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Production of Hypernuclei

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   Λ, Σ^-

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   - Study of the Σ-nucleus potentials Σ^--^{5}He
   - Production of the neutron-rich Λ hypernucleus $^{6}_{\Lambda}$H

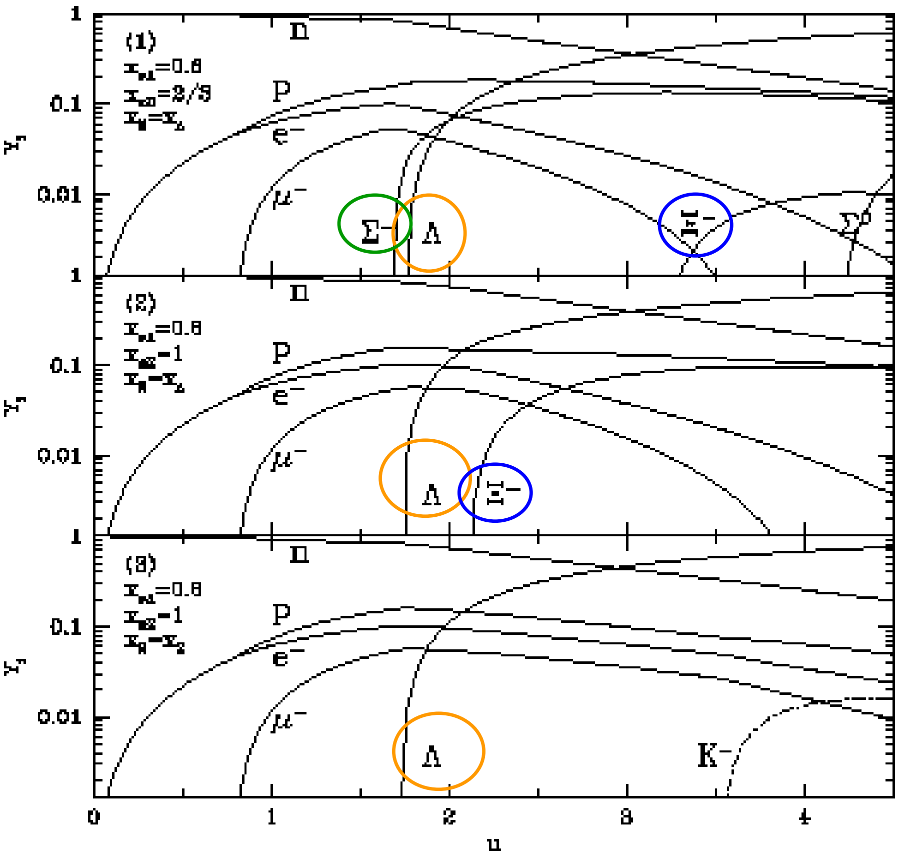
4. Summary

ΛΣ coupling ***Hyperon-mixing***

Neutron star core

= “An interesting neutron-rich hypernuclear system”

Coupling constant ratio; $x_{iY} = g_{iY}/g_{iN} \ (i = \sigma, \omega, \rho)$

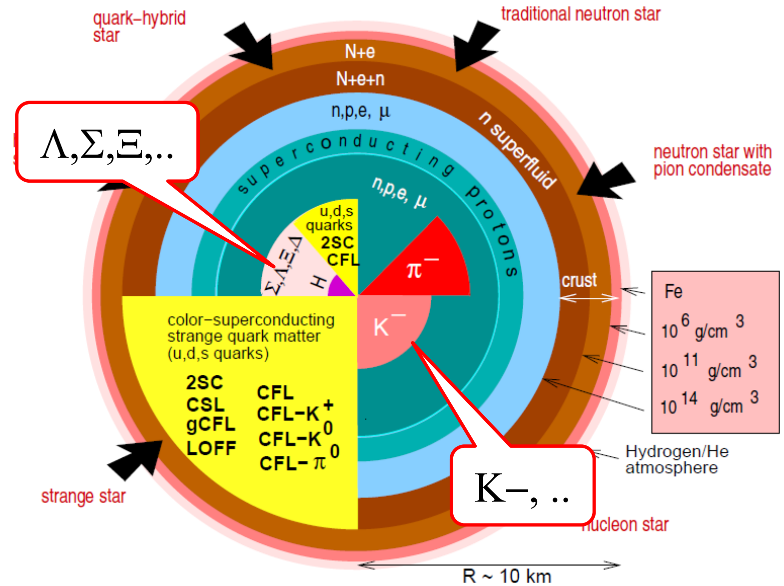


$U_\Sigma < 0$
$U_\Xi < 0$

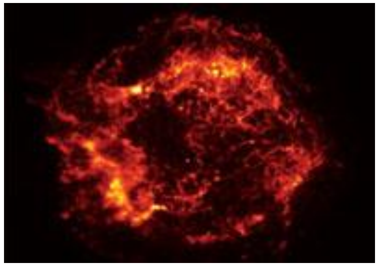
$U_\Sigma > 0$
$U_\Xi < 0$

$U_\Sigma > 0$
$U_\Xi > 0$

[R. Knorren, M. Prakash, P.J.Ellis, PRC52(1995)3470]



[F. Weber, PPNP 54(2005)193]



Cassiopeia A nebula
NASA/CXC/SAO.

Hyperon-mixing

In this talk,

I will discuss production and spectroscopy of the Λ and Σ hypernuclei, within the distorted-wave impulse approximation (DWIA) in a theoretical viewpoint.

I will focus on

i. DWIA calculations of the Λ (Σ) production spectra with effective number and Green's function descriptions.
ii. application of coupled-channel Green's function to neutron-rich Λ (Σ) hypernuclei via Σ^- doorways in the nuclear (π^-, K^+) reaction.

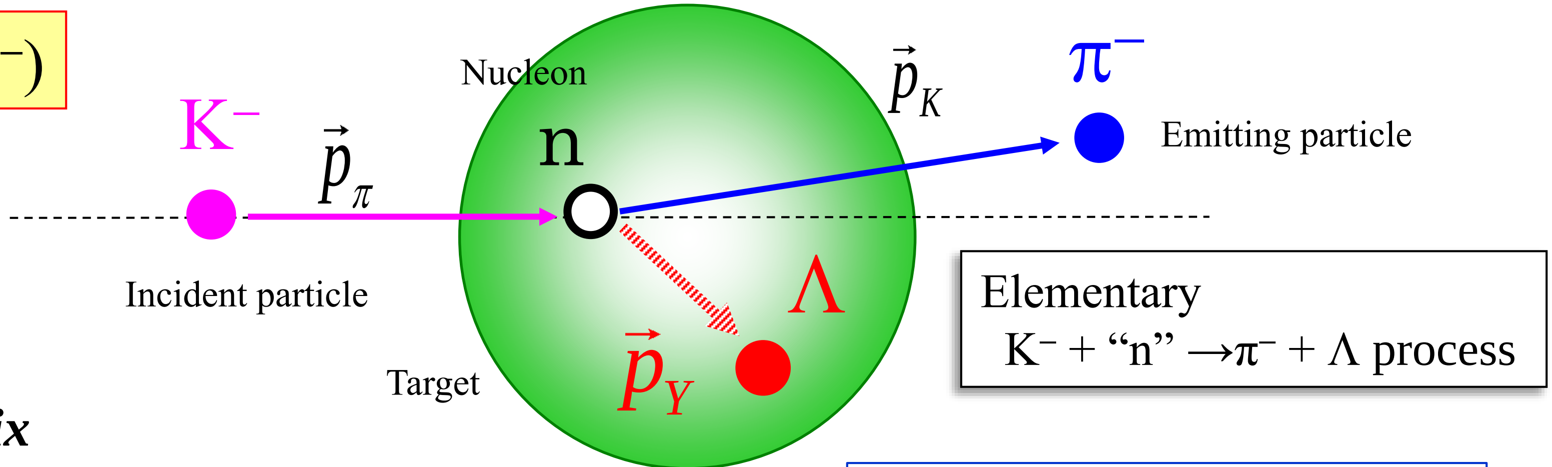
Hyperon-mixing

Distorted-wave impulse approximation (DWIA)

Distorted-wave Impulse Approximation (DWIA)

J. Hufner et al, NPA234 (1974) 429; E.H. Auerbach et al., Ann. Phys. (N.Y.) 148 (1983) 381; C.B. Dover et al.,PRC22 (1980) 2073.

(K^-, π^-)



T-matrix

Hypernuclear w.f.

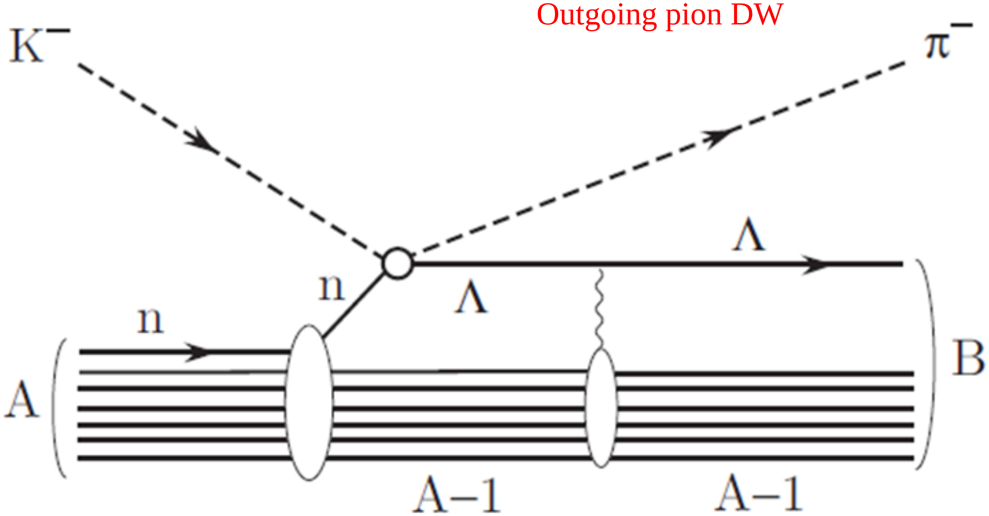
Elementary t-matix including medium effects

$$T_{fi} \simeq \langle \Psi_B(\boldsymbol{r}) | \langle \chi_b^{(-)}(\boldsymbol{r}_3) | \sum t_j(\boldsymbol{r}_4, \boldsymbol{r}_3, \boldsymbol{r}_2, \boldsymbol{r}_1) | \chi_a^{(+)}(\boldsymbol{r}_2) \rangle | \Psi_A(\boldsymbol{r}') \rangle$$

Outgoing pion DW

Incoming kaon DW

Nuclear target w.f.



Distorted Waves e.g. eikonal app.

$$\chi_a^{(+)}(\boldsymbol{r}) = \exp\left\{ i\boldsymbol{p}_a \cdot \boldsymbol{r} - i v_a^{-1} \int_{-\infty}^{z} U_a(\boldsymbol{b}, z')dz' \right\}$$

$$\chi_b^{(-)*}(\boldsymbol{r}) = \exp\left\{ -i\boldsymbol{p}_b \cdot \boldsymbol{r} + i v_b^{-1} \int_{z}^{\infty} U_b^{\dagger}(\boldsymbol{b}, z')dz' \right\}$$

(Complex) effective number description

for (quasi-)bound states

(Complex) effective number description in DWIA

Integrated differential Lab Cross Sections

$$\left(\frac{d\sigma}{d\Omega_b}\right) = \bar{\alpha}\left\langle\frac{d\sigma}{d\Omega}\right\rangle_{\rm lab}^{aN\to bY} {\rm Re}\, N_{\rm eff}^{(j_N^{-1}j_Y)J_B}$$

Real ← bound state
Complex ←quasibound states

Effective number of a nucleon (Closed-shell target $J^p = 0^+$)

$$N_{\rm eff}^{(j_N^{-1}j_Y)J_B} = (2J_B+1)(2j_Y+1)(2j_N+1)\begin{pmatrix} j_Y & j_N & J_B \\ \frac{1}{2} & -\frac{1}{2} & 0\end{pmatrix}^2 F(q)F^{\dagger}(q)$$

Form factor

$$F(q) = \int_0^\infty r^2 dr (\tilde{\varphi}_{j_Y}(r))^* \tilde{j}_L(p_a,p_b;r)\varphi_{j_N}^{(N)}(r)$$

biorthgonal set

$$H\varphi_{n\ell} = E_{n\ell}\varphi_{n\ell}$$

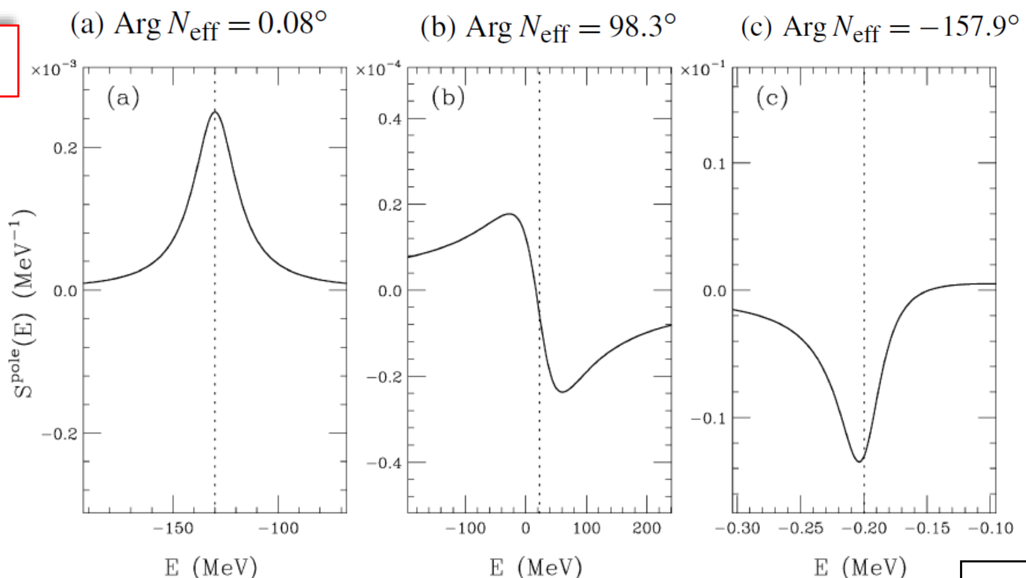
$$H^{\dagger}\tilde{\varphi}_{n\ell} = E_{n\ell}^*\tilde{\varphi}_{n\ell}$$

$$E_{n\ell} = (k_{n\ell}^{(\rm pole)})^2/2\mu = -B_{n\ell} - i\,\Gamma_{n\ell}/2$$

c-product

$$\int_0^\infty r^2\,dr\,(\tilde{\varphi}_{n\ell}(r))^*\varphi_{n\ell}(r) = \int_0^\infty r^2\,dr\,(\varphi_{n\ell}(r))^2 = 1$$

Arg $N_{\rm eff}$



Single-particle states for N and Λ

Schrodinger equations

$$\Big[-\frac{\hbar^2}{2M_N}\Big\{\frac{1}{r^2}\frac{d}{dr}\Big(r^2\frac{d}{dr}\Big)-\frac{l(l+1)}{r^2}\Big\}+U_N(r)\Big]R_{nlj}(r)=\varepsilon_N R_{nlj}(r)$$

Wave functions of a nucleon in the target

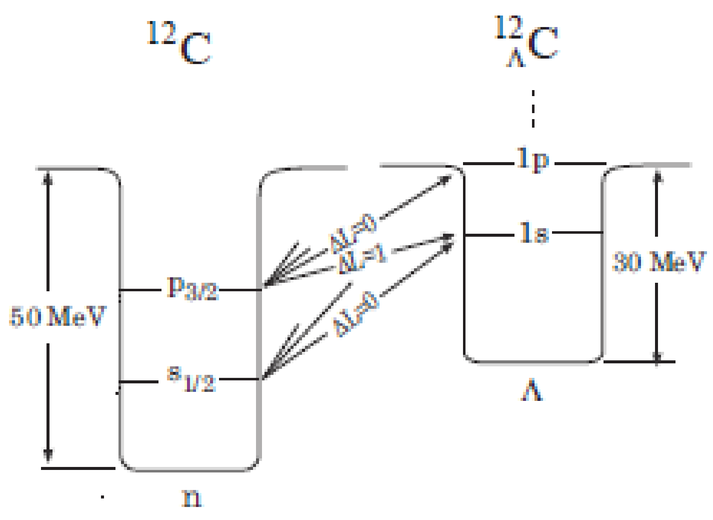
$$U_N(r) = V_0^N\, f(r) + V_{ls}^N(\boldsymbol{l}\cdot\boldsymbol{s})r_0^2\frac{1}{r}\frac{d}{dr}f(r)$$

$$(\boldsymbol{l}\cdot\boldsymbol{s}) = \tfrac{1}{2}[j(j+1)-l(l+1)-s(s+1)]$$

$$f(r) = \Big[1+\exp\Big(\frac{r-R}{a}\Big)\Big]^{-1}, \quad R = r_0 A^{1/3}$$

$$V_0^N = \Big(-51+33\frac{N-Z}{A}\Big)\,\mathrm{MeV},$$

$$V_{ls}^N = -0.44V_0^N, \quad r_0 = 1.27\,\mathrm{fm}, \quad a = 0.67\,\mathrm{fm}.$$



Wave functions of a Λ hyperon

$$U_\Lambda(r) = V_0^\Lambda\, f(r) + V_{ls}^\Lambda(\boldsymbol{l}\cdot\boldsymbol{s})r_0^2\frac{1}{r}\frac{d}{dr}f(r),$$

$$V_0^\Lambda = -30\,\mathrm{MeV}, \quad V_{ls}^\Lambda = 4\,\mathrm{MeV},$$

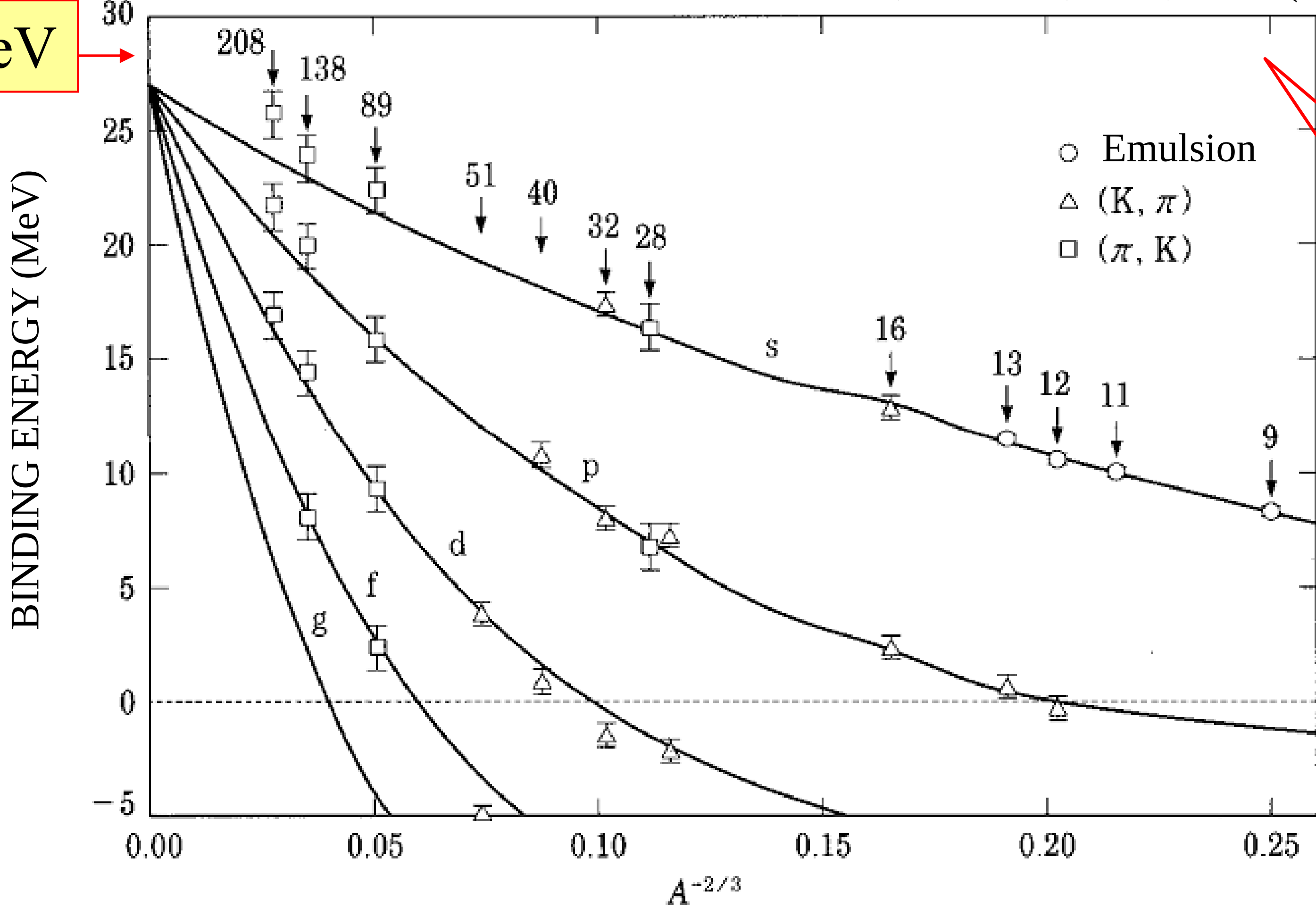
$$R = r_0(A-1)^{1/3}, \quad r_0 = 1.1\,\mathrm{fm}, \quad a = 0.6\,\mathrm{fm}$$

Binding energies of Λ single-particle states

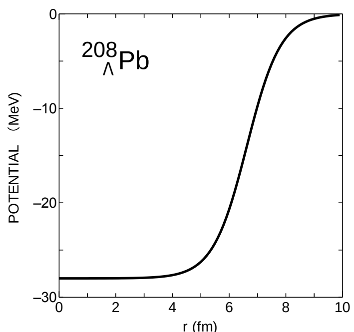
D.J. Millener, C.B.Dover, A.Gal, PRC38(1988)2700

~30 MeV

$\approx \frac{2}{3} U_N$



V_Λ ?

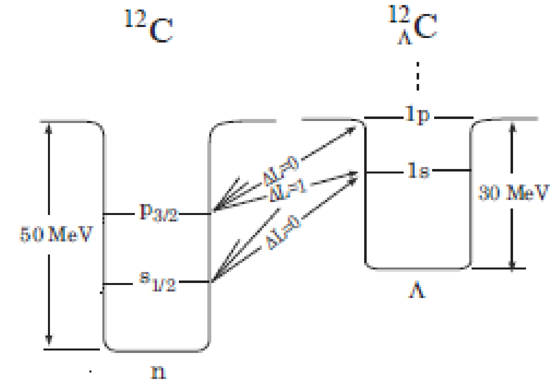


Woods-Saxon form

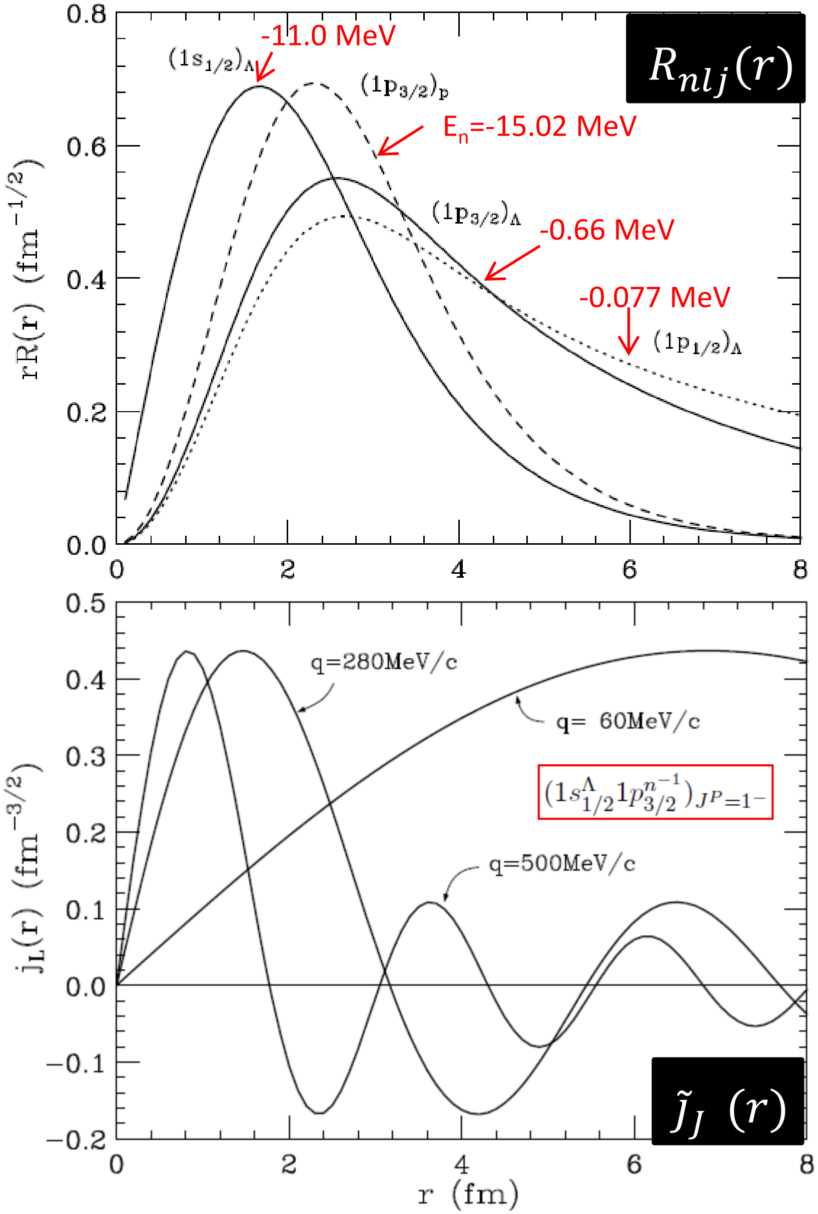
$$U_\Lambda = -U_\Lambda^0 / (1 + \exp[(r - R)/a]) \qquad R = r_0 (A-1)^{1/3} \text{fm}$$

$$U_\Lambda^0 = 28 \text{ MeV} \qquad a = 0.6 \text{ fm} \qquad r_0 = 1.128 + 0.439 A^{-2/3} \text{fm}$$

Calculations for $^{12}C \rightarrow {}^{12}C_{\Lambda}$ transitions

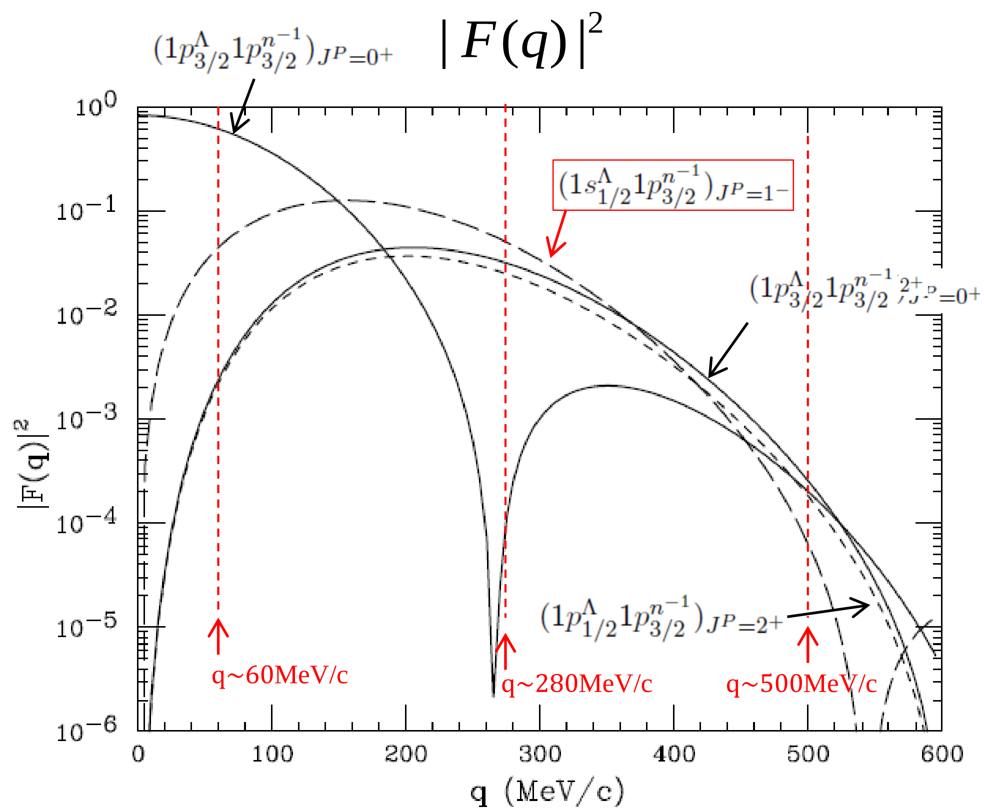


A=12 (n-^{11}C, Λ-^{11}C)



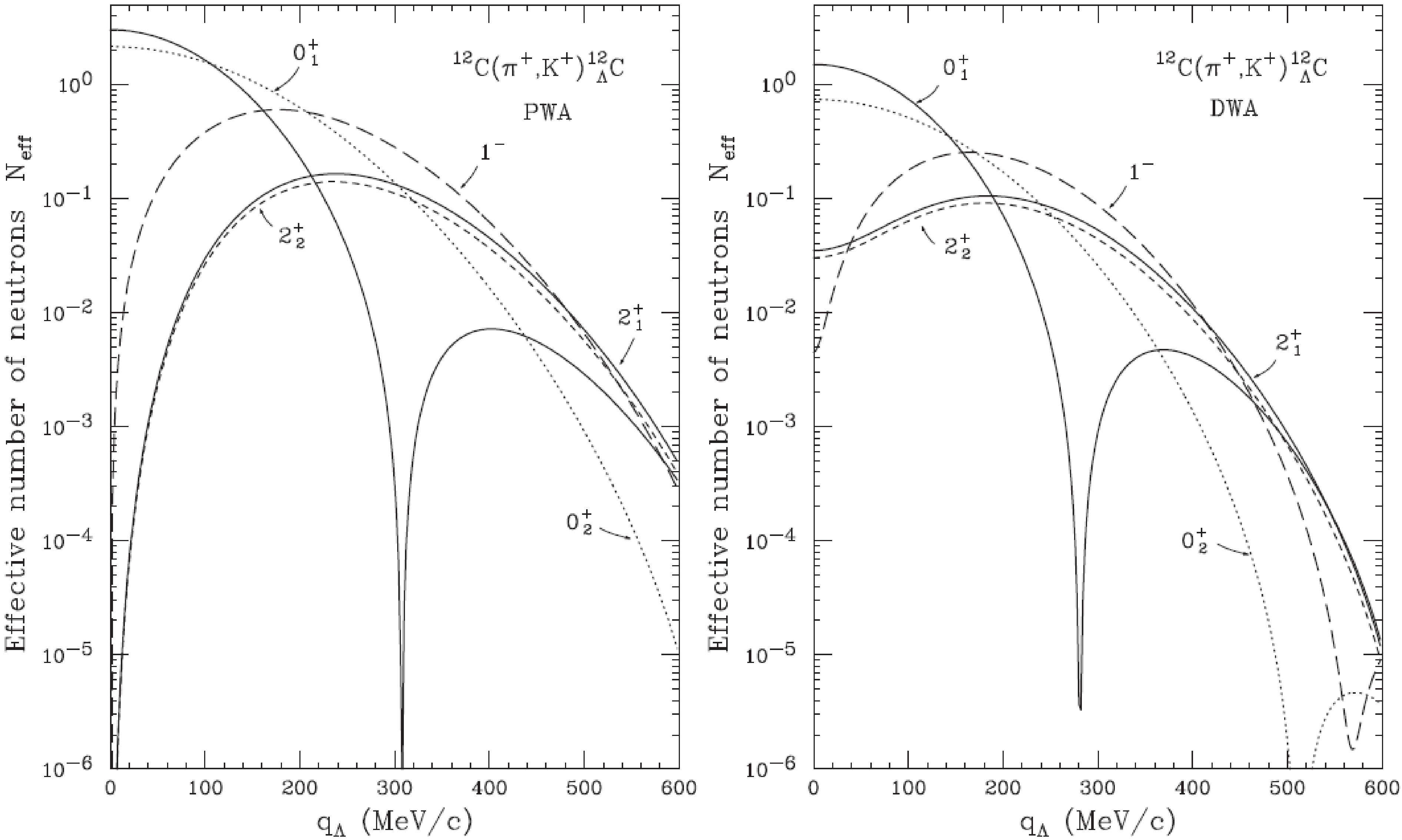
Form factor

$$F(q) = \int_0^\infty r^2 dr\, R^{(\Lambda)}_{n_\Lambda l_\Lambda j_\Lambda}(r) \tilde{j}_J(p_1, p_3; r) R^{(n)}_{n_n l_n j_n}(r)$$



Differential cross sections (PWA/DWA)

WS w.f.



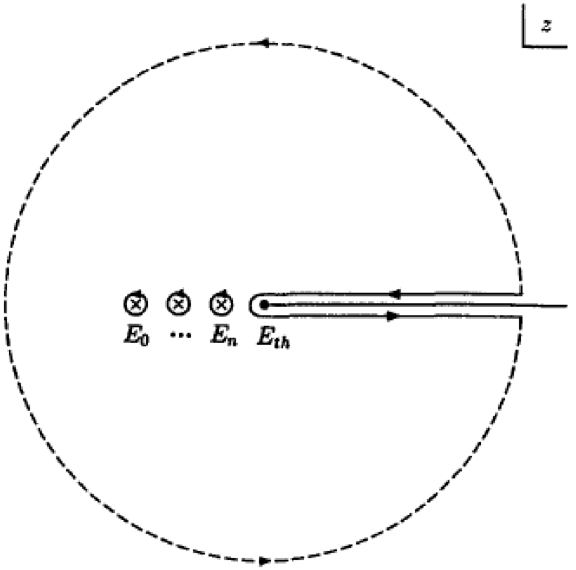
Green's function description

for continuum states

Green's function description

Green's function method

Morimatsu, Yazaki, NPA483 (1988) 493.



Double-differential Lab Cross Sections

$$\left(\frac{d^2\sigma}{dE_b\, d\Omega_b}\right) = \beta \left(\frac{d\sigma}{d\Omega_b}\right)^{aN \to bY}_{\text{lab}} S(E)$$

Strength functions

$$S(E_B) = \sum_B |\langle \Psi_B | \hat{F} | \Psi_A \rangle|^2 \delta(E_\pi + E_B - E_K - E_A)$$

$$= (-)\frac{1}{\pi}\mathrm{Im} \sum_{\alpha\alpha'} \int d\mathbf{R} d\mathbf{R}' F_\alpha^\dagger(\mathbf{R}) G_{\alpha\alpha'}(E_B; \mathbf{R}, \mathbf{R}') F_{\alpha'}(\mathbf{R}')$$

Green's function

$$\sum_B |\Psi_B\rangle \delta(E - E_B) \langle \Psi_B| = (-)\frac{1}{\pi}\mathrm{Im}\left[\frac{1}{E - H_B + i\epsilon}\right]$$

Completeness relation

$$G^{(+)}(E; \boldsymbol{r}, \boldsymbol{r}') = \sum_n \frac{\varphi_n(\boldsymbol{r})(\tilde{\varphi}_n(\boldsymbol{r}'))^*}{E - E_n + i\epsilon} + \frac{2}{\pi}\int_0^\infty dk \frac{k^2 S(k) u(k, \boldsymbol{r})(\tilde{u}(k, \boldsymbol{r}'))^*}{E - E_k + i\epsilon}$$

bound states,
quasibound states

Continuum states,
resonance states

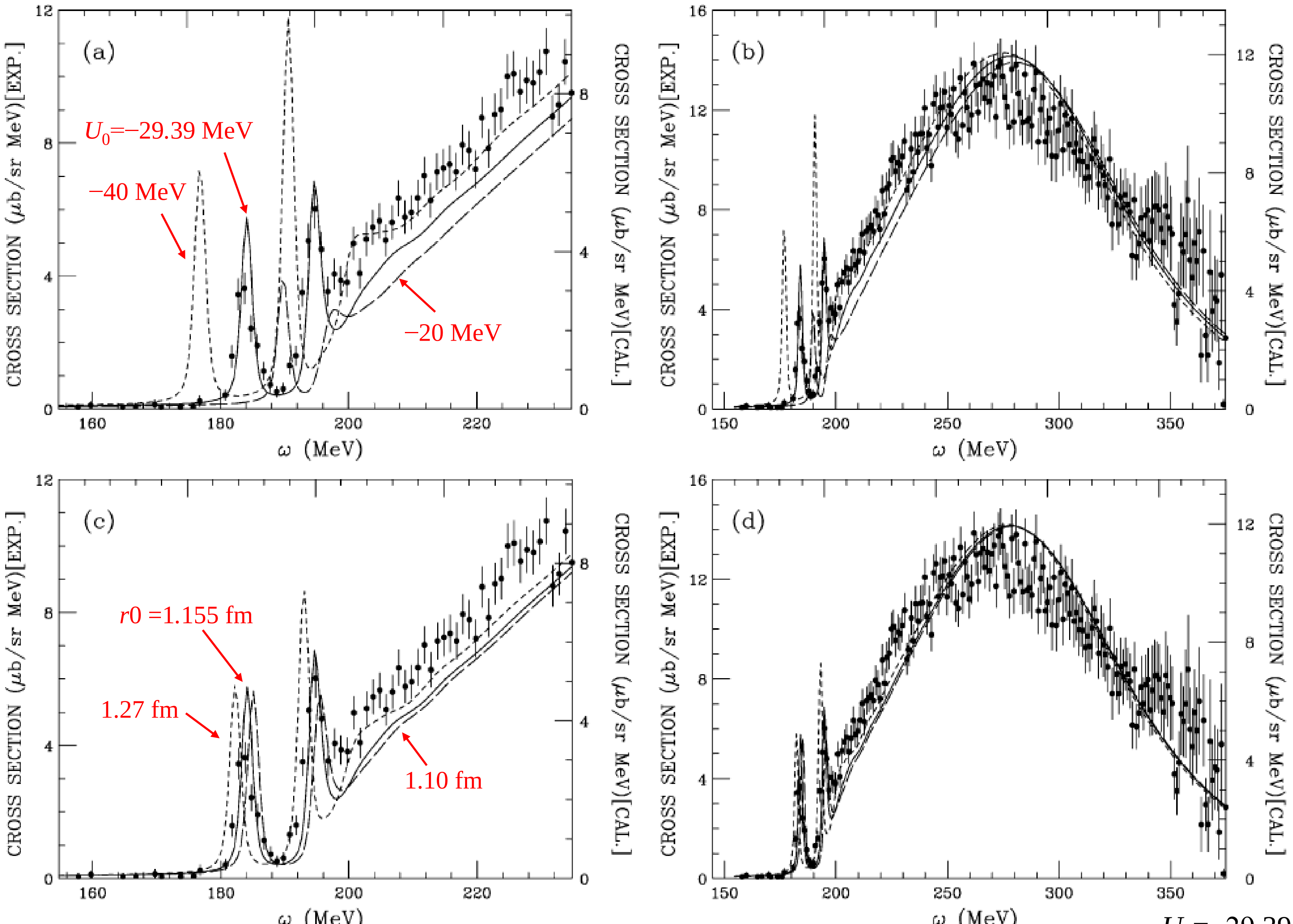
^{12}C

Λ spectrum by (π^+,K^+) reaction at 1.2 GeV/c (6°)

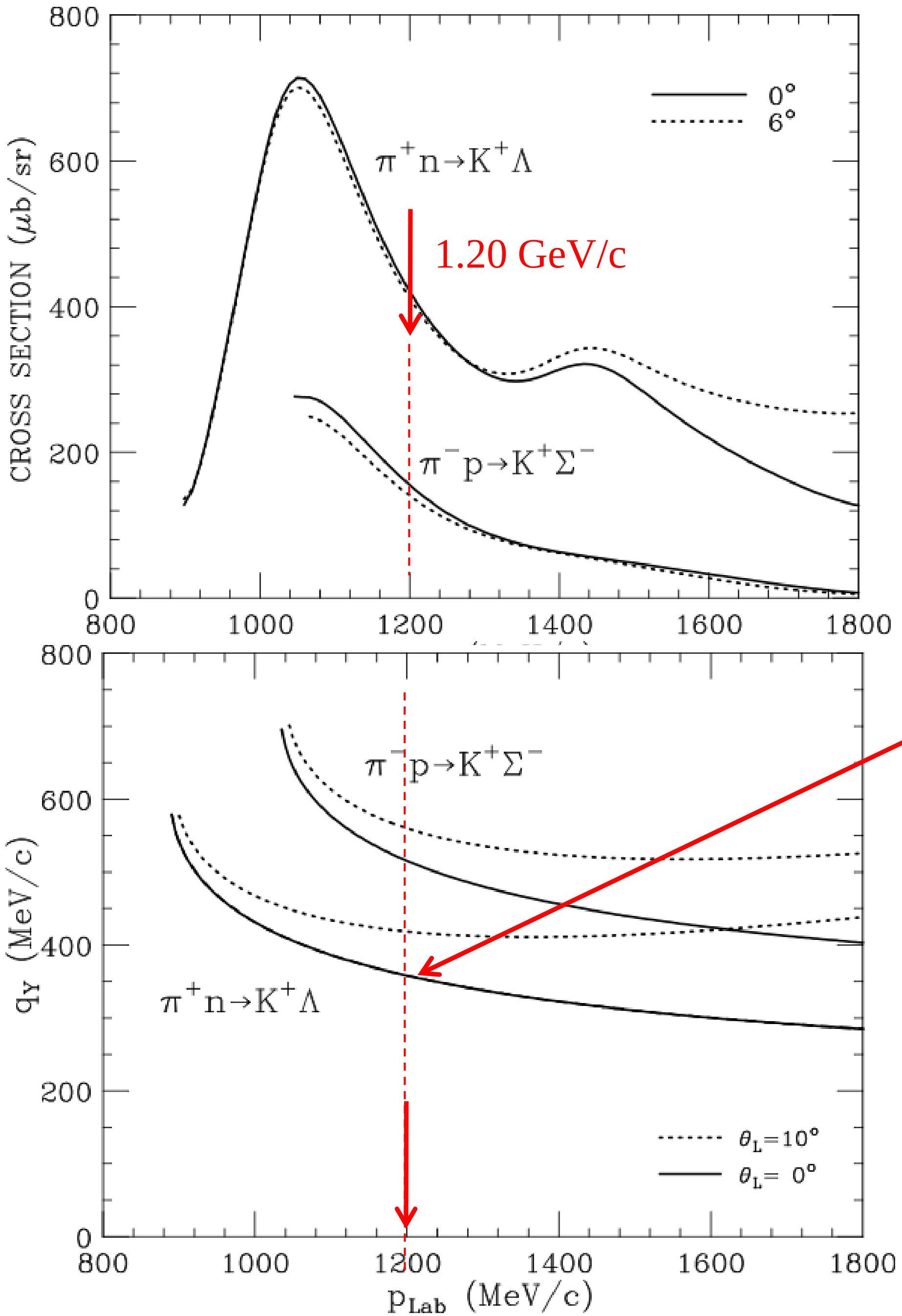
Harada, Hirabayashi, NPA744 (2004) 323.

Sensitivity of the spectrum to the Λ–nucleus potential parameters

r_0 = 1.155 fm



U_0=−29.39 MeV



Elementary processes of the $\pi^+p \rightarrow K^+\Lambda$ and $\pi^-p \rightarrow K^+\Sigma^-$ reactions

Large momentum transfer

$q_\Lambda \approx 400$ MeV/c

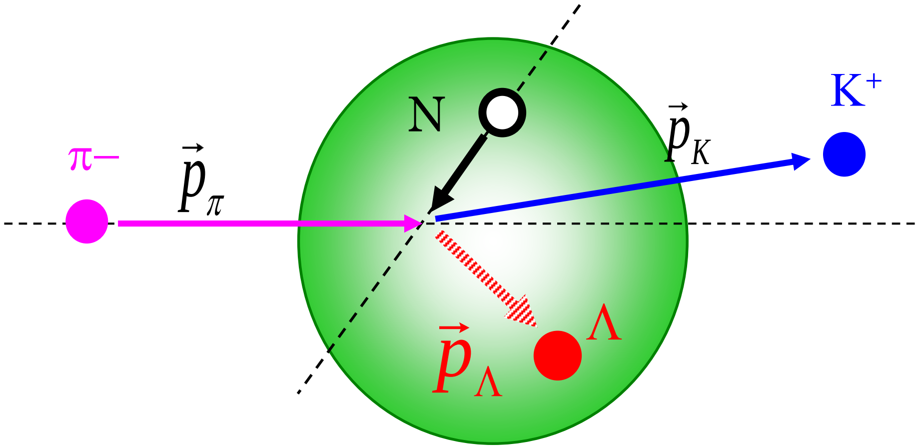
Optimal Fermi-averaging for an elementary T-matrix

T. Harada and Y.Hirabayashi, NPA744 (2004) 323.

"Optimal" cross section

$$\left[\frac{d\sigma}{d\Omega}\right]^{\rm opt}_{\pi^+ p\to K^+\Lambda} = \left|\overline{f}_{\pi^+ p\to K^+\Lambda}\right|^2$$

$$= \frac{k_K E_K}{(2\pi)^2 v_\pi}\left|\hat{t}^{\rm opt}(p_\pi;\omega,\mathbf{q})\right|^2$$



Optimal Fermi-averaged T-Matrix

On-shell T-matrix

$$\hat{t}^{\rm opt}(p_\pi;\omega,\mathbf{q}) = \left.\frac{\int_0^\pi \sin\theta d\theta \int_0^\infty p_N^2 dp_N \hat{t}_{Lab}(E_{\pi N};p_\pi,p_N)\rho(p_N)}{\int_0^\pi \sin\theta d\theta \int_0^\infty p_N^2 dp_N \rho(p_N)}\right|_{\mathbf{p}_N=\mathbf{p}_N^*}$$

given

"On-energy-shell" equation $\omega = E_{\rm f} - E_{\rm i} = \sqrt{(\mathbf{p}_N^* + \mathbf{q})^2 + m_\Sigma} - \sqrt{\mathbf{p}_N^{*\,2} + m_N}$

given

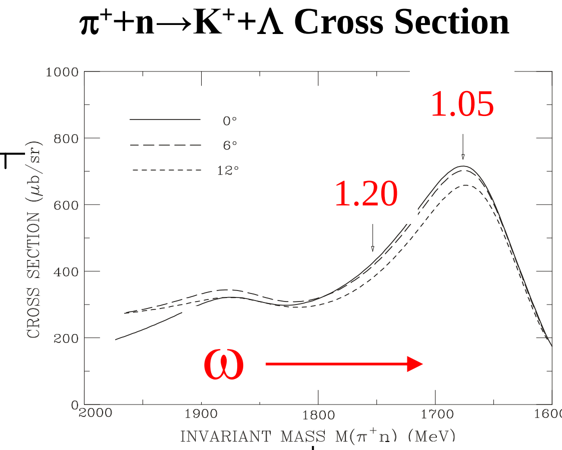
$$\mathbf{p}_\pi + \mathbf{p}_N^* = \mathbf{p}_K + \mathbf{p}_\Sigma$$

S,A.Gurvitz, PRC33(1986)422: Optimal factorization

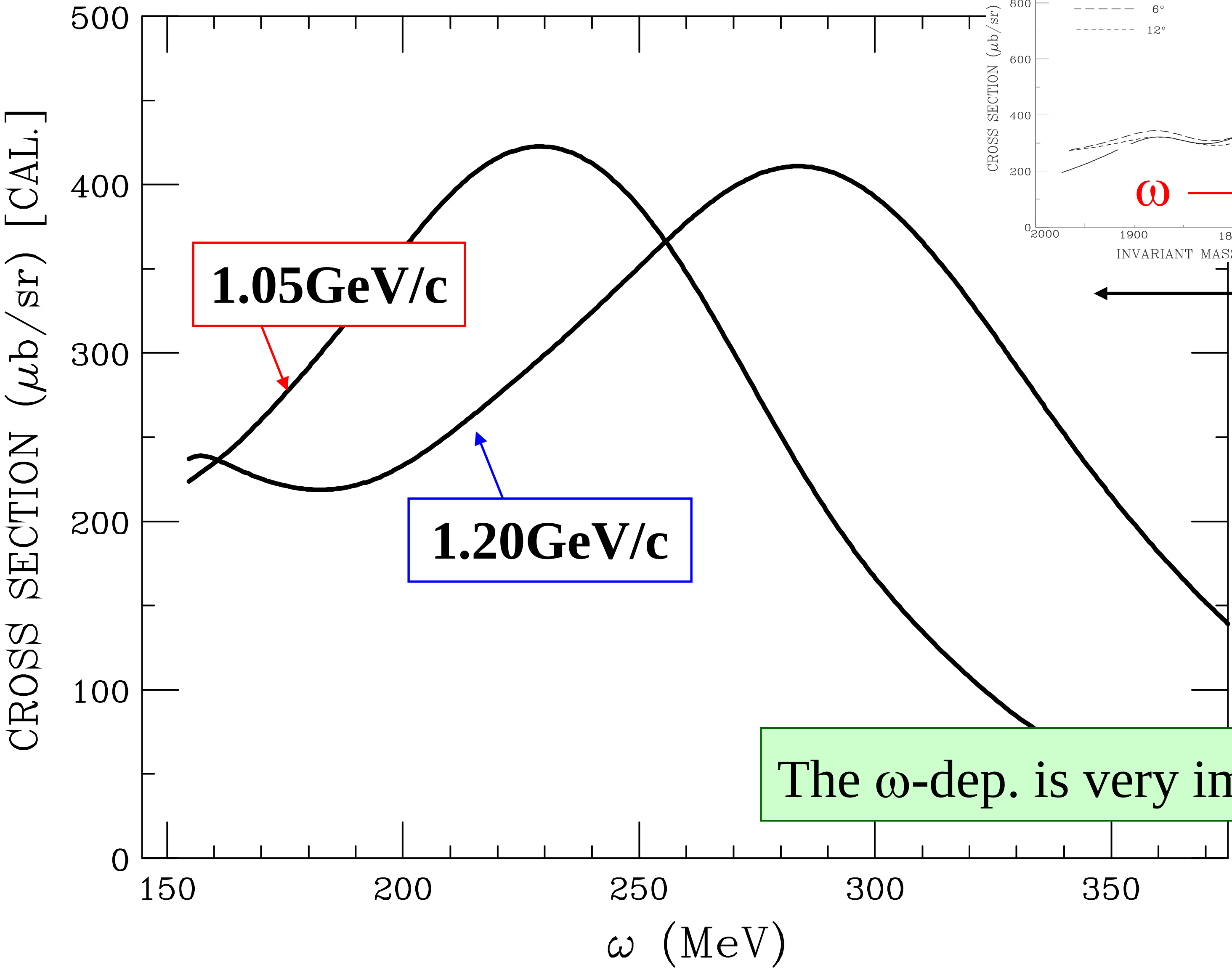
Elementary cross section of $\pi^+ + n \rightarrow K^+ + \Lambda$ reactions in nuclei

Optimal Fermi-averaging

$$\left(\frac{d\sigma}{d\Omega_b}\right)^{aN\to bY}_{\rm lab}$$

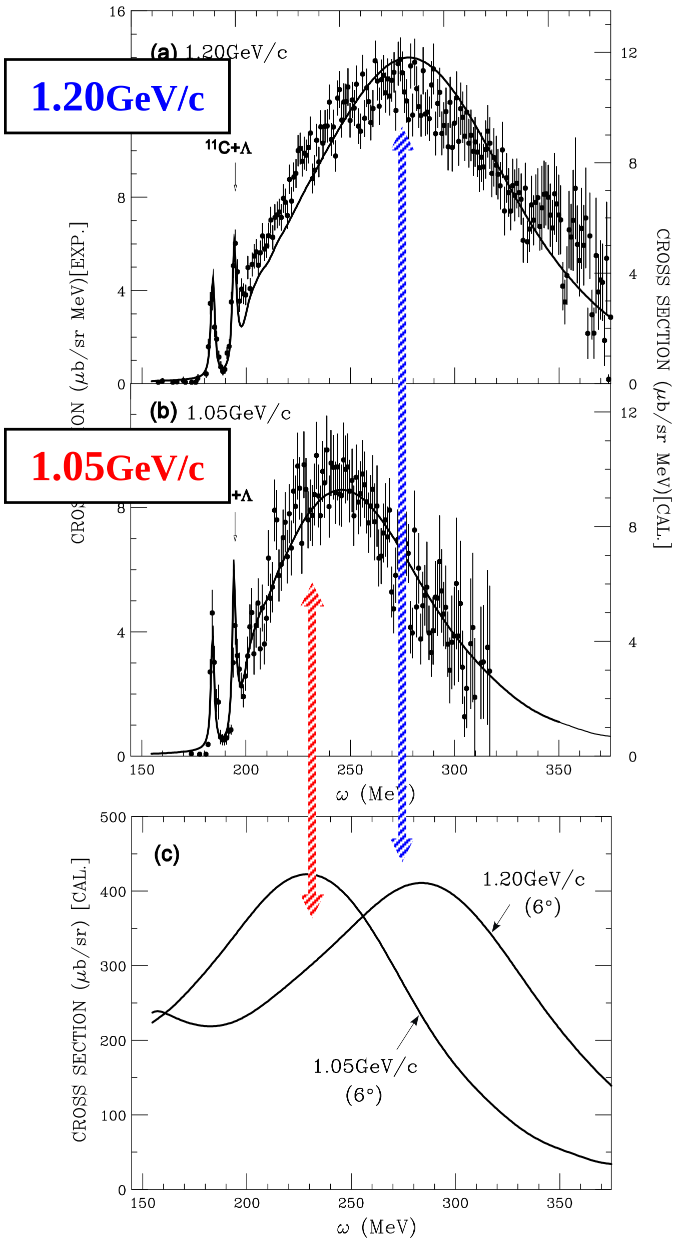


M(π^+n)



The ω-dep. is very important !!

$^{12}C(\pi^+,K^+)$ Reactions



Remarks

The calculated spectra in the QF region can explain the experimental data at 1.20 and 1.05GeV/c.

make the width look narrow

The ω energy-dependence originates from the nature of the "optimal Fermi-averaging" t-matrix.

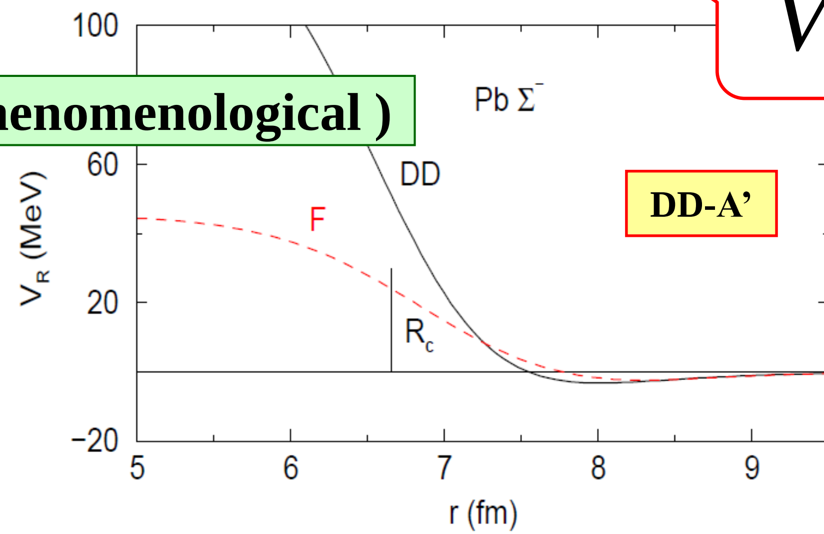
Need careful consideration for energy-dependent of the elementary cross section.

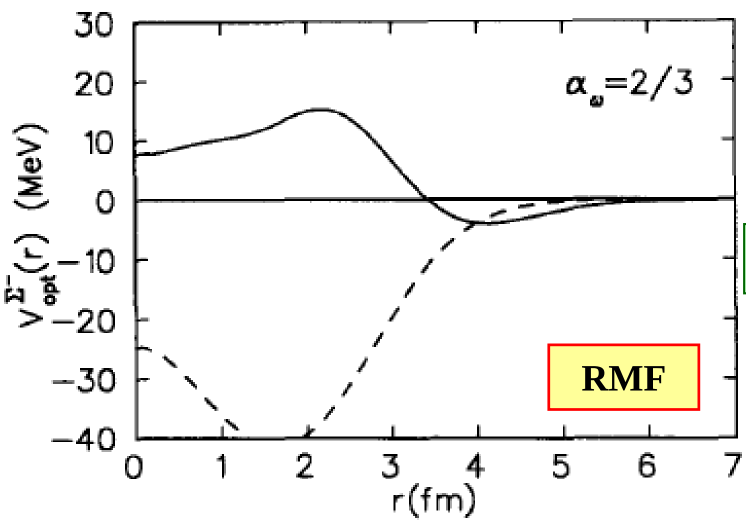
Σ^- s.p. potentials (fitted to the Σ^- atomic data)

V_Σ ?

Density-dependent (DD) potential (Phenomenological)

C.J.Batty et al., Phys.Rep.287(1997)385,
E. Friedman and A. Gal, Phys. Rep. 452 (2007)89.





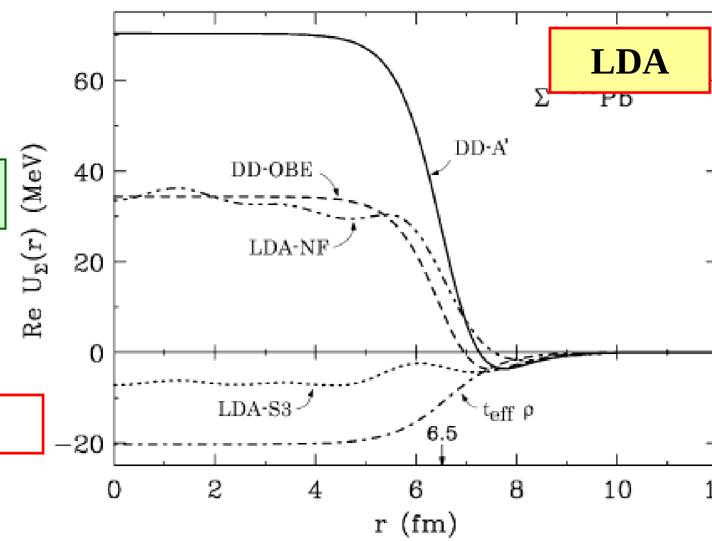
Relativistic mean-field (RMF) potential

J. Mares et al., NPA594(1995)311.
K.Tsubakihara et al., EPJA33(2007)295

Folding-model potential for LDA with G-matrix

D. Halderson, Phys. Rev. C40(1989)2173.
T.Yamada and Y.Yamamoto, PTP. Suppl. 117(1994)241
J. Dabrowski, Acta Phys. Pol. B31(2001)2179
T.Harada, Y.Hirabayashi, NPA759 (2005) 143; 767(2006)206

YNG-F

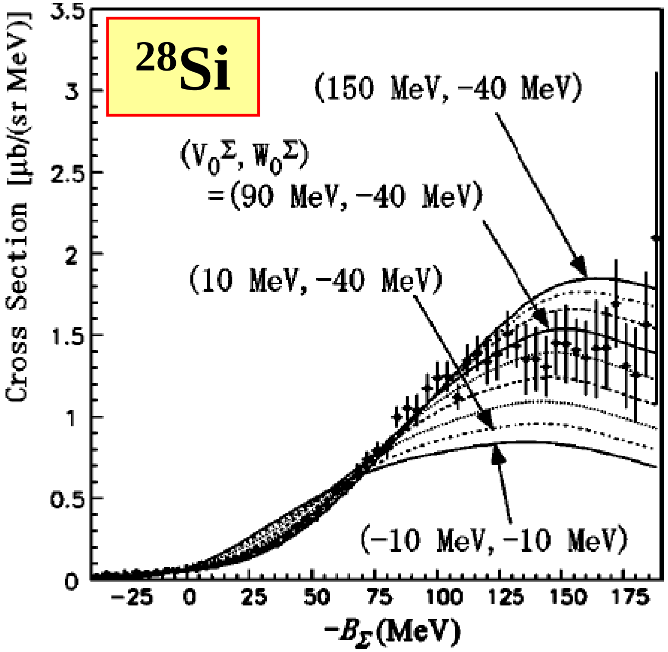


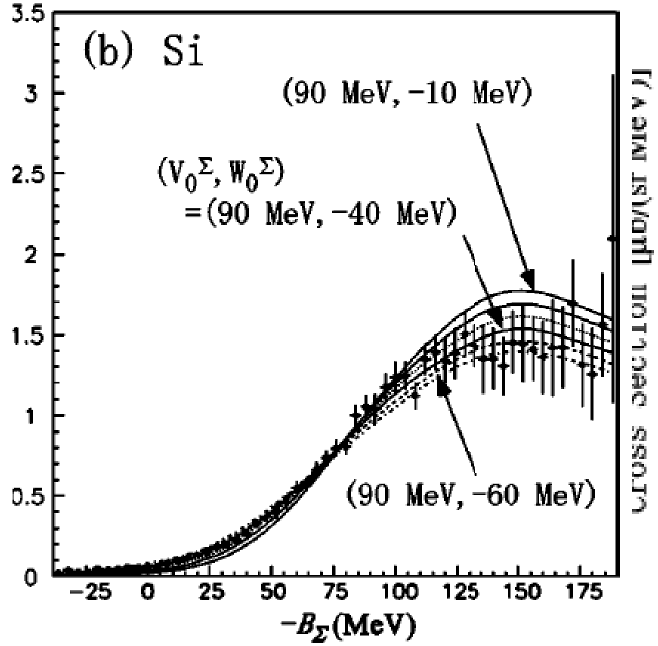
- It suggests that Σ-nucleus potentials have a strong repulsion in the real part.

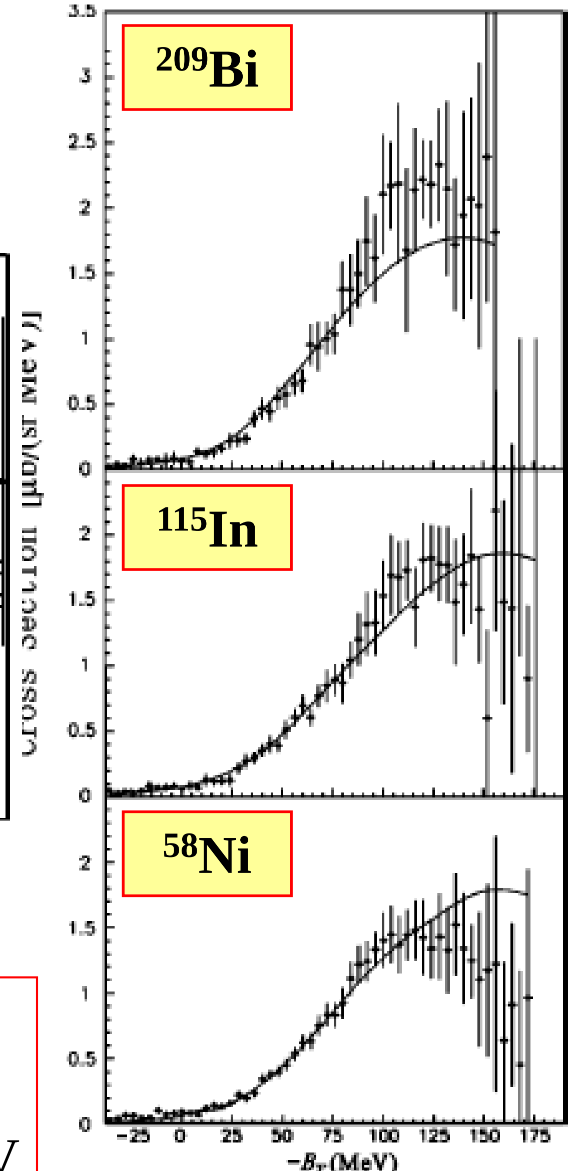
Σ^- spectrum by (π^-,K^+) reaction at 1.2GeV/c

Study of Σ s.p. potentials for havier targets

[H.Noumi, et al. PRL89(2002)072301]
[P.K.Saha, et al., PRC70(2004)044613]







Woods-Saxon form

$$U_\Sigma = \frac{V_\Sigma + iW_\Sigma}{1+\exp[(r-R)/a]}$$

$$R = r_0(A-1)^{1/3}\,\text{fm}$$

$$a = 0.67\ \text{fm} \qquad r_0 = 1.1\text{fm}$$

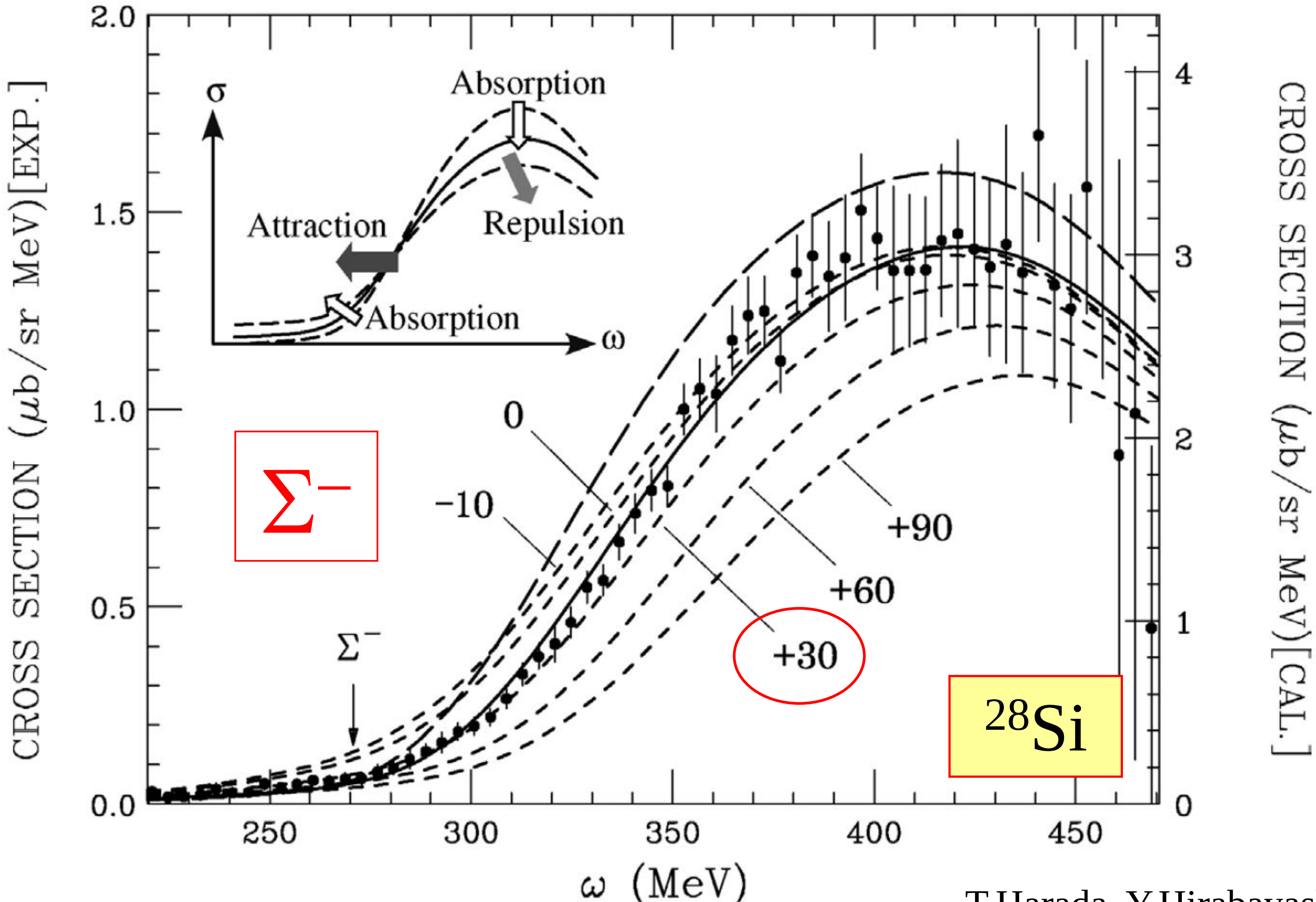
$$V_\Sigma = +90\ \text{MeV}$$

$$W_\Sigma = -40\ \text{MeV}$$

Strong repulsion with large imaginary

Inclusive spectrum in $^{28}Si(\pi^-, K^+)$ reaction at 1.2GeV/c

Exp. Data from P.K.Saha, H. Noumi, et al., PRC70(2004)044613



(V_Σ, W_Σ) = (+30, –40) MeV by χ^2/N-fitting

T.Harada, Y.Hirabayashi, NPA759 (2005) 143

Remarks

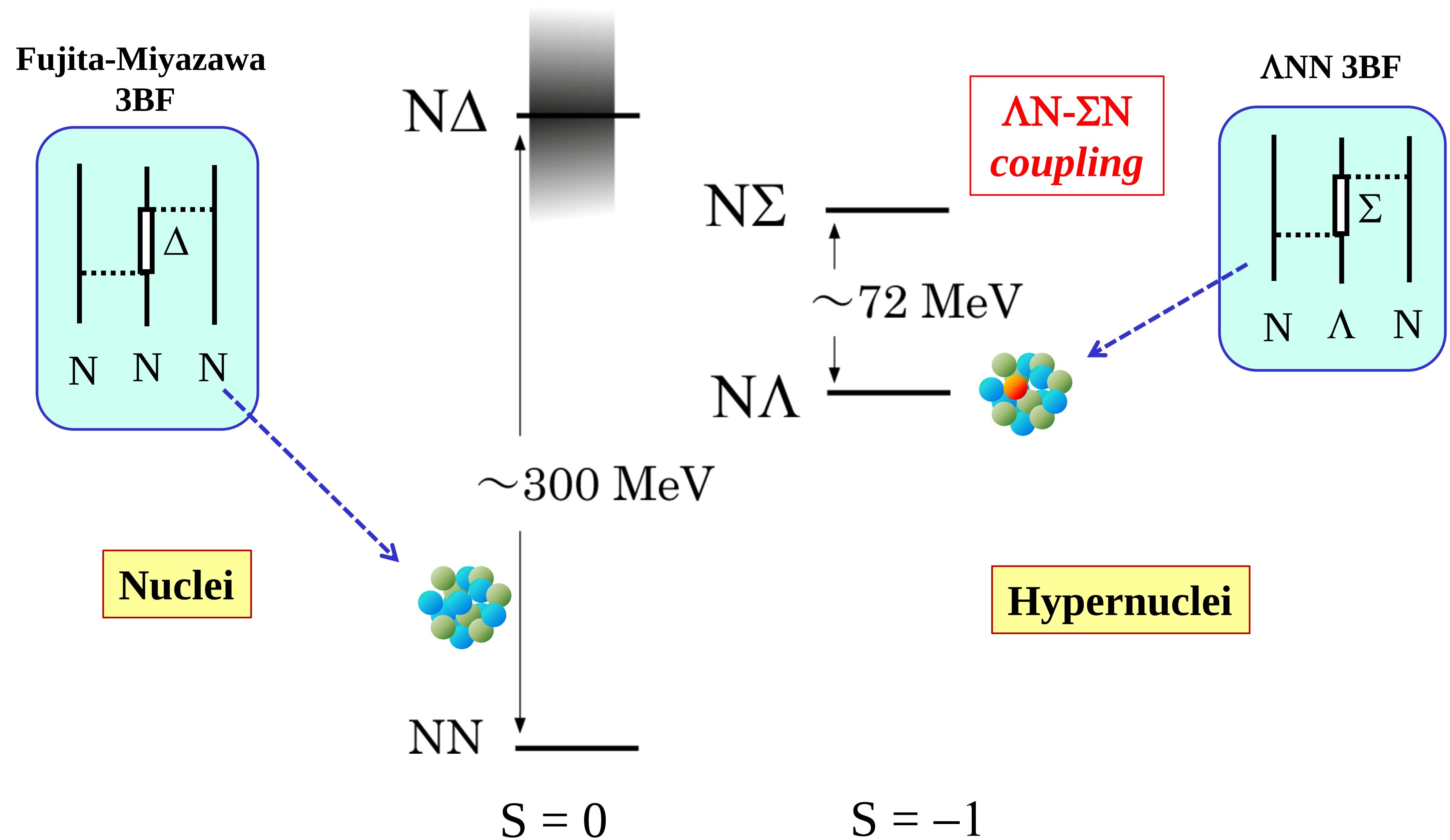
- The resultant spectra for (π^+,K^+) reactions can reproduce fully the experimental data, including the Λ bound and QF region.
  - Λ potential U_0 → Neutron star
  - Fermi-averaging cross sections

- We can explain simultaneously the data of the Σ^- atoms and (π^-,K^+) reactions, including the Σ bound and QF region.
  - Σ potential U_0 → Neutron star
  - Repulsion inside the nuclear surface and an attraction outside the nucleus ← Quark exchange in BB interaction

- The DWIA calculations work very well in Λ, Σ production spectra.
  - (quasi-)bound states and continuum states → Green's function

Production of a neutron-rich hypernucleus by $^{6}Li(\pi^{-},K^{+})$ reactions

Dynamics in Hypernuclear Systems



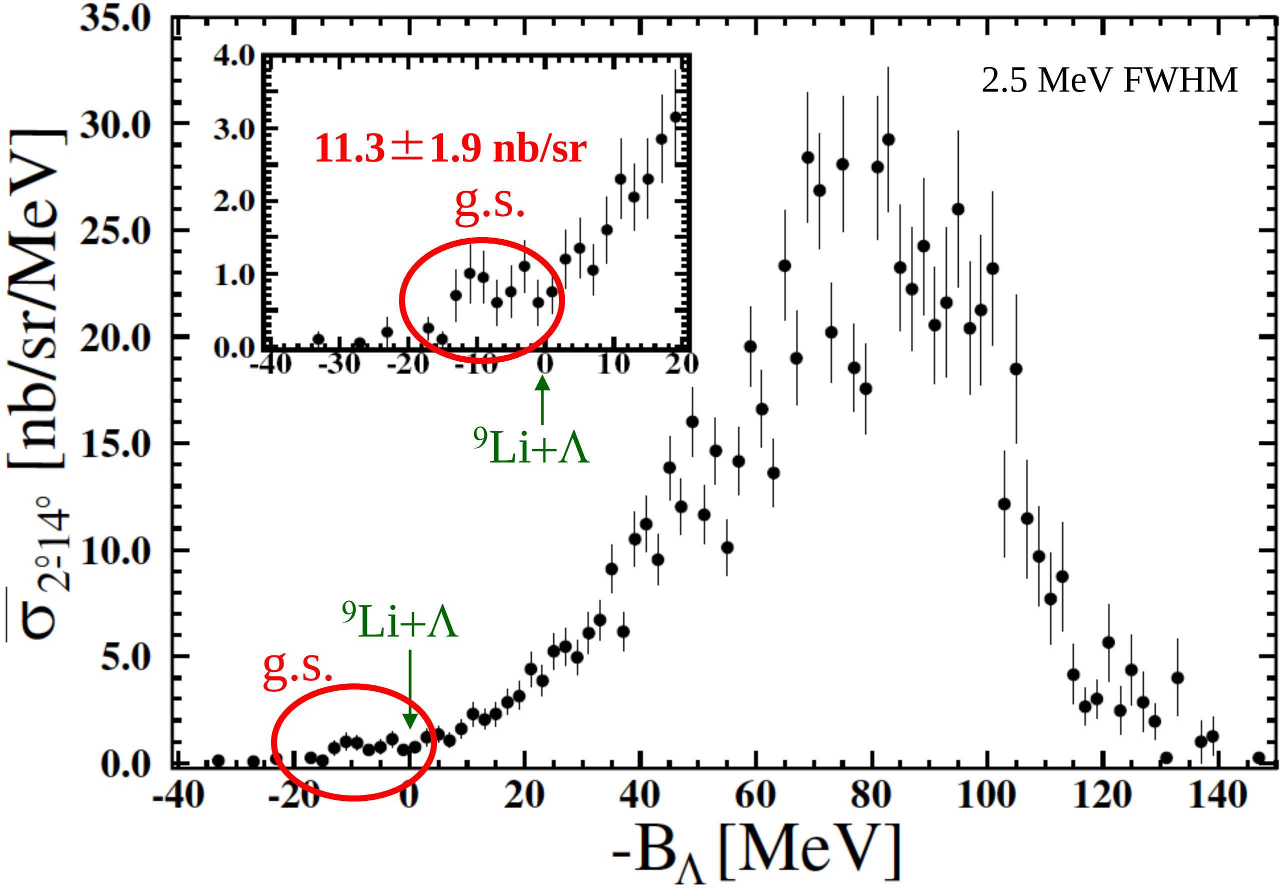
- Various effects on the hyperon mixing
- Related to the 3BF in nuclei

First production of neutron-rich Λ hypernuclei

$^{10}\mathrm{B}(\pi^-, K^+)^{10}_{\Lambda}\mathrm{Li}$

Λ spectrum by DCX (π⁻,K⁺) reaction at 1.2GeV/c

KEK-PS-E521 P. K. Saha, et al., PRL94(2005)052502



Cross sections

- p_π=1.20 GeV/c
  $\frac{d\sigma}{d\Omega_L} \approx 11.3 \pm 1.9$ nb/sr

- p_π=1.05 GeV/c
  $\frac{d\sigma}{d\Omega_L} \approx 5.8 \pm 2.2$ nb/sr

～1/1000

$^{12}\mathrm{C}(\pi^+, K^+)^{12}_{\Lambda}\mathrm{C}$ (1.2 GeV/c)
17.5±0.6 μb/sr

The DCX (π⁻,K⁺) *reaction at 1.2GeV/c can produce the neutron-rich Λ hypernucler states, whereas the cross section is as small as 1/1000 of the* (π⁺,K⁺) *reaction.*

(π^-, K^+) —Double Charge Exchange (DCX) Reaction

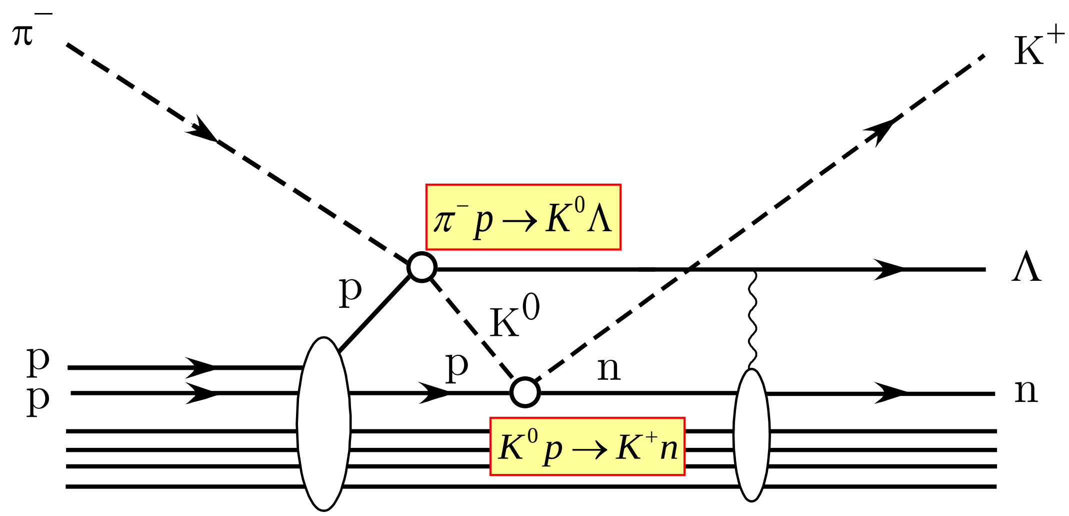
Two-step process:

$$\pi^- p \to K^0 \Lambda$$

$$K^0 p \to K^+ n$$

$$\pi^- p \to \pi^0 n$$

$$\pi^0 p \to K^+ \Lambda$$



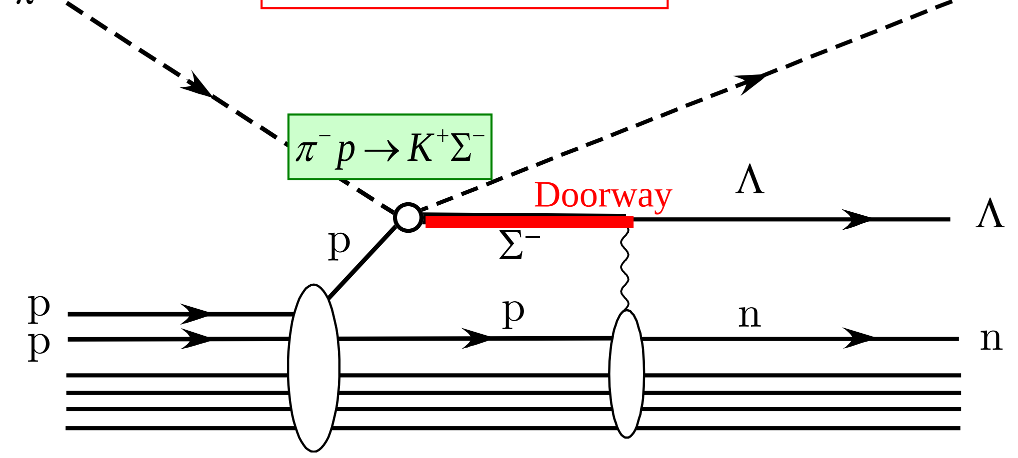
One-step process:

$$\pi^- p \to K^+ \Sigma^-$$

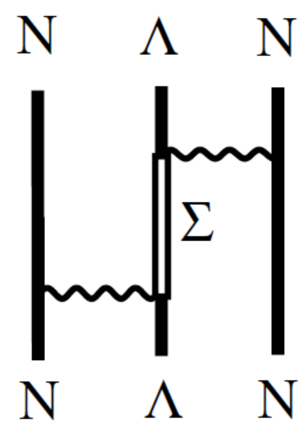
$$\Sigma^- p \leftrightarrow \Lambda n$$

via Σ^- doorways caused
by ΛN-ΣN coupling

Hyperon-mixing



Coherent Λ-Σ coupling



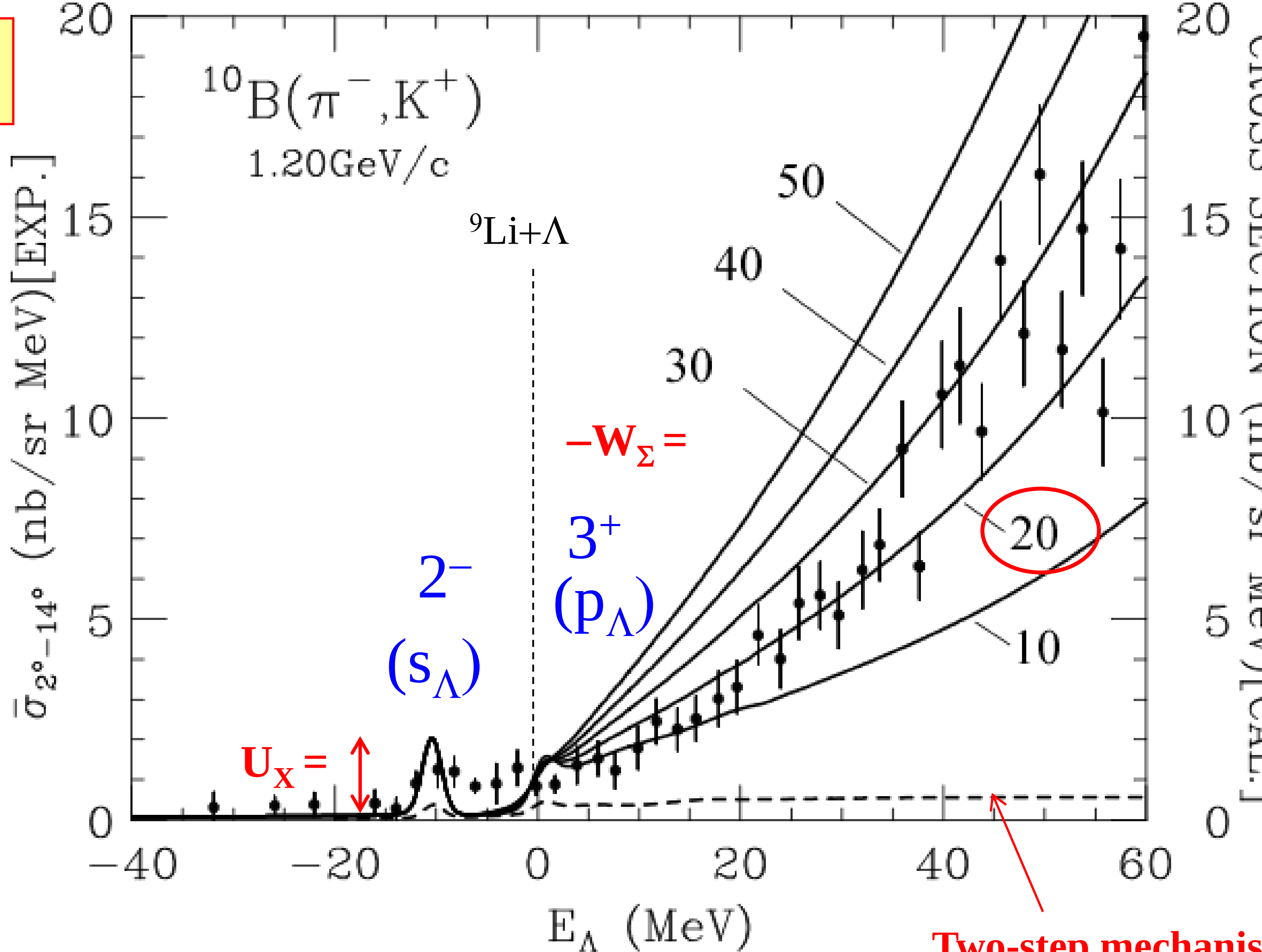
Λ spectrum by DCX (π^-,K^+) reactions at 1.2GeV/c

Harada, Umeya, Hirabayashi, PRC79(2009)014603

Spreading potential dep. W_Σ

U_X =11 MeV is fixed. P_{Σ^-}=0.57%

^{10}B



Status

- ${}^{6}_{\Lambda}$H is one of the most interesting candidates to investigate neutron-rich hypernuclei; $B[{}^{6}_{\Lambda}\mathrm{H}(0^{+}_{\mathrm{g.s.}})]$ = 5.8 MeV caused by the coherent ΛΣ coupling.

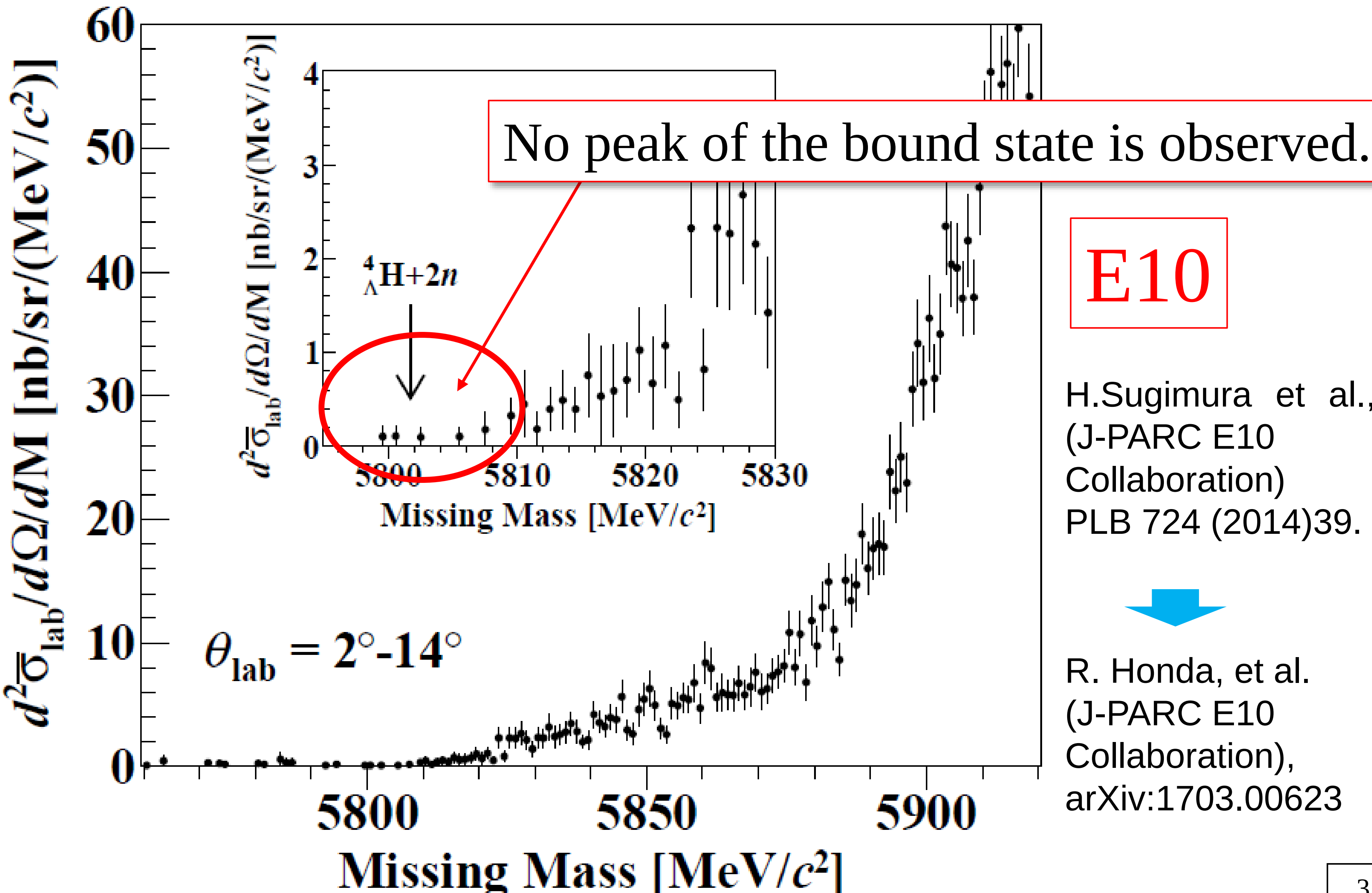
  Y. Akaishi, Khin Swe Myint, AIP Conf. Proc. 1011 (2008) 277.

- FINUDA collaboration reported a binding energy of $B({}^{6}_{\Lambda}\mathrm{H})$= 4.5±1.2 MeV in ${}^{6}\mathrm{Li}(K^{-}_{\mathrm{stop}}, \pi^{+})$ reactions. M. Agnello, et al., PRL. 108 (2012) 042501.

- No peak is observed around the ${}^{4}_{\Lambda}\mathrm{H} + 2n$ threshold in the ${}^{6}\mathrm{Li}(\pi^{-}, \mathrm{K}^{+})\,{}^{6}_{\Lambda}\mathrm{H}$ reaction at $p_{\pi-}$=1.2 GeV/c by J-PARC E10 collaboration.

  H. Sugimura, Phys. Lett. B729 (2014) 39.
  R. Honda, Ph.D. thesis, Tohoku University (2014).

Search for the $^{6}_{\Lambda}H$ hypernucleus by $^{6}Li(\pi^{-}, K^{+})$ reactions 1.2GeV/c@J-PARC E10



E10

H.Sugimura et al., (J-PARC E10 Collaboration) PLB 724 (2014)39.

R. Honda, et al. (J-PARC E10 Collaboration), arXiv:1703.00623

Our Purpose

- We theoretically demonstrate the inclusive spectra of the ^{6}Li(π^{-}, K^{+}) reaction within a distorted-wave impulse approximation, using a coupled (^{5}H-Λ) +(^{5}He-Σ^{-}) model with a spreading potential by the ***one-step mechanism*** via Σ^{-} doorways.

(1) To extract valuable information on the Σ-nucleus potential for Σ^{-}-^{5}He from the data of the J-PARC E10 experiments. Σ- regions

(2) To study the $\Sigma\Lambda$ coupling effects related to the Σ-mixing and the strengths of the Λ-^{5}H potential in $^{6}_{\Lambda}$H($1^{+}_{\mathrm{exc.}}$). [not $^{6}_{\Lambda}$H($0^{+}_{\mathrm{g.s.}}$)] Λ regions

Coupled-channel calculations in the DWIA

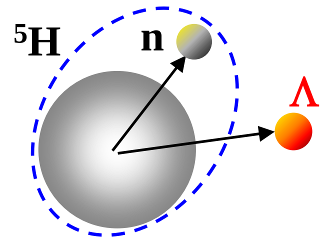
with the optimal Fermi-averaged *t*-matrix

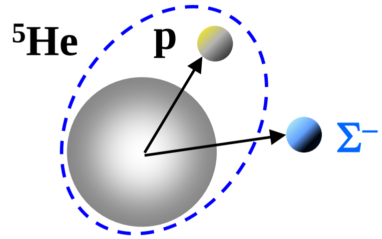
^{6}Li(π^{-}, K^{+}) reactions

Model for final states of the hypernucleus

Single-particle shell model wf.

$$|\Psi_{J_B}({}^{6}_{Y}\mathrm{H})\rangle = \sum_{JJ''j_nj_\Lambda} [\Phi_{J''}({}^5\mathrm{H}), \varphi^{(\Lambda)}_{j_\Lambda}(\boldsymbol{r}_\Lambda)]_{J_B} + \sum_{JJ'j_pj_\Sigma} [\Phi_{J'}({}^5\mathrm{He}), \varphi^{(\Sigma^-)}_{j_\Sigma}(\boldsymbol{r}_\Sigma)]_{J_B}$$





Σ^- mixing probability

$$P_{\Sigma^-} = \sum_{j_2} \langle \varphi^{(\Sigma^-)}_{j_2} | \varphi^{(\Sigma^-)}_{j_2} \rangle$$

Hyperon-nucleus potentials

spreading potential for excited states

$$U_Y(r) = V_Y f(r, R, a) + iW_Y f(r, R', a')$$

$V_\Lambda = -19$ MeV is assumed

(V_Σ, W_Σ) determined as fitting parameters

zero-range interaction:
Woods-Saxon form

$$f(r, R, a) = [1 + \exp((r - R)/a)]^{-1}$$

Coupling Λ-Σ folding potential

Shell-model w.f. with (s^3p^2) configuration

$$U_X(r) = \left\langle [\Phi_{J'}({}^5\mathrm{He}) \otimes \mathcal{Y}^{(\Sigma^-)}_{j'\ell's'}(\hat{\boldsymbol{r}})]_{J_B} \middle| \sum_i v_{\Sigma,\Lambda}(\boldsymbol{r}'_i, \boldsymbol{r}) \middle| [\Phi_J({}^5\mathrm{H}) \otimes \mathcal{Y}^{(\Lambda)}_{j\ell s}(\hat{\boldsymbol{r}})]_{J_B} \right\rangle$$

$$= \sum_{LSK} v^S_{\Sigma N,\Lambda N} C^{J_B}_{LSK}(J'J'') \mathcal{F}^{J'J''}_{LSK}(r)$$

zero-range interaction:

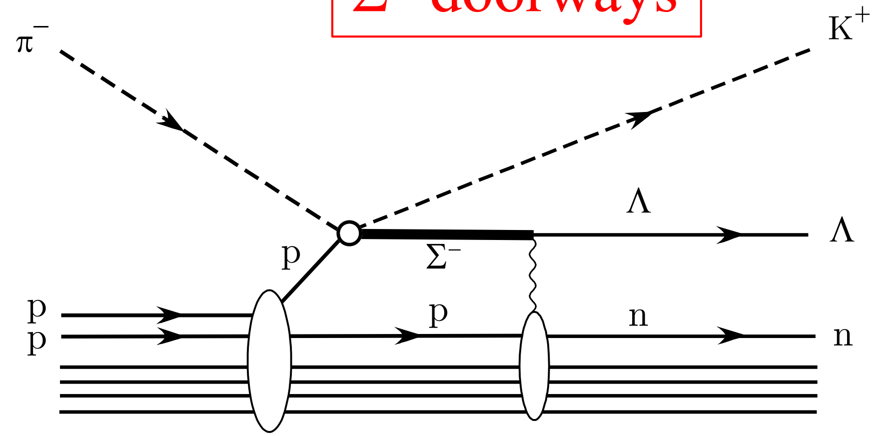
$$v_{\Lambda\Lambda\text{-}\Xi N} = v^0_{\Lambda\Lambda\text{-}\Xi N} \delta(\mathbf{r} - \mathbf{r}')$$

volume integral:

Coupled-channels DWIA calculation for one-step mechanism

Coupled-channel Green's function

Σ^- doorways



$$\boldsymbol{G}(\omega) = \boldsymbol{G}^{(0)}(\omega) + \boldsymbol{G}^{(0)}(\omega)\boldsymbol{U}\boldsymbol{G}(\omega)$$

$$\boldsymbol{G}(\omega) = \begin{pmatrix} G_\Lambda(\omega) & G_X(\omega) \\ G_X(\omega) & G_\Sigma(\omega) \end{pmatrix}, \quad \boldsymbol{U} = \begin{pmatrix} U_\Lambda & U_X \\ U_X & U_\Sigma \end{pmatrix}$$

DWIA+CCGFM

Inclusive cross sections

T. Harada, NPA672(2000)181

$$\left(\frac{d^2\sigma}{d\Omega_K dE_K}\right)_{\rm lab} = \beta \frac{1}{[J_A]} \sum_{M_z} \sum_{\alpha'\alpha} \left(-\frac{1}{\pi}\right) {\rm Im}\left[\int d\boldsymbol{r}' d\boldsymbol{r}\, F_\Sigma^{\alpha'\dagger}(\boldsymbol{r}')\, G_\Sigma^{\alpha'\alpha}(\omega, \boldsymbol{r}', \boldsymbol{r}) F_\Sigma^{\alpha}(\boldsymbol{r})\right]$$

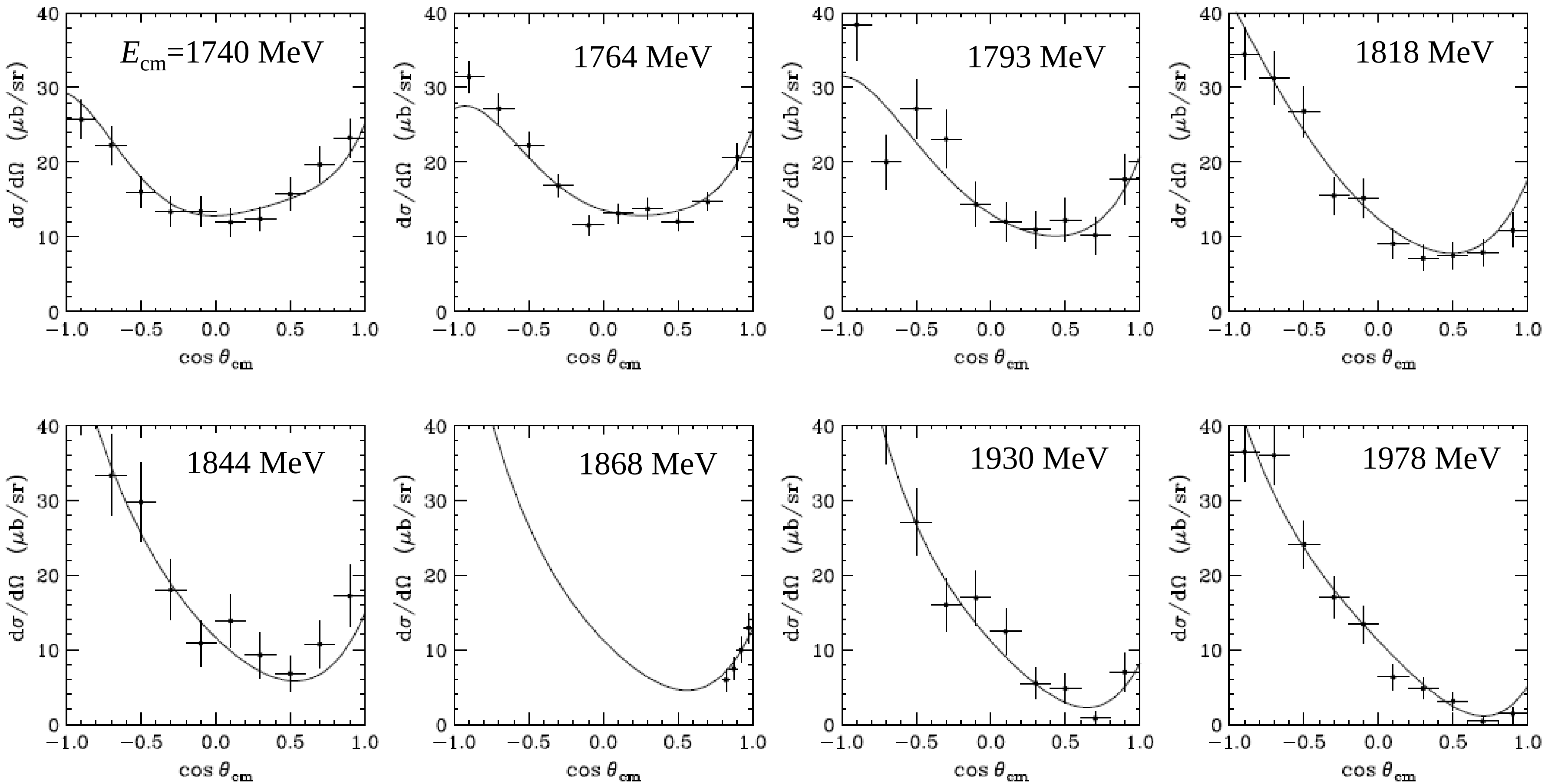
$$F_\Sigma^\alpha = \beta^{\frac{1}{2}} \underline{\overline{f}_{\pi^- p \to K^+ \Sigma^-}} \chi_{\boldsymbol{p}_K}^{(-)*} \chi_{\boldsymbol{p}_\pi}^{(+)} \langle \alpha | \hat{\psi}_p | \Psi_A \rangle$$

Fermi-averaged amplitudes

Decomposition of the inclusive spectrum into components

$$\mathrm{Im}\, \hat{G}_\Sigma = \underbrace{\hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\, \hat{G}_\Lambda^{(0)}\}\hat{\Omega}^{(-)}}_{\Lambda\ \text{escape}} + \underbrace{\hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\, \hat{G}_\Sigma^{(0)}\}\hat{\Omega}^{(-)}}_{\Sigma^-\ \text{escape}} + \underbrace{\hat{G}^\dagger\{W_{Y,T}\}\hat{G}}_{\text{Spreading (nuclear-core breakup)}}$$

Differential cross sections for the $\pi^- p \to K^+ \Sigma^-$ reactions



$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm cm}^{\rm elem} = \lambda\!\!\!^{-2} \sum_{\ell=0}^{\ell_{\max}} A_\ell(E_{\rm cm}) P_\ell(\cos\theta_{\rm cm})$$

Exp. data

O.I. Dahl, et al., Phys. Rev. 163 (1967) 1430.

T.O. Binford, et al., Phys. Rev. 183 (1969) 1134.

W. Langbein, F. Wagner, Nucl. Phys. B53 (1973) 251, and references therein.

R. Honda, Ph.D. thesis, Tohoku University (2014).

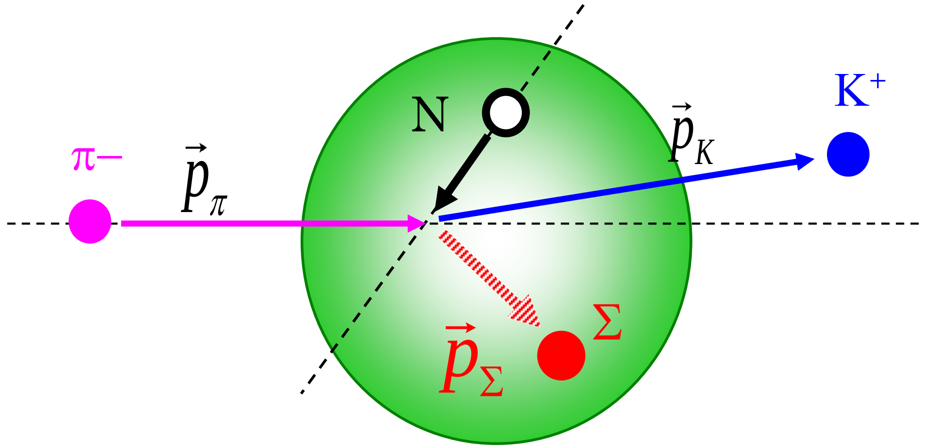
Optimal Fermi-averaging for an elementary T-matrix

T. Harada and Y.Hirabayashi, NPA744 (2004) 323.

“Optimal” cross section

$$\left[\frac{d\sigma}{d\Omega}\right]^{\rm opt}_{\pi^- p\to K^+\Sigma^-} = \left|\overline{f}_{\pi^- p\to K^+\Sigma^-}\right|^2$$

$$= \frac{k_K E_K}{(2\pi)^2 v_\pi}\left|\hat{t}^{\rm opt}(p_\pi;\omega,\mathbf{q})\right|^2$$



Optimal Fermi-averaged T-Matrix

On-shell T-matrix

$$\hat{t}^{\rm opt}(p_\pi;\omega,\mathbf{q}) = \left.\frac{\int_0^\pi \sin\theta d\theta \int_0^\infty p_N^2 dp_N \hat{t}_{Lab}(E_{\pi N};p_\pi,p_N)\rho(p_N)}{\int_0^\pi \sin\theta d\theta \int_0^\infty p_N^2 dp_N \rho(p_N)}\right|_{\mathbf{p}_N=\mathbf{p}_N^*}$$

given

“On-energy-shell” equation $\omega = E_{\rm f} - E_{\rm i} = \sqrt{(\mathbf{p}_N^* + \mathbf{q})^2 + m_\Sigma} - \sqrt{\mathbf{p}_N^{*\,2} + m_N}$

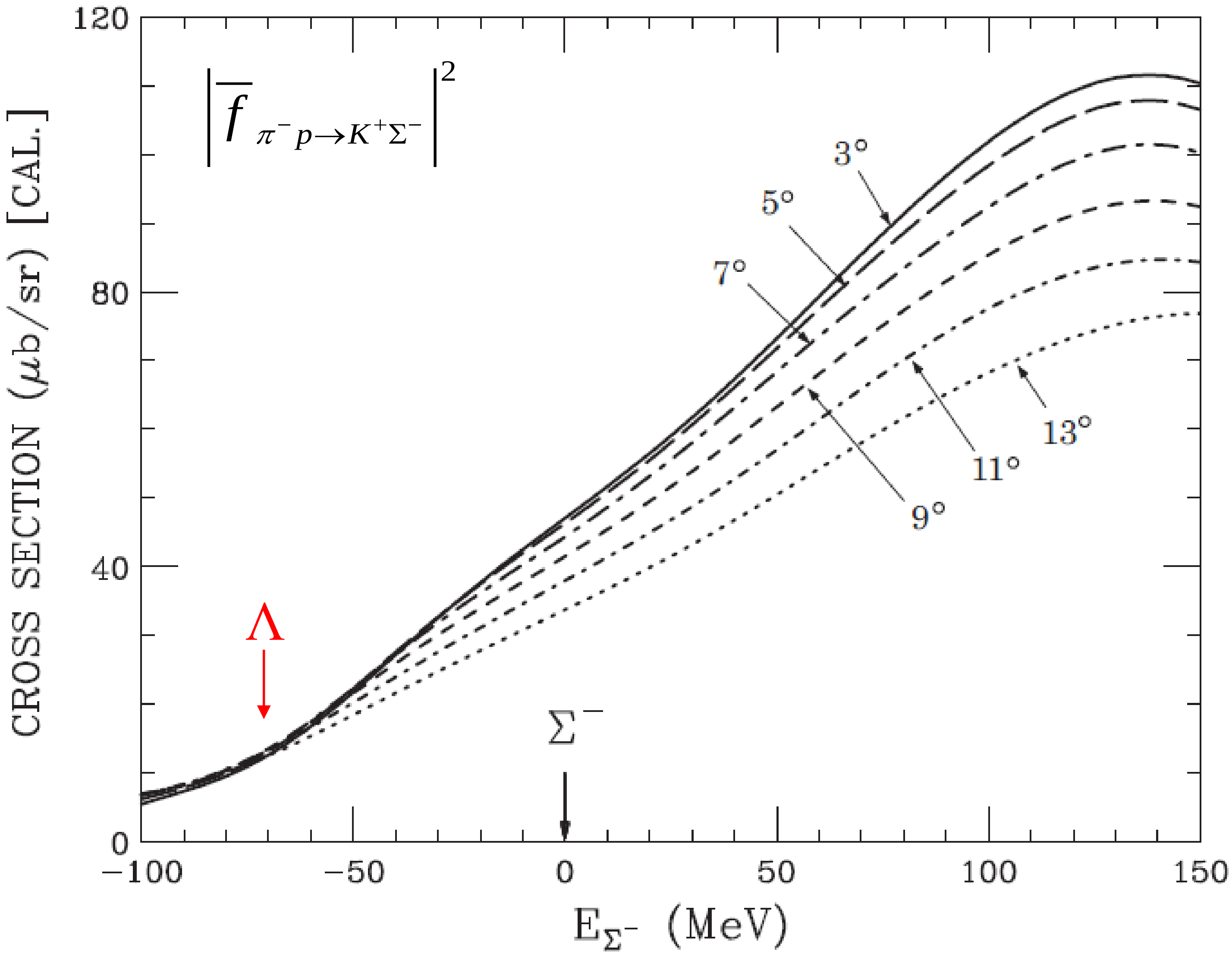
given

$$\mathbf{p}_\pi + \mathbf{p}_N^* = \mathbf{p}_K + \mathbf{p}_\Sigma$$

S,A.Gurvitz, PRC33(1986)422: Optimal factorization

Angular dependence of the optimal Fermi-av. cross section

“$\pi^- p \to K^+ \Sigma^-$ reactions” in the nucleus



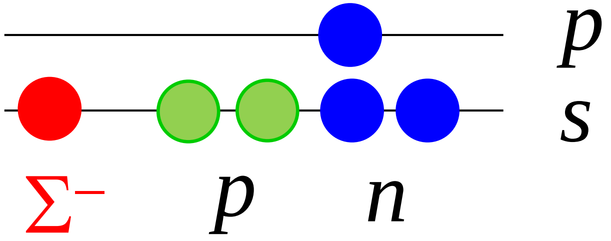
- There exists a strong energy dependence in the amplitudes.

Results and discussion

Part I

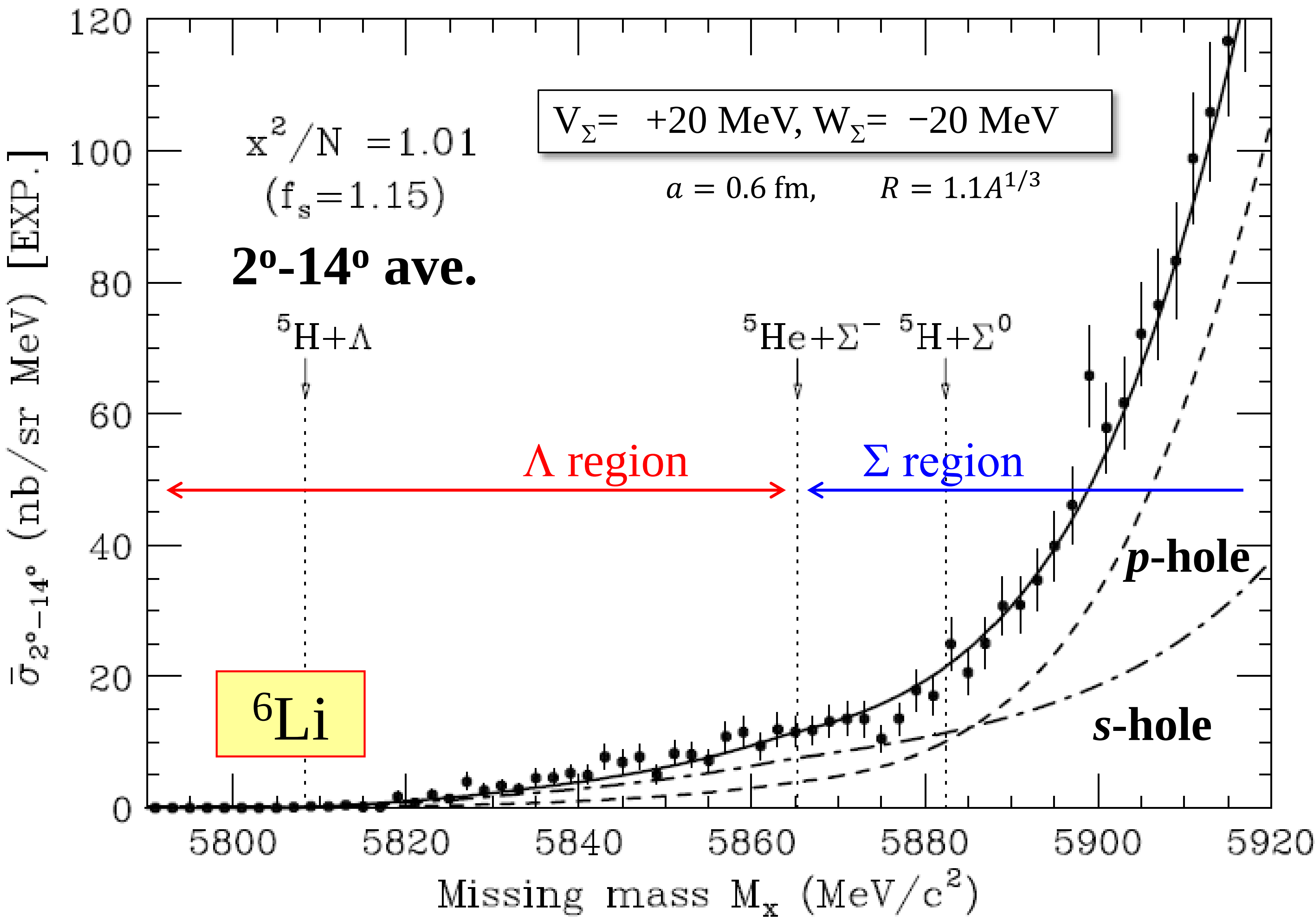
Study of the Σ-nucleus potentials

Σ^{-}-^{5}He



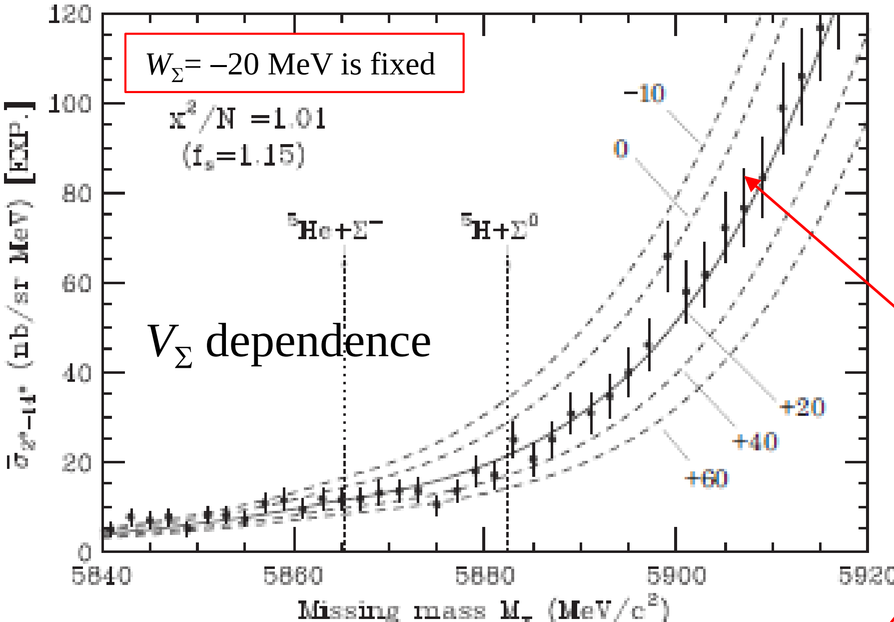
Inclusive spectrum in ^{6}Li(π^{-}, K^{+}) reaction at 1.2GeV/c

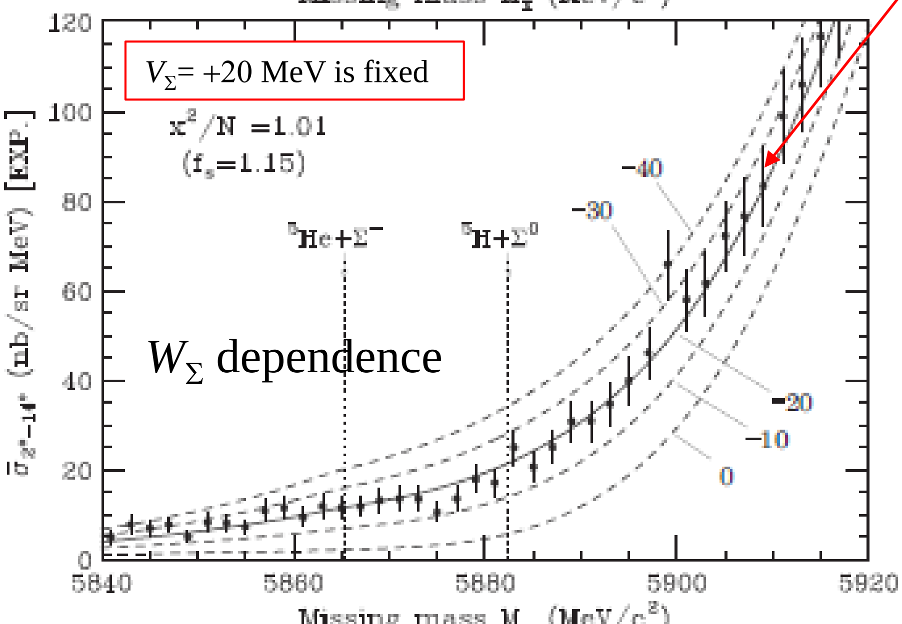
R. Honda, et al. (J-PARC E10 Collaboration), arXiv:1703.00623



Dependence of the calculated spectra for the ^{6}Li(π^{-},K^{+}) reaction

R. Honda, et al. (J-PARC E10 Collaboration), arXiv:1703.00623





3.0 MeV FWHM

$p_{\pi-}$=1.2 GeV/c

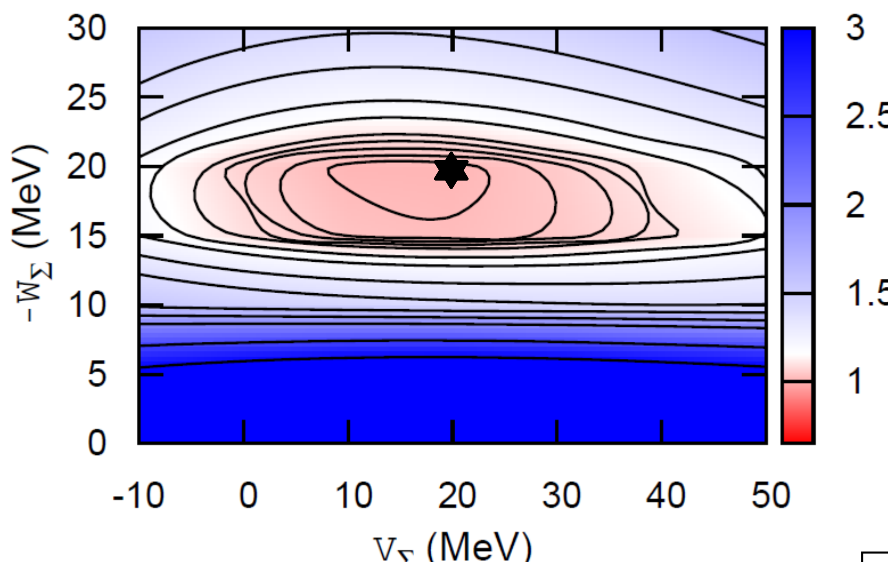
WS potential

The shape and magnitude of the spectrum are sensitive to the strengths of (V_Σ, W_Σ).

(V_Σ, W_Σ) = (+20, –20) MeV

χ^2/N = 1.01 with f_s (N=65)

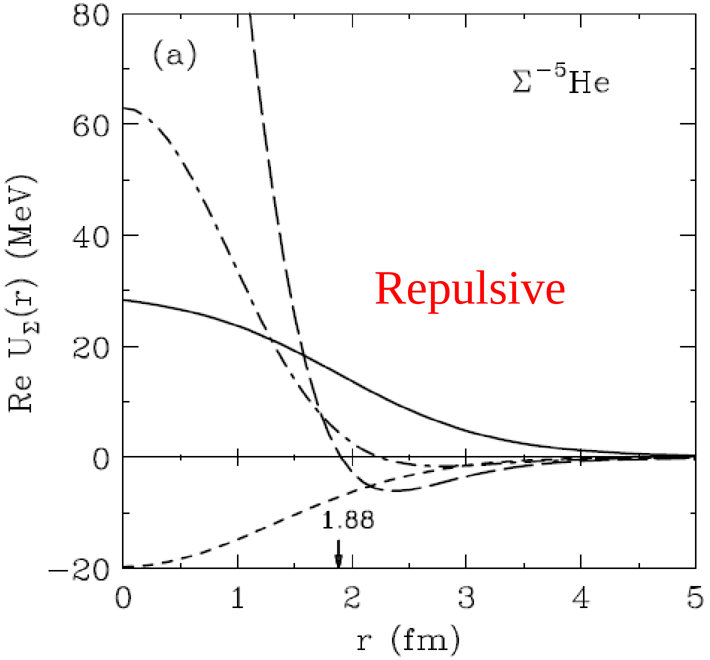
The χ^2/N-value distribution in V_Σ, W_Σ



Remarks

- The calculated DWIA spectrum can fully explain the experimental data of the $^{6}Li(\pi^{-}, K^{+})$ reaction at 1.2 GeV/c.
- The optimal Fermi-averaged *t*-matrix of $\pi^{-}p \rightarrow K^{+}\Sigma^{-}$ reactions is essential to describe the energy and angular dependence of the data.
- The results show that the Σ^{-}-^{5}He potential has the repulsive and absorptive components with (V_{Σ}, W_{Σ})= (+20 MeV, −20 MeV) with the WS potential.

⇨ (V_{Σ}, W_{Σ}) = (+34, –34) MeV for N.M.

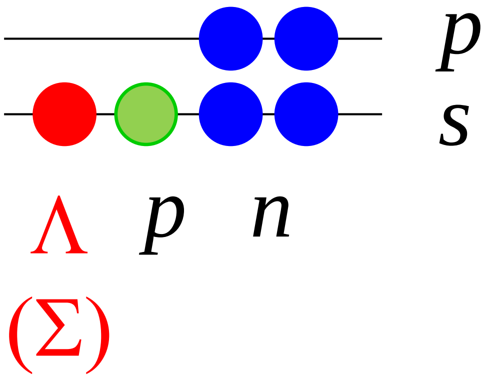


⇔ (V_{Σ}, W_{Σ}) = (+30, –40) MeV for ^{28}Si

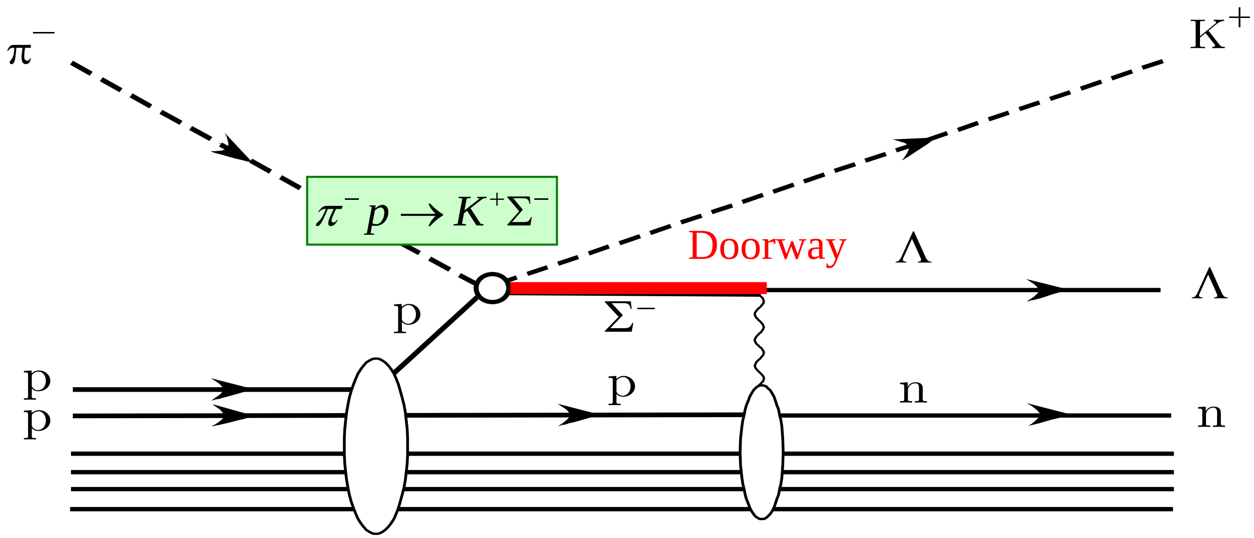
Part II

Production of the neutron-rich Λ hypernucleus

$^{6}_{\Lambda}H$



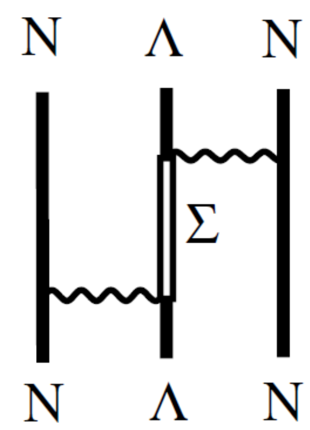
One-step process



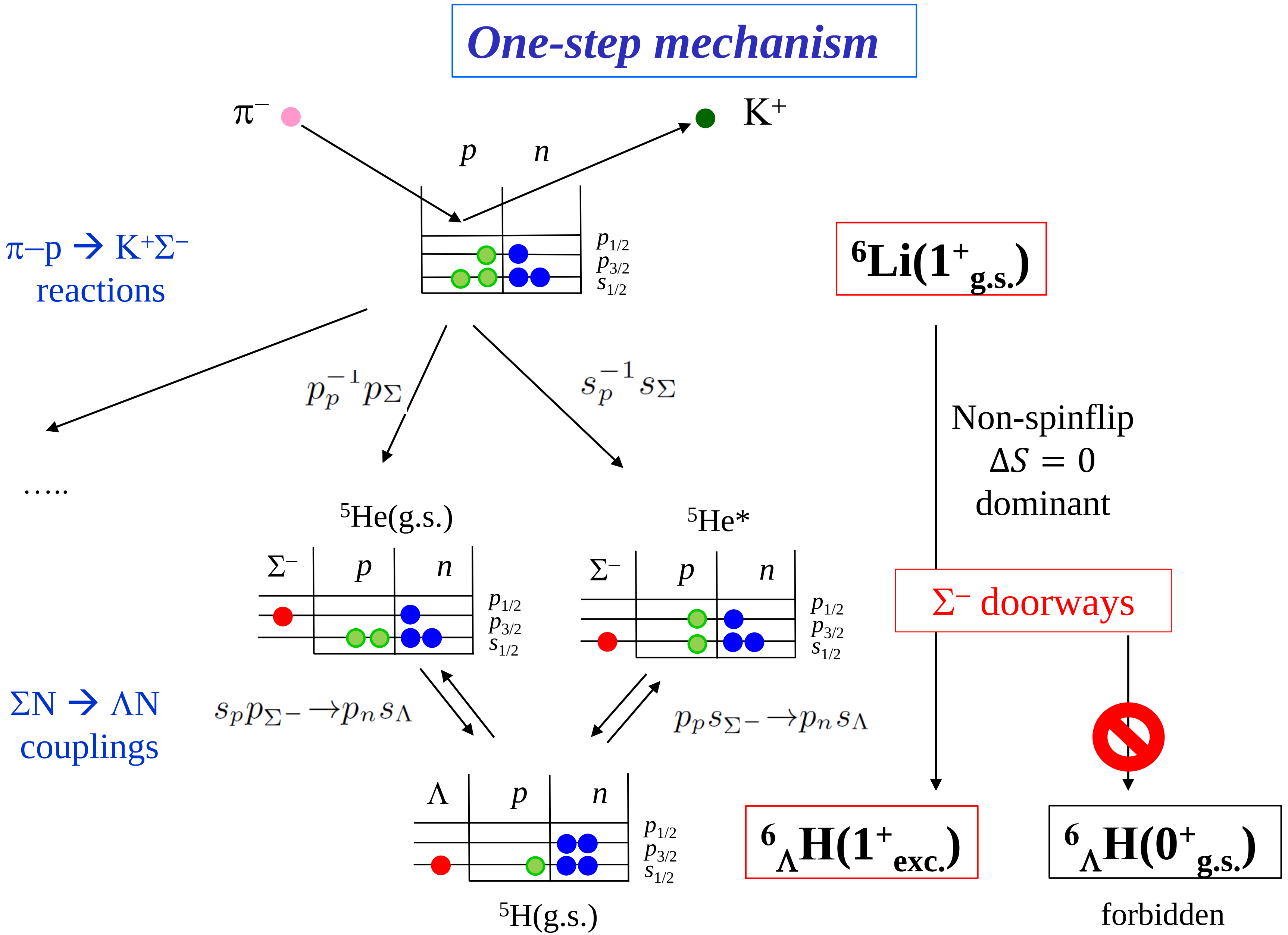
$$\pi^- p \to K^+\Sigma^-$$

$$\Sigma^- p \leftrightarrow \Lambda n$$

via Σ^- doorways caused
by ΛN-ΣN coupling



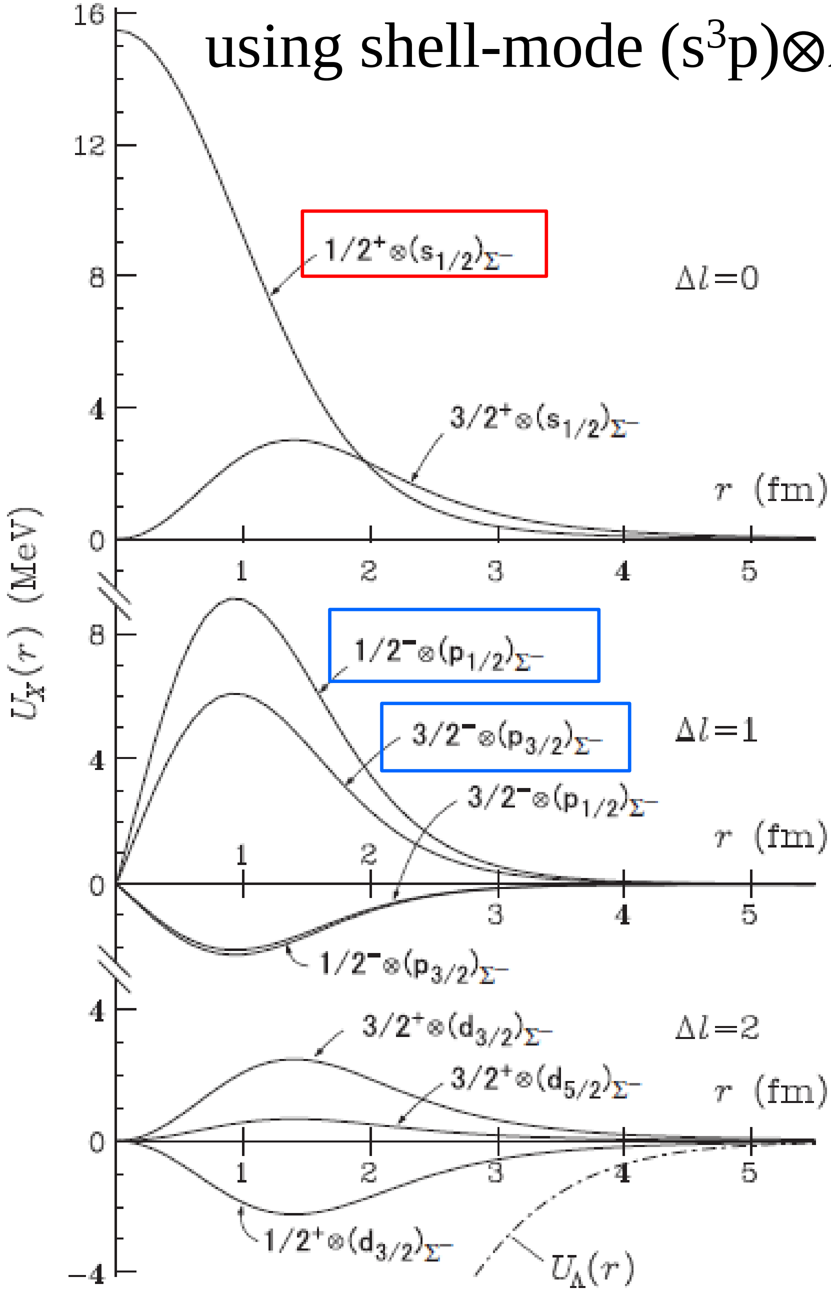
Schematic illustration of $^{6}_{\Lambda}$H production in the ^{6}Li(π^{-},K^{+}) reaction

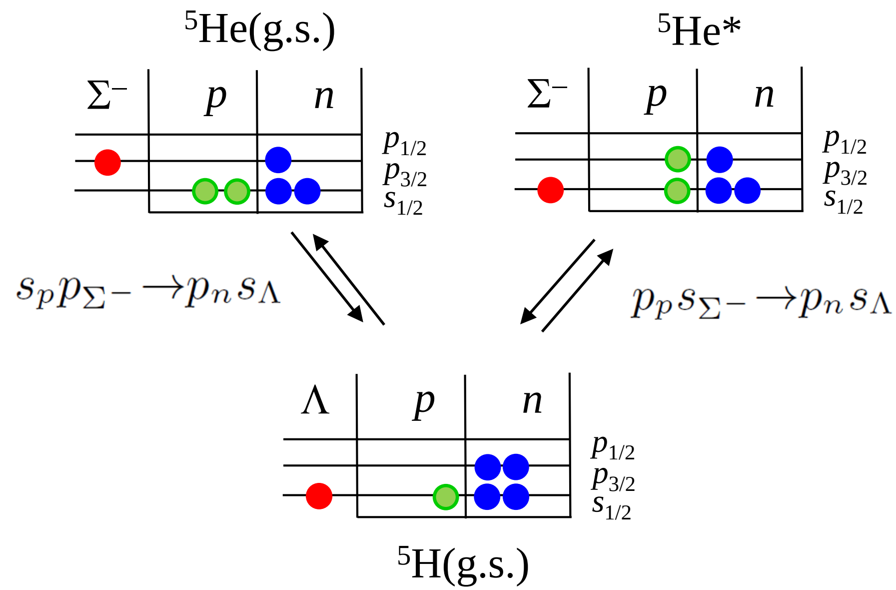


Calculated ΣΛ coupling folding potentials in $^{6}_{\Lambda}$H(1^+)

using shell-mode (s^3p)⊗L configurations for the core nucleus.

$(v^{1}_{\Sigma N,\Lambda N}, v^{0}_{\Sigma N,\Lambda N})$ = (-900 MeV・fm³, 500 MeV・fm³)



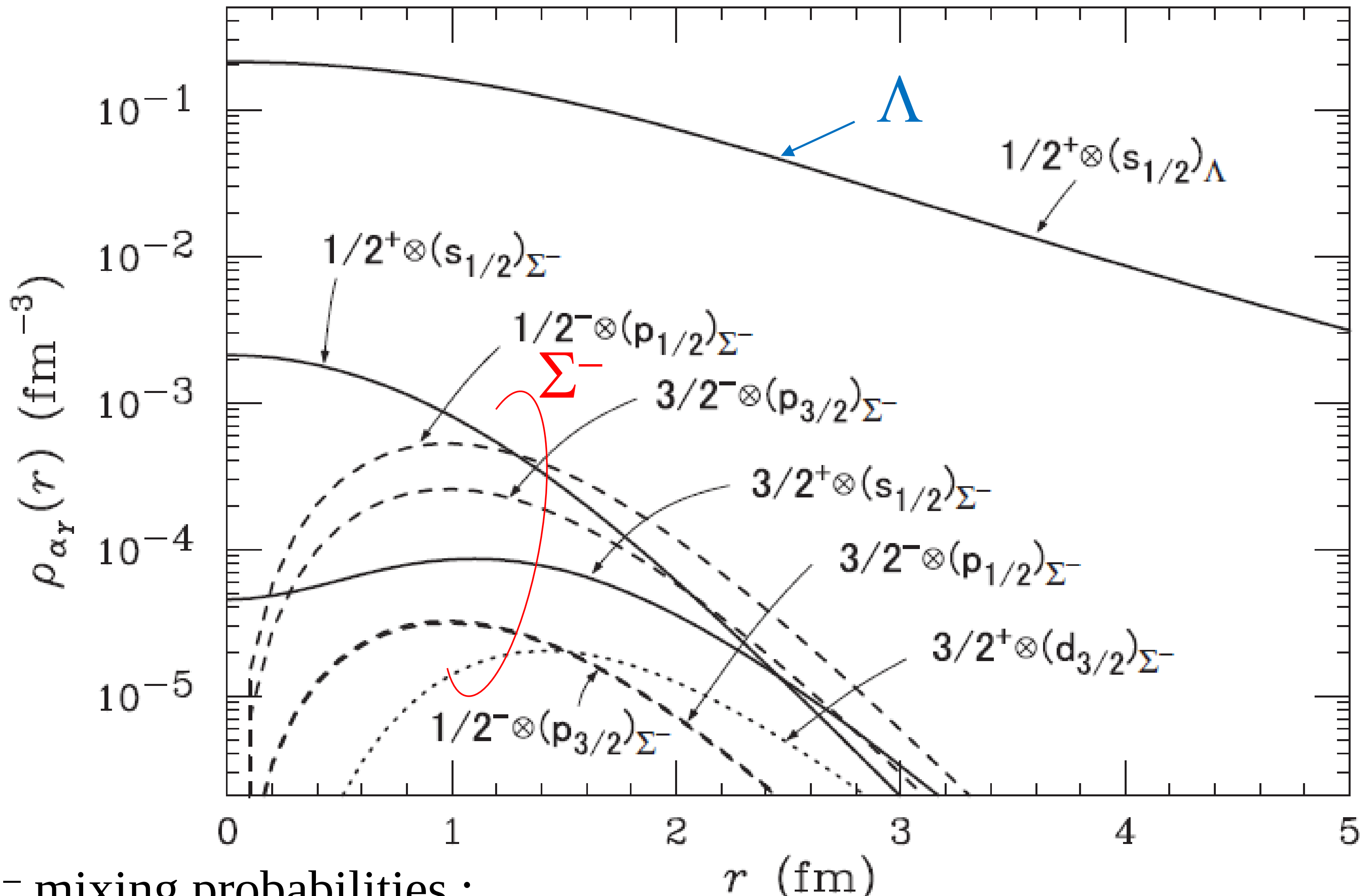


- Shell-model with *spsd* model space
- Central effective *YN* interaction (D2'g)

➢ The coupling strengths of $p_\Sigma \leftrightarrow s_\Lambda$ are so large, as well as $s_\Sigma \leftrightarrow s_\Lambda$.

Single-particle density distributions of Λ and Σ^- in ${}^{6}_{\Lambda}$H(1^+)

$(v^{1}_{\Sigma N,\Lambda N}, v^{0}_{\Sigma N,\Lambda N})$ = (-900 MeV・fm^3, 500 MeV・fm^3)



Σ^- mixing probabilities :

$$P_{\Sigma^-}(\text{tot}) = 0.32\% \quad [P_{\Sigma^-}(s_\Sigma) = 0.13\%,\ P_{\Sigma^-}(p_\Sigma) = 0.17\%]$$

Integrated cross section and Σ^- mixing probability

$^{6}_{\Lambda}H(1^+)$

Case	$\tilde{v}^S_{\Sigma N,\Lambda N}$ (MeV) $S=1$	$S=0$	$B_\Lambda(^6_\Lambda H)$ (MeV)	P_{Σ^-} (%) s_Σ	p_Σ	d_Σ	total
	0	0	1.492	0.00	0.00	0.00	0.00
D	−450	250	1.576	0.03	0.04	0.00	0.07
C	−900	500	1.841	0.13	0.17	0.02	0.32
B	−1350	750	2.328	0.34	0.41	0.04	0.79
A	−1800	1000	3.100	0.68	0.82	0.08	1.58

good agreement

Case	$\tilde{v}^S_{\Sigma N,\Lambda N}$ (MeV) $S=1$	$S=0$	$B_\Lambda(^6_\Lambda H)$ (MeV)	$d\sigma/d\Omega$ (nb/sr) s_p^{-1}	p_p^{-1}	total
	0	0	1.492	0.00	0.00	0.00
D	−450	250	1.576	0.03	0.01	0.04
C	−900	500	1.841	0.16	0.06	0.22
B	−1350	750	2.328	0.44	0.15	0.59
A	−1800	1000	3.100	1.00	0.32	1.32

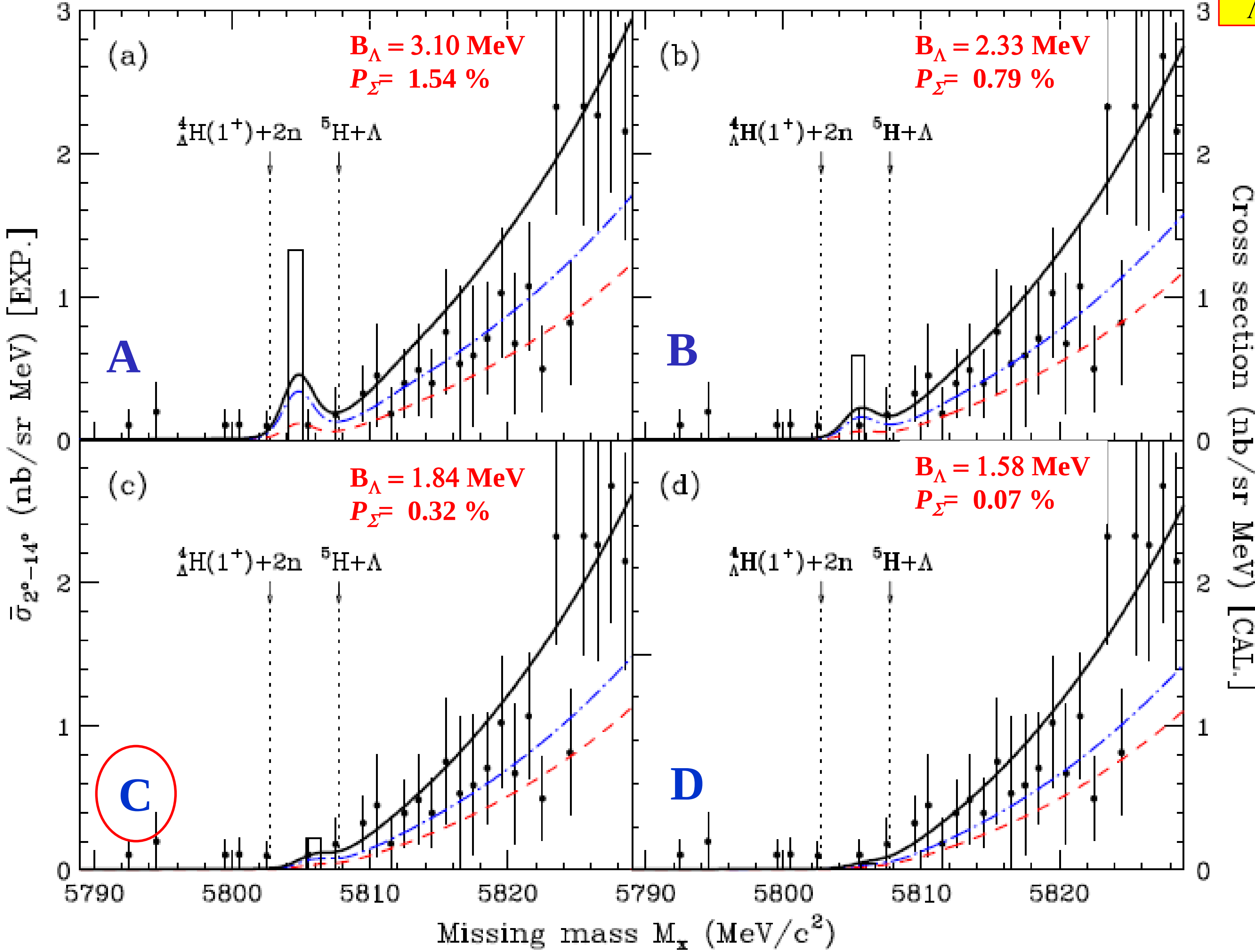
Cross sections: $d\sigma/d\Omega = 0.22$ nb/sr

Σ^- mixing probabilities: $P_{\Sigma^-}(\text{tot}) = 0.32\%$ $[P_{\Sigma^-}(s_\Sigma) = 0.13\%,\ P_{\Sigma^-}(p_\Sigma) = 0.17\%]$

Production cross section of ^{6}Li(π^{-},K^{+}) reactions

Hyperon-mixing

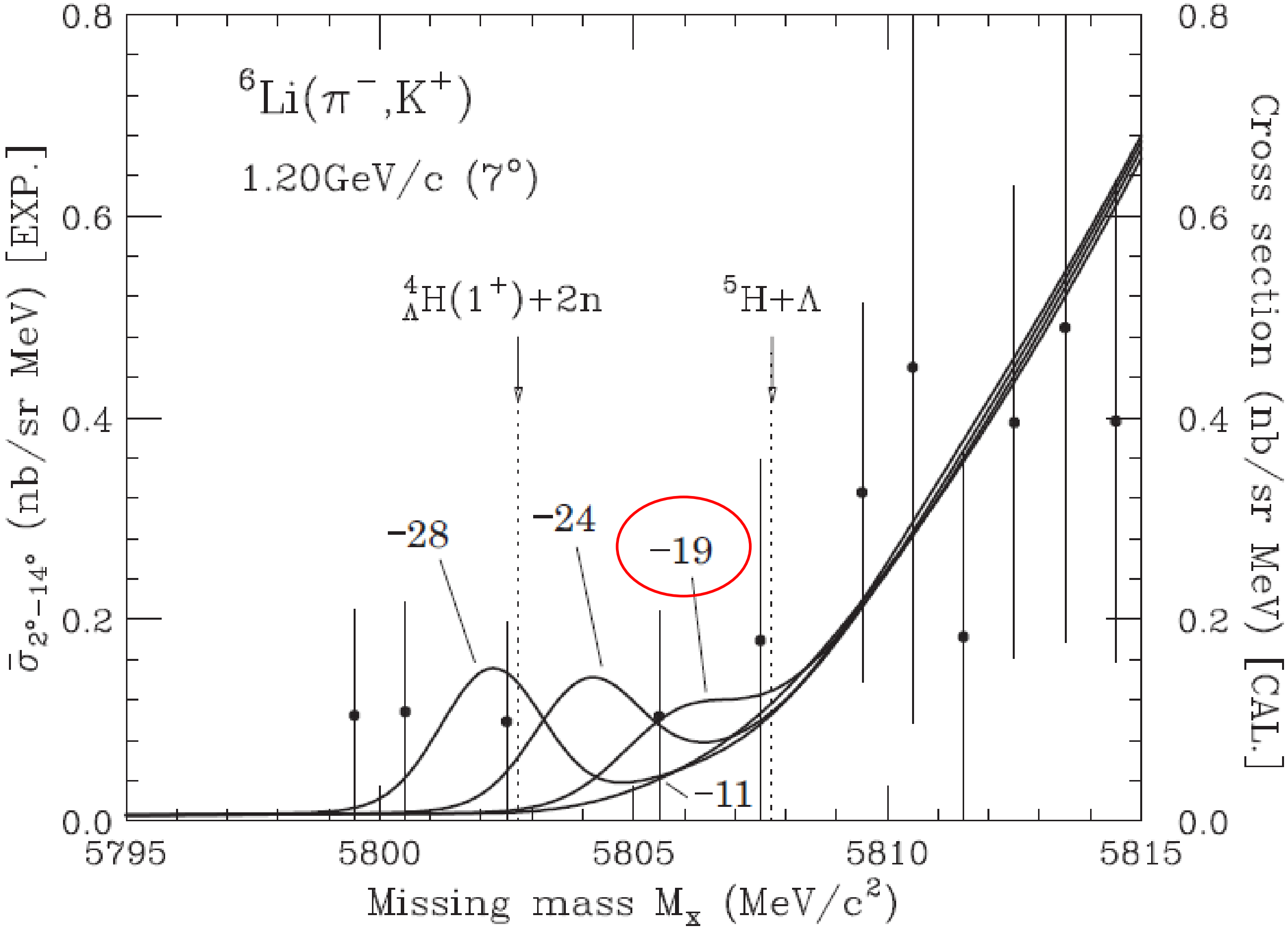
$^{6}_{\Lambda}$H(1^{+})



Dependence of the spectrum on V_Λ in the Λ-^{5}H potential

- $V_\Lambda = -19, -24, -28$ MeV because the structure of ^{5}H is still uncertain experimentally.

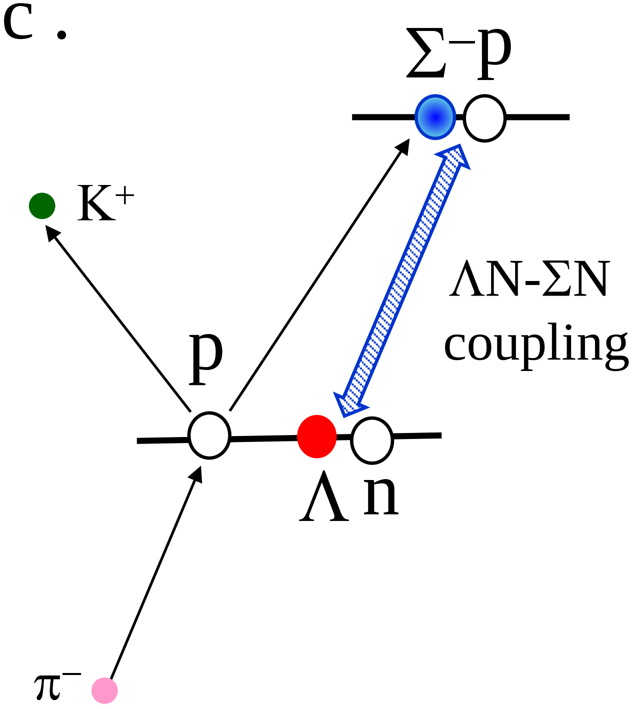
$^6_\Lambda$H(1^+)



- The shallow potential $V_\Lambda = -19$ MeV is favored to be compared with the data.
- The shape of the spectrum is so sensitive to the structure of the ^{5}H resonance.

Summary

- The calculated spectrum of the $^{6}_{\Lambda}$H by the one-step mechanism via Σ^{-} doorways can explain the data of the DCX ^{6}Li(π^{-}, K^{+}) reaction at 1.20GeV/c .
  - (V_{Σ}, W_{Σ})= (+20 MeV, −20 MeV)
  - Σ^{-} mixing probability
    $P_{\Sigma} \sim 0.4$ % for $^{6}_{\Lambda}$H($1^{+}_{\text{exc.}}$).
    $[P_{\Sigma^-}(s_{\Sigma}) = 0.11\%,\ P_{\Sigma^-}(p_{\Sigma}) = 0.17\%]$
  - Shallow Λ potential for ^{5}H$_{\text{res}}$
    ($V_{\Lambda} \simeq -19$ MeV) is favored.



- Our phenomenological calculations provide the ability to extract the production mechanism from the data.

Conclusion

- We have investigated the calculated spectrum of the neutron-rich ${}^{6}_{\Lambda}$H hypernuclei by the one-step mechanism via Σ^- doorways in the ^{6}Li(π^-,K$^+$) reaction at 1.20 GeV/c.

The results suggest that

- $(V_\Sigma, W_\Sigma) = (+20\ \mathrm{MeV}, -20\ \mathrm{MeV})$
- $P_\Sigma\,(\mathrm{tot}) \simeq 0.4\ \%$ for ${}^{6}_{\Lambda}\mathrm{H}(1^+_{\mathrm{exc.}})$
- $V_\Lambda \simeq -19$ MeV (shallow) for the ^{5}H resonance

- The sensitivity to the coupling potential parameters implies that the nuclear (π^-,K$^+$) reactions provide a high ability for the theoretical analysis of precise wave functions in the neutron-rich Λ hypernuclei.

Thank you very much for your attention.