

Charm hadron interactions from lattice QCD

Yoichi Ikeda (RCNP, Osaka University)

HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

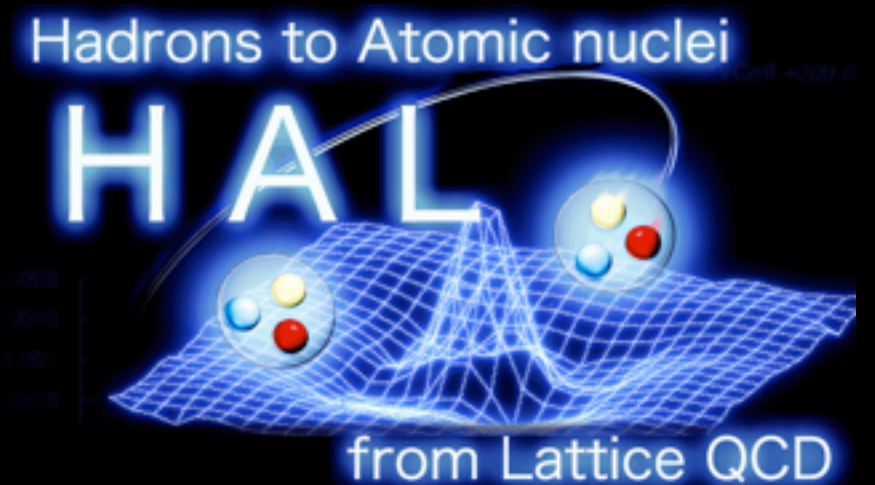
S. Aoki, T. Aoyama, D. Kawai, T. Miyamoto, K. Sasaki (YITP, Kyoto Univ.)

T. Doi, S. Gongyo, T. Hatsuda, T. Iritani (RIKEN)

Y. Ikeda, N. Ishii, K. Murano (RCNP, Osaka Univ.)

T. Inoue (Nihon Univ.)

H. Nemura (Univ. Tsukuba)



International Workshop on Strangeness Nuclear Physics (SNP2017)

@ Osaka Electro-Communication University, Osaka, Mar. 12--14, 2017.

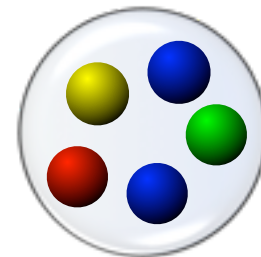
Why charm quark?

★ Good testing ground to understand nature of non-Abelian interactions

- **Exotic hadrons** have not been established yet, but many candidates are experimentally reported in heavy (charm & bottom) quark sectors



▶ tetraquarks
(X, Y, Z_c, Z_b, \dots)



▶ pentaquarks
(P_c, \dots)

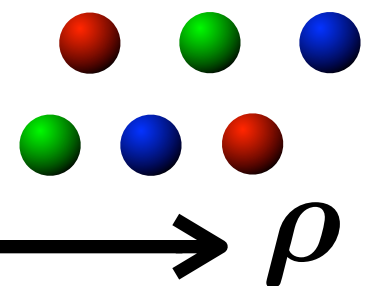
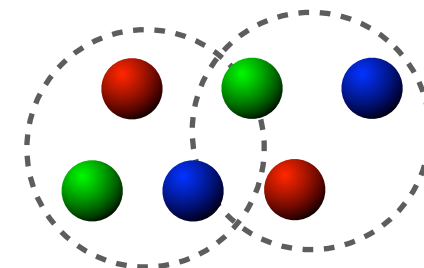
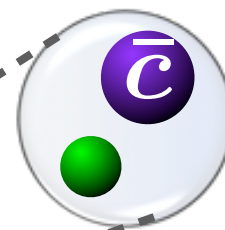
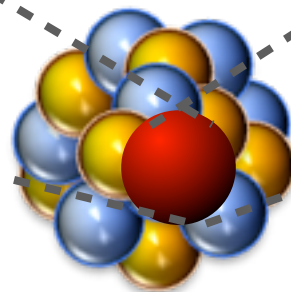
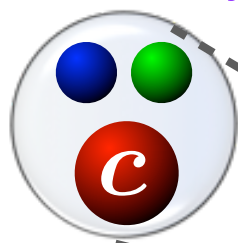
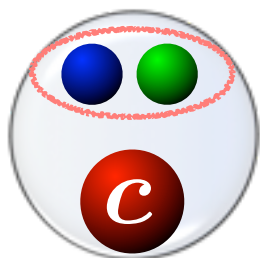
- **Charm baryons & nuclei** may tell nature of non-Abelian interactions (color, spin, isospin)

- Λ_c -nuclei (spin)

talk by Miyamoto (Mon.)

- D^{bar} -nuclei (isospin)

- $\Lambda_c^{(*)}, \Sigma_c^{(*)}$ spectra (color)



- ▶ heavy quark as color source
(diquark DoF?)

talk by Noumi (Mon.)

- ▶ heavy hadrons as impurity
(emergence of Kondo effect?)

talk by Yasui (Mon.)

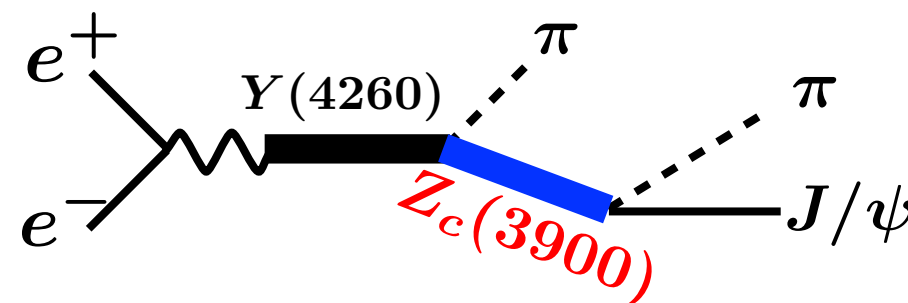
- ▶ property of quark matter?

Topics covered in this talk

❖ Tetraquark candidate $Z_c(3900)$

[Y. Ikeda et al., \(HAL QCD\), PRL117, 242001 \(2016\).](#)

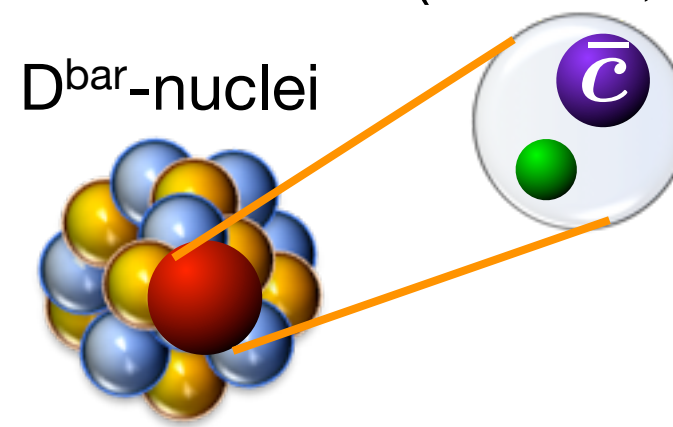
- ✓ How to analyze exotic hadrons on the lattice
- ✓ Coupled-channel HAL QCD method
- ✓ Nature of $Z_c(3900)$
- ✓ Comparison with experiments



❖ D^{bar} -nucleon interaction & D^{bar} -nuclei

- ✓ D^{bar} -nucleon interaction from LQCD
- ✓ Possible structure of D^{bar} -nuclei

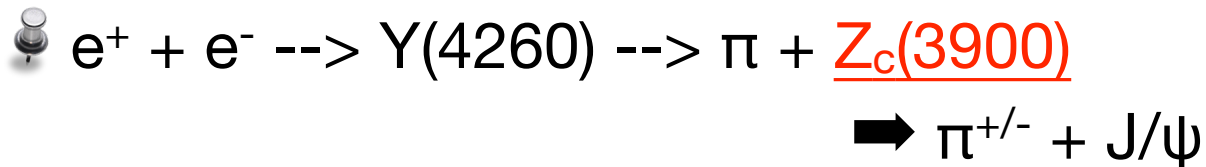
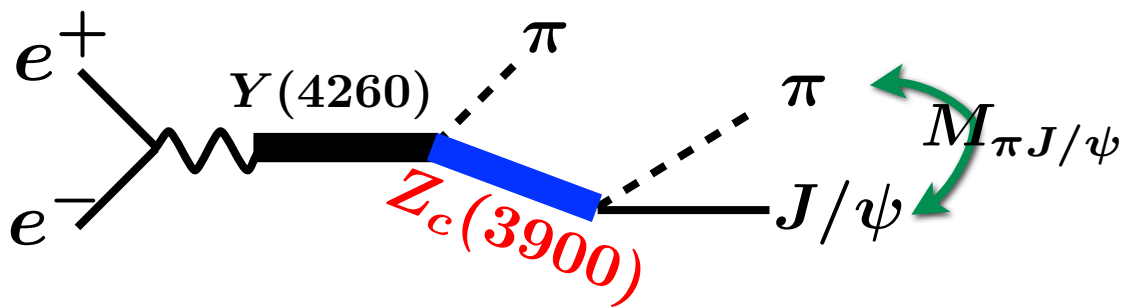
$$D^{\text{bar}} = (D^0 = u\bar{c}, D^- = d\bar{c})$$



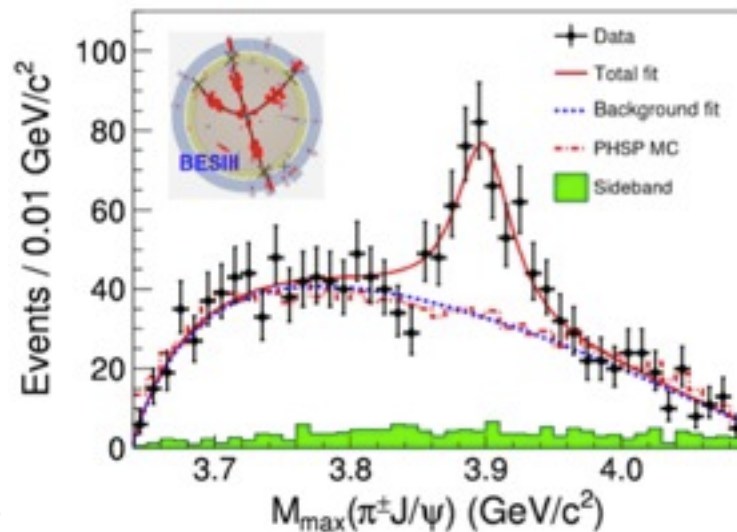
❖ Summary

What is $Z_c(3900)$?

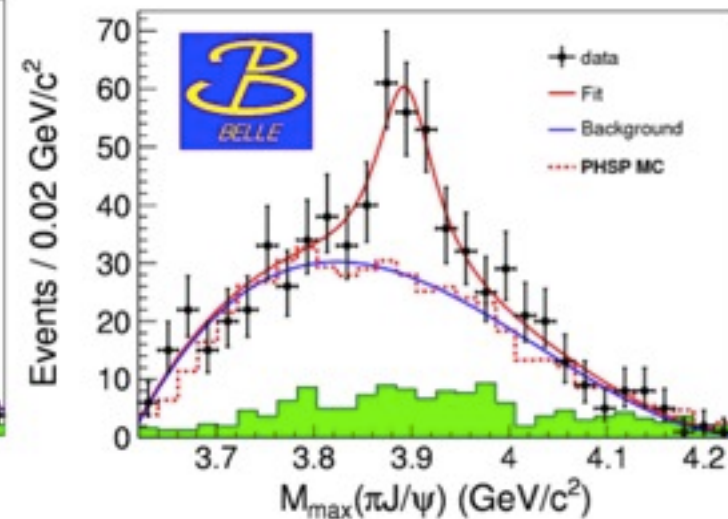
● Tetraquark $Z_c(3900)$?



BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).



- peak in $\pi^{+/-} J/\psi$ invariant mass (minimal quark content $cc^{\text{bar}} ud^{\text{bar}}$ \leftrightarrow tetraquark candidate)
- $M \sim 3900$, $\Gamma \sim 60$ MeV (Breit-Wigner) \rightarrow just above $D^{\text{bar}} D^*$ threshold
 $(J^P=1^+ \leftrightarrow$ couple to **s-wave** meson-meson continuum)

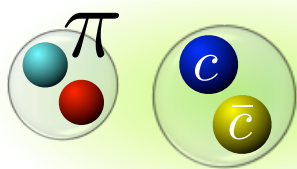
★ structure of $Z_c(3900)$ from models

tetraquark



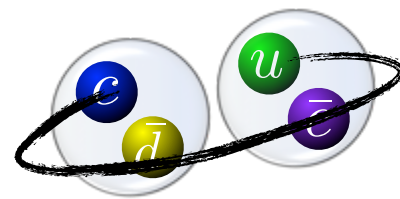
Maiani et al.('13)

$J/\psi + \pi$ atom



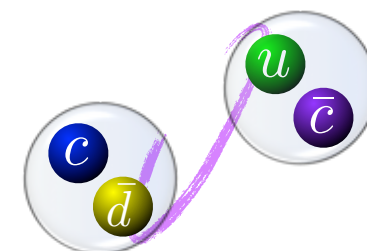
Voloshin('08)

$D^{\text{bar}} D^*$ molecule



Nieves et al.('11) + many others

$D^{\text{bar}} D^*$ threshold effect

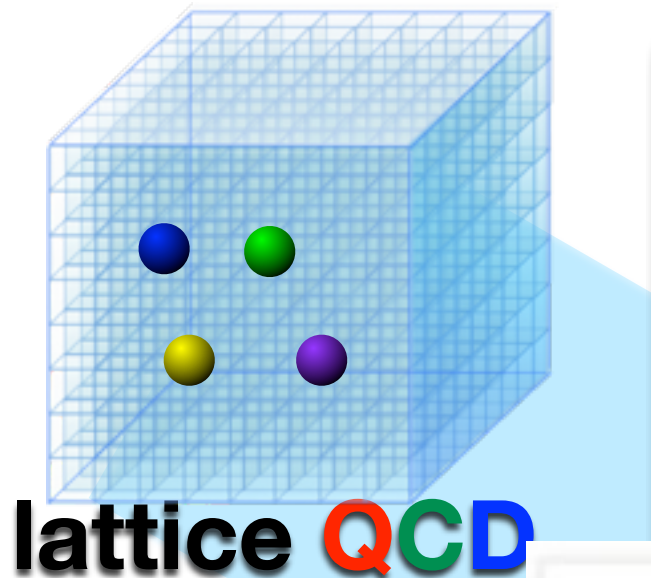


Chen et al.('14), Swanson('15)

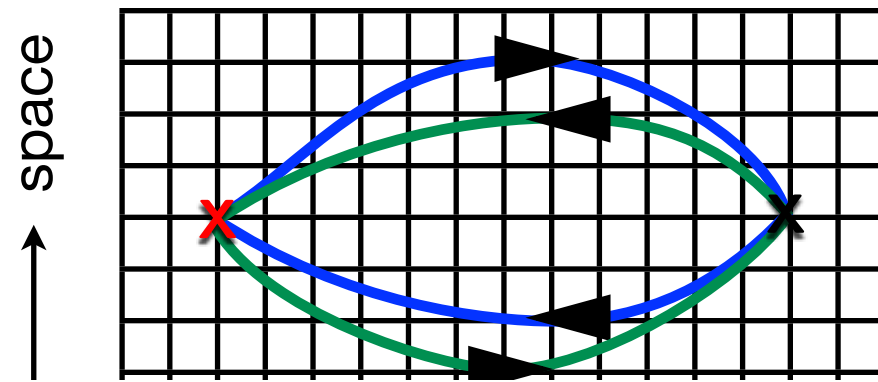
genuine state expected

kinematical origin

Exotic hadrons on the lattice



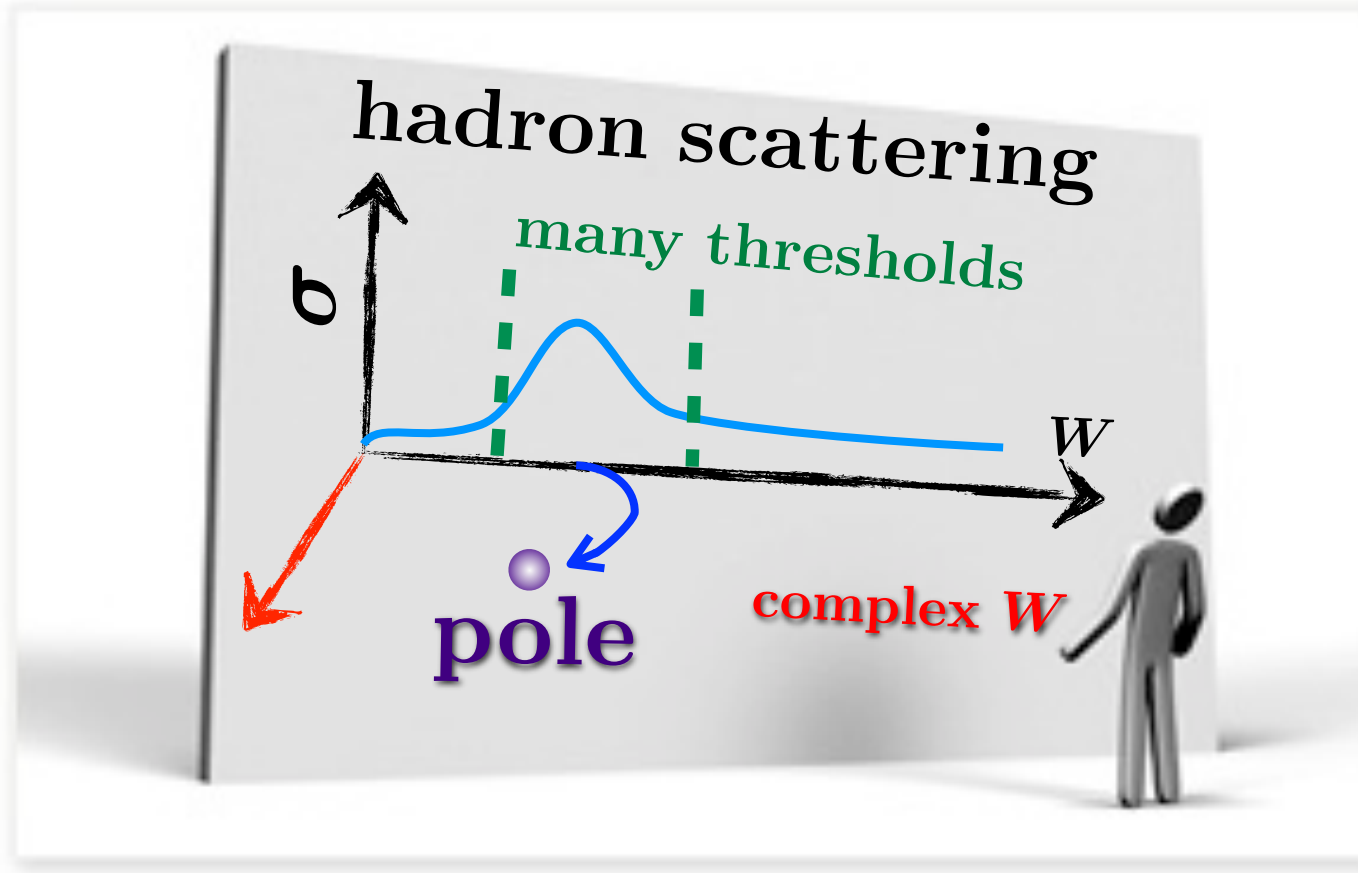
Lattice QCD spectroscopy by conventional approach



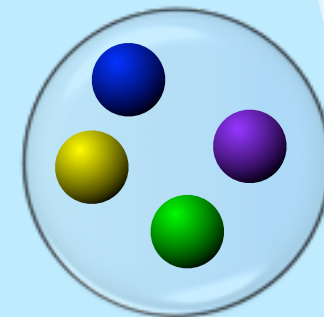
temporal corr. = $\langle 4q(\tau) 4q(0) \rangle$

$$= A_1 e^{-W_1 \tau} + A_2 e^{-W_2 \tau} + \dots$$

(W_1, W_2, \dots ; QCD eigen-energies)



Exotic hadrons



- ★ Exotic hadrons are **resonance** in coupled-channel scatterings
- ★ **Resonance energies** are **NOT** QCD eigen-energies (W_1, W_2, \dots), but **determined by pole of S-matrix on complex energy plane**

How to search for resonances?

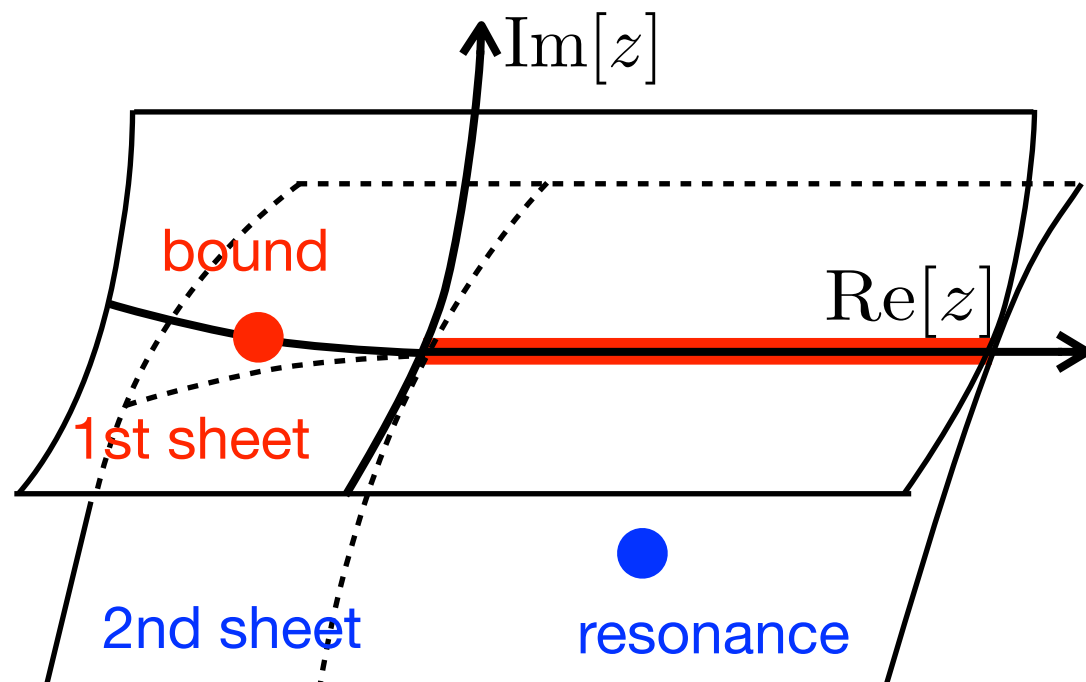
If **S-matrix based on QCD** (in local energy regime) **obtained...**

$S(W)$

← **partial wave analysis of expt. data**

- ▶ cross sections ($d\sigma/d\Omega$)
- ▶ spin polarization observables
- ▶ etc.

Analyticity of S-matrix is **uniquely** determined (**S-matrix theory**)



bound state (1st sheet)

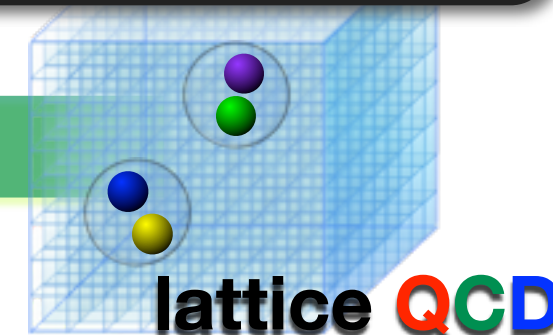
- pole position --> binding energy
- residue --> coupling to scattering state

resonance (2nd sheet)

- analytic continuation onto 2nd sheet
- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

How to search for resonances **on the lattice**

S-matrix based on QCD (in local energy regime)

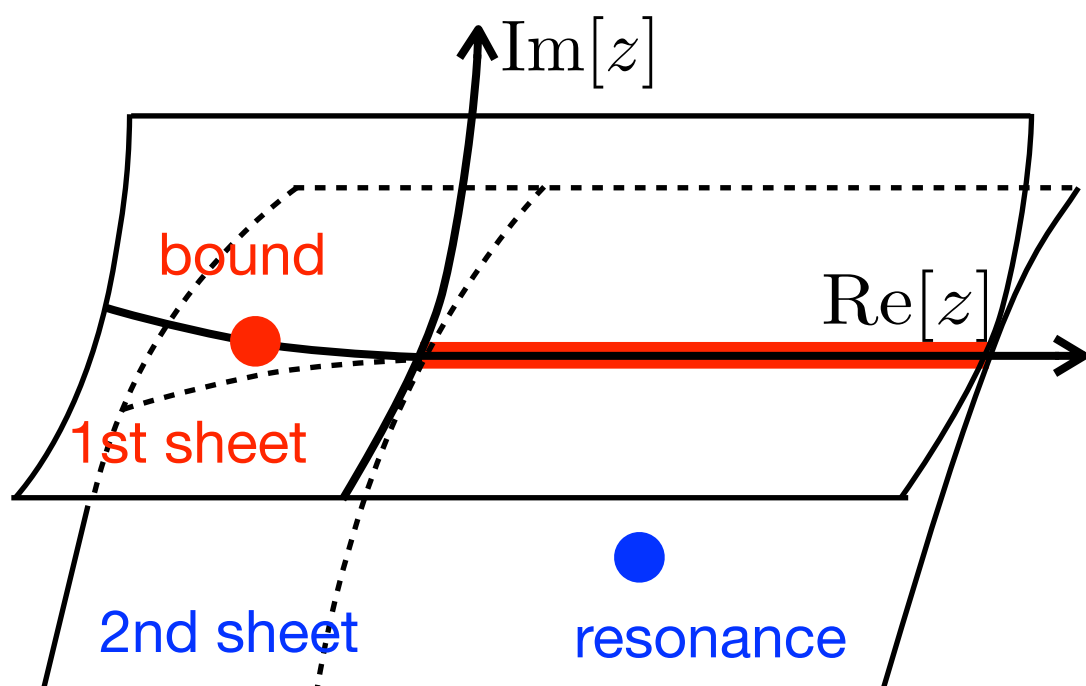


$S(W)$

two identical methods

- ▶ **Lüscher**: temporal correlation --> eigen-energy W_n --> $S(W)$
(difficulty in coupled-channel scattering & large volume simulation)
- ▶ **HAL QCD**: space-time correlation --> “potentials” --> $S(W)$

Analyticity of S-matrix is **uniquely** determined (**S-matrix theory**)



bound state (1st sheet)

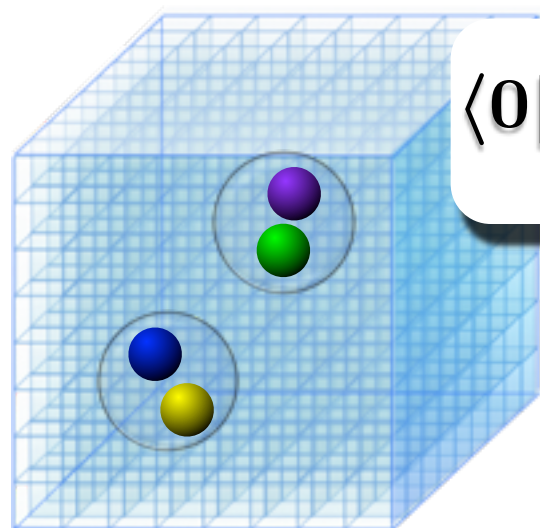
- pole position --> binding energy
- residue --> coupling to scattering state

resonance (2nd sheet)

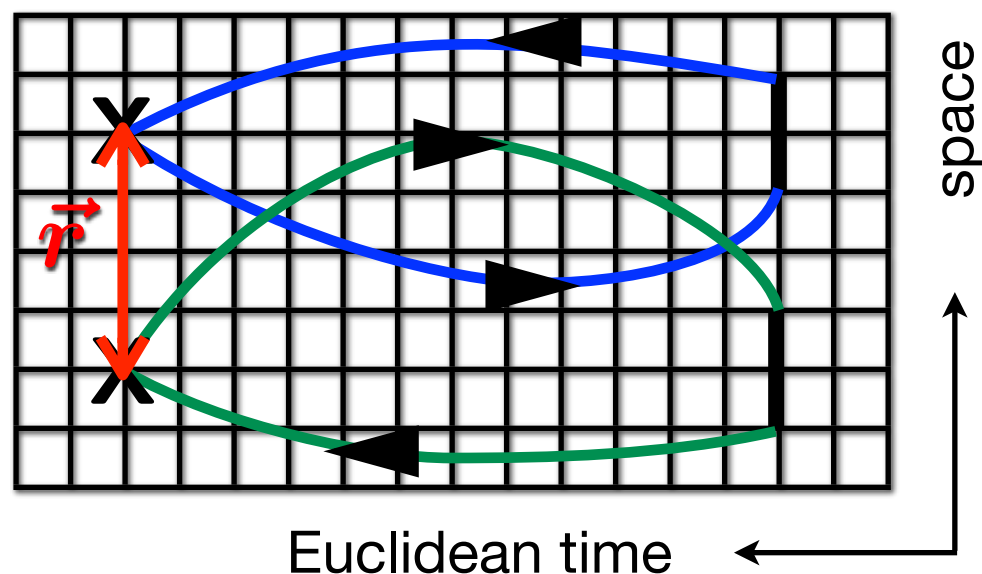
- analytic continuation onto 2nd sheet
- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

HAL QCD method “potentials as representation of S-matrix”

◆ measure not only temporal but also **spatial** correlation



$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1 Z_2} \sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$



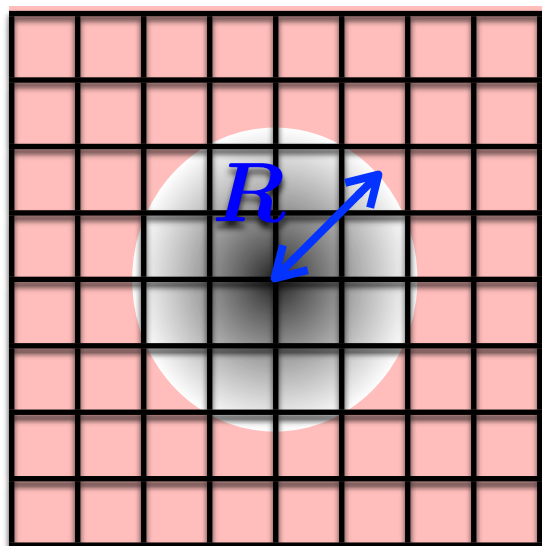
Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. (HAL QCD), PLB712, 437(2012).

★ **Nambu-Bethe-Salpeter (NBS) wave function: $\psi_n(\mathbf{r})$**

▶ NBS wave functions ($r > |R|$) satisfy **Helmholtz equation**

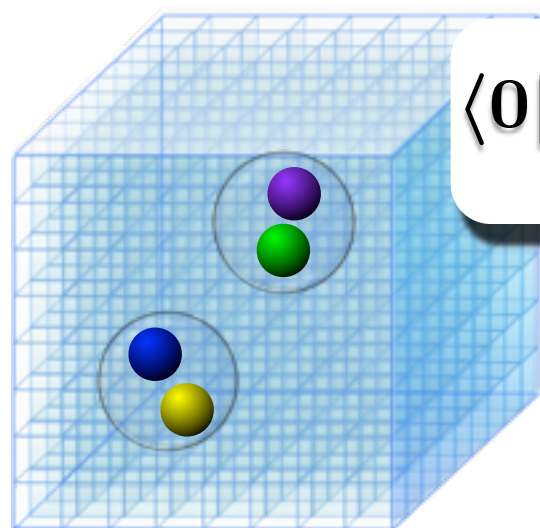
$$\left(\nabla^2 + \vec{k}_n^2 \right) \psi_n(\vec{r}) = 0 \quad (|\vec{r}| > R)$$

➔ **faithful to S-matrix** ($S(W) = e^{2i\delta(W)}$)

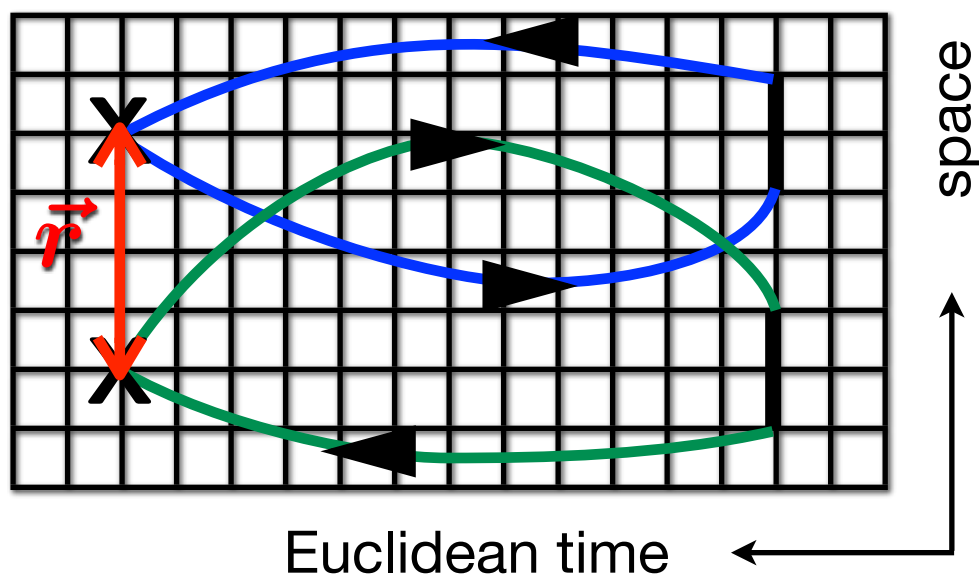


HAL QCD method “potentials as representation of S-matrix”

◆ measure not only temporal but also **spatial** correlation



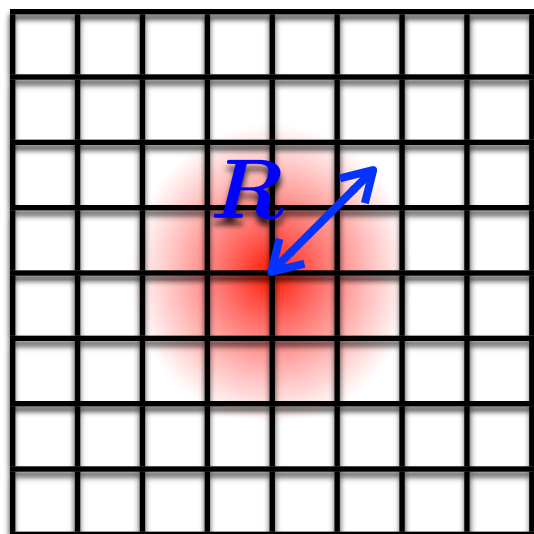
$$\langle 0 | \phi_1(\vec{x} + \vec{r}, \tau) \phi_2(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1 Z_2} \sum_n A_n \psi_n(\vec{r}) e^{-W_n \tau}$$



Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
 Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
 Ishii et al. (HAL QCD), PLB712, 437(2012).

★ **NBS wave function** ($r < |R|$) --> “**potential U(r,r')**”

$$\left(\nabla^2 + \vec{k}_n^2 \right) \psi_n(\vec{r}) = 2\mu \int d\vec{r}' U(\vec{r}, \vec{r}') \psi_n(\vec{r}')$$

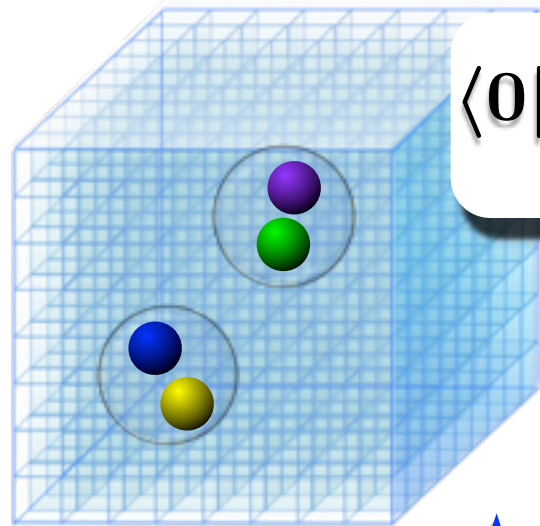


- U(r,r') is faithful to **S-matrix**
- U(r,r') is **energy independent** (until new threshold opens)
- U(r,r') contains all 2PI contributions

(Non-relativistic approximation is **NOT** necessary)

Coupled-channel HAL QCD method

◆ measure not only temporal but also **spatial** correlation



$$\langle 0 | \phi_1^a(\vec{x} + \vec{r}, \tau) \phi_2^a(\vec{x}, \tau) \Phi^\dagger(0) | 0 \rangle = \sqrt{Z_1^a Z_2^a} \sum_n A_n \psi_n^a(\vec{r}) e^{-W_n \tau}$$

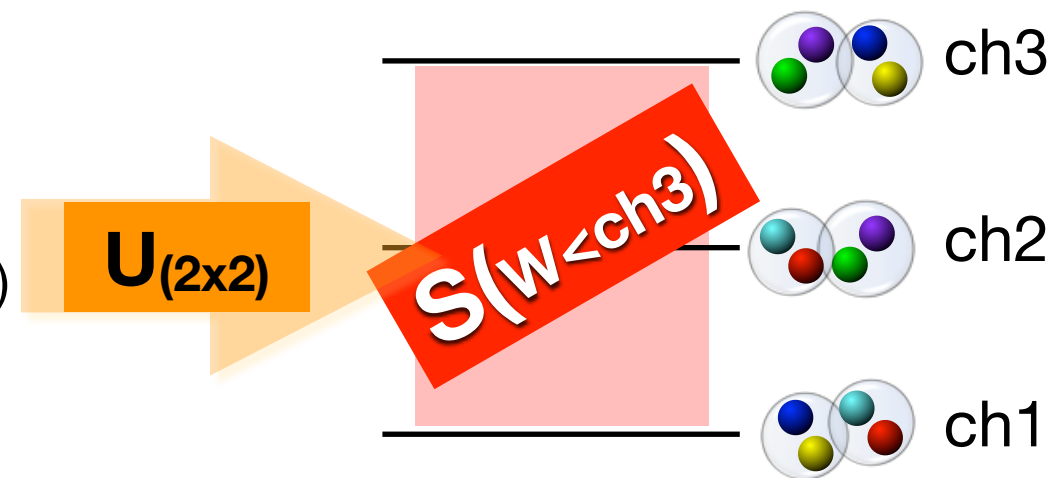
Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
 Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
 Ishii et al. (HAL QCD), PLB712, 437(2012).

★ spatial correlation --> identify **channel** wave function

$$\left(\nabla^2 + (\vec{k}_n^a)^2 \right) \psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}' U^{ab}(\vec{r}, \vec{r}') \psi_n^b(\vec{r}')$$

★ **coupled-channel potential** $U^{ab}(r, r')$:

- $U^{ab}(r, r')$ is faithful to **coupled-channel S-matrix**
- $U^{ab}(r, r')$ is **energy independent** (until new threshold opens)
- Non-relativistic approximation is not necessary
- $U^{ab}(r, r')$ contains all 2PI contributions



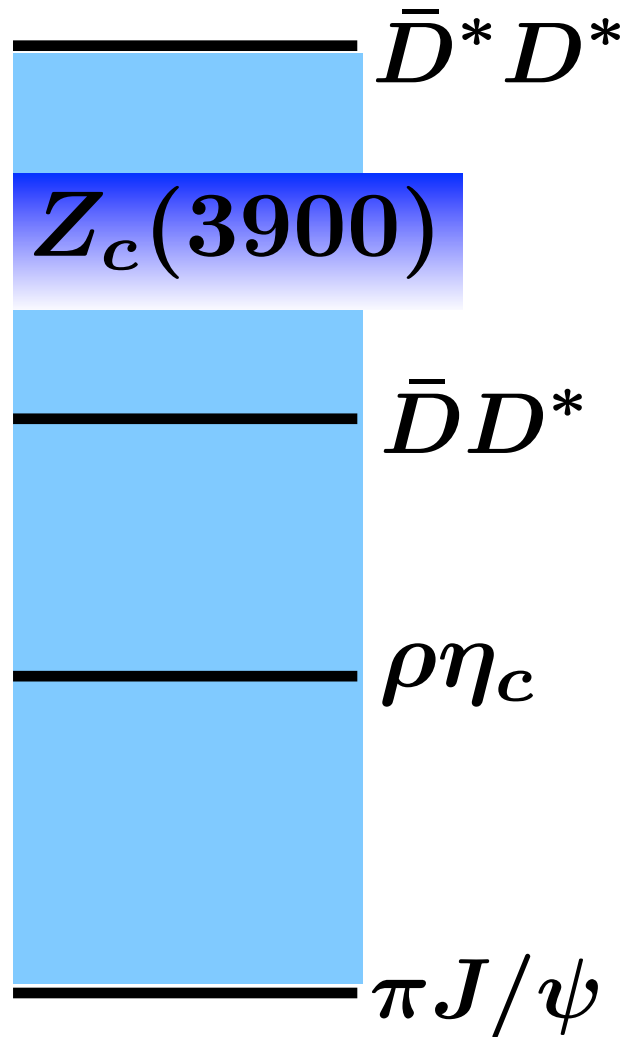
➡ **potential derived from time-dependent HAL QCD method** talk by Sasaki (Sun.)

Full details, Aoki et al. (HAL QCD), PTEP 2012, 01A105 (2012); Proc. Jpn. Acad., Ser. B, 87 (2011).

$Z_c(3900)$ in $1^G(J^{PC})=1^+(1^{+-})$

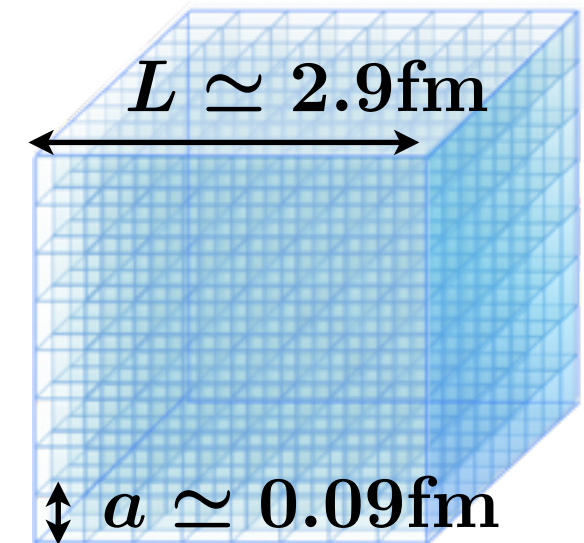
-- $\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$ coupled-channel --

[Y. Ikeda et al., \[HAL QCD\], PRL117, 242001 \(2016\).](#)



❖ $N_f=2+1$ full QCD

- Iwasaki gauge
- clover Wilson quark
- $32^3 \times 64$ lattice



❖ Relativistic Heavy Quark (charm)

- remove leading cutoff errors $O((m_c a)^n)$, $O(\Lambda_{\text{QCD}} a)$, ...
- ➔ We are left with $O((a\Lambda_{\text{QCD}})^2)$ syst. error (\sim a few %)

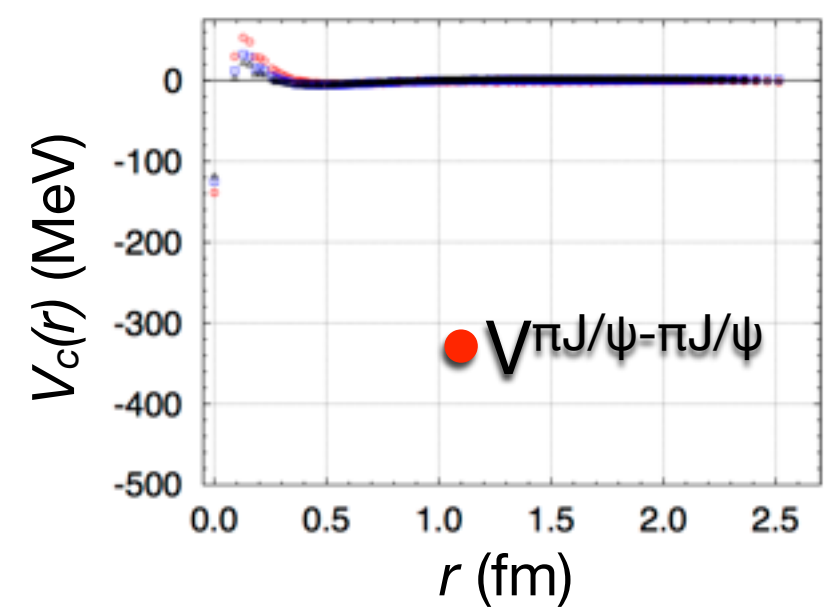
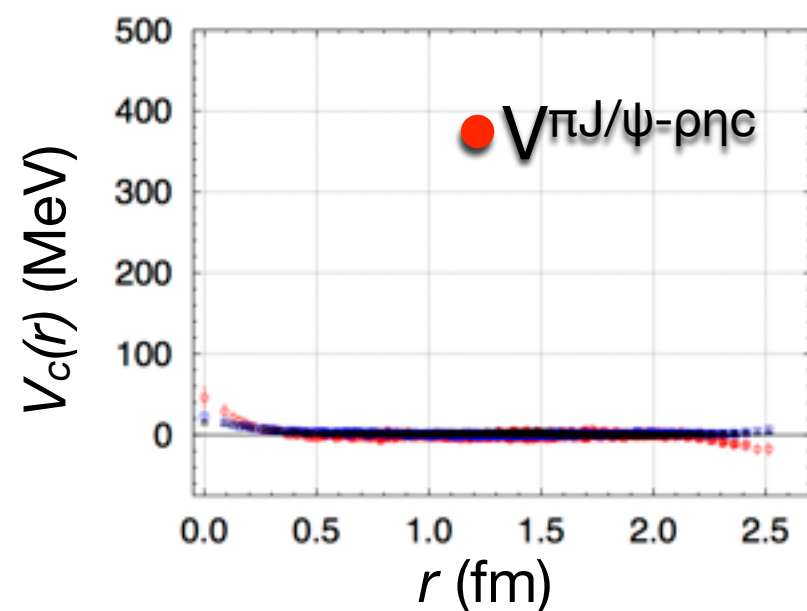
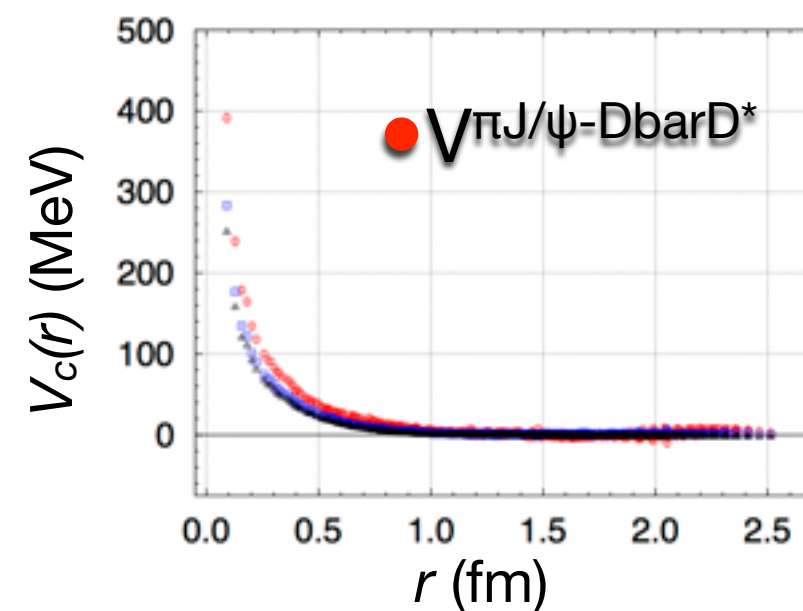
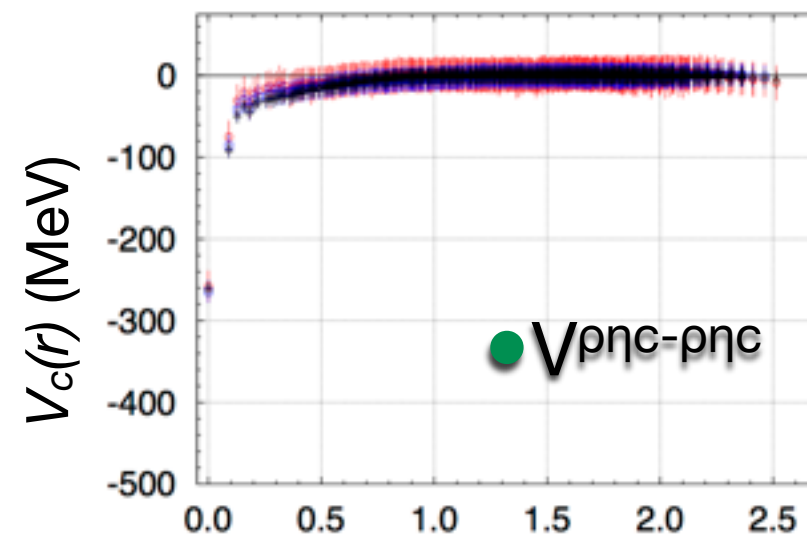
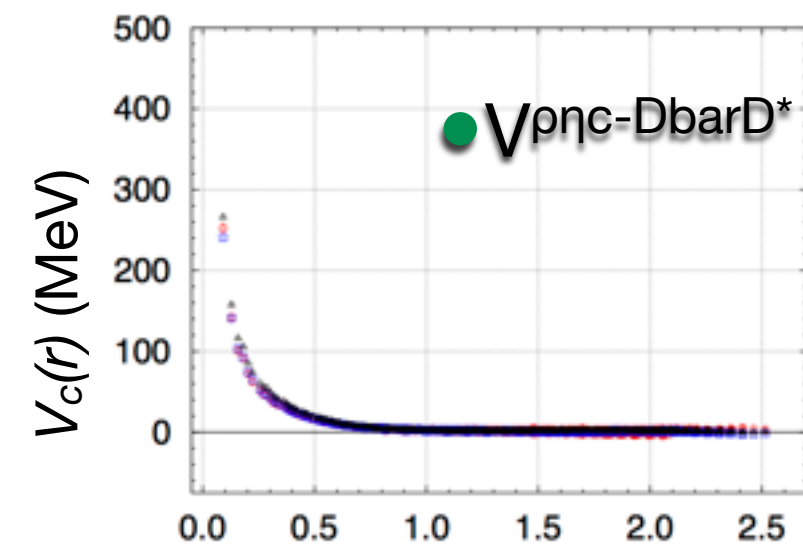
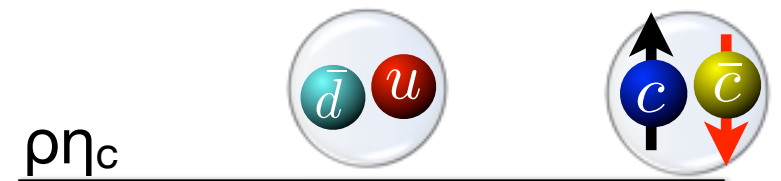
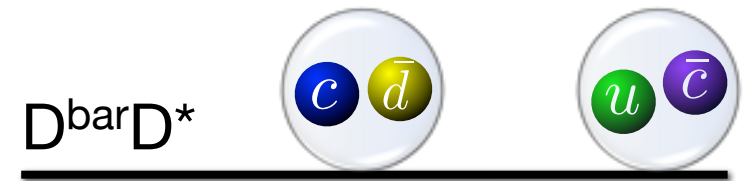
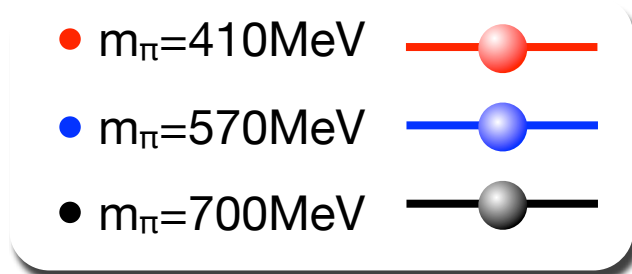
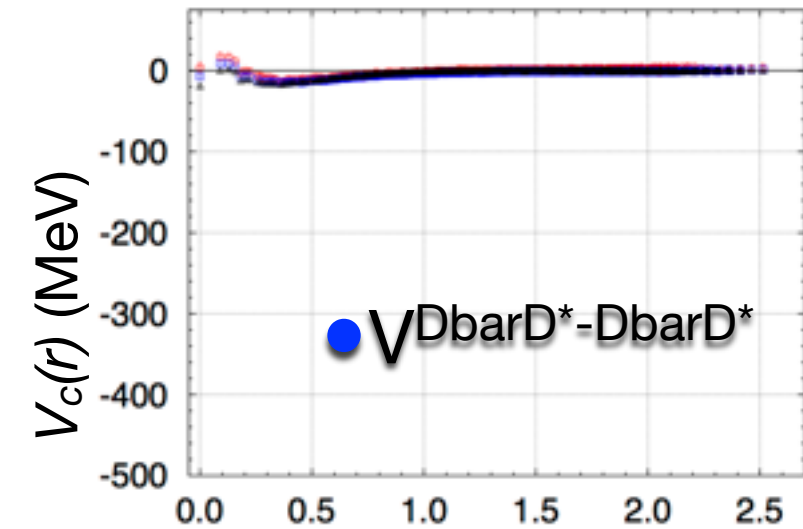
light meson mass (MeV)

$m_\pi = 411(1), 572(1), 701(1)$
 $m_\rho = 896(8), 1000(5), 1097(4)$

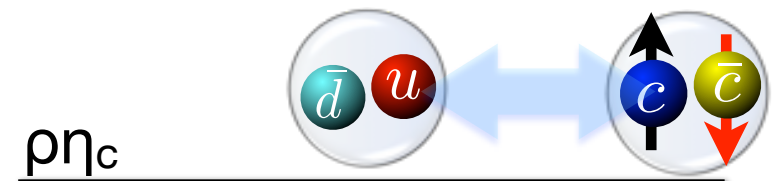
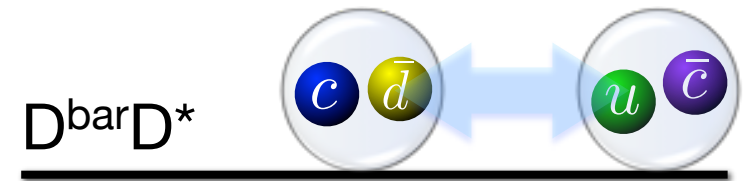
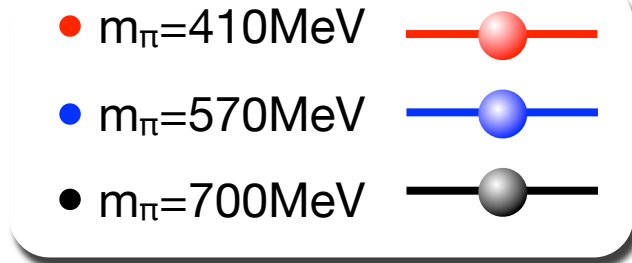
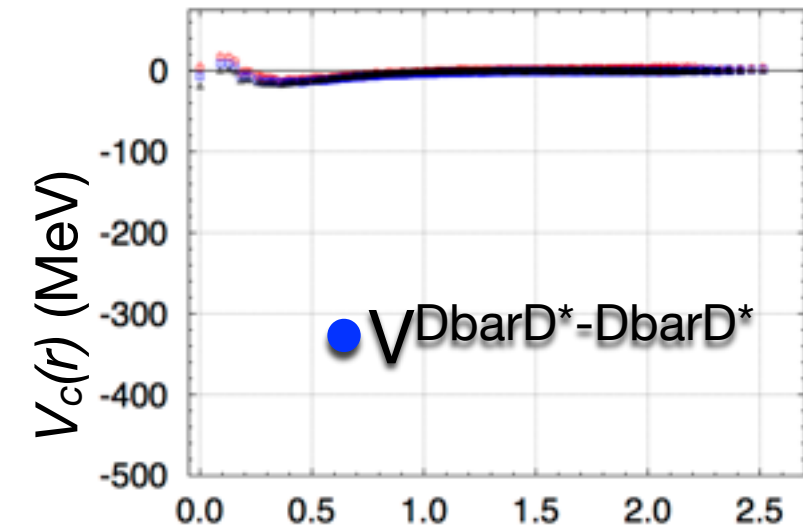
charm meson mass (MeV)

$m_{\eta_c} = 2988(1), 3005(1), 3024(1)$
 $m_{J/\psi} = 3097(1), 3118(1), 3143(1)$
 $m_D = 1903(1), 1947(1), 2000(1)$
 $m_{D^*} = 2056(3), 2101(2), 2159(2)$

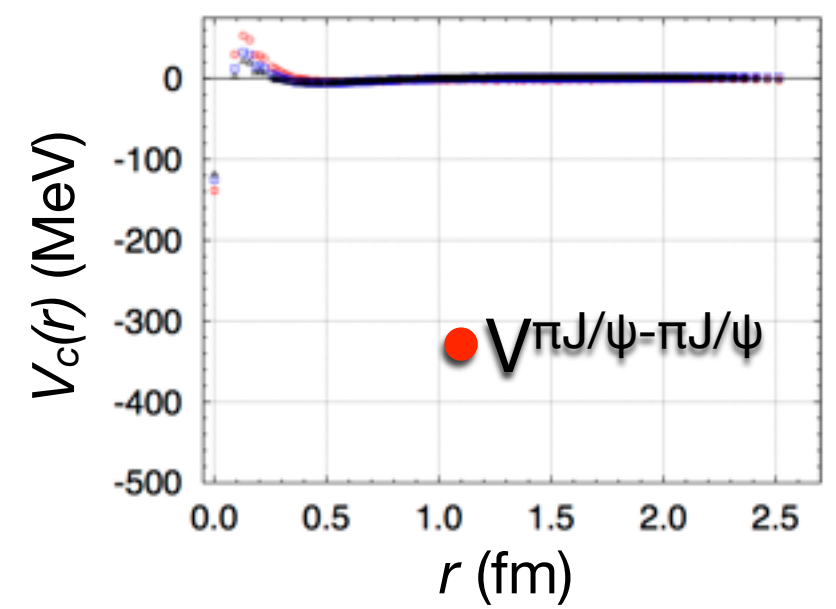
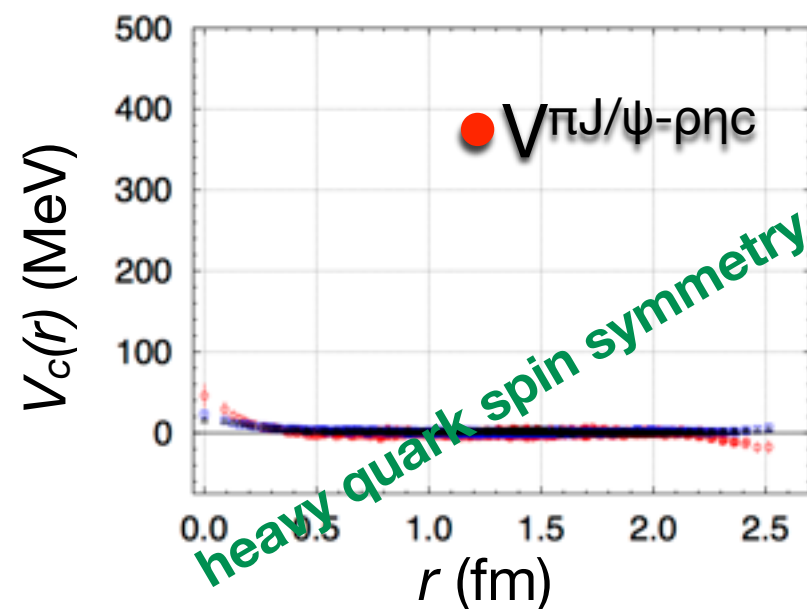
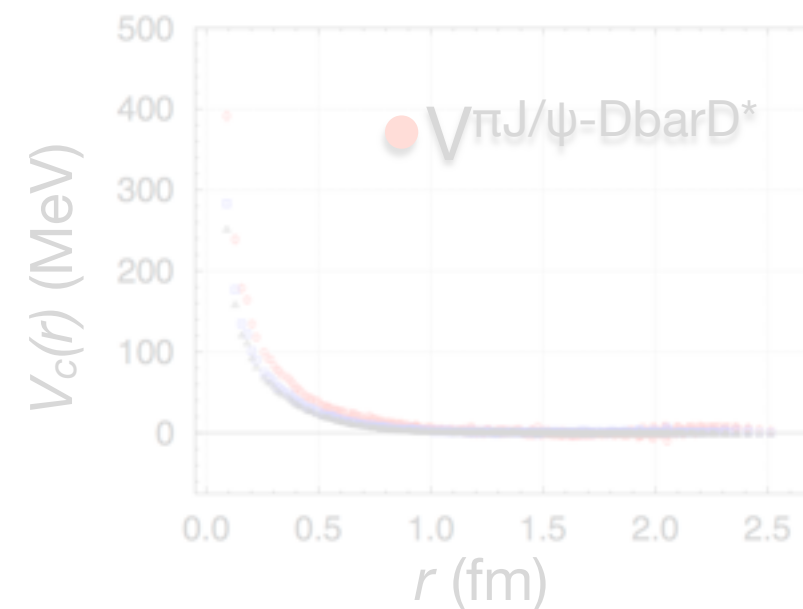
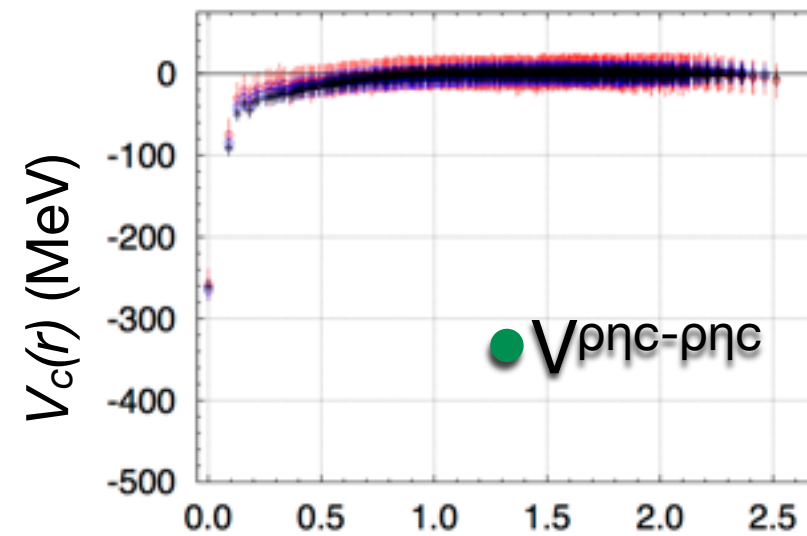
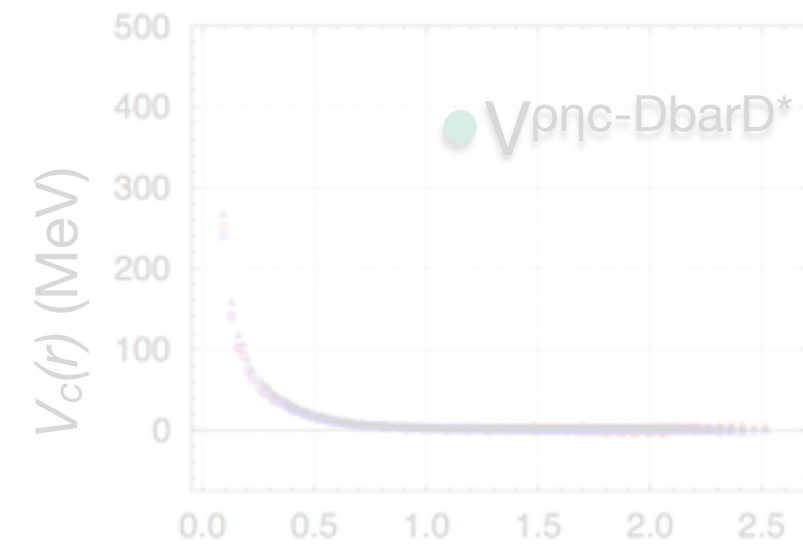
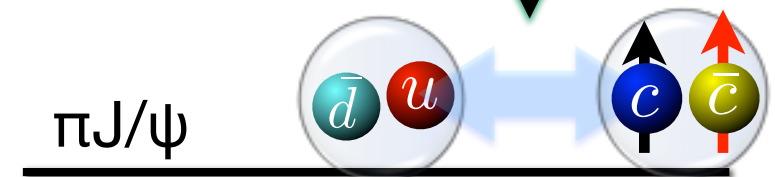
3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



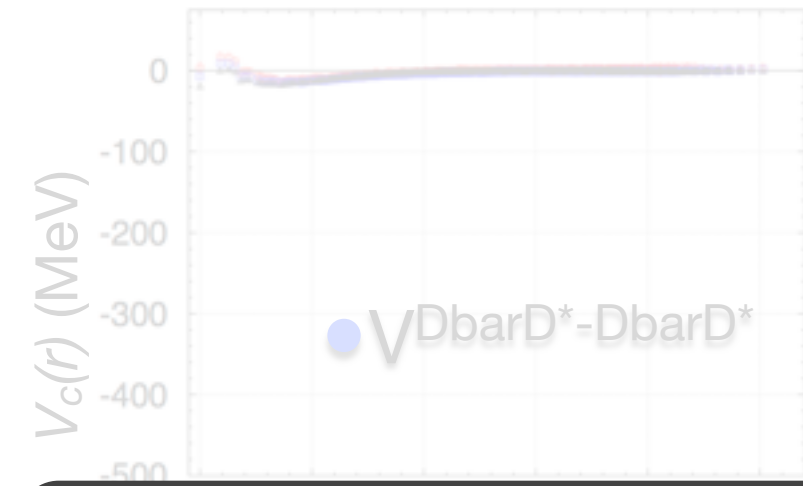
3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



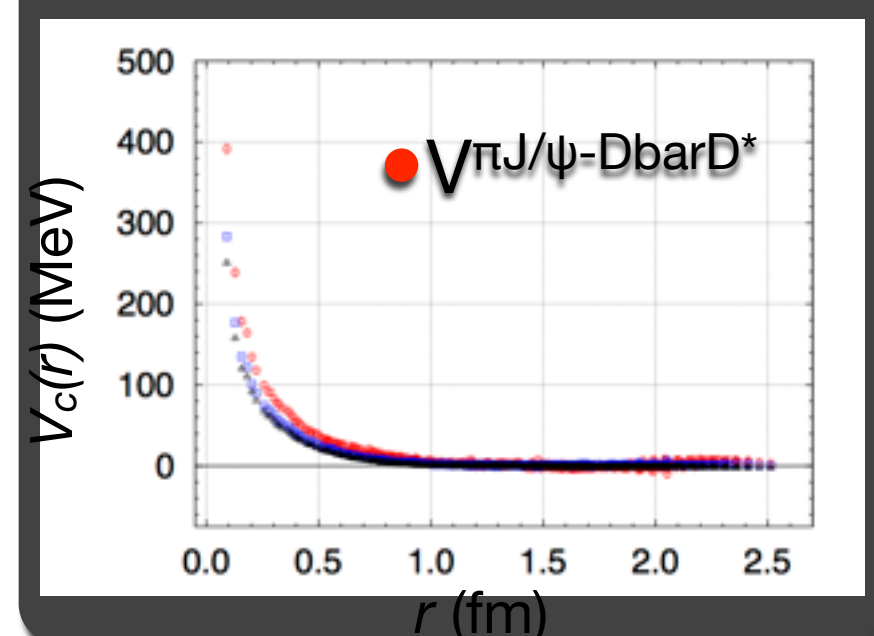
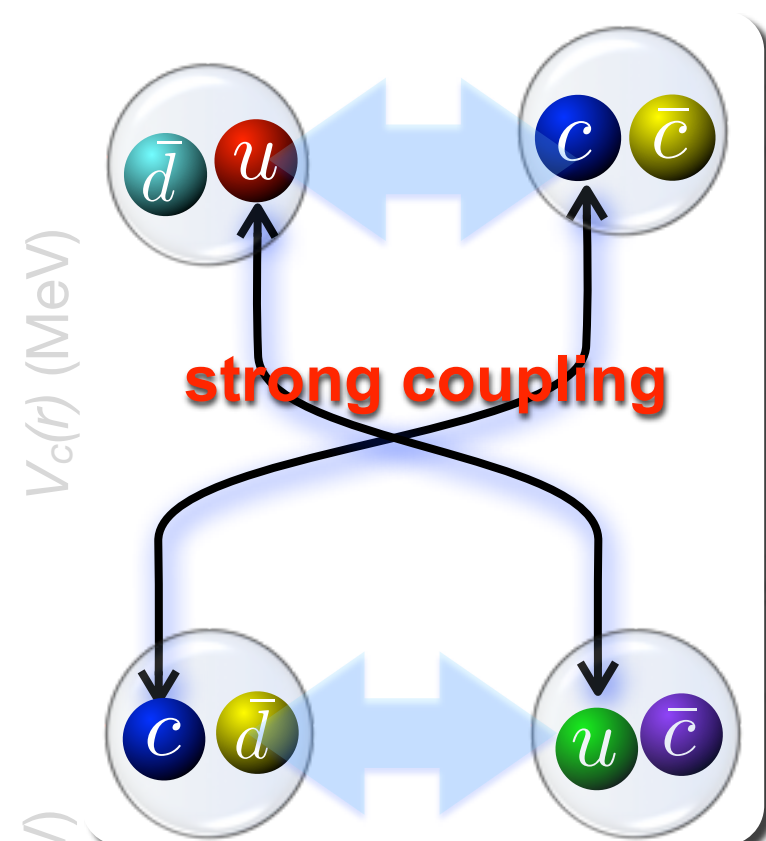
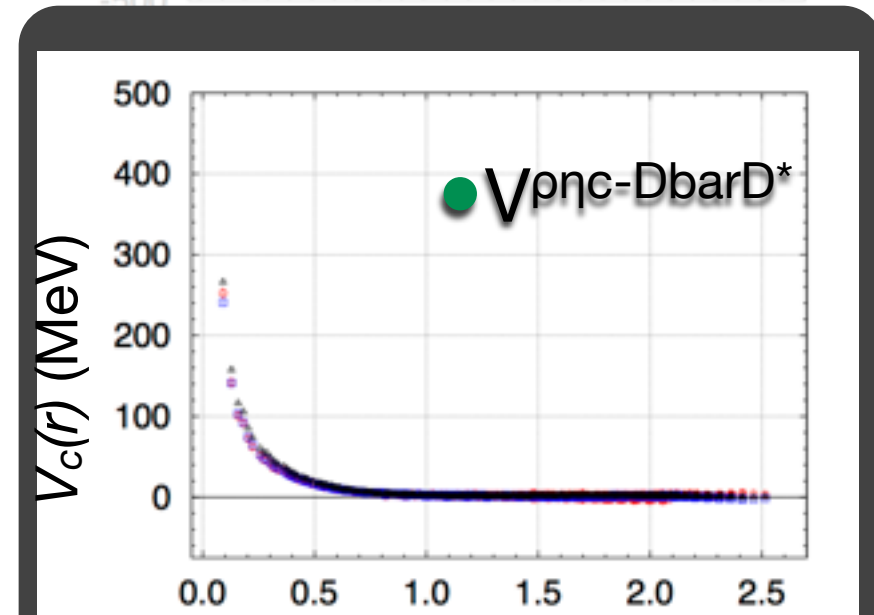
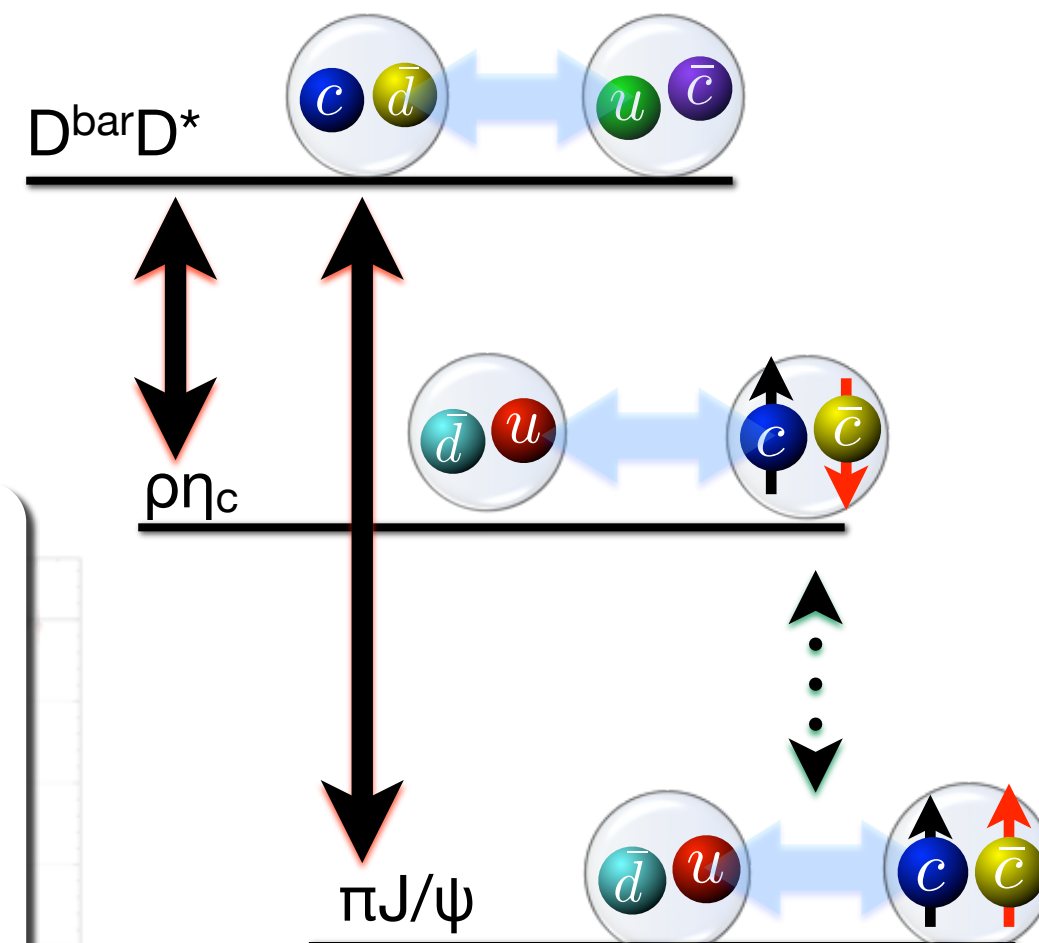
heavy quark spin symmetry



3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{\text{bar}}D^*$)



- $m_\pi = 410 \text{ MeV}$ (red)
- $m_\pi = 570 \text{ MeV}$ (blue)
- $m_\pi = 700 \text{ MeV}$ (black)



- strong $V_{\pi J/\psi, D^{\text{bar}}D^*}$ & $V_{\rho\eta_c, D^{\text{bar}}D^*}$
- ➔ charm quark exchange process



Structure of $Z_c(3900)$

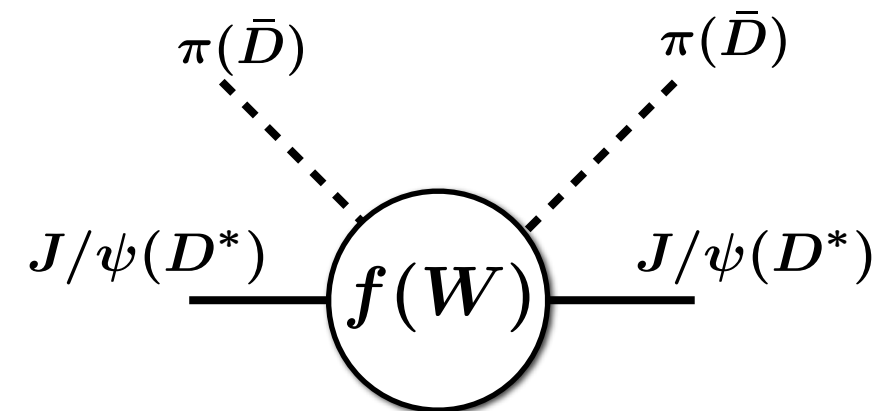
studied by **the most ideal scattering process**

- **S-wave $\pi J/\psi - \rho \eta_c - D^{\text{bar}} D^*$ coupled-channel scattering**
- ➔ **$Z_c(3900)$ is observed in $\pi J/\psi$ & $D^{\text{bar}} D^*$ -> 2-body scattering is the most ideal reaction**

1. invariant mass distribution of 2-body scattering

of scat. particles proportional to imaginary part of amplitude

$$N_{\text{sc}} \propto (\text{flux}) \cdot \sigma(W) \propto \text{Im} f(W)$$



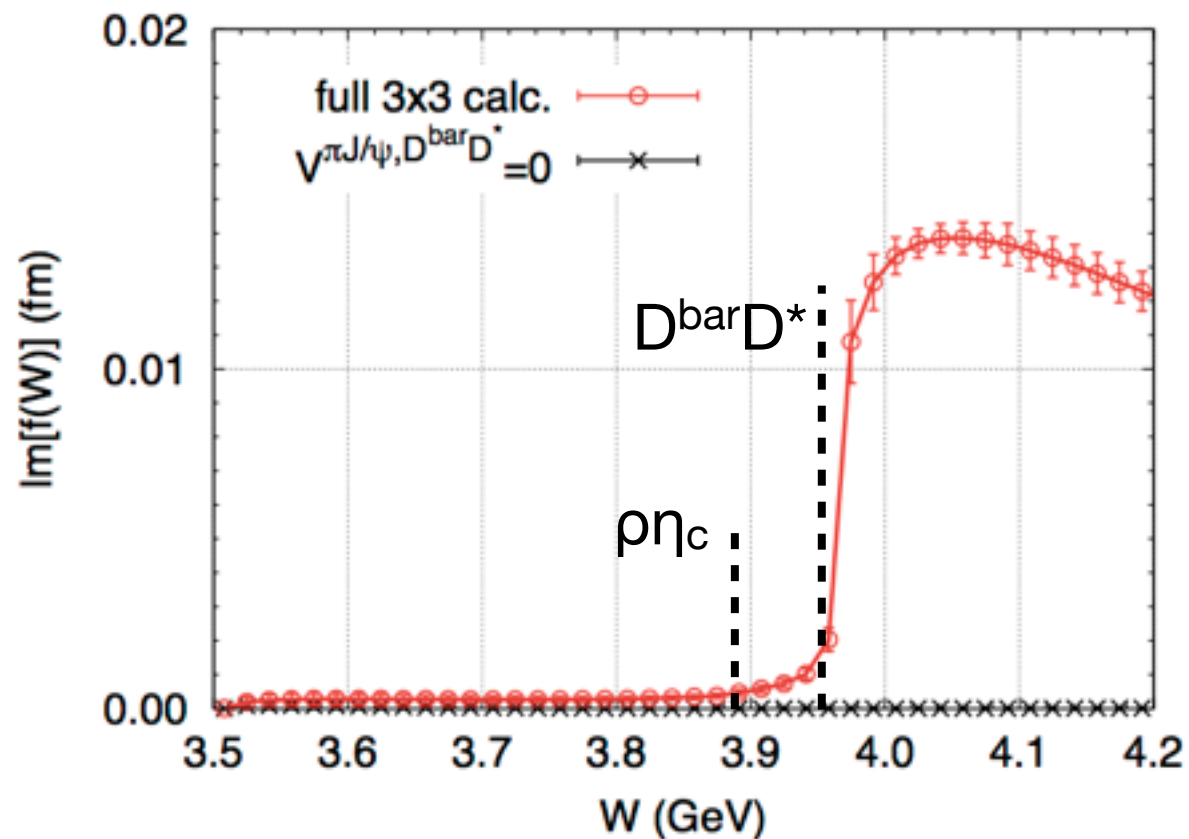
2. pole position of S-matrix

- ▶ analytic continuation of c.c. S-matrix onto complex energy plane
- ▶ understand nature of $Z_c(3900)$

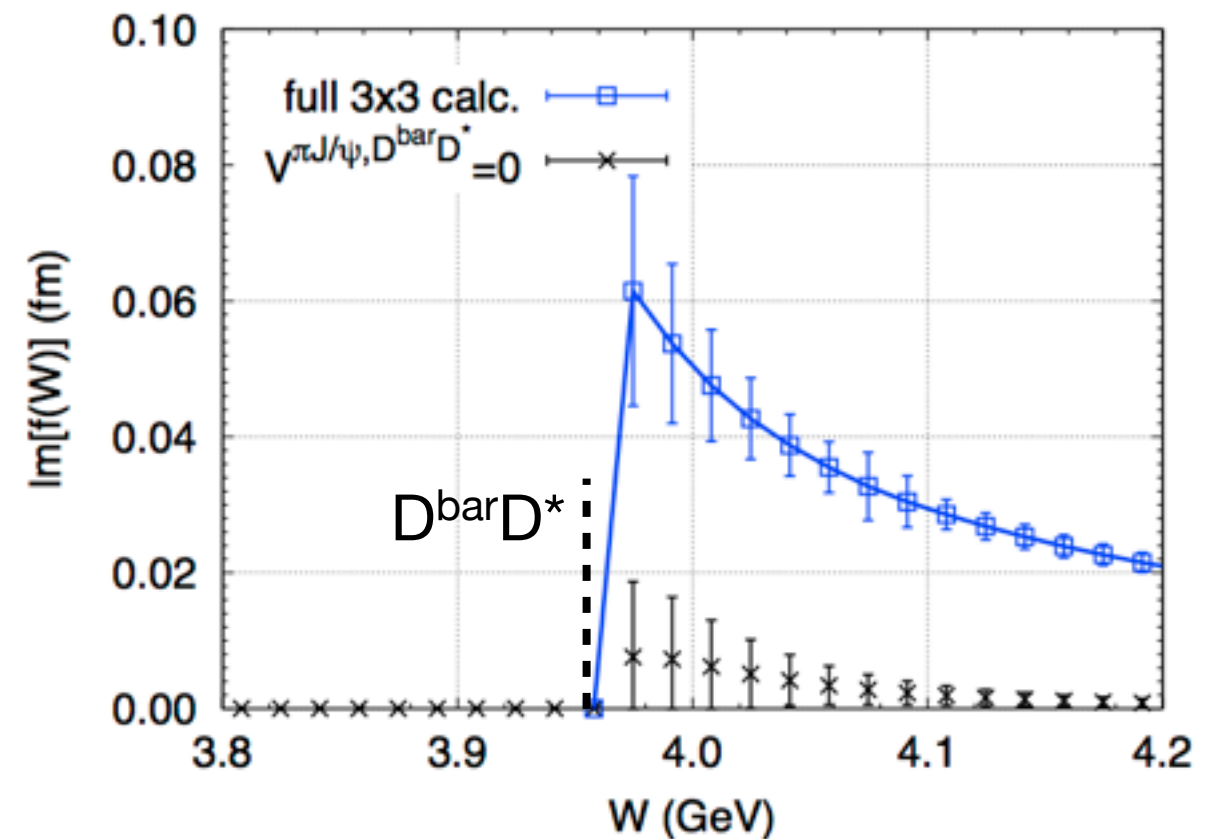
- Results w/ $m_\pi=410\text{MeV}$ are shown. (**weak quark mass dependence observed**)

Invariant mass of $\pi J/\psi$ & $D^{\text{bar}} D^*$ (2-body scat.)

● $\pi J/\psi$ invariant mass



● $D^{\text{bar}} D^*$ invariant mass



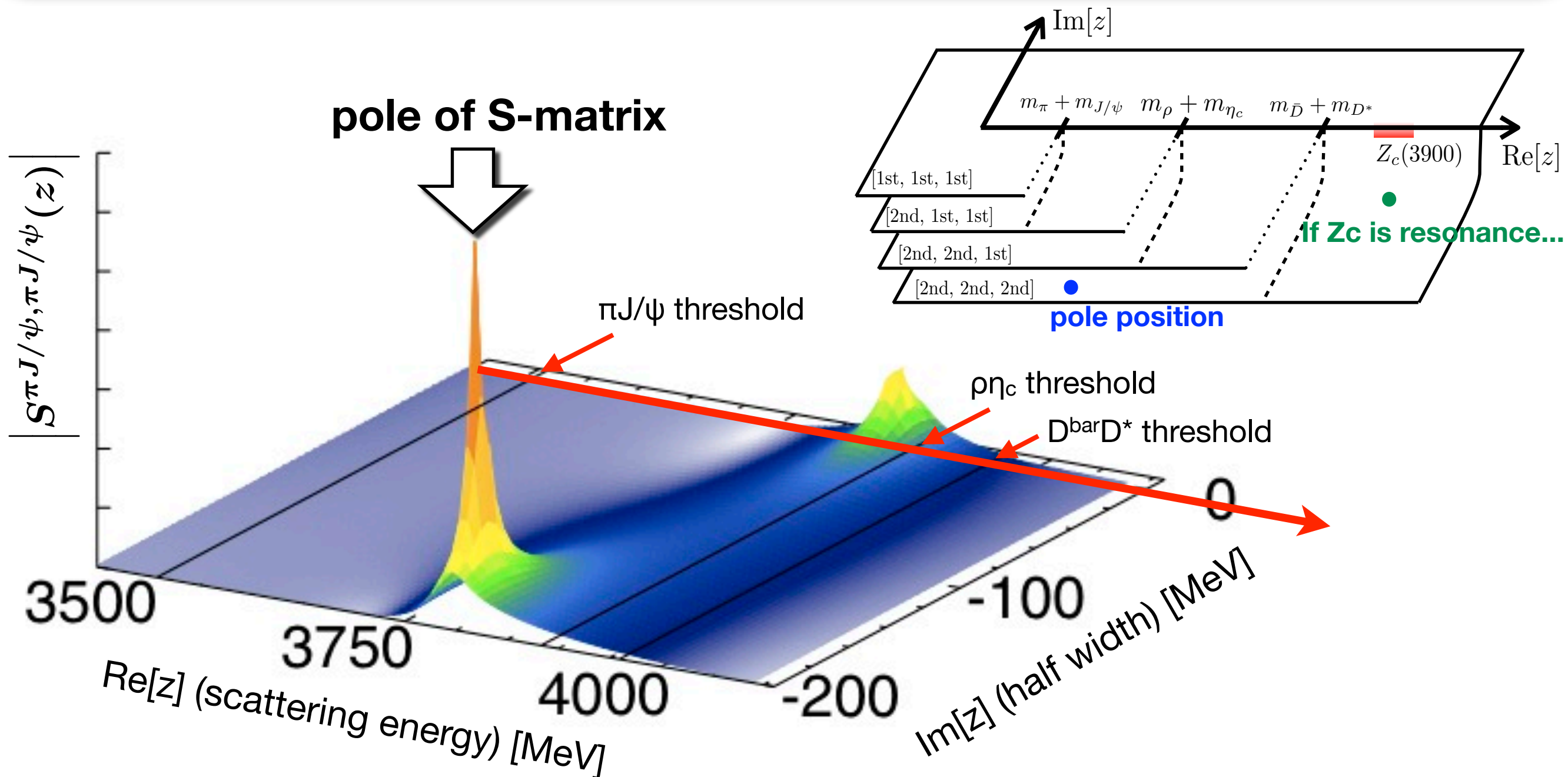
✓ **Enhancement near $D^{\text{bar}} D^*$ threshold** in both amplitudes

➔ **effect of strong $V^{\pi J/\psi, D^{\text{bar}} D^*}$** (black $\rightarrow V^{\pi J/\psi, D^{\text{bar}} D^*} = 0$)

● peak in $\pi J/\psi$ inv. mass (not Breit-Wigner line shape)

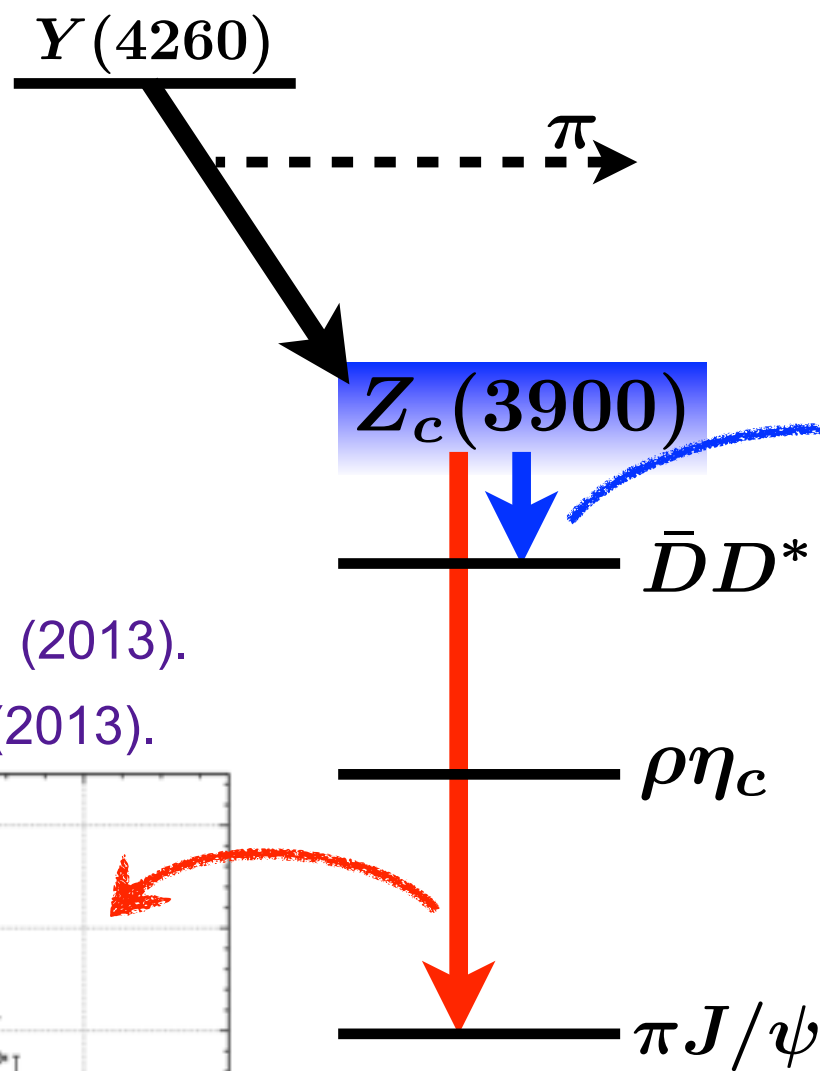
✓ **Is $Z_c(3900)$ a conventional resonance? \rightarrow pole of S-matrix**

Pole of S-matrix ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{\text{bar}}D^*$:2nd)

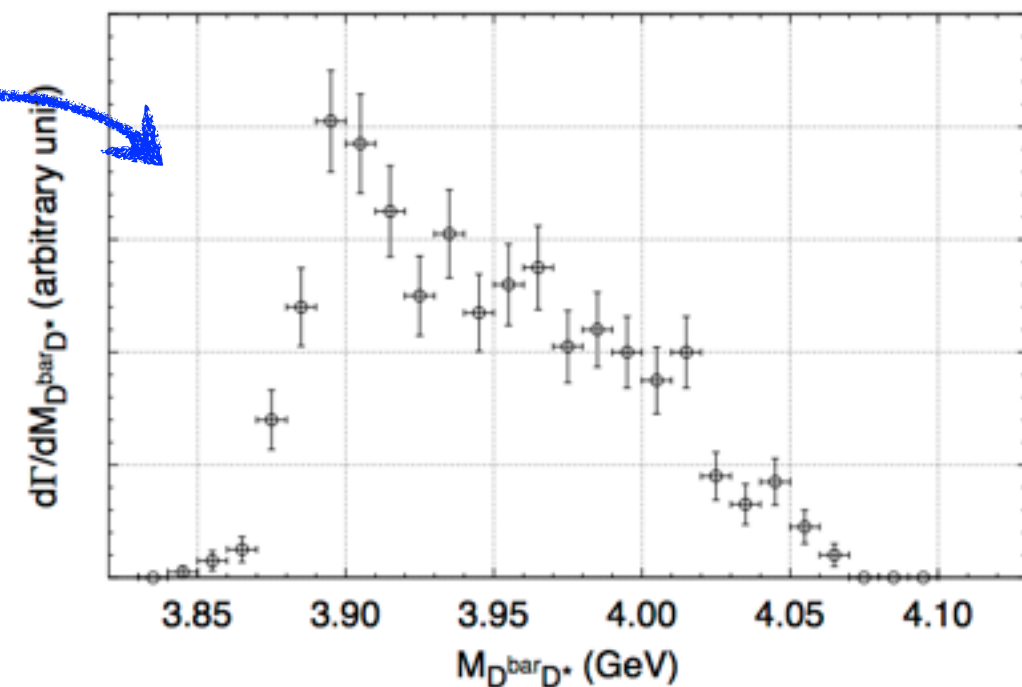


- Pole corresponding to “**virtual state**”
- Pole contribution to scat. observables is small (far from scat. axis)
- $Z_c(3900)$ is not a resonance but “**threshold cusp**” induced by strong $V^{\pi J/\psi, D^{\text{bar}}D^*}$

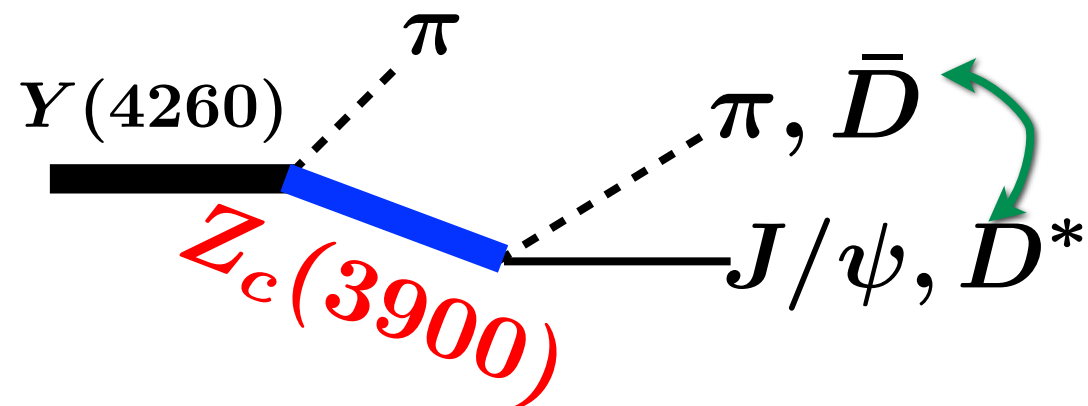
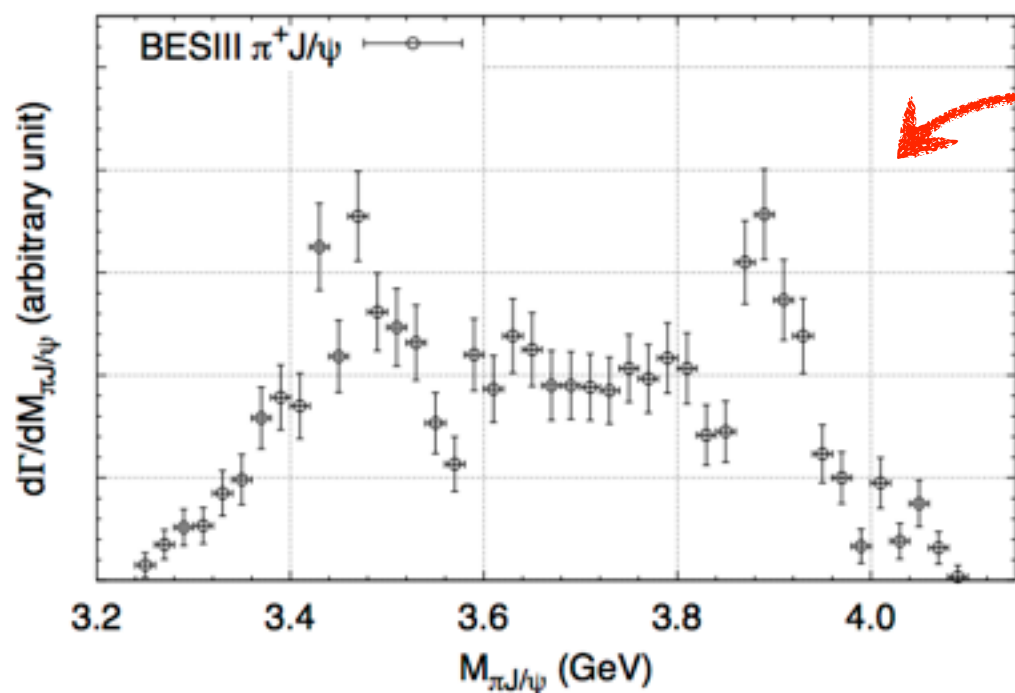
Comparison with expt. data: -- spectrum of $Y(4260)$ 3-body decay --



BESIII Coll., PRL112, 022001, (2014).
Wang (BESIII Coll.), MENU2016 talk



BESIII Coll., PRL110, 252001, (2013).
Belle Coll., PRL110, 252002, (2013).

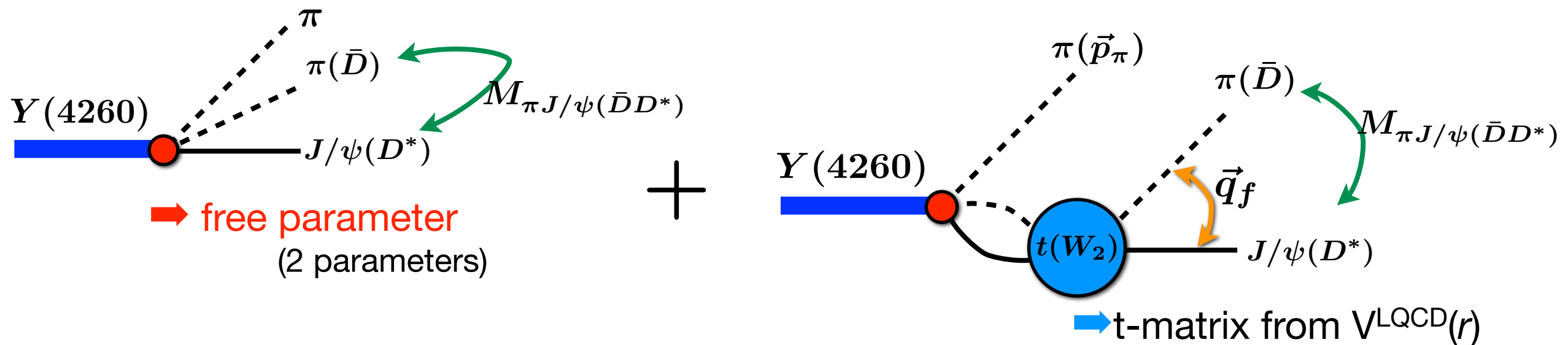


$Y(4260) \rightarrow \pi\pi J/\psi$ & $\pi D^{\text{bar}} D^*$

$$d\Gamma_{Y \rightarrow \pi+f} = (2\pi)^4 \delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f)) d^3 p_\pi d^3 q_f |T_{Y \rightarrow \pi+f}(\vec{p}_\pi, \vec{q}_f; W_3)|^2$$

✓ 3-body T-matrix: $T_{Y \rightarrow \pi+f}(W_3=4260\text{MeV})$

$$T_{Y \rightarrow \pi+f}(\vec{p}_\pi, \vec{q}_f; W_3) = \sum_{n=\pi J/\psi, \bar{D} D^*} C^{Y \rightarrow \pi+n} \left[\delta_{nf} + \int d^3 q' \frac{t_{nf}(\vec{q}', \vec{q}_f, \vec{p}_\pi; W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q}', \vec{p}_\pi) + i\epsilon} \right]$$



employ physical hadron masses to compare w/ expt. data

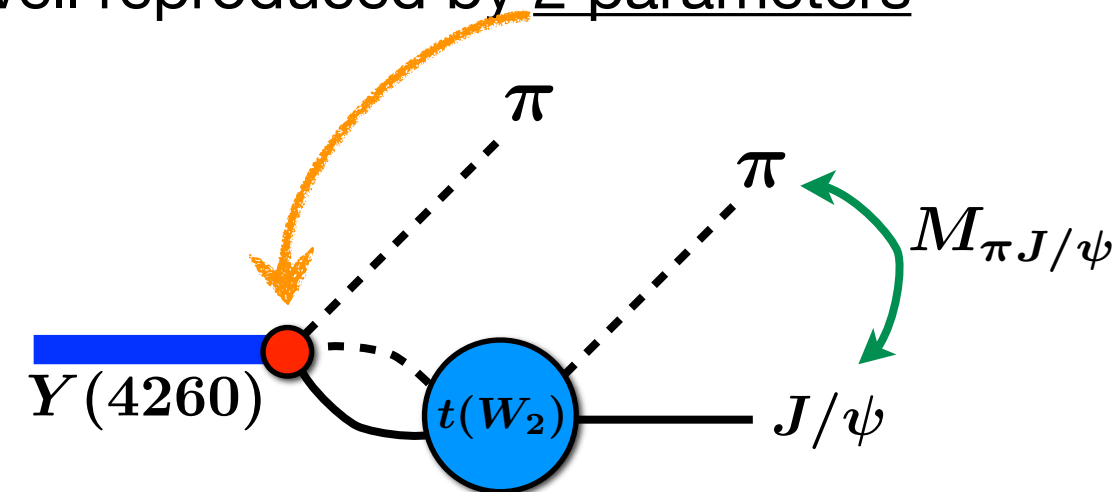
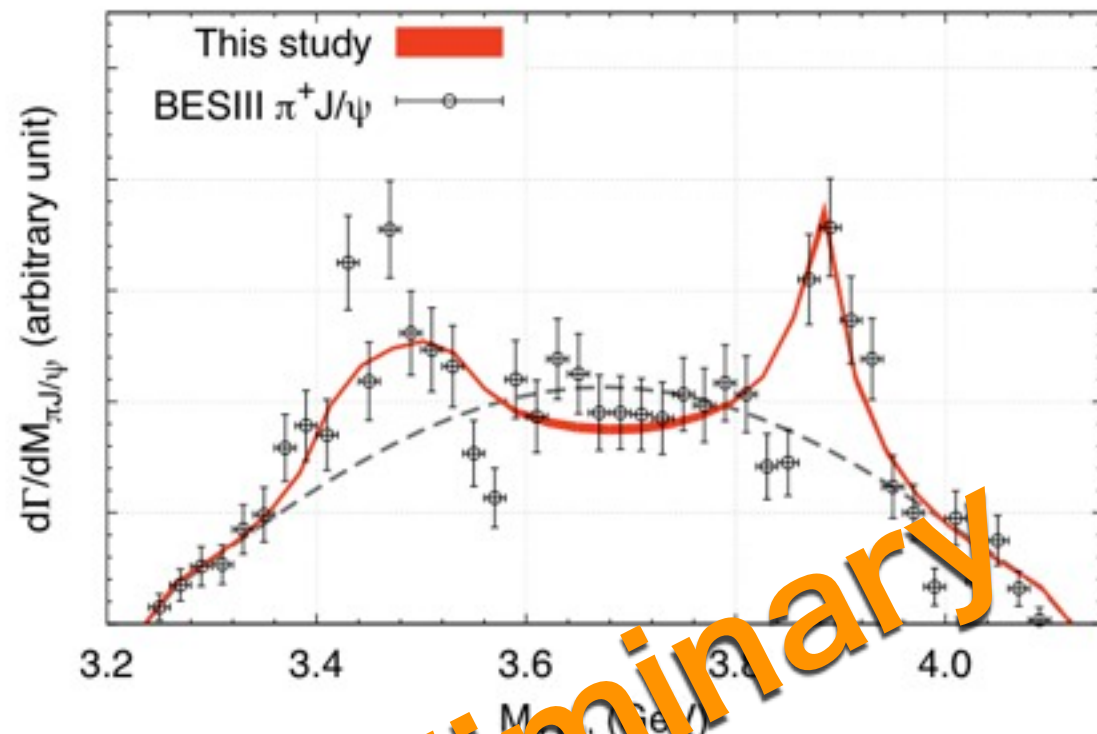
✓ LQCD $V(r)$ is taken into account

✓ fix free parameters by fitting $Y(4260) \rightarrow \pi\pi J/\psi$ expt. data

Invariant mass of 3-body decay (relativistic dispersion)

[Y. Ikeda et al., \[HAL QCD\], PRL117 & in preparation](#)

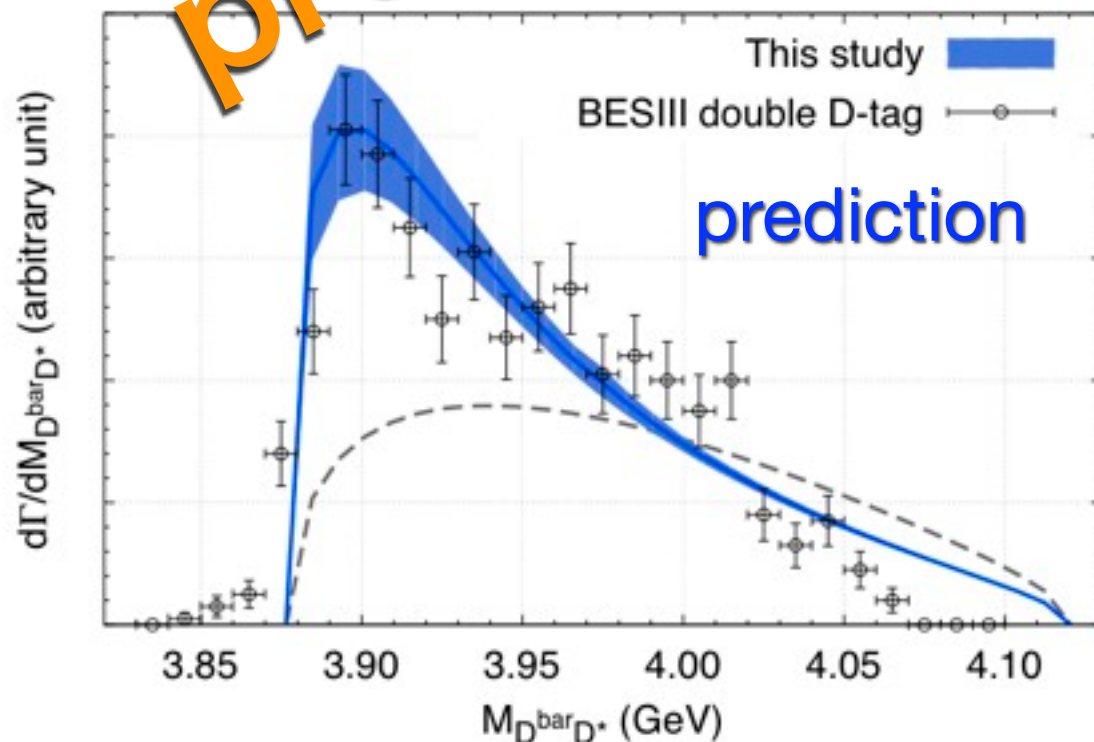
- Expt. data well reproduced by 2 parameters



- When $V^{\pi J/\psi, D\bar{D}^*} = 0$ (dashed curve), expt. data are not reproduced.

- predicted curve well agrees with expt. data**

- When $V^{\pi J/\psi, D\bar{D}^*} = 0$ (dashed curve), peak disappears

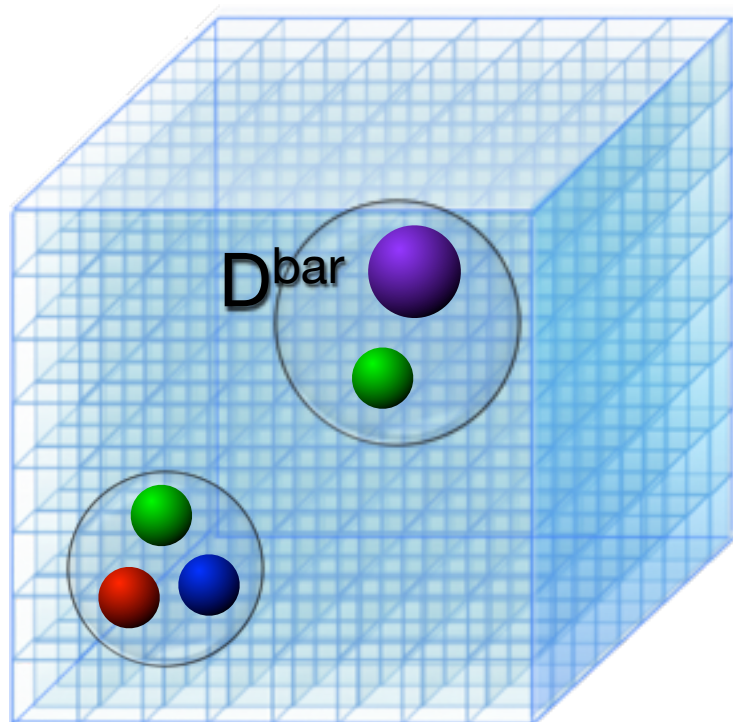
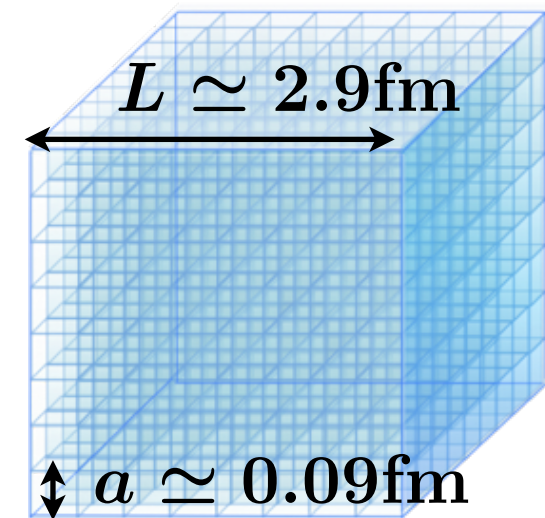


conclusion: $Z_c(3900)$ is threshold cusp caused by strong $V^{\pi J/\psi, D\bar{D}^*}$

S-wave D^{bar} -nucleon interaction and D^{bar} -nuclei

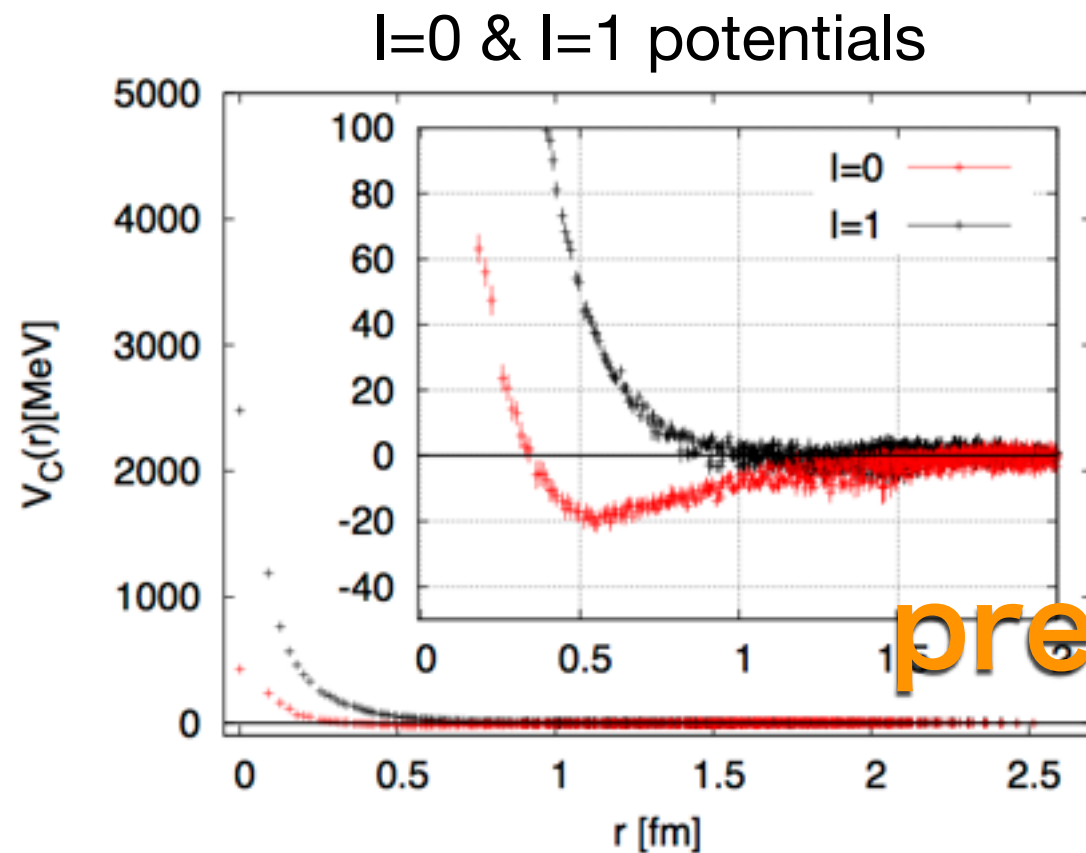
- ▶ $N_f=2+1$ full QCD configs. (PACS-CS collaboration)
- ▶ RHQ action for charm quark

$m_\pi = 411, 572, 701$
 $m_N = 1215, 1411, 1583$
 $m_D = 1903, 1947, 2000$
 $m_{\Lambda_c} = 2434, 2584, 2710$



- $D^{\text{bar}}N$ interaction in $l=0$ & 1 by HAL QCD method
- Naive expectation to D^{bar} -nuclei by simple folding potential model

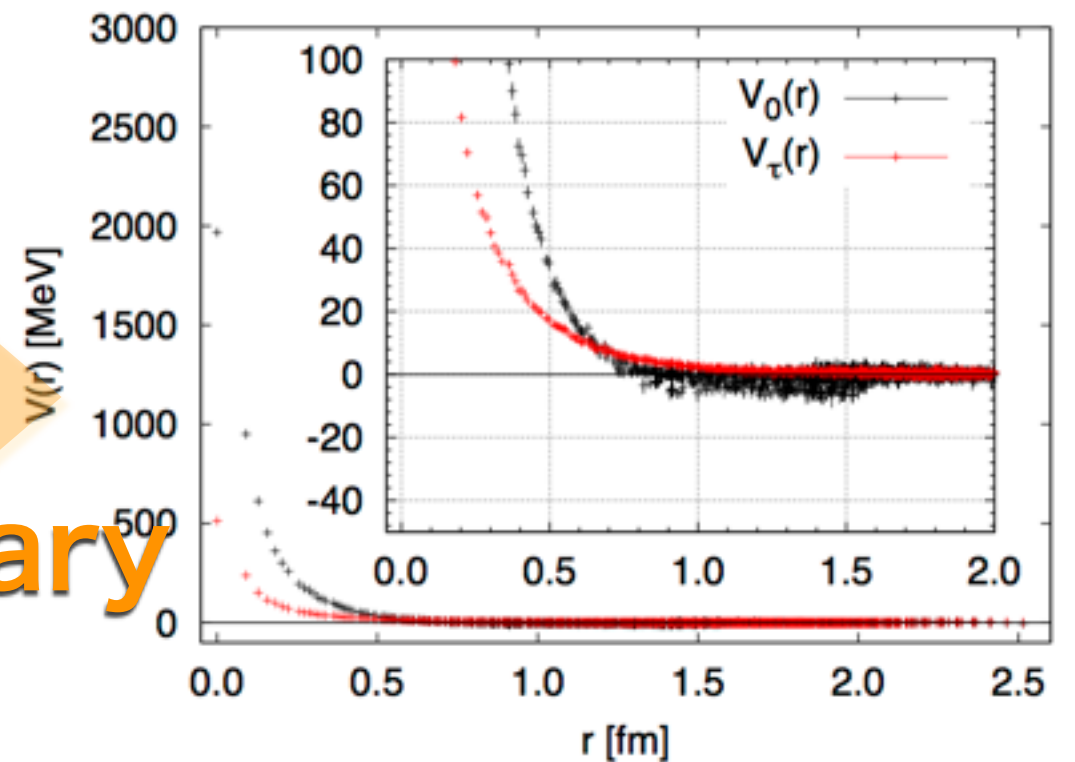
S-wave $D^{\text{bar}}N$ interaction in $l=0$ & 1 @ $m_\pi=410\text{MeV}$



$$a_{\bar{D}N} = \begin{cases} 0.79(42) & (I = 0) \\ -0.20(1) & (I = 1) \end{cases}$$

preliminary

$$V_{\bar{D}N}(\vec{r}) = V_0(\vec{r}) + V_\tau(\vec{r})(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_N)$$



► $V_0(r)$ is repulsive, $V_\tau(r)$ is positive

[Y. Ikeda \[HAL QCD\], in preparation](#)

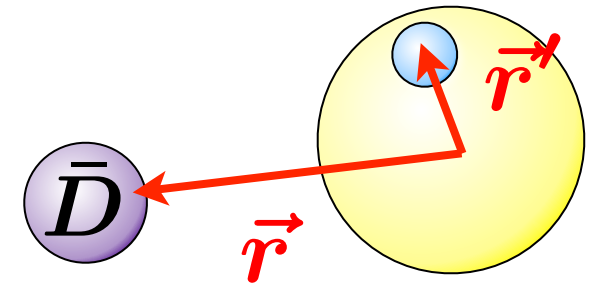
$D^{\text{bar}}N$ interaction

- (1) attractive in $l=0$ & repulsive in $l=1$
- (2) $V_0(r)$ is repulsive, $V_\tau(r)$ is positive

\bar{D} -nuclei (naive expectation from $\bar{D}A$ folding potential model)

✓ What is possible structure of \bar{D} -nuclei?

$$V_{\bar{D}N}(\vec{r}) = V_0(\vec{r}) + V_\tau(\vec{r})(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_N)$$



$$V_{\bar{D}A}(\vec{r}) = \sum_{i=1}^A \int d^3r' \rho_{N_i}(\vec{r}') [V_0(\vec{r} - \vec{r}') + V_\tau(\vec{r} - \vec{r}')(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_{N_i})]$$

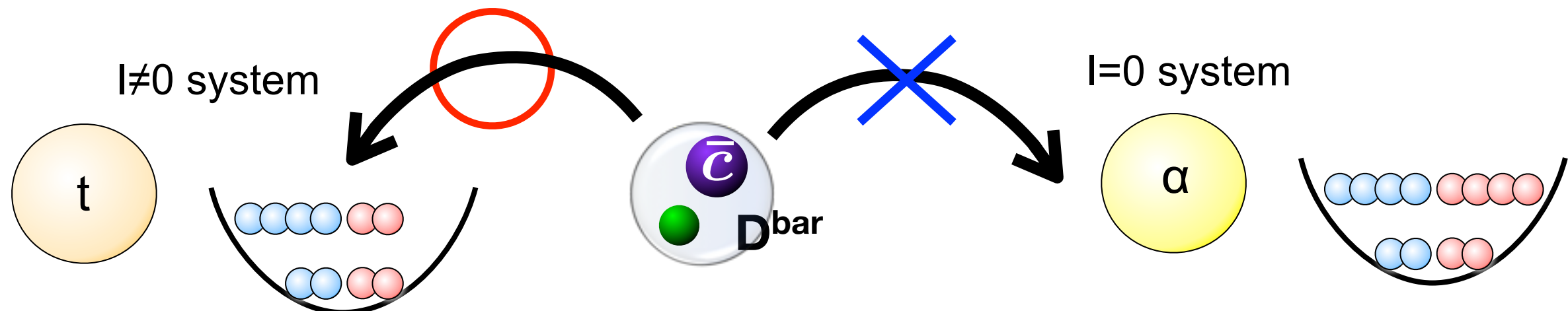
contact interaction : $V_{\bar{D}N}(\vec{r}) \rightarrow \{g_0 + g_\tau(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_N)\} \delta(\vec{r})$

g_0 : repulsive, g_τ : positive

Energy shift

$$\Delta E = \int d^3r V_{\bar{D}A}(\vec{r}) = A [g_0 + g_\tau(\vec{\tau}_{\bar{D}} \cdot \vec{\tau}_A)], \quad \vec{\tau}_A = \sum_{i=1}^A \vec{\tau}_{N_i}$$

✓ Isospin correlation plays crucial role to make \bar{D} bound into nuclei



Summary

◆ Coupled-channel HAL QCD method

- ▶ derive potential faithful to S-matrix from BS wave function
- ▶ a solution to coupled-channel problems from LQCD

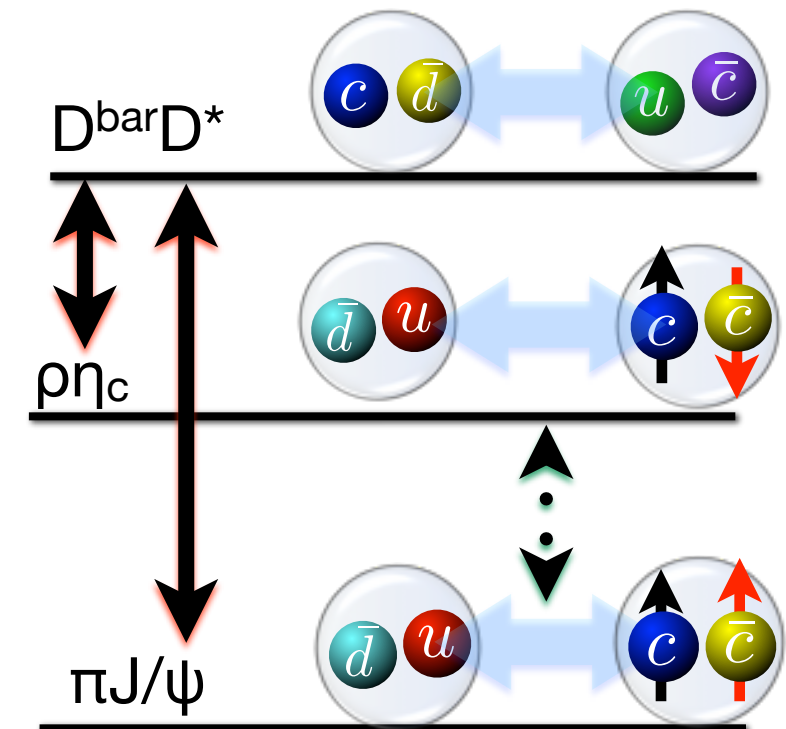
✿ Tetra-quark candidate $Z_c(3900)$

- ▶ $Z_c(3900)$ is threshold cusp induced by strong $V^{D\bar{D}^*, \pi J/\psi}$
 - pole position very far from scat. axis
 - expt. data of $Y(4260)$ decay well reproduced
 - no peak structure when $V^{D\bar{D}^*, \pi J/\psi}=0$

✿ $D^{\bar{}}$ -nuclei

- ▶ $D^{\bar{}}$ may be bound into neutron or proton rich nuclei
 - repulsive isospin-independent & positive sign of isospin-dependent interactions
 - $D^{\bar{}}$ is good probe to explore isospin correlation inside nuclei

✿ Future: physical point calculations



Thank you for your attention!!