Kaon-Nucleon systems and their interaction in the Skyrme model

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arXiv:1605.01203

Contents

1. Introduction

Introduction

Kaon nucleon systems have been receiving a lot of attention

・Strong attraction between the anti-kaon and the nucleon
Y. Akaishi and T. Yamazaki, Phys. Rev. C **65** (2002)
・The anti-kaon nucleon ($\bar{K}N$) bound state = $\Lambda(1405)$

・few body nuclear systems with $\bar{K}$

Y. Akaishi and T. Yamazaki, Phys. Rev. **C 65** (2002)
Y. Ikeda and T. Sato, Phys. Rev. **C 76** (2007)
A. Doté, T. Hyodo and W. Weise, Phys. Rev. **C 79** (2009) etc

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The properties of these systems are under debate

Introduction・Purpose

the $\bar{K}N$ interaction is important
to investigate the few body systems with $\bar{K}$

Theoretical studies of $\bar{K}N$ interaction

・Phenomeonlogical approach

Y. Akaishi and T. Yamazaki, Phys. Rev. **C 65** (2002) etc

・Chiral theory: based on a 4-point local interaction

Y. Ikeda, T. Hyodo and W. Weise, Nucl. Phys. **A 881** (2012)

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Introduction・Purpose

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Investigate the KN system in the Skyrme model
where the nucleon is described as a soliton.

2. The Skyrme model

The Skyrme model 1

T.H.R. Skyrme, Nucl. Phys. **31** (1962);
Proc. Roy. Soc. A **260** (1961)

- Describe the interaction between mesons and baryons by mesons
- Baryon emerges as a soliton of meson fields.

$$\phi = \frac{1}{\sqrt{2}}\lambda_a\phi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$U = \exp\left[i\frac{2}{F_\pi}\lambda_a\phi_a\right]$$

λ_a: Gell-Mann matrices (a = 1, 2, … ,8)

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$$U = \exp\left[i\frac{2}{F_\pi}\lambda_a \phi_a\right]$$

λ_a: Gell-Mann matrices (a = 1, 2, … ,8)

- **<u>For SU(2)</u>**

$$L = \underbrace{\frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right)}_{\text{The kinetic term}} + \underbrace{\frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2}_{\text{The interaction term (the Skyrme term)}}$$

F_π, e: parameters

The Skyrme model 2

・Hedgehog ansatz

π has three degrees of freedom(π^0, π^+, π^-)

・two of these: the angles of the radial vector, θ, φ

・the rest: a function depending on r

⇒a special configuration called the hedgehog ansatz

$$\boxed{\text{Hedgehog ansatz:}\,U_H = \exp\left[i\boldsymbol{\tau}\cdot\hat{r}F(r)\right]}$$

・Quantization

The hedgehog ansatz is classical

→without spin or isospin

→become a physical state by quantization

$A(t)$: 2×2 isospin rotation matrix

$R_{ab}(t)$: 3×3 spatial rotation matrix

$$U_H(\boldsymbol{x}) \to U_H(t,\boldsymbol{x}) = A(t)\exp\left[i\tau_a R_{ab}(t)\hat{r}_b F(r)\right]A^\dagger(t)$$

The baryon with $I=J$ from the symmetry

of the hedgehog soliton

3. Method

Method

SU(3) symmetry is broken → $m_u = m_d = 0, m_s \neq 0$

Callan-Klebanov approach (CK approach)

- Introduce the kaon as fluctuations around the hedgehog soliton
- Form a bound state of the kaon and the hedgehog soliton
- rotate the system to generate hyperons
- Follow the $1/N_c$ counting rule

C.G. Callan and I. Klebanov, Nucl. Phys. **B 262** (1985)

C .G.Callan, K .Hornbostel and I. Klebanov, Phys. Lett. **B 202** (1988)

Our approach

- Rotate the hedgehog soliton to generate the nucleon
- Introduce the kaon as fluctuations around the nucleon
- describe kaon-nucleon systems
- Violate the $1/N_c$ counting rule

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Lagrangian and ansatz

- **Expand to the SU(3) Skyrme model**

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

- **Ansatz**

$$U = \begin{cases} A(t)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger(t) & \text{: Callan-Klebanov ansatz} \\ A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t) & \text{: } \textbf{Our ansatz} \end{cases}$$

$$U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

Hedgehog ansatz (2×2 matrix)

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 3, 4, 5, 6$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

Derivation 1

・Substitute our ansatz for the Lagrangian

Ansatz

$$U = A(t)\sqrt{U_\pi}A^\dagger(t)U_K A(t)\sqrt{U_\pi}A^\dagger(t)$$

$$U_K = \exp\left[i\frac{2}{F_\pi}\lambda_a K_a\right], \quad a = 3,4,5,6 \qquad U_\pi = \begin{pmatrix} U_H & 0 \\ 0 & 1 \end{pmatrix}$$

$$\lambda_a K_a = \sqrt{2}\begin{pmatrix} 0_{2\times 2} & K \\ K^\dagger & 0 \end{pmatrix} \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = \begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$$

Lagrangian

$$\begin{aligned} L \quad = \quad & \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 \\ & + L_{SB} + L_{WZ} \end{aligned}$$

Obtaining Lagrangian

$$L = L_{SU(2)} + L_{KN}$$

$$L_{SU(2)} = \frac{1}{16}{F_\pi}^2 \mathrm{tr}\left[\partial_\mu \tilde{U}^\dagger \partial^\mu \tilde{U}\right] + \frac{1}{32e^2}\mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2$$

$$\begin{aligned} L_{KN} \;=\;& (D_\mu K)^\dagger D^\mu K - K^\dagger a_\mu^\dagger a^\mu K - m_K^2 K^\dagger K \\ &+ \frac{1}{(eF_\pi)^2}\left\{ -K^\dagger K \mathrm{tr}\left[\partial_\mu U_H U_H^\dagger, \partial_\nu U_H U_H^\dagger\right]^2 - 2\,(D_\mu K)^\dagger D_\nu K \mathrm{tr}\,(a^\mu a^\nu) \right. \\ &\qquad \left. -\frac{1}{2}\,(D_\mu K)^\dagger D^\mu K \mathrm{tr}\left(\partial_\nu U_H^\dagger \partial^\nu U_H\right) + 6\,(D_\nu K)^\dagger\,[a^\nu, a^\mu]\, D_\mu K \right\} \\ &+ \frac{3i}{F_\pi^2} B^\mu \left[(D_\mu K)^\dagger K - K^\dagger (D_\mu K)\right] \end{aligned}$$

$$\tilde{U} = A(t) U_H A^\dagger(t), \quad \tilde{\xi} = A(t)\sqrt{U_H}A^\dagger(t) \qquad D_\mu K = \partial_\mu K + v_\mu K$$

$$v_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} + \tilde{\xi}\partial_\mu \tilde{\xi}^\dagger\right)$$

$$a_\mu = \frac{1}{2}\left(\tilde{\xi}^\dagger \partial_\mu \tilde{\xi} - \tilde{\xi}\partial_\mu \tilde{\xi}^\dagger\right)$$

$$B^\mu = -\frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2}\mathrm{tr}\left[\left(U_H^\dagger \partial_\nu U_H\right)\left(U_H^\dagger \partial_\alpha U_H\right)\left(U_H^\dagger \partial_\beta U_H\right)\right]$$

G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228** (1983)

Derivation 2

・Decompose the kaon filed

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} = \psi_I K(t, \boldsymbol{r}) \to \underline{\psi_I}\ \underline{K(\boldsymbol{r})}\, e^{-iEt}$$

Spatial wave function

Isospin wave function

・Expand the $K(r)$ by the spherical harmonics

$$K(\boldsymbol{r}) = \sum_{l,m} C_{lm\alpha} Y_{lm}(\theta, \phi)\, k_l^{\alpha}(r)$$

$Y_{lm}(\theta, \varphi)$: Spherical harmonics

l : orbital angular momentum

m : the 3rd component of l

α : the other quantum numbers

・Take a variation with respect to the kaon radial function

⇒Obtain the equation of motion for the kaon around the nucleon

4. Results and Discussions

Results (Equation of motion)

・Equation of motion(E.o.M)

$$\boxed{-\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^{\alpha}(r)}{dr}\right) - E^2 f(r) k_l^{\alpha}(r) + \left(m_K^2 + V(r)\right) k_l^{\alpha}(r) = 0}$$

$h(r), f(r)$: functions depending on r

m_K : the mass of the kaon, E : the kaon energy

$V(r)$: the kaon nucleon interaction term

Investigate KN systems by solving this E.o.M

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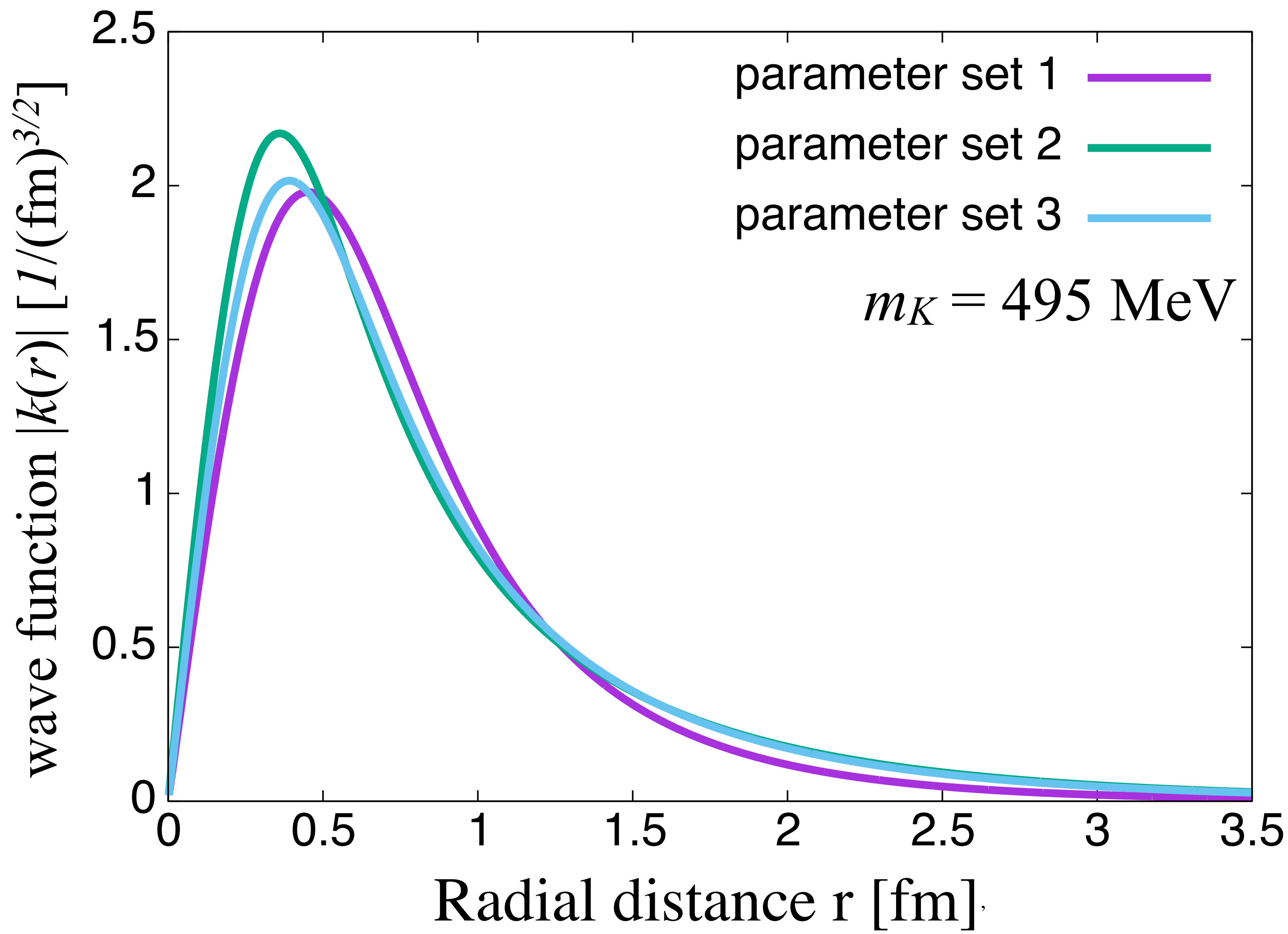
Concentrate on the $\bar{\mathrm{K}}$N(I=0) bound state

$\bar{K}N$ Bound state



- **$\bar{K}N$ bound states with $I_{tot} = 0, l = 0$ (Binding Energy: B.E.)**

	F_π [MeV]	e	B.E. [MeV]
parameter set 1	129	5.45	82.9
parameter set 2	186	5.45	27.2
parameter set 3	186	4.82	32.9

when F_π is the exp. value, **of order ten MeV**

Bound state properties

• Root mean square radii for N and K

$$\langle r_N^2 \rangle = \int_0^\infty dr\ r^2 \rho_B(r)\,,\quad \rho_B(r) = -\frac{2}{\pi}\sin^2 F F'$$

G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228** (1983)

$$\langle r_K^2 \rangle = \int dV\ r^2 \left[Y_{00}(\hat{r})\, k_0^0(r)\right]^2 = \int_0^\infty dr\ r^4 k^2(r) \qquad Y_{00} = \frac{1}{\sqrt{4\pi}}$$

• properties of the $\bar{\mathrm{K}}$N($I = 0$) bound states

	F_π [MeV]	e	B.E. [MeV]	$\langle r_N^2 \rangle^{1/2}$ [fm]	$\langle r_K^2 \rangle^{1/2}$ [fm]
parameter set 1	129	5.45	82.9	0.59	0.99
parameter set 2	186	5.45	27.2	0.41	1.19
parameter set 3	186	4.82	32.9	0.46	1.18

The anti-kaon is **weakly binding** to the nucleon

Potential

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^\alpha(r)}{dr}\right) - E^2 f(r) k_l^\alpha(r) + \left(m_K^2 + V(r)\right) k_l^\alpha(r) = 0$$

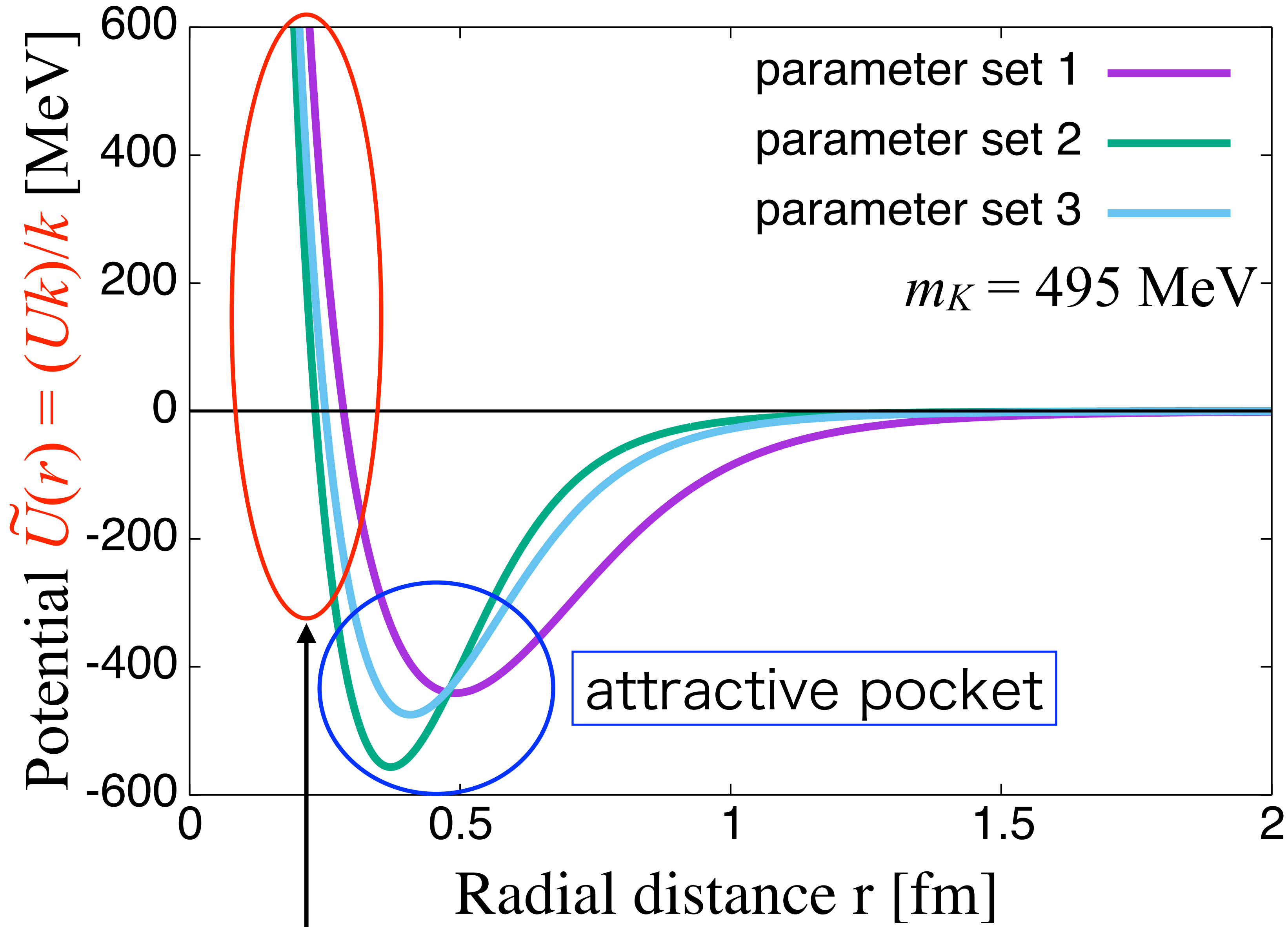
$$\longrightarrow -\frac{1}{m_K + E}\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dk_l^\alpha(r)}{dr}\right) + U(r) k_l^\alpha(r) = \varepsilon k_l^\alpha(r)$$

$$\boxed{\begin{aligned} U(r) &= -\boxed{\frac{1}{m_K+E}}\left[\frac{h(r)-1}{r^2}\boxed{\frac{d}{dr}\left(r^2\frac{d}{dr}\right)} + \frac{dh(r)}{dr}\boxed{\frac{d}{dr}}\right] - \boxed{\frac{(f(r)-1)E^2}{m_K+E}} \\ &\quad + \boxed{\frac{V(r)}{m_K+E}} \end{aligned}}$$

・properties of resulting potential *U*

1. Nonlocal and depend on the kaon energy
2. Contain isospin dependent and independent **central forces** and the similar **spin-orbit(LS) forces**
3. In the short range, behave as a repulsive force proportional to $1/r^2$

$\bar{K}N$ ($L = 0$, $I_{tot} = 0$) potential



Repulsion proportional to $1/r^2$

• Equivalent local potential

$$\tilde{U}(r) = \frac{U(r)k(r)}{k(r)}$$

	F_π [MeV]	e
parameter set 1	129	5.45
parameter set 2	186	5.45
parameter set 3	186	4.82

5. Summary

Summaries

Construct the new method to investigate the kaon-nucleon systems in the Skyrme model and apply to a channel

・Results

1. $\bar{K}N$(I = 0) bound states exist with B.E. of order ten MeV
2. The obtaining potential is nonlocal and depends on the kaon energy
3. Contain **central and LS terms with and without isospin dependence**
4. **repulsion proportional to $1/r^2$** for small r

・Future works

1. Scattering states of the KN system (ongoing)
2. The $\pi\Sigma$ system
3. The properties of Λ(1405)
4. few body nuclear system with kaon

Thank you for
your attention

back-up

The Skyrme model 2

・Hedgehog ansatz

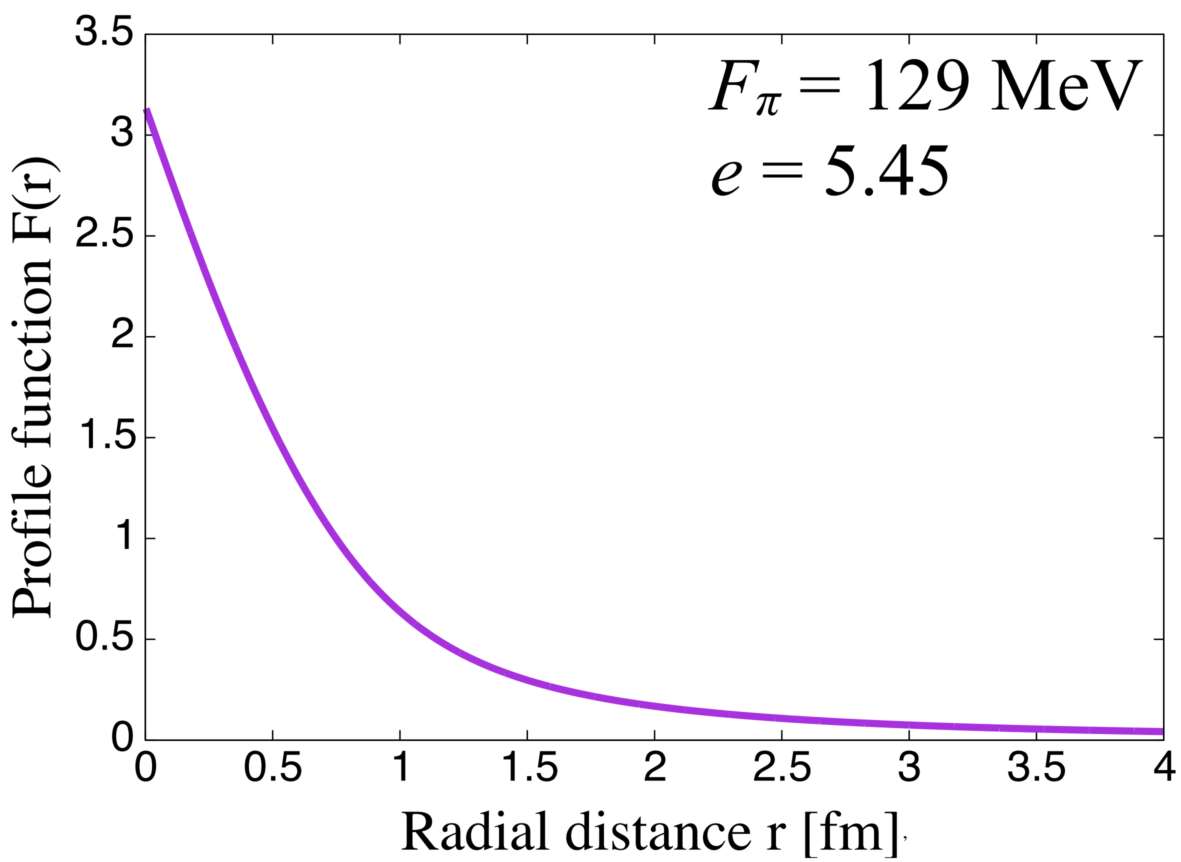
π has three degrees of freedom(π^0, π^+, π^-)

・two of these: the angles of the radial vector, θ, φ

・the rest: a function depending on r

⇒a special configuration called the hedgehog ansatz

Hedgehog ansatz: $U_H = \exp\left[i\boldsymbol{\tau}\cdot\hat{r}\boxed{F(r)}\right]$



minimize the mass of the soliton

with B.C. for $F(r)$: $\underline{F(\infty) = 0,\ F(0) = \pi}$

G. S. Adkins, C. R. Nappi and E. Witten, Nucl. Phys. B **228** (1983)

The Skyrme model 3

・Quantization

The hedgehog ansatz is classical

→without spin or isospin

→become a physical state by quantization

$$U_H(\boldsymbol{x}) \to U_H(t, \boldsymbol{x}) = A(t) \exp\left[i\tau_a R_{ab}(t) \hat{r}_b F(r)\right] A^{\dagger}(t)$$

$A(t)$: 2×2 isospin rotation matrix

$R_{ab}(t)$: 3×3 spatial rotation matrix

The baryon with $I = J$ from the symmetry which the hedgehog ansatz has

・Quantized Hamiltonian

$$H = M_{sol} + \boxed{\frac{J(J+1)}{2\Lambda}}$$

the rotation energy

M_{sol}: soliton mass

J: spin or isospin value

Λ: moment of inertia

$1/N_c$ expansion

Callan-Klebanov approach

Hyperon

Hedgehog soliton	$\rightarrow$	Soliton + Kaon	$\rightarrow$	di-quark + S-quark
$O(N_c)$		$O(N_c^0)$		$O(1/N_c)$

Our approach

Hedgehog soliton	$\rightarrow$	Nucleon	$\rightarrow$	Nucleon + Kaon
$O(N_c)$		$O(1/N_c)$		$O(1/N_c)$

$O(N_c^0)$ is missing

Lagrangian and ansatz

・Expand to the SU(3) Skyrme model

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

・Ansatz

the kaon around the hedgehog soliton

$$U = \begin{cases} A(t)\boxed{\sqrt{U_\pi}U_K\sqrt{U_\pi}}A^\dagger(t) & \text{: Callan-Klebanov ansatz} \\ \boxed{A(t)\sqrt{U_\pi}A^\dagger(t)}U_K\boxed{A(t)\sqrt{U_\pi}A^\dagger(t)} & \text{: \textbf{Our ansatz}} \end{cases}$$

the kaon around the **rotating** hedgehog soliton

Interaction term

$$V(r) = V_{nor}(r) + V_{WZ}(r)$$

$$\begin{aligned}
V_{nor}(r) \quad = \quad & -\frac{1}{4}\left(2\frac{\sin^2 F}{r^2} + F'^2\right) + 2\frac{s^4}{r^2} - \frac{1}{(eF_\pi)^2}\left[2\frac{\sin^2 F}{r^2}\left(\frac{\sin^2 F}{r^2} + 2F'^2\right) - 2\frac{s^4}{r^2}\left(F'^2 + \frac{\sin^2 F}{r^2}\right)\right] \\
& + \frac{1}{(eF_\pi)^2}\frac{6}{r^2}\left[\frac{s^4 \sin^2 F}{r^2} + \frac{d}{dr}\left\{s^2 \sin F F'\right\}\right] \\
& + \frac{2E}{\Lambda}s^2\left[1 + \frac{1}{(eF_\pi)^2}\left(F'^2 + \frac{5}{r^2}\sin^2 F\right)\right] + \frac{8E}{3\Lambda}s^2\boxed{I_{KN}} + \frac{1}{(eF_\pi)^2}\frac{8Es^2}{3\Lambda}\left[F'^2 + \frac{4}{r^2}\sin^2 F\right]\boxed{I_{KN}} \\
& + \frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\frac{4}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\boxed{I_{KN}} + \frac{3}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\right)\right] \\
& + \left[1 + \frac{1}{(eF_\pi)^2}\left(\frac{\sin^2 F}{r^2} + F'^2\right)\right]\frac{l(l+1)}{r^2} - \left[1 + \frac{1}{(eF_\pi)^2}\left(4\frac{\sin^2 F}{r^2} + F'^2\right)\right]\frac{16s^2}{3r^2}\boxed{J_{KN}}\boxed{I_{KN}} \\
& + \frac{1}{(eF_\pi)^2}\frac{2E\sin^2 F}{\Lambda r^2}\boxed{J_{KN}} - \frac{1}{(eF_\pi)^2}\frac{8}{r^2}\frac{d}{dr}\left(\sin F F'\right)\boxed{J_{KN}}\boxed{I_{KN}}
\end{aligned}$$

$$V_{WZ}(r) = \frac{3E}{(\pi F_\pi)^2}\frac{\sin^2 F}{r^2}F' - \frac{3}{(\pi F_\pi)^2}\frac{\sin^2 F s^2}{\Lambda r^2}F' + \frac{3}{(\pi F_\pi)^2}\frac{\sin^2 F}{\Lambda r^2}F'\boxed{J_{KN}}$$

$$s = \sin(F/2) \quad \boxed{I_{KN} = \boldsymbol{I}^K \cdot \boldsymbol{I}^N}, \quad \boxed{J_{KN} = \boldsymbol{L}^K \cdot \boldsymbol{J}^N}$$

Results 1 E.o.M with I = 0

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2 h(r)\frac{dk_l^\alpha(r)}{dr}\right) - E^2 f(r)k_l^\alpha(r) + \left(m_K^2 + V(r)\right)k_l^\alpha(r) = 0$$

$$\boxed{-\frac{1}{m_K+E}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dk}{dr}\right) + \frac{2}{m_K+E}\frac{1}{r^2}k \simeq 0, \quad (r\sim 0)}$$

$$h(r) = 1 + \frac{1}{(eF_\pi)^2}\frac{2}{r^2}\sin^2 F \qquad f(r) = 1 + \frac{1}{(eF_\pi)^2}\left(\frac{2}{r^2}\sin^2 F + F'^2\right) \qquad V(r) = V_{eff}(r) \pm V_{WZ}(r)$$

$$s = \sin(F/2)$$

$$\begin{aligned} V_{nor}(r) \quad = \quad & -\frac{1}{4}\left(2\frac{\sin^2 F}{r^2} + F'^2\right) + 2\frac{s^4}{r^2} - \frac{1}{(eF_\pi)^2}\left[2\frac{\sin^2 F}{r^2}\left(\frac{\sin^2 F}{r^2} + 2F'^2\right) - 2\frac{s^4}{r^2}\left(F'^2 + \frac{\sin^2 F}{r^2}\right)\right] \\ & + \frac{1}{(eF_\pi)^2}\frac{6}{r^2}\left[\frac{s^4\sin^2 F}{r^2} + \frac{d}{dr}\left\{s^2\sin F F'\right\}\right] \\ & + \frac{2E}{\Lambda}s^2\left[1 + \frac{1}{(eF_\pi)^2}\left(F'^2 + \frac{5}{r^2}\sin^2 F\right)\right] + \frac{8E}{3\Lambda}s^2 I_{KN} + \frac{1}{(eF_\pi)^2}\frac{8Es^2}{3\Lambda}\left[F'^2 + \frac{4}{r^2}\sin^2 F\right]I_{KN} \\ & + \frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\frac{4}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}I_{KN} + \frac{3}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\right)\right] \end{aligned}$$

$$V_{WZ}(r) = \frac{3E}{(\pi F_\pi)^2}\frac{\sin^2 F}{r^2}F' - \frac{3}{(\pi F_\pi)^2}\frac{\sin^2 F s^2}{\Lambda r^2}F' \qquad I_{KN} = \boldsymbol{I}^K \cdot \boldsymbol{I}^N$$

Bule: dominant in r ~ 0

Red: $O(1/N_C)$ contributions

Results 1 potential for I = 0

$$U(r) = -\frac{1}{m_K+E}\left[\frac{h(r)-1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{dh(r)}{dr}\frac{d}{dr}\right]-\frac{(f(r)-1)\,E^2}{m_K+E} + \frac{V(r)}{m_K+E}$$

$$h(r) = 1+\frac{1}{(eF_\pi)^2}\frac{2}{r^2}\sin^2 F \qquad f(r) = 1+\frac{1}{(eF_\pi)^2}\left(\frac{2}{r^2}\sin^2 F + F'^2\right) \qquad s = \sin(F/2)$$

$$\begin{aligned}V_{nor}(r) = & -\frac{1}{4}\left(2\frac{\sin^2 F}{r^2}+F'^2\right)+2\frac{s^4}{r^2}-\frac{1}{(eF_\pi)^2}\left[2\frac{\sin^2 F}{r^2}\left(\frac{\sin^2 F}{r^2}+2F'^2\right)-2\frac{s^4}{r^2}\left(F'^2+\frac{\sin^2 F}{r^2}\right)\right]\\ & +\frac{1}{(eF_\pi)^2}\frac{6}{r^2}\left[\frac{s^4\sin^2 F}{r^2}+\frac{d}{dr}\left\{s^2\sin F F'\right\}\right]\\ & +\frac{2E}{\Lambda}s^2\left[1+\frac{1}{(eF_\pi)^2}\left(F'^2+\frac{5}{r^2}\sin^2 F\right)\right]+\frac{8E}{3\Lambda}s^2 I_{KN}+\frac{1}{(eF_\pi)^2}\frac{8Es^2}{3\Lambda}\left[F'^2+\frac{4}{r^2}\sin^2 F\right]I_{KN}\\ & +\frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\frac{4}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}I_{KN}+\frac{3}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\right)\right]\end{aligned}$$

$$V_{WZ} = \frac{3E}{(\pi F_\pi)^2}\frac{\sin^2 F}{r^2}F' - \frac{3}{(\pi F_\pi)^2}\frac{\sin^2 F s^2}{\Lambda r^2}F' \qquad I_{KN} = \boldsymbol{I}^K\cdot\boldsymbol{I}^N$$

Results 2 potential for l ≠ 0

$$
\begin{aligned}
V_{nor}(r) \quad = \quad & -\frac{1}{4}\left(2\frac{\sin^2 F}{r^2}+F'^2\right)+2\frac{s^4}{r^2}-\frac{1}{(eF_\pi)^2}\left[2\frac{\sin^2 F}{r^2}\left(\frac{\sin^2 F}{r^2}+2F'^2\right)-2\frac{s^4}{r^2}\left(F'^2+\frac{\sin^2 F}{r^2}\right)\right] \\
& +\frac{1}{(eF_\pi)^2}\frac{6}{r^2}\left[\frac{s^4\sin^2 F}{r^2}+\frac{d}{dr}\left\{s^2\sin F F'\right\}\right] \\
& +\frac{2E}{\Lambda}s^2\left[1+\frac{1}{(eF_\pi)^2}\left(F'^2+\frac{5}{r^2}\sin^2 F\right)\right]+\frac{8E}{3\Lambda}s^2 \underline{I_{KN}}+\frac{1}{(eF_\pi)^2}\frac{8Es^2}{3\Lambda}\left[F'^2+\frac{4}{r^2}\sin^2 F\right]\underline{I_{KN}} \\
& +\frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\frac{4}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\underline{I_{KN}}+\frac{3}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\right)\right] \\
& +\left[1+\frac{1}{(eF_\pi)^2}\left(\frac{\sin^2 F}{r^2}+F'^2\right)\right]\frac{l\,(l+1)}{r^2}-\left[1+\frac{1}{(eF_\pi)^2}\left(4\frac{\sin^2 F}{r^2}+F'^2\right)\right]\frac{16s^2}{3r^2}\underline{J_{KN}I_{KN}} \\
& +\frac{1}{(eF_\pi)^2}\frac{2E\sin^2 F}{\Lambda r^2}\underline{J_{KN}}-\frac{1}{(eF_\pi)^2}\frac{8}{r^2}\frac{d}{dr}\left(\sin F F'\right)\underline{J_{KN}I_{KN}}
\end{aligned}
$$

$$
V_{WZ}(r)=\frac{3E}{(\pi F_\pi)^2}\frac{\sin^2 F}{r^2}F'-\frac{3}{(\pi F_\pi)^2}\frac{\sin^2 F s^2}{\Lambda r^2}F'+\frac{3}{(\pi F_\pi)^2}\frac{\sin^2 F}{\Lambda r^2}F'\underline{J_{KN}}
$$

$$
s=\sin(F/2) \qquad\qquad I_{KN}=\boldsymbol{I}^K\cdot\boldsymbol{I}^N,\quad J_{KN}=\boldsymbol{L}^K\cdot\boldsymbol{J}^N
$$

Red: New contributions for l ≠ 0

Results 2 potential for l ≠ 0 (r ~ 0)

$$
\begin{aligned}
V_{nor}(r) \;=\; & -\frac{1}{4}\left(2\frac{\sin^2 F}{r^2}+F'^2\right)+\boxed{2\frac{s^4}{r^2}}-\frac{1}{(eF_\pi)^2}\left[2\frac{\sin^2 F}{r^2}\left(\frac{\sin^2 F}{r^2}+2F'^2\right)\boxed{-2\frac{s^4}{r^2}\left(F'^2+\frac{\sin^2 F}{r^2}\right)}\right]\\
&+\frac{1}{(eF_\pi)^2}\frac{6}{r^2}\left[\frac{s^4\sin^2 F}{r^2}+\frac{d}{dr}\left\{s^2\sin F F'\right\}\right]\\
&+\frac{2E}{\Lambda}s^2\left[1+\frac{1}{(eF_\pi)^2}\left(F'^2+\frac{5}{r^2}\sin^2 F\right)\right]+\frac{8E}{3\Lambda}s^2 I_{KN}+\frac{1}{(eF_\pi)^2}\frac{8Es^2}{3\Lambda}\left[F'^2+\frac{4}{r^2}\sin^2 F\right]I_{KN}\\
&+\frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\frac{4}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}I_{KN}+\frac{3}{(eF_\pi)^2}\frac{EF'\sin F}{\Lambda}\right)\right]\\
&+\boxed{\left[1+\frac{1}{(eF_\pi)^2}\left(\frac{\sin^2 F}{r^2}+F'^2\right)\right]\frac{l(l+1)}{r^2}-\left[1+\frac{1}{(eF_\pi)^2}\left(4\frac{\sin^2 F}{r^2}+F'^2\right)\right]\frac{16s^2}{3r^2}J_{KN}I_{KN}}\\
&+\frac{1}{(eF_\pi)^2}\frac{2E\sin^2 F}{\Lambda r^2}J_{KN}\boxed{-\frac{1}{(eF_\pi)^2}\frac{8}{r^2}\frac{d}{dr}\left(\sin F F'\right)J_{KN}I_{KN}}
\end{aligned}
$$

Bule: dominant for r ~ 0

$$\Big\downarrow\; F(r\sim 0)\simeq \pi - ar$$

$$
\begin{aligned}
V(r) \;=\; & \boxed{\frac{2}{r^2}+\frac{a^2}{(eF_\pi)^2}\frac{4}{r^2}+\left[1+\frac{2a^2}{(eF_\pi)^2}\right]\frac{l(l+1)}{r^2}} \leftarrow \text{Repulsion}\\
& \boxed{-\left[1+\frac{5a^2}{(eF_\pi)^2}\right]\frac{16}{3r^2}J_{KN}I_{KN}-\frac{a^2}{(eF_\pi)^2}\frac{8}{r^2}J_{KN}I_{KN}}
\end{aligned}
$$

Repulsion or Attraction

(No contribution from WZ term)

Short range behavior of V_{eff}

Repulsive or Attractive depending on l

	$J_{tot} = l - 1/2$, $J_{NK} = -(l+1)/2$	$J_{tot} = l + 1/2$, $J_{NK} = l/2$
$I_{tot} = 0$, $I_{NK} = -3/4$	$J_{NK}I_{NK} = 3(l+1)/8$	$J_{NK}I_{NK} = -3l/8$
$I_{tot} = 1$, $I_{NK} = 1/4$	$J_{NK}I_{NK} = -(l+1)/8$	$J_{NK}I_{NK} = l/8$

Analytically,
attraction ($1 \leq l \leq 3$)
repulsion ($l=0$, $4 \leq l$)

Attractive for small r

Comparisons with the chiral theory 1

• Weinberg-Tomozawa intereaction

$$L_{WT} = \frac{2}{F_\pi^2}\{\bar{N}\boldsymbol{I}^N\gamma^\mu N\cdot(\partial_\mu K^\dagger \boldsymbol{I}^K K - K^\dagger \boldsymbol{I}^K \partial_\mu K)\} \propto \frac{1}{F_\pi^2}$$

$\downarrow$

S. Weinberg, Phys. Rev. Lett. **17** (1966)
Y. Tomozawa, Nuovo Cim. A **46** (1966)

The strength of $L_{WT} \propto 1/F_\pi^2$

→For F_π = 129 MeV and 186 MeV,

$$1/129^2 : 1/186^2 \sim \boxed{15:7}$$

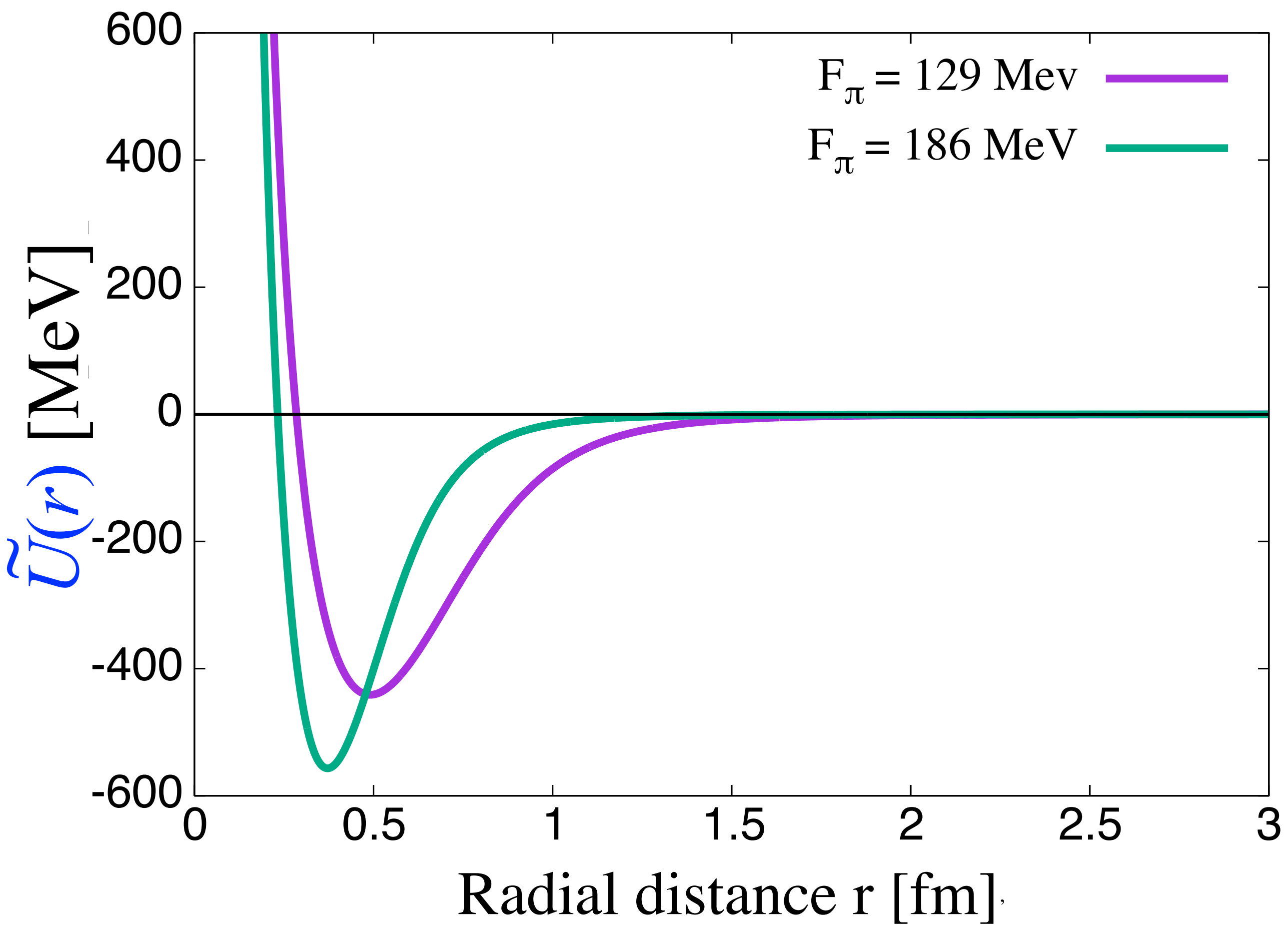
• The interaction for KN($I = 0$) bound state

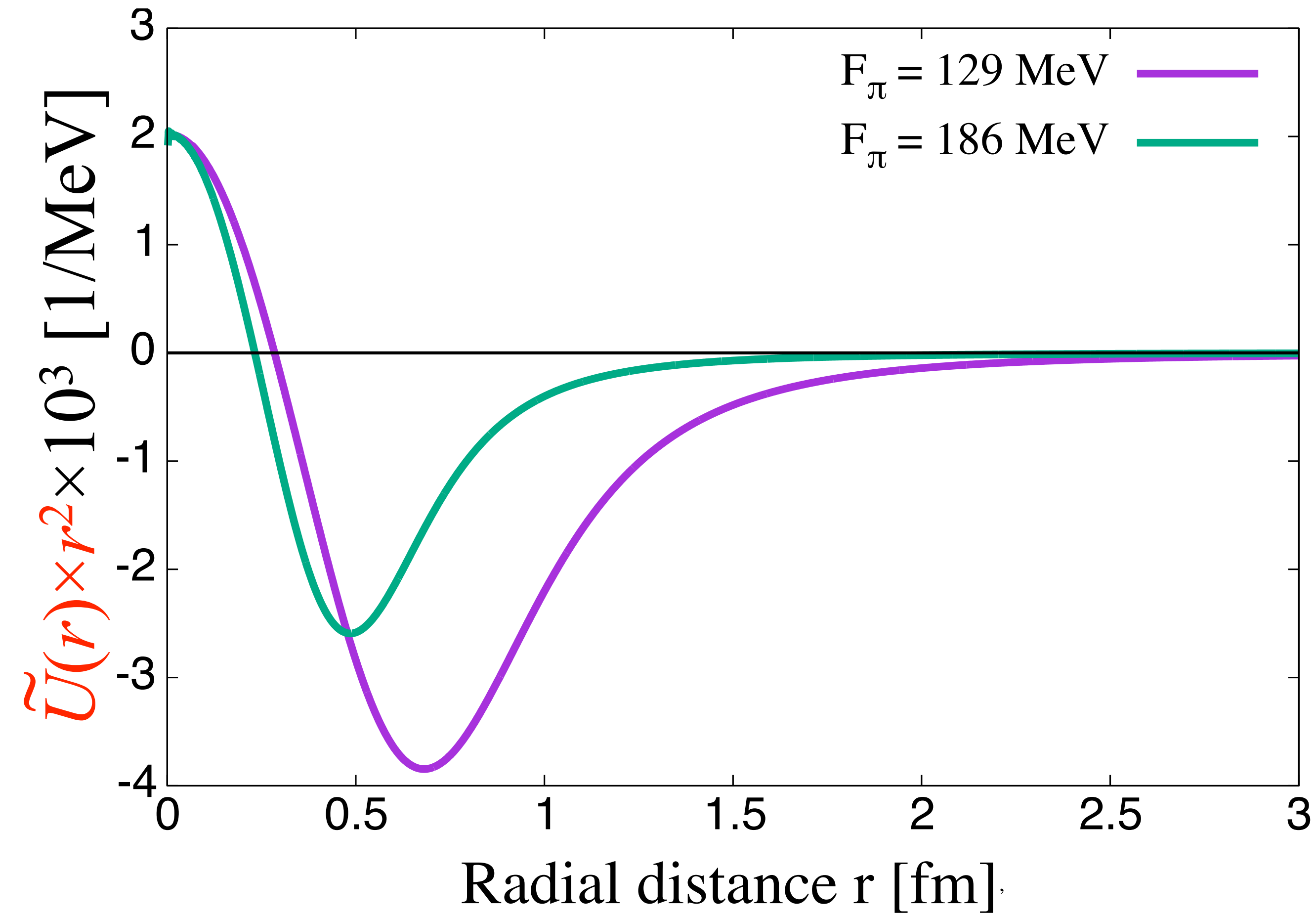
$$W \equiv 4\pi \int r^2 dr \tilde{U}(r)$$

F_π [MeV]	e	$-W \times 10^5$ [1/MeV ^{2}]
129	5.45	$1.2 \times 4\pi$
186	5.45	$0.48 \times 4\pi$

$\rightarrow$ $\boxed{5:2}$

Comparisons with the chiral theory 2





• The interaction for KN($I = 0$) bound state

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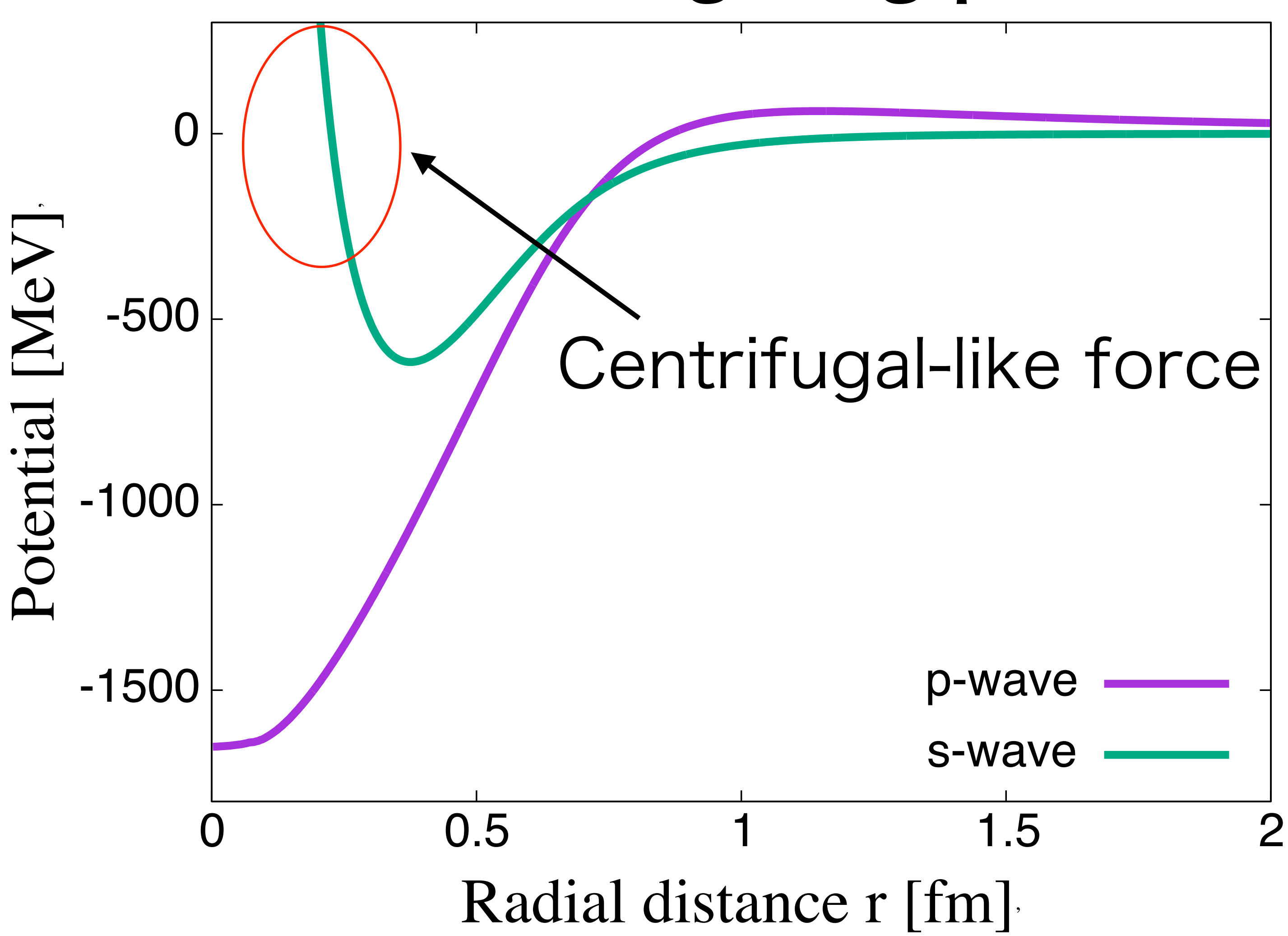
F_π [MeV]	e	$-W \times 10^5$ [1/MeV ^{2}]
129	5.45	$1.2 \times 4\pi$
186	5.45	$0.48 \times 4\pi$

Comparisons with the CK approach

• Comparisons between the CK and our approach

Callan-Klebanov approach				Our approach			Physical state
l	l_{eff}	B.E. [MeV]	$\langle r_K^2 \rangle^{1/2}$ [fm]	l	B.E. [MeV]	$\langle r_K^2 \rangle^{1/2}$ [fm]	
0	1	61.7	0.93	0	32.9	1.18	$\Lambda(1405)$
1	0	326.6	0.54	—	—	—	$\Lambda(1116)$

• Anti-kaon hedgehog potential



$$l_{eff}\left(l_{eff}+1\right) = l\left(l+1\right) + 4\boldsymbol{I}\cdot\boldsymbol{L} + 2$$

C.G. Callan and I. Klebanov, Nucl. Phys. B **262** (1985)

Parameters:

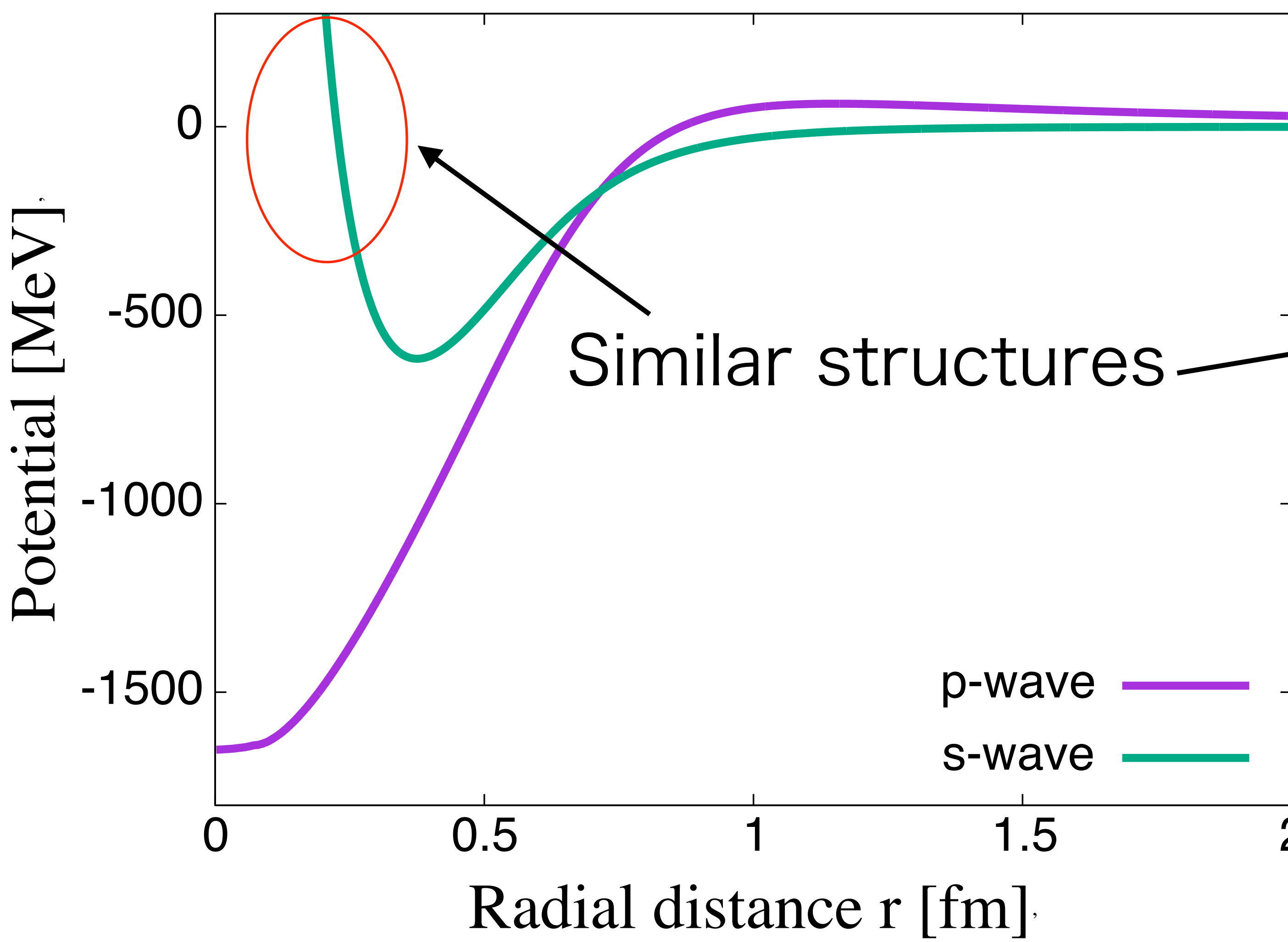
$$F_\pi = 186 \text{ MeV}\ , \ \ e = 4.82$$

Comparisons with the CK approach

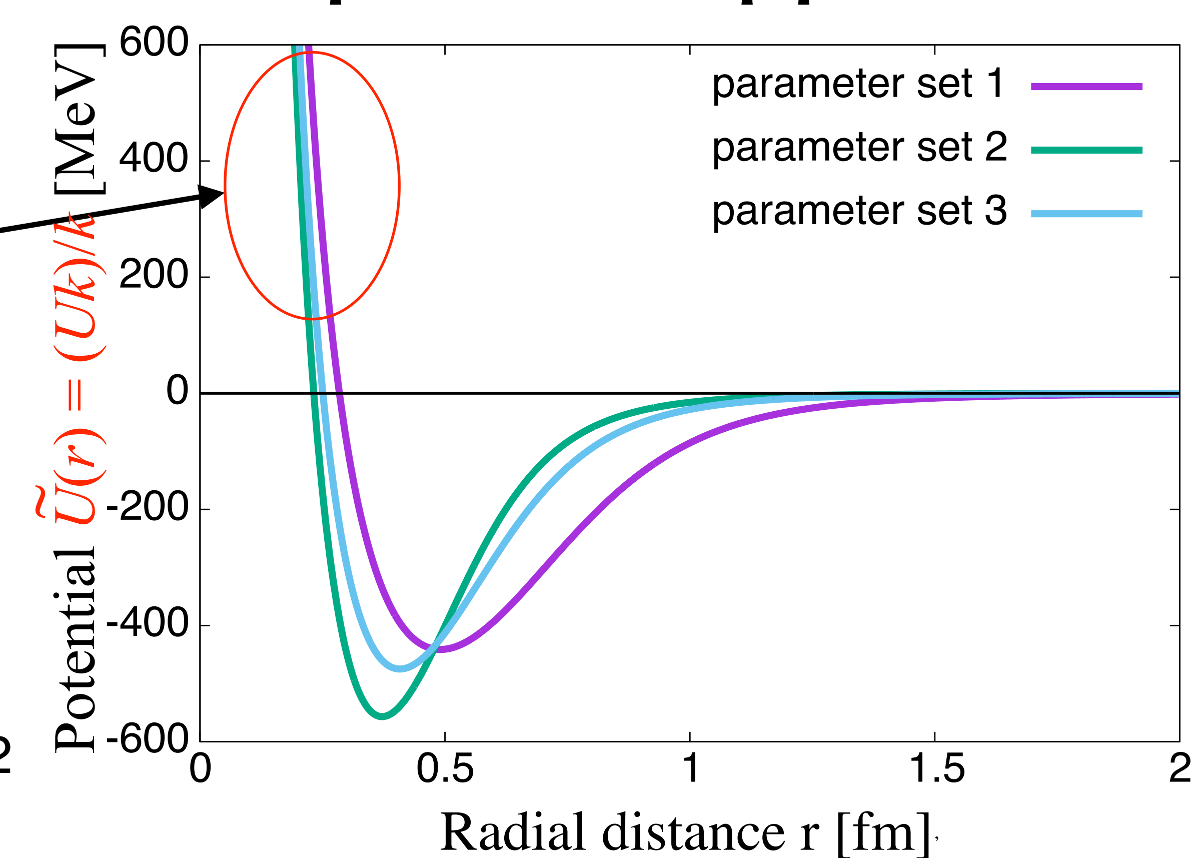
• Comparisons between the CK and our approach

Callan-Klebanov approach				Our approach			Physical state
l	l_{eff}	B.E. [MeV]	$\langle r_K^2 \rangle^{1/2}$ [fm]	l	B.E. [MeV]	$\langle r_K^2 \rangle^{1/2}$ [fm]	
0	1	61.7	0.93	0	32.9	1.18	$\Lambda(1405)$
1	0	326.6	0.54	—	—	—	$\Lambda(1116)$

• The CK approach



• The present approach



Potential

$$U(r) = U_{nor}(r) + U_{WZ}(r)$$

parameter set 1: $(F_\pi, e) = (129\mathrm{MeV}, 5.45)$