C. P. T symmetry and operation

define operation U_c, U_p, U_T are the operation operator for C, P, T

Then

- **P operation**

P-invariance : $|\alpha_p\rangle = U_p|\alpha\rangle \Rightarrow \langle\alpha|\beta\rangle = \langle\alpha_p|\beta_p\rangle$

$J_p = \gamma^0$

$J_p \gamma^\mu J_p^{-1} = -(-1)^{\delta_{\mu,0}} \gamma^\mu$

$$U_p \psi(x_0, \vec{x}) U_p^{-1} = \gamma_0 \psi(x_0, -\vec{x})$$

$$U_p A_\mu(x_0, \vec{x}) U_p^{-1} = A^\mu(x_0, -\vec{x})$$

$$U_p |\vec{p}, \vec{s}\rangle = |-\vec{p}, \vec{s}\rangle$$

$U_p^+ = U_p^{-1}$

$x^\mu \xrightarrow{P} x_\mu$

$p^\mu \xrightarrow{P} p_\mu$

$\partial^\mu \xrightarrow{P} \partial_\mu$

$F^{\mu\nu} \xrightarrow{P} F_{\mu\nu}$

$\vec{p} \xrightarrow{U_p} -\vec{p}$

$\vec{s} \xrightarrow{U_p} \vec{s}$ (spin)

- **T operation**

T invariance: $|\alpha_T\rangle = U_T|\alpha\rangle \Rightarrow \langle\alpha|\beta\rangle = \langle\beta_T|\alpha_T\rangle$

$$U_T \psi(x_0, \vec{x}) U_T^{-1} = J^+ \psi(-x_0, \vec{x})$$

$J^+ = J = J^{-1} = i\gamma^1\gamma^3$

$J(\gamma^\mu)^* J^{-1} = \gamma_\mu$

$$U_T A_\mu(x_0, \vec{x}) U_T^{-1} = A^\mu(-x_0, \vec{x})$$

$$U_T |\vec{p}, \vec{s}\rangle = |-\vec{p}, -\vec{s}\rangle$$

$x^\mu \xrightarrow{T} -x_\mu$

$\partial^\mu \xrightarrow{T} -\partial_\mu$

$p^\mu \xrightarrow{T} p_\mu$

$F^{\mu\nu} \xrightarrow{T} -F_{\mu\nu}$

$\vec{p} \xrightarrow{U_T} -\vec{p}$

$\vec{s} \xrightarrow{U_T} -\vec{s}$ (spin)

$$U_T (c\#) U_T^{-1} = (c\#)^*$$

- **C operation**

$U_c^+ = U_c^{-1}$

$J_c = i\gamma^2\gamma^0$

transpose

$J_c^{-1} = J_c^+ = J_c^t = -J_c$

$[J_c \gamma_\mu J_c^{-1} = -(\gamma_\mu)^t]$

$$U_c \bar{\psi}(x_0, \vec{x}) U_c^{-1} = -\psi(x_0, \vec{x}) J_c^{-1}$$

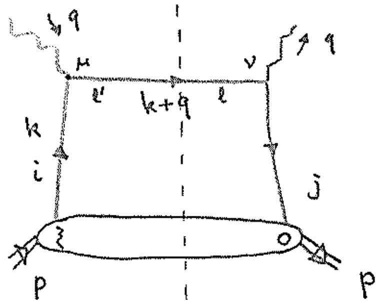
$$U_c \psi(x_0, \vec{x}) U_c^{-1} = J_c \bar{\psi}(x_0, \vec{x})$$

$$U_c A_\mu(x_0, \vec{x}) U_c^{-1} = -A_\mu(x_0, \vec{x})$$

$$U_c |(\vec{p}, \vec{s})^{(-)}\rangle = |(\vec{p}, \vec{s})^{(+)}\rangle$$

[The operation of charge conjugation reverses particle and anti particle states, while leaving spins and momenta unchanged]

How many correlation function / parton distribution function do we need to characterize the structure of a spin-½ proton ?



$$\sim \langle PS|\ \tilde{\bar{\psi}}_j\ \gamma^{\nu}_{jl}\ (\not{k}+\not{q})_{ll'}\ \gamma^{\mu}_{l'i}\ \tilde{\psi}_i\ |PS\rangle$$

$$\sim \boxed{\langle PS|\ \tilde{\bar{\psi}}_j\ \tilde{\psi}_i\ |PS\rangle} * \left[\gamma^{\nu}(\not{k}+\not{q})\gamma^{\mu}\right]_{ji}$$

$$\Downarrow$$

- generic two-quark correlation function to characterize the nucleon structure
- So far in momentum space
- in Coordinate space we have

$$\Phi_{ij}(k, P, S) = \int \frac{d^4\xi}{(2\pi)^4}\ e^{ik\cdot\xi}\ \langle PS|\ \bar{\psi}_j(0)\ \psi_i(\xi)\ |PS\rangle$$

NOTE: In general we also need gauge link to render the above definition gauge invariant, we'll talk about that later

Q: how is Φ_{ij} related to the parton distribution function, as well as many other quantities like transversity, Sivers function, etc ?

To proceed, let's study what requirements do we have from QCD'

- Hermiticity

$$\Phi^{\dagger}(k,P,S) = \gamma^0 \Phi(k,P,S)\gamma^0$$

- Parity

$$\Phi(k,P,S) = \gamma^0 \Phi(\bar{k},\bar{P},-\bar{S})\gamma^0$$

- Time reversal

$$\Phi^{*}(k,P,S) = (-i\gamma^5 C)\,\Phi(\bar{k},\bar{P},\bar{S})\,(-i\gamma^5 C)$$

where $C = i\gamma^2\gamma^0$, $-i\gamma^5 C = i\gamma^1\gamma^3$, and $\bar{k} = (k^0, -\vec{k})$

To show this, we need some background how fields transform under "C, P, T", which you might find some limited information from any standard textbook on Quantum Field Theory; for extended discussion, see

CP violation by Branco, Lavoura, Silva

- Hermiticity

$$(\Phi^\dagger)_{ij} = \Phi^*_{ji} = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{-ik\cdot\xi}\, \langle PS|\, \overline{\psi}_i(0)\, \psi_j(\xi)\,|PS\rangle^*$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{-ik\cdot\xi}\, \langle PS|\, \psi^\dagger_\ell(0)\, \gamma^0_{\ell i}\, \psi_j(\xi)\,|PS\rangle^*$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{-ik\cdot\xi}\, \langle PS|\, \psi^\dagger_j(\xi)\, \gamma^0_{i\ell}\, \psi_\ell(0)\,|PS\rangle$$

$\Downarrow$ Note $\quad \gamma^0_{j'\ell'}\gamma^0_{\ell' j} = \delta_{jj'}$

$$\psi^\dagger_j(\xi) = \psi^\dagger_{j'}(\xi)\,\delta_{jj'}$$

$$= \underbrace{\psi^\dagger_{j'}(\xi)\,\gamma^0_{j'\ell'}}\,\gamma^0_{\ell' j}$$

$$= \overline{\psi}_{\ell'}(\xi)\,\gamma^0_{\ell' j}$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{-ik\cdot\xi}\, \langle PS|\, \overline{\psi}_{\ell'}(\xi)\, \gamma^0_{\ell' j}\,\gamma^0_{i\ell}\, \psi_\ell(0)\,|PS\rangle$$

$\Downarrow$ Change variable $\quad \xi \to -\xi$

$$d^4\xi = d^4(-\xi)$$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ik\cdot\xi}\, \langle PS|\, \overline{\psi}_{\ell'}(-\xi)\, \gamma^0_{\ell' j}\,\gamma^0_{i\ell}\, \psi_\ell(0)\,|PS\rangle$$

translational invariance

$\Downarrow$ $\quad \langle PS|\, \overline{\psi}(-\xi) \cdots \psi(0)\,|PS\rangle = \langle PS|\, \overline{\psi}(0) \cdots \psi(\xi)\,|PS\rangle$

$$= \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ik\cdot\xi}\, \langle PS|\, \overline{\psi}_{\ell'}(0)\, \gamma^0_{\ell' j}\,\gamma^0_{i\ell}\, \psi_\ell(\xi)\,|PS\rangle$$

$$= \gamma^0_{i\ell}\,\Phi_{\ell\ell'}\,\gamma^0_{\ell' j} = [\gamma^0\,\Phi\,\gamma^0]_{ij}$$

$$\Rightarrow \quad \boxed{\Phi^\dagger(k,P,S) = \gamma^0\,\Phi(k,P,S)\,\gamma^0}$$

- Parity

① under parity $\quad p^\mu \to p_\mu$

$$p^\mu = (p^0, \vec{p}) \qquad p_\mu = (p^0, -\vec{p}) \Rightarrow \bar{p}$$

- momentum change
- spin does not change

$$S^\mu = (0, \vec{S}) \longrightarrow S^\mu = (0, \vec{S})$$

use notation $\quad \bar{S} \Rightarrow S_\mu = (0, -\vec{S})$

$$-\bar{S} = (0, \vec{S})$$

$$(K, P, S) \to (\bar{K}, \bar{P}, -\bar{S})$$

$$\Phi_{ij}(K, P, S) = \frac{1}{(2\pi)^4} \int d^4\xi \; e^{iK\cdot\xi} \langle PS | \bar{\psi}_j(0) \; \psi_i(\xi) | PS \rangle$$

$$\Downarrow \quad U_p U_p^{-1} = \mathbb{1}$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{iK\cdot\xi} \langle PS | U_p^{-1} U_p \bar{\psi}_j(0) U_p^{-1} U_p \psi_i(\xi) U_p^{-1} U_p | PS \rangle$$

$$U_p | PS \rangle = | \bar{P}, -\bar{S} \rangle$$

$$\langle PS | U_p^{-1} = \langle \bar{P}, -\bar{S} |$$

$$U_p \psi(\xi) U_p^{-1} = \gamma^0 \psi(\bar{\xi})$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{iK\cdot\xi} \langle \bar{P}, -\bar{S} | \bar{\psi}_l(0) \gamma^0_{lj} \quad \gamma^0_{ii'} \psi_{i'}(\bar{\xi}) | \bar{P}, -\bar{S} \rangle$$

NOTE: $k\cdot\xi = k^0\xi^0 - \vec{k}\cdot\vec{\xi}$ $\quad k = (k^0, \vec{k})$ $\quad \xi = (\xi^0, \vec{\xi})$

$\bar{k}\cdot\bar{\xi} = k^0\xi^0 - \vec{k}\cdot\vec{\xi}$ $\quad \bar{k} = (k^0, -\vec{k})$ $\quad \bar{\xi} = (\xi^0, -\vec{\xi})$

Thus $k\cdot\xi = \bar{k}\cdot\bar{\xi}$

$$\Phi_{ij}(k,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi \; e^{i\bar{k}\cdot\bar{\xi}} \langle \bar{P}, -\bar{S}|\,\bar{\psi}_\ell(0)\,\gamma^0_{\ell j}\;\gamma^0_{ii'}\,\psi_{i'}(\xi)\,|\bar{P}, -\bar{S}\rangle$$

$$\Downarrow \qquad d^4\xi = d^4\bar{\xi}$$

$$= \frac{1}{(2\pi)^4}\int d^4\bar{\xi}\; e^{i\bar{k}\cdot\bar{\xi}} \langle \bar{P}, -\bar{S}|\,\bar{\psi}_\ell(0)\,\gamma^0_{\ell j}\;\gamma^0_{ii'}\,\psi_{i'}(\bar{\xi})\,|\bar{P}, -\bar{S}\rangle$$

$$= \gamma^0_{ii'}\;\Phi_{i'\ell}(\bar{k}, \bar{P}, -\bar{S})\;\gamma^0_{\ell j}$$

$$\boxed{\Phi(k,P,S) = \gamma^0\,\Phi(\bar{k},\bar{P},-\bar{S})\,\gamma^0}$$

• Time reversal

anti-unitary operator

$\langle \alpha | = \langle P, S | \hat{O}^\dagger \quad \Rightarrow | \alpha \rangle = \hat{O} | P, S \rangle$

$| \beta \rangle = | P, S \rangle \quad \Rightarrow \langle \beta | = \langle P, S |$

under time

$P \to \bar{P}$

$S \to \bar{S}$

$\xi \to -\bar{\xi}$

$\hookrightarrow (t, \vec{\xi})$

Then time-reversal invariance indicates

$$\langle \alpha | \beta \rangle = \langle \beta_T | \alpha_T \rangle$$

"state" after T-operation

$$| \beta_T \rangle = | \bar{P}, \bar{S} \rangle \quad \Rightarrow \quad \langle \beta_T | = \langle \bar{P}, \bar{S} |$$

$$| \alpha_T \rangle = U_T \, \hat{O} \, U_T^{-1} \, | \bar{P}, \bar{S} \rangle$$

Thus we have

$$\langle P S | [\bar{\Psi}_j(0) \, \Psi_i(\xi)]^\dagger | P S \rangle = \langle \bar{P} \bar{S} | U_T \, [\bar{\Psi}_j(0) \, \Psi_i(\xi)] \, U_T^{-1} | \bar{P} \bar{S} \rangle$$

NOTE:

$$\Phi^*_{ij}(k, P, S) = \frac{1}{(2\pi)^4} \int d^4\xi \; e^{-ik\cdot\xi} \langle P S | [\bar{\Psi}_j(0) \, \Psi_i(\xi)]^\dagger | P S \rangle$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{-ik\cdot\xi} \langle \bar{P} \bar{S} | U_T [\bar{\Psi}_j(0) \, \Psi_i(\xi)] U_T^{-1} | \bar{P} \bar{S} \rangle$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \; e^{-ik\cdot\xi} \langle \bar{P} \bar{S} | U_T \bar{\Psi}_j(0) U_T^{-1} \, U_T \Psi_i(\xi) U_T^{-1} | \bar{P} \bar{S} \rangle$$

$$U_T \psi(\xi) U_T^{-1} = -i\gamma^5 C \psi(-\bar{\xi})$$

$$(-i\gamma^5 C)^\dagger = -i\gamma^5 C$$

$$\Phi^*_{ij}(K, P, S) = \frac{1}{(2\pi)^4}\int d^4\xi \, e^{-iK\cdot\xi} \langle \bar{P}\bar{S} | \bar{\psi}_\ell(0)(-i\gamma^5 C)_{\ell j} \quad (-i\gamma^5 C)_{ii'} \; \psi_{i'}(-\bar{\xi}) | \bar{P}\bar{S}\rangle$$

$$\Downarrow \qquad K\cdot\xi = \bar{K}\cdot\bar{\xi}$$

$$= \frac{1}{(2\pi)^4}\int d^4(-\bar{\xi}) \, e^{i\bar{K}\cdot(-\bar{\xi})} \langle \bar{P}\bar{S} | \bar{\psi}_\ell(0)(-i\gamma^5 C)_{\ell j} (-i\gamma^5 C)_{ii'} \psi_{i'}(-\bar{\xi}) | \bar{P}$$

$$= \frac{1}{(2\pi)^4} \int d^4\xi \, e^{i\bar{K}\cdot\xi} \langle \bar{P}\bar{S} | \bar{\psi}_\ell(0)(-i\gamma^5 C)_{\ell j} (-i\gamma^5 C)_{ii'} \psi_{i'}(\xi) | \bar{P}\bar{S}\rangle$$

$$= (-i\gamma^5 C)_{ii'} \; \Phi_{i'\ell}(\bar{K}, \bar{P}, \bar{S}) \; (-i\gamma^5 C)_{\ell j}$$

Independent 4X4 matrix basis

$$1$$

$$\gamma^\mu$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

$$\gamma^\mu \gamma^5$$

$$i\gamma^5$$

Now perform expansion

assume proton is moving in +z direction

$$p^\mu = [p^0, 0, 0, p^z]$$

$$v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^z)$$

$$p^\mu = p^+ \bar{n}^\mu$$

$$\bar{n}^\mu = [1^+, 0^-, 0_\perp]$$

$$n^\mu = [0^+, 1^-, 0_\perp]$$

spin of proton

$$S^\mu = \lambda \frac{p^+}{m} \bar{n}^\mu + S_T^\mu$$

helicity

$$\Phi_{ij}(k,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ik\cdot\xi} \langle PS| \bar{\psi}_j(0)\, \psi_i(\xi) |PS\rangle$$

- consider purely collinear case

In other words, integrate over k_T, k^- components and set $k^+ = x p^+$

$$\Phi_{ij}(x) = \int d^2k_T\, dk^- \, \Phi_{ij}(k,P,S)\Big|_{k^+ = xp^+}$$

$$= \int \frac{d\xi^-}{2\pi}\, e^{ik\cdot\xi} \langle PS| \bar{\psi}_j(0)\, \psi_i(\xi) |PS\rangle_{\xi^+ = \xi_T = 0}$$

In other words $\Phi_{ij}(x)$ should depend on $\not{P}$ only (as well as Spin "S" vector) since $k \approx xP$

- What about TMD — k_T-dependent parton distribution

$$\Phi_{ij}(x, k_T) = \int dk^- \, \Phi_{ij}(k,P,S)\Big|_{k^+ = xp^+}$$

$$= \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_T}{(2\pi)^2}\, e^{ik\cdot\xi} \langle PS| \bar{\psi}_j(0)\psi_i(\xi) |PS\rangle_{\xi^+ = 0}$$

famous mistake — Sivers function vanishes ?!

$$f_{q/p\uparrow}(x, k_T, S_T) = f_{q/p}(x, k_T^2) + \vec{S}_T \cdot (\vec{k}_T \times \hat{p}) \frac{1}{M} f_{1T}^{\perp}(x, k_T)$$

$$\to \int \frac{d\xi^-}{2\pi} \frac{d^2\xi_T}{(2\pi)^2} e^{ik\cdot\xi} \langle PS | \bar{\psi}(0) \frac{\gamma^+}{2} \psi(\xi) | PS \rangle_{\xi^+=0}$$

$$f_{1T}^{\perp}(x, k_T) \quad \propto \quad \boxed{f_{q/p\uparrow}(x, k_T, S_T) - f_{q/p\uparrow}(x, k_T, -S_T)}$$

Apply both P and T invariance, see what happens

you'll find

$$f_{q/p\uparrow}(x, k_T, S_T) = f_{q/p\uparrow}(x, k_T, -S_T)$$

$\Rightarrow$ Vanish ?!

$\Rightarrow$ gauge link !

$\langle \alpha | = \langle \vec{p}, \vec{s} | \hat{O}$

$|\beta\rangle = |\vec{p}, \vec{s}\rangle$

T-invariance $\Rightarrow \quad \langle \alpha | \beta \rangle = \langle \beta_T | \alpha_T \rangle$

$$\langle \vec{p}, \vec{s} | \hat{O} | \vec{p}, \vec{s} \rangle = \langle -\vec{p}, -\vec{s} | U_T \, \hat{O}^+ \, U_T^{-1} | -\vec{p}, -\vec{s} \rangle$$

$$= \langle -\vec{p}, -\vec{s} | U_P^{-1} U_P \, U_T \, \hat{O}^+ \, U_T^{-1} \, U_P^{-1} U_P \, | -\vec{p}, -\vec{s} \rangle$$

$$= \langle \vec{p}, -\vec{s} | \, U_P \, U_T \, \hat{O}^+ \, U_T^{-1} \, U_P^{-1} \, | \vec{p}, -\vec{s} \rangle$$

$\hat{O} = \bar{\Psi}(0) \, \Gamma \, \psi(\xi) \qquad$ with $\Gamma = \frac{\gamma^+}{2}$

$\hat{O}^+ = \psi^+(\xi) \, \Gamma^+ \, \gamma^0 \, \psi(0)$

$$U_P \, U_T \, \hat{O}^+ \, U_T^{-1} \, U_P^{-1} = U_P \, U_T (\psi^+(\xi)) U_T^{-1} U_P^{-1} \; \Gamma^+ \gamma^0 \, U_P U_T \psi(0) U_T^{-1} U_P^{-1}$$

$$= U_P \, \psi^+(-\xi^0, \vec{\xi}) J U_P^{-1} \; \Gamma^+ \gamma^0 \; U_P \, J \, \psi(0) \, U_P^{-1}$$

$$= \psi^+(-\xi^0, -\vec{\xi}) \, \gamma^0 J \; \Gamma^+ \gamma^0 \; J \gamma^0 \psi(0)$$

$$= \bar{\Psi}(-\xi) \; J \; \Gamma^+ \; \gamma^0 J \; \gamma^0 \; \psi(0)$$

$$\Downarrow \quad \begin{aligned} J &= -i \gamma^5 C = i \gamma^1 \gamma^3 \\ \gamma^0 J &= J \gamma^0 \end{aligned}$$

$$= \bar{\Psi}(-\xi) \; J \, \Gamma^+ J \; \psi(0)$$

$$= \bar{\Psi}(-\xi) \; \Gamma^+ \; \psi(0)$$

Thus with translational invariance we'll have

$$f_{q/p\uparrow}(x, k_T, \vec{S}_T) = f_{q/p\uparrow}(x, k_T, -\vec{S}_T)$$

$\Rightarrow$ Sivers function vanish ?!

Not Really

- Start with a case when spin "S" is not observed (or spin-0 particle like pions)

Simpler

$$\Gamma = \{1,\ \gamma^\mu,\ \gamma^\mu\gamma^5,\ \sigma^{\mu\nu},\ i\gamma^5,\ \sigma^{\mu\nu} i\gamma^5\}$$

show $\gamma^0\gamma^\mu\gamma^0 = \gamma_\mu$

$$\left.\begin{aligned} \mu = 0:&\ \gamma^0\gamma^0\gamma^0 = \gamma^0 \\ \mu \neq 0:&\ \gamma^0\gamma^i\gamma^0 = \gamma^0(-\gamma^0\gamma^i) = -\gamma^i \end{aligned}\right\} \quad (\gamma^0, -\gamma^i) = \gamma_\mu$$

$$\gamma^0\{1,\ \gamma^\mu,\ \gamma^\mu\gamma^5,\ \sigma^{\mu\nu},\ i\gamma^5,\ \sigma^{\mu\nu} i\gamma^5\}\gamma^0$$

$$= \{1,\ \gamma_\mu,\ -\gamma_\mu\gamma^5,\ \sigma_{\mu\nu},\ -i\gamma^5,\ -\sigma_{\mu\nu} i\gamma^5\}$$

$$\boxed{\begin{aligned} \Phi^\dagger(P,k) &= \gamma^0\,\Phi(P,k)\,\gamma^0 \\ \Phi(P,k) &= \gamma^0\,\Phi(\bar{P},\bar{k})\,\gamma^0 \\ \Phi^*(P,k) &= J\,\Phi(\bar{P},\bar{k})\,J \qquad \text{where } J = -i\gamma^5 C = i\gamma^1\gamma^3 \end{aligned}}$$

$$k^\mu = x P^\mu + k_T^\mu$$

Two possible momenta : P, K

$\mathbb{1}$: A_1

γ^μ : $\{P_\mu, K_\nu\}$ $\Rightarrow$ A_2, A_3

$\gamma^\mu\gamma^5$: $\{P_\mu, K_\nu\}$ $\Rightarrow$ A_5, A_6

$\sigma^{\mu\nu}$: $K_\mu P_\nu$ $\Rightarrow$ A_4

$i\gamma^5$: A_7

$\sigma^{\mu\nu} i\gamma^5$: $K_\mu P_\nu$ $\Rightarrow$ A_8

$$\Phi(P,K) = \left[M A_1 + A_2 \not{P} + A_3 \not{K} + A_4 \frac{\sigma^{\mu\nu} K_\mu P_\nu}{M} \right]$$

$$+ \left[A_5 \not{P}\gamma^5 + A_6 \not{K}\gamma^5 + M A_7 (i\gamma^5) + A_8 \frac{\sigma^{\mu\nu} i\gamma^5 K_\mu P_\nu}{M} \right]$$

- Hermiticity $\Rightarrow$ All "A" should be real

e.g. $A_5 \not{P}\gamma^5$

$$(A_5 \not{P}\gamma^5)^\dagger = \gamma^0 (A_5 \not{P}\gamma^5)\gamma^0$$

Note $(\gamma^0)^2 = 1$, $\gamma^0 (\gamma^\mu)^\dagger \gamma^0 = \gamma^\mu$

$\Rightarrow$ LHS $= A_5^* (\gamma^\mu\gamma^5)^\dagger P_\mu = A_5^* \gamma^5 (\gamma^\mu)^\dagger P_\mu = A_5^* \gamma^5 \gamma^0\gamma^0 (\gamma^\mu)^\dagger \gamma^0\gamma^0 P_\mu$

$= A_5^* \gamma^5 \gamma^0 \gamma^\mu \gamma^0 P_\mu$

$= A_5^* (-\gamma^0 \gamma^5 \gamma^\mu \gamma^0) P_\mu$

$= A_5^* (\gamma^0 \gamma^\mu \gamma^5 \gamma^0) P_\mu$

used $\{\gamma^\mu, \gamma^5\} = 0$

RHS $= A_5 P_\mu \gamma^0 (\gamma^\mu\gamma^5)\gamma^0$

$\Rightarrow$ $\boxed{A_5^* = A_5}$

• parity $\Rightarrow$ $A_5 = A_6 = A_7 = A_8 = 0$

$$\Phi(P, K) = \gamma^0 \, \Phi(\bar{P}, \bar{K}) \, \gamma^0$$

e.g. $A_8 \dfrac{\sigma^{\mu\nu} i\gamma^5 K_\mu P_\nu}{M}$

$$\text{LHS} = A_8 \frac{\sigma^{\mu\nu} i\gamma^5 K_\mu P_\nu}{M}$$

$$\text{RHS} = \gamma^0 \left[A_8 \frac{\sigma^{\mu\nu} i\gamma^5 \bar{K}_\mu \bar{P}_\nu}{M} \right] \gamma^0$$

NOTE: $\gamma^0 (\sigma^{\mu\nu} i\gamma^5) \gamma^0 = -\sigma_{\mu\nu} i\gamma^5$

$\bar{P}_\mu = P^\mu$

$\bar{K}_\mu = K^\mu$

$$= A_8 \left[\frac{-\sigma_{\mu\nu} i\gamma^5 \; K^\mu P^\nu}{M} \right]$$

$$= -A_8 \frac{\sigma_{\mu\nu} i\gamma^5 K^\mu P^\nu}{M}$$

$$= -A_8 \frac{\sigma^{\mu\nu} i\gamma^5 K_\mu P_\nu}{M}$$

LHS = RHS $\Rightarrow$ $A_8 = -A_8$

$\Rightarrow$ $\boxed{A_8 = 0}$

$$p^\mu = p^+ \bar{n}^\mu \qquad \bar{n}^\mu = [1^+, 0^-, 0_\perp]$$

$$k^\mu = x p^\mu + k_T^\mu$$

$$\Phi(P, k) = M A_1 + A_2 \not{P} + A_3 (x \not{P} + \not{k}_T) + A_4 \frac{\sigma^{\mu\nu}(xP_\mu + k_{T\mu}) P_\nu}{M}$$

$\Downarrow \quad \sigma^{\mu\nu} P_\mu P_\nu = 0$

$$= M A_1 + A_3 \not{k}_T$$

$$+ (A_2 + A_3 x) \not{P}$$

$$+ A_4 \frac{\sigma^{\mu\nu} k_{T\mu} P_\nu}{M}$$

$\Downarrow$ NOTE $P \gg k_T \sim M$ Thus we can drop terms like $M A_1$, $A_3 \not{k}_T$

keep only the largest contribution (twist analysis)

$$= (A_2 + A_3 x) \not{P}$$

$$+ A_4 \frac{\sigma^{\mu\nu} k_{T\mu} P_\nu}{M}$$

$\Downarrow$ give them better names

$$= \frac{1}{2} \left[f_1(x, k_T^2) \not{P} + h_1^\perp(x, k_T^2) \frac{\sigma^{\mu\nu} k_{T\mu} P_\nu}{M} \right]$$

unpolarized PDFs

Boer - Mulders function

transversely polarized quark inside unpolarized nucleon/pion