Evidence for three-nucleon interaction in isotope shifts of $Z = $ magic nuclei

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Contents :

H.N. & T. Inakura, P.R.C 91, 021302(R) ('15)

H.N., P.R.C 92, 044307 ('15)

I. Introduction

Shell structure ($\to$ **magic number**)

   — fundamental concept for nuclear structure
     astrophysical importance

$$\begin{cases} \text{waiting point in } s\text{- \& } r\text{-processes} \\ \text{constraint on EoS} \quad \leftarrow \text{ subtracting shell effects} \\ \text{clusters in } n\text{-star inner crust } (\leftrightarrow \textbf{\textit{e.g.}} \text{ QPO}) \end{cases}$$

★ Z- & N-dep.!   ("shell evolution")

★ central $+\ell s$ potential

  ℓs **pot.** $\leftrightarrow$ ℓs **splitting**   ($\to$ **magic #'s in** $Z, N > 20$) $\cdots$ **origin?**

   $\circ$ $2N$ **LS int.** — insufficient

          ($\Rightarrow$ **strong LS int.** used in phenomenology)

   $\circ$ tensor int. $(+\alpha) \to Z$- & N-dep. (1st-order effects)

     contribution to overall strength (2nd-order effects)? $\cdots$ small

       comprehension of its origin
         $\to$ **correct prediction of shell structure**

★ $3N$ int. (χEFT) → ρ-dep. LS int.

stronger LS int. at higher ρ (→ stronger ℓs pot.)

$k_F = 1.35$ fm^{-1}	AV18	NSC97	CD-B	N^{3}LO	N^{3}LO+3NF
$B_S(T=0)$	2.0	1.9	3.1	2.5	7.0
$B_S(T=1)$	86.4	86.7	90.2	84.6	116.2
$B_S(\bar{q})$	88.4	88.6	93.3	87.1	123.2
$k_F = 1.07$ fm^{-1}	AV18	NSC97	CD-B	N^{3}LO	N^{3}LO+3NF
$B_S(T=0)$	1.4	1.3	2.3	1.6	4.1
$B_S(T=1)$	88.1	88.7	92.2	86.5	106.7
$B_S(\bar{q})$	89.5	90.0	94.5	88.1	110.8

—— complementary to $2N$ LS int !

⇒ experimental evidence independent of ℓs splitting ?

(→ good reliability)

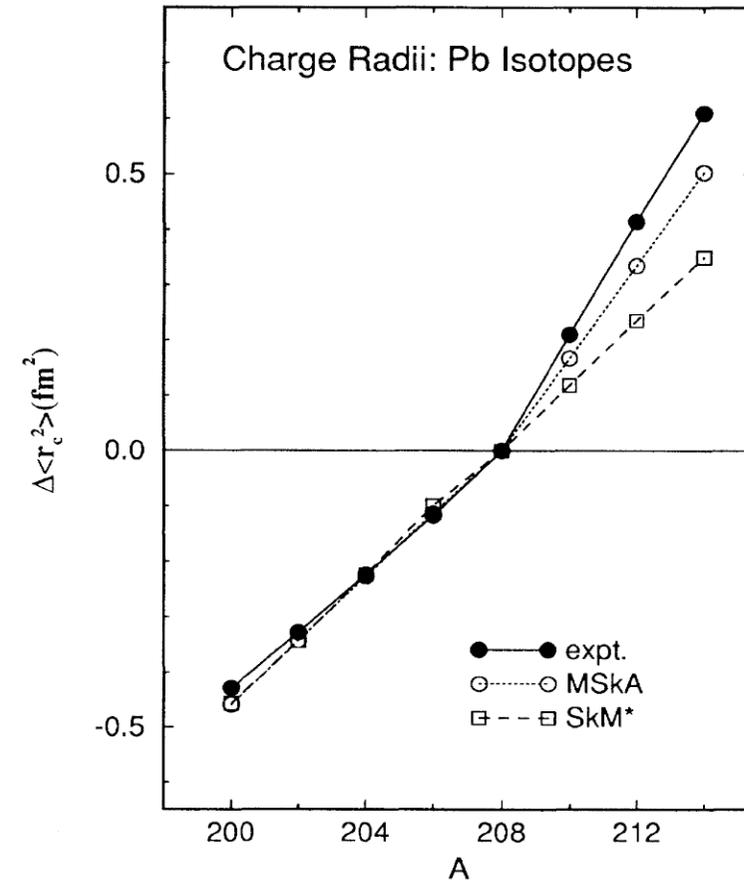**Isotope shifts in Pb nuclei** $\Delta\langle r^2\rangle_p(^A\mathbf{Pb}) := \langle r^2\rangle_p(^A\mathbf{Pb}) - \langle r^2\rangle_p(^{208}\mathbf{Pb})$

exp. $\Rightarrow$ **kink at** $N = 126$ **!**

$\begin{cases} \text{electron scatt.} \\ \textbf{X-ray freq. difference} \\ \quad (\mu^- \textbf{ atom, } K\alpha, \textbf{ OIS}) \end{cases}$

Charge Radii: Pb Isotopes

Ref.: M.M. Sharma *et al.*, P.R.L. 74, 3744

reproduced by RMF, but not by Skyrme EDF up to '95
  → dep. on isospin content of LS int. (→ extension of Skyrme EDF)

··· but cannot be a complete solution!

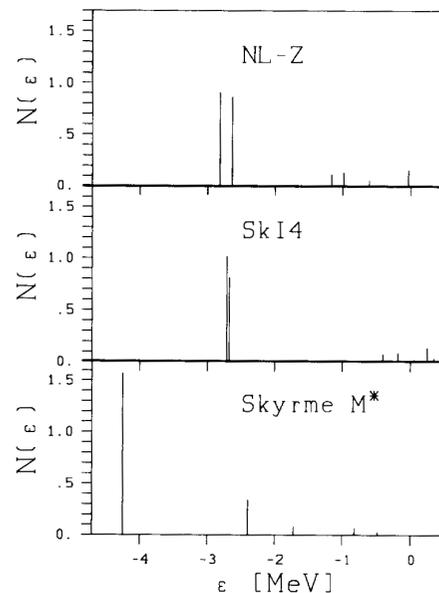**kink in $\Delta \langle r^2 \rangle_p (^A\text{Pb})$ at $N = 126$** $\longleftarrow$ $n0i_{11/2}$ **occupation**

$\Uparrow$

p-n **attraction**

$\left\{ \begin{array}{l} \text{larger } \langle r^2 \rangle \\ \qquad \text{than neighboring orbits} \\ N < 126 \text{ --- } \textbf{unocc.} \\ N > 126 \text{ --- } \textbf{sizable occ. prob.} \\ \qquad\qquad\qquad (\because \textbf{pairing}) \end{array} \right.$
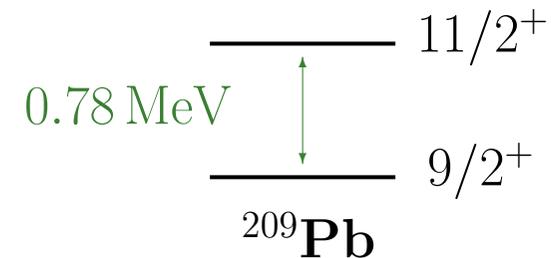
Ref.: P.-G. Reinhard & H. Flocard, N.P.A 584, 467

'understanding' in '90's $\cdots$ $\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2})$ **is a key**

$\leftrightarrow$ **isospin content of LS int.**

However, $\varepsilon_n(0i_{11/2}) \approx \varepsilon_n(1g_{9/2})$ **required!** ($\leftrightarrow$ **equal occ. prob.**)

on the contrary $\cdots$

$0.78\,\text{MeV}$

$\underline{\qquad\qquad}\ 11/2^+$

$\underline{\qquad\qquad}\ 9/2^+$

^{209}Pb

II. Mean-field approaches with semi-realistic interaction

"Semi-realistic" nucleonic interaction $\longleftarrow$ microscopic $2N\,(+3N)$ int.

$$\uparrow$$

phenomenological modification

$$\begin{cases} \text{saturation properties} \\ \ell s \text{ splitting (?)} \end{cases}$$

$\Rightarrow$ MF (HF, HFB) & RPA calculations

nuclear reactions $\cdots$ future project

Effective Hamiltonian $H = H_N + V_C - H_{\text{c.m.}}$; $\quad H_N = \sum_i \dfrac{\boldsymbol{p}_i^2}{2M} + \sum_{i<j} \hat{v}_{ij}$

$$\textbf{(rotational \& translational inv.)}$$

$$\hat{v}_{ij} = \hat{v}_{ij}^{(\text{C})} + \hat{v}_{ij}^{(\text{LS})} + \hat{v}_{ij}^{(\text{TN})} + \hat{v}_{ij}^{(\text{C}\rho)} \left(+ \hat{v}_{ij}^{(\text{LS}\rho)} \right);$$

$$\hat{v}_{ij}^{(\text{C})} = \sum_n \left(t_n^{(\text{SE})} P_{\text{SE}} + t_n^{(\text{TE})} P_{\text{TE}} + t_n^{(\text{SO})} P_{\text{SO}} + t_n^{(\text{TO})} P_{\text{TO}} \right) f_n^{(\text{C})}(r_{ij}),$$

$$\left(\begin{array}{ll} P_{\text{SE}} := \left(\dfrac{1 - P_\sigma}{2} \right) \left(\dfrac{1 + P_\tau}{2} \right), & P_{\text{TE}} := \left(\dfrac{1 + P_\sigma}{2} \right) \left(\dfrac{1 - P_\tau}{2} \right), \\[3mm] P_{\text{SO}} := \left(\dfrac{1 - P_\sigma}{2} \right) \left(\dfrac{1 - P_\tau}{2} \right), & P_{\text{TO}} := \left(\dfrac{1 + P_\sigma}{2} \right) \left(\dfrac{1 + P_\tau}{2} \right) \end{array} \right)$$

$$\hat{v}_{ij}^{(\text{LS})} = \sum_n \left(t_n^{(\text{LSE})} P_{\text{TE}} + t_n^{(\text{LSO})} P_{\text{TO}} \right) f_n^{(\text{LS})}(r_{ij}) \, \boldsymbol{L}_{ij} \cdot (\boldsymbol{s}_i + \boldsymbol{s}_j),$$

$$\hat{v}_{ij}^{(\text{TN})} = \sum_n \left(t_n^{(\text{TNE})} P_{\text{TE}} + t_n^{(\text{TNO})} P_{\text{TO}} \right) f_n^{(\text{TN})}(r_{ij}) \, r_{ij}^2 S_{ij}$$

$$\hat{v}_{ij}^{(\text{C}\rho)} = \left(C^{(\text{SE})}[\rho(\boldsymbol{r}_i)] P_{\text{SE}} + C^{(\text{TE})}[\rho(\boldsymbol{r}_i)] P_{\text{TE}} \right) \delta(\boldsymbol{r}_{ij});$$

$$\boldsymbol{L}_{ij} := \boldsymbol{r}_{ij} \times \boldsymbol{p}_{ij}, \quad S_{ij} := 3(\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{ij})(\boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ij}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \quad f_n(r) = e^{-\mu_n r}/\mu_n r$$

$$C^{(\text{Y})}[\rho] = t_\rho^{(\text{Y})} \rho^{\alpha^{(\text{Y})}} \quad (\text{Y} = \text{SE } \textbf{or } \text{TE}; \quad \alpha^{(\text{SE})} = 1, \, \alpha^{(\text{TE})} = 1/3)$$

M3Y int. $\cdots$ Yukawa function $\to$ fit to G-matrix (at $\rho \approx \rho_0/3$)

- **OPEP $\to$ longest part of** $\hat{v}_{ij}^{(\mathrm{C})}$ ($\equiv \hat{v}_{\mathrm{OPEP}}^{(\mathrm{C})}$)
- popular in reaction problems
- no saturation (without modification) $\quad \to$ **add** $\hat{v}_{ij}^{(\mathrm{C}\rho)}$

'**M3Y-Pn**'

- modifying M3Y-Paris $\left\{ \begin{array}{l} \text{replace short-range part of } \hat{v}^{(\mathrm{C})} \text{ by } \hat{v}^{(\mathrm{C}\rho)} \\ \text{enhance } \hat{v}^{(\mathrm{LS})} \quad (\leftrightarrow \ell s \text{ splitting}) \end{array} \right.$

- keeping $\hat{v}_{\mathrm{OPEP}}^{(\mathrm{C})}$
- **no change for** $\hat{v}^{(\mathrm{TN})}$ **from M3Y-Paris** in **M3Y-P5 to P7**

(— realistic tensor force)

$\cdots$ leading order of **chiral dynamics**

basic formulae : **H.N., P.R.C 68, 014316 ('03)**

M3Y-P6, **P7** : **H.N., P.R.C 87, 014336 ('13)**

$\Rightarrow$ **nuclear matter & finite nuclei**

Numerical methods for finite nuclei — Gaussian expansion method

- spherical HF $\cdots$ H.N. & M. Sato, N.P.A 699, 511 ('02); 714, 696 ('03)
- spherical HFB $\cdots$ H.N., N.P.A 764, 117 ('06); 801, 169 ('08)
- axial HF & HFB $\cdots$ H.N., N.P.A 808, 47 ('08)
- spherical RPA $\cdots$ H.N., K. Mizuyama, M. Yamagami & M. Matsuo,

N.P.A 828, 283 ('09)

Advantages of the method

(i) ability to describe ε-dep. exponential/oscillatory asymptotics

(ii) tractability of various 2-body interactions

(iii) basis parameters insensitive to nuclide

(iv) exact treatment of Coulomb & c.m. Hamiltonian

★ **Energies & radii of doubly magic nuclei :**

		SLy5	D1S	M3Y-P6	M3Y-P7	CCSD	Exp.
^{16}O	$-E$	128.6	129.5	126.3	125.9	107.5	127.6
	$\sqrt{\langle r^2 \rangle}$	2.59	2.61	2.59	2.57	—	2.61
^{40}Ca	$-E$	344.3	344.6	335.9	334.3	308.8	342.1
	$\sqrt{\langle r^2 \rangle}$	3.29	3.37	3.37	3.35	—	3.47
^{48}Ca	$-E$	416.0	416.8	413.8	414.9	355.2	416.0
	$\sqrt{\langle r^2 \rangle}$	3.44	3.51	3.51	3.49	—	3.57
^{90}Zr	$-E$	782.4	785.9	781.1	780.8	—	783.9
	$\sqrt{\langle r^2 \rangle}$	4.22	4.24	4.23	4.22	—	4.32
^{208}Pb	$-E$	1635.2	1639.0	1634.5	1635.5	—	1636.4
	$\sqrt{\langle r^2 \rangle}$	5.52	5.51	5.53	5.51	—	5.49

CCSD $\cdots$ **G. Hagen** *et al.*, **P.R.L. 101, 092502 ('08)**

(chiral N^{3}LO without 3NF)

★ Separation energies of proton- / neutron-magic nuclei :

$\cdots$ **important in astrophysics**

$$S_n(Z, N) := E(Z, N-1) - E(Z, N)$$

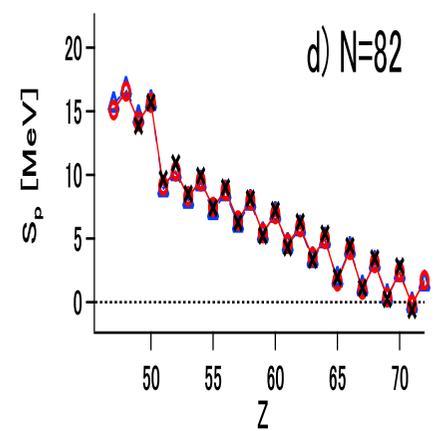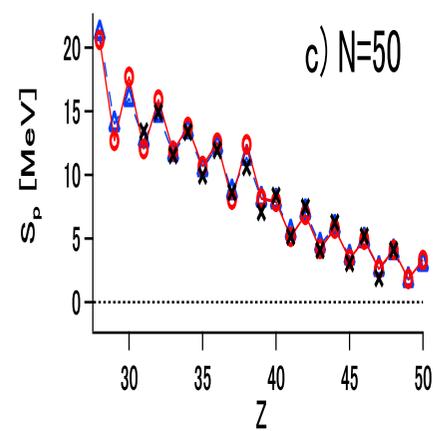$$S_{2n}(Z, N) := E(Z, N-2) - E(Z, N) = S_n(Z, N) + S_n(Z, N-1)$$

$$\Delta_n^{\mathrm{mass}}(Z, N-1) := E(Z, N-1) - \frac{1}{2}\big[E(Z, N-2) + E(Z, N)\big]$$

$$= \frac{1}{2}\big[S_n(Z, N) - S_n(Z, N-1)\big] \quad (\textbf{for } N-1 = \mathrm{odd})$$

$\leftrightarrow$ **pairing**

$(Z, N-2)$

$(Z, N-1)$

S_{2n}

Δ_n^{mass}

S_n

(Z, N)
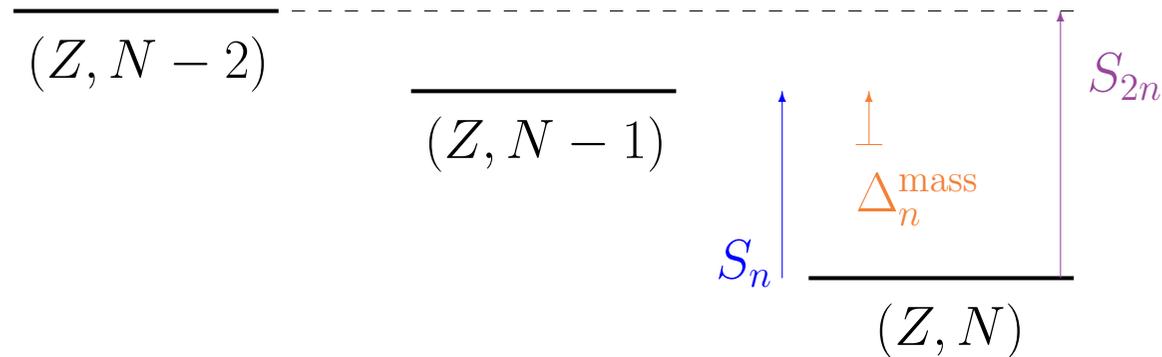
$\big(S_p,\ S_{2p},\ \Delta_p^{\mathrm{mass}} \to$ **obtained analogously**$\big)$

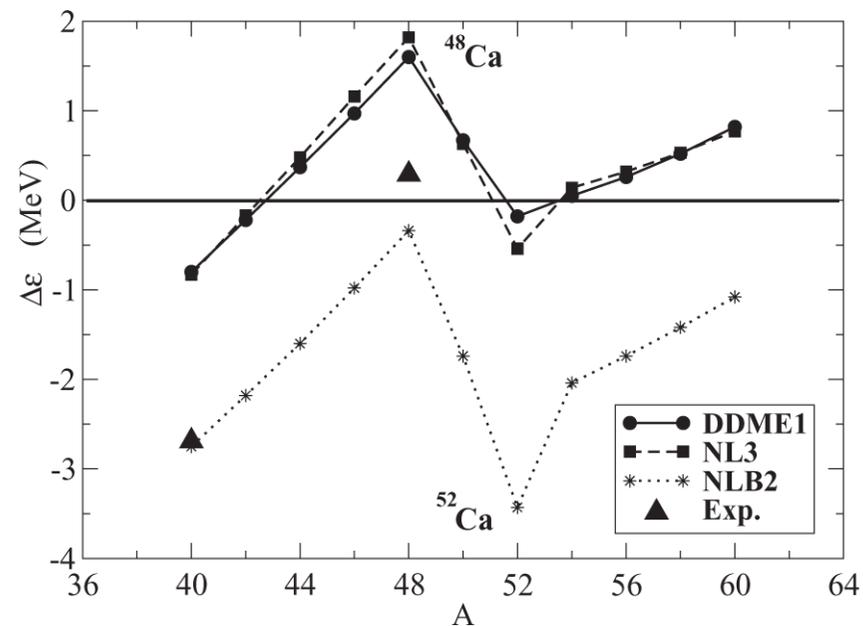Δ_n^{mass} **for** $Z = 50$ **&** Δ_p^{mass} **for** $N = 82\ \cdots$ **fitted**
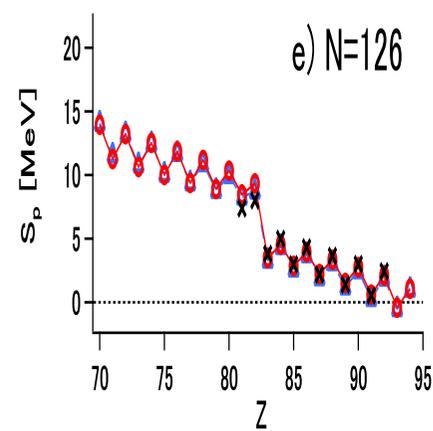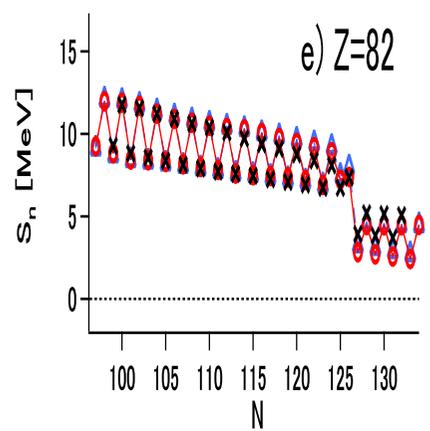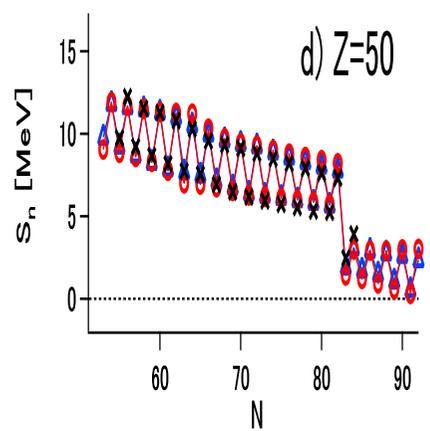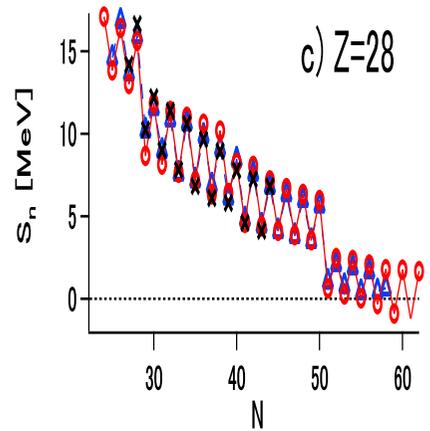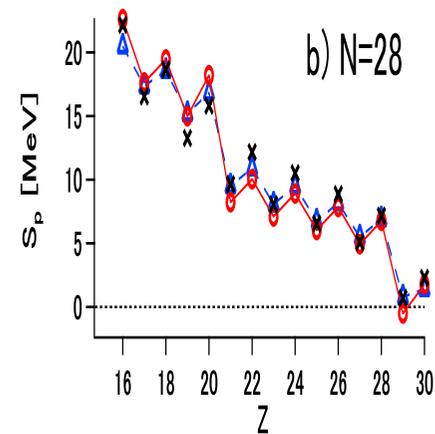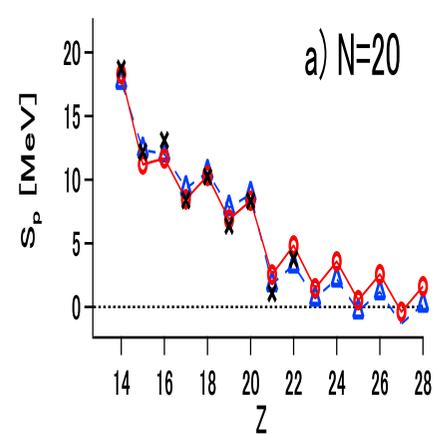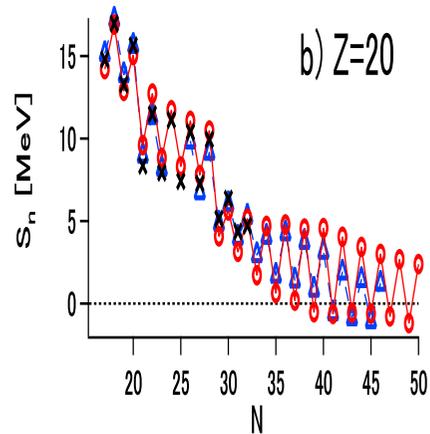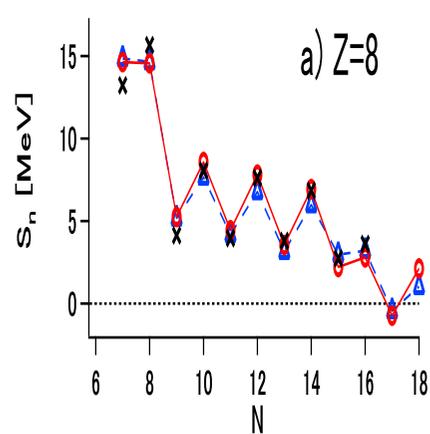
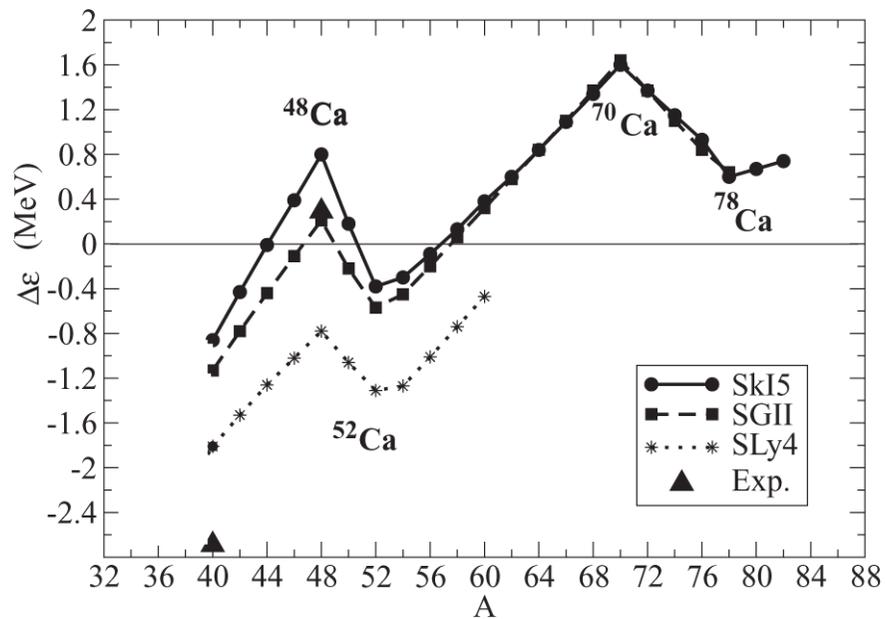$\triangle$: D1M　　● : M3Y-P7　　× : Exp.

★ N-dependence of single-proton energies below $Z = 20$
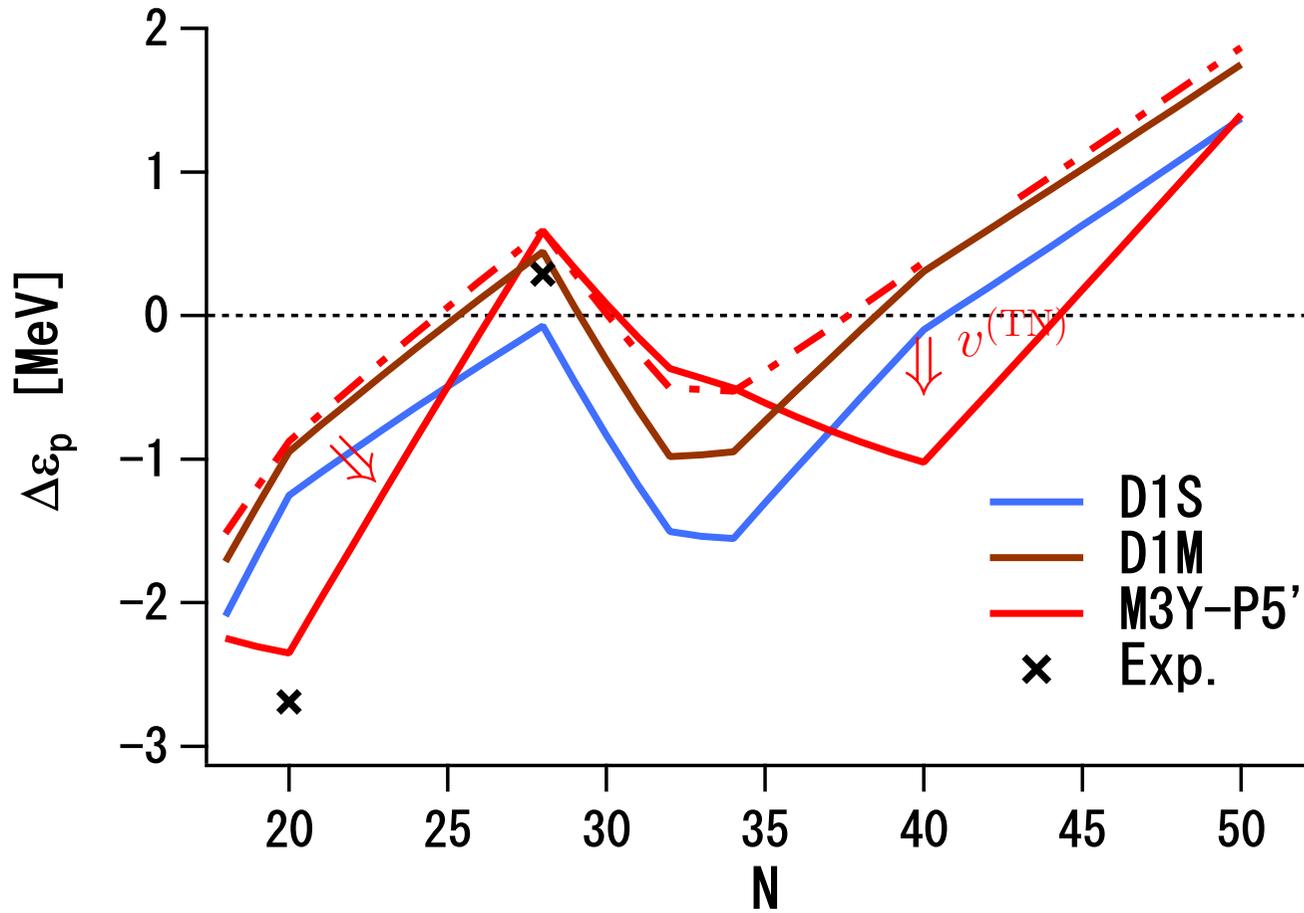
— $1s_{1/2}$-$0d_{3/2}$ inversion

$$\Delta\varepsilon_p = \varepsilon_p(1s_{1/2}) - \varepsilon_p(0d_{3/2})$$

Ref: M. Grasso *et al.*, P.R.C 76, 044319 ('07)

(Exp.: average weighted by spectroscopic factor)

Case of semi-realistic int. (M3Y-P5′)

realistic $\hat{v}^{(\mathrm{TN})} \rightarrow$ correct N-dep. of $\Delta\varepsilon_p$ (in $N = 20-28$) !

★ **Magic numbers**

M3Y-P6

□ Z magic
□ submagic (0.5)
□ (0.8)

■ N magic
■ submagic (0.5)
■ (0.8)

126
120
92
$Z=82$
64
58
$Z=50$
40
$Z=28$
$Z=20$
14
$Z=8$

$N=184$
164
$N=126$
$N=82$
$N=50$
28

Z
N
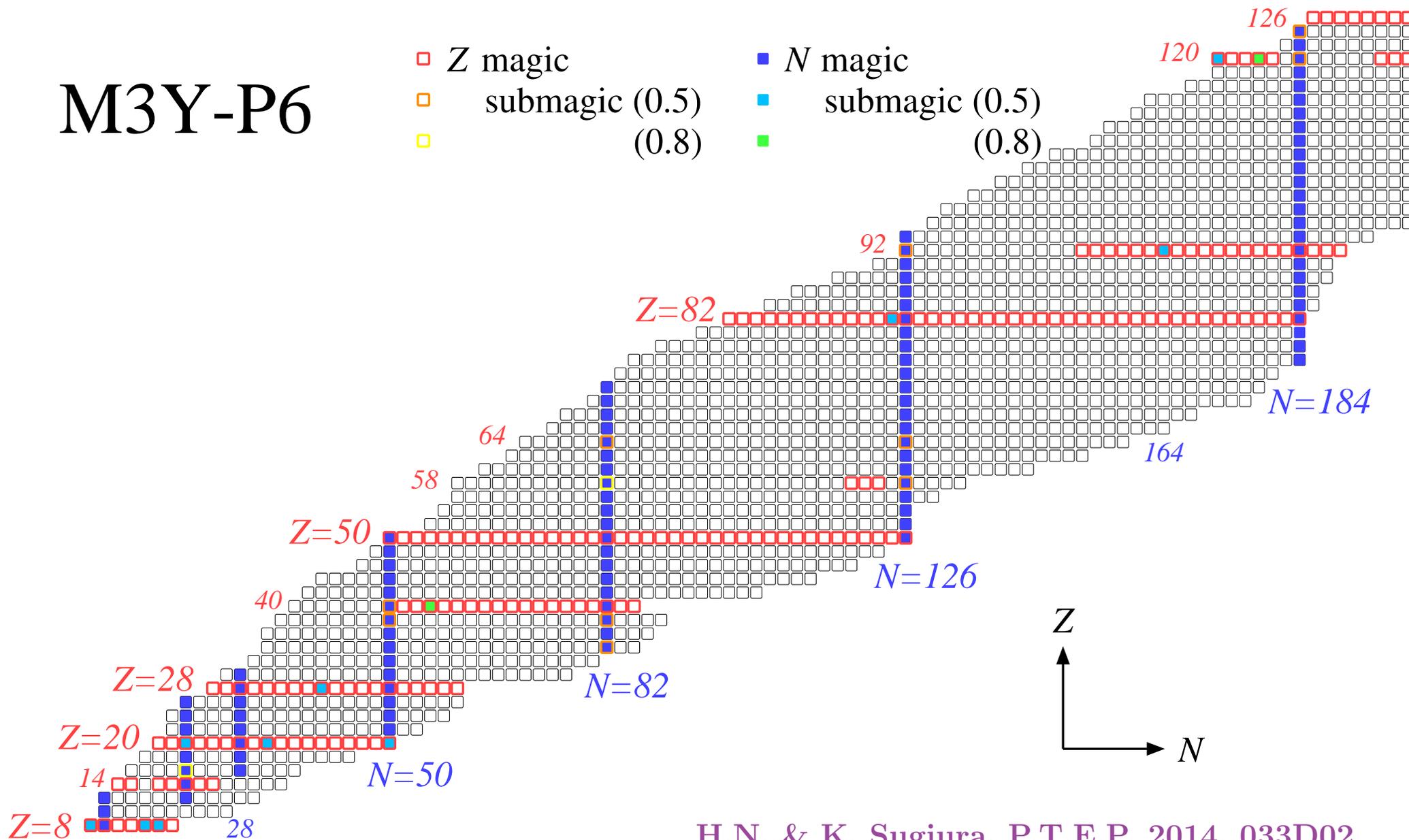
III. Incorporating $3N$ LS interaction

Semi-realistic M3Y-P6 int. $\cdots$ **reasonable shell structure** $\Rightarrow$ **yardstick**

$3N$ **LS int.** $\leftrightarrow$ ρ**-dep. LS int.** $\left(\hat{v}^{(\mathrm{LS}\rho)}\right)$ **Ref.**: M. Kohno, P.R.C 86, 061301(R)

$$\Downarrow$$

M3Y-P6 — $\hat{v}_{\mathrm{M3Y}}^{(\mathrm{LS})} \times 2.2$

vs. (**equate** $\varepsilon_n(0i_{11/2}) - \varepsilon_n(0i_{13/2})$ **at** ^{208}**Pb**)

M3Y-P6a — $\hat{v}_{\mathrm{M3Y}}^{(\mathrm{LS})} + \hat{v}^{(\mathrm{LS}\rho)}$

$$v^{(\mathrm{LS}\rho)} = 2i\, D[\rho(\boldsymbol{R}_{ij})]\, \boldsymbol{p}_{ij} \times \delta(\boldsymbol{r}_{ij})\, \boldsymbol{p}_{ij} \cdot (\boldsymbol{s}_i + \boldsymbol{s}_j)\,;$$

$$D[\rho(\boldsymbol{r})] = -w_1 \frac{\rho(\boldsymbol{r})}{1 + d_1\rho(\boldsymbol{r})} \left(\approx -w_1\, \rho(\boldsymbol{r}) \right)$$

$$d_1 = 1.0\,\mathrm{fm}^3\ (\textbf{prefix}),\ w_1\,(>0)\colon \textbf{fitted}$$

★ Local current representation of contribution to HF energy

$$E_{ph}^{(\text{LS}\rho)} = \frac{1}{4} \int d^3r \, D[\rho(\mathbf{r})]$$

$$\times \left\{ \rho(\mathbf{r}) \, \nabla \cdot \mathbf{J}(\mathbf{r}) + \sum_{\tau=p,n} \rho_\tau(\mathbf{r}) \, \nabla \cdot \mathbf{J}_\tau(\mathbf{r}) + i \, \mathbf{J}(\mathbf{r}) \cdot \mathbf{j}^*(\mathbf{r}) + i \sum_{\tau=p,n} \mathbf{J}_\tau(\mathbf{r}) \cdot \mathbf{j}_\tau^*(\mathbf{r}) \right.$$

$$\left. -i \, \mathbf{J}^*(\mathbf{r}) \cdot \mathbf{j}(\mathbf{r}) - i \sum_{\tau=p,n} \mathbf{J}_\tau^*(\mathbf{r}) \cdot \mathbf{j}_\tau(\mathbf{r}) - 2 \, \mathbf{Q}(\mathbf{r}) \cdot \mathbf{s}(\mathbf{r}) - 2 \sum_{\tau=p,n} \mathbf{Q}_\tau(\mathbf{r}) \cdot \mathbf{s}_\tau(\mathbf{r}) \right\} ;$$

$$\rho(\mathbf{r}) = \sum_{\tau=p,n} \rho_\tau(\mathbf{r}) \,, \qquad \rho_\tau(\mathbf{r}) = \sum_{\alpha,\beta \in \tau} \varrho_{\alpha\beta} \, \phi_\beta^\dagger(\mathbf{r}) \, \phi_\alpha(\mathbf{r}) \,,$$

$$\mathbf{j}(\mathbf{r}) = \sum_{\tau=p,n} \mathbf{j}_\tau(\mathbf{r}) \,, \qquad \mathbf{j}_\tau(\mathbf{r}) = -i \sum_{\alpha,\beta \in \tau} \varrho_{\alpha\beta} \, \phi_\beta^\dagger(\mathbf{r}) \, \nabla \phi_\alpha(\mathbf{r}) \,,$$

$$\mathbf{Q}(\mathbf{r}) = \sum_{\tau=p,n} \mathbf{Q}_\tau(\mathbf{r}) \,, \qquad \mathbf{Q}_\tau(\mathbf{r}) = i \sum_{\alpha,\beta \in \tau} \varrho_{\alpha\beta} \, \nabla \phi_\beta^\dagger(\mathbf{r}) \times \nabla \phi_\alpha(\mathbf{r}) \,,$$

$$\mathbf{s}(\mathbf{r}) = \sum_{\tau=p,n} \mathbf{s}_\tau(\mathbf{r}) \,, \qquad \mathbf{s}_\tau(\mathbf{r}) = \sum_{\alpha,\beta \in \tau} \varrho_{\alpha\beta} \, \phi_\beta^\dagger(\mathbf{r}) \, \mathbf{s} \, \phi_\alpha(\mathbf{r}) \,,$$

$$\mathbf{J}(\mathbf{r}) = \sum_{\tau=p,n} \mathbf{J}_\tau(\mathbf{r}) \,, \qquad \mathbf{J}_\tau(\mathbf{r}) = 2i \sum_{\alpha,\beta \in \tau} \varrho_{\alpha\beta} \, \phi_\beta^\dagger(\mathbf{r}) \, \mathbf{s} \times \nabla \phi_\alpha(\mathbf{r}) \,.$$

spherical $\rightarrow \mathbf{Q}_\tau(\mathbf{r}) = \mathbf{s}_\tau(\mathbf{r}) = 0 \,, \; i[\mathbf{j}_\tau(\mathbf{r}) - \mathbf{j}_\tau^*(\mathbf{r})] = \nabla \rho_\tau(r) \,, \; \mathbf{J}_\tau(\mathbf{r}) = \mathbf{J}_\tau^*(\mathbf{r})$

★ **Contribution to ℓs potential**

$$-\frac{1}{r}\left[D[\rho(r)]\,\frac{d}{dr}\Big(\rho(r)+\rho_\tau(r)\Big) + \frac{1}{2}\frac{\delta D}{\delta\rho}[\rho(r)]\Big(\rho(r)+\rho_\tau(r)\Big)\frac{d\rho(r)}{dr}\right]\boldsymbol{\ell}\cdot\mathbf{s}\,.\quad (\tau=p,n)$$

★ **Influence on s.p. wave functions:**

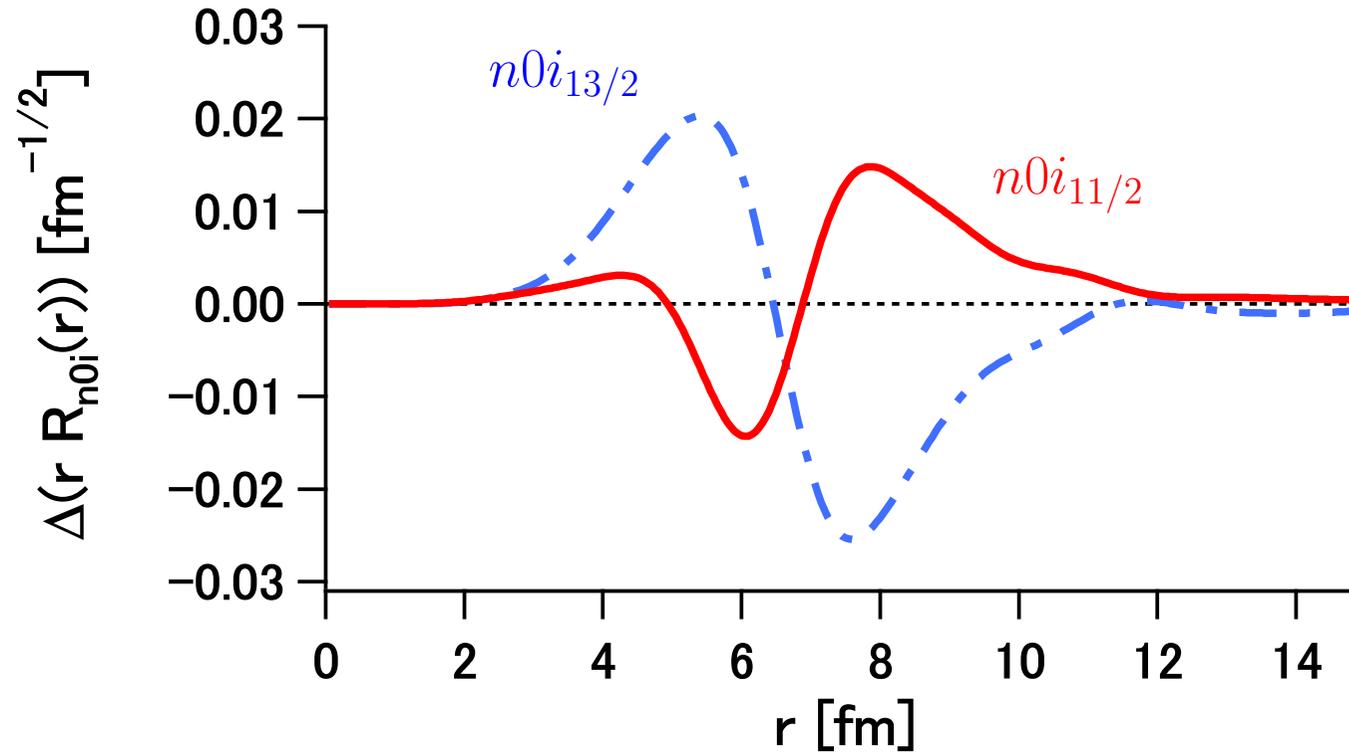**presence of $D[\rho]$** $\rightarrow$ $\begin{cases} \textbf{stronger LS for interior} \ \ (\text{higher } \rho) \\ \textbf{weaker LS for extetior} \ \ \ (\text{lower } \rho) \end{cases}$

$\rightarrow$ $\begin{cases} j=\ell+1/2 \ \textbf{orbitals shrink} \\ j=\ell-1/2 \ \textbf{orbitals extends} \end{cases}$ $(\because \textbf{variation})$

e.g. **larger $\langle r^2\rangle$ for $n0i_{11/2}$ for Pb**

$\rightarrow$ **influence on isotope shifts of Pb ?**

★ Confirming effects on s.p. functions

$$\Delta[r\,R_j(r)] := [r\,R_j(r)]_{\text{M3Y}-\text{P6a}} - [r\,R_j(r)]_{\text{M3Y}-\text{P6}} \qquad @\ ^{208}\text{Pb}$$
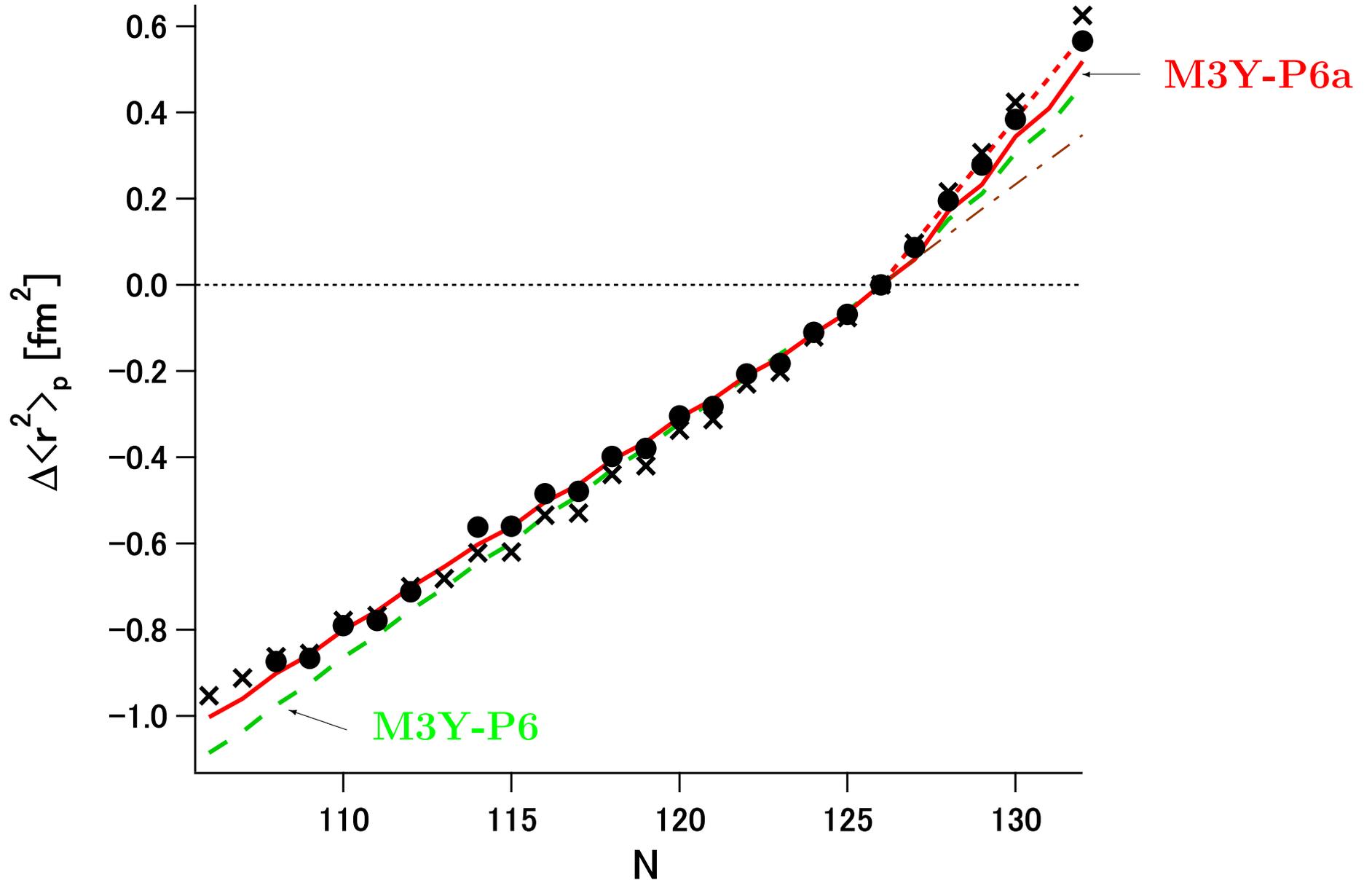

$\begin{cases} j = \ell + 1/2 \ \textbf{orbitals shrink} \\ j = \ell - 1/2 \ \textbf{orbitals extends} \end{cases}$

$\langle r^2 \rangle_j\ [\text{fm}^2]$:	**M3Y-P6**	**M3Y-P6a**
$n1g_{9/2}$	32.3	31.8
$n0i_{11/2}$	40.2	40.7

IV. $3N$ **LS interaction & isotope shifts**

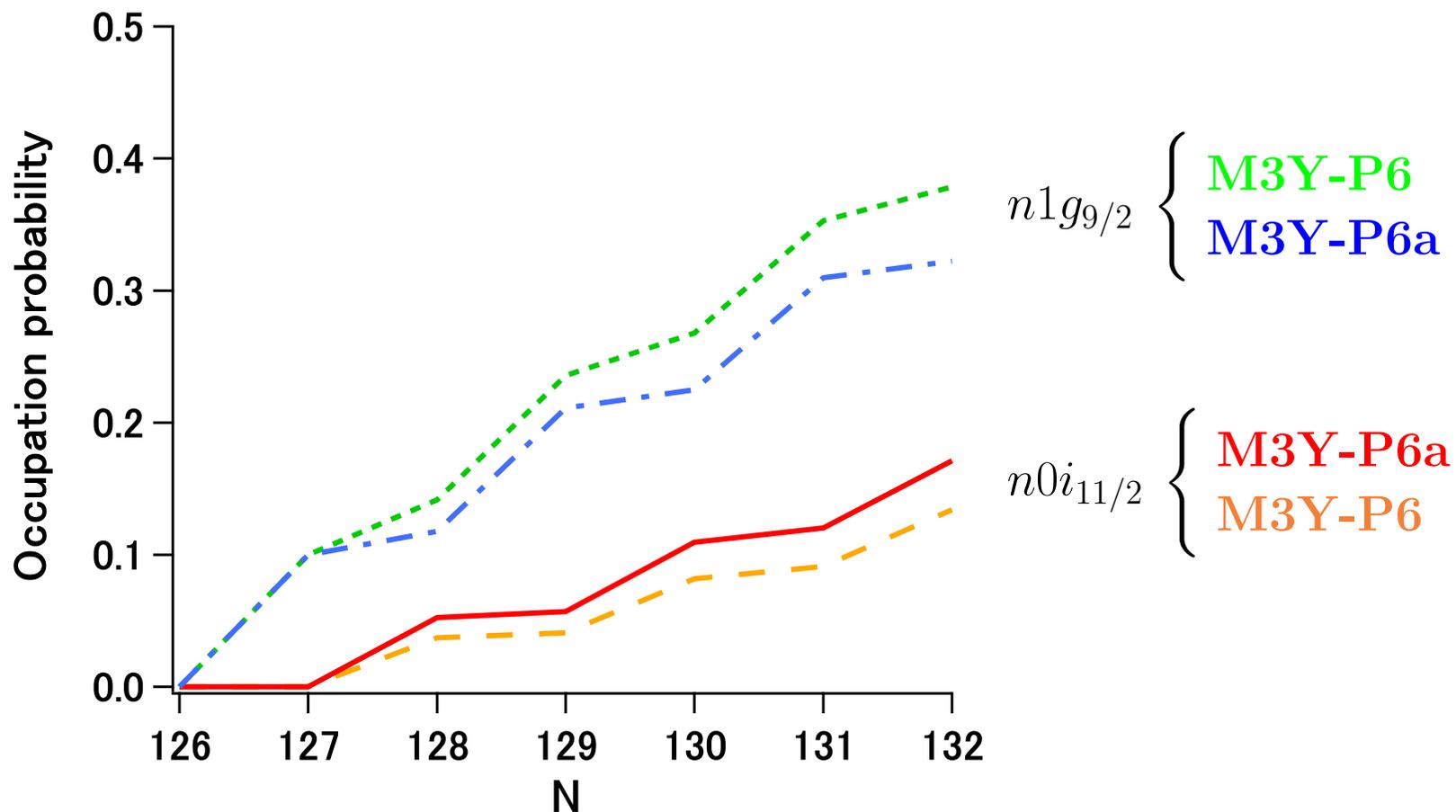★ **Isotope shifts of Pb nuclei** $\Delta\langle r^2\rangle_p(^A\mathbf{Pb}) := \langle r^2\rangle_p(^A\mathbf{Pb}) - \langle r^2\rangle_p(^{208}\mathbf{Pb})$

M3Y-P6a

M3Y-P6

$\Delta\langle r^2\rangle_p$ [fm^2]
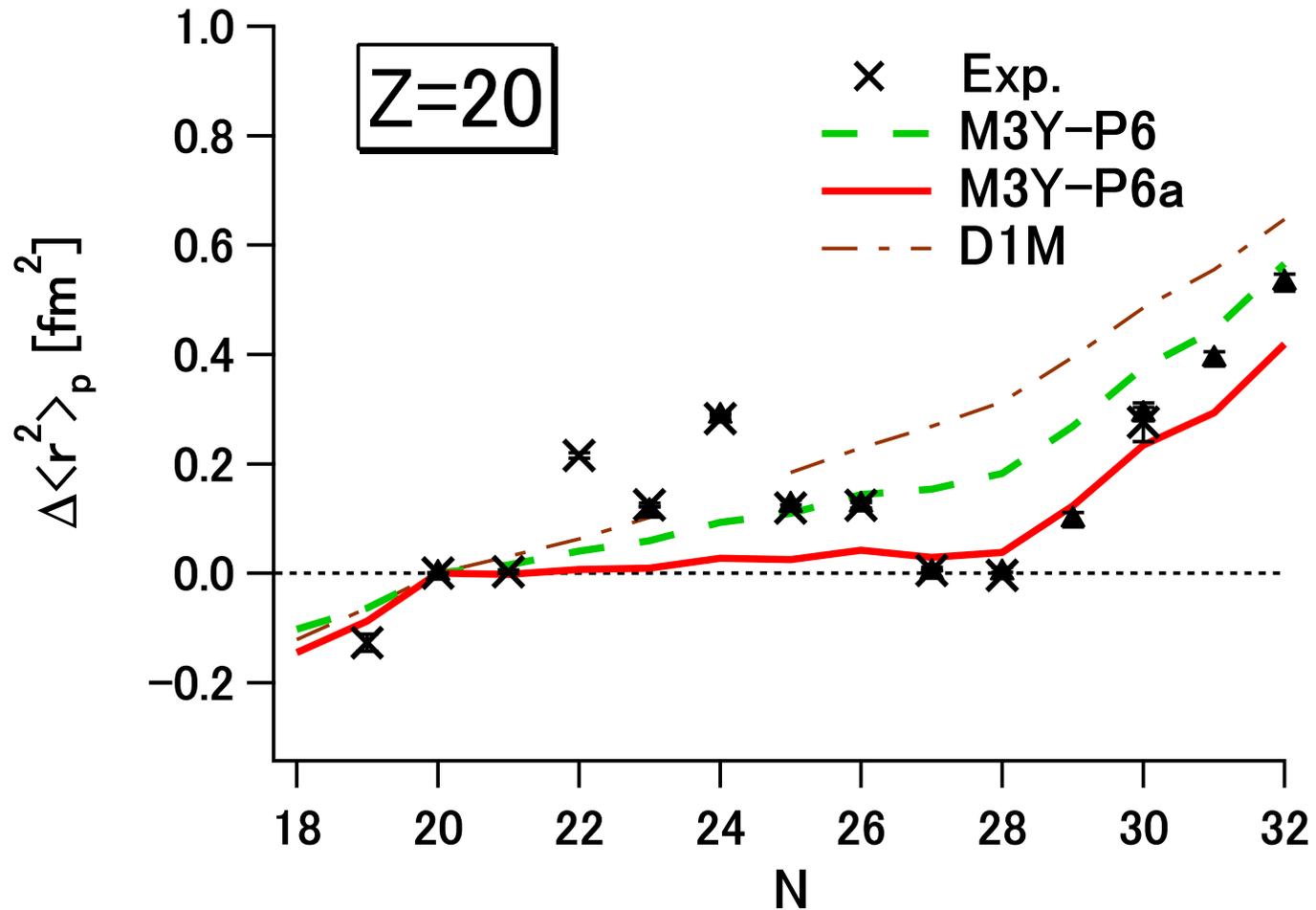
N

★ **S.p. energies & occ. prob.**

$$\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2}): \begin{cases} \textbf{exp. @ } ^{209}\textbf{Pb} \rightarrow 0.78\,\text{MeV} \\ \textbf{M3Y-P6a} \quad\rightarrow 0.72\,\text{MeV} \end{cases}$$

occ. prob.

$n1g_{9/2} \begin{cases} \textbf{\textcolor{green}{M3Y-P6}} \\ \textbf{\textcolor{blue}{M3Y-P6a}} \end{cases}$

$n0i_{11/2} \begin{cases} \textbf{\textcolor{red}{M3Y-P6a}} \\ \textbf{\textcolor{orange}{M3Y-P6}} \end{cases}$

$\Rightarrow$ **kink at** $N = 126$ **reproduced without** $n1g_{9/2}$-$n0i_{11/2}$ **degeneracy !**

★ **Isotope shifts of Ca nuclei** $\quad \Delta\langle r^2\rangle_p(^A\mathbf{Ca}) := \langle r^2\rangle_p(^A\mathbf{Ca}) - \langle r^2\rangle_p(^{40}\mathbf{Ca})$

$$\sqrt{\langle r^2\rangle_p}(^{40}\mathbf{Ca}) \approx \sqrt{\langle r^2\rangle_p}(^{48}\mathbf{Ca})\,!$$

$\cdots$ difficult to be reproduced so far

(▲: new exp., Nat. Phys. AOP 3645)

★ **Isotope shifts of Ni nuclei** $\quad \Delta \langle r^2 \rangle_p(^A\text{Ni}) := \langle r^2 \rangle_p(^A\text{Ni}) - \langle r^2 \rangle_p(^{60}\text{Ni})$
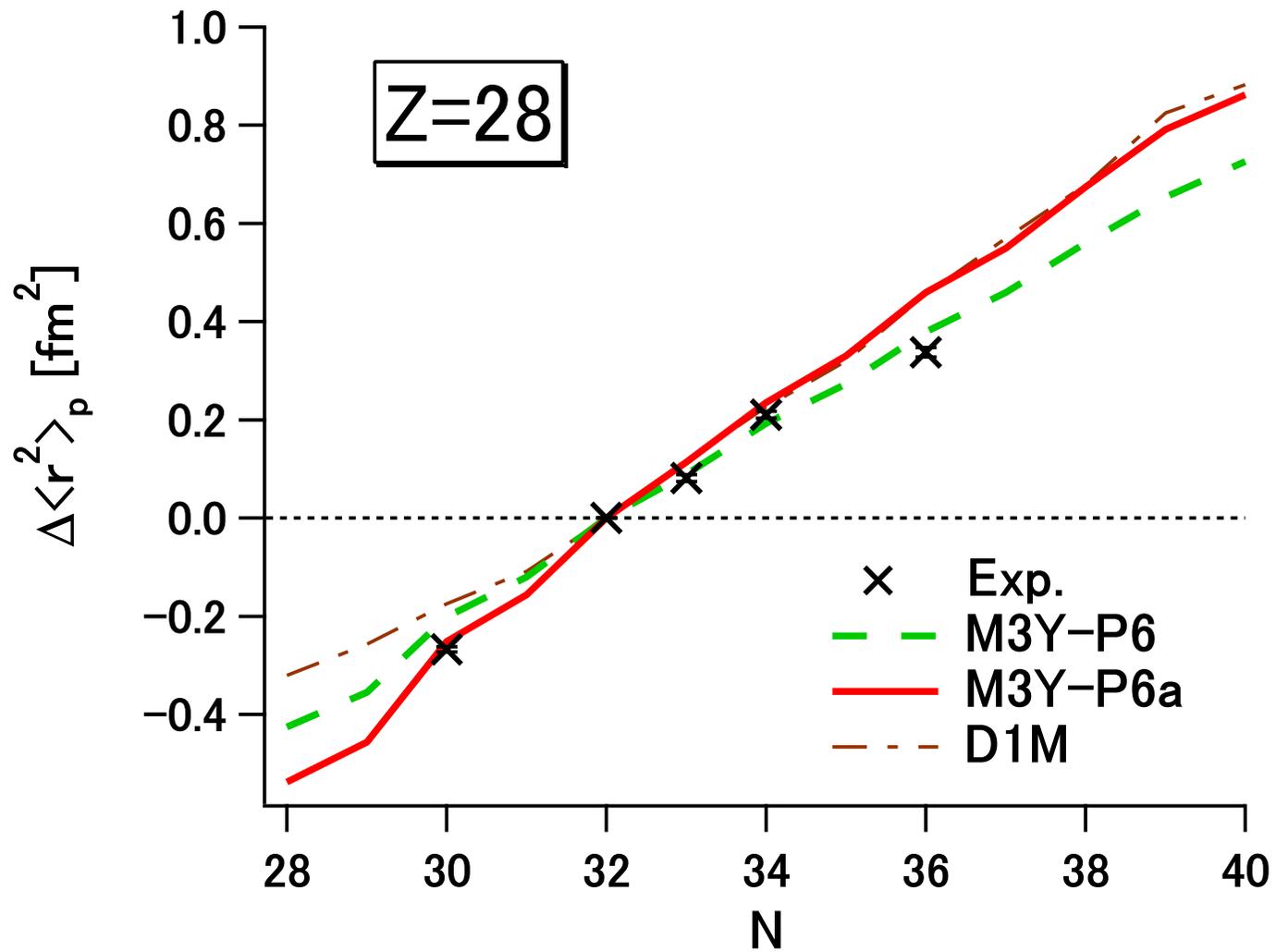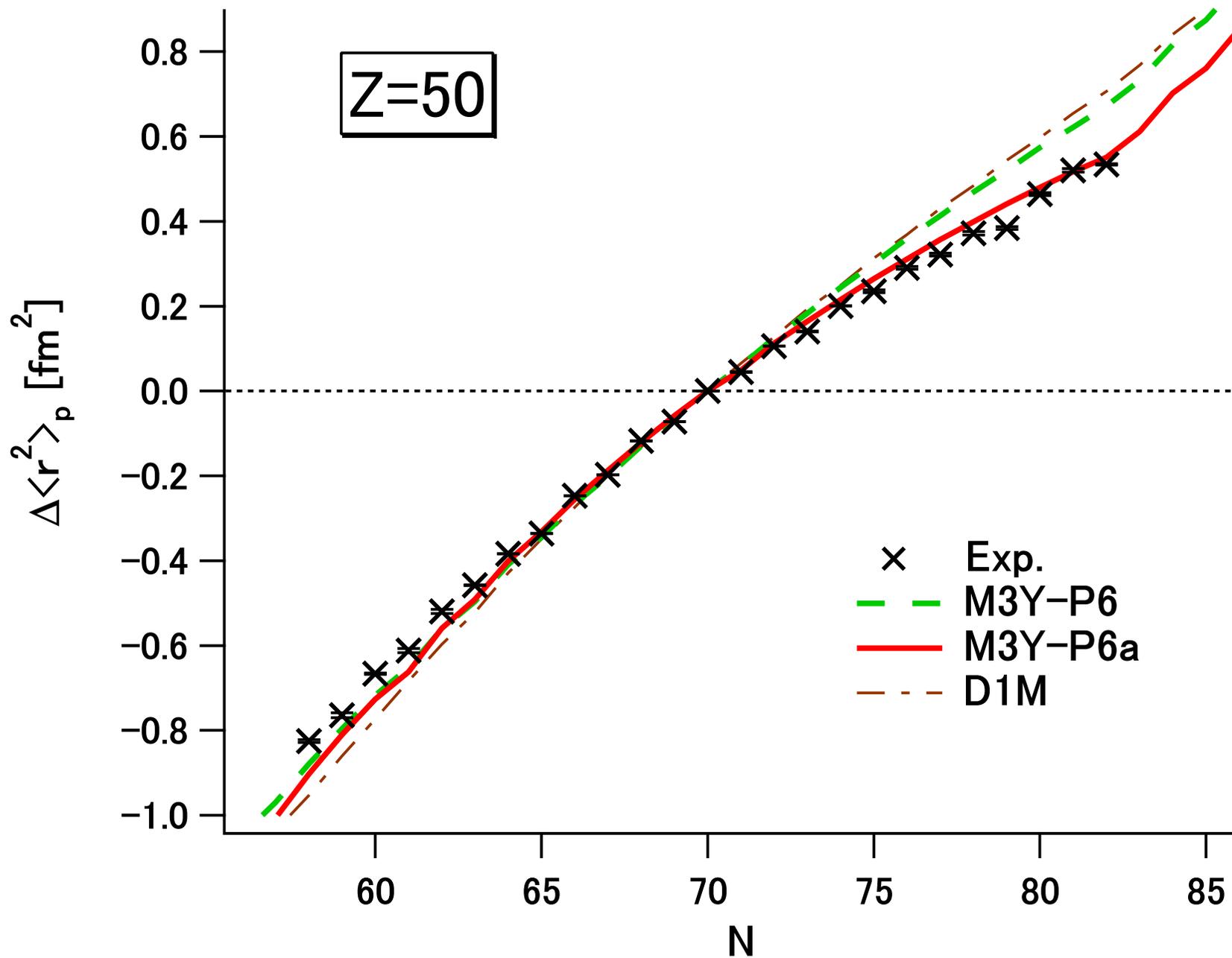

Data on ^{56}Ni ?

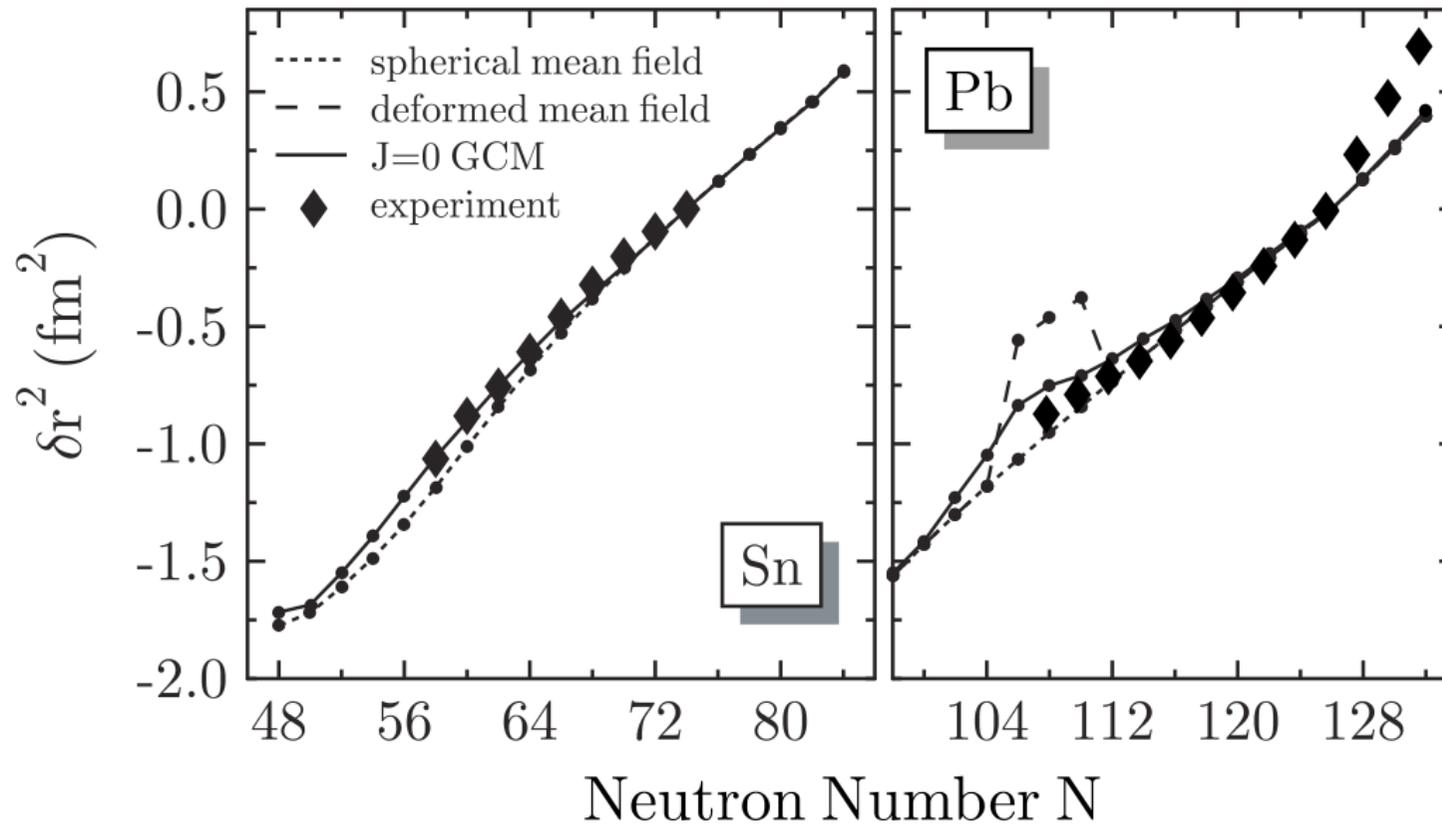★ **Isotope shifts of Sn nuclei** $\Delta\langle r^2\rangle_p({}^A\mathbf{Sn}) := \langle r^2\rangle_p({}^A\mathbf{Sn}) - \langle r^2\rangle_p({}^{120}\mathbf{Sn})$

A kink predicted at $N = 82$!

— in sharp contrast to int. without ρ-dep. LS $\quad \Rightarrow$ **data ?**

cf. Influence of deformation & correlations beyond mean field (SLy4)

The plot shows δr^2 (fm^2) on the y-axis versus Neutron Number N on the x-axis, with two panels for Sn and Pb isotopes. Legend: spherical mean field, deformed mean field, J=0 GCM, experiment.

Ref.: M. Bender *et al.*, P.R.C 73, 034322

- **No improvement on the kink at $N = 126$ for Pb isotopes**
- Not in good agreement at $N \approx 80$ for Sn isotopes (not shown)

  — **no kink at $N = 82$ (?)**

V. Summary

1. We have investigated effects of $3N$ LS int. on isotope shifts of nuclei.

   ← sph. HFB with semi-realistic int. M3Y-P6 & its variant M3Y-P6a

2. With M3Y-P6a which contains ρ-dep. LS channel,

   - isotope shifts of the Pb nuclei are described fairly well

     without fictitious $n1g_{9/2}$-$n0i_{11/2}$ degeneracy,

   - almost equal charge radii between ^{40}Ca and ^{48}Ca are reproduced,

   - isotope shifts of the Sn nuclei are in agreement with available data,

     and a kink is predicted at $N = 82$.   → data ?

3. Results may be regarded as evidence for $3N$ LS interaction

   based on χEFT, indep. of ℓs splitting.

   — qualitative evidence for $3N$ interaction !

Perspectives :

- Influence on drip line, shell structure ?

- Effects on deformation & excitation ($e.g.$ spin excitation) ?