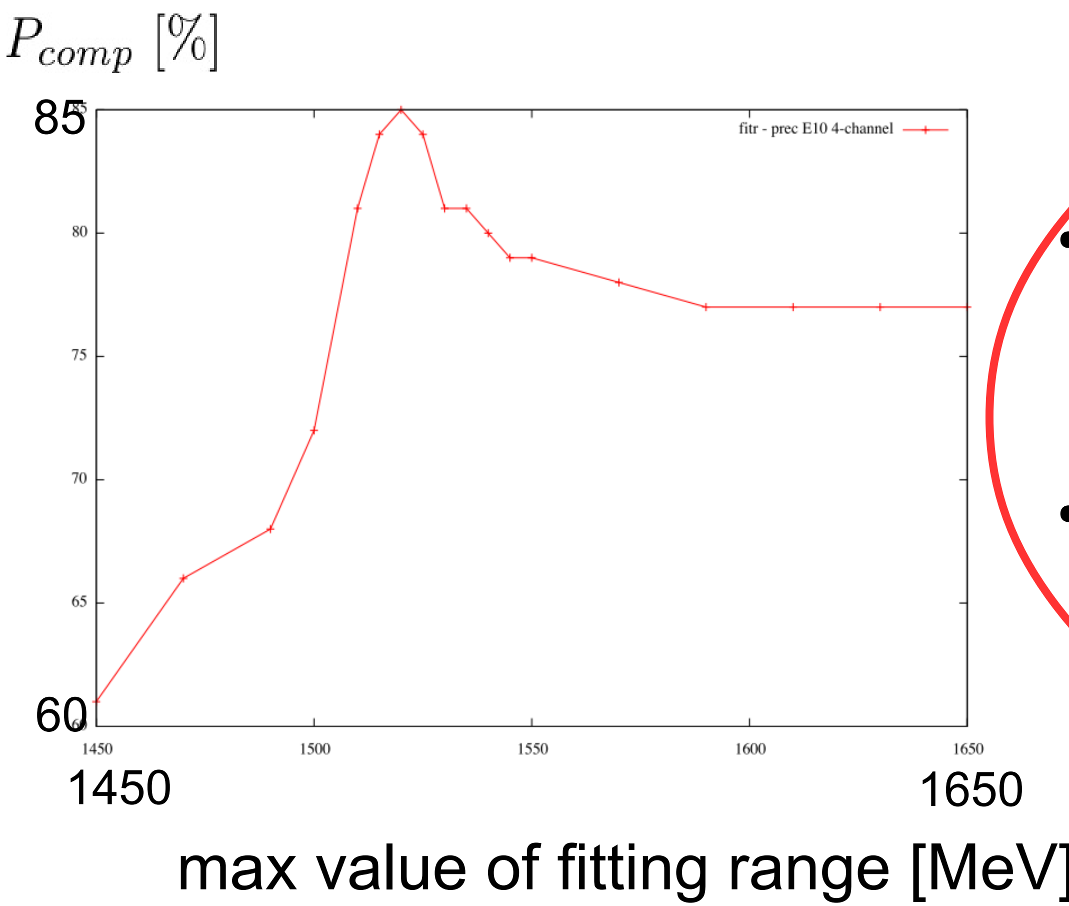


Second pole position (HNJH)



“precise area” against fit range (HNJH)

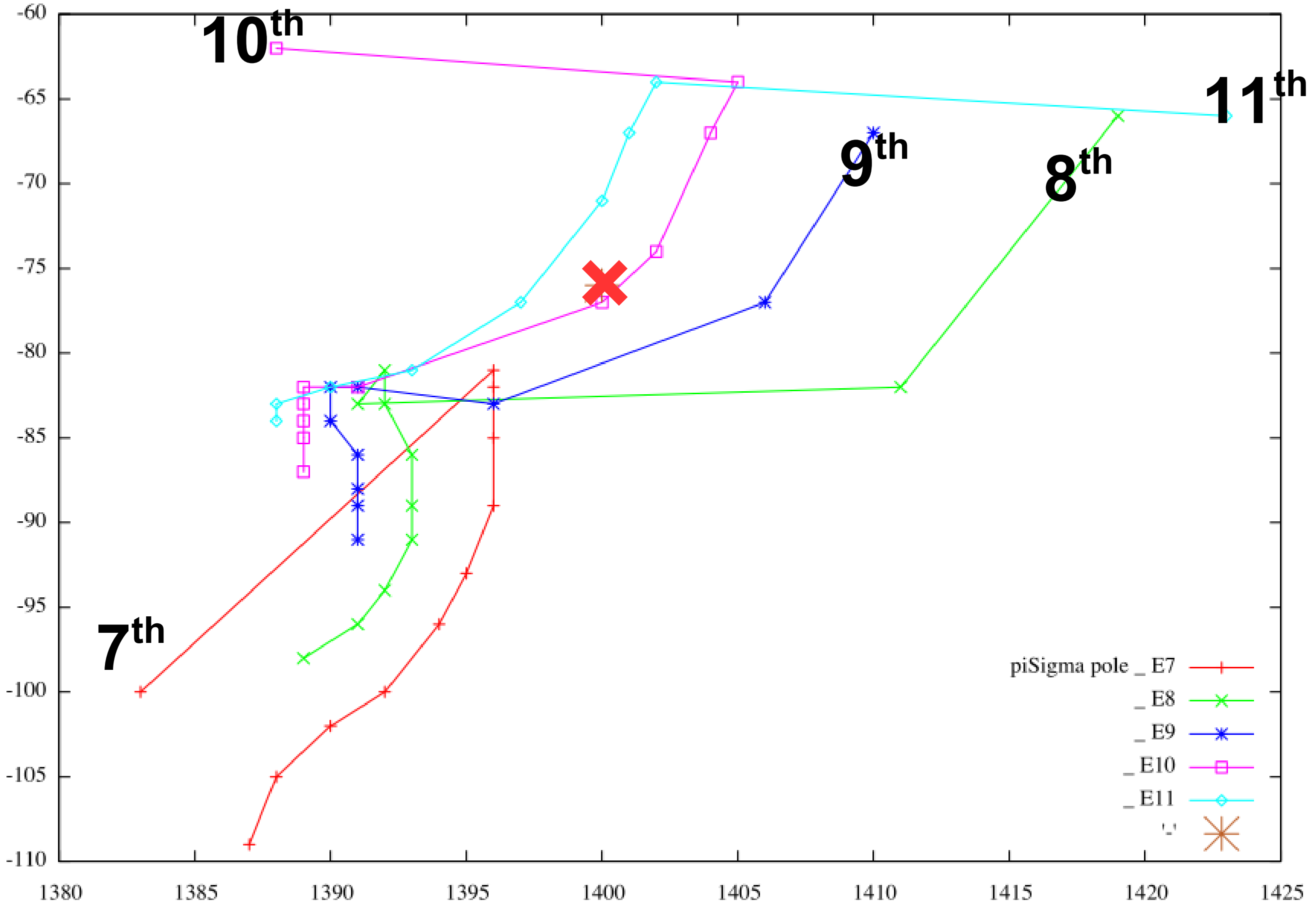
- We get second pole
- P_{comp} changes depending on fitting range



- P_{comp} tends to increase as fitting range is broadened
- P_{comp} reaches a maximum at $\sqrt{s}_{max} = 1520\ \mathrm{MeV}$

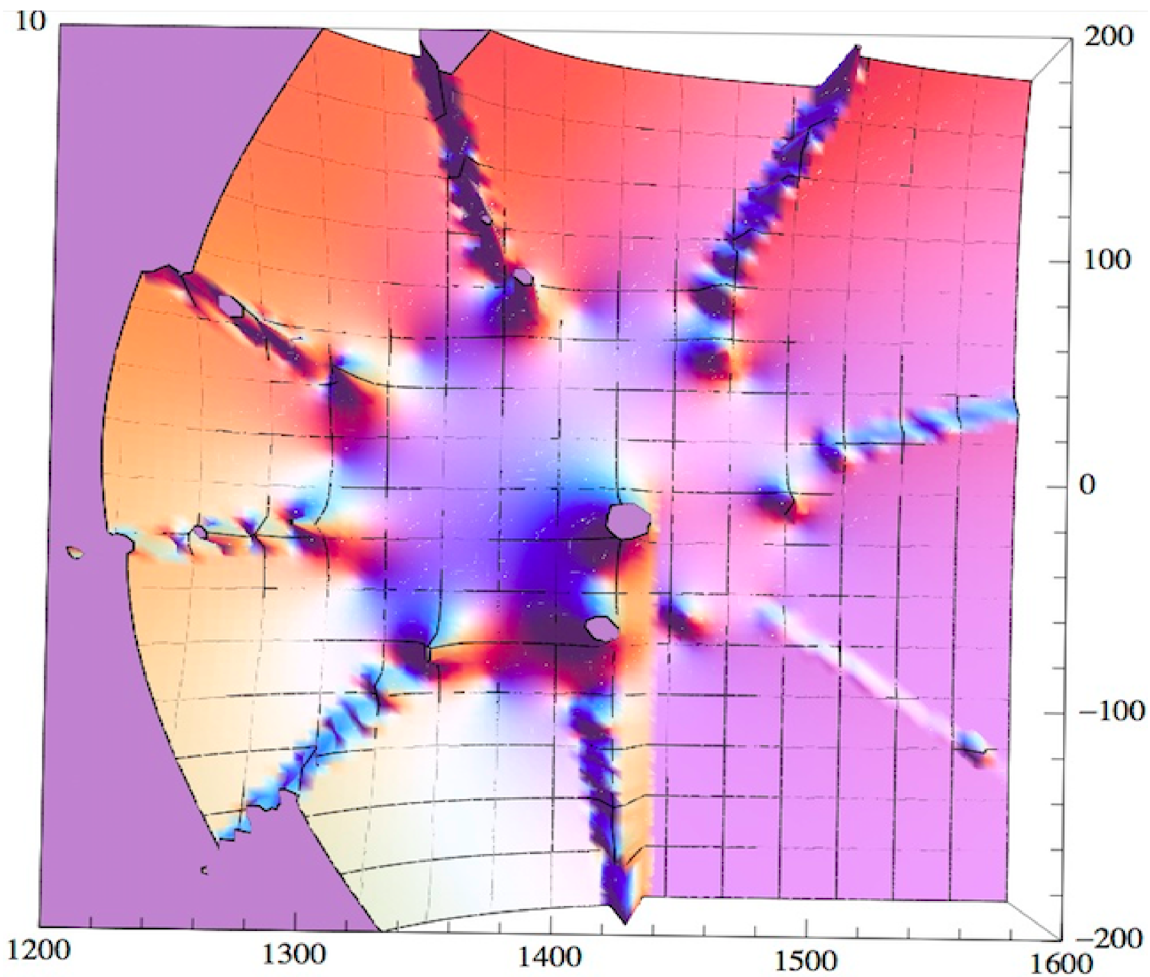
Pole position with several polynomial types (HNJH)

fit range : 1332~1450 (1470 for 11th) -1650 MeV

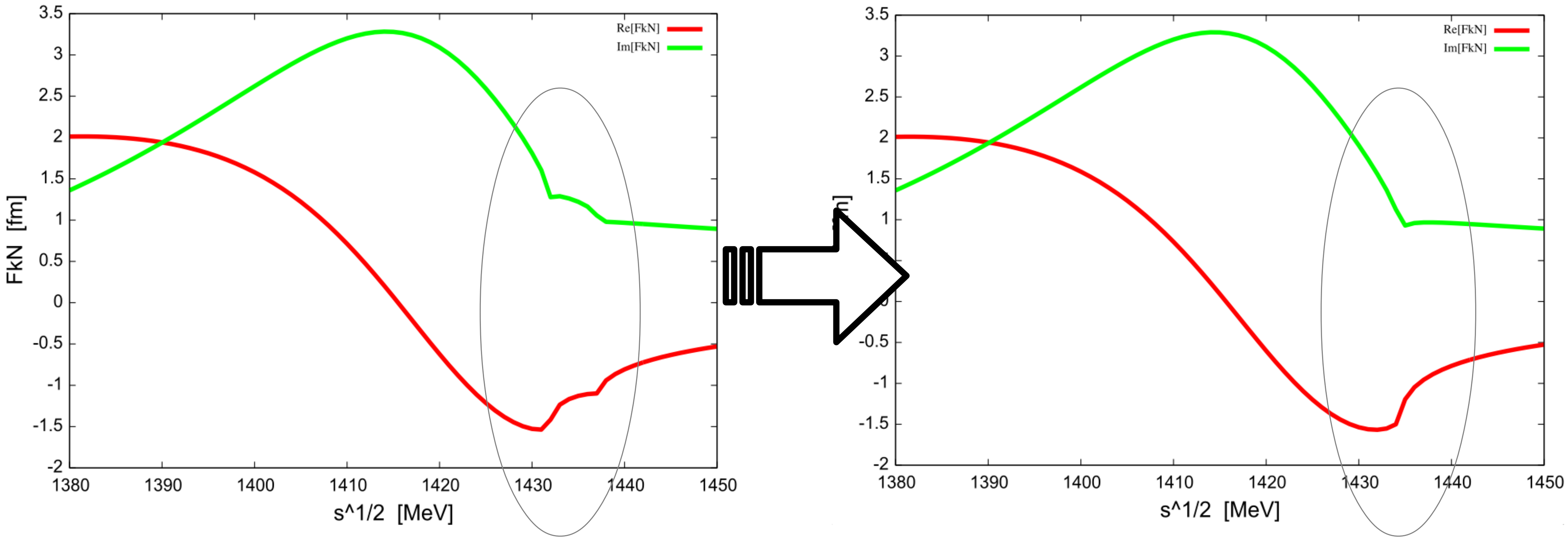


Pole position (HNJH)

fit function : 8th order in E
fit range : 1332~1450 MeV



➢ isospin average of $F_{\bar{K}N}$

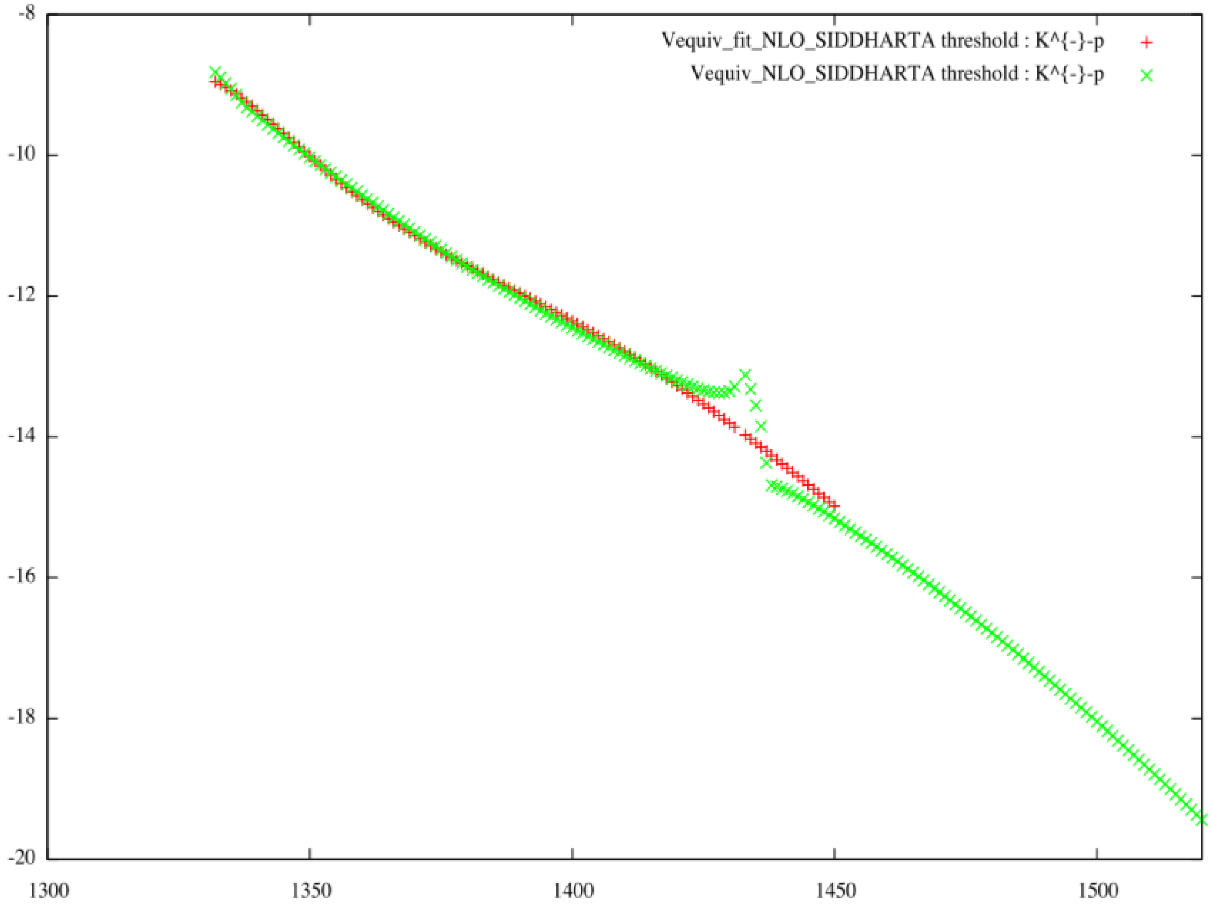


$F_{\bar{K}N}$ with physical mass (isosipin breaking)

$F_{\bar{K}N}$ with average mass (isosipin average)

- Two $\overline{\text{K}}$N thresholds coincide
- Subthreshold behavior is not affected

V^{equiv} with physical mass (SIDDHARTA)



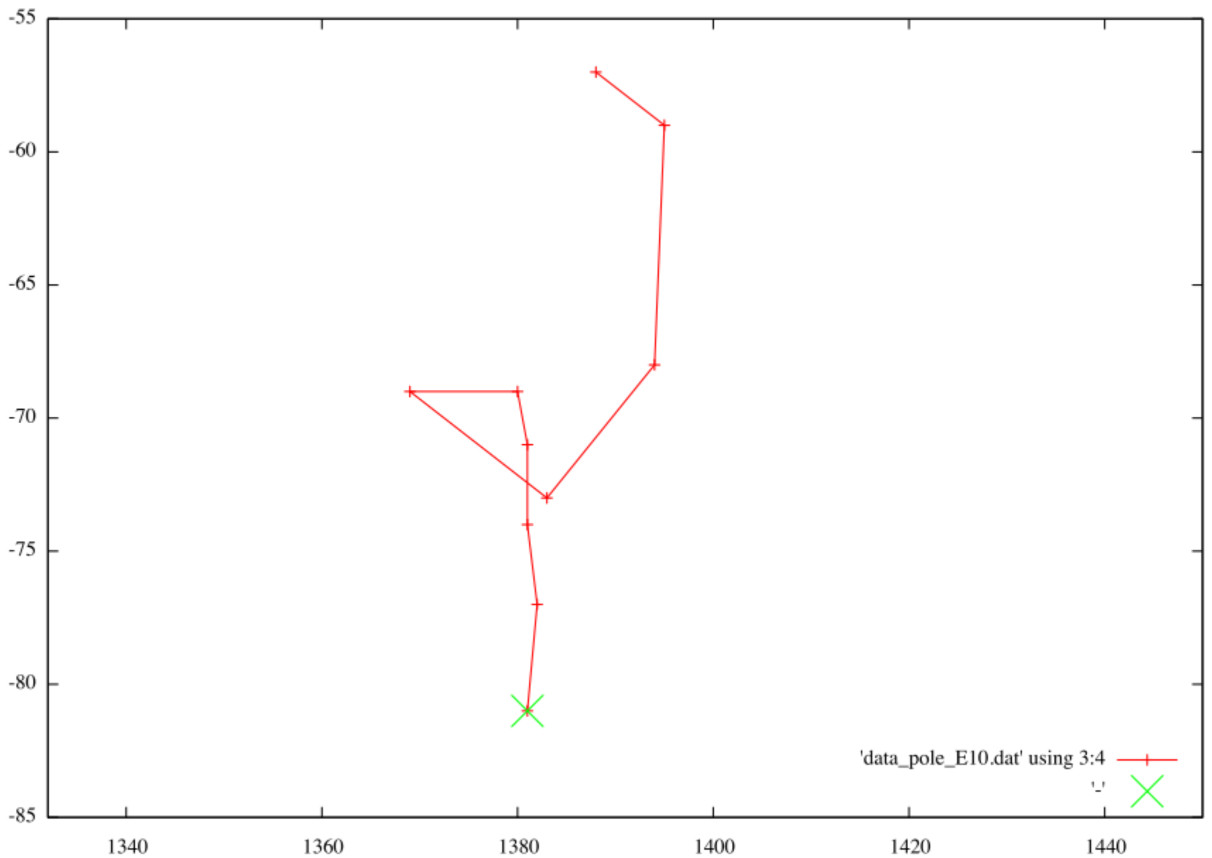
green : before fitting
red : after fitting

Isospin breaking

$(m_{\overline{K}}=m_{K-}\ ,\ \ M_N=M_p)$

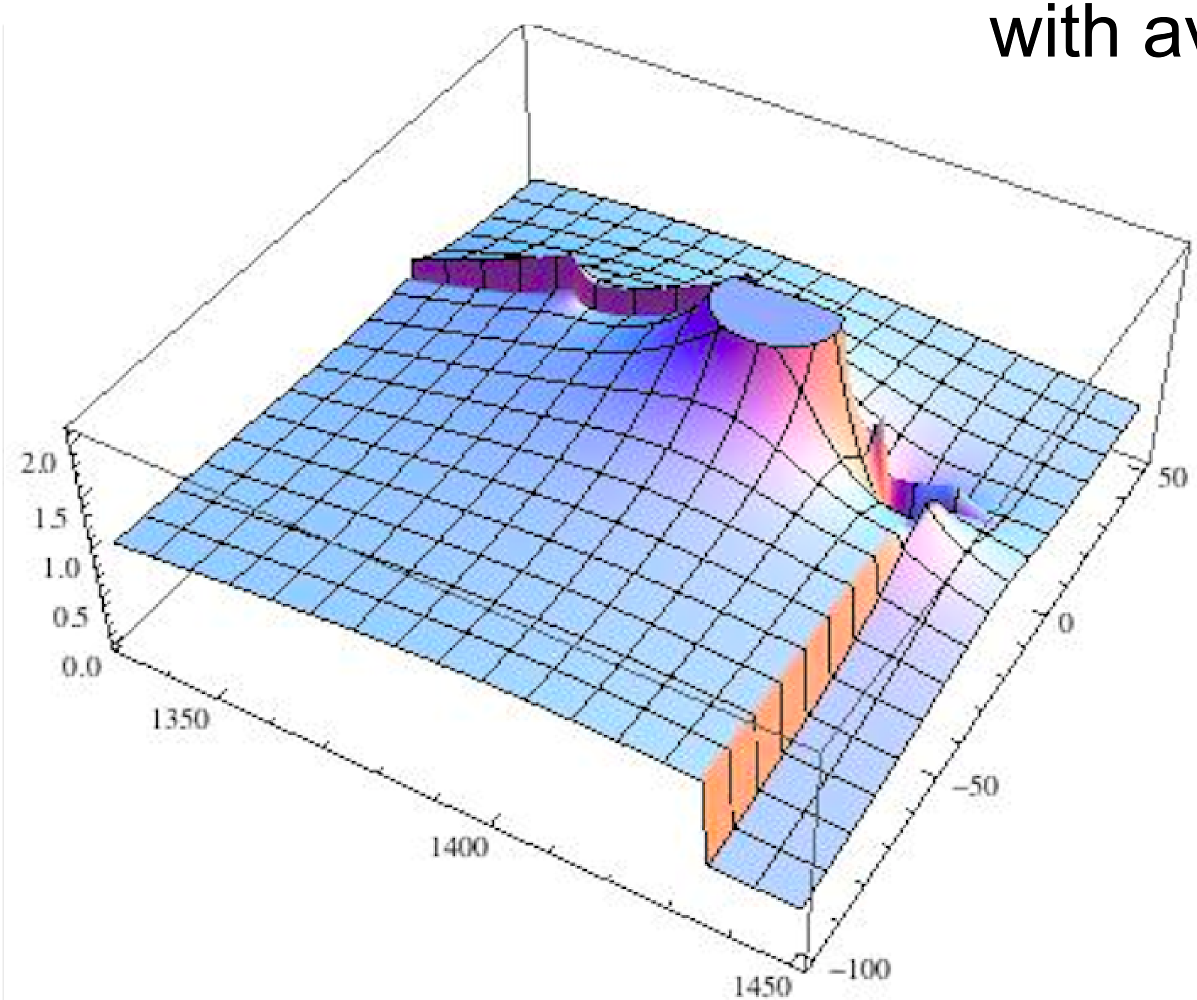
Pole position (SIDDHARTA)

fit function : 10^{th} order in E
fit range : 1332~1450-1650 MeV



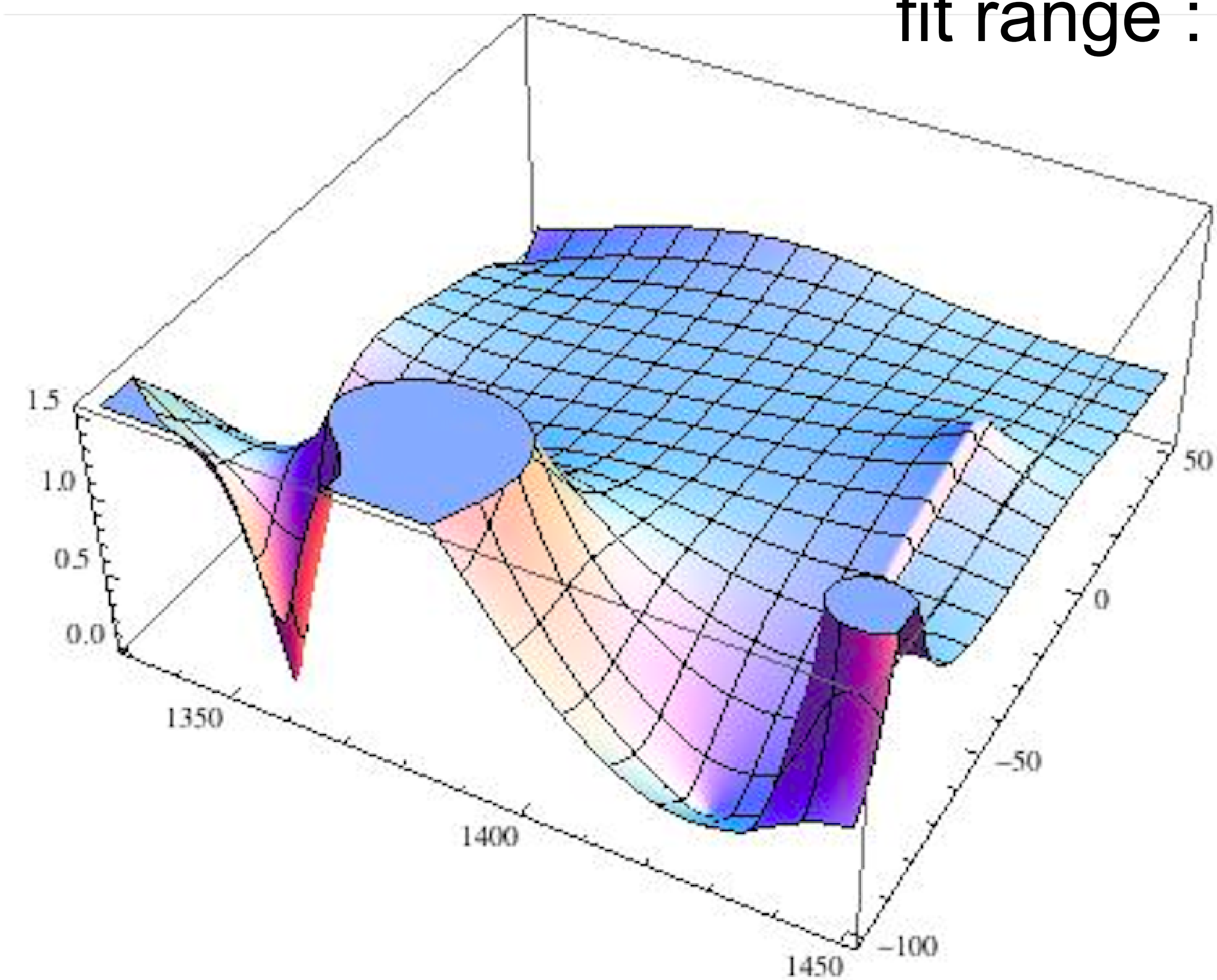
FkNCh in I=1 sector (SIDDHARTA)

with averaged mass

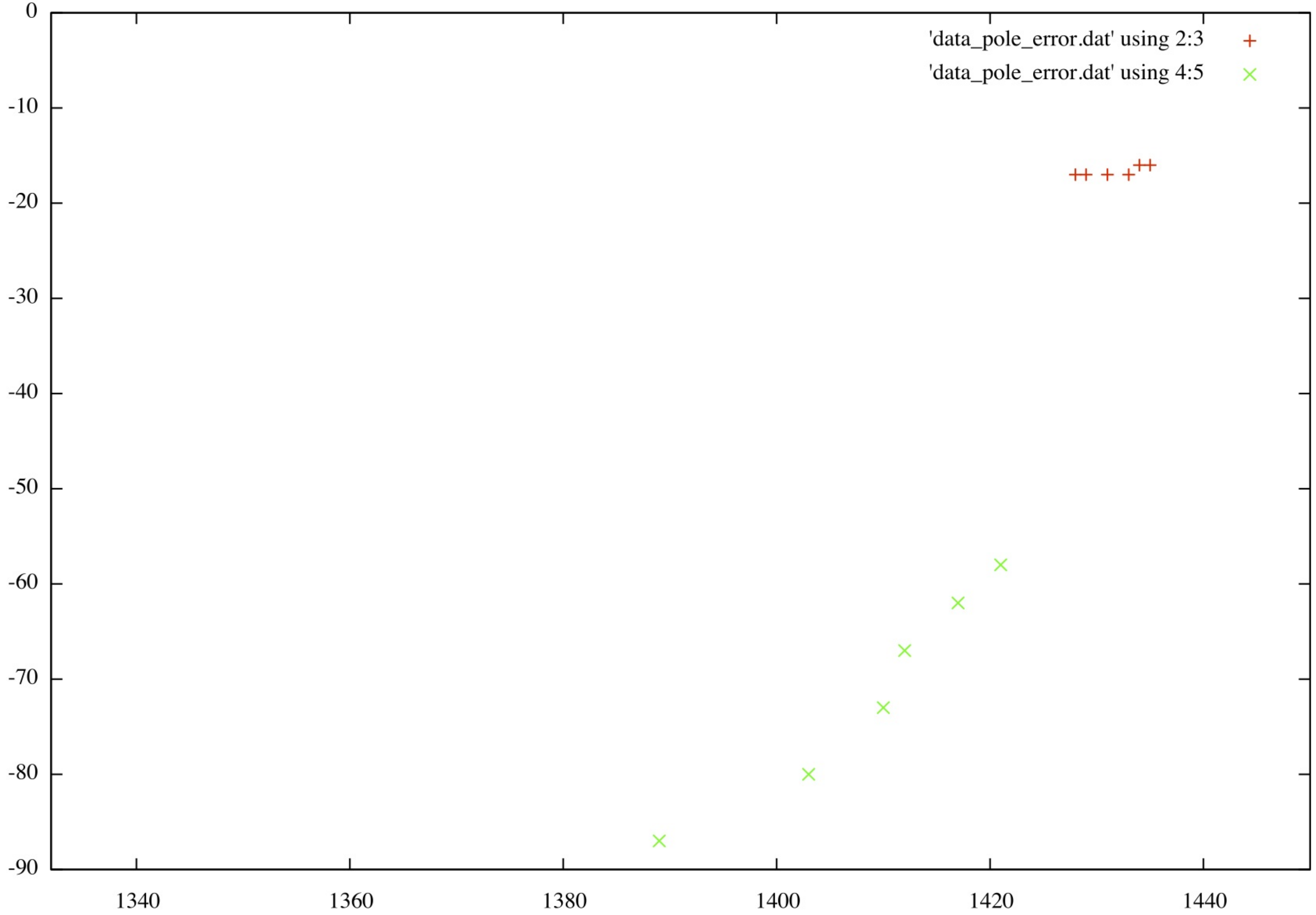


FkN in I=1 sector (SIDDHARTA)

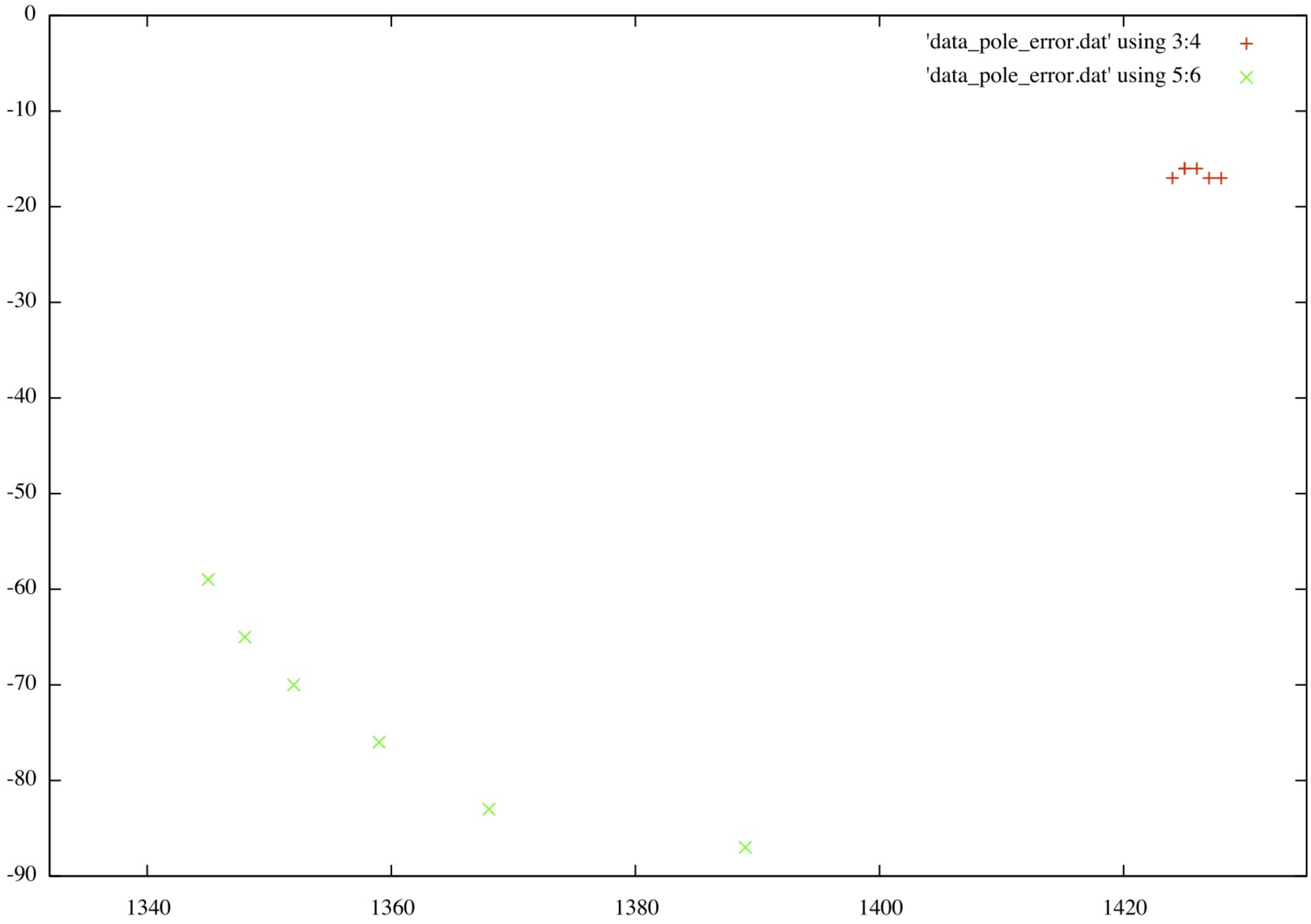
fit function : 10th order in E
fit range : 1332~1650 MeV



Pole uncertainty against the deviation on real axis (1)

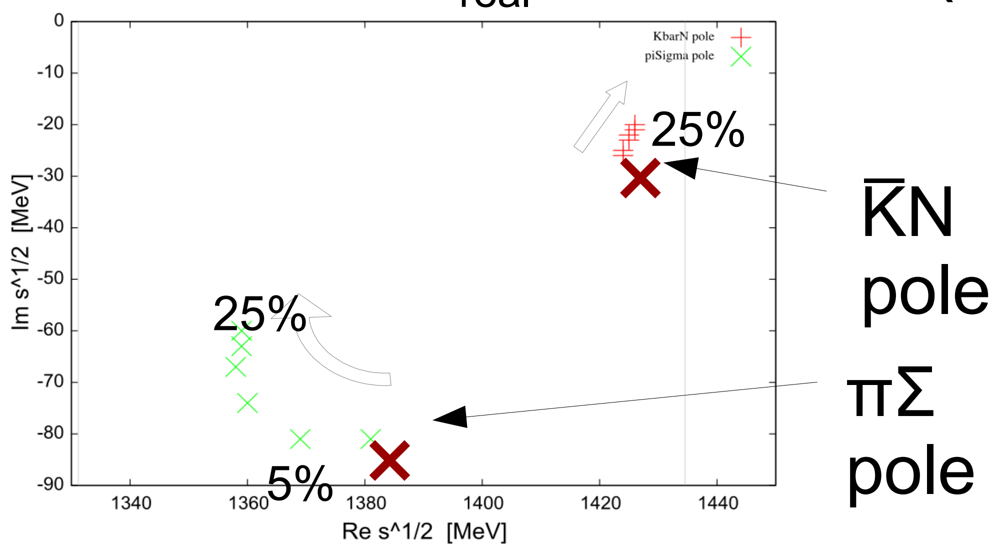


Pole uncertainty against the deviation on real axis (2)



➢ Stability of poles

Change ΔF_{real} by 0~25% $\left(\Delta F_{real} = \frac{1}{N} \sum_i \left| \frac{F^{Ch}_{\bar{K}N} - F_{\bar{K}N}}{F^{Ch}_{\bar{K}N}} \right| \times 100 \right)$

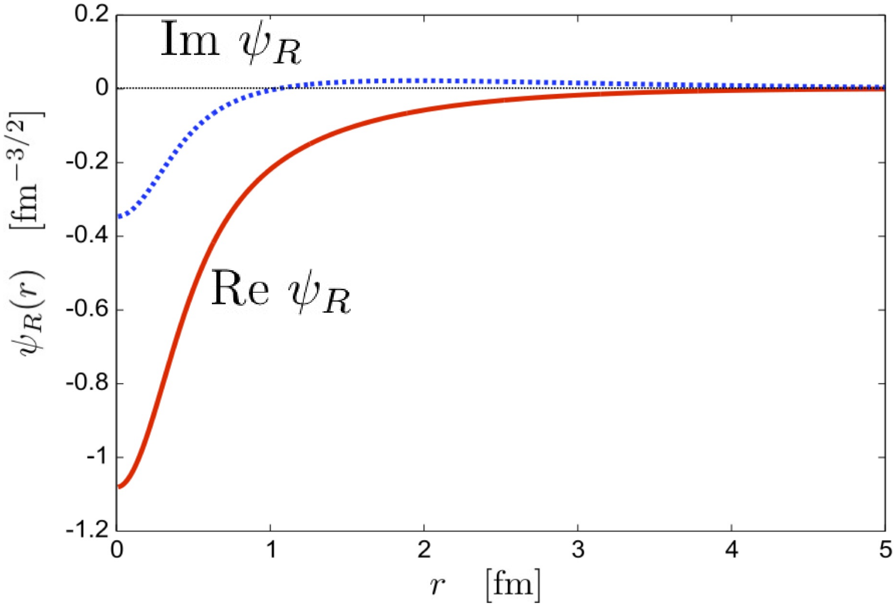


Deviation on real axis [%]	K̅N pole [MeV]	πΣ pole [MeV]	RMS (K̅N pole) → abs [fm]	RMS (πΣ pole) → abs [fm]
0	1424-26i	1381-81i	1.11-0.57i → 1.25	0.75-0.26i → 0.80
10	1425-23i		1.16-0.60i → 1.31	
25	1426-20i	1359-60i	124-0.64i → 1.39	0.79-0.19i → 0.81

Discussion

Hokkyo, Progress of Theoretical Physics, 33, 6 (1965)

➢ Wave function



$$\sqrt{\langle r^2 \rangle} = 1.06 - 0.57i \text{ fm}$$

$$\left|\sqrt{\langle r^2 \rangle}\right| = 1.20 \text{ fm}$$

cf.

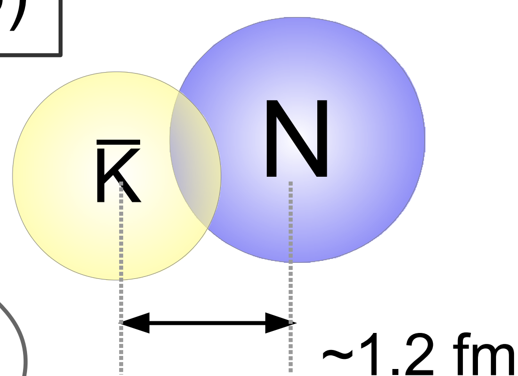
- potential from Weinberg Tomozawa term
  Dote, Myo, Nucl. Phys. A 930,86 (2014)

$$\sqrt{\langle r^2 \rangle} = 1.22 - 0.47i \text{ fm}$$

- response to the external current
  Sekihara, Hyodo, Phys.Rev.C 87,045202 (2013)

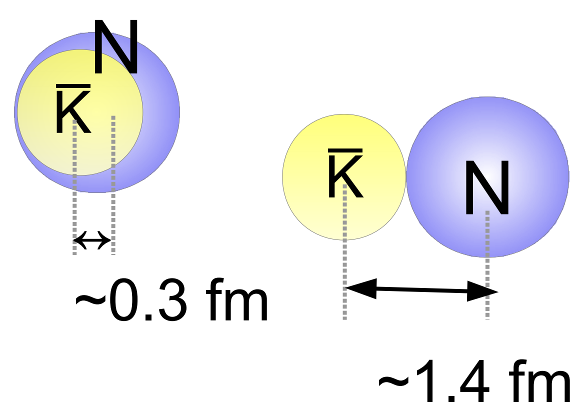
$$\sqrt{\langle r^2 \rangle} = 1.22 - 0.63i \text{ fm}$$

Λ(1405)

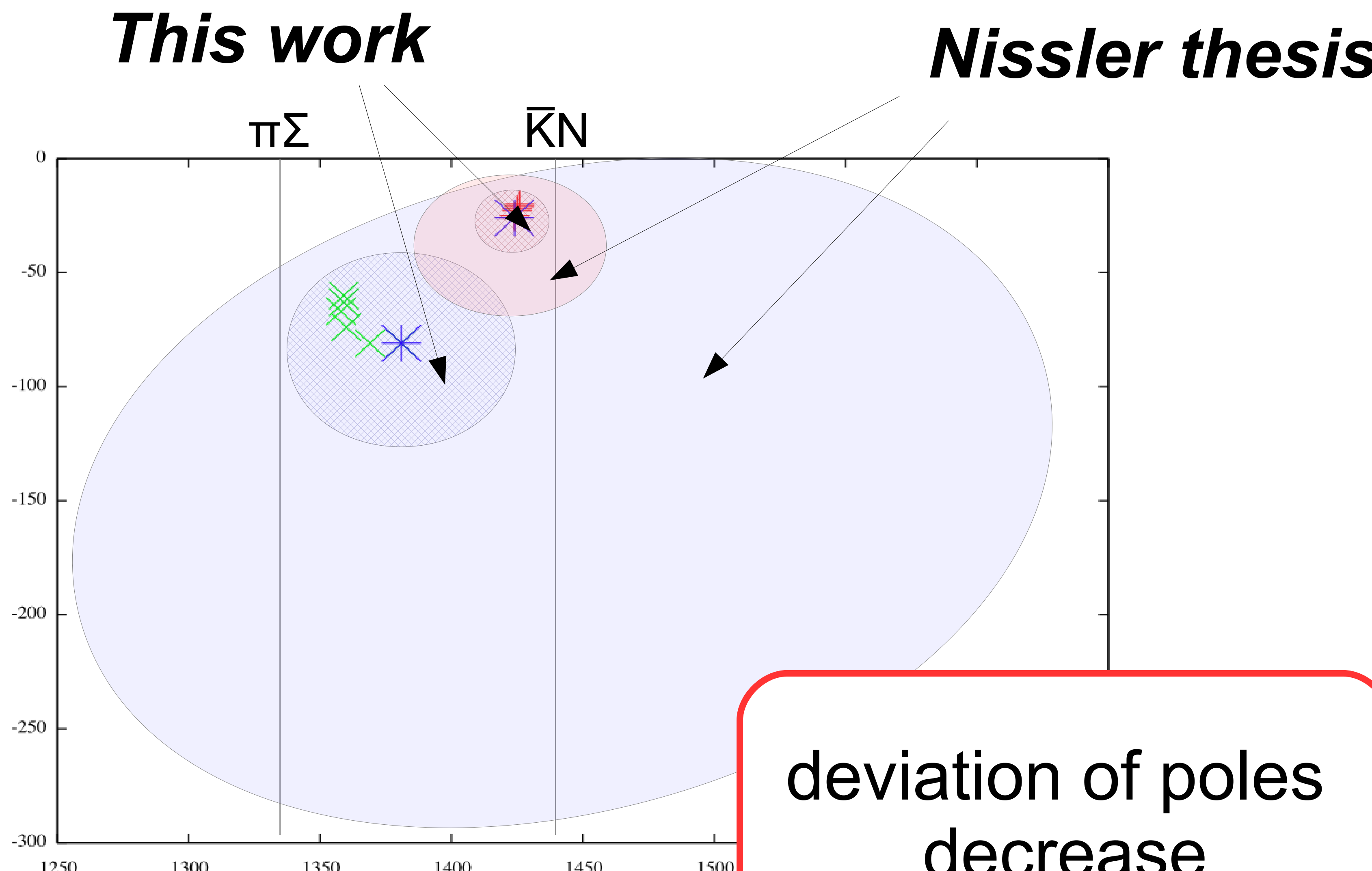


p : ~0.85fm
K^- : ~0.55fm

cf.



➢ Deviation of poles



deviation of poles decrease

Normalization of wave function

usual normalization (complex potential) :

$$\frac{\partial}{\partial t}(\Psi_m^*\Psi_n) = (\frac{\partial}{\partial t}\Psi_m)^*\Psi_n + \Psi_m^*(\frac{\partial}{\partial t}\Psi_n)$$
$$= i(-\frac{1}{2}\nabla^2\Psi_m + V\Psi_m)^*\Psi_n - i\Psi_m^*(-\frac{1}{2}\nabla^2\Psi_n + V\Psi_n)$$
$$= -\nabla\cdot\boldsymbol{j} + 2(\mathrm{Im}V)\Psi_m^*\Psi_n,$$

$$\left(\ \boldsymbol{j} \equiv -\frac{i}{2}[\Psi_m^*\nabla\Psi_n - (\nabla\Psi_m)^*\Psi_n]\ \right)$$

$$\frac{\partial}{\partial t}(\Psi_m^*\Psi_n) = i(E_m^* - E_n)\Psi_m^*\Psi_n.$$

$$i(E_m^* - E_n)\int d\boldsymbol{r}\Psi_m^*\Psi_n = 2\int d\boldsymbol{r}\Psi_m^*(\mathrm{Im}V)\Psi_n.$$

Usual normalization doesn't satisfy the orthogonality.

Normalization of wave function

Complex potential (E-indep.) :

$$\frac{\partial}{\partial t}({\Psi^{\dagger}_m}^*\Psi_n) = (\frac{\partial}{\partial t}\Psi^{\dagger}_m)^*\Psi_n + {\Psi^{\dagger}_m}^*(\frac{\partial}{\partial t}\Psi_n)$$

$$= i(-\frac{1}{2}\nabla^2\Psi^{\dagger}_m + V^*\Psi^{\dagger}_m)^*\Psi_n - i{\Psi^{\dagger}_m}^*(-\frac{1}{2}\nabla^2\Psi_n + V\Psi_n)$$

$$= -\nabla\cdot\boldsymbol{j}, \qquad \left(\boldsymbol{j} \equiv -\frac{i}{2}[{\Psi^{\dagger}_m}^*\nabla\Psi_n - (\nabla\Psi^{\dagger}_m)^*\Psi_n]\right)$$

$$\frac{\partial}{\partial t}({\Psi^{\dagger}_m}^*\Psi_n) = i(E_m - E_n)\Psi^*_m\Psi_n = i(E_m - E_n)e^{-i(E_n-E_m)t}{\psi^{\dagger}_m}^*\psi_n$$

$$= i(E_m - E_n)e^{-i(E_n-E_m)t}\psi_m\psi_n$$

$$i(E_m - E_n)e^{-i(E_n-E_m)t}\int \boldsymbol{r}\psi_m\psi_n = 0.$$

Orthogonality is satisfied with Gamow vector for complex potential.

Normalization of wave function

E-dep. Complex potential :

$$\frac{\partial}{\partial t}\Psi^{\dagger *}_{E'}\Psi_E = \frac{\partial \Psi^{\dagger *}_{E'}}{\partial t}\Psi_E + \Psi^{\dagger *}_{E'}\frac{\partial \Psi_E}{\partial t}$$

$$= \left[-i\left\{-\frac{1}{2}\nabla^2 + V^*(E')\right\}\Psi^{\dagger}_{E'}\right]^* \Psi_E + \Psi^{\dagger *}_{E'}\left[-i\left\{-\frac{1}{2}\nabla^2 + V(E)\right\}\Psi_E\right]$$

$$= -\nabla\cdot\boldsymbol{j}^G + i\Psi^{\dagger *}_{E'}\left(V(E') - V(E)\right)\Psi_E,$$

$$\left(\ \boldsymbol{j}^G = -\frac{i}{2}\left[\Psi^{\dagger *}_{E'}(\boldsymbol{r},t)\nabla\Psi_E(\boldsymbol{r},t) - \left\{\nabla\Psi^{\dagger *}_{E'}(\boldsymbol{r},t)\right\}\Psi_E(\boldsymbol{r},t)\right]\ \right)$$

$$\int d\boldsymbol{r}\ \psi_{E'}(\boldsymbol{r})\left[1 - \frac{V(\boldsymbol{r},E') - V(\boldsymbol{r},E)}{E' - E}\right]\psi_E(\boldsymbol{r}) = 0,$$

$$(E' \neq E).$$

Orthogonality is satisfied with Gamow vector and the additional term.

K^-p correlation