

# Production and Structure of (*p*-shell) $\Xi$ -Hypernuclei

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**SNP 2017**

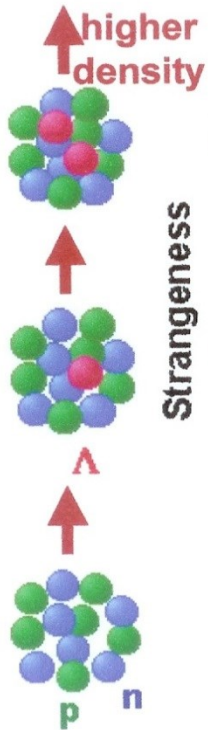
March 12-14, 2017, Neyagawa, Osaka

**Ξ-Hypernuclei are not well known, but its study is an entrance to the S=-2 world.**

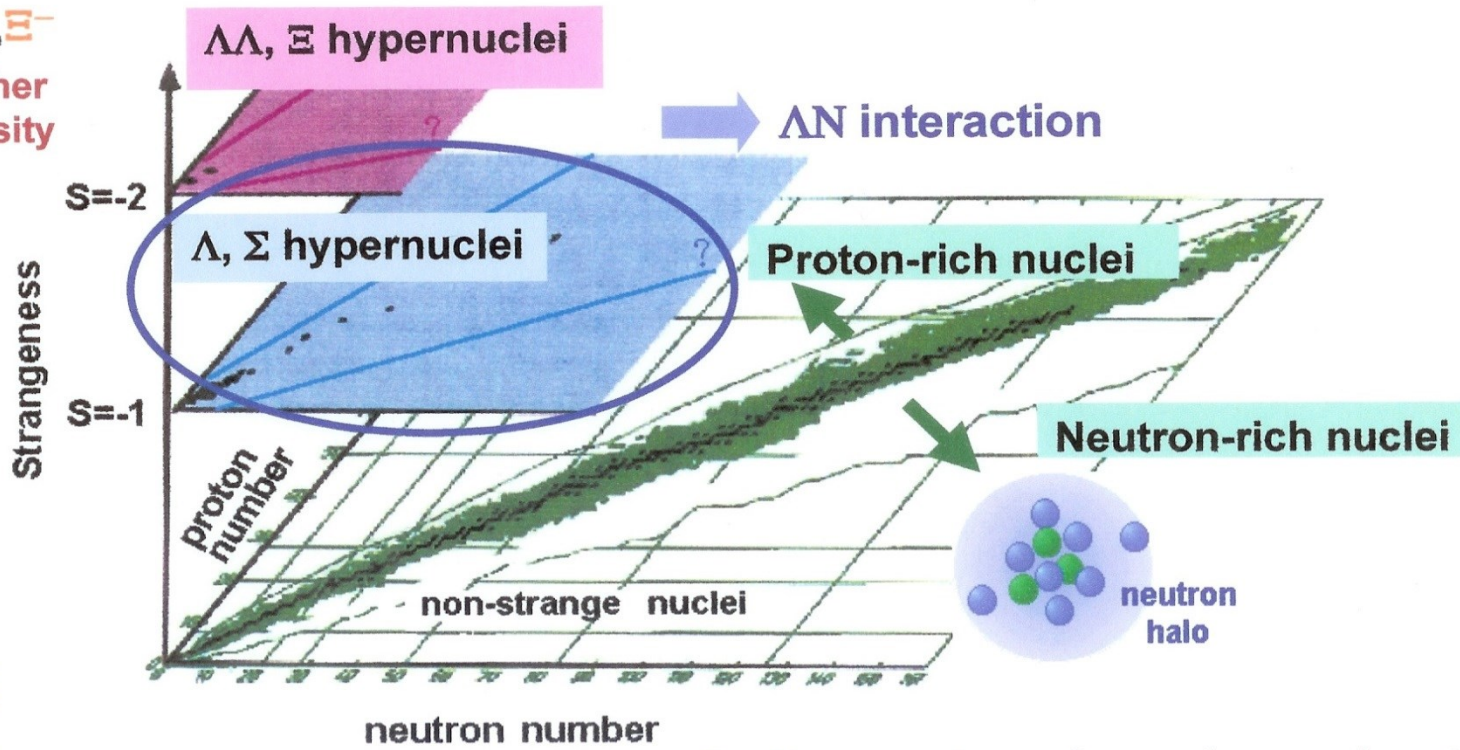
$N_u \sim N_d \sim N_s$



$p, n, \Lambda, \Xi^0, \Xi^-$



$S=-\infty$  Strangeness in neutron stars ( $\rho > 3 - 4 \rho_0$ )  
 Strange hadronic matter ( $A \rightarrow \infty$ )

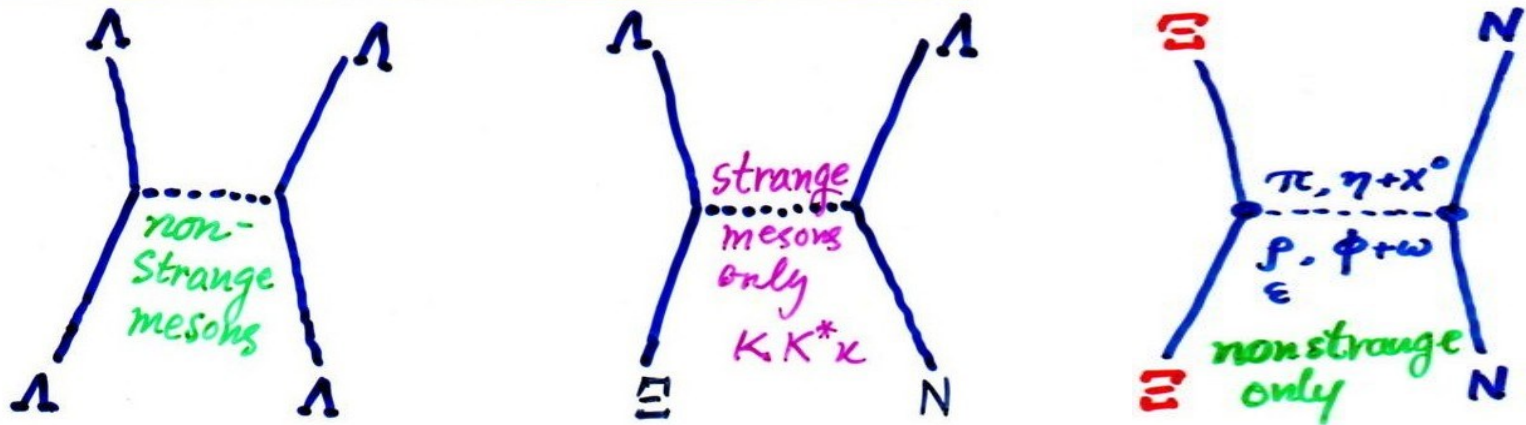


**3-dimensional nuclear chart**

(taken from H. Tamura)

# Why $\Xi$ -hypernuclei ?

1) They provide unique information on the  $S=-2$  B-B interactions inaccessible otherwise.



2) High-priority experiment at J-PARC

E-05:  $^{12}\text{C}(K^-, K^+)_{\Xi}^{12}\text{Be}$  reaction data have appeared

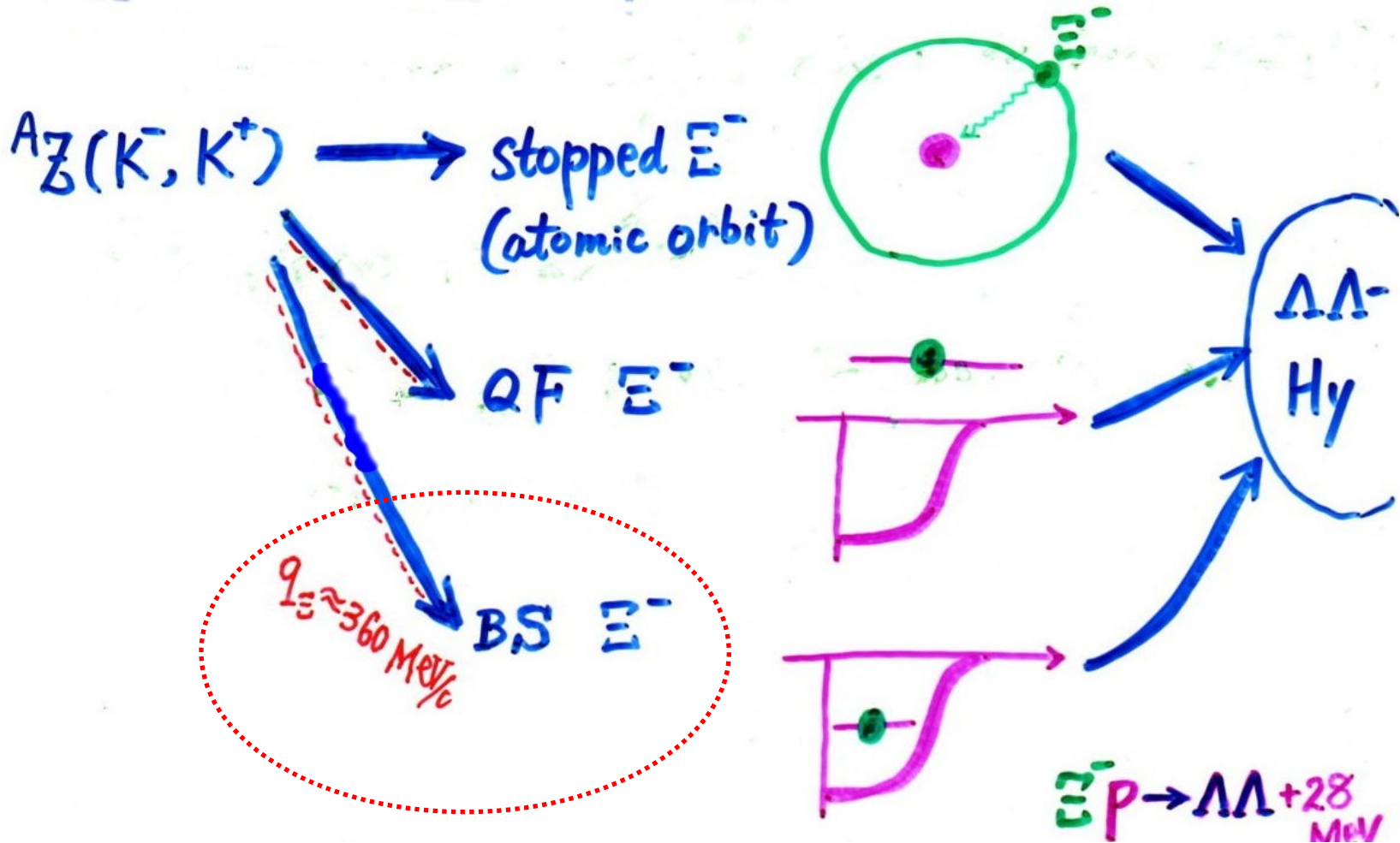
→ Realistic Calculations are required.

# CONTENTS

Focus on the theoretical status of  
 $\Xi$ -hypernuclear productions

1. DWIA based on the one-body motion of  $\Xi$  in an average potential (WS)
2. DWIA with use of many-body  $\Xi$  hypernuclear W.F. based on the available  $\Xi$ -N interactions
3. Possible few other targets next to  $^{12}\text{C}$
4. Summary

# Three cases of $\Xi$ production on nuclear targets:



# 1. DWIA cross sections with $\Xi$ one-body treatment

- (1)  $\Xi$  one-body motion in an average nuclear potential such as W-S and/or some folding potentials.
- (2) Nuclear core-excitations are not taken into account.
- (3)  $K^-$  and  $K^+$  distorted waves are obtained by solving the Klein-Gordon Eq. (OR one may take the eikonal approximation.)

**DWIA Treatment** within Kapur-Peierls method  
for  ${}^A\mathbf{Z}(K^-, K^+) {}^A_{\Xi}\mathbf{Z}'$  reaction cross section

$$\frac{d^2\sigma}{d\Omega dE_Y} = \xi \left[ \frac{d\sigma(\theta)}{d\Omega} \right]_{\text{elem}} S(E_Y, \theta),$$

$$S(E_Y, \theta) = -\frac{1}{\pi} \sum_f \text{Im} \left[ \frac{N_f(E_Y, \theta)}{E_Y - \epsilon_f(E_Y)} \right].$$

$\xi$  = kinematical factor for 2-body to A-body frame transformation

$$N_f(E_Y, \theta) = \langle \Phi_0 | \hat{O}^\dagger(\theta) | \Psi_f(E_Y) \rangle \langle \tilde{\Psi}_f(E_Y) | \hat{O}(\theta) | \Phi_0 \rangle.$$

$$\hat{O}^{(K^-, K^+)} = \int d^3\mathbf{r} \chi_{K^+}^*(\mathbf{k}_f, a\mathbf{r}) \chi_{K^-}(\mathbf{k}_i, \mathbf{r}) \sum_{\nu=1}^A V_{-2}^{-2}(\nu) \delta\left(\mathbf{r} - \frac{M_c}{M_A} \mathbf{r}_\nu\right),$$

$$\chi_{K^+}^*(\mathbf{k}_f, a\mathbf{r}) \chi_{K^-}(\mathbf{k}_i, \mathbf{r}) = \sum_{LM} i^L \sqrt{4\pi[L]} \tilde{j}_{LM}(k_i, k_f, \theta; a, r) Y_{LM}(\hat{\mathbf{r}}).$$

$$\sigma_{K-p} = 32.5 \text{ mb}, \quad \sigma_{K-n} = 25.5 \text{ mb}, \quad \sigma_{K+p} = 19.6 \text{ mb} \quad \text{and} \quad \sigma_{K+n} = 20.1 \text{ mb},$$

$\varepsilon(E)$  = solutions of the following Hamiltonian, depending on the final hypernuclear excitation energy  $E$  (given)

$$\mathcal{H} = (H_N + T_Y + U_Y(r) + \sum v_{YN}) + \frac{\hbar^2}{2m_Y} \delta(r - r_c) \left( \frac{d}{dr} - \frac{b}{r_c} \right),$$

$$b = \frac{d\psi_l(k_Y; r)/dr}{\psi_l(k_Y; r)/r} \Big|_{r=r_c}, \quad \psi_l(k_Y; r) \equiv r \{ j_l(k_Y r) + i n_l(k_Y r) \}.$$

Continuum: Boundary condition for each  $l$  at channel radius  $r_c$ ,

For the case of bound states ( $E < 0$ ), if any,  $S(E_Y; )$  tends to the effective number  $Z_{\text{eff}}$ .

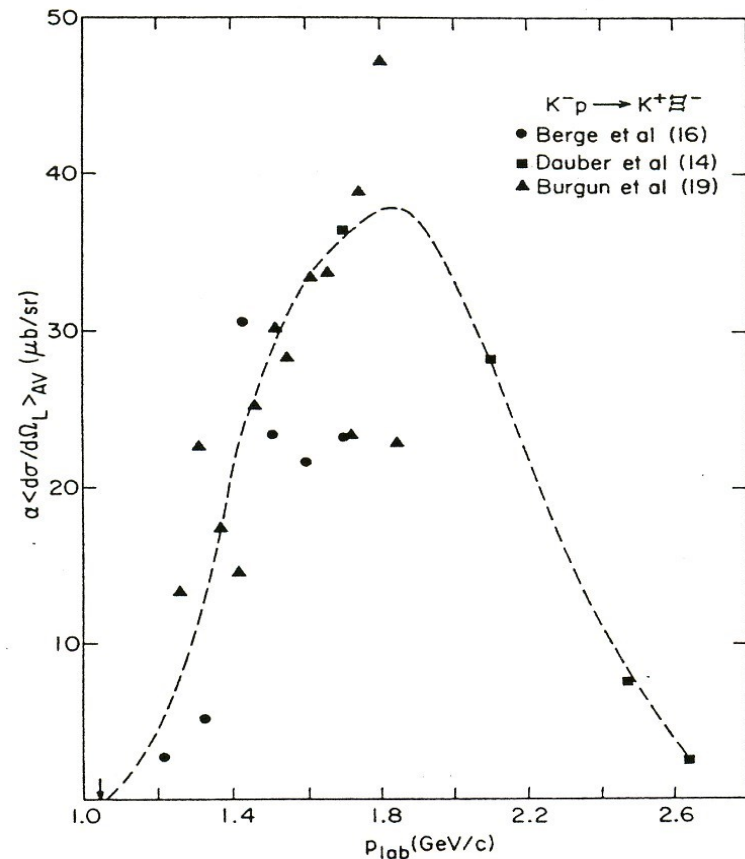
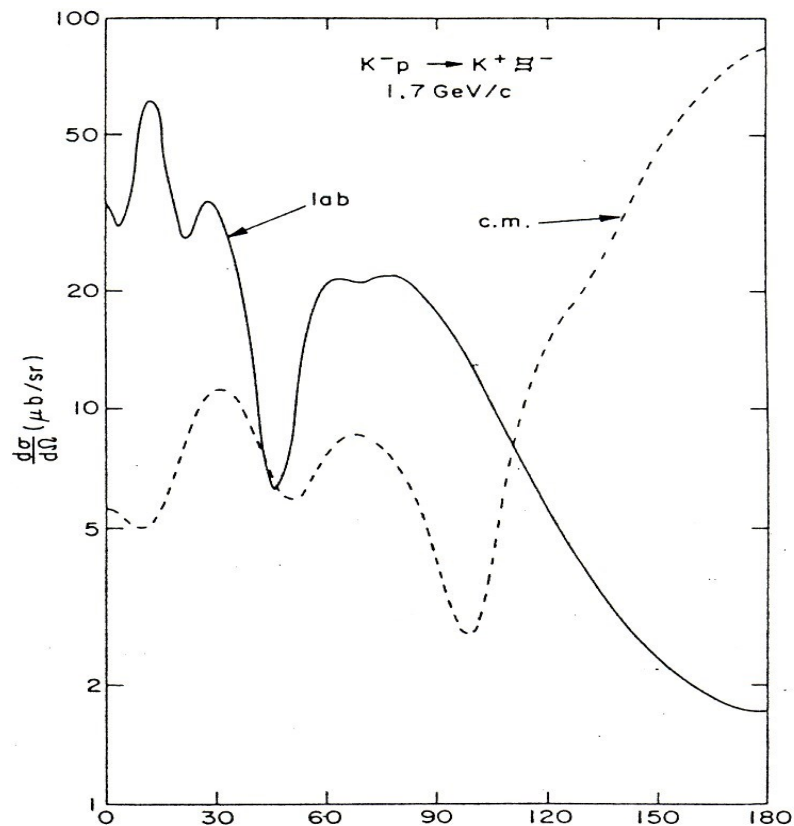
$$\frac{d\sigma(\theta)}{d\Omega_L} = \xi \left[ \frac{d\sigma(\theta)}{d\Omega_L} \right]_{K-p \rightarrow S-K^+} Z_{\text{eff}}(i \rightarrow f; \theta).$$



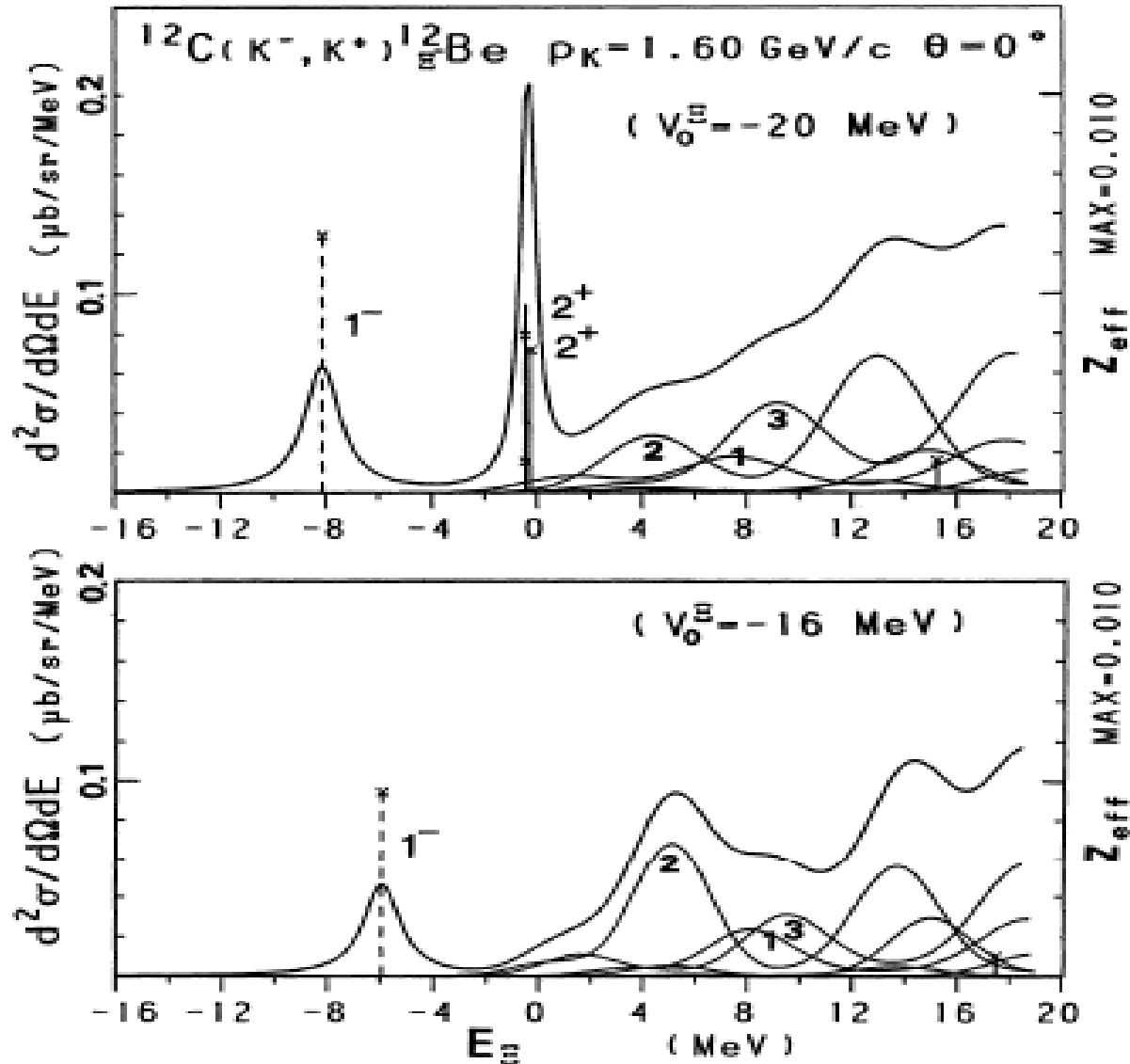
# Use the empirical $\Xi$ -production cross section, angular distribution and $p_K$ -dependence

Data from V.Flamino et a, CERN-HERA Report 79-02 (1979)

Pioneering work by C.B.Dover and A.Gal, Annals of Phys. **146** (1983)



One-body potential model naturally leads to the single-hyperon states such as  $s_{\Xi}$  and  $p_{\Xi}$ , combined with a proton hole



Arbitrary smearing widths are used in these figures

*E-N interactions are not taken up to this stage (except W-S pot.)*

→ **Use basic Xi-N interactions**

**Typical meson theoretical models:**

Nijmegen model-D, Ehime (Ueda)

Nijmegen ESC04d, ESC08a, ESC08c, ESC2016  
(Gmatrix: 2 or 3-range Gaussian expressions, YNG)

→ dynamical nuclear core excitations are taken into account ( $^{11}\text{B}^* + \text{E}^-$ ),

This theory works quite nice as proved in ( $\pi^+, K^+$ ).

CAL (Itonaga et al 1994) VS. EXP (Hotchi et al, 2001)

$$f + ig(\sigma \cdot \hat{n})$$

DWIA

*K. Itonaga, T. Motoba, O. Richter, M. Sotona,  
Phys. Rev. C49 (1994) 1045*

93-12-14

DASHED (J-) FULL (J+)

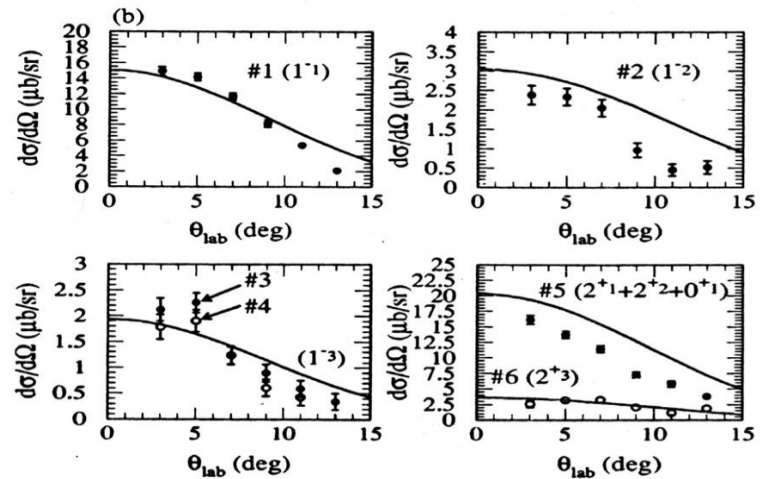
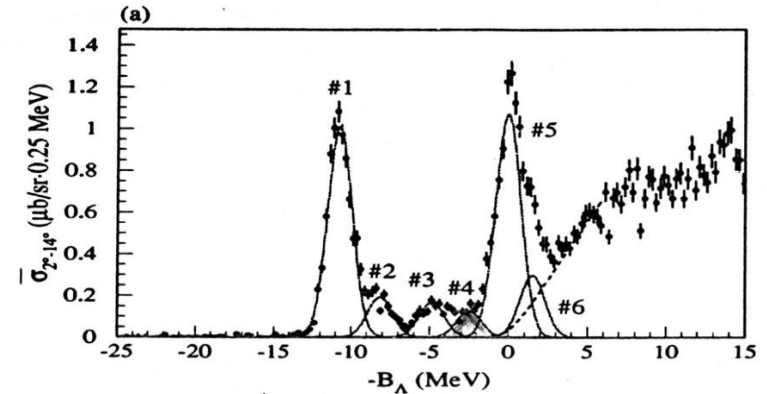
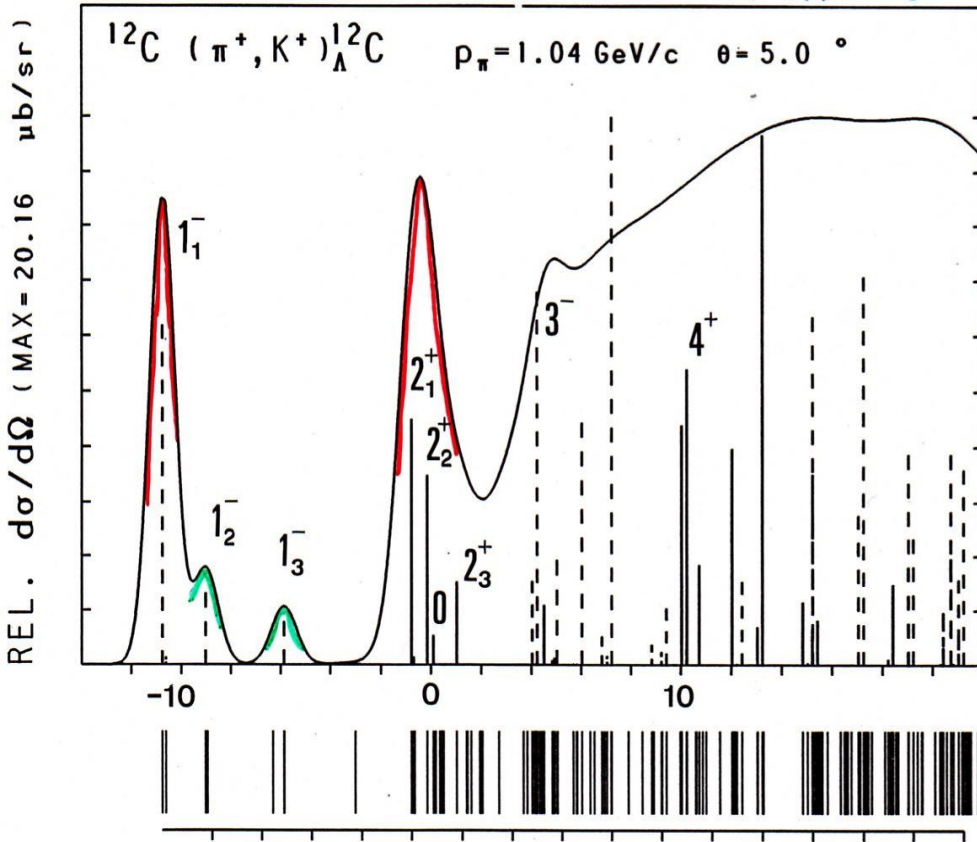
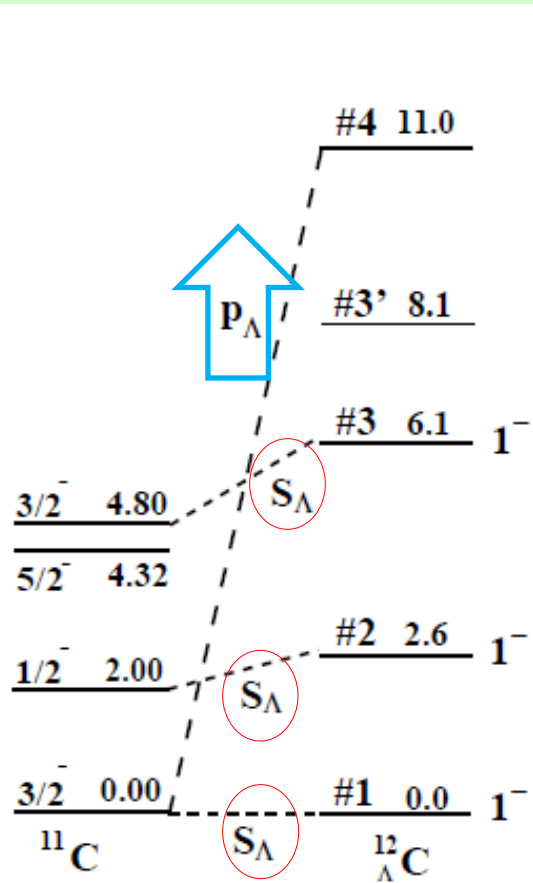


FIG. 8. (a)  $^{12}\text{C}$  spectrum obtained with the thick carbon target. (b) Angular distributions of kaons leading to the observed peaks for the  $^{12}\text{C}(\pi^+ K^+)$  reaction, derived from the above high-statistics

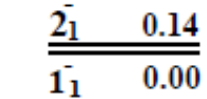
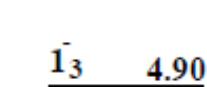
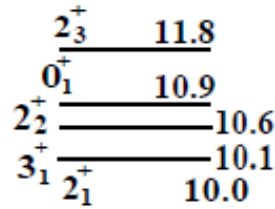
Hypernuclear low-lying states are composed of the coupling between s-state  $\Lambda$  with the  $^{11}\text{C}$  excited states.



F. Ajzenberg-selove  
NPA506,N1(1990)1

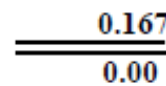
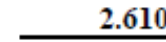
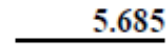
E336 Exp.

← Exp.



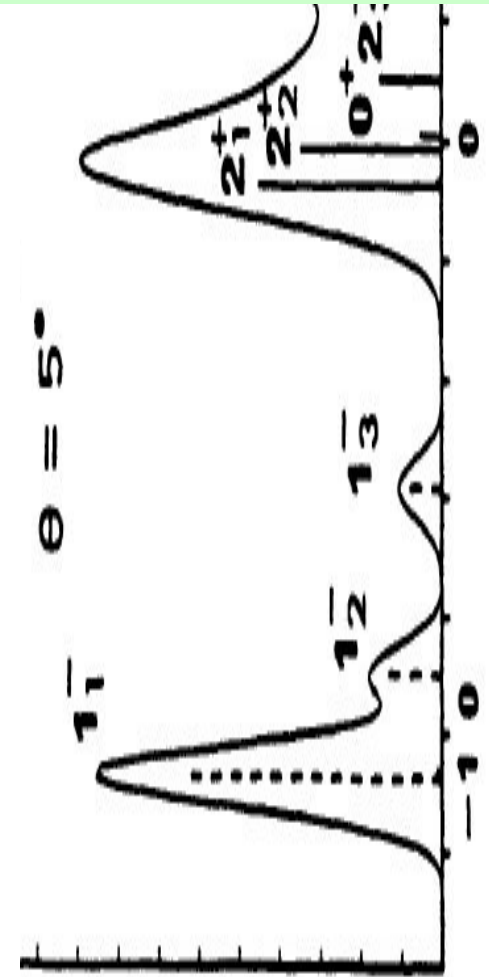
K. Itonaga *et al.*  
PRC 49,N2(1994)1045

→ Theory



D.J. Millner  
HYP2000, NPA691(2001)93c

(Unit : MeV)



$\frac{d\sigma}{d\Omega}$  (MAX=20.16  $\mu\text{b/s}$ )  $\theta = 5^\circ$  13

## 2. Many-body calculations

Take account of :

1) dynamical nuclear core excitations

$(^{11}\text{B}^* + \Xi^-)$ ,  $V(\text{NN}) = \text{conventional}$

2) NN and  $\Xi\text{N}$  effective interactions on equal footing,

3) hyperon tail (radial) behavior of  $\phi_{\Xi}(r)$   
carefully:  $0s+1s+2s$ ,  $0p+1p+2p$ , .....

- $H_{\text{total}} = H^{(\text{Cohen-Kurath})} + t_Y + \sum V^{(\text{YNG})}$

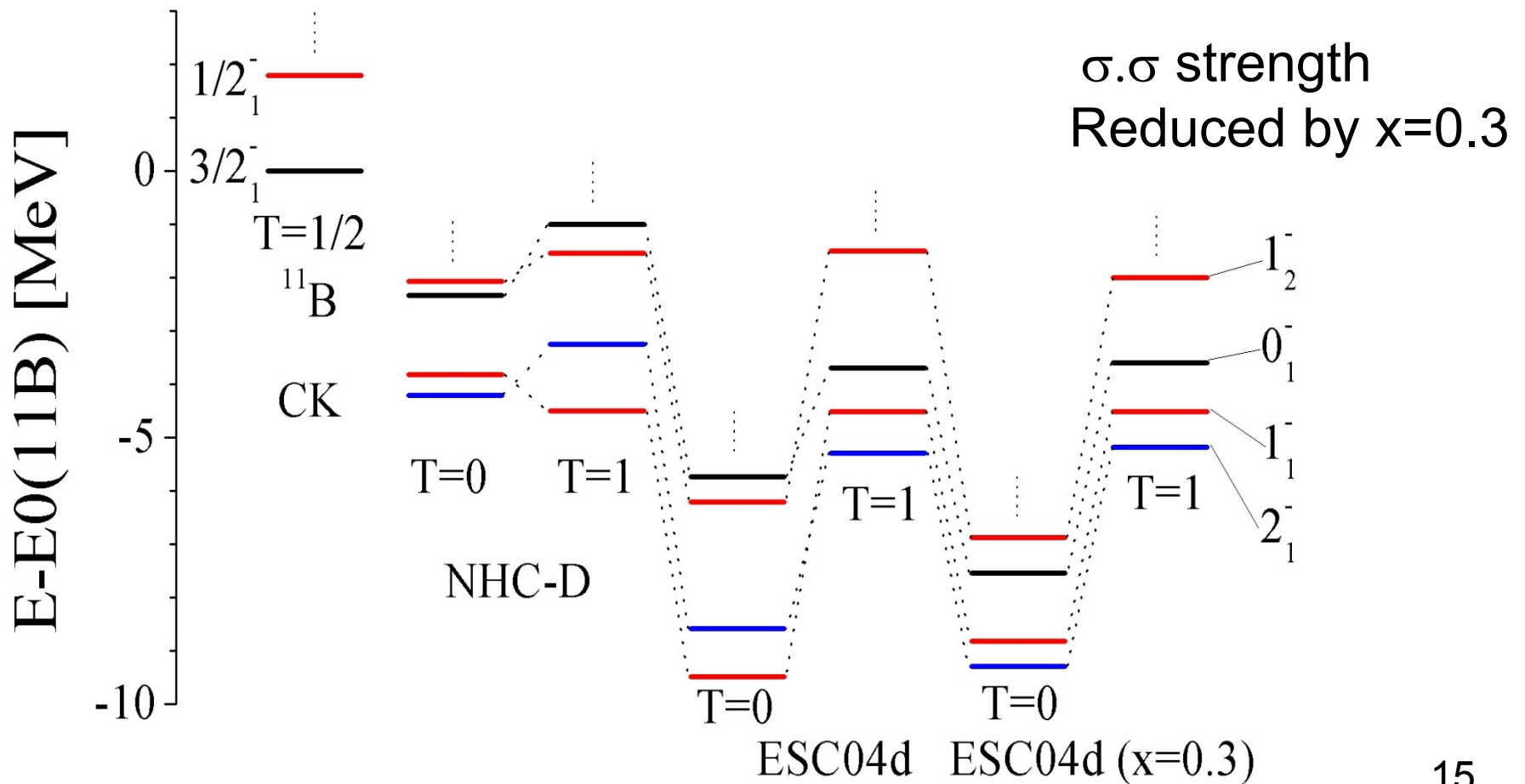
- Interaction property is essential to form the XS spectrum

$V^{(\text{YNG})}$  : NHC-D, Ehime, ESC04d, ESC08, ESC08c, ESC2016

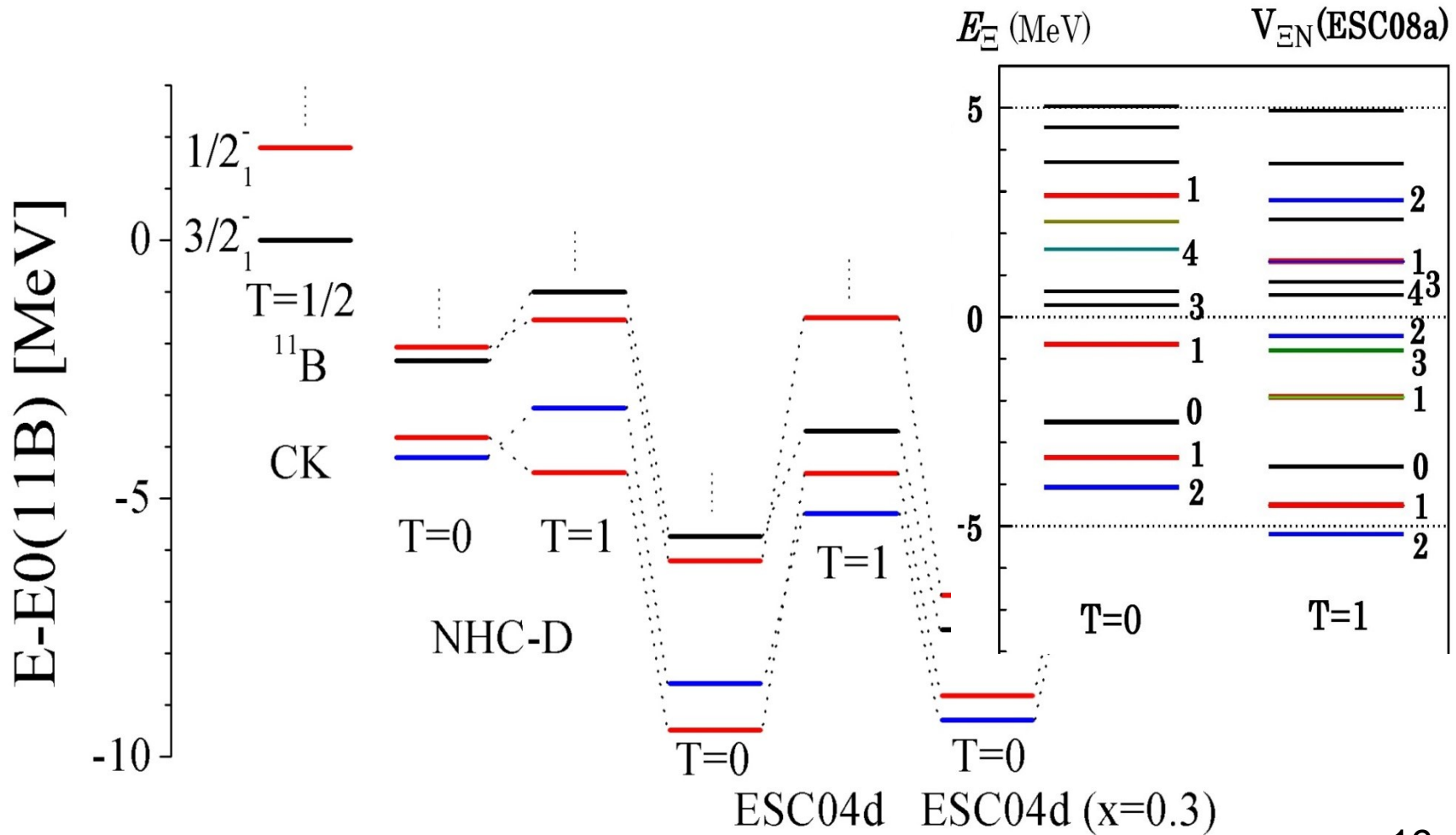
Show how different they are:

NHC-D: T=0 & 1 states appear in the similar E region.

ESC04d: deep T=0 states.

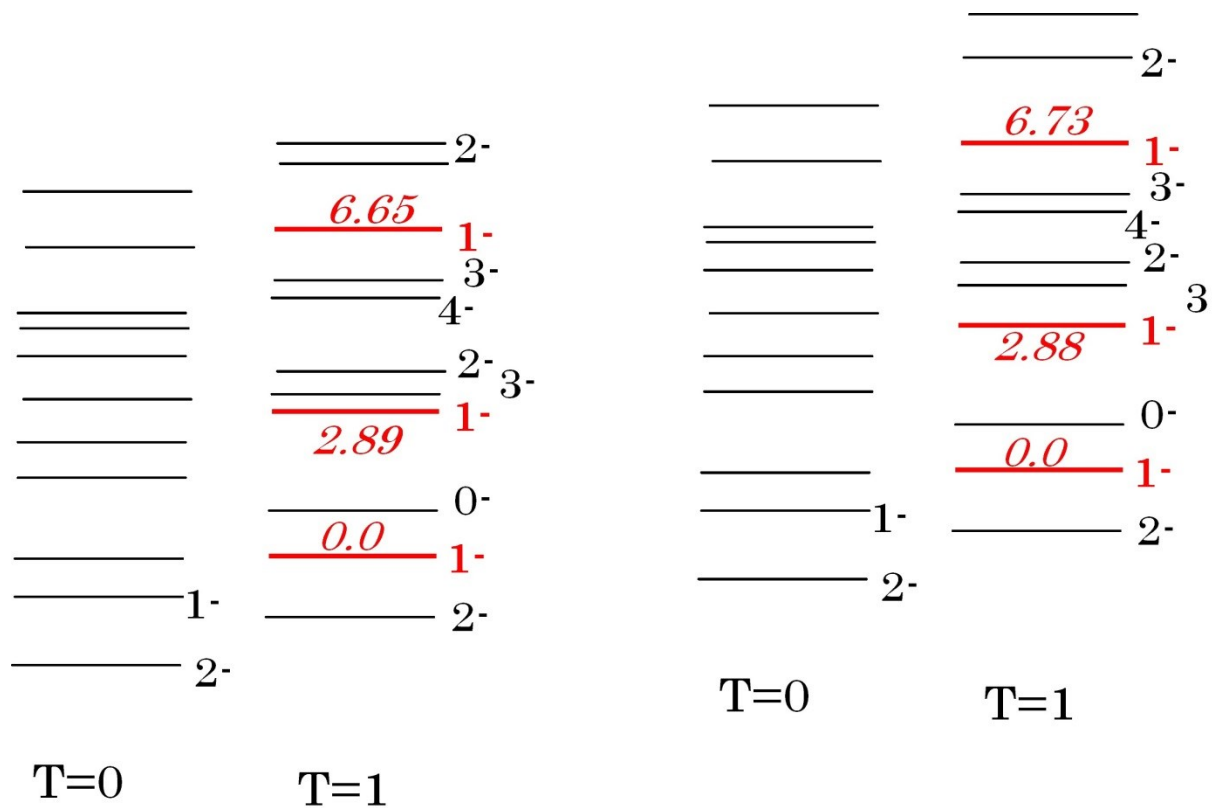


ESC04d:  $T=0$  states are deep,  
 ND, ESC08:  $T=0$  &  $T=1$  states appear in the  
 similar energy region.  $T=1$  is a little deeper.





Most recent versions of ESC08 model do not lead to big change in *relative level structure concerned in (K-,K+) reaction*

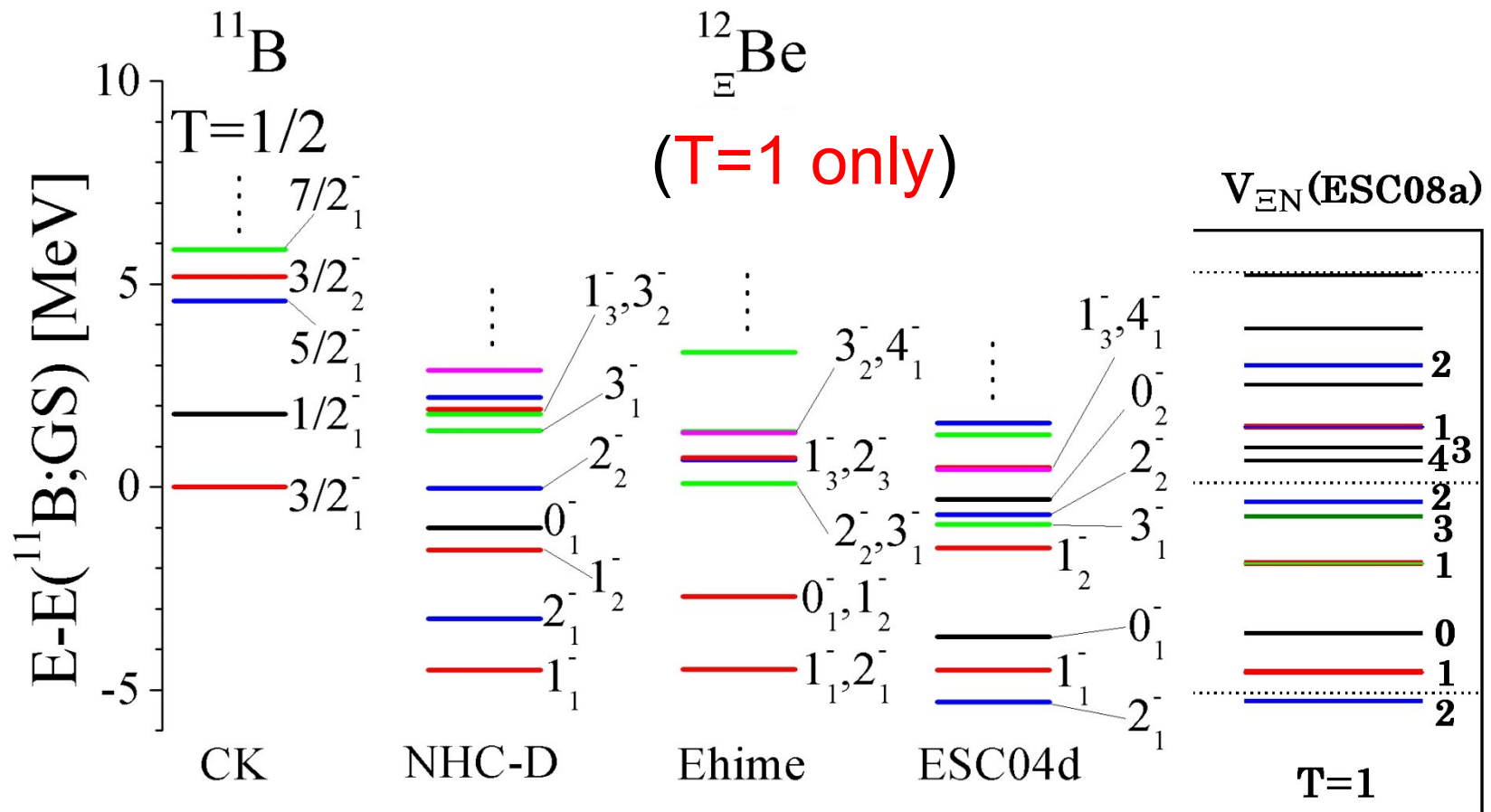


ESC08c2  
ESC08c1

ESC2016  
ESC16MPP

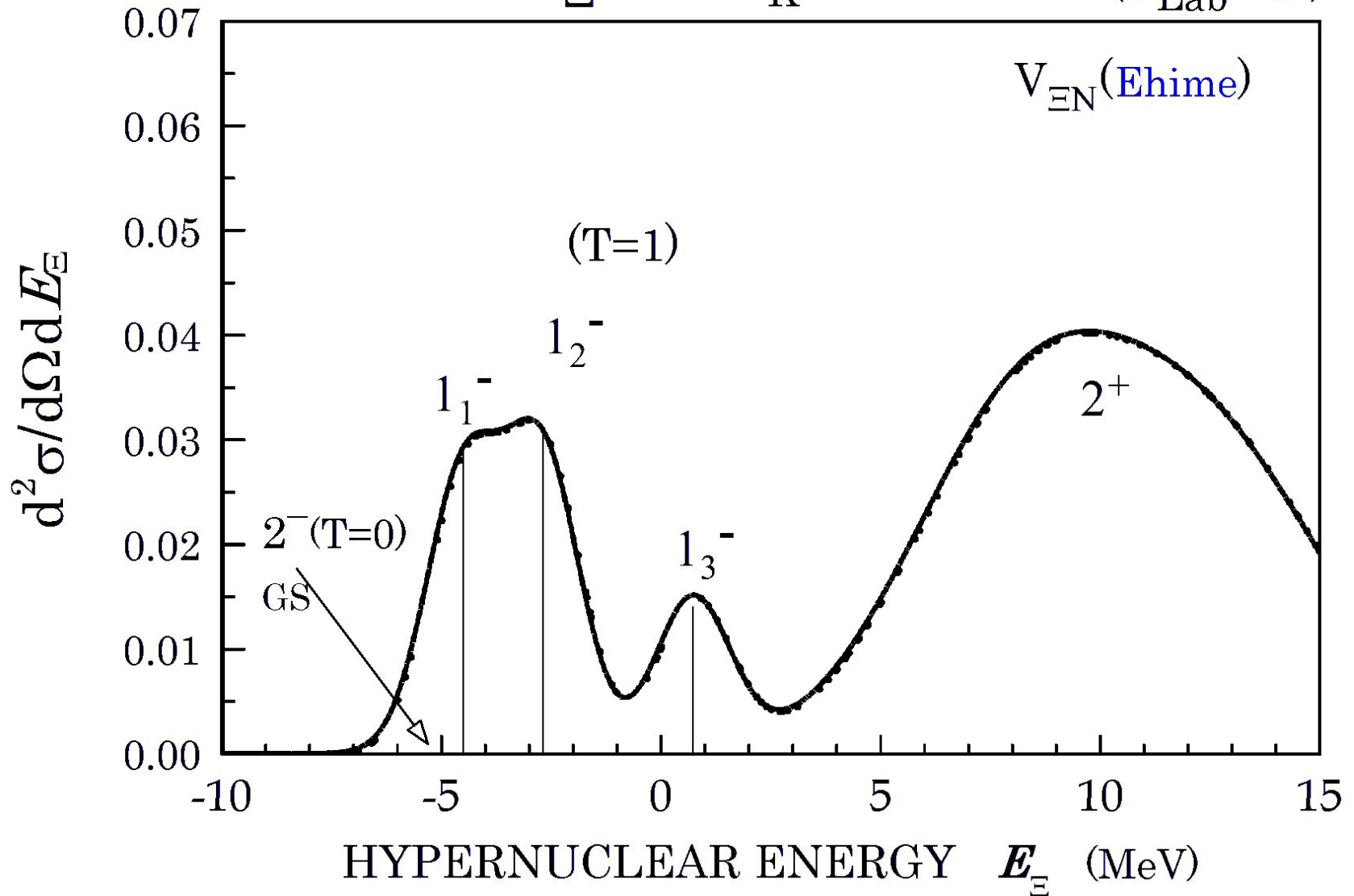
# Reference position of $J=1-(T=1)$ states.

The relative positions of  $J_n$  show the different  $\sigma.\sigma$  interaction nature of  $V_{EN}$ .



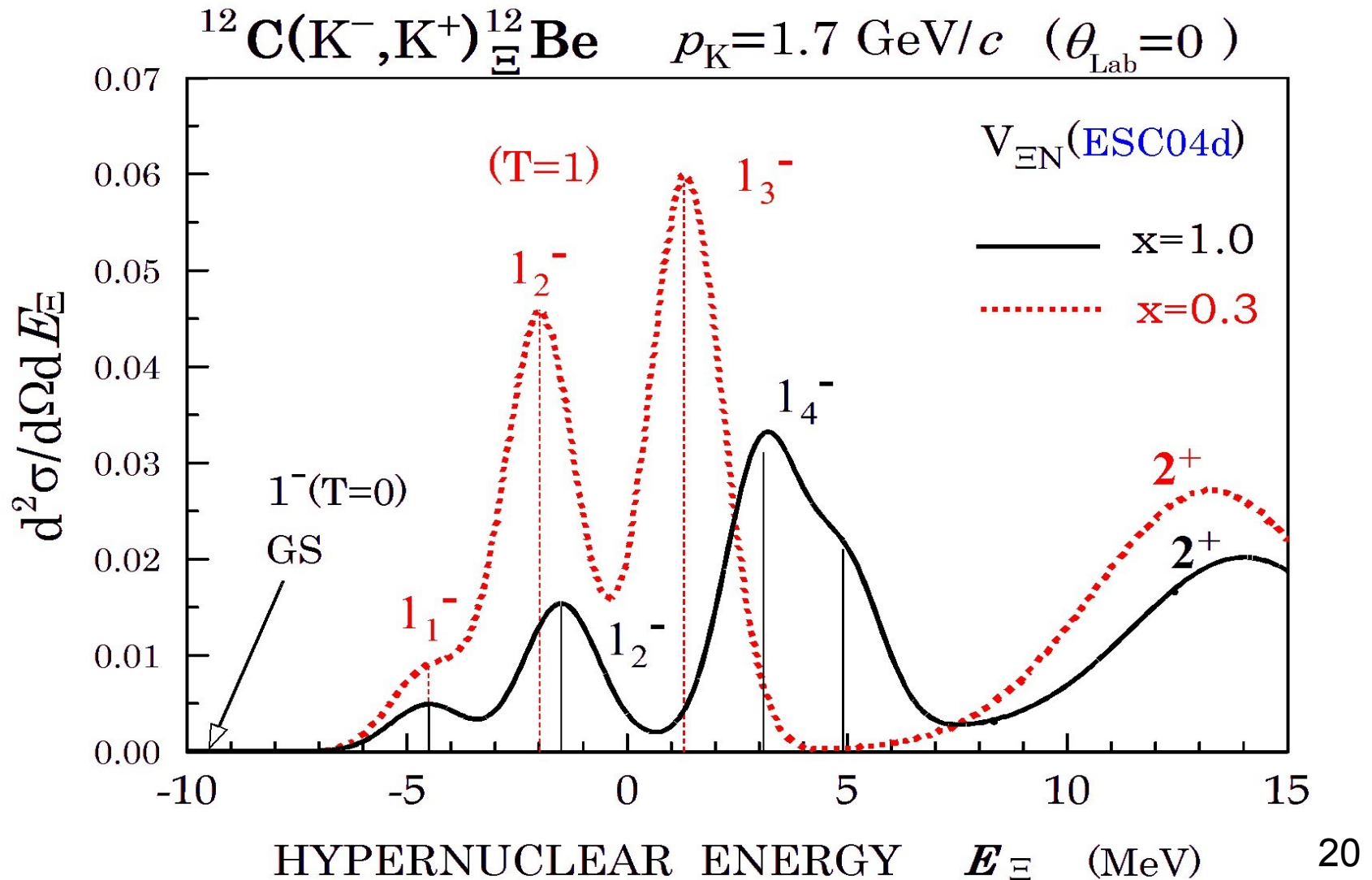
# Spectrum (1): Ehime case.

$^{12}\text{C}(\text{K}^-, \text{K}^+)_{\Xi}^{12}\text{Be}$   $p_{\text{K}}=1.7 \text{ GeV}/c$  ( $\theta_{\text{Lab}}=0$ )



# Spectrum (2): ESC04d case

(large S- & T-dependence)



# Why so sensitive $V(\text{Xi-N})$ -dependence

## (1) Nijmegen NHC-D vs. ESC04d

- **Different partial-wave contributions**  
NHC-D (large p-state attraction) vs. ESC04d(s-state)
- **ESC04d (quite large spin- & isospin-dependence)**

Table 1:  $\Xi$  single particle energies  $U_{\Xi}$  and conversion widths  $\Gamma_{\Xi}$  at normal density calculated with ESC04d and NHC-D.  $S$ -state contributions in  $(TSLJ)$  states and total  $P$ -state contributions are also given. All entries are in MeV.

	$^{11}S_0$	$^{13}S_1$	$^{31}S_0$	$^{33}S_1$	$P$	$U_{\Xi}$	$\Gamma_{\Xi}$
ESC04d( $\alpha = 0$ )	6.4		6.4	-5.0	-6.9	-18.7	11.4
ESC04d( $\alpha = .18$ )	6.3		7.2	-1.7	-5.6	-12.1	12.7
NHC-D	-2.6	0.7	-2.3	-0.4		-21.4	1.1

( From Y. Yamamoto)

## (2) Core- $\Xi$ coupling: ESC04d vs. ND

The spatial symmetry of target GS does not persist due to large spin-dep., when a proton is replaced by  $\Xi$ .

### Mixing probability in $^{12}_{\Xi}\text{Be}$

ESC04d		$J\pi$ of the core ( $^{11}\text{B}$ )		
T=1 $J\pi$	BE( $\Xi$ )	3/2- 1st (GS)	1/2- 1st (1.8MeV)	3/2- 2nd (5.2MeV)
1st 1-	4.5MeV	58.5%	39.0%	1%
2nd 1-	1.5MeV	13.3%	34.0%	48.7%

NHC-D		$J\pi$ of the core ( $^{11}\text{B}$ )		
T=1 $J\pi$	BE( $\Xi$ )	3/2- 1st (GS)	1/2- 1st (1.8MeV)	3/2- 2nd (5.2MeV)
1st 1-	4.5MeV	85.5%	12.3%	2%
2nd 1-	3.0MeV	13.1%	85.8%	1%

- Probability for  $0s_{1/2}(\Xi)$   $\otimes$   $J\pi(\text{Core})$
- ESC04d induce large mixing.
- It probably reflects on the spectra of ( $K^+, K^-$ ) reaction.

# Trying to use the most recent $\Xi$ -N interaction from Nijmegen (ESC08)

Table 1: Partial wave contributions to  $U_{\Xi}(\rho_0)$

model	$T$	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	$U_{\Xi}$	$\Gamma_{\Xi}$
ESC08	0	4.0	-2.7	0.2	-2.2	0.6	-1.1		
	1	7.0	-19.5	-0.3	0.1	-3.6	-0.7	-18.0	6.0
ESC04d	0	6.4	-19.6	1.1	1.2	-1.3	-2.0		
	1	6.4	-5.0	-1.0	-0.6	-1.4	-2.8	-18.7	11.4

$V(\Xi\text{-N})$ .

# Both come from strong S-T-dependence

Table 2:  $U_{\Xi}(\rho_0)$  and partial wave contributions for ESC08c, ESC08c<sub>1</sub><sup>+</sup> and ESC08c<sub>2</sub><sup>+</sup> calculated with the CON choice.  $\Gamma_{\Xi}^c$  denotes  $\Xi N$ - $\Lambda$  conversion width. All entries are in MeV.

	$T$	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	$U_{\Xi}$	$\Gamma_{\Xi}^c$
ESC08c	0	1.1	-6.9	-0.2	1.6	1.5	-1.8	-12.6	7.0
	1	9.0	-16.9	1.2	0.8	-2.3	0.3		
ESC08c <sub>1</sub> <sup>+</sup>	0	2.0	-5.5	-0.1	1.5	1.6	-1.5	-4.2	6.9
	1	9.8	-13.5	1.6	0.9	-1.9	0.9		
ESC08c <sub>2</sub> <sup>+</sup>	0	1.7	-5.9	-0.1	1.6	1.6	-1.5	-6.4	6.6
	1	9.3	-14.6	1.6	0.9	-1.9	0.9		

	$T$	$^1S_0$	$^3S_1$	$^1P_1$	$^3P_0$	$^3P_1$	$^3P_2$	$U_{\Xi}$	$\Gamma_{\Xi}^c$
ESC2016	0	1.7	-7.6	-0.2	0.5	1.5	-1.9	-6.6	6.7
	1	9.1	-9.8	1.2	0.8	-2.2	0.3		
ESC2016 +MPP	0	2.5	-5.8	-0.1	1.2	1.6	-1.6	+2.3	6.6
	1	9.7	-6.9	1.6	0.9	-1.8	0.9		



# $\Xi$ -N interactions used in Shell Model

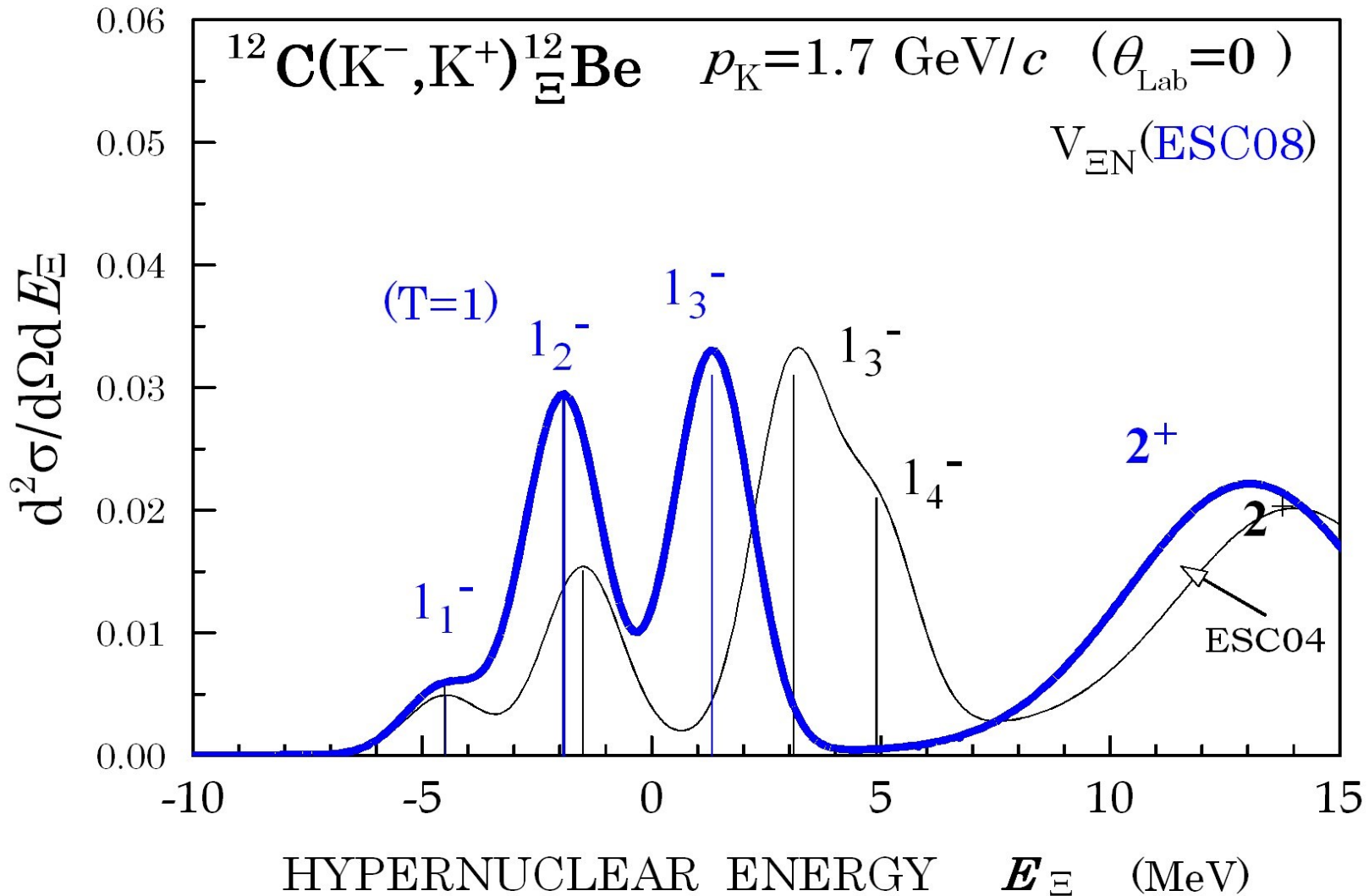
Comparison in the form of  $V = -V_0 + \Delta(\sigma \cdot \sigma)$

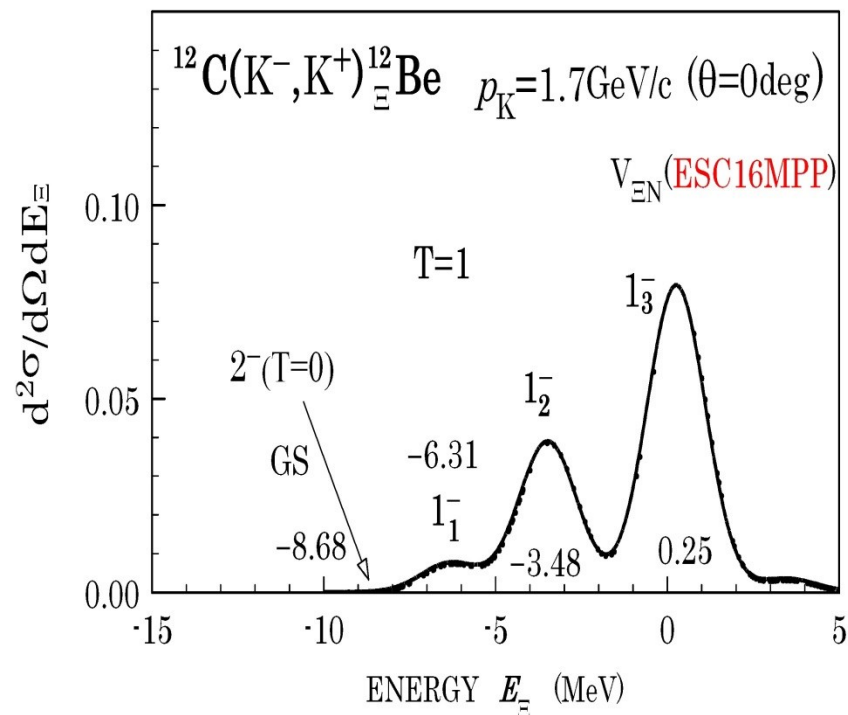
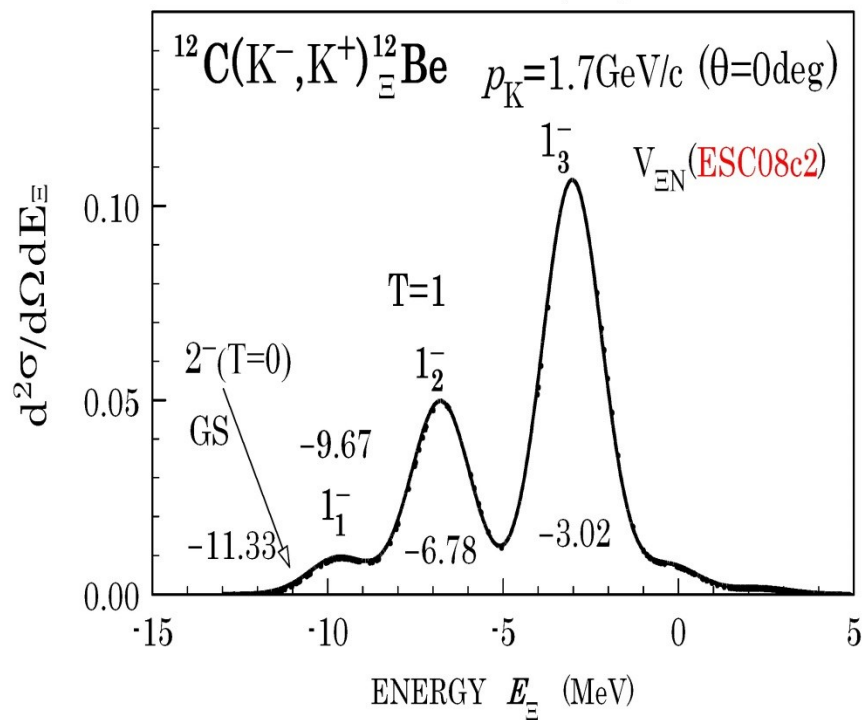
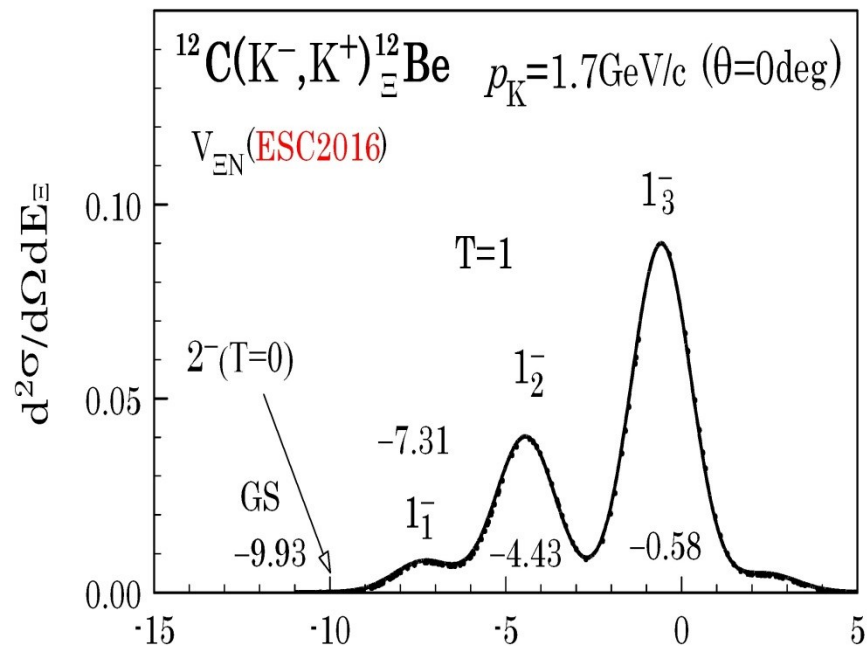
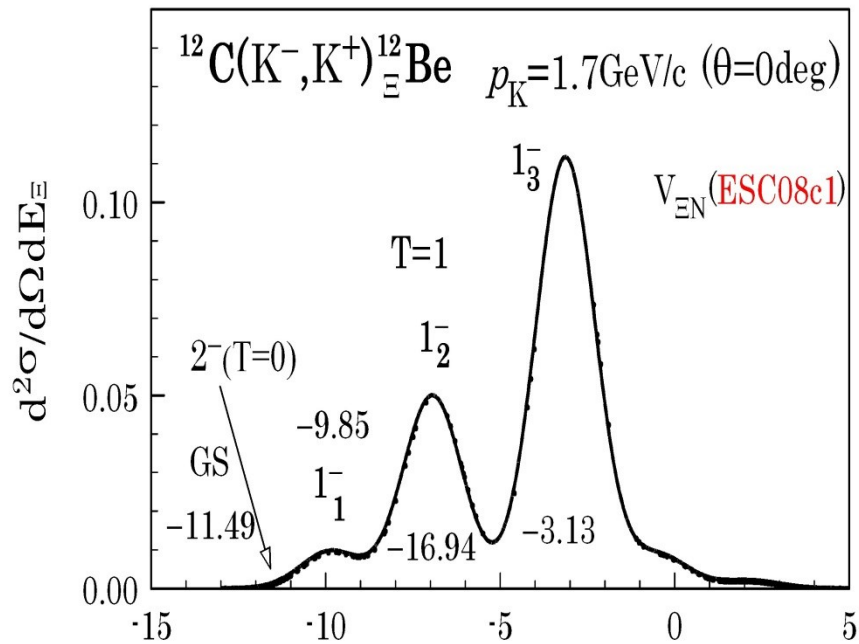
		$V_0$	$\Delta$	$\Delta/V_0$
NE(ESC04d)	T=0	4.98	-15.81	-3.18
	T=1	0.30	-2.96	-9.88
NE(NHC-D)	T=0	2.14	4.75	2.23
	T=1	1.55	0.79	0.51
N $\Lambda$ (NSC97f)	T=1/2	1.36	1.16	0.86
N $\Lambda$ (Millener)		1.49	0.50	0.33

$\sigma \cdot \sigma$  strengths are quite different between ESC and ND, so further trials and improvements are required.

# ESC04 modified to be ESC08 !

(continuum bump should be larger)



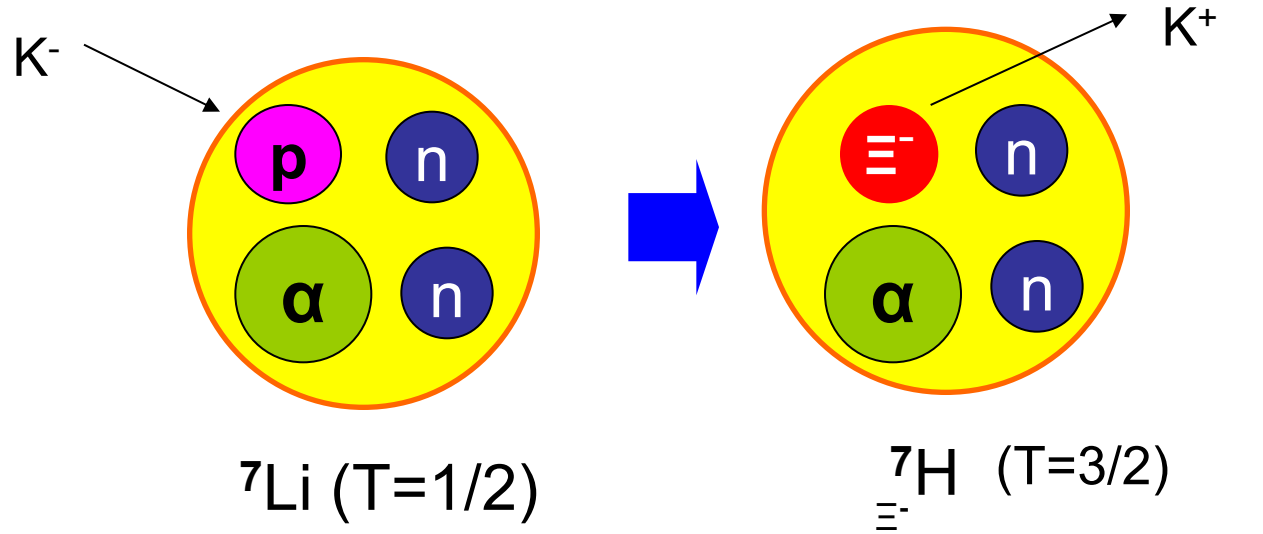


- The patterns of (K-,K+) spectrum --- “two peak structure” ----- obtained from the most recent versions (ESC08c1, ESC08c2, ESC2016 and ESC16MPP) are not much different from each other. ( $\Xi$  in **s**-state)
  - Relative strength of these peaks might change depending on the spin-isospin properties of the Xi-N interactions.
- ←-----→ waiting for the analysis of E05 exp.

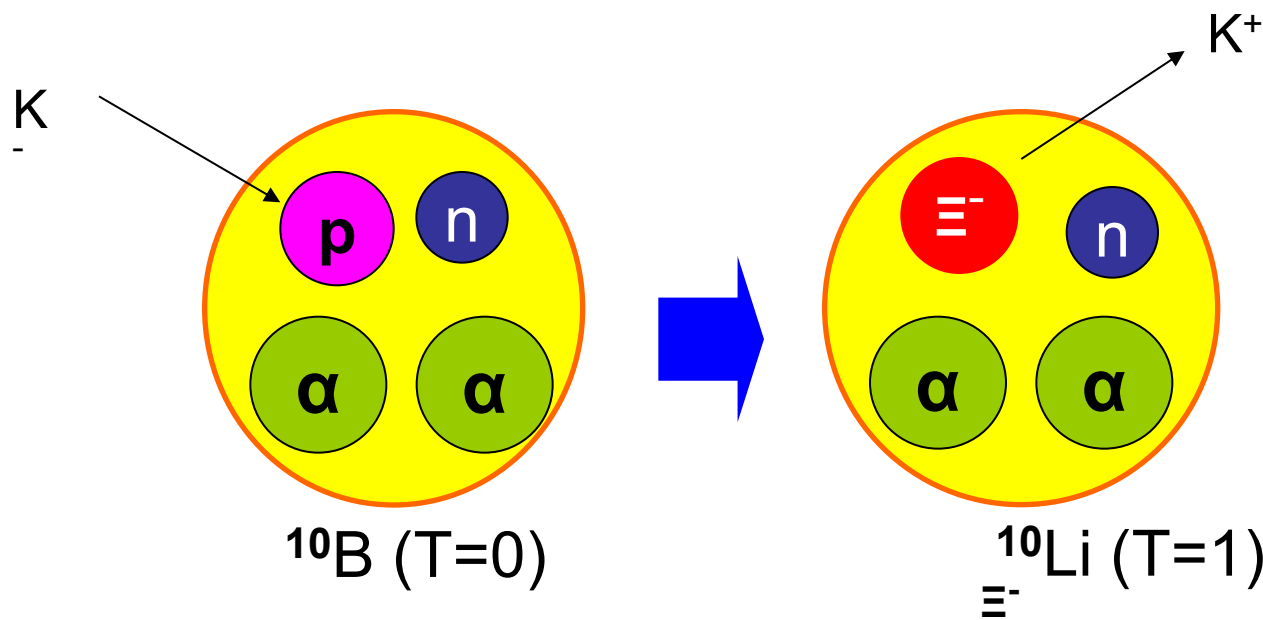
# Summary

1. Shell-Model many-body WF have been applied to  $^{12}\text{C}(K^-, K^+)_{\Xi} ^{12}\text{B}$  in DWIA. +  $^{10}\text{B}$
2. Low-lying levels and the reaction strength functions are shown to be quite sensitive to the choice of available  $\Xi$ -N interactions.
3. Analyses are in progress, especially, to clarify the effect of  $\Xi$ -N spin-dependence on the reaction spectrum so as to improve it.
4. It is interesting if the recent E05 experiment surely provides us with good restriction of discriminating the existing  $\Xi$ -N interactions.<sub>29</sub>

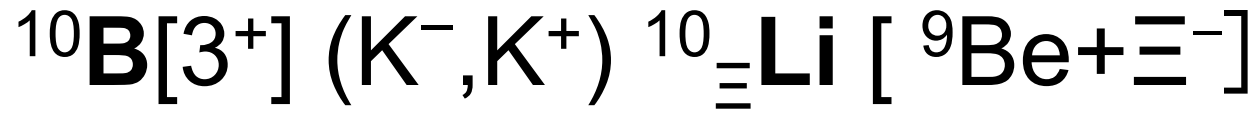
As the second best candidates to extract information about the spin-, isospin-independent term  $V_0$ , we propose to perform...



Why they are suited for investigating  $V_0$ ?



Almost spin-saturated case (Pictures by Hiyama)



The lowest J=2- state gets the largest cross section. ( $\Delta L=1-$ )

