"Probabilistic aftershock forecasting:
A Bayesian statistical approach

Takahiro Omi

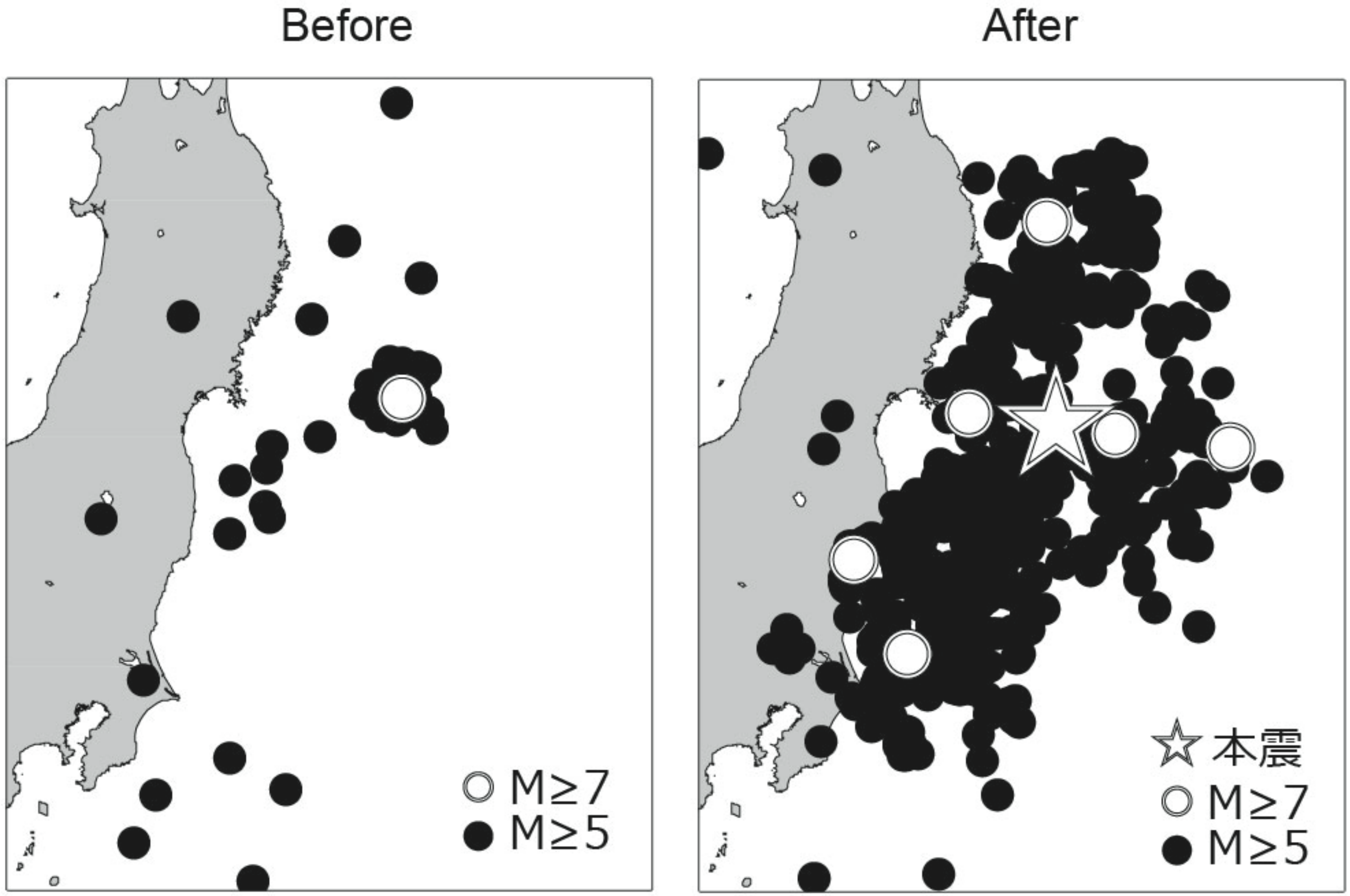
(Institute of Industrial Science, The University of Tokyo)

New Horizon of Mathematical Science @RIKEN

Outline

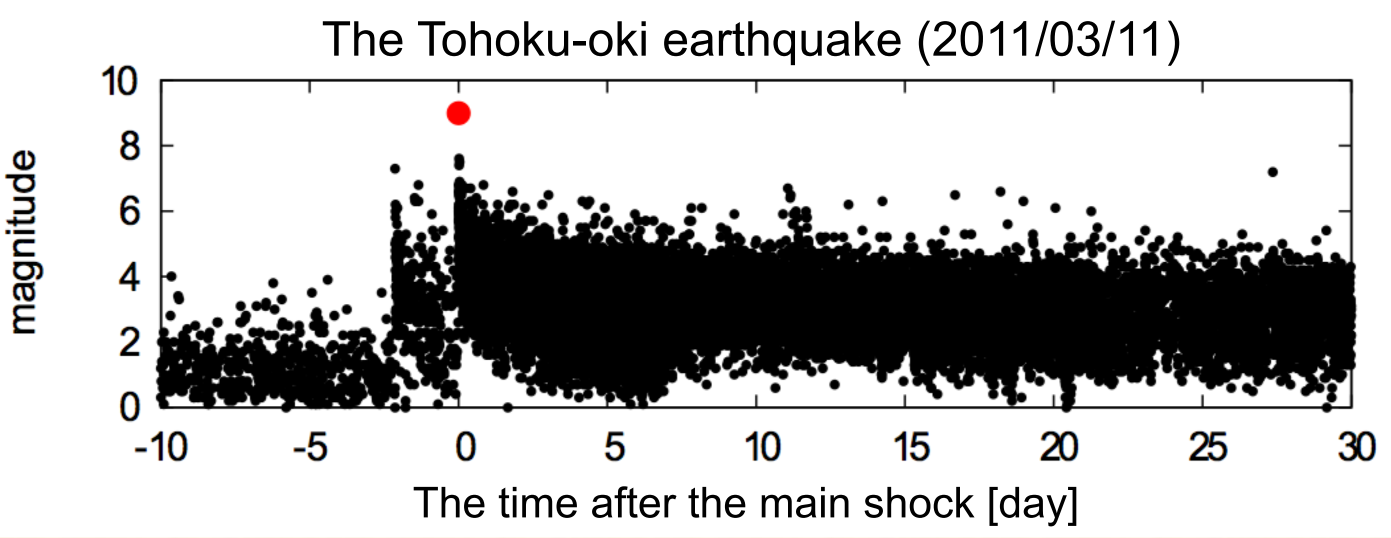
- Aftershock forecasting, and its problems
- Immediate forecasting of large aftershocks within one day after the main shock

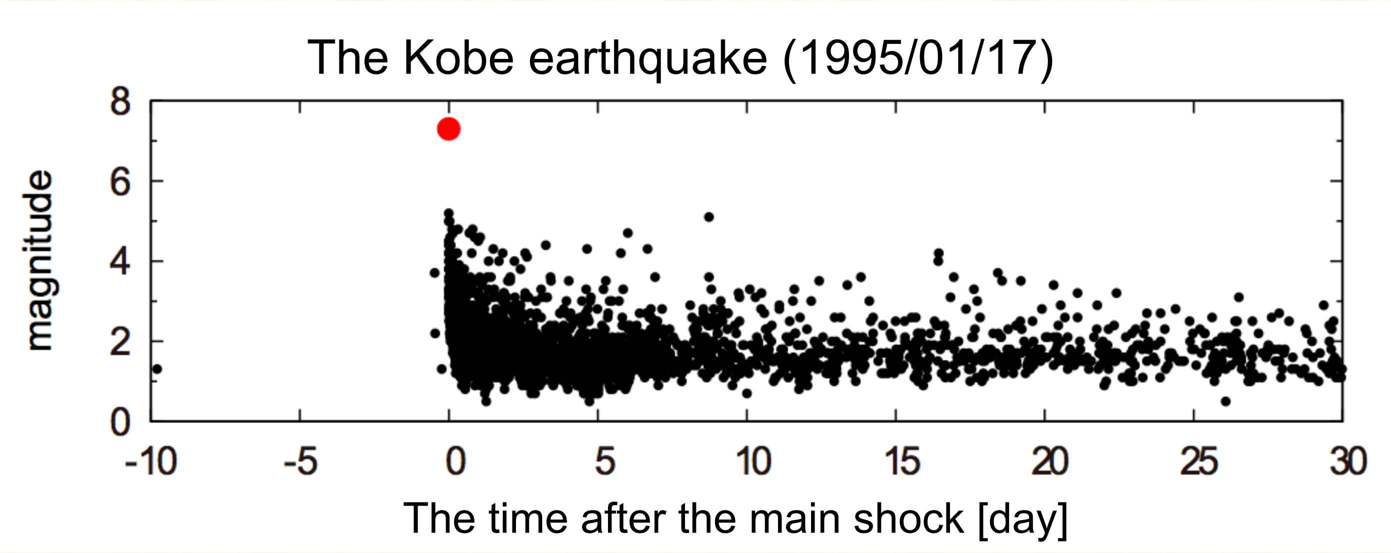
Mainshock, Aftershocks



Eqrthaueks during 1-year period before and after
the Tohoku-oki (M9.0) earthquake

Mainshock, Aftershocks





Importance of Aftershock forecasting after the main shock

- Large aftershocks can cause additional damages in the affected area.
- Aftershock forecasting has been carried out to reduce the seismic risks.

Q6(2)　**〔回答票6〕**　（２）それでは、地震発生後数時間が経過した時、あなたは特にどのようなことを知りたいと思いますか。この中から３つまであげてください。（３ M. A. ）（２）数時間経過後知りたいこと

(12.2) (ア) 自分の住んでいる地域の震度
(16.2) (イ) 震源や地震の規模
(4.2) (ウ) 津波襲来の有無
(35.1) (エ) 今後の余震の見通しなど地震活動の推移
(30.1) (オ) 家族や親戚などの安否情報や被害情報
(33.7) (カ) 知人・友人などの安否情報や被害情報
(47.2) (キ) 水道・電気・ガス・電話の被害状況と復旧の見通し
(39.1) (ク) 道路・電車など交通関係の被害状況と復旧の見通し
(16.0) (ケ) 学校や医療機関など公共施設の被害状況
(13.3) (コ) 会社など職場の被害状況
(13.6) (サ) 国、県や市区町村が実施している（実施する）対策
(0.3) その他
(1.8) 特にない
(1.4) わからない

Aftershock

People's safety

Damage situation and recovery period of Infrastructure

Public opinion research in Japan (1995)

What kind of information do you want to know a few hours after the main shock?

Aftershock forecasting by Japan Meteorological Agency

作業概要

地震発生直後
　この段階では、まだ余震の状況は正確に把握できず、過去における類似の地震を検索し、過去の活動事例を情報に活用する場合があります。

2011/03/11/14:46 Tohoku-oki earthquake

数時間後

数時間後
　「本震ー余震型」であることが確認され、余震域をほぼ把握し、余震の状況を発表します。

○ 余震活動の状況
　強い揺れを伴う多数の余震が観測されています。しばらくはこのような余震活動が続くと考えられます。

2011/03/11/16:00

約1日後

約1日後
　観測結果から余震の確率の計算式に入れる数値が一部求まり、3日間程度の短期間の余震の確率を計算し、それを活用して作成した余震の見通しを発表します。

	マグニチュード7以上
3月 13 日 10 時から3日間以内	70%
3月 16 日 10 時から3日間以内	50%

2011/03/13/12:55

4日後以降

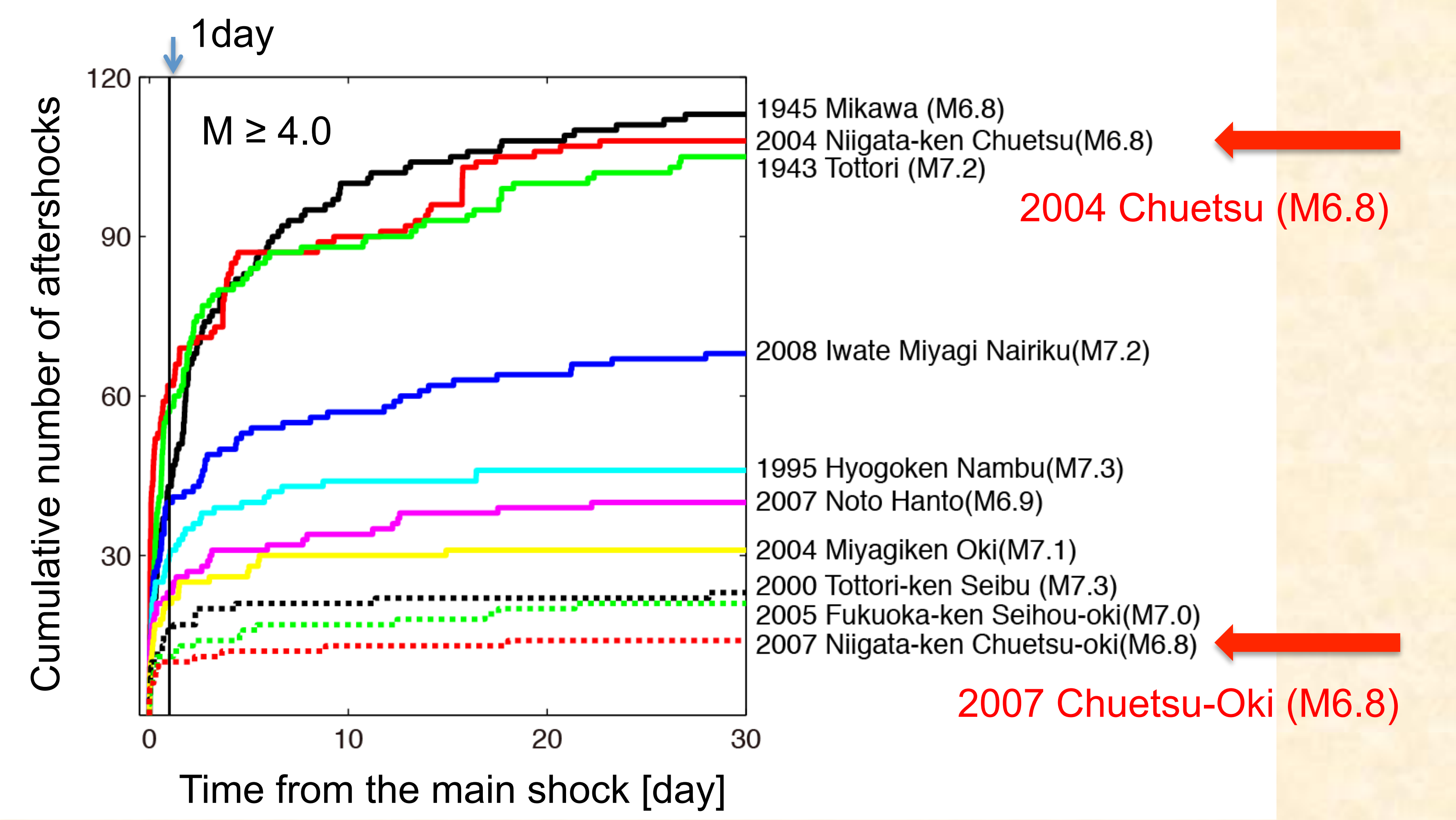
地震調査委員会開催時
　気象庁などの関係機関から提出される資料を検討し、発生した大地震の特徴や今後の余震活動などを総合的に評価し、それに基づき広報します。

4日間後以降（必要に応じて）
　計算式に入れる必要数値が次々と求まり、比較的長い期間の余震の確率と余震発生回数の予測を計算し、それを活用して作成した余震の見通しを発表します。

地震調査研究推進本部パンフレットより

Forecasting aftershocks after the main shock

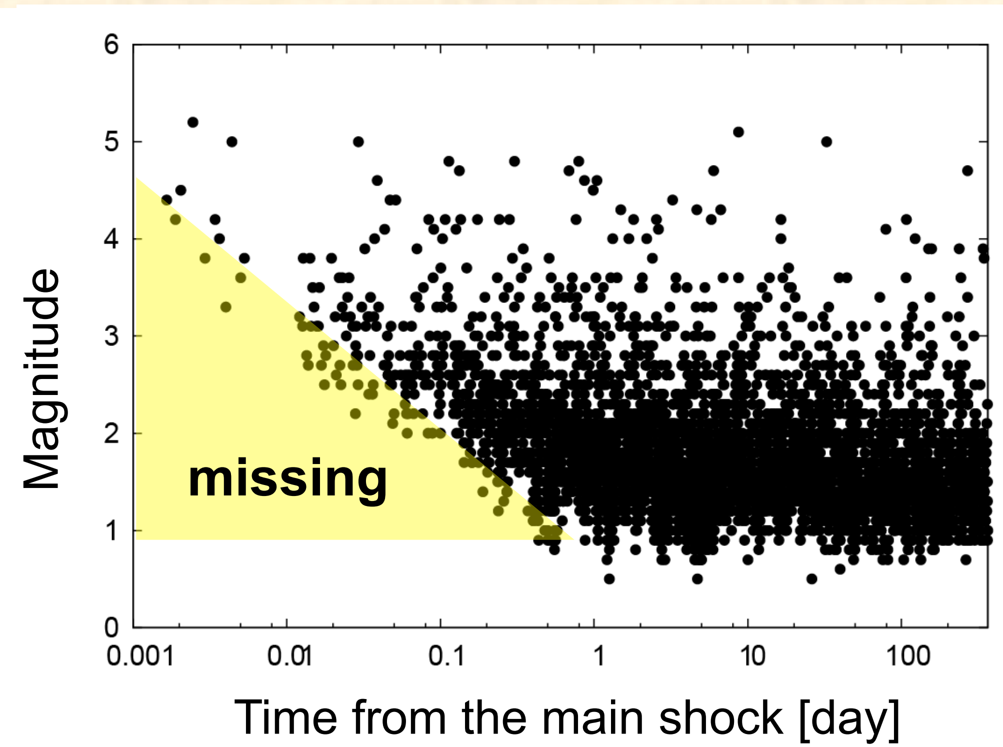
- Immediate forecast of aftershock is strongly required.
- We need to tailor a forecast model to each aftershock sequence.

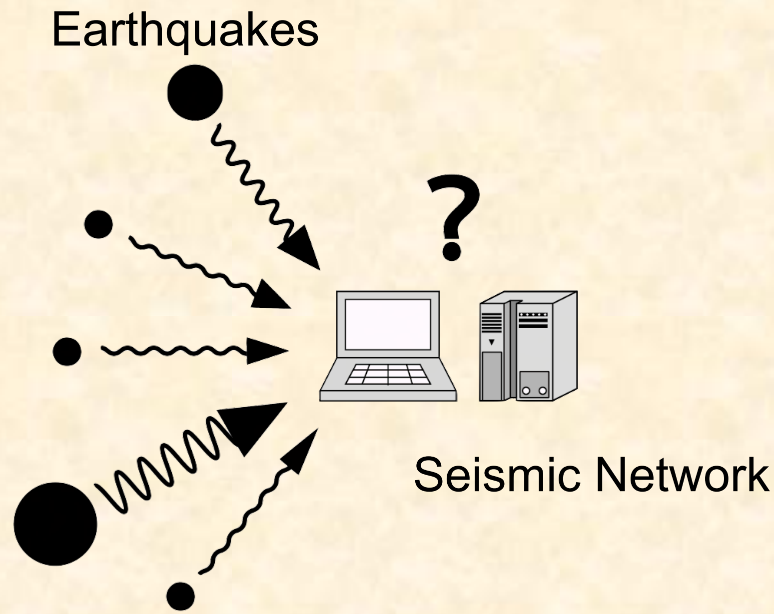


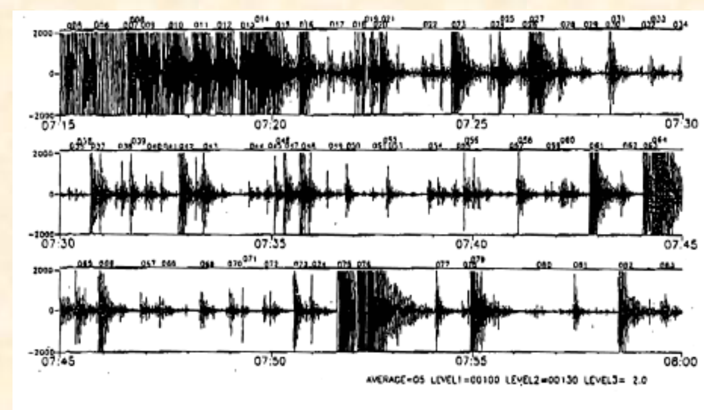
Why is immediate aftershock forecasting difficult?

The data in the early period is highly deficient.

1995 Kobe earthquake of M 7.3







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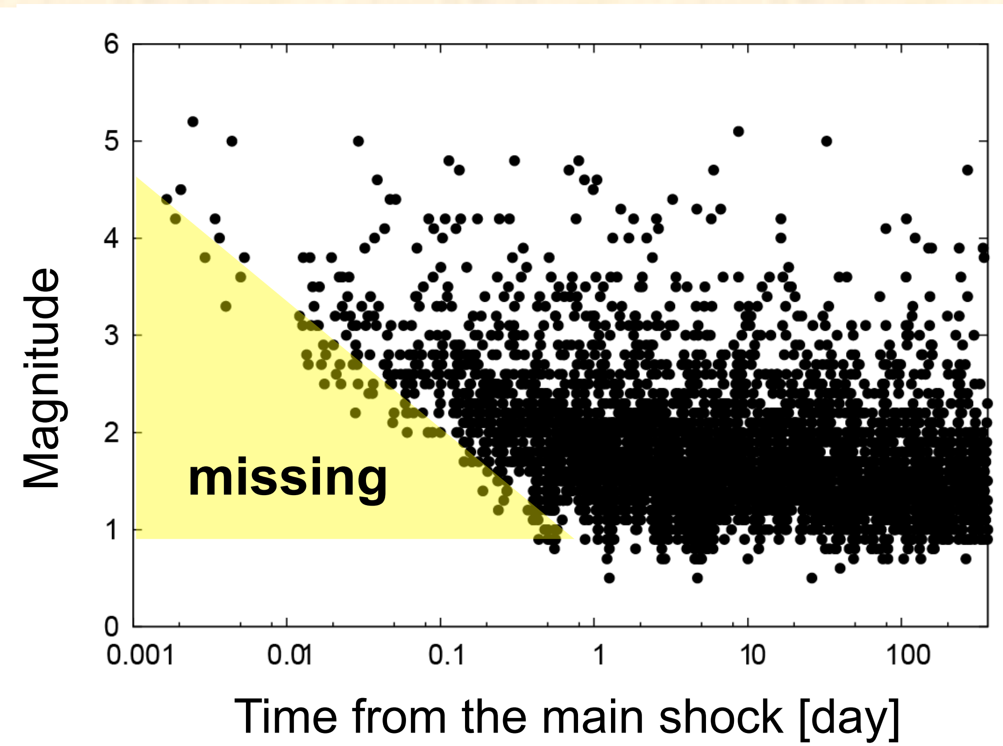

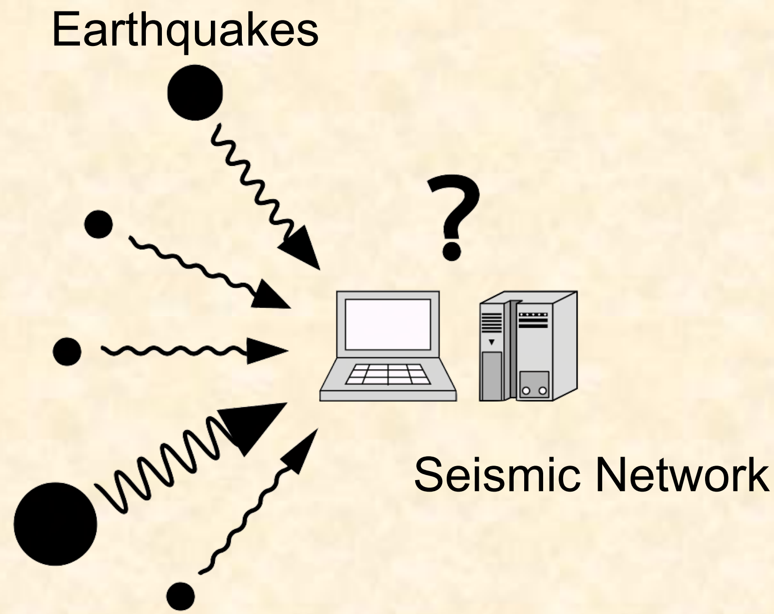

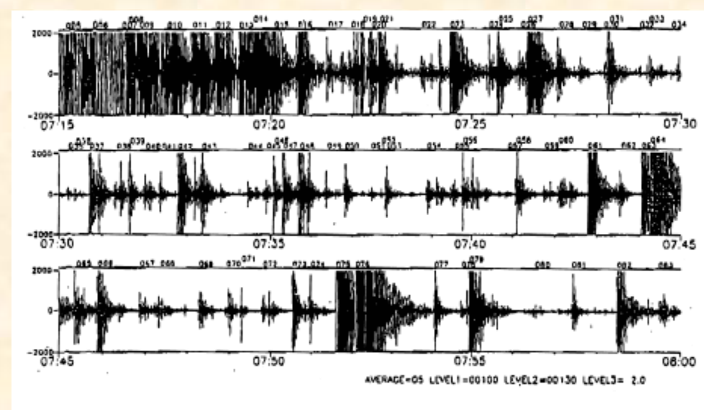

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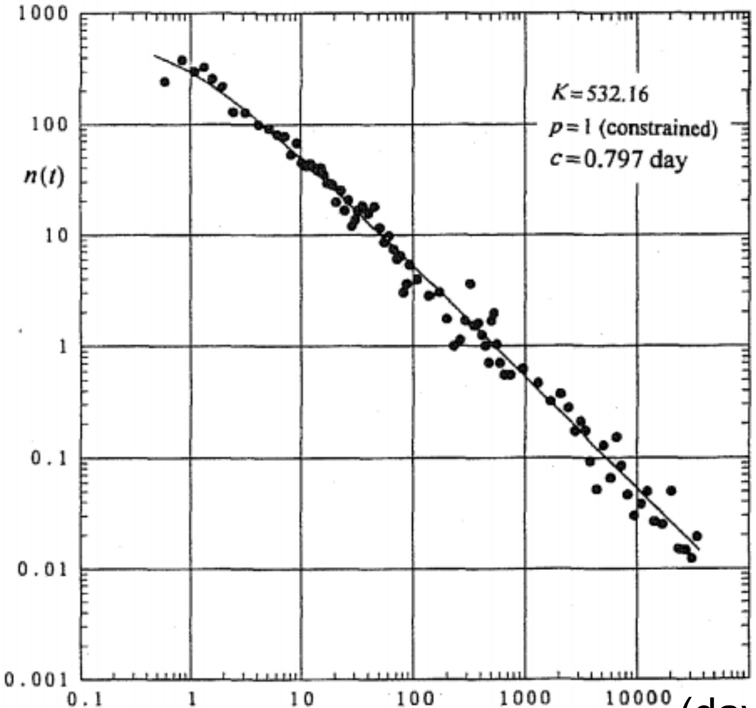
Empirical law of aftershocks:

- time evolution of aftershock activity

The Nobi earthquake M8.0 (1891)



Utsu, Ogata & Matsu'ura (1995)

Omori-law

$$\lambda(t) = \frac{k}{t+c}$$

F. Omori (1894)

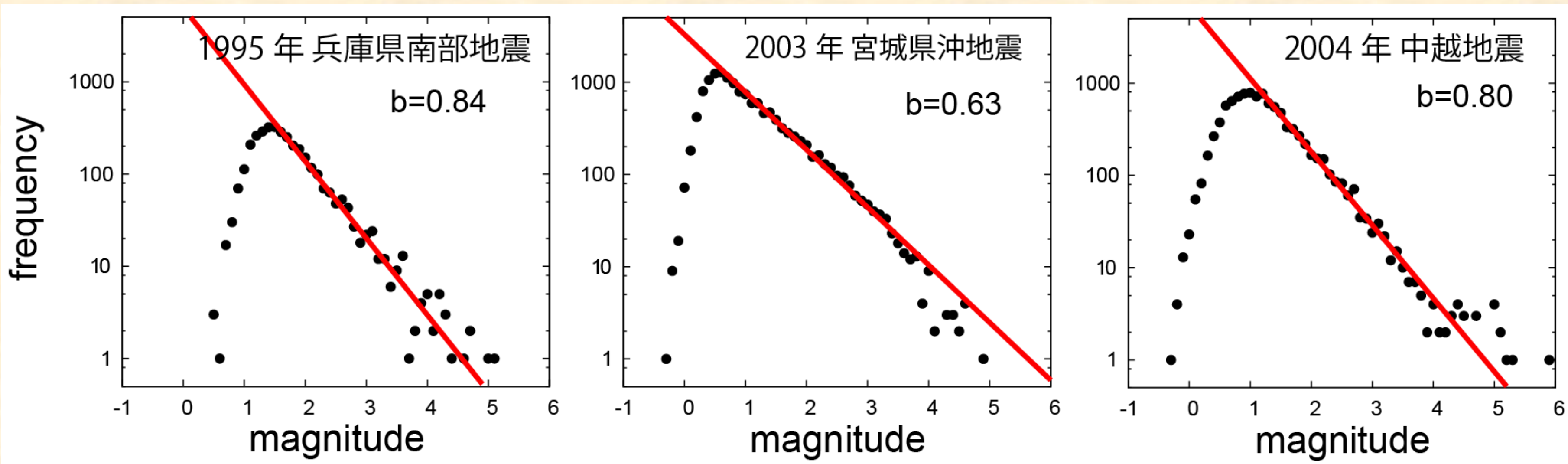
Omori-Utsu law

$$\lambda(t) = \frac{k}{(t+c)^p}$$

T. Utsu(1961)

Empirical law of aftershocks:

- magnitude-frequency relation

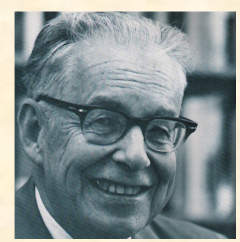


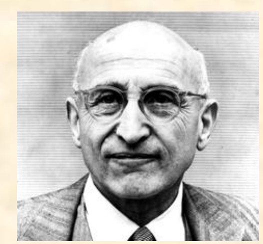
Gutenberg-Richter law (1944)

$$logN = a - bM$$

Exponential distribution

$$P(M) \propto e^{-\beta M} \quad (\ \beta = b \log 10\ )$$





Forecast model

(Reasenberg and Jones 1989)

The rate of earthquakes of the magnitude *M* at time *t*

$$\lambda(t, M) = \underbrace{\frac{K}{(t+c)^p}}_{\text{Omori-Utu}} \underbrace{e^{-\beta M}}_{\text{Gutenberg-Richter}}$$

<u>*Parameter Estimation*</u>

Estimate the parameters *K*, *c*, *p* and *β*.

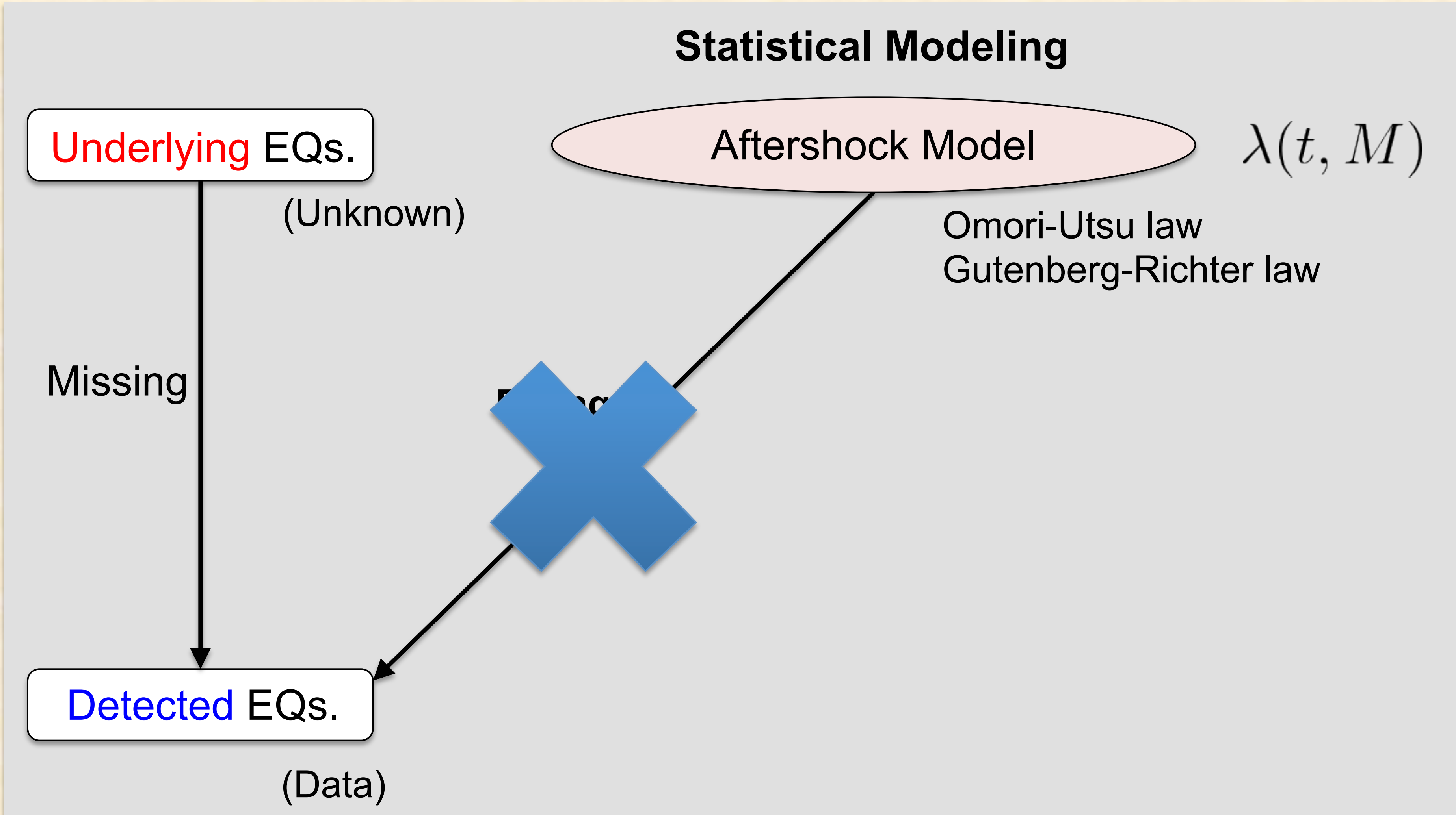
<u>*Forecast the probability*</u>

The expected number of earthquakes with magnitude $M > M_c$ in the time interval $[t_1, t_2]$

$$n = \int_{t_1}^{t_2} dt \int_{M_c}^{\infty} dM \lambda(t, M)$$

Schematic illustration of our procedure

Underlying Eqs: detected EQs. + missing EQs.



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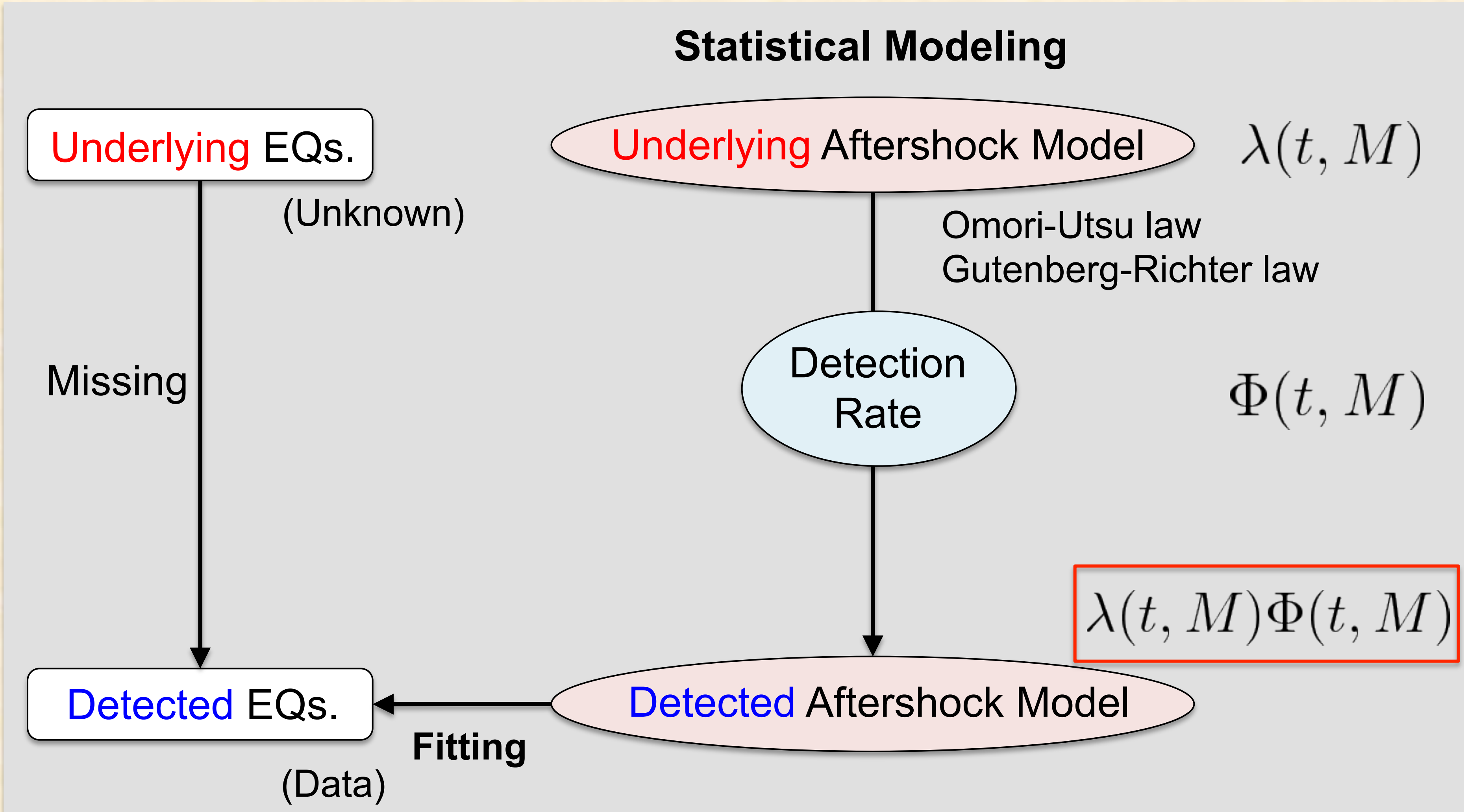
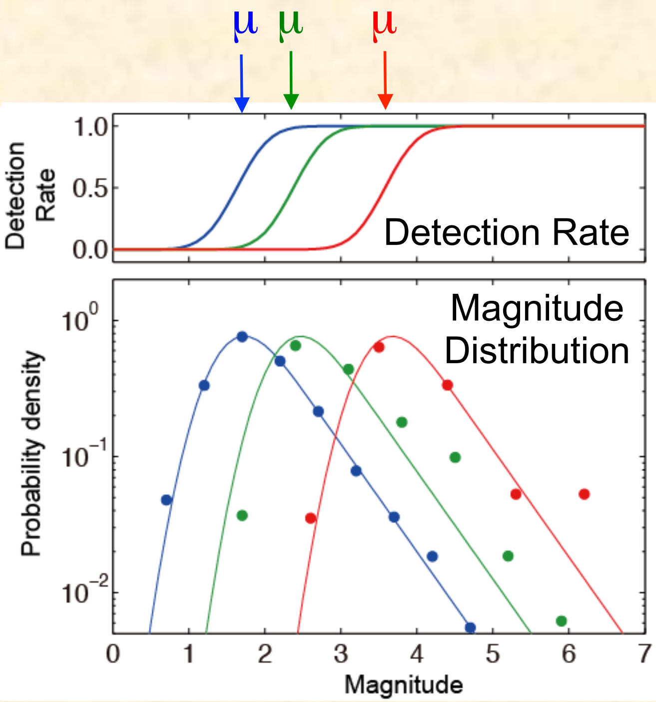


Estimating the detection rate



Magnitude dependence of the detection rate

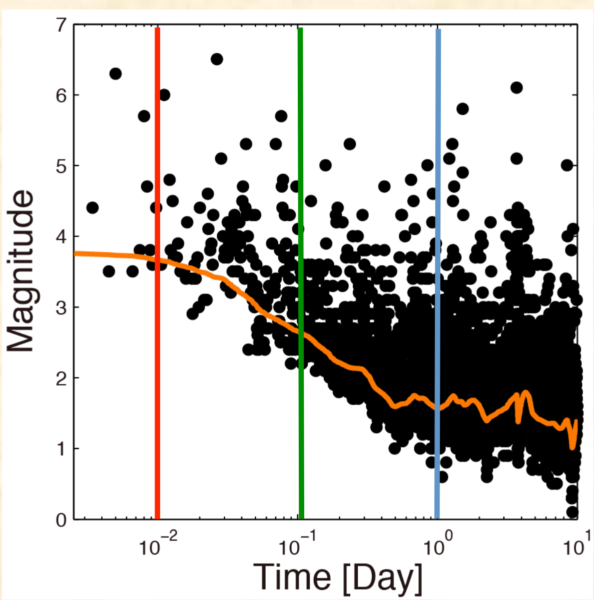
$$\Phi(M|\mu,\sigma) = \int_{-\infty}^{M} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

μ: the magnitude with 50 % detection rate

Observed magnitude distribution:

(G-R law) × (Detection Rate function)

Ogata & Katsura (1993).



Time dependence of the detection rate

- Assume the parameter μ is a function of the time.
- Estimate μ(t) by using Bayesian smoothing method

Omi et al., (2013, 2014, 2015).

2004 Chuetsu aftershock sequence

Objective Bayesian Estimation

Estimate $\boldsymbol{\mu} = \{\mu_1, \mu_2, \cdots, \mu_N\}$ β and σ from $\boldsymbol{M} = \{M_1, M_2, \cdots, M_N\}$

Likelihood: $$P_{\beta,\sigma}(\boldsymbol{M}|\boldsymbol{\mu}) = \prod_{i=1}^{N} \beta e^{-\beta(M_i-\mu_i)-\frac{\beta^2\sigma^2}{2}} \Phi(M_i|\mu_i,\sigma)$$

Prior: $$P_V(\boldsymbol{\mu}) = P(\mu_1,\mu_2) \prod_{i=1}^{N-2} \frac{1}{\sqrt{2\pi V}} e^{-\frac{(\mu_{i+2}-2\mu_{i+1}+\mu_i)^2}{2V}}$$

State Estimation: Find $\boldsymbol{\mu}$ that maximizes the posterior p.d.f.

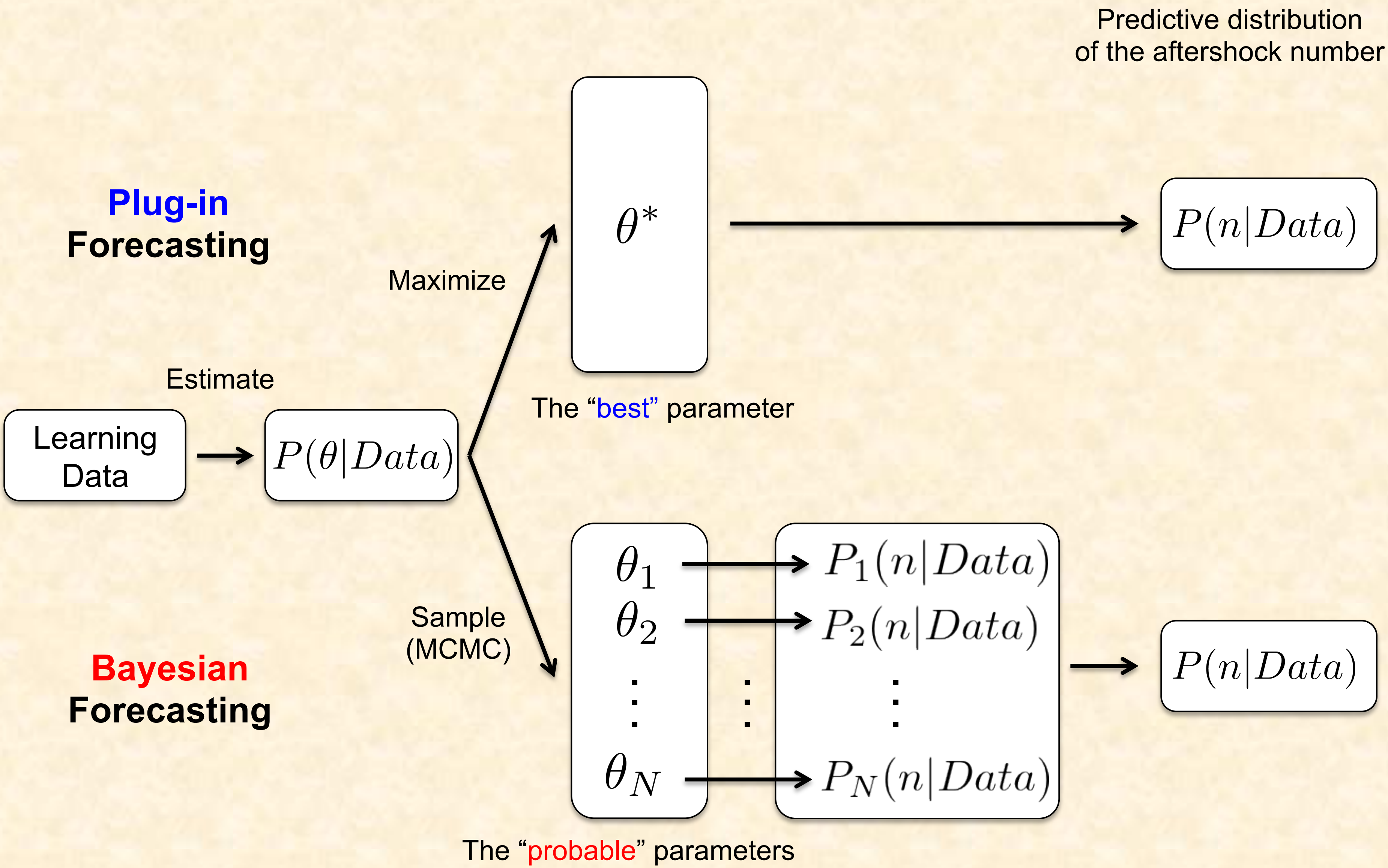
Posterior: $$P_{\beta,\sigma,V}(\boldsymbol{\mu}|\boldsymbol{M}) \propto \underbrace{P_{\beta,\sigma}(\boldsymbol{M}|\boldsymbol{\mu})}_{\text{Likelihood}} \underbrace{P_V(\boldsymbol{\mu})}_{\text{Prior}}$$

Parameter Estimation:

Find β, σ and V that maimizes the posterior p.d.f.

$$P(\beta,\sigma,V|\boldsymbol{M}) \propto \underbrace{P_{\beta,\sigma,V}(\boldsymbol{M})}_{\text{Marginal Likelihood}} \underbrace{P(\beta,\sigma,V)}_{\text{Prior}}$$

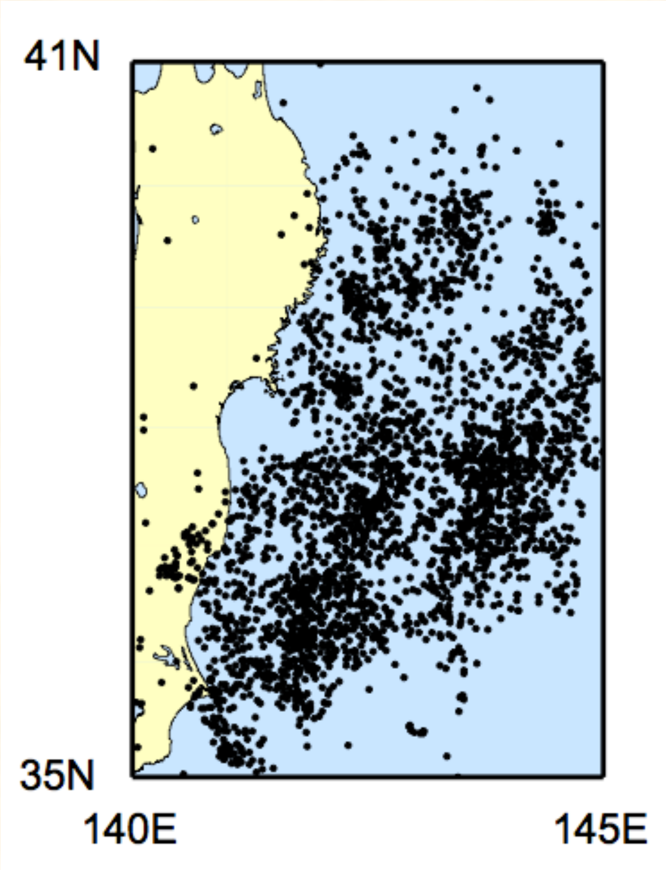
Bayesian forecasting: considering the estimation uncertainty



(Omi et al., JGR 2015)

Forecasting experiment

The aftershocks of the 2011 Tohoku-oki earthquake of M9.0
The NEIC PDE Catalog (provided by USGS)

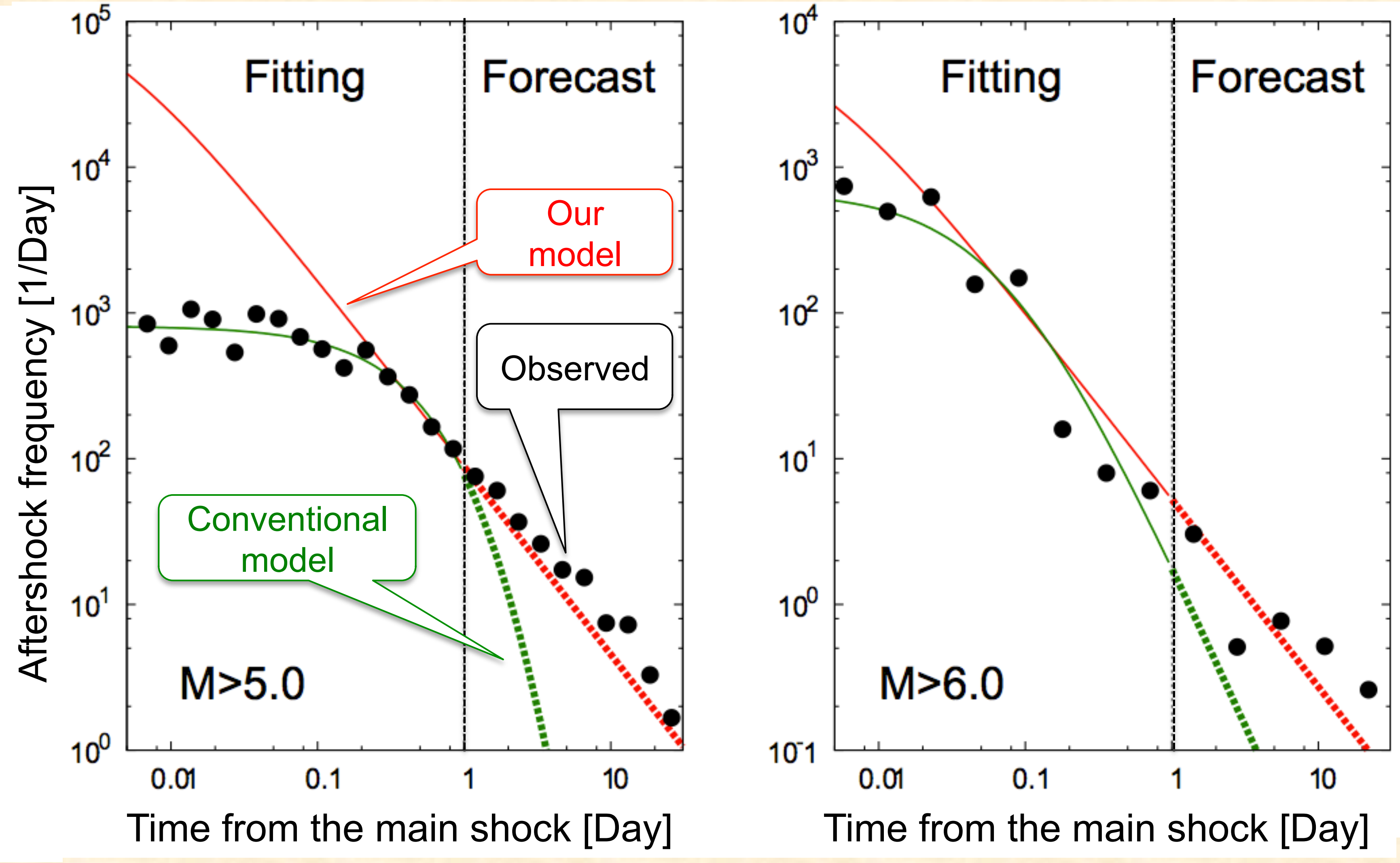


Learning period	Forecast period
0 – 1 [day]	1-31 [day]
0 – 3 [hour]	3 – 6 [hour]
0 – 6 [hour]	6 – 12 [hour]
0 – 12 [hour]	12 – 24 [hour]
0 – 24 [hour]	24 – 48 [hour]

Forecast experiment with the 2011 Tohoku sequence

Learning period	0 - 1 day
Forecasting period	1 - 31 day

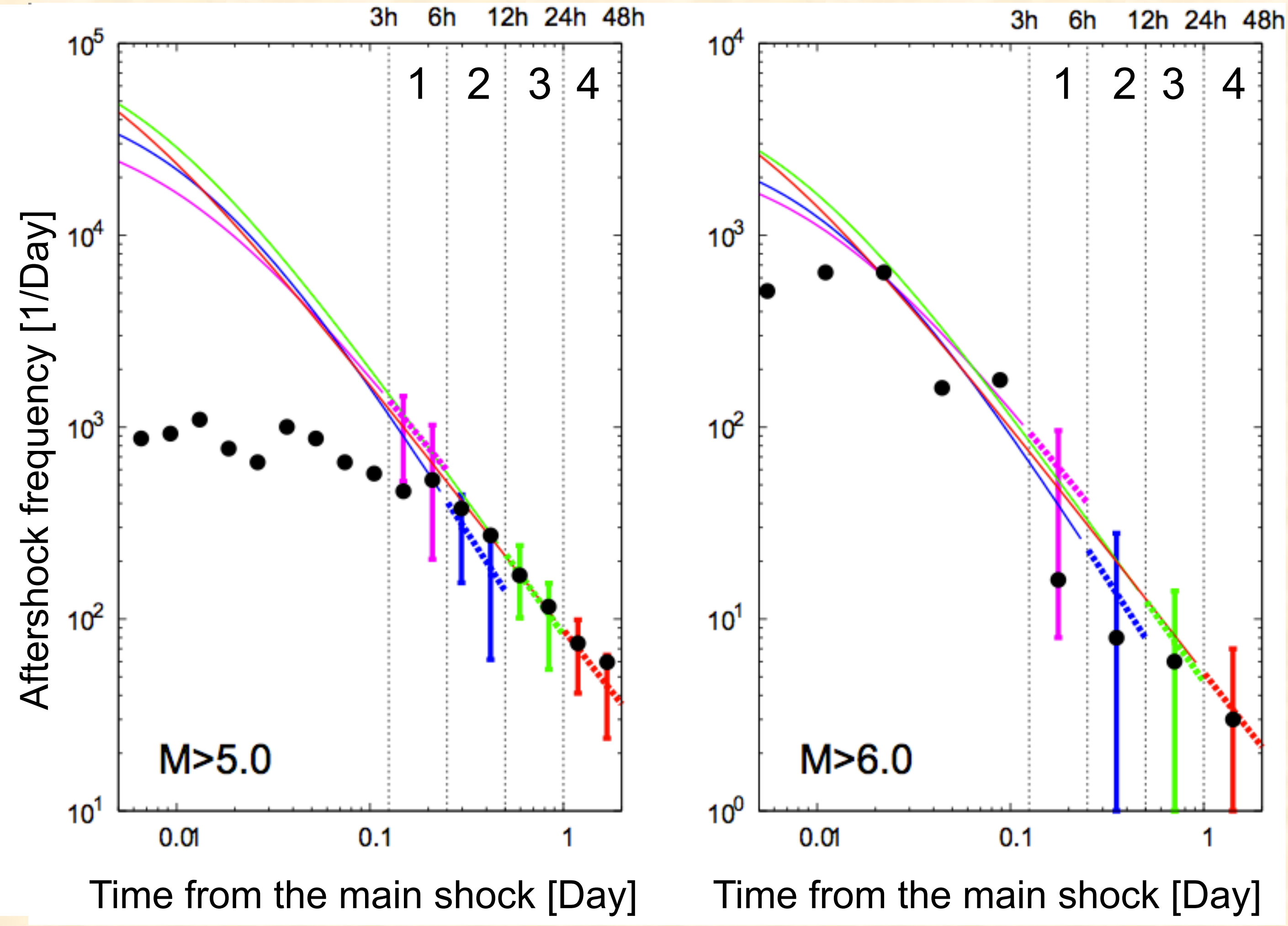
Omi et al., Scientific Reports (2013)



Immediate forecast with the 2011 Tohoku sequence

	1	2	3	4
Learning period	**0 - 3 h**	**0 - 6 h**	**0 - 12 h**	**0 - 24 h**
Forecast period	**3 - 6 h**	**6 - 12 h**	**12 - 24 h**	**24 - 48 h**

Bar: 95% interval



Omi et al.,
Sci. Rep. (2013)

Conclusion

- We proposed a method for estimating the underlying aftershock model from the deficient data of early aftershocks.

- Our method well forecasts the aftershock activity from the deficient data available a few hours after a main shock.

References

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"Estimating the ETAS model from an early aftershock sequence"
Geophysical Research Letters (2014).

T. Omi, Y. Ogata, Y. Hirata & K. Aihara,
"Intermediate-term forecasting of aftershocks from an early aftershock sequence: Bayesian and ensemble forecasting approaches"
Journal of Geophysical Research (Accepted after minor revision).

General Discussion

- Point process models can be applicable to diverse phenomena such as earthquakes, neural spiking activity, human communication, crime occurrence (predictive policing).

- Mathematical methods are useful to solve problems in the society.

Acknowledgement

- Yosihiko Ogata
- Kazuyuki Aihara
- Yoshito Hirata

Thank you for attention !