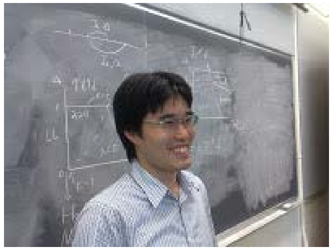
Quantum many-body dynamics under continuous observation

Masahito Ueda
RIKEN CEMS, University of Tokyo

collaborators

Yuto Ashida
U. Tokyo

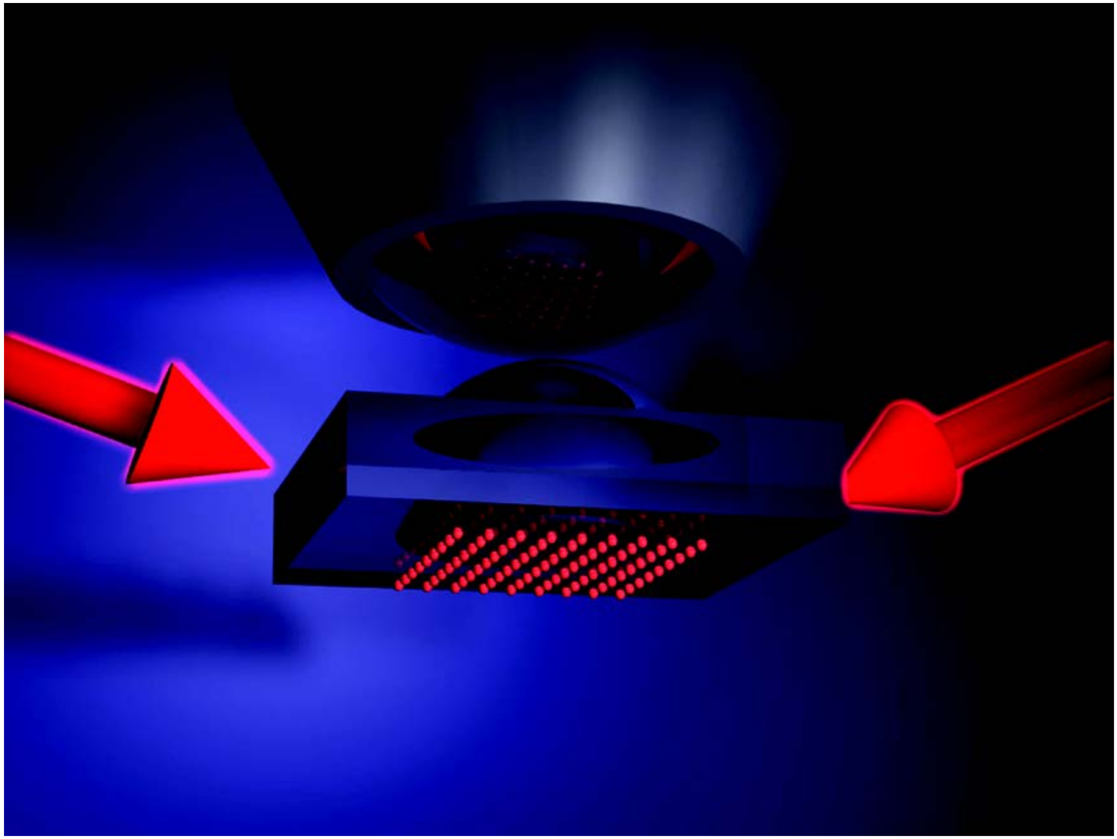


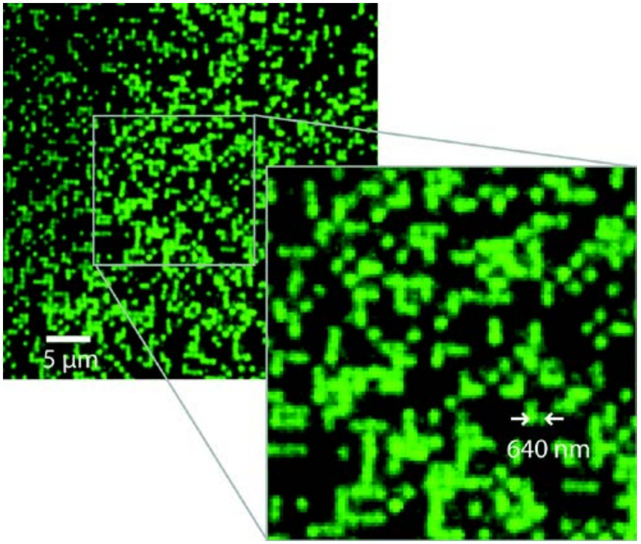
Shunsuke Furukawa
U. Tokyo

Motivation of this work

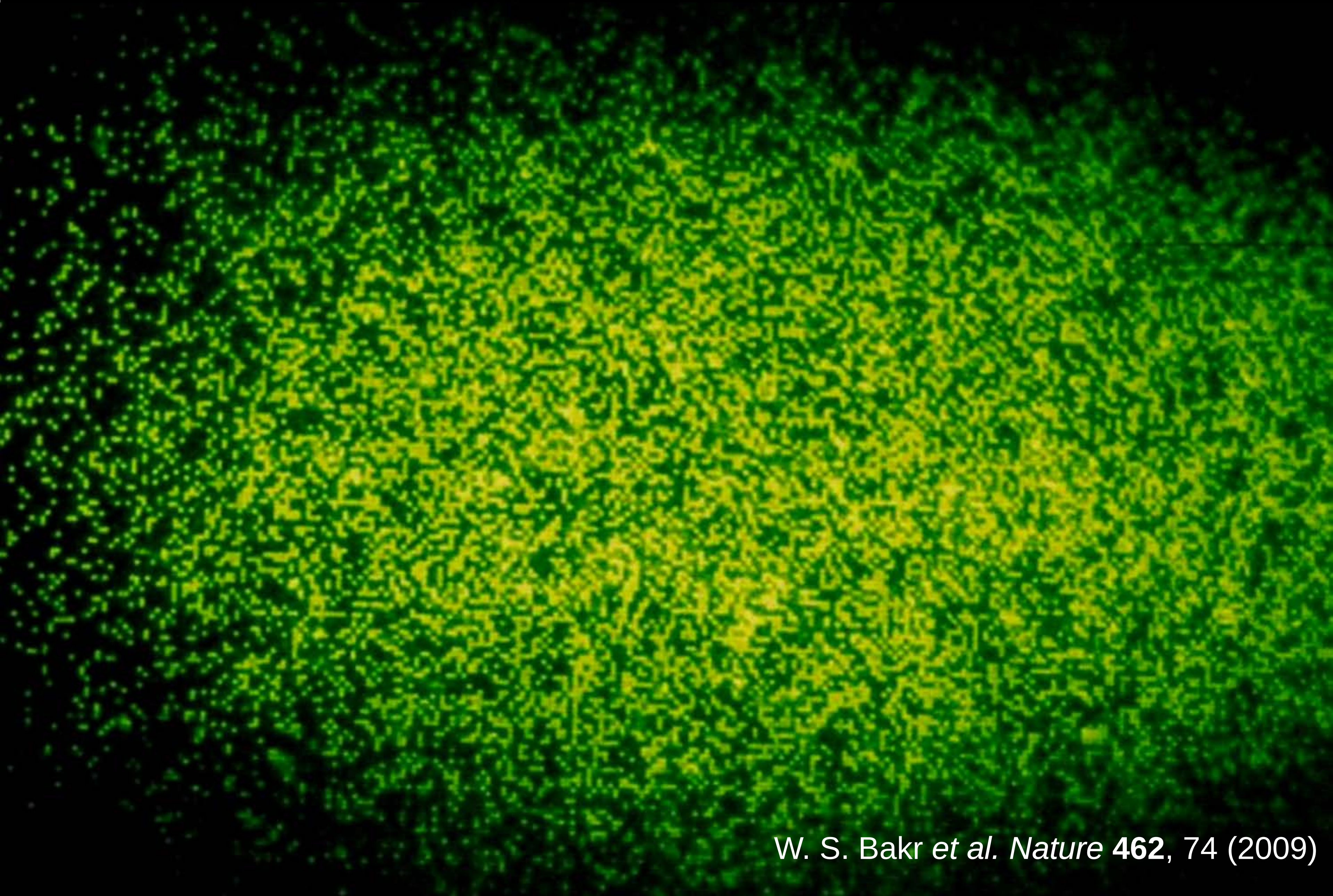
Experimental breakthrough
-- quantum gas microscopy –

Markus Greiner (Harvard)
Immanuel Bloch (LMU)





Snapshot of many-body wavefunction

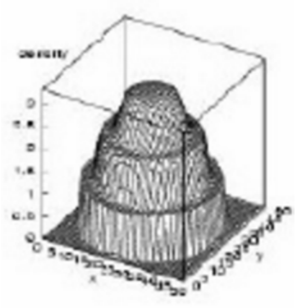


W. S. Bakr *et al. Nature* **462**, 74 (2009)

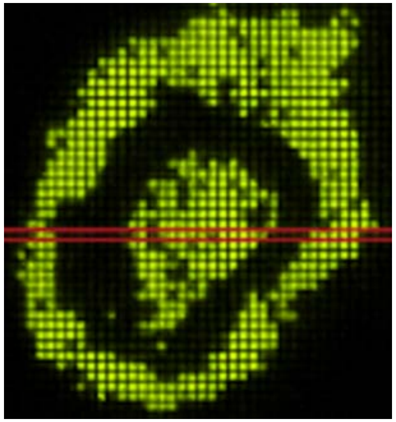
Superfluid-Mott insulator transition

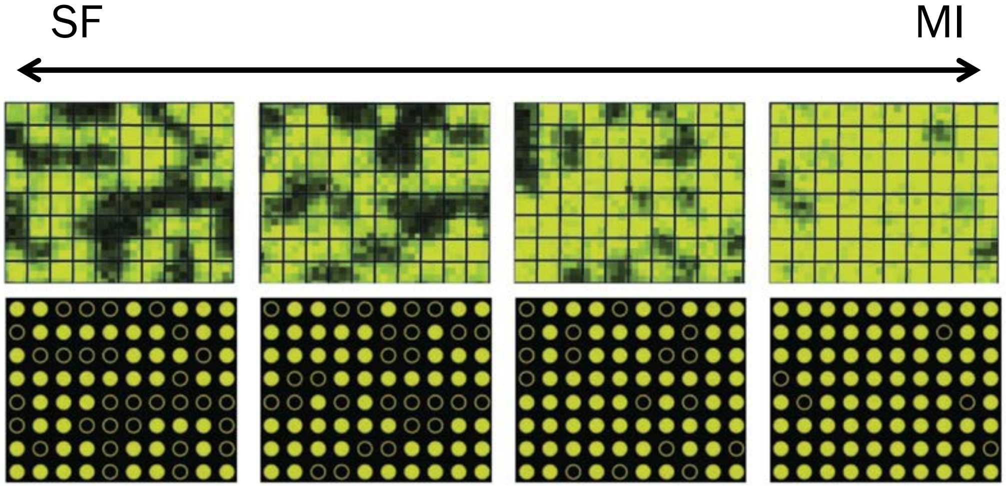
- SF-MI transition has been observed at the **single-particle** level by verifying suppression of on-site atom-number fluctuations.
- Predicted “wedding” cake pattern has experimentally been confirmed.

Theory



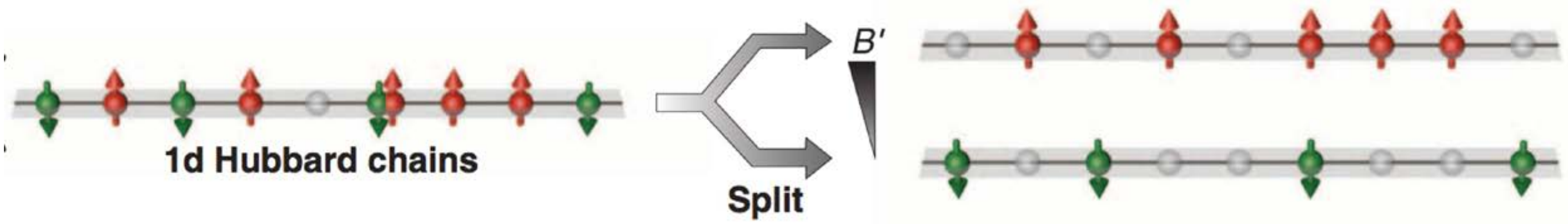
Experiment

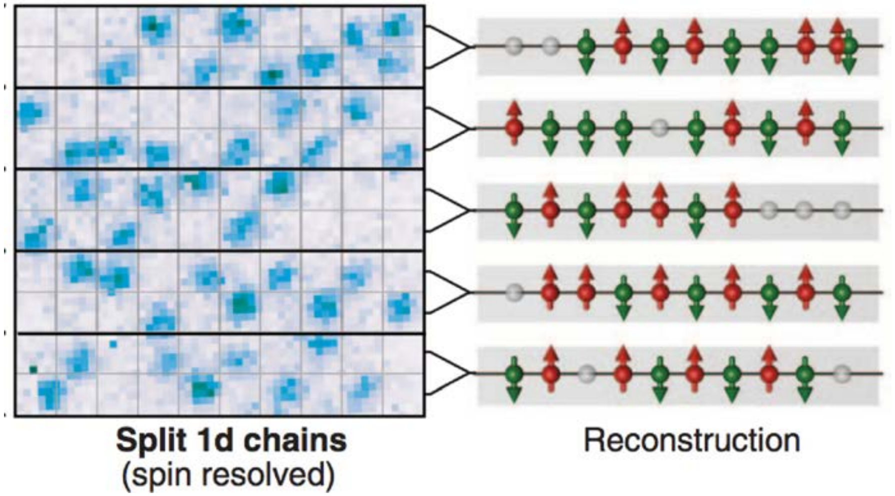




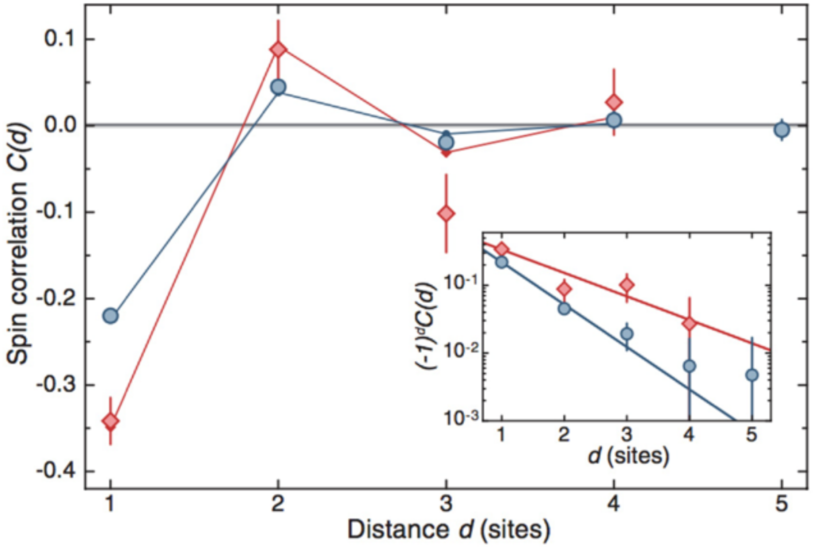
W. S. Bakr et al., Science 329, 547 (2010).

Site-resolved quantum gas microscopy: Antiferromagnetic order of Fermi-Hubbard model





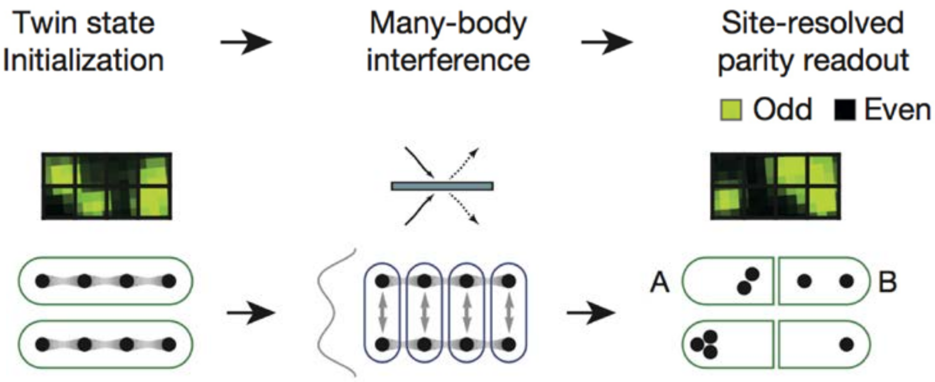
$$C(d) \propto \langle \hat{S}_0^z \hat{S}_d^z \rangle - \langle \hat{S}_0^z \rangle \langle \hat{S}_d^z \rangle$$



M. F. Parsons et al., Science 353, 1253 (2016),
M. Boll et al., Sceience 353, 1257 (2016),
L. W. Cheuk et al., Science 353, 1260 (2016).

Staggered behavior against distance directly reveals the **antiferromagnetic** nature of the spin-spin correlation.

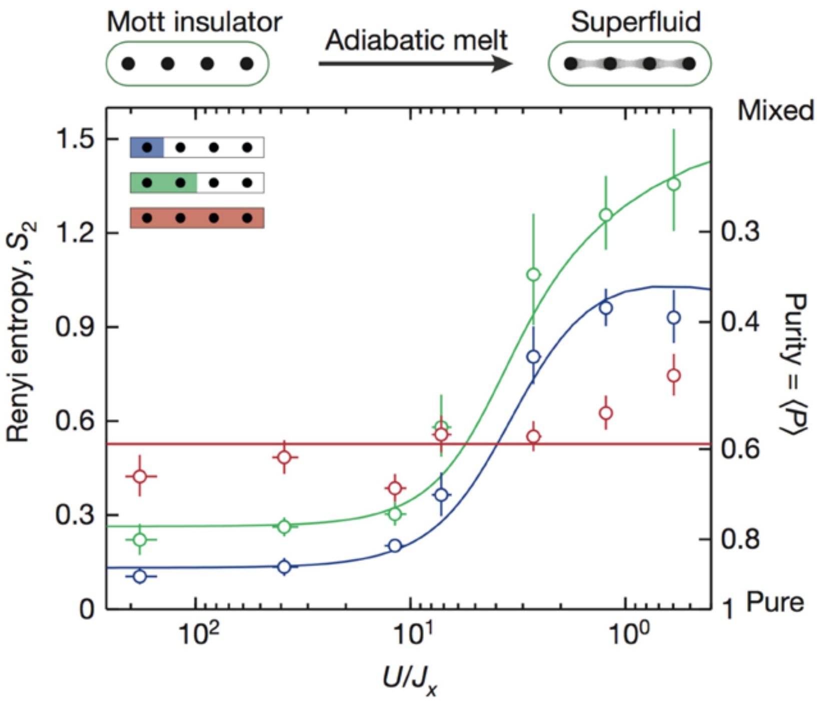
Measurement of entanglement entropy

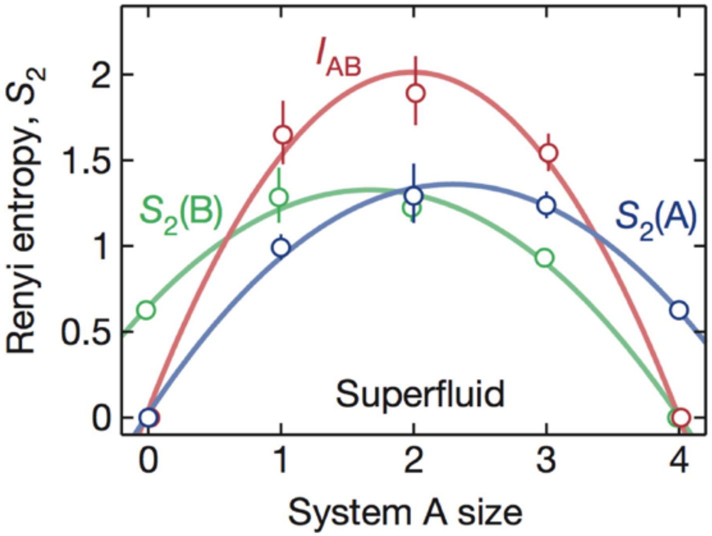


Entanglement generation across QPT has been experimentally demonstrated

entanglement of bosons: bunching

Predicted size dependence of Renyi entropy (*n*=2) has been observed.

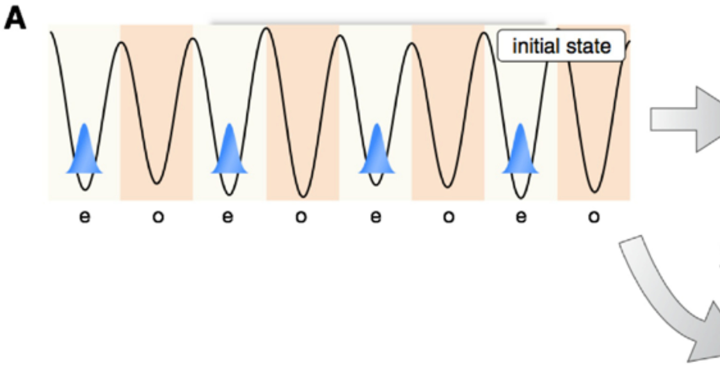


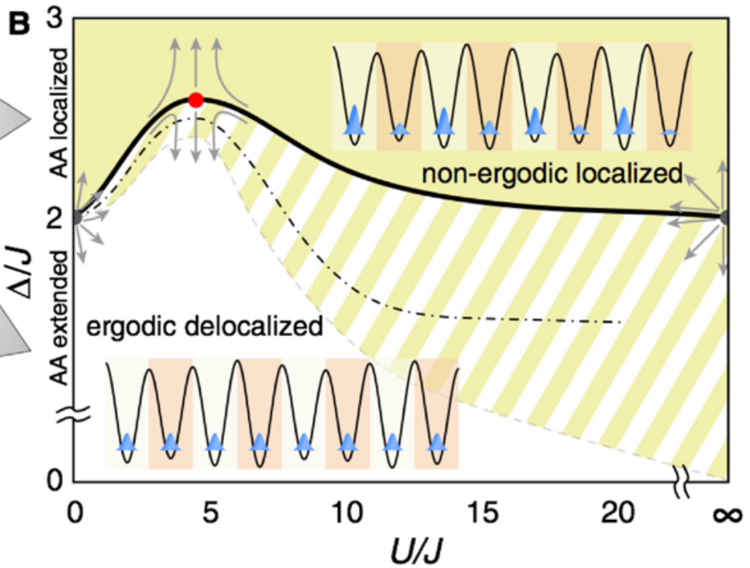


R. Islam et al., Nature 528, 77 (2015).

Many-body localization

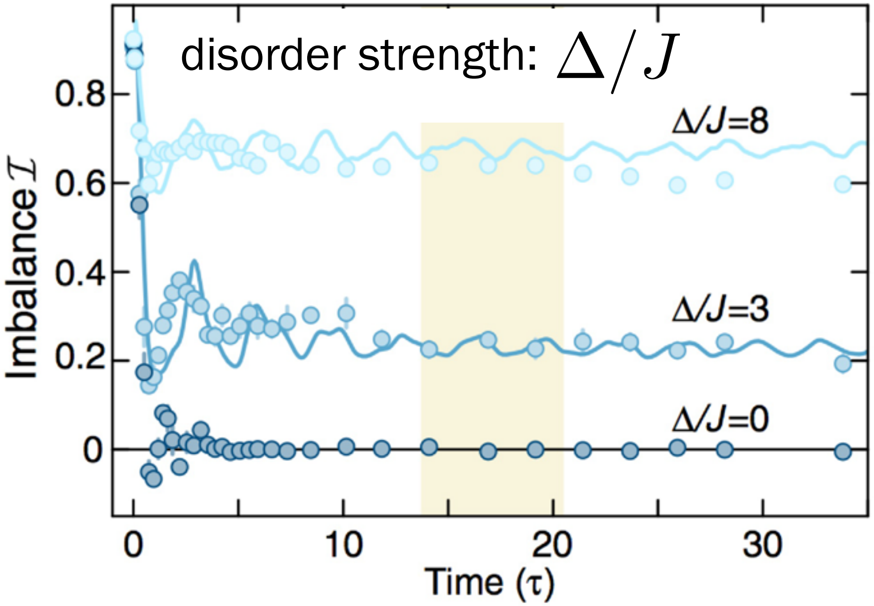
Setup





Imbalance of atom-number occupation:

$$\mathcal{I} = \frac{N_{\text{even}} - N_{\text{odd}}}{N_{\text{even}} + N_{\text{odd}}}$$



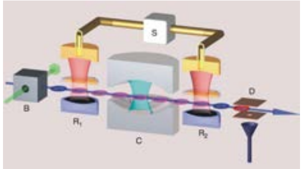
disorder strength: Δ/J

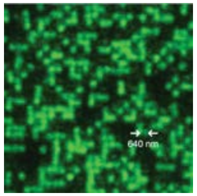
- An initially prepared charge-density-wave order remains in a strong **disorder.**

→ **Non-ergodic** nature of **many-body localized** phase has been confirmed.

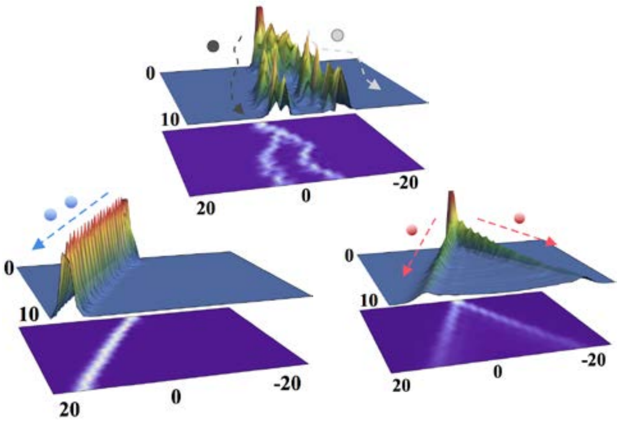
M. Schreiber et al., Science 349, 842 (2015).

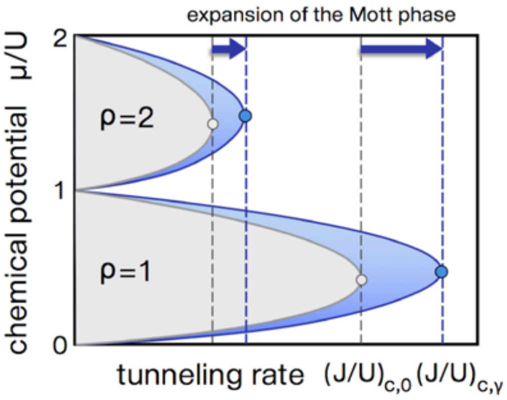
Outline of this talk

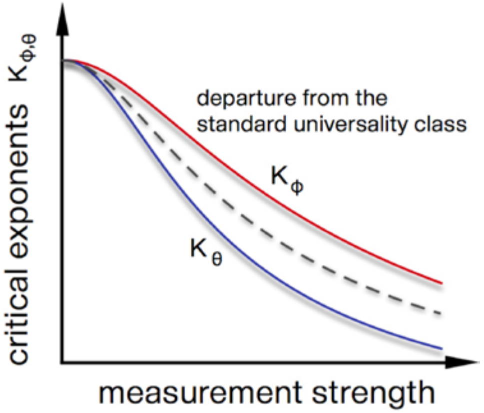




- **Introduction**
  - Continuous monitoring of quantum systems
  - Quantum gas microscopy as a new tool to observe quantum many-body dynamics
- **Two-particle quantum dynamics under continuous observation**
  - Compare quantum transport for distinguishable and indistinguishable particles.
- **Quantum critical phenomena influenced by measurement backaction**
  - Measurements change the quantum critical point and critical exponents.
  - A unique 1D universality class beyond the conventional paradigm of TLL.





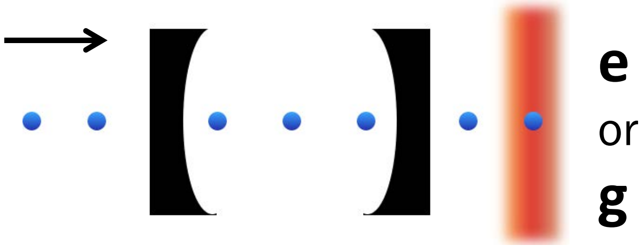


Continuous Monitoring of Quantum Systems

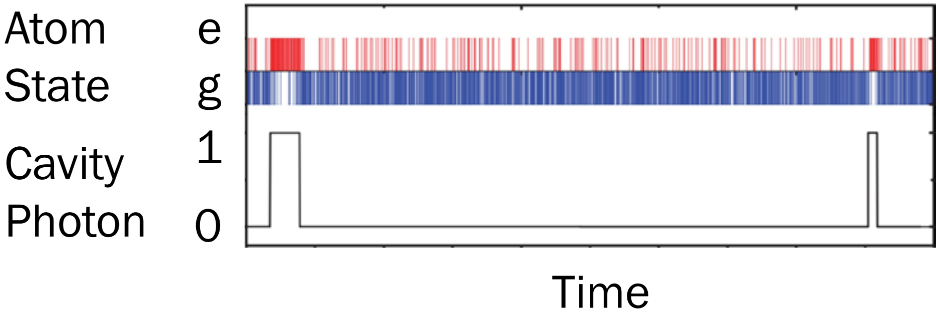
- **A seminal experiment: cavity-QED experiment**

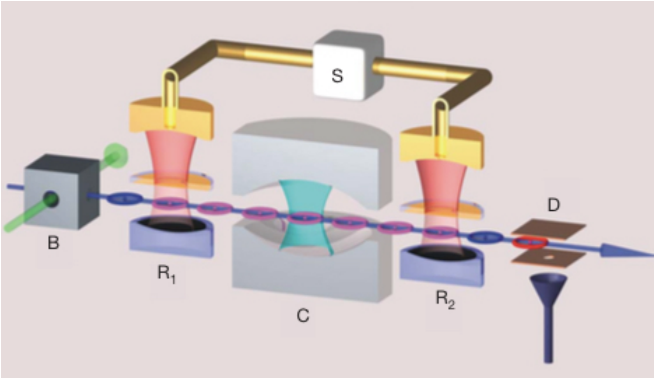
Haroche, 2012

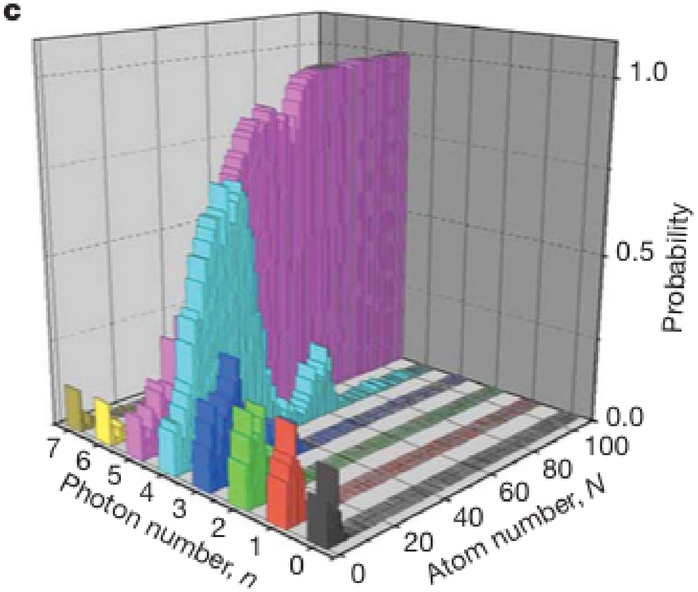
Measure the cavity photon number through monitoring of atoms.



S. Gleyzes et al., Nature 446, 297 (2007)





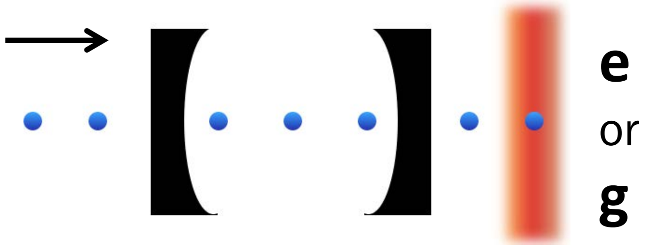


Continuous Monitoring of Quantum Systems

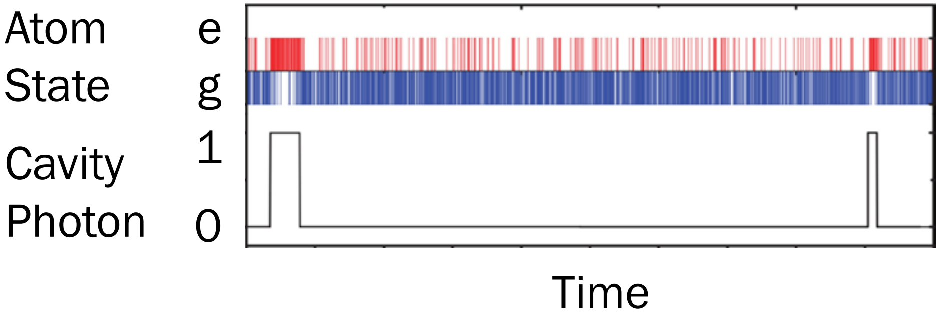
- **A seminal experiment: cavity-QED experiment**

Measure the cavity photon number through monitoring of atoms.

Haroche, 2012



S. Gleyzes et al., Nature 446, 297 (2007)



- **Application: measurement-based control of quantum systems**

Prepare an arbitrary state of cavity photons.

C. Sayrin et al., Nature 477, 73 (2011)

- **Other systems:** trapped ions, superconducting qubits, quantum dots, etc.

→ Restricted to quantum systems with small degrees of freedom

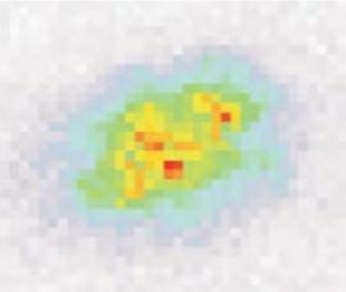
Quantum Gas Microscopy

A new approach to quantum many-body systems via *in-situ* imaging of ultracold atoms

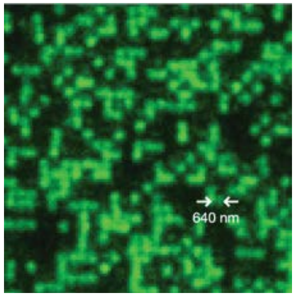
weak resolved imaging

single-site resolved imaging

Destructive single-shot measurement

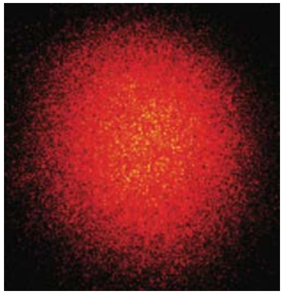


N. Gemelke et al., Nature 460, 995 (2009)



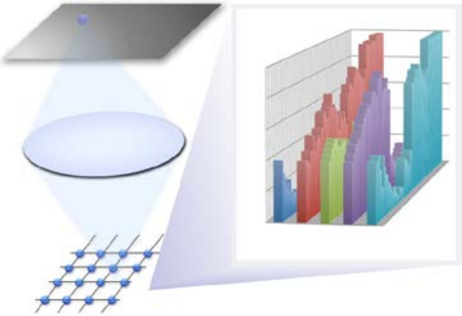
W. S. Bakr et al., Nature 462, 74 (2009)

Nondestructive continuous imaging of atoms



Y. Patil et al., PRA 90 033422 (2014)

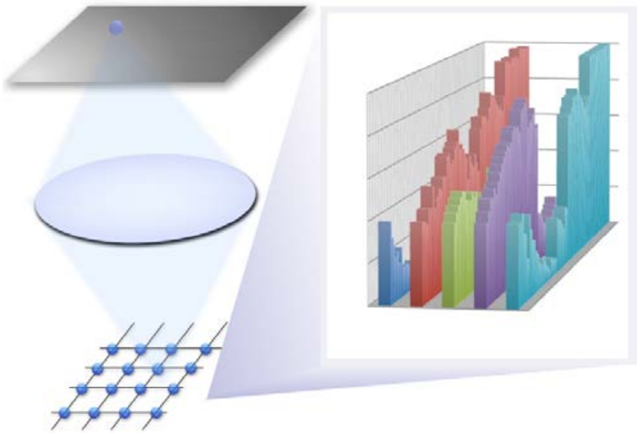
no single-site resolution



Y. Ashida and MU, PRL 115,095301 (2015)

continuous monitoring of atoms with single-site resolution

Our scheme dual to ENS experiments



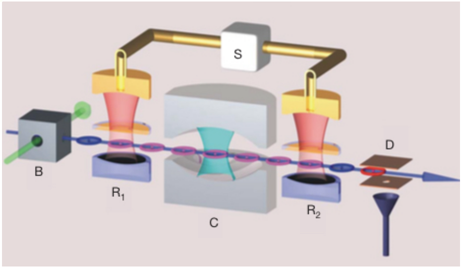
Atom

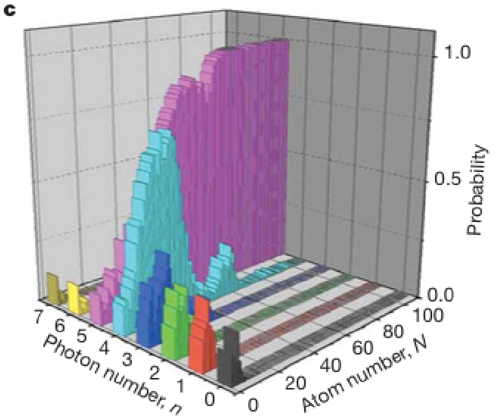
↕

Photon

Y. Ashida and MU, PRL 115,095301 (2015)

2012 Nobel Prize in Physics





S. Gleyzes *et al.*, *Nature* **446** 297 (2007)
C. Guerlin *et al.*, *Nature* **448** 889 (2007)

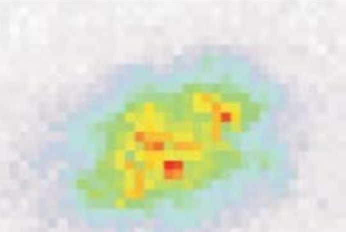
Quantum Gas Microscopy

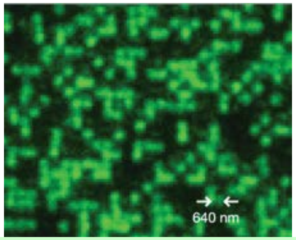
A new approach to quantum many-body systems via *in-situ* imaging of ultracold atoms

weak resolved imaging

single-site resolved imaging (quantum gas microscopy)

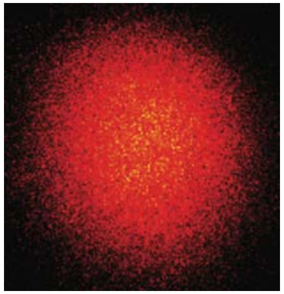
Destructive single-shot measurement

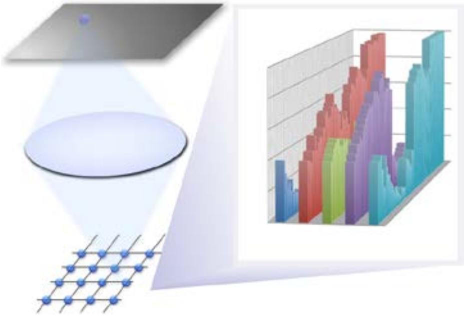




How does measurement backaction alter quantum many-body dynamics under continuous observation?

Nondestructive continuous Imaging of atoms





Y. Patil et al., PRA 90 033422 (2014)

Y. Ashida and MU, PRL 115,095301 (2015)

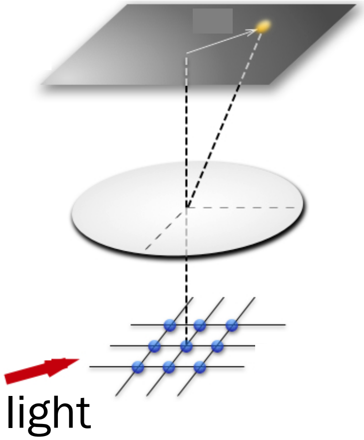
no single-site resolution

continuous monitoring of atoms with single-site resolution

Multi-Particle Quantum Dynamics under Continuous Observation

Our Model

To continuously monitor atoms in an optical lattice by spatially resolved measurement.



- **Measurement operator:**

measurement rate

$$\hat{M}(X) = \sqrt{\gamma}\sum_{m} f\left(X - md\right)\hat{n}_m$$

point spread function atom number operator

- **Dynamics:**

$$d|\psi\rangle = -\frac{i}{\hbar}\hat{H}|\psi\rangle dt$$

unitary evolution under Bose-Hubbard Hamiltonian

$$-\frac{1}{2}\int dX\left(\hat{M}^\dagger(X)\hat{M}(X) - \langle\hat{M}^\dagger(X)\hat{M}(X)\rangle\right)|\psi\rangle dt$$

non-unitary evolution without photodetection

$$+\int dX\left(\frac{\hat{M}(X)|\psi\rangle}{\sqrt{\langle\hat{M}^\dagger(X)\hat{M}(X)\rangle}} - |\psi\rangle\right)dN(X)$$

with photodetection Poisson stochastic process

The last term describes the jump process associated with photodetection, and can be modeled by the Poisson stochastic process: $dN(X)$.

Y. Ashida and M. Ueda, arXiv: 1510. 04001

Continuous-Measurement Limit

1. frequent $\gamma \gg J$

   measurement rate hopping rate

2. weak $\sigma \gg d$

   spatial resolution lattice constant

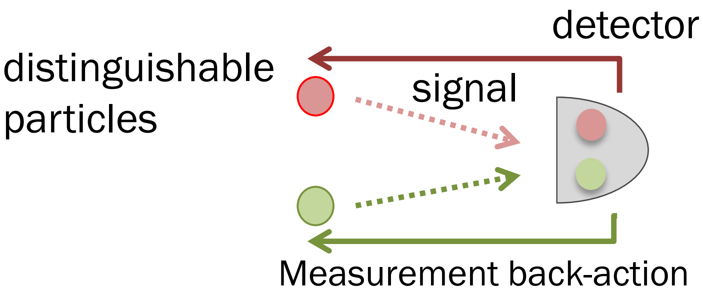
while keeping the ratio γ/σ^2 finite.

1. From the central limit theorem, the Poisson process replaced by the Wiener process.

$$dN(X) = \langle \hat{M}^\dagger(X)\hat{M}(X)\rangle dt + \sqrt{\langle \hat{M}^\dagger(X)\hat{M}(X)\rangle} dW(X)$$

2. Take the low resolution limit (σ→large) to keep the disturbance of measurement small.

Distinguishable Particles



The continuous monitoring model of distinguishable particles can be obtained by a simple generalization of the single particle model due to Diosi (1989).

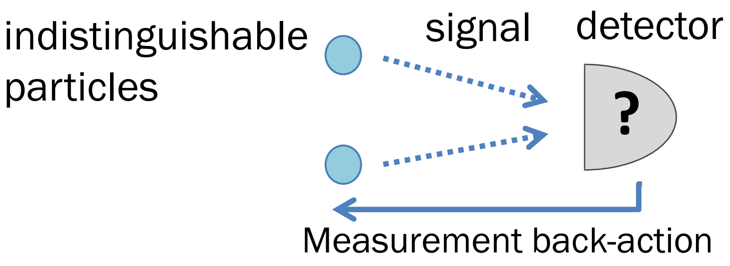
$$d\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]dt \quad \underbrace{-\frac{\gamma_{\mathrm{tot}}d^2}{4\sigma^2}\left[\hat{X}_{CM},\left[\hat{X}_{CM},\hat{\rho}\right]\right]dt+\sqrt{\frac{\gamma_{\mathrm{tot}}d^2}{2\sigma^2}}\left\{\hat{X}_{CM}-\langle\hat{X}_{CM}\rangle,\hat{\rho}\right\}dW}_{\text{center-of-mass decoherence}}$$

$$\underbrace{-\frac{d^2}{4\sigma^2}\sum_{i=1}^{N}\gamma_i\left[\hat{r}_i,\left[\hat{r}_i,\hat{\rho}\right]\right]dt+\sum_{i=1}^{N}\sqrt{\frac{\gamma_i d^2}{2\sigma^2}}\left\{\hat{r}_i-\langle\hat{r}_i\rangle,\hat{\rho}\right\}dW_i}_{\text{relative positional decoherence}}$$

The measurement back-action leads to the decoherence of the center-of-mass coordinate and that of the relative coordinates.

Y. Ashida and M. Ueda, arXiv: 1510. 04001

Indistinguishable Particles


indistinguishable particles

signal

detector

?

Measurement back-action

Indistinguishable particles should not recognize "relative positions." Mathematically, the relative coordinate part vanishes due to two-particle interference!

$$d\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]dt \quad \underbrace{-\frac{\gamma_{\mathrm{tot}}d^2}{4\sigma^2}\left[\hat{X}_{CM},\left[\hat{X}_{CM},\hat{\rho}\right]\right]dt+\sqrt{\frac{\gamma_{\mathrm{tot}}d^2}{2\sigma^2}}\left\{\hat{X}_{CM}-\langle\hat{X}_{CM}\rangle,\hat{\rho}\right\}dW}_{\text{center-of-mass decoherence}}$$

$$\underbrace{-\frac{d^2}{4\sigma^2}\sum_{i=1}^{N}\gamma_i\left[\hat{r}_i,\left[\hat{r}_i,\hat{\rho}\right]\right]dt+\sum_{i=1}^{N}\sqrt{\frac{\gamma_i d^2}{2\sigma^2}}\left\{\hat{r}_i-\langle\hat{r}_i\rangle,\hat{\rho}\right\}dW_i}_{\text{relative positional decoherence}}$$

Quantum interference suppresses the relative positional decoherence, and only the center-of-mass decoherence term remains.

Y. Ashida and M. Ueda, arXiv: 1510. 04001

Three Distinct Regimes for the Dynamics

$$d\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]dt \quad \underline{- \frac{\gamma_{\mathrm{tot}} d^2}{4\sigma^2}\left[\hat{X}_{CM},\left[\hat{X}_{CM},\hat{\rho}\right]\right]dt + \sqrt{\frac{\gamma_{\mathrm{tot}} d^2}{2\sigma^2}}\left\{\hat{X}_{CM} - \langle \hat{X}_{CM}\rangle,\hat{\rho}\right\}dW}$$

center-of-mass decoherence

(i) *Center-of-mass collapse regime*

A superposition between different center-of-mass states is decohered, and therefore the center-of-mass coordinate localizes.

(ii) *Inertial regime*

Indistinguishable particles move balistically.

(iii) *Diffusive regime*

The center-of-mass motion exhibits diffusive behavior due to the random, probabilistic nature of the measurement backaction.

Y. Ashida and M. Ueda, arXiv: 1510. 04001

Numerical Simulations: Quantum Walks of Two Particles

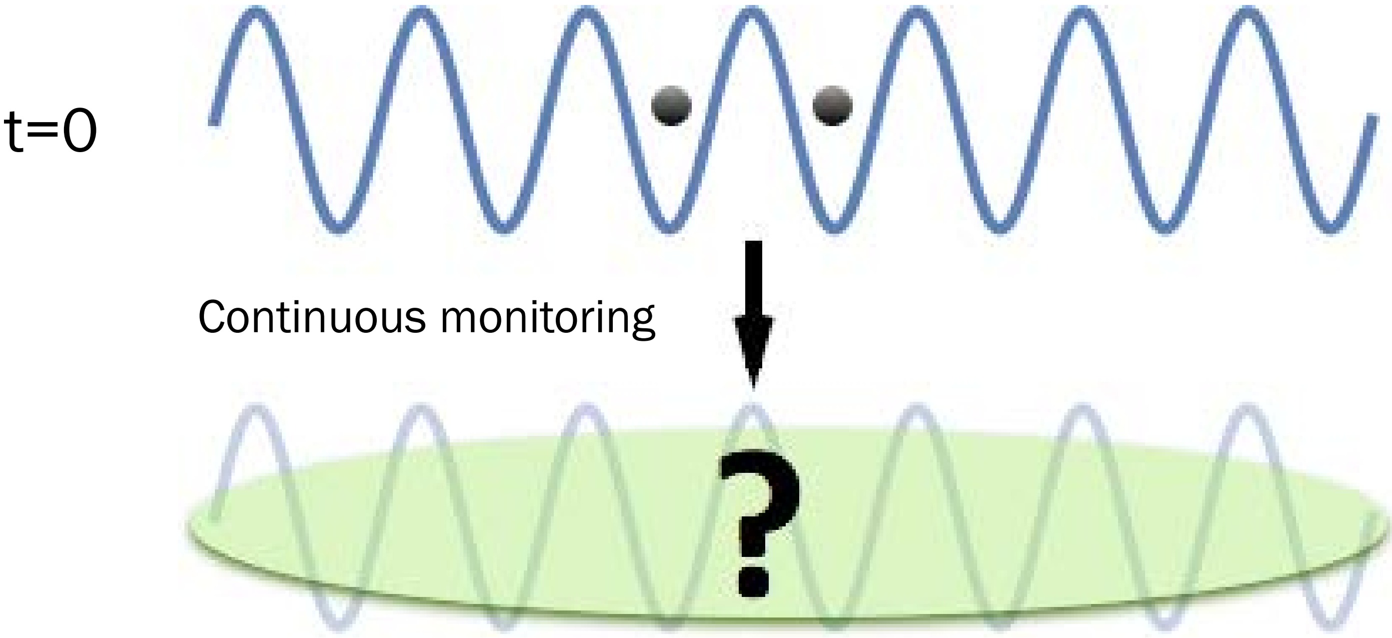
Initial condition: two particles at adjacent sites

t=0

Y. Ashida and M. Ueda, arXiv: 1510. 04001

Numerical Simulations: Quantum Walks of Two Particles

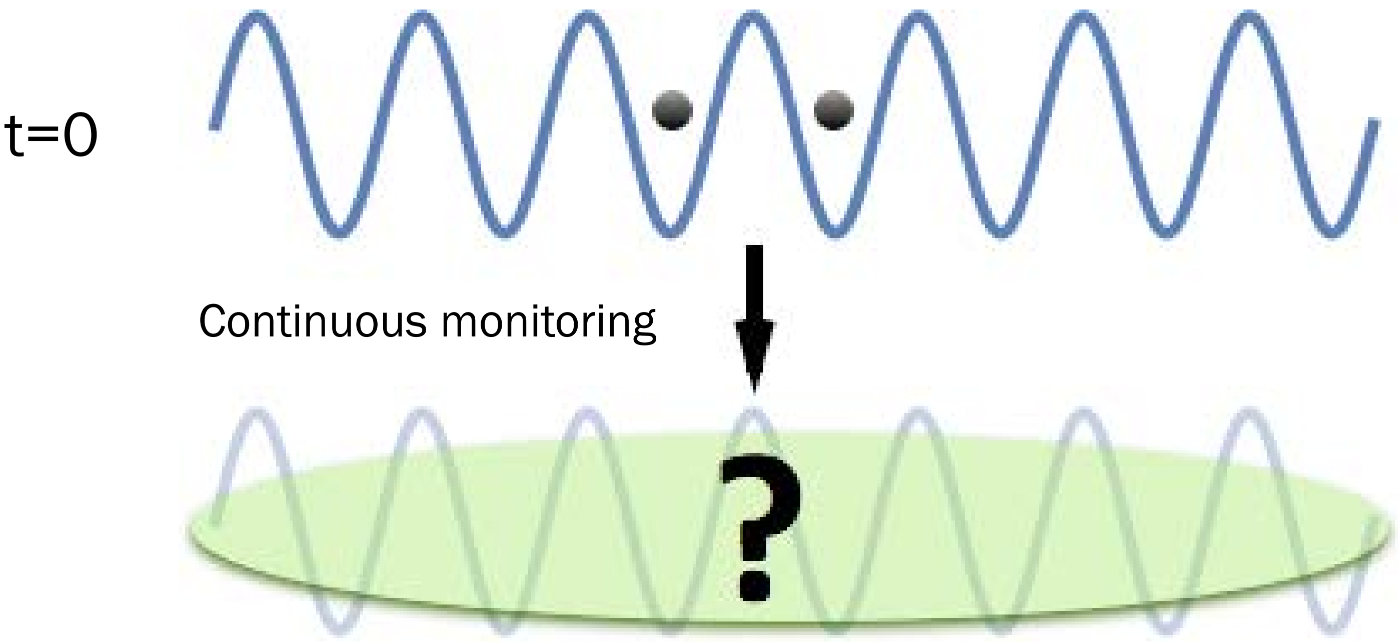
Initial condition: two particles at adjacent sites



Y. Ashida and M. Ueda, arXiv: 1510. 04001

Numerical Simulations: Quantum Walks of Two Particles

Initial condition: two particles at adjacent sites

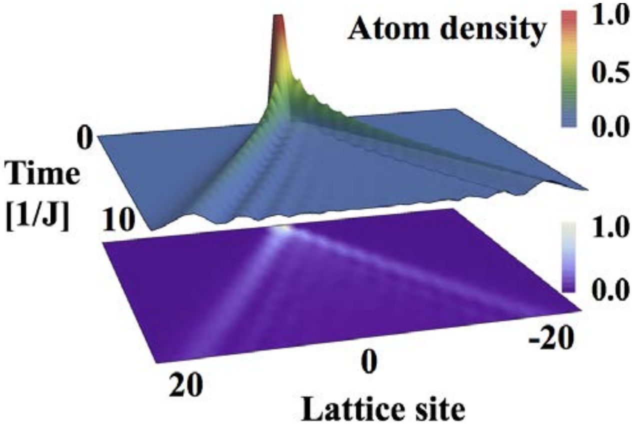


We compare quantum transport for
- distinguishable particles
- fermions
- bosons

Y. Ashida and M. Ueda, arXiv: 1510. 04001

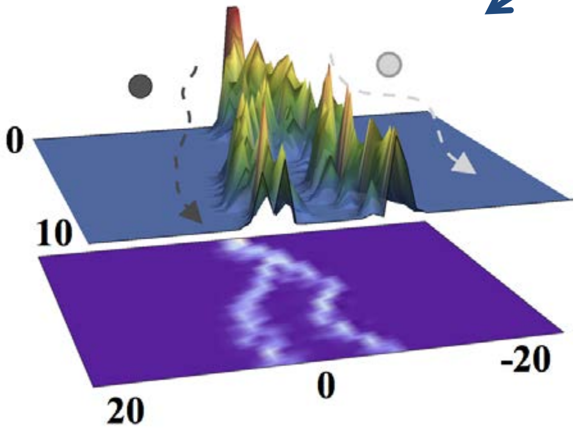
Distinguishable vs. Indistinguishable Quantum Transport

Without measurement:
unitary evolution

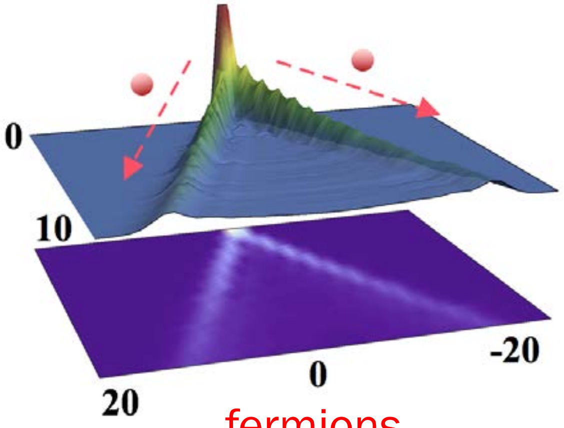


The two particles move away in opposite directions, and the density profile is almost independent of quantum statistics of particles.

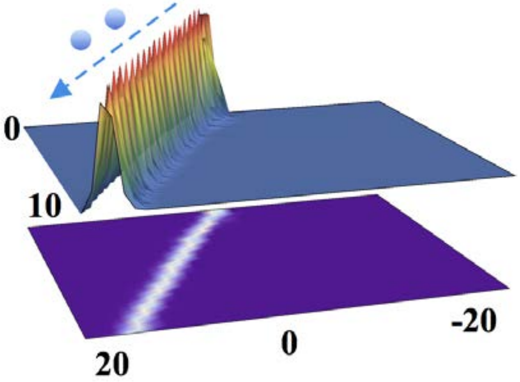
Under continuous observation



distinguishable



fermions



bosons

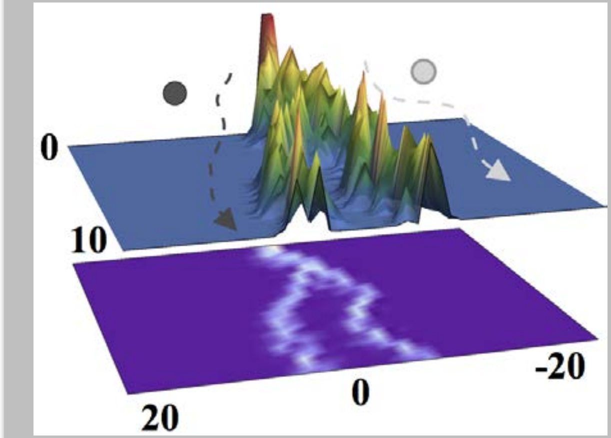
Y. Ashida and M. Ueda, arXiv: 1510. 04001

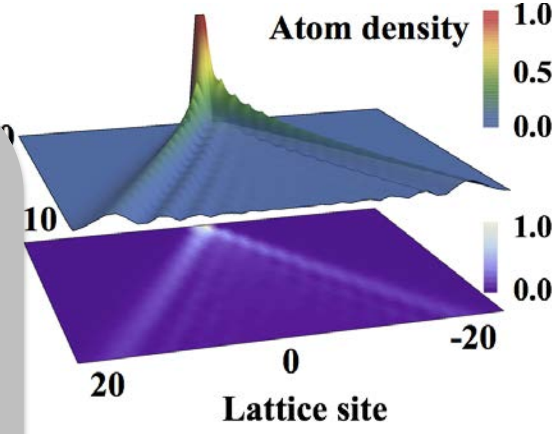
Distinguishable Particles

Distinguishable

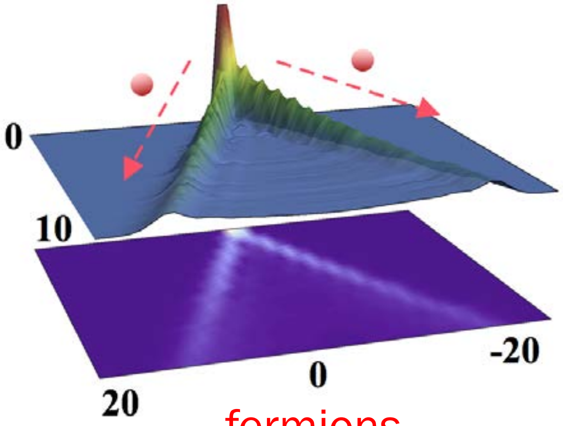
Measurement backaction
→ CM+relative positional decoherence

- uncorrelated
- diffusive random walk

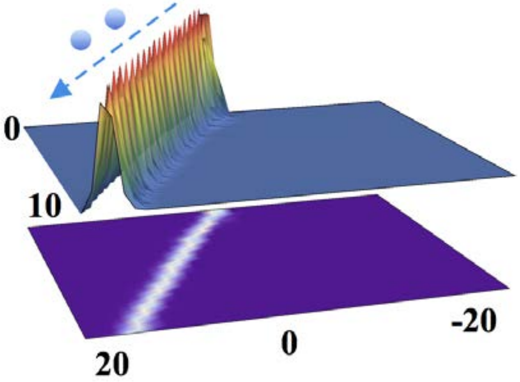




The density profile is almost independent of the species and distinguishability of particles.



fermions



bosons

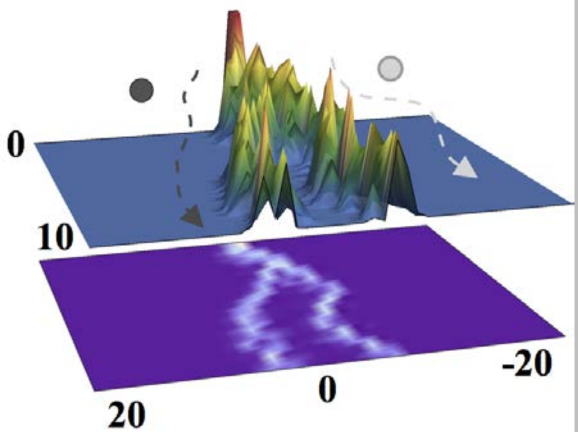
Y. Ashida and M. Ueda, arXiv: 1510. 04001

Indistinguishable Particles

Distinguishable

Measurement backaction
→ CM+relative positional
decoherence

- uncorrelated
- diffusive random walk

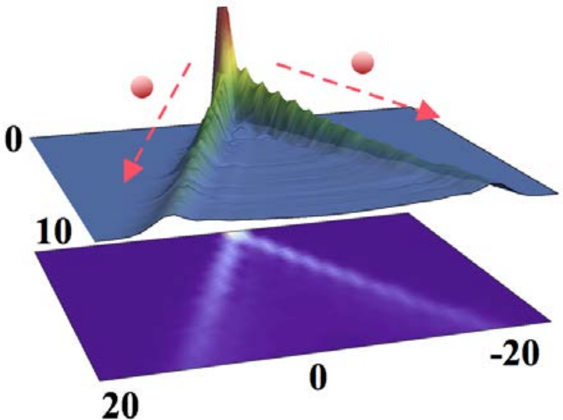


Indistinguishable

CM+relative position decoherence
↑
(no relative positional decoherence)

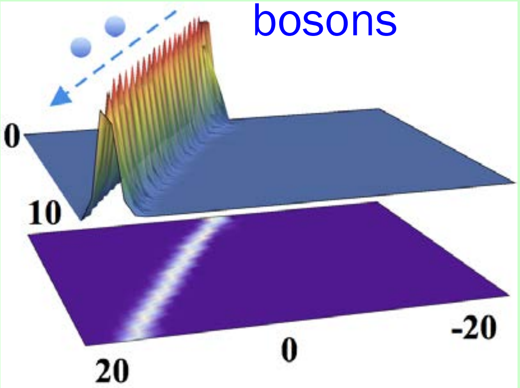
CM
- localized
- strongly correlated
- ballistic transport (inertial regime)

fermions



antibunching

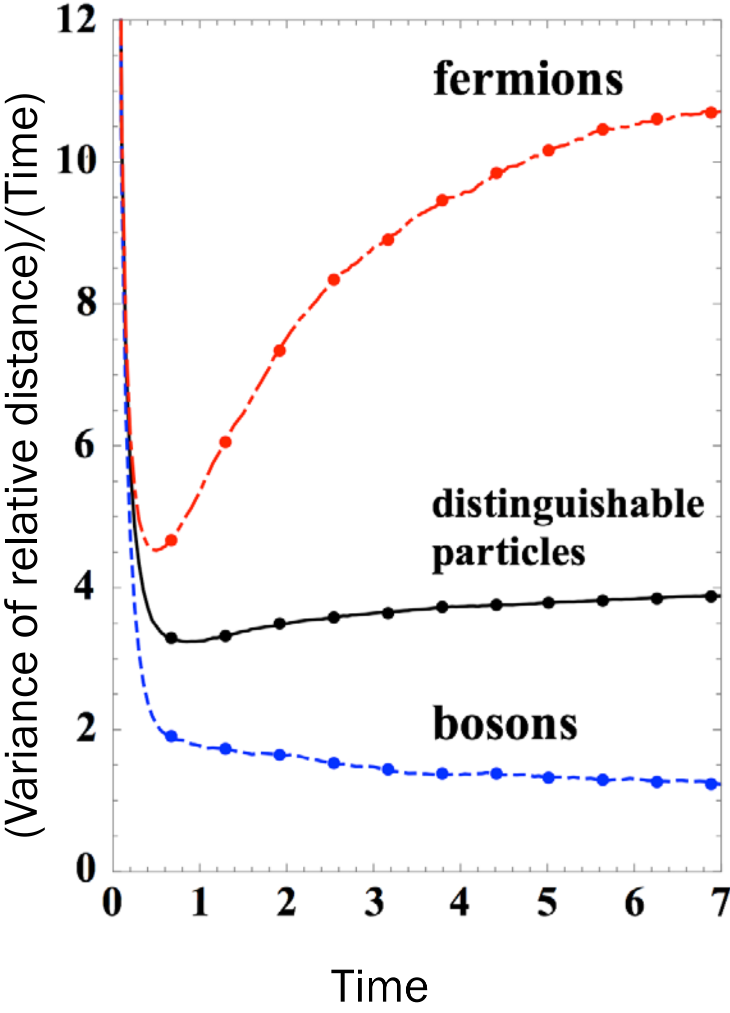
bosons

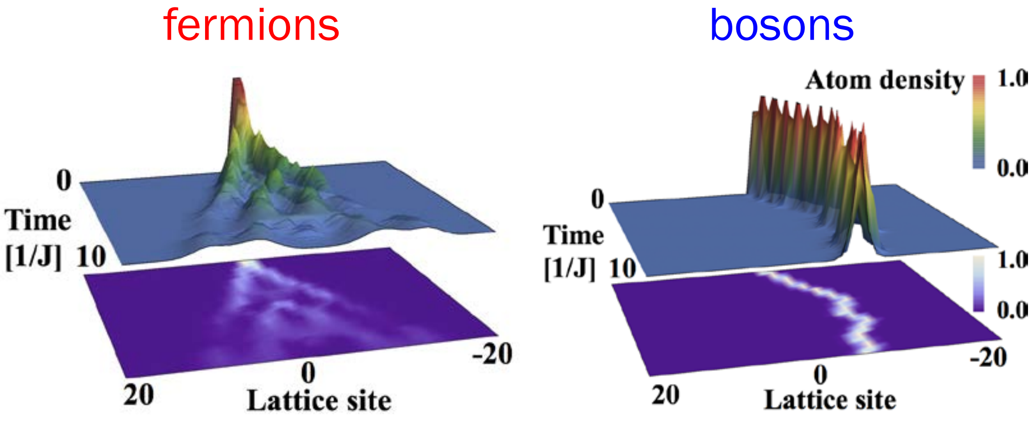


bunching

Y. Ashida and M. Ueda, arXiv: 1510. 04001

Dynamics for Strong Measurement: Diffusive Transport





classical diffusion constant: $2D_c = \dfrac{32J^2\sigma^2}{\gamma\hbar^2 d^2}$

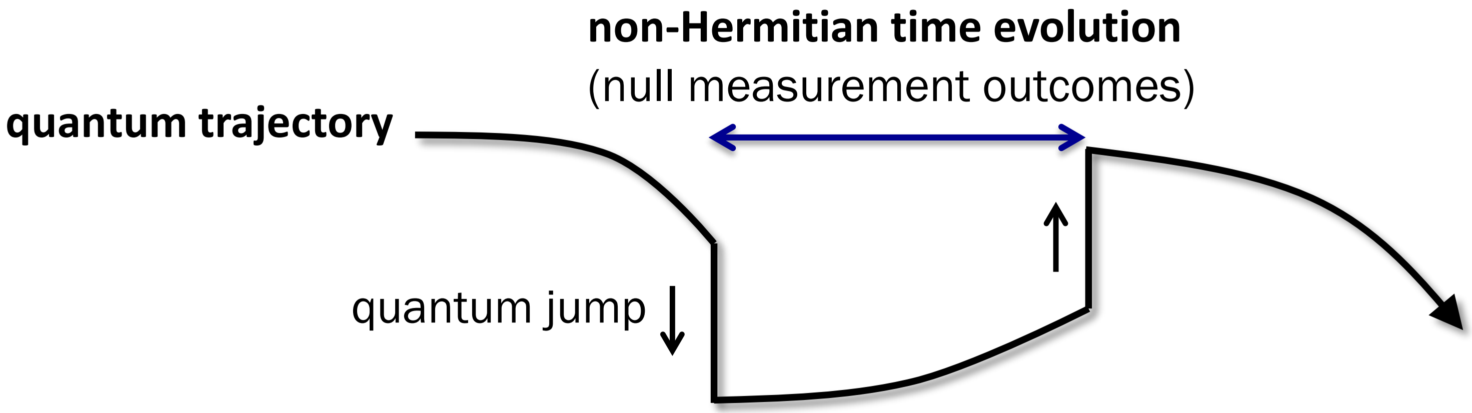
- **Quantum statistics** serves as effective repulsion for fermions and attraction for boson.
- Fermions show faster diffusion and bosons show slower diffusion than distinguishable particles.

Y. Ashida and M. Ueda, arXiv: 1510. 04001

Quantum Critical Phenomena under Continuous Observation

The Model

— continuously monitored quantum many-body system —



effective Hamiltonian: $\hat{H}_{\text{eff}} = \hat{H} \underline{- \frac{i\gamma}{2} \sum_i \hat{M}_i^\dagger \hat{M}_i}$ $\hat{M}_i$: jump operators

measurement backaction for null measurement outcomes

*Ensemble average over all outcomes ⟶ Dynamics described by the Lindblad eq.
⟶ Quantum correlations and criticality are smeared out.

J. Schachenmayer et al., PRA 89, 011601 (2014).
Y. Yanay and E. J. Mueller, PRA 90, 023611 (2014).

Properties and Interpretation of non-Hermitian Hamiltonian

- **effective Hamiltonian:** $\hat{H}_{\text{eff}} = \hat{H} - \frac{i\gamma}{2}\sum_i \hat{M}_i^\dagger \hat{M}_i$ $\hat{H}$ exhibits quantum criticality.

- **complex eigenvalues:** $E_\lambda - \frac{i\Gamma_\lambda}{2}$

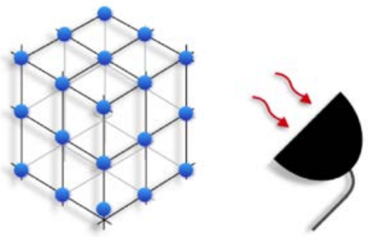
E_λ	real part: effective **energy** of the system
Γ_λ	imaginary part: **decay rate** into outside of the Hilbert space

- **effective ground state:** $|\Psi_{\text{GS}}\rangle$ eigenstate of $\hat{H}_{\text{eff}}$ having the **lowest** eigenenergy E_λ

In our models, $|\Psi_{\text{GS}}\rangle$ also has the **minimal** Γ_λ (the **longest** lifetime).

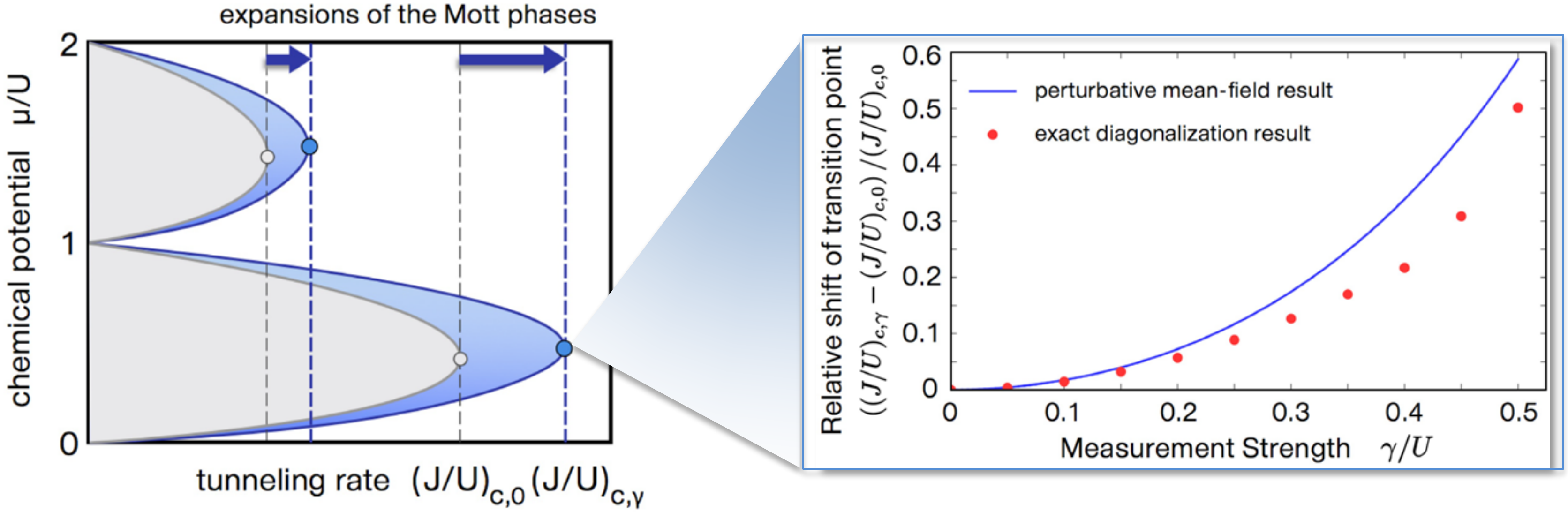
Superfluid-Mott Insulator Transition in Bose-Hubbard Model

- **jump process: two-body loss**



$$\hat{H}_{\mathrm{eff}} = \hat{H}_{\mathrm{BH}} - \frac{i\gamma}{2}\sum_i \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i \qquad \hat{M}_i = \hat{b}_i^2$$

The transition point is identified as the point at which an energy gap of the ground state vanishes.



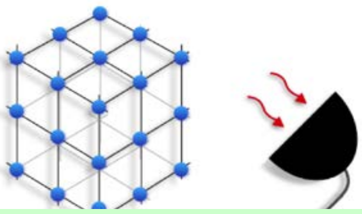
The measurement backaction shifts the transition point, so that the Mott lobes expand.

Y. Ashida, S. Furukawa, and M. Ueda, Phys. Rev. A **94**, 053615 (2016)

Superfluid-Mott Insulator Transition in Bose-Hubbard Model

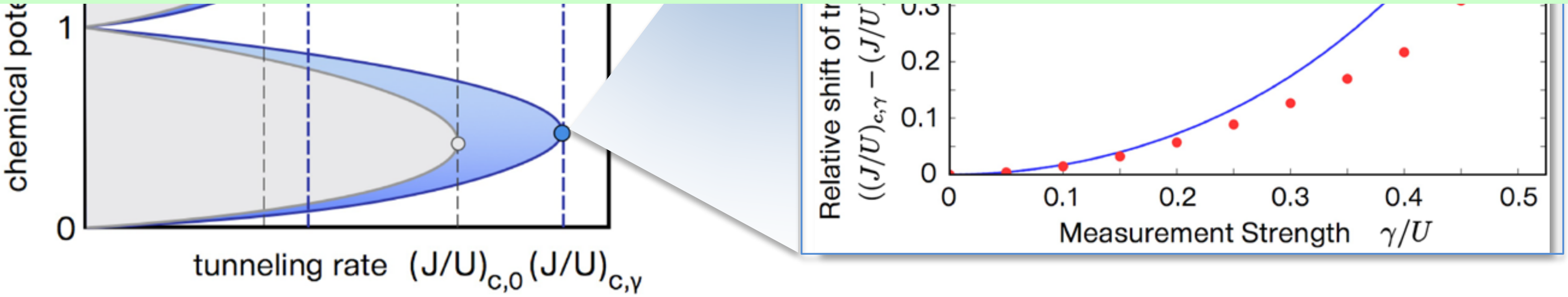
- **jump process: two-body loss**

$$\hat{H}_{\mathrm{eff}} = \hat{H}_{\mathrm{BH}} - \frac{i\gamma}{2}\sum_i \hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i \hat{b}_i \qquad \hat{M}_i = \hat{b}_i^2$$



The transition point is identified as the point at which an energy gap of the ground state vanishes.

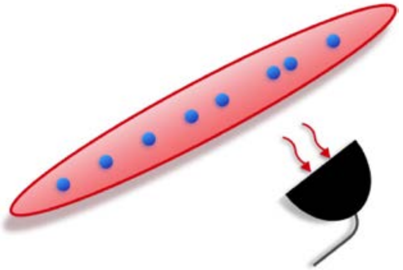
Expansion of Mott lobes can be interpreted as suppression of the hopping rate due to continuous quantum Zeno effect



The measurement backaction shifts the transition point, so that the Mott lobes expand.

Y. Ashida, S. Furukawa, and M. Ueda, Phys. Rev. A **94**, 053615 (2016)

Quantum Critical Phase of a 1D Bose Gas



- **Jump process: two-body loss**
  → modified Lieb-Liniger model

inelastic scattering

$$\hat{H}_{\mathrm{LL}} = \int dx \left[\hat{\Psi}^{\dagger}(x)\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\hat{\Psi}(x) + \frac{g-i\gamma}{2}\hat{\Psi}^{\dagger}(x)\hat{\Psi}^{\dagger}(x)\hat{\Psi}(x)\hat{\Psi}(x)\right]$$

- **Low-energy Hamiltonian = non-Hermitian Tomonaga-Luttinger liquid**

$$\hat{H}_{\mathrm{eff}} = \frac{\hbar}{2\pi}\int_{-\infty}^{\infty} dx \left[v_J(\partial_x\hat{\phi})^2 + \tilde{v}_N e^{-i\delta_\gamma}(\partial_x\hat{\theta})^2\right]$$

non-Hermiticity: $\delta_\gamma \to 0 \ \ (\gamma \to 0)$

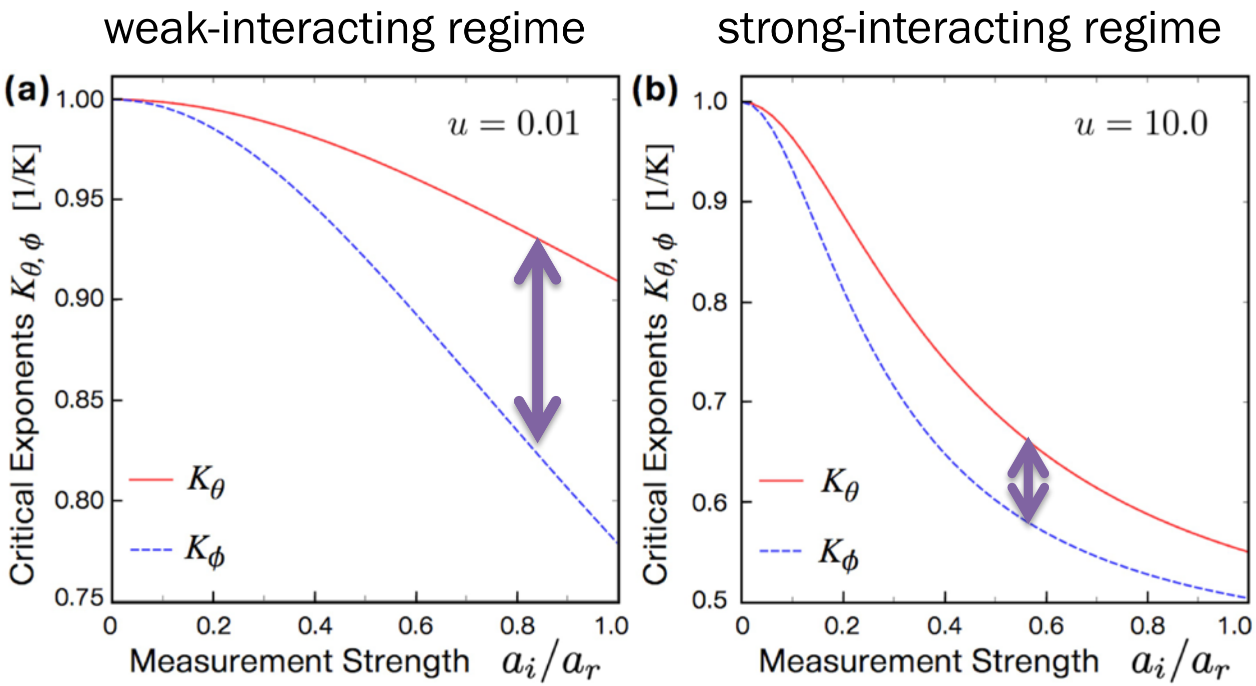
Y. Ashida, S. Furukawa, and M. Ueda, Phys. Rev. A **94**, 053615 (2016)

Bifurcation of critical exponent: a unique universality class beyond TLL class

- **Measurement backaction bifurcates the critical exponent into two.**

one-particle correlation: $$\langle \hat{\Psi}^\dagger(r)\hat{\Psi}(0)\rangle \propto \left(\frac{1}{r}\right)^{\underline{\frac{1}{2K_\phi}}}$$

density-density correlation: $$\langle \hat{\rho}(r)\hat{\rho}(0)\rangle - \rho_0^2 = -\frac{K_\theta}{2\pi^2 r^2} + \text{const.} \times \frac{\cos(2\pi\rho_0 r)}{r^{\underline{2K_\theta}}}$$



$u \equiv mg/(\hbar^2\rho_0)$

Y. Ashida, S. Furukawa, and M. Ueda, Phys. Rev. A **94**, 053615 (2016)

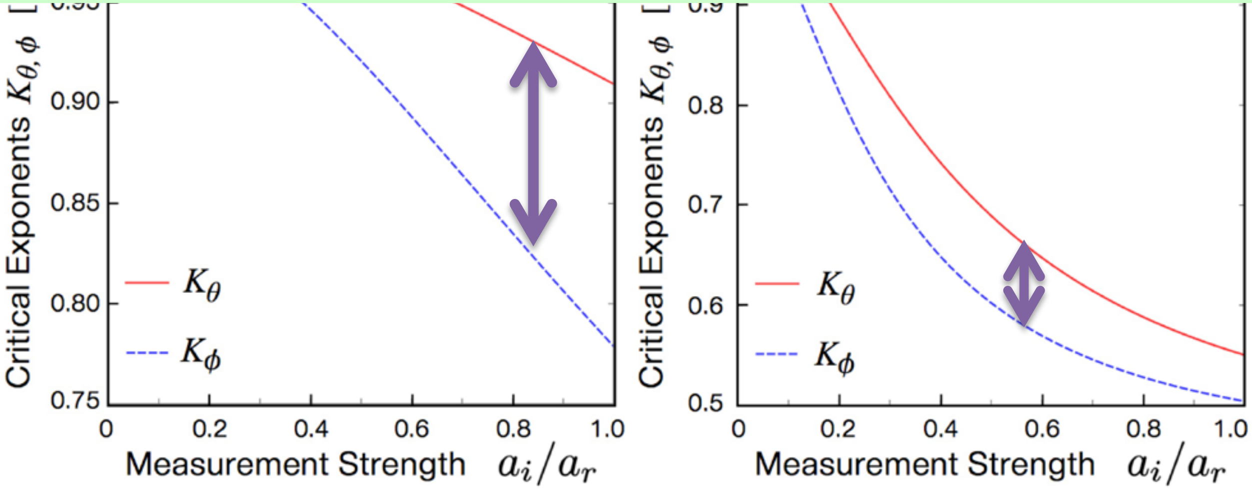
Bifurcation of critical exponent: a unique universality class beyond TLL class

- **Measurement backaction bifurcates the critical exponent into two.**

one-particle correlation: $\langle \hat{\Psi}^\dagger(r)\hat{\Psi}(0)\rangle \propto \left(\frac{1}{r}\right)^{\underline{\frac{1}{2K_\phi}}}$

density-density correlation: $\langle \hat{\rho}(r)\hat{\rho}(0)\rangle - \rho_0^2 = -\frac{K_\theta}{2\pi^2 r^2} + \mathrm{const.} \times \frac{\cos(2\pi\rho_0 r)}{r^{\underline{2K_\theta}}}$

Emergence of two different exponents indicates the unique critical behavior beyond TLL universality class.



Y. Ashida, S. Furukawa, and M. Ueda, Phys. Rev. A **94**, 053615 (2016)

Summary

Multi-particle dynamics under continuous observation

- We have derived a continuous position measurement model of indistinguishable particles.
- In contrast to distinguishable particles, indistinguishability protects the system from relative positional decoherence.
  → decoherence-free subspace

Quantum critical behavior influenced by measurement backaction

- By analyzing non-Hermitian many-body Hamiltonians, we show that the measurement backaction shifts the quantum critical point and leads to a unique 1D quantum critical phase beyond the standard universality class of the Tomonaga-Luttinger liquid.

Y. Ashida and M. Ueda, arXiv: 1510. 04001

Y. Ashida, S. Furukawa, and M. Ueda, Phys. Rev. A **94**, 053615 (2016)