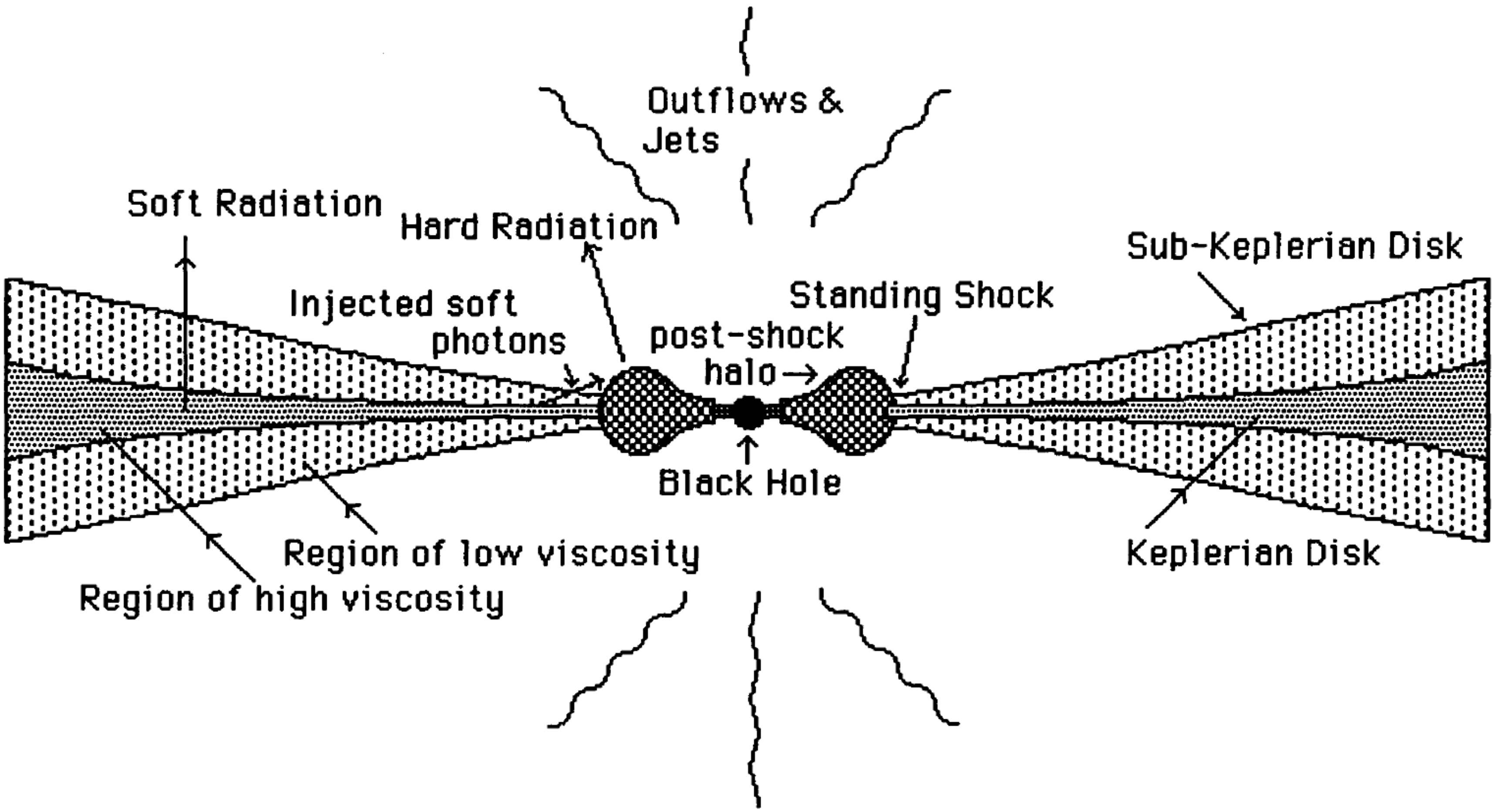
Accretion Flow Properties of three MAXI Black Hole Candidates: Analysis with TCAF Solution

Dipak Debnath

Indian Centre for Space Physics, Kolkata, India

Collaborators: Prof. Sandip K. Chakrabarti, Dr. Sanatnu Mondal, Md. Aslam Ali Molla, Mr. Arghajit Jana, Mr. Debjit Chatterjee, Mr. Shreeram Nagarkoti, Mr. Ayan Bhattacharjee, Mr. Anuvab Banerjee, Dr. Indrani Banerjee, Ms. Bandy Shang, Prof. H. K. Chang

Two Component Advective Flow (TCAF) Model



A Schematic diagram of the accretion flow around the black hole.

Chakrabarti & Titarchuk 1995

Generating TCAF model fits file :

Took Chakrabarti & Titarchuk 1995 (CT95) original code as a basic program to generate TCAF model fits file to include as a local addtitive table model for fitting black hole spectra.

To fit spectra from all possible spectral states, several modifications are made in original CT95 code

i)Variation of compression ratio R is allowed from 4 (strong) to 1 (weak). CT95 assumed only strong shocks for illustration purpose.

ii) Computation of temperature of post-shock region using this R.

iii) Radial velocity of a rotating flow as in Chakrabarti (1997).

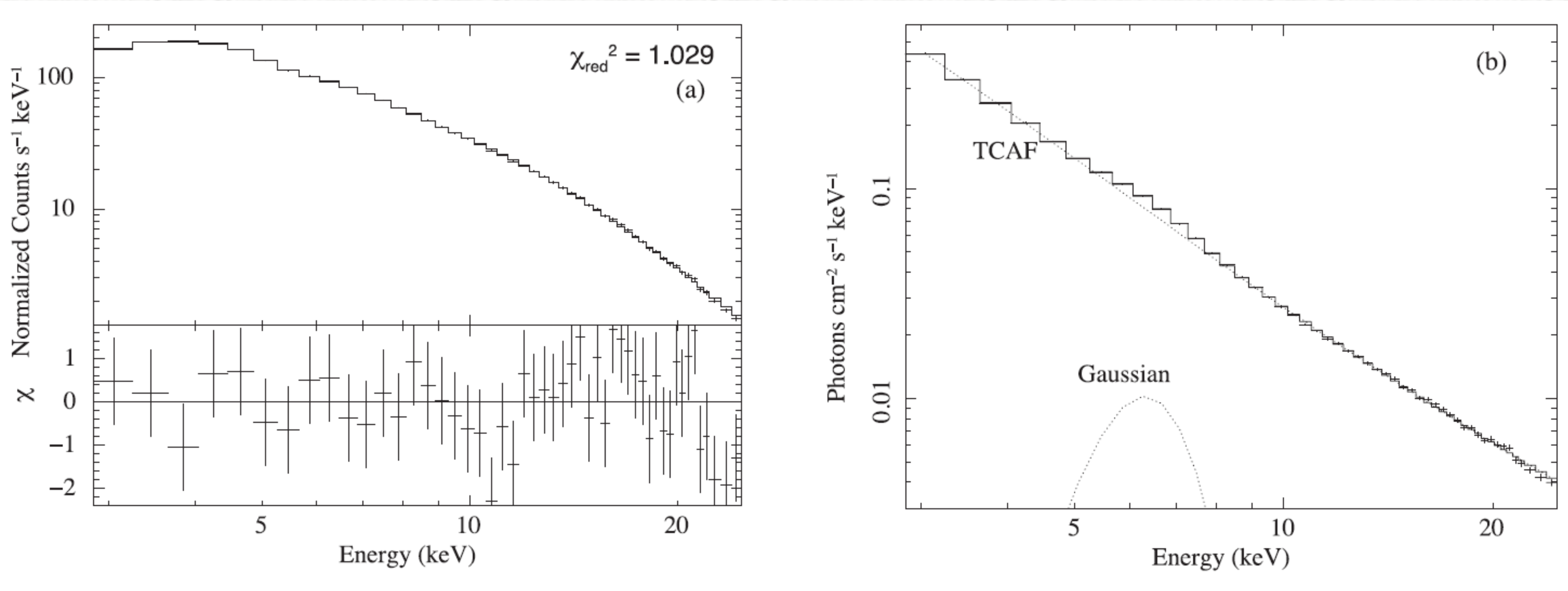
iv) Spectral hardening correction of Shimura & Takahara (1995), which depends on the accretion flow rate. We uniformly consider the correction factor (f) to be 1.8 to calculate effective temperature in emitted spectrum.

➢ **"TCAF_V0.3.fits"** file was created by using around 1 million (~10^6) model spectra. The spectra were generated by varying five model input parameters.

➢ For spectral with current TCAF model fits file, one needs to supply 6 initial input parameters:

i) M_{BH} (black hole mass in solar mass unit),
ii) $\dot{m}_d$ (Keplerian or disk rate),
iii) $\dot{m}_h$ (sub-Keplerain or halo rate),
iv) X_s (shock location in Schwarzschild radii r_g,
v) R (shock compression ratio,
vi) Normalization (which is fraction of $(r_g^2/4\pi D^2)\cos(i)$, where r_g ($=2GM_{BH}/c^2$) is the Schwarzschild radius, '*D*' is source distance in 10 kpc unit and '*i*' is disk inclination angle with line of sight).

GX 339-4 spectra fitted with TCAF Model:



(a) TCAF model fitted 2.5–25 keV PCA spectrum of GX 339-4 (observation ID = 95409-01-14-04; MJD = 553 00, UT Date = 14/04/2010) with variation of $\Delta\chi$ is shown. (b) The unfolded model components of the spectral fit are show.

Debnath, Chakrabarti & Mondal, 2014, MNRAS, 440, L121

Introduction:

MAXI J1659-152:

- MAXI/GSC discovered on 2010 September 25 (Negoro et al. 2010)
- Shortest orbital period BH binary of the period = 2.414 ± 0.005 hr (Kuulkers 2010, 2013; Kennea 2011)
- Distance and disk inclination angle are 5.3– 8.6 kpc and 60° – 80° (Yamaoka 2012; Kuulkers et al. 2013)
- Mass the source = 3-8 $M_\odot$ (Yamaoka 2012) and the companion = 0.15-0.25 $M_\odot$ (Kuulkers et al. 2013)

MAXI J1836-194:

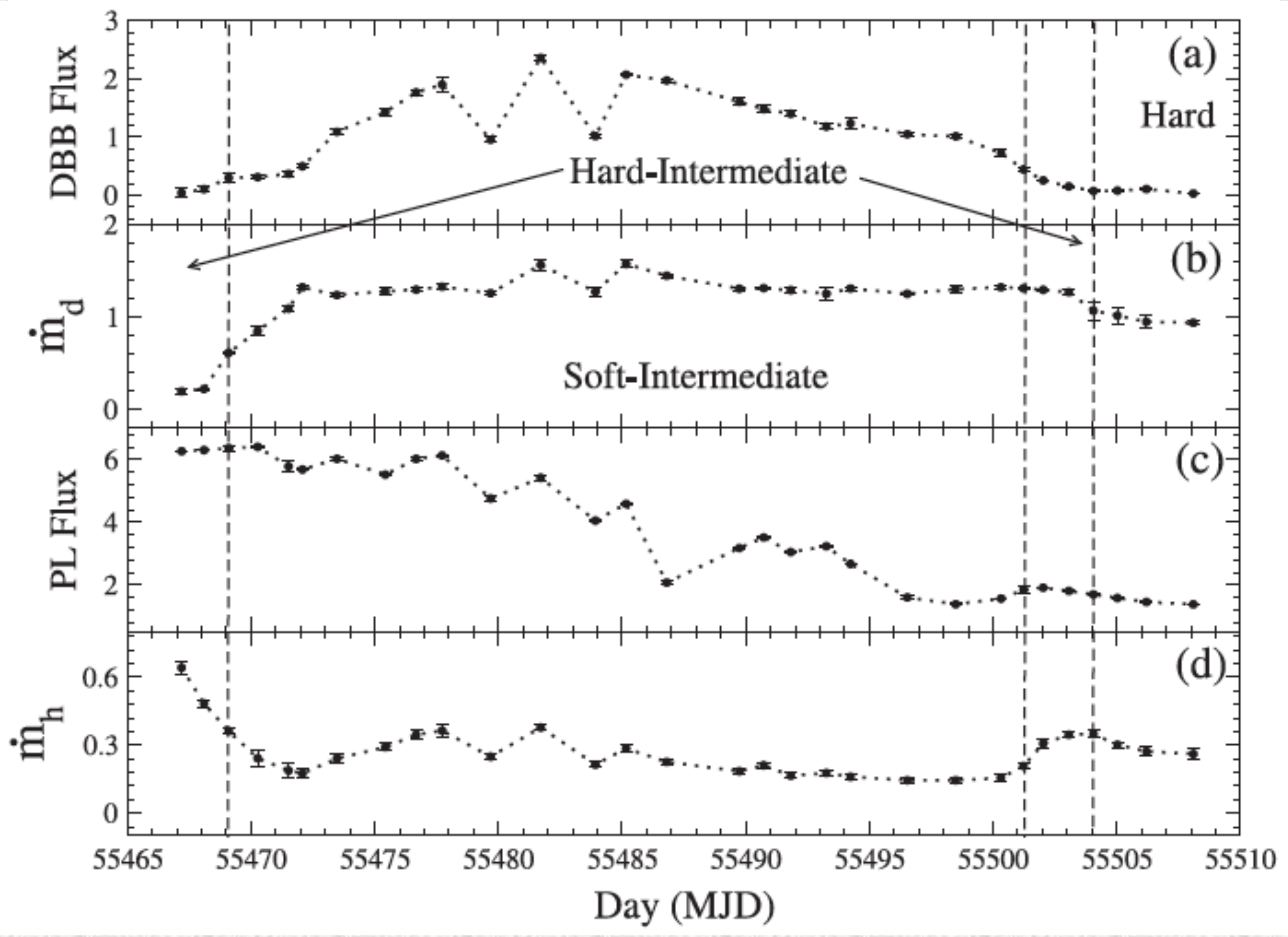
- MAXI/GSC discovered 2011 August 29 (Negoro et al. 2011)
- Short orbital period of < 4.9 hr, low disk inclination angle of 4°– 15° and highly spinning (a=0.88 ± 0.03) object (Russell et al. 2014)
- Companion is assumed to be a high massive B[e] star (Cenko et al. 2011)
- Mass and the distance of the source to be 4– 12$M_\odot$ and 4– 10 kpc, respectively (Russell et al. 2014)
- Evolving radio jets were observed by Russell et al. (2013)

MAXI J1543-564:

- MAXI/GSC discovered 2011 May 8(Negoro et al. 2011)
- The nature of the companion (or companions) is not confirmed
- Distance of the source to be 8.5 kpc (Stiele et al. 2012)
- We first estimated mass of this object.

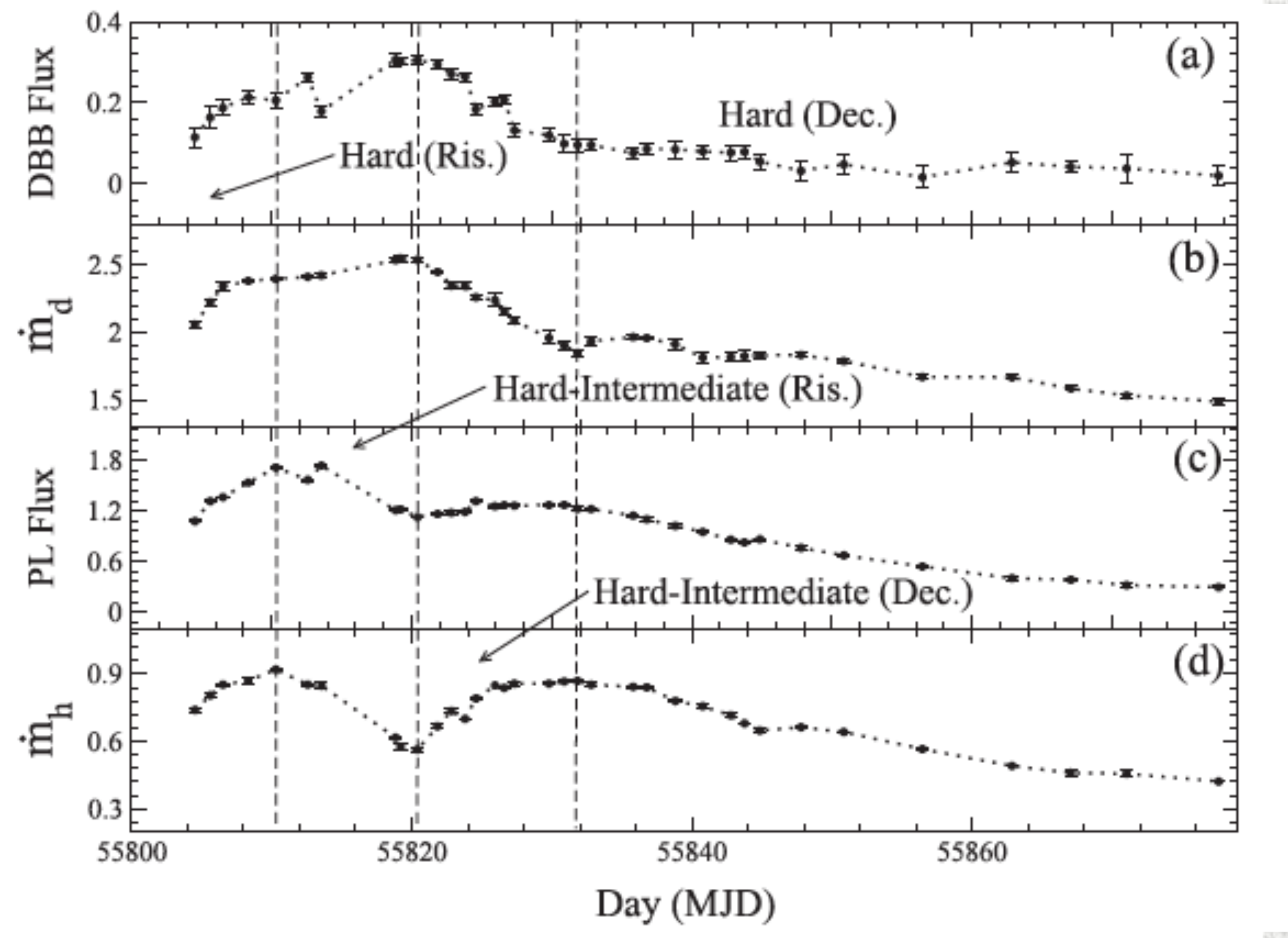
Characterization of BHCs during their X-ray outbursts with TCAF Solution

2010 Outburst of MAXI J1659-162 with TCAF Model:



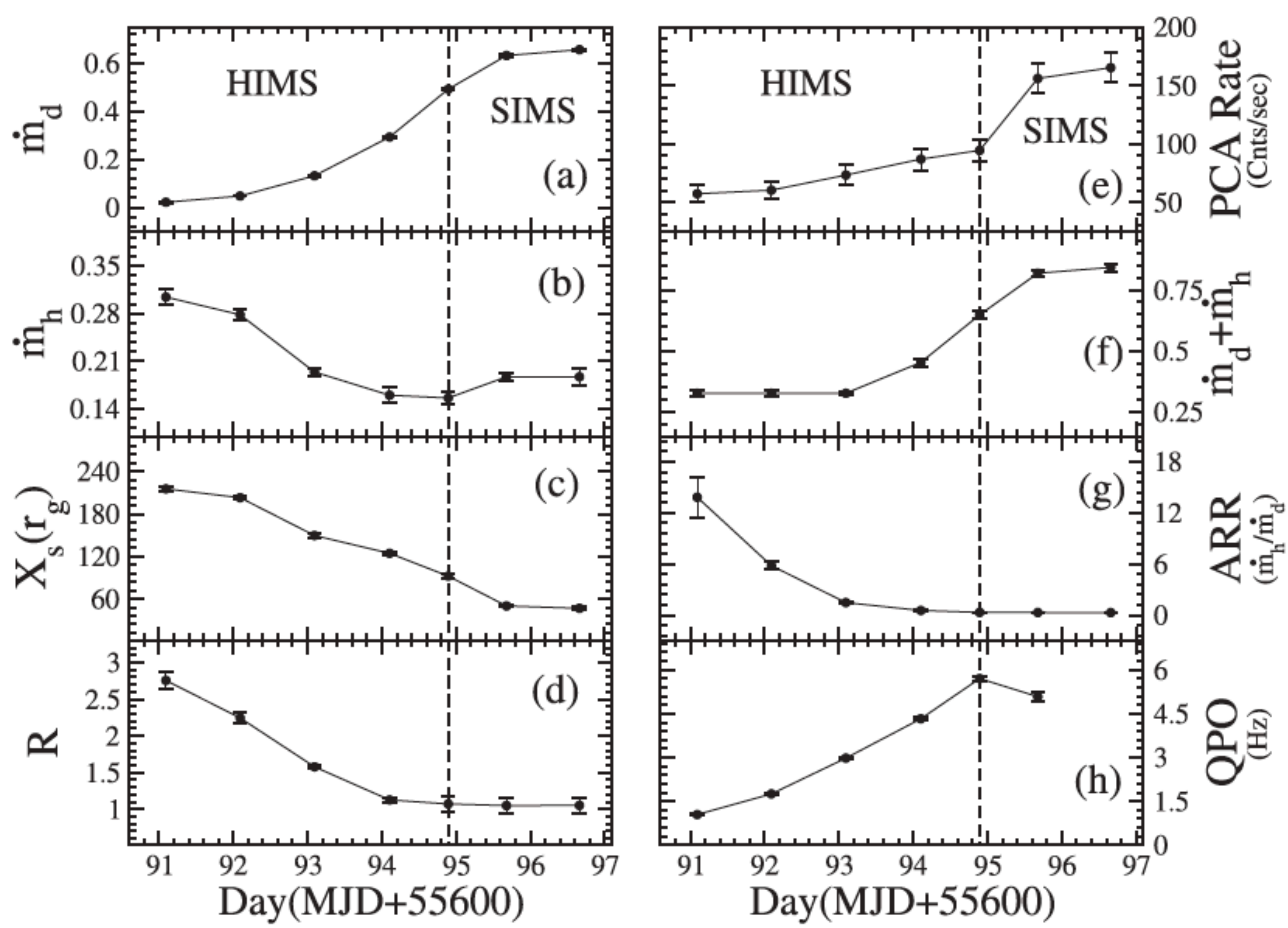
Debnath, Molla, Chakrabarti & Mondal, 2015, ApJ, 803, 59

2011 Outburst of MAXI J1836-194 with TCAF Model:



Jana, Debnath, Chakrabarti, et al., 2016, ApJ, 819, 107

2011 Outburst of MAXI J1543-564 with TCAF Model:



Chatterjee, Debnath, Chakrabarti, et al., 2016, ApJ, 827, 88

Accretion Rate Intensity Diagram (ARRID) : A correlation to spectro-temporal properties

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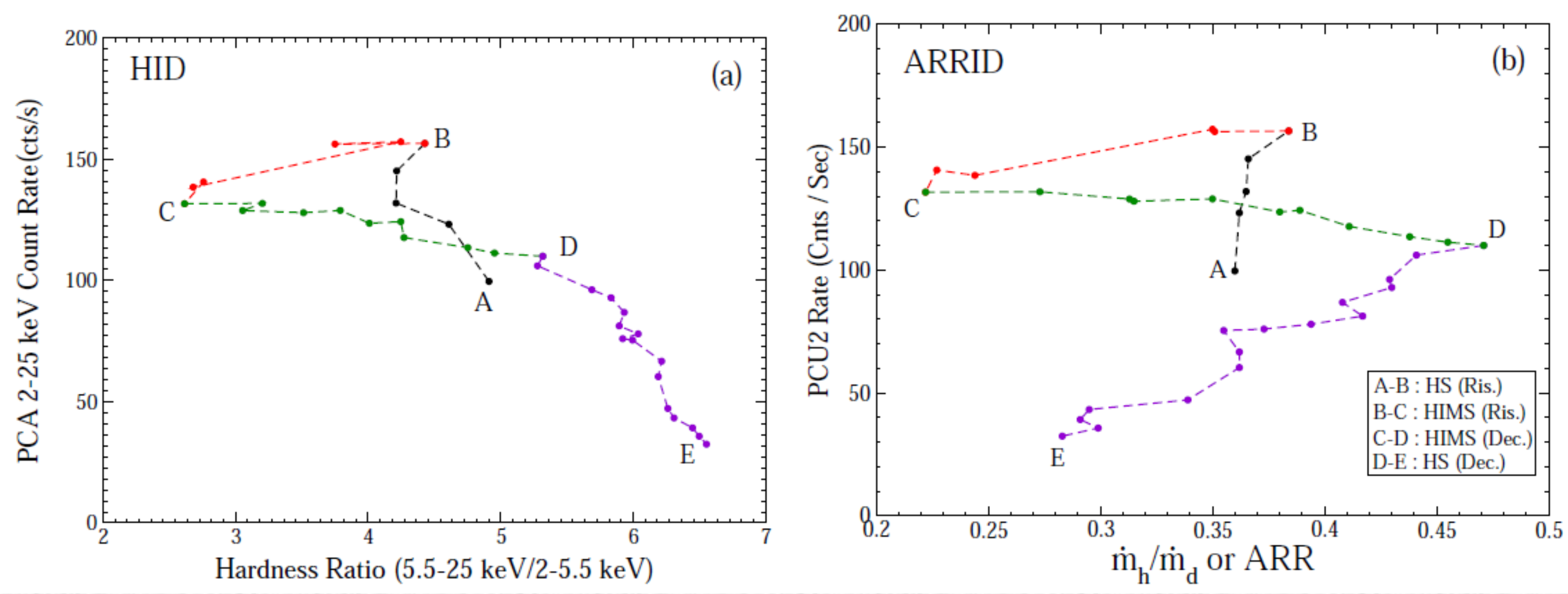
ACCRETION FLOW DYNAMICS OF MAXI J1836-194 DURING ITS 2011 OUTBURST FROM TCAF SOLUTION

ARGHAJIT JANA[1], DIPAK DEBNATH[1], SANDIP K. CHAKRABARTI[1,2], SANTANU MONDAL[1], AND ASLAM ALI MOLLA[1]
[1] Indian Center for Space Physics, 43 Chalantika, Garia St. Rd., Kolkata, 700084, India;
argha@csp.res.in, dipak@csp.res.in, chakraba@bose.res.in, santanu@csp.res.in, aslam@csp.res.in
[2] S. N. Bose National Centre for Basic Sciences, Salt Lake, Kolkata, 700098, India
Received 2015 August 30; accepted 2016 January 8; published 2016 March 3

ARRID for 2011 outburst of MAXI J1836-194:



Jana, Debnath, Chakrabarti et al., 2016, ApJ, 819, 107

Prediction of dominating QPO frequency : Using TCAF model fitted shock parameters

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

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Advance Access publication 2014 March 12

Implementation of two-component advective flow solution in XSPEC

Dipak Debnath,[1★] Sandip K. Chakrabarti[1,2] and Santanu Mondal[1]

[1] *Indian Centre For Space Physics, 43 Chalantika, Garia Station Road, Kolkata 700084, India*
[2] *S. N. Bose National Center for Basic Sciences, JD-Block, Salt Lake, Kolkata 700098, India*

Accepted 2014 February 3. Received 2014 January 24; in original form 2013 August 22

Prediction of dominating QPO frequency from TCAF model fitted shock parameters:

From TCAF model fit, location (r_s) and compression ratio (R) of the sock wave can be extracted.

$\boldsymbol{\nu_{QPO} = C/ [R\ r_s(r_s-1)^{1/2}]}$, where C is a constant = $M_{BH} \times 10^{-5}$.

Source	Obs. Id	$\dot{m}_d$ ($\dot{M}_{Edd}$)	$\dot{m}_h$ ($\dot{M}_{Edd}$)	X_s (r_g)	R	χ^2/DOF	ν^*_{QPO} (Obs.)	ν^*_{QPO} (Predic.)
H 1743−322	X-02-01	0.516 ±0.013	0.189 ±0.081	320.0 ±20.04	1.250 ±0.012	60.1/42	1.045 ±0.007	1.228 ±0.293
GX 339−4	Y-14-04	6.883 ±0.003	6.087 ±0.349	147.9 ±1.13	4.000 ±0.075	43.2/42	2.374 ±0.006	2.356 ±0.265
GRO J1655−40	Z-01-00	6.987 ±0.273	1.733 ±0.232	153.8 ±13.36	3.449 ±0.433	64.78/41	2.313 ±0.010	2.172 ±0.529

Notes. Here, X = 95360-14, Y = 95409-01 and Z = 90704-04. DOF means degrees of freedom.* Only frequency of the primary dominating QPOs (in Hz) in mentioned.

Debnath, Chakrabarti & Mondal, 2014, MNRAS, 440, L121

Prediction of dominating QPO frequency of MAXI J1543-564 (2011 outburst) using TCAF & POS models:

Table 2: QPO evolution in initial rising phase: Fitted with POS Model

Obs.	Id.	MJD	ν_{Obs} (Hz)	ν_{POS} (Hz)	X_s (r_g)	V (cm/s)	R	ν_{TCAF} (Hz)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	X-01-00	55691.09	$1.05^{\pm0.02}$	1.04	210.0	2450.0	2.44	$0.85^{\pm0.06}$
2	X-01-01	55692.09	$1.75^{\pm0.02}$	1.65	163.4	2105.7	2.24	$1.12^{\pm0.05}$
3	X-01-02	55693.09	$2.98^{\pm0.02}$	2.82	132.0	1760.1	1.81	$2.49^{\pm0.14}$
4	X-02-00	55694.10	$4.38^{\pm0.05}$	4.56	115.9	1411.5	1.36	$4.67^{\pm0.27}$
5	X-02-01	55694.89	$5.70^{\pm0.09}$	5.89	114.1	1139.0	1.08	$7.68^{\pm1.11}$
6*	X-02-02	55695.67	$5.08^{\pm0.17}$	—-	—-	—-	—-	—-
7	X-02-03	55696.68	——	—-	—-	—-	—-	—-

Here 'X'=96371-02 signifies the initial part of an observation Id.

Chatterjee, Debnath, Chakrabarti, Mondal & Jana, 2016, ApJ, 827, 88

Mass Estimation : Using TCAF & POS Models

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Advance Access publication 2016 April 15

Estimation of the mass of the black hole candidate MAXI J1659−152 using TCAF and POS models

Aslam Ali Molla,[1] Dipak Debnath,[1★] Sandip K. Chakrabarti,[1,2] S. Mondal[1] and A. Jana[1]

[1] *Indian Centre For Space Physics, 43 Chalantika, Garia Station Road, Kolkata 700084, India*
[2] *S. N. Bose National Center for Basic Sciences, JD-Block, Salt Lake, Kolkata 700098, India*

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QPO Freq. - Photon Index Correlation Method

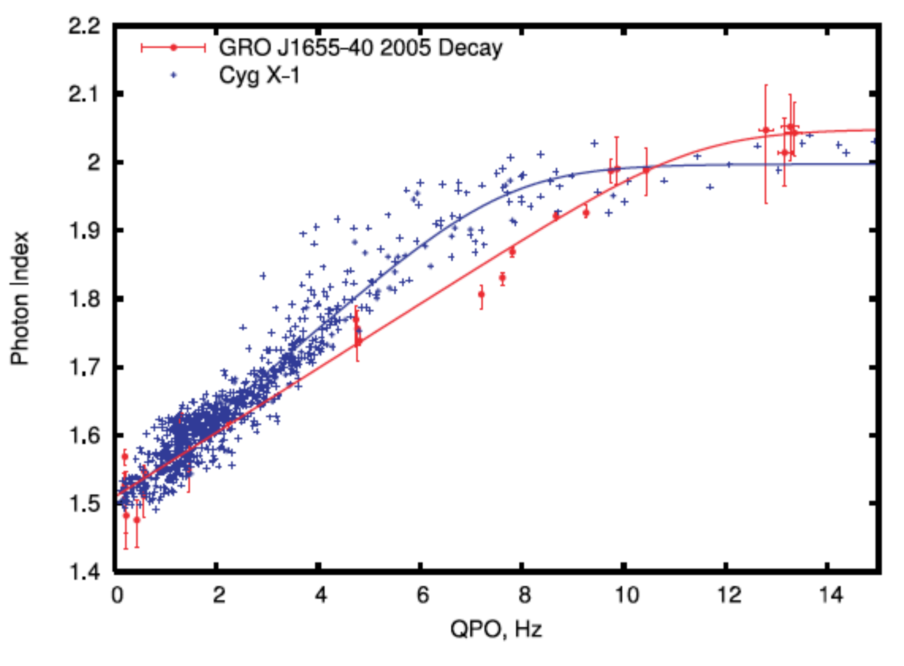
$$f(\nu) = A - DB \ln\left[\exp\left(\frac{\nu_{tr} - \nu}{D}\right) + 1\right]. \qquad (1)$$

where

$f(\nu) = A + B(\nu - \nu_{tr})$ for $\nu < \nu_{tr}$

$f(\nu) = A$ for $\nu > \nu_{tr}$

Here 'A' is a value of the index saturation level, 'B' is a slope of the low-frequency part of the data, and 'ν_{tr}'□ is the frequency at which index-QPO dependence levels off. The parameter 'D' controls how fast the transition occurs.

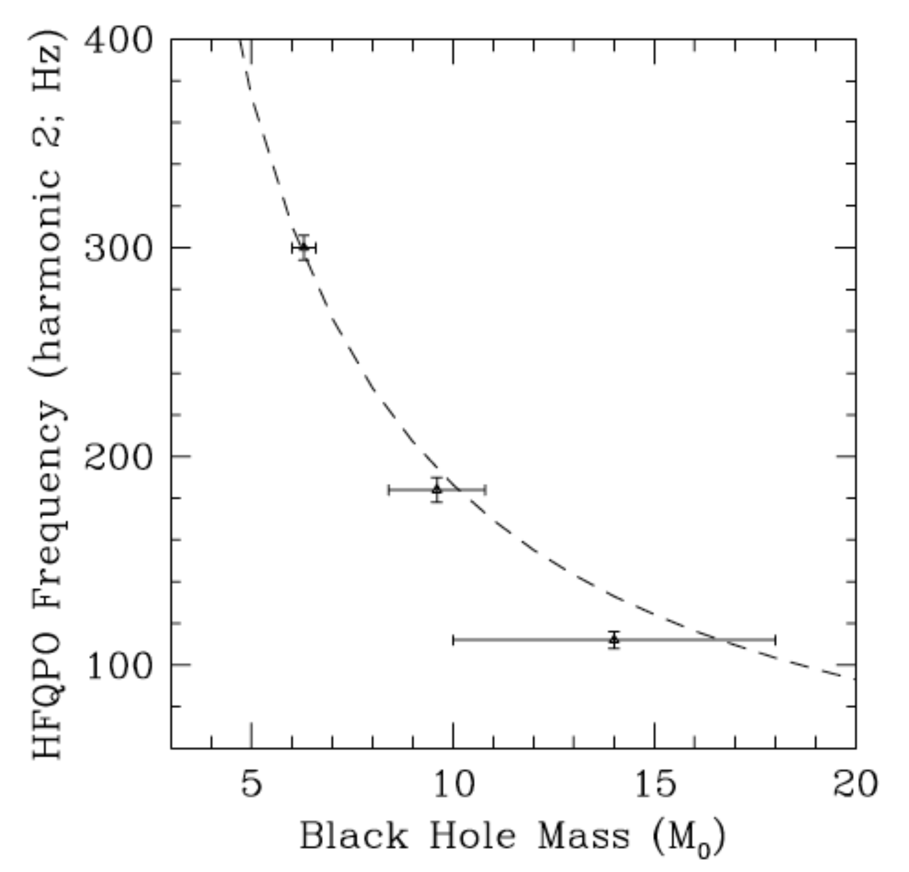


$$M_{\mathrm{Cyg\,X\text{-}1}} = B_{\mathrm{Cyg\,X\text{-}1}} \frac{M_{J1655}}{B_{J1655}} = 8.7 \pm 0.8\ M_{\odot}.$$

By considering reference BH GRO J1655-40 mass = 6.3 $M_{\odot}$

Shaposnikov & Titarchuk, 2007, ApJ, 663, 445

High Freq. QPO Freq. - BH Mass Correlation Method



High Frequency QPOs for different black holes are found to be correlated with BH mass in M^{-1}.

Relationship between HFQPO frequency and BH mass for XTE J1550–564, GRO J1655–40, and GRS 1915+105.. The dashed line shows a relation, ν_0 (Hz) = 931 $(M/M_\odot)^{-1}$.

McClintock & Remillard, 2006

In Molla, Debnath, Chakrabarti et al. (2016), we introduce two independent methods to determine the mass (M_{BH}) of the BH sources.

i) Keeping TCAF fitted normalization parameter in a narrow range

ii) Studying evolution of the Quasi-Periodic Oscillation frequency with time, fitted with the propagating oscillatory shock (POS) model

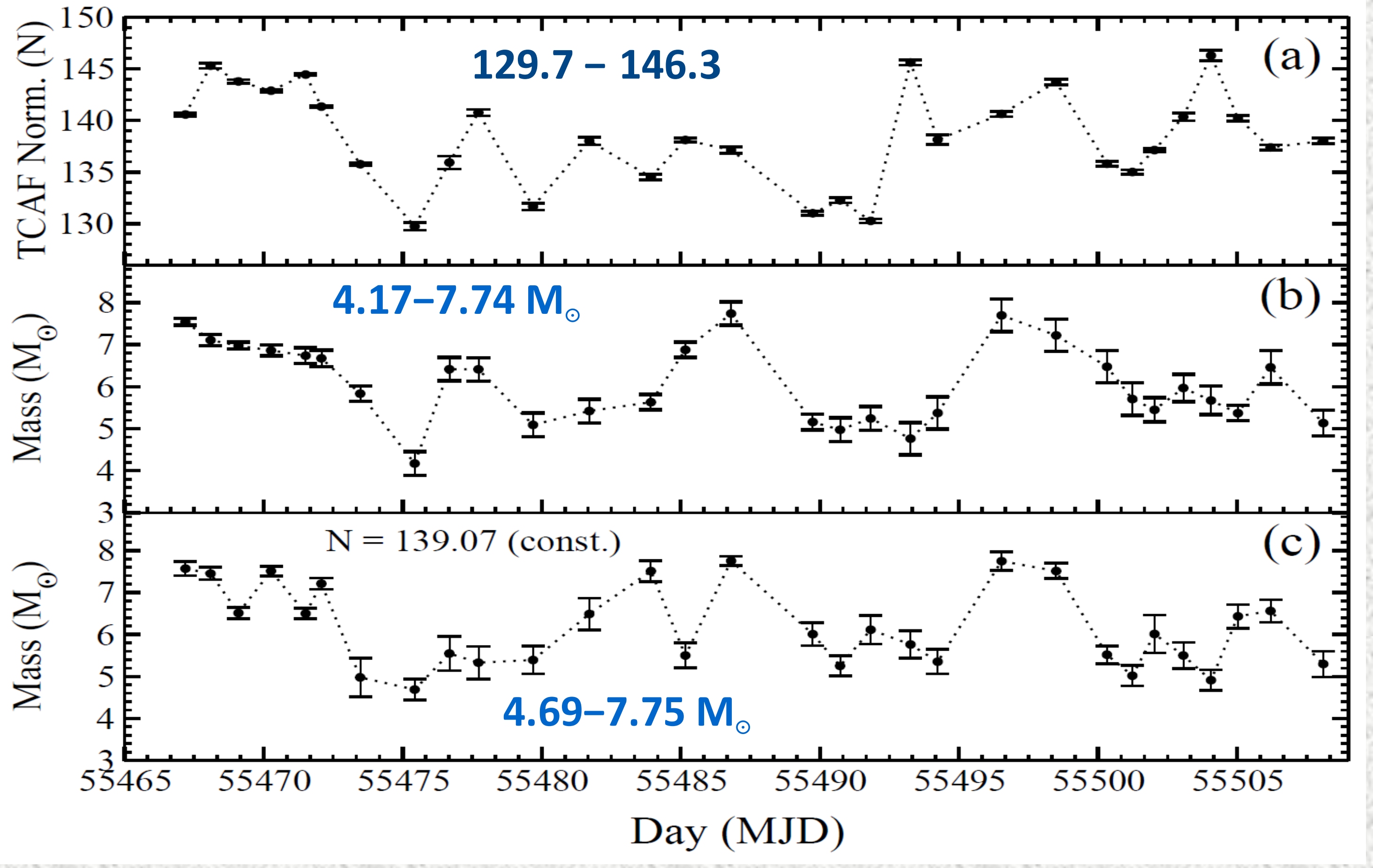
TCAF model normalization (N) should not vary observation to observations, since it only depends on mass (M_{BH}), distance (D), and disk inclination angle (i) of the source, unless there is a precession in the disk or change in the projected surface of effective emission area along the line of sight or there are significant outflow activities that are not included in the TCAF model fits file.

We found high dependence of the mass of BH (M_{BH}) with N, while fitting BH spectra. It helps us to estimate mass of different BHCs.

➢ While studying evolutions of QPOs with POS model, one can calculate frequency of the observed QPOs with following equation, where mass of the BH is an important parameter.

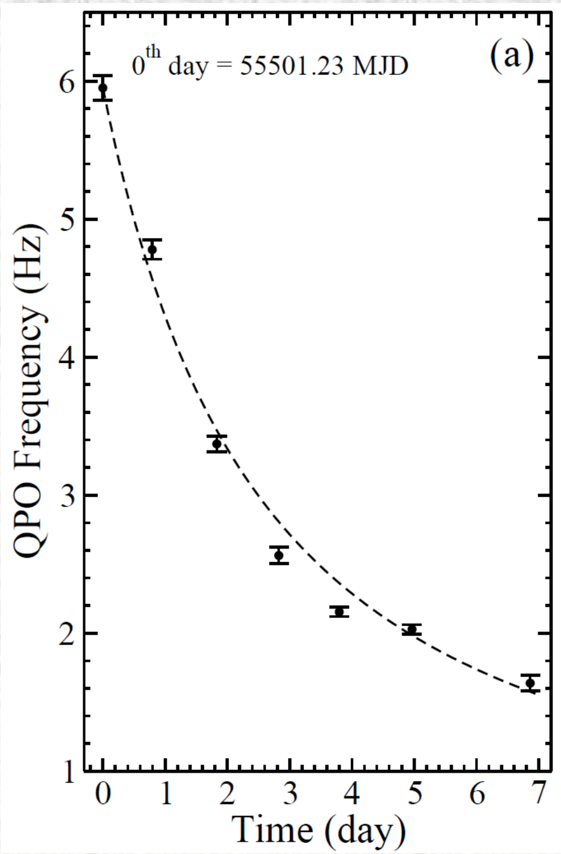
$\boldsymbol{\nu_{QPO} = C/ [R\ r_s(r_s-1)^{1/2}]}$, where C is a constant = $M_{BH} \times 10^{-5}$.

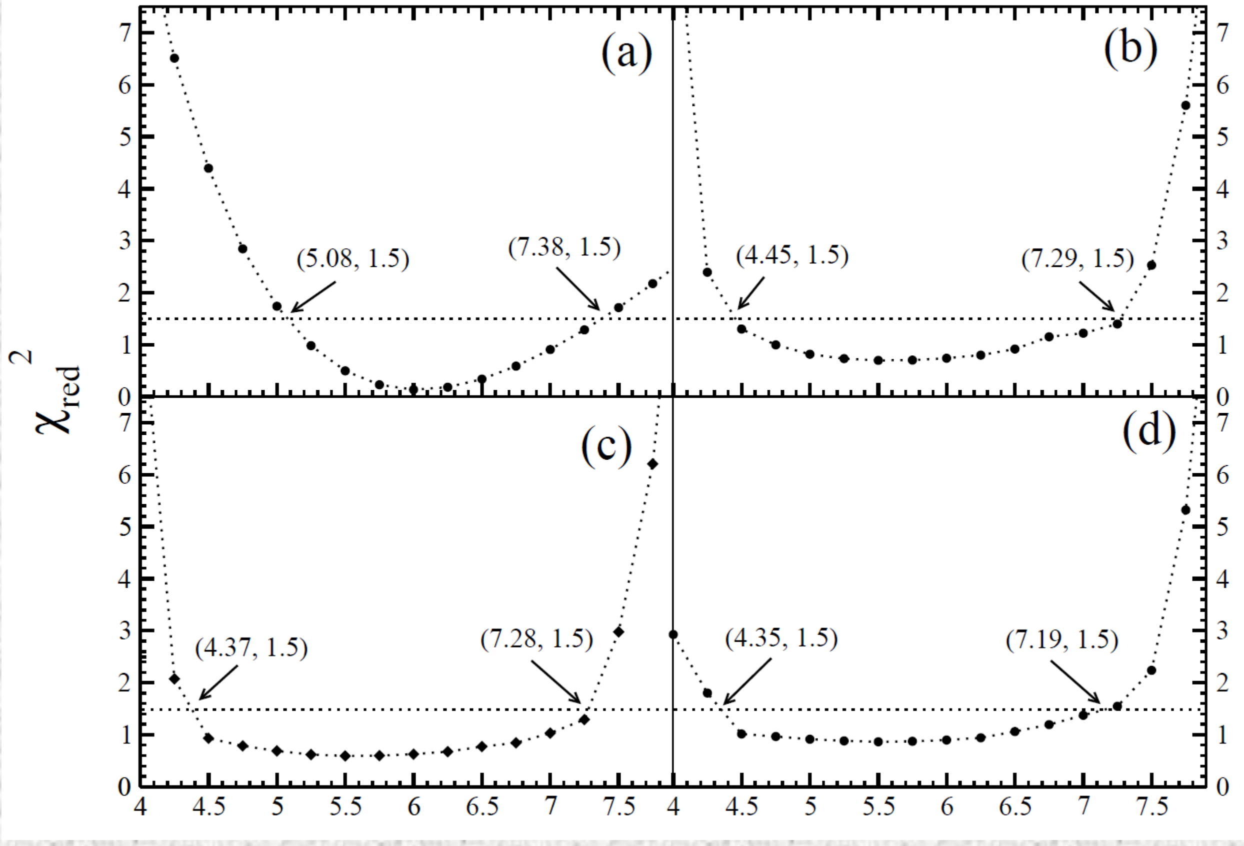
MAXI J1659-152: Mass prediction using TCAF Model fitted Constant Normalization Method



Molla, Debnath, Chakrabarti, et al., 2016, MNRAS, 460, 3163

MAXI J1659-152: Mass prediction using POS Model fitted & Reduced χ^2 Methods





POS Model Fitted Method: **5.08–7.38 $M_\odot$**

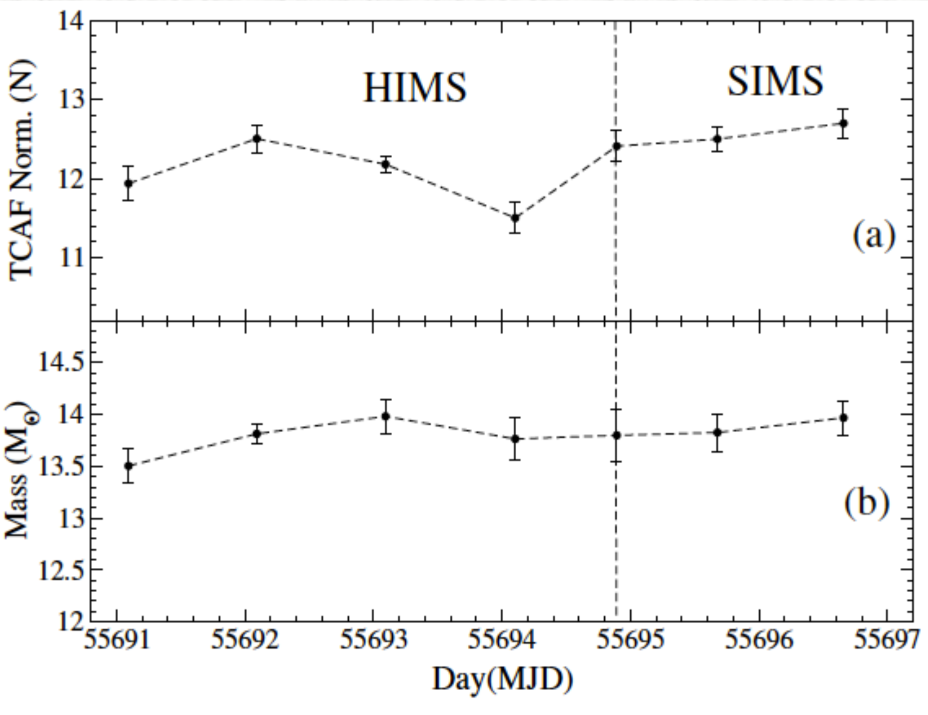
M_{BH}-χ_{red}^2 Method : **4.35–7.29 $M_\odot$**

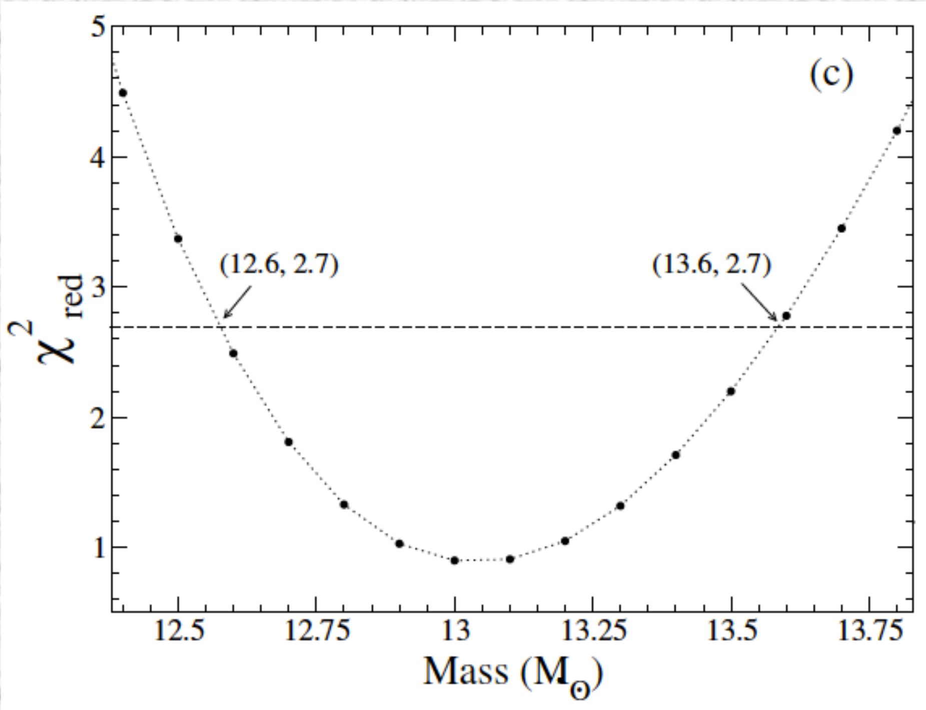
Combining three methods: **4.35 – 7.75 $M_\odot$**

or $6^{+1.75}_{-1.65}\ M_\odot$

Molla, Debnath, Chakrabarti, et al., 2016, MNRAS, 460, 3163

MAXI J1543-564: Mass Estimation using TCAF & POS models





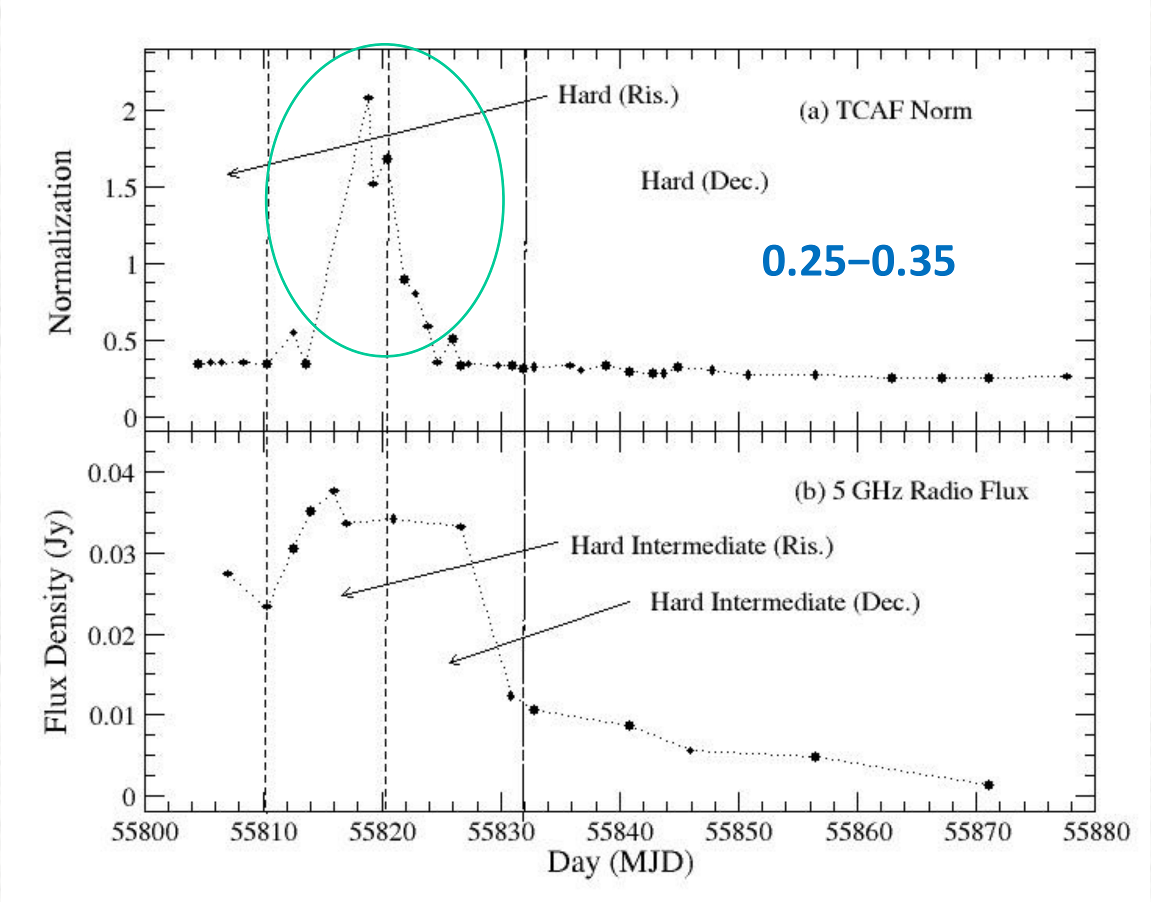
TCAF Model Fitted Method: **13.5–14.0 M☉**

POS Model Fitted Method: **12.6–13.6 M☉**

Combining two methods : **12.6–14.0 M☉ or** $13^{+1.0}_{-0.4}\ M_\odot$

Chatterjee, Debnath, & Chakrabarti et al., 2016, ApJ, 827, 88

MAXI J1836-194: Mass prediction using TCAF Model fitted Constant Normalization Method



diskbb model normalization varies in the range of 0.34−291

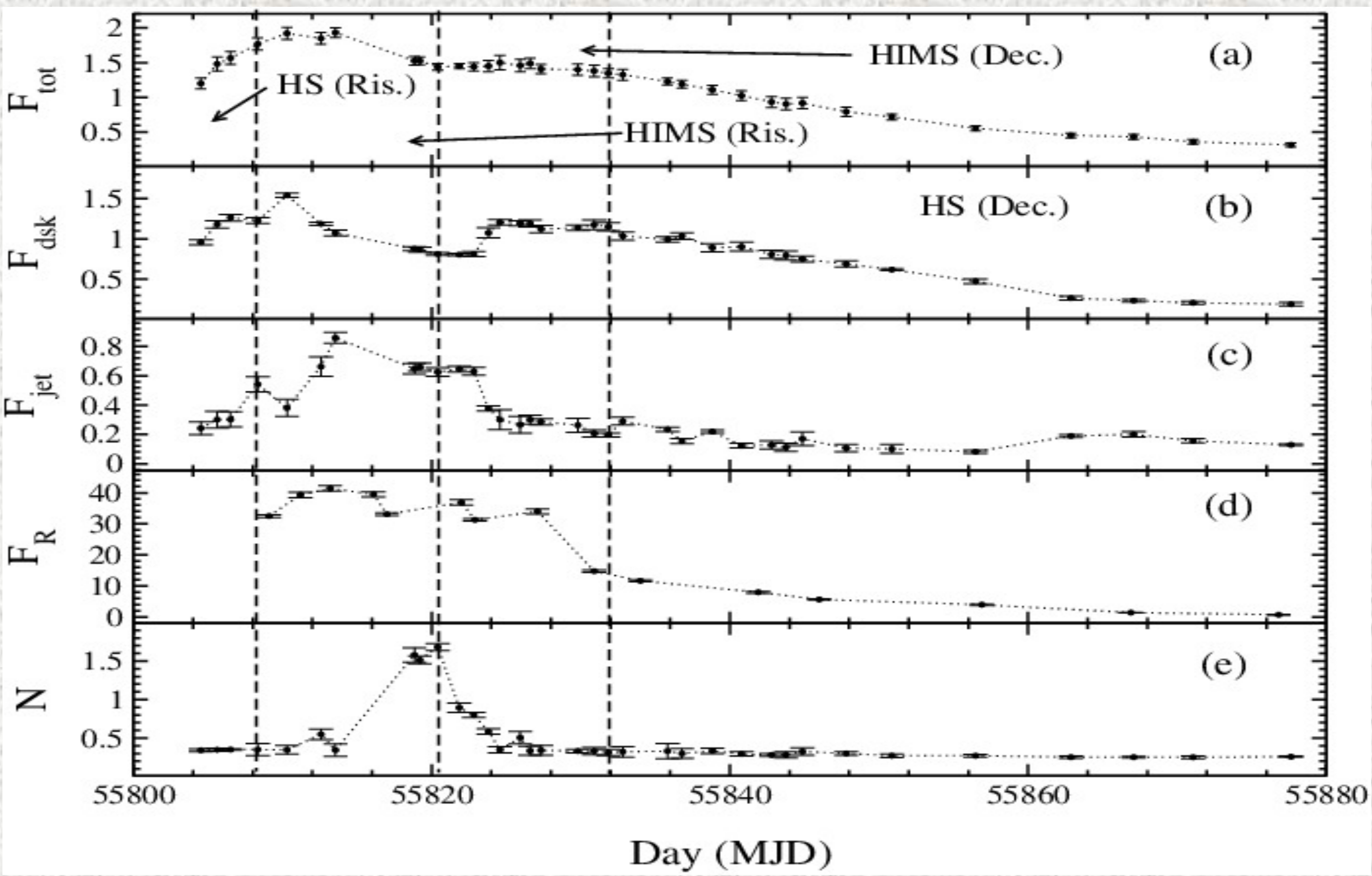
M_{BH} is found in the range of ***7.5 – 11 $M_{\odot}$***.

Jana, DebnathChakrabarti, et al., 2016, ApJ, 819, 107

X-ray Jets: Detection and estimation from spectral analysis with the TCAF solution

MAXI J1836-194: Estimation of X-ray Jet fluxes

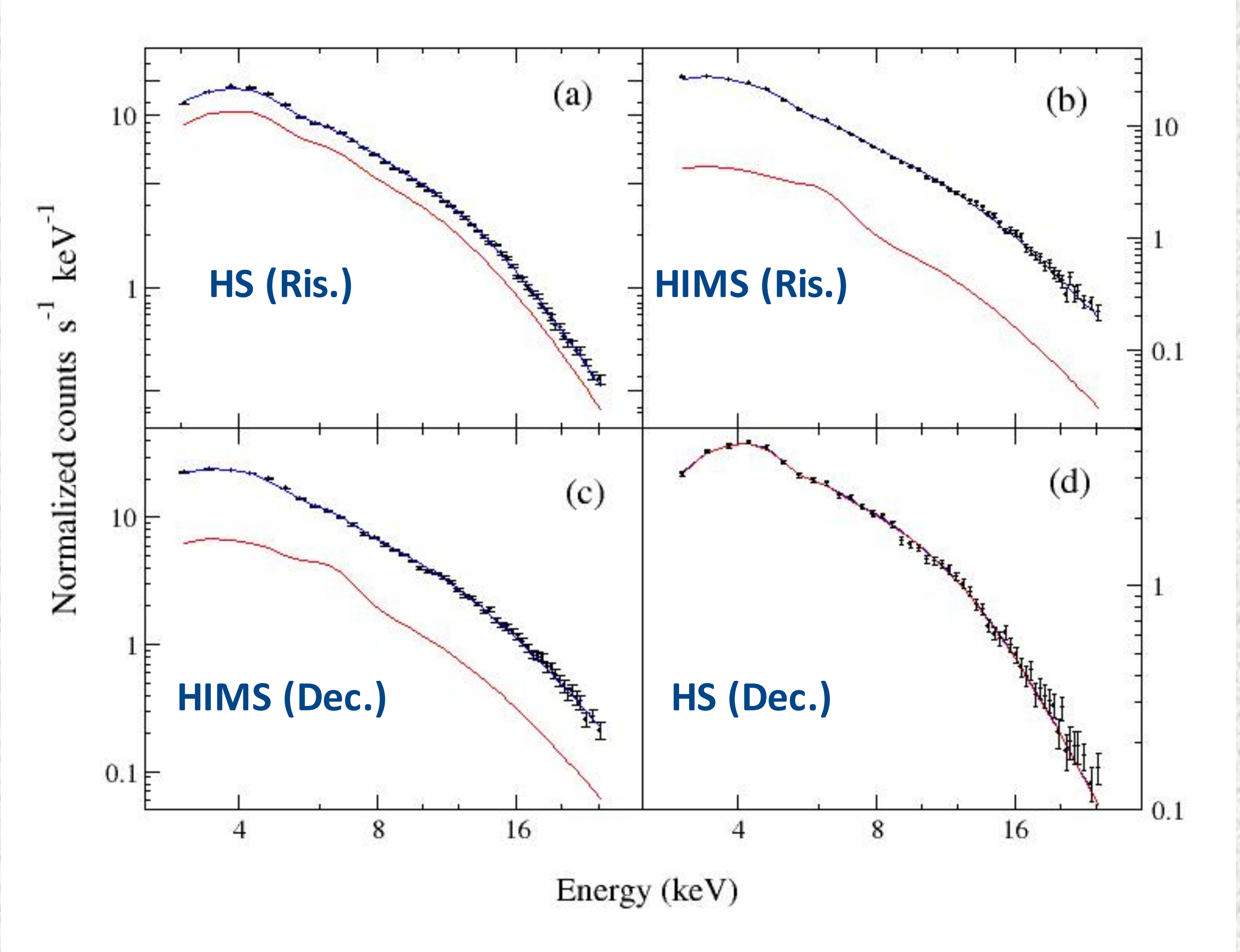
$$F_{jet} = F_{tot} - F_{dsk}, \qquad (1)$$



Jana, Debnath, Chakrabarti, et al., 2016, ApJ (submitted)

Refer to Poster no. P-03

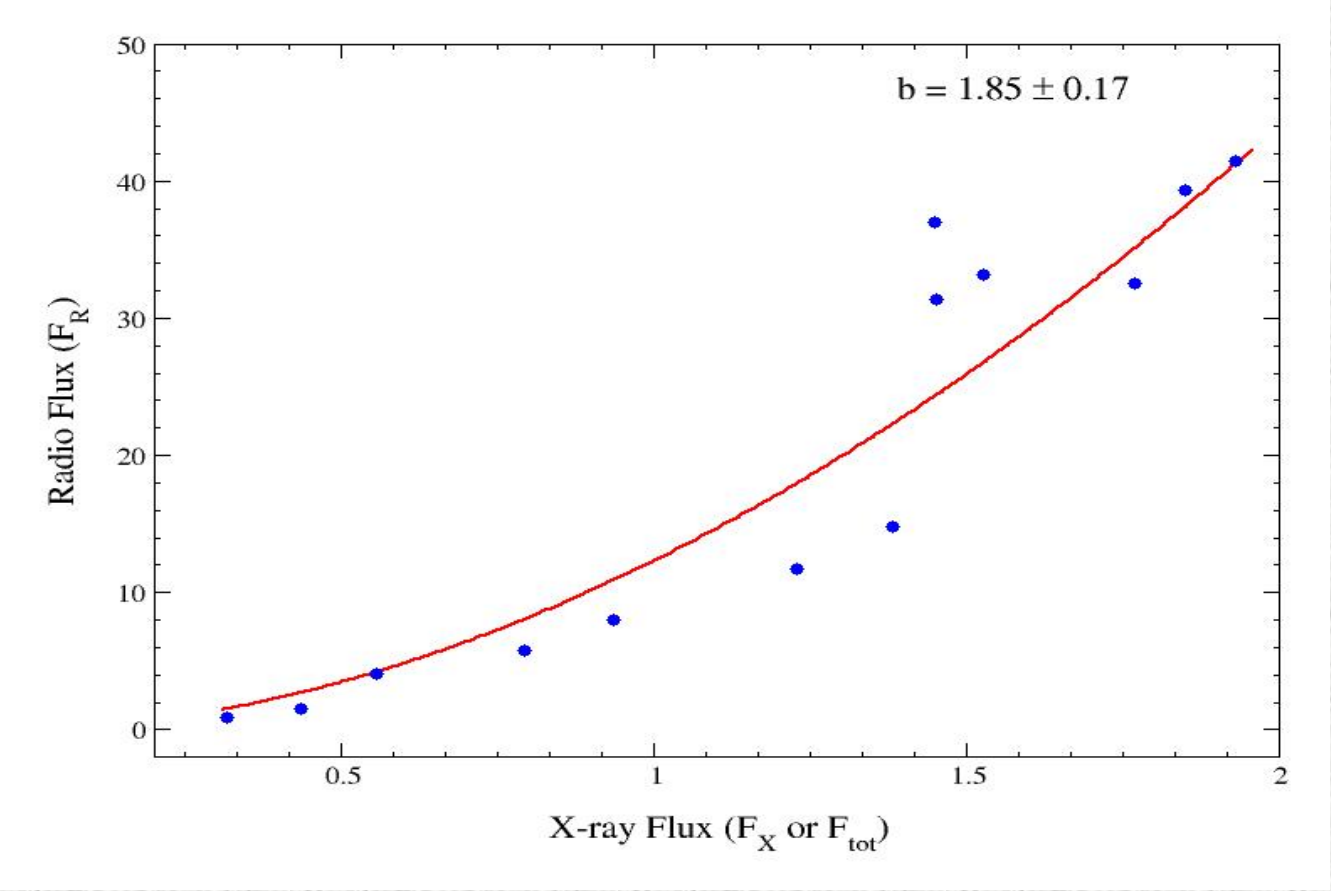
MAXI J1836-194: Fitted with TCAF to separate Jet part



Jana, Debnath, Chakrabarti, et al., 2016, ApJ (submitted)

Refer to Poster no. P-03

MAXI J1836-194: Correlation between X-ray and Radio fluxes



Radio and X-ray fluxes follows correlation relation

$$F_R \sim F_X^b$$

During the 2011 outburst of MAXI J1836-194, average jet contribution in total X-ray is about 25 %, where as on the day of strongest jet, the contribution is rised upto ~ 44% of the total flux.

Russel et al. (2015) also found similar correlation of b ~ 1.8.

Jana, Debnath, Chakrabarti, et al., 2016, ApJ (submitted)

Refer to Poster no. P-03

Viscous Time: Using peak differences between disk & halo rates

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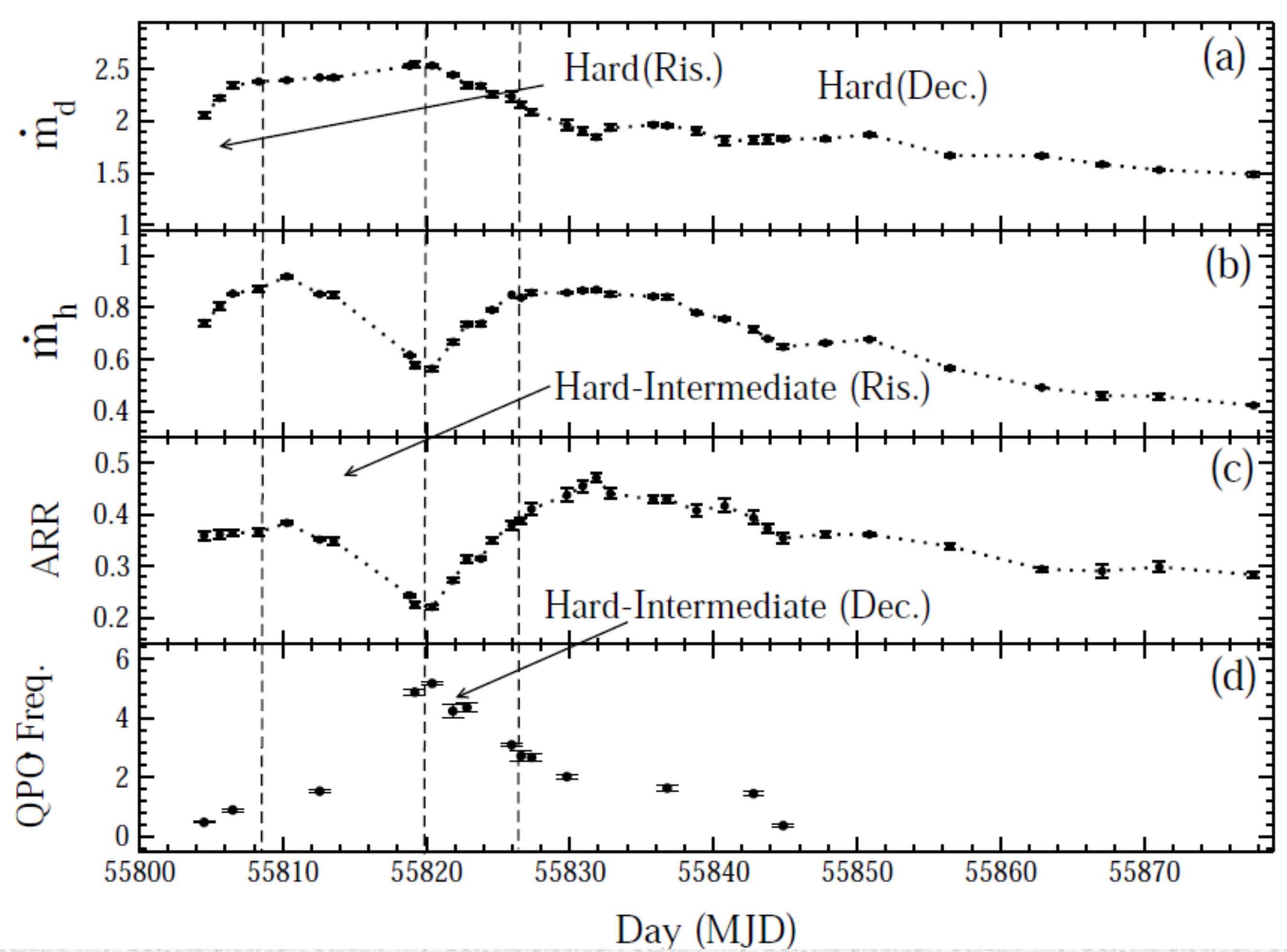
ACCRETION FLOW DYNAMICS OF MAXI J1836-194 DURING ITS 2011 OUTBURST FROM TCAF SOLUTION

ARGHAJIT JANA[1], DIPAK DEBNATH[1], SANDIP K. CHAKRABARTI[1,2], SANTANU MONDAL[1], AND ASLAM ALI MOLLA[1]
[1] Indian Center for Space Physics, 43 Chalantika, Garia St. Rd., Kolkata, 700084, India;
argha@csp.res.in, dipak@csp.res.in, chakraba@bose.res.in, santanu@csp.res.in, aslam@csp.res.in
[2] S. N. Bose National Centre for Basic Sciences, Salt Lake, Kolkata, 700098, India
Received 2015 August 30; accepted 2016 January 8; published 2016 March 3

MAXI J1836-194: Prediction of Viscous Time Scale during its 2011 outburst



halo peaks are nearly at MJDs 55810 & 55826

disk peaks are nearly at MJDs 55820 & 55836

Peak time differences between halo & disk rates in rising & declining phases are around 10 days, which is the viscous time scale.

Jana, Debnath, Chakrabarti et al., 2016, ApJ, 819, 107

Origin of LFQPOs and their evolution

Origin of QPOs in black hole X-ray binaries:

- Low and Intermediate frequency (0.01 – 30 Hz) QPOs are generally observed in hard, intermediate spectral states of BHCs.

- For few BHCs, QPOs are also observed to increase monotonically in rising phase (hard and hard-intermediate spectral states)of the outburst and opposite scenario is observed in declining phase of the outburst (Nandi et al. 2012; Debnath et al. 2013).

- Sporadic QPOs are observed during soft-intermediate spectral states of the outburst (Debnath et al. 2010, 2013; Nandi et al. 2012).

- Many models, such as: perturbation inside a Keplerian disk (Trudolyubov et al. 1999), global disk oscillation (Titarchuk & Osherovich 2000), oscillation of warped disk (Shirakawa & Lai 2002), accretion ejection instability at the inner radius of a Keplerian disk (Rodriguez et al. 2000), tries to find origin of these LFQPOs.

Shock Oscillation Model (SOM) to find origin BHCs :

According to Shock Oscillation Model (SOM) by Chakrabarti and his collaborators (Molteni et al. 1996; Mondal, Chakrabarti & Debnath, 2015) QPOs are caused due to shock oscillation.

There are mainly two reasons behind oscillation of shock wave in an accretion flow:

i) **<u>Resonance Oscillation</u>**: **when cooling time scale of the flow is comparable to the infall time scale** (Molteni, Sponholtz &Chakrabarti 1996), this type of oscillation occurs. Such cases can be identified by the fact that when accretion of the Keplerian disk is steadily increased, **QPOs may occur in a range of the accretion rates, and the frequency should go up with accretion rate.**

ii) **<u>Non-Steady Solution</u>**: In this case, flow has two saddle type sonic points, but **Rankine-Hugoniot conditions** which were used to study standing shocks in Chakrabarti (1989) **are not satisfied to form a steady shock** (Ryu, Chakrabarti & Molteni 1997).

Propagating Oscillatory Shock (POS) Model : Governing Equations

Infall time is denoted by,

$$t_{infall} \sim r_s/v \sim Rr_s(r_s - 1)^{1/2}, \quad (1)$$

where $r_g = 2GM/c^2$ is the Schwarzschild radius, and compression ratio $R = \rho_+/\rho_-$, where ρ_+ and ρ_- are the densities of the post and pre-shock flows.

R may vary as $1/R \rightarrow 1/R_0 + \alpha(t_d)^2$, where α is a constant which determines how shock (strength) becomes weaker with time and reaches its lowest possible value when R .

QPO frequency which is inversely proportional to infall time is given by,

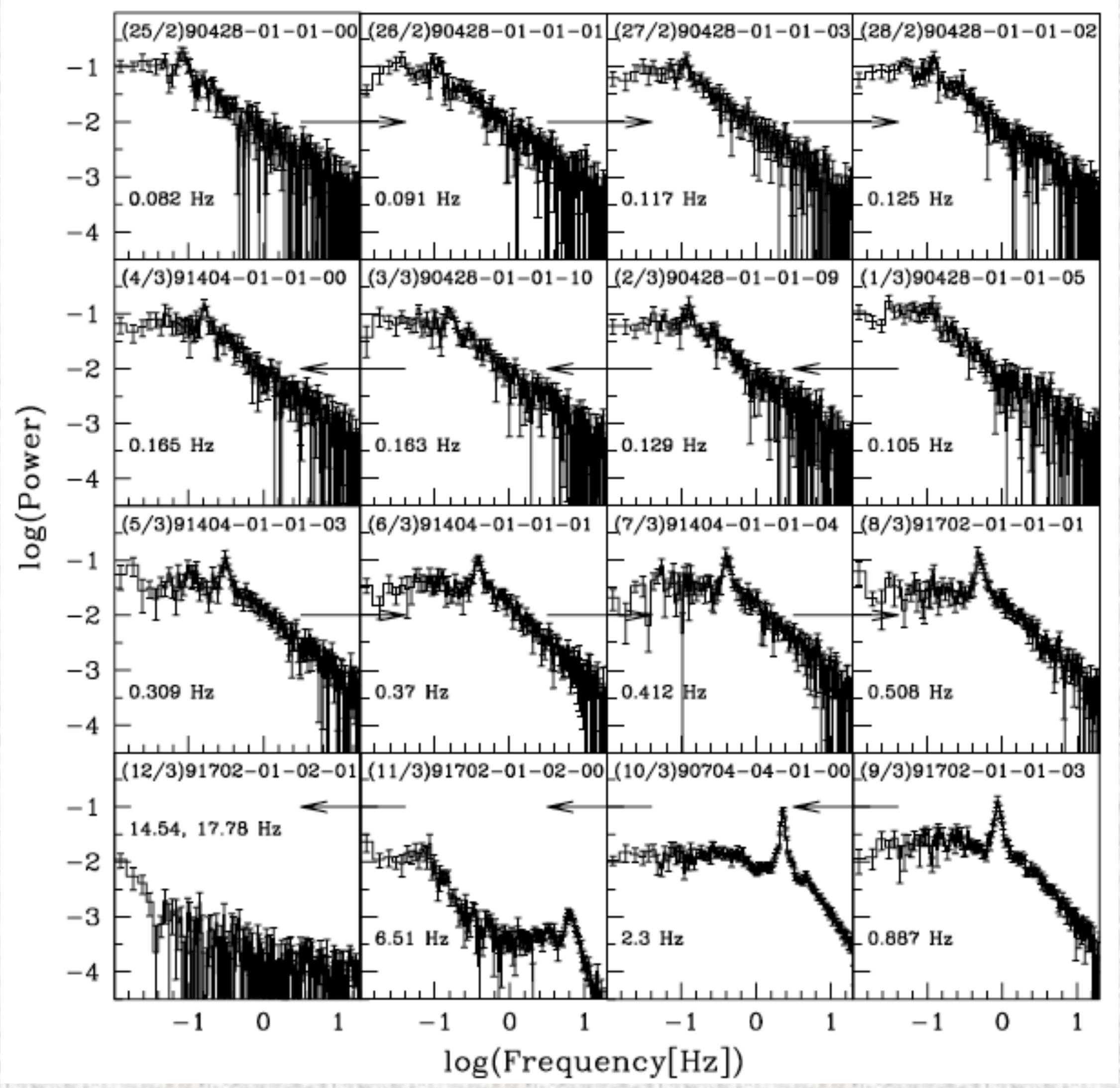
$$\nu_{QPO} = \nu_{s0}/t_{infall} = \nu_{s0}\Big/\left[Rr_s(r_s - 1)^{1/2}\right]. \quad (2)$$

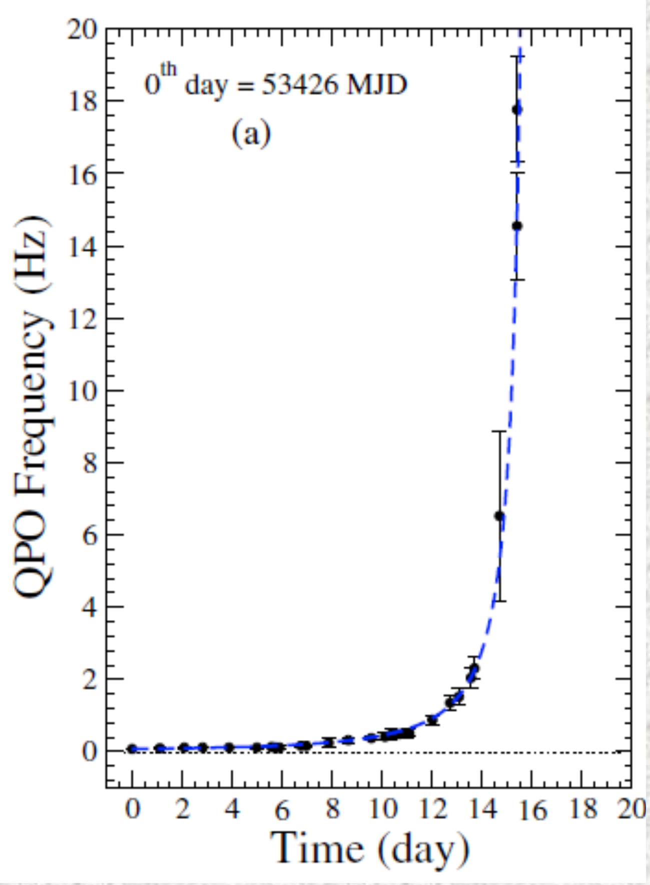
Shock may move towards ('-' ve sign) or away ('+'ve sign) from the black hole, expressed as

$$r_s(t) = r_{s0} \pm v_0 t/r_g, \quad (3)$$

Chakrabarti, Nandi, Debnath, Sarkar & Dutta, 2005, IJP, 78, 841

GRO J1655-40 during its 2005 outburst: QPO frequency Evolution (Rising Phase; hard and hard-intermediate States)

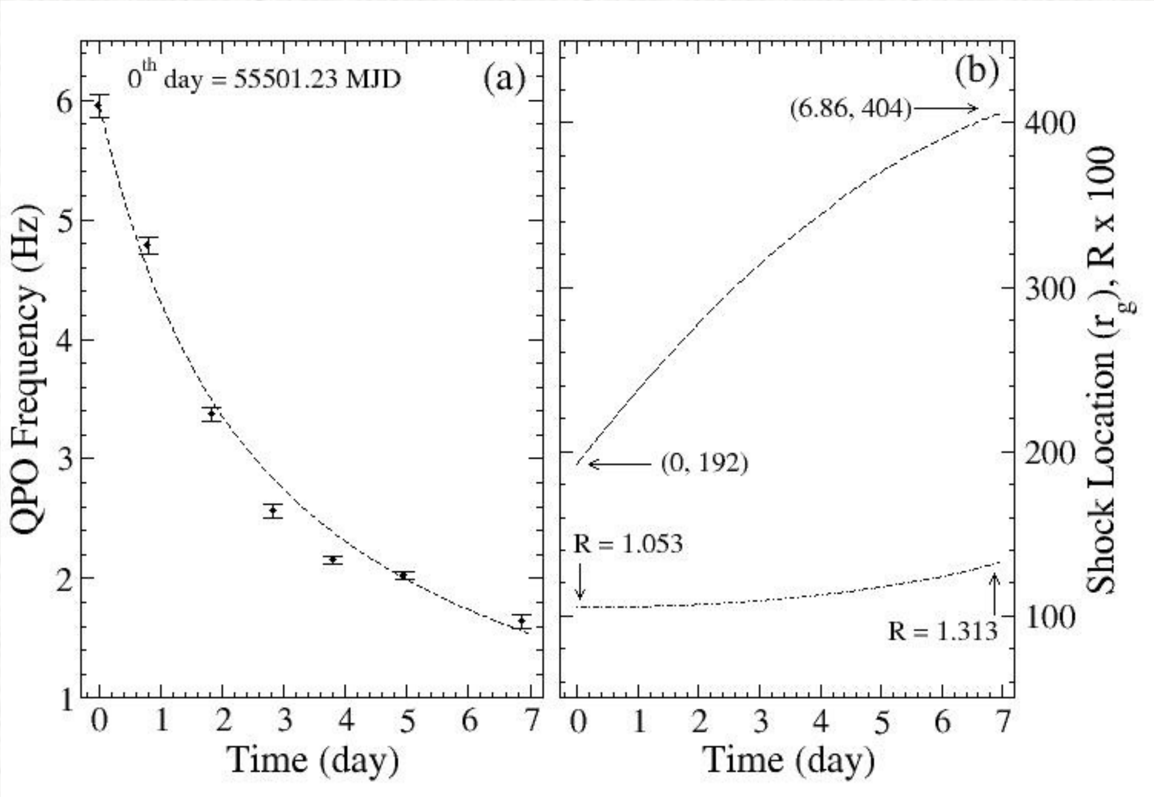




Chakrabarti, Debnath, Nandi & Pal, 2008, A&A, 489, L41

Debnath, Chakrabarti, Nandi & Mandal, 2008, BASI, 36, 151

MAXI J1659-152 : POS model fitted QPO Evolution



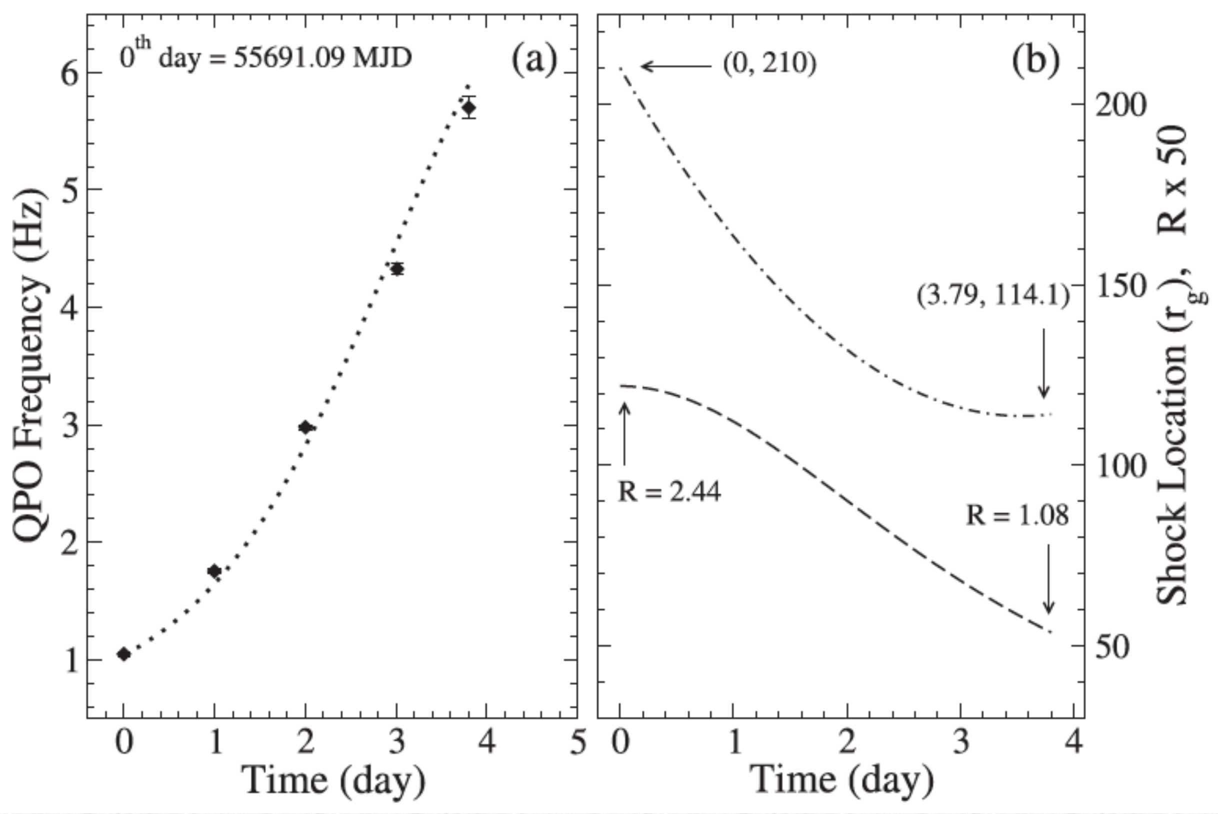
Declining Phase of 2010 outburst of MAXI J1659-152:

Shock moved away from the black hole in decelerating way.

ν_i (Hz)	ν_f (Hz)	r_{si} (r_g)	r_{sf} (r_g)	V_i (cm/s)	V_f (cm/s)	a (cm/s/d)	R_i	R_f	α
5.951	1.638	192.3	404.2	1000	643	-52.0	1.053	1.313	0.004

Molla, Debnath, Chakrabarti, Mandal & Jana, 2016, MNRAS, 460, 3163

MAXI J1543-564 : POS model fitted QPO Evolution



Rising Phase of 2011 outburst of MAXI J1543-564:

Shock moved towards the black hole in decelerating way.

ν_i (Hz)	ν_f (Hz)	r_{si} (r_g)	r_{sf} (r_g)	V_i (cm/s)	V_f (cm/s)	a (cm/s/d)	R_i	R_f	α
1.05	5.70	210.0	114.1	2450	1139	-345	2.44	1.08	0.037

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Summary and Concluding Remarks:

1) **Accretion flow dynamics** during outbursts of transient BHCs well understood with spectral studies with Two-Component Advective Flow (**TCAF**) model, as an additive table model in XSPEC.

2) **Classification of spectral states** during outbursts can be well understood based on TCAF model fitted physical parameters (variation of accretion rate ratios, i.e., **ARRs**; ratio between TCAF model fitted sub-Keplerian halo to Kelerian disk rates) and **nature of QPOs** (if present).

3) **Prediction of dominating QPO** frequencies could be done with the TCAF model shock parameters (**location and compression ratio**).

4) **Estimation of mass** of an unknown BH from TCAF & POS model fits.

5) Idea about the **viscous time scale** from peak time differences of disk & halo rates.

6) **ARRID** to find correlation between timing and spectral properties.

7) **Detection of Jets in X-rays** and **estimation** of flux contributions and find its correlation with radio.

8) In future, we will **extend our work to other transient** as well as persistent BHCs(for e.g., GRO J1655-40, 4U 1630-272, XTE J1550-564, GRS 1915+105, IGR J17091-3654, Cyg X-1, etc.) and will also try predict mass of some unknown objects.

9) In future, we also have plan to use **MAXI** and **ASTROSAT data** of transient BHCs to study detailed temporal and spectral properties of BHCs.

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Thank you