# Hypernuclear structure with antisymmetrized molecular dynamics

#### Structure of Be hypernuclei ( ${}^{10}_{\Lambda}$ Be, ${}^{11}_{\Lambda}$ Be and ${}^{12}_{\Lambda}$ Be)

Based on

M. Isaka and M. Kimura, PRC92, 044326 (2015)

H. Homma, M. Isaka and M. Kimura, PRC91, 014314(2015)

#### Effects of core structure on $B_{\Lambda}$ values: many-body force effects/density dependence

Based on

M. Isaka, Y. Yamamoto and Th.A. Rijken, PRC**94,** 044310(2016) M. Isaka, Y. Yamamoto and Th.A. Rijken, PRC in print

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# Grand challenges of hypernuclear physics

#### Interaction: To understand baryon-baryon interaction

- 2 body interaction between baryons (nucleon, hyperon)

  - hyperon-nucleon (YN)
    hyperon-hyperon (YY)
    A major issue in hypernuclear physics

#### Structure: To understand many-body system of nucleons and hyperon

#### Structure changes by hyperon(s)

- No Pauli exclusion between N and Y "Hyperon as an impurity in nuclei"
- YN interaction is different from NN



#### Today's talk: "Structure change by a $\Lambda$ particle"

# Interaction: recent achievements

#### $\Lambda$ hypernuclei observed so far

#### • Concentrated in light $\Lambda$ hypernuclei with A $\lesssim$ 16

- Structure study based on accurate calc. of few-body problems
- Increases of experimental information

Hiyama & Yamada, PPNP **63** (2009) 339. Hashimoto, & Tamura, PPNP **57** (2006), 564.

• Increasing knowledge of  $\Lambda N$  two-body interaction • Development of  $\Lambda N$  interaction models



Figs: taken from Hashimoto and Tamura, PPNP 57 (2006), 564.

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**Developments of effective interactions** 

In this study,

#### G-matrix interaction derived from Nijmegen potential (YNG)

- Nijmegen potential: a meson exchange model
- G-matrix calculation takes into account medium effects

YNG interaction has density (Fermi momentum  $k_{\rm F}$ ) dependence coming from  $\Lambda N\mbox{-}\Sigma N$  coupling effects

**k**<sub>F</sub> can be calculated from density e.g. Averaged Density Approximation (ADA)  $k_F = \left(\frac{3\pi^2 \langle \rho \rangle}{2}\right)^{1/3}$ ,  $\langle \rho \rangle = \int dr^3 \rho_N(\mathbf{r}) \rho_\Lambda(\mathbf{r})$ 

Y. Yamamoto, T. Motoba and Th.A. Rijken, PTPS185(2010)72.

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# Structure of $\Lambda$ hypernuclei

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- Most of them have well pronounced cluster structure



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Structure study of such hypernuclei becomes one of interesting topics

# Structure in hypernuclei: impurity effects

### What is interesting and important?

#### **Change of cluster structure**

• "How does a  $\Lambda$  particle modify cluster states?"



Experiment: Tanida, et al., PRL 86, 1982(2001).

How about other cluster states? Dependence on degrees of clustering



### Deformation changes by $\Lambda$

etc.

"How do nuclear deformations change if  $\Lambda$  particle is added?"

$$+ \Lambda \Rightarrow \Lambda ?$$

# Deformation change by $\Lambda$ in *s*-orbit

ullet Many authors predict the deformation change by  $\Lambda$  in s-orbit

#### AMD





#### Skyrme-Hartree-Fock (SHF)

X.R. Zhou, et al., PRC76, 034312('07)

J.W. Cui, X.R. Zhou, H.J. Schulze, PRC**91**,054306('15)





#### Relativistic mean-field (RMF)



**RMF & SHF** H. J. Schulze, et al., PTP**123**, 569('10)

<sup>13</sup><sub>A</sub>C (+11.0 MeV) <sup>14</sup><sub>AA</sub>C (+21.5 MeV)

 $\beta = \sqrt{5\pi/3} Q_z/ZR_0^2$ 

-0.1 0 0.1 0.2 0.3 0.4



J.W. Cui, X.R. Zhou, H.J. Schulze, PRC**91**,054306('15)

H. Mei, K. Hagino, J.M. Yao, T. Motoba, PRC91, 064305('15)

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etc.

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$$+ \Lambda \rightarrow \Lambda$$

**Today's talk: "change of cluster structure" Example: Be hypernuclei** 

# Structure of neutron-rich nuclei

Ex.) Be isotopes

- Exotic cluster structure exists in the ground state regions
- Be isotopes have a  $2\alpha$  cluster structure
  - $2\alpha$  cluster structure is changed depending on the neutron number



Today's talk: "change of cluster structure" changes of  $2\alpha$  cluster structure by  $\Lambda$  and dependence on states Example:  ${}^{10}_{\Lambda}Be$ ,  ${}^{11}_{\Lambda}Be$ , and  ${}^{12}_{\Lambda}Be$ 

#### We extended the AMD to hypernuclei

HyperAMD (Antisymmetrized Molecular Dynamics for hypernuclei)

Hamiltonian

$$\hat{H} = \hat{T}_{\scriptscriptstyle N} + \hat{V}_{\scriptscriptstyle NN} + \hat{T}_{\scriptscriptstyle \Lambda} + \hat{V}_{\scriptscriptstyle \Lambda N} - \hat{T}_{\scriptscriptstyle g}$$

NN: Gogny D1S

 $\Lambda$ N: YNG interactions (ESC08c, NSC97f, NF)

#### Wave function

- Nucleon part: Slater determinant Spatial part of single particle w.f. is described as Gaussian packet
- Single-particle w.f. of Λ hyperon: Superposition of Gaussian packets
- Total w.f.:

$$\psi(\vec{r}) = \sum_{m} c_{m} \varphi_{m}(r_{\Lambda}) \otimes \frac{1}{\sqrt{A!}} \det[\varphi_{i}(\vec{r}_{j})]$$

$$\begin{split} \varphi_{N}(\vec{r}) &= \frac{1}{\sqrt{A!}} \det[\varphi_{i}(\vec{r}_{j})] \\ \varphi_{i}(r) &\propto \exp\left[-\sum_{\sigma=x,y,z} v_{\sigma}(r-Z_{i})_{\sigma}^{2}\right] \chi_{i} \eta_{i} \quad \chi_{i} = \alpha_{i} \chi_{\uparrow} + \beta_{i} \chi_{\downarrow} \\ \varphi_{\Lambda}(r) &= \sum_{m} c_{m} \varphi_{m}(r) \\ \varphi_{m}(r) &\propto \exp\left[-\sum_{\sigma=x,y,z} \mu v_{\sigma}(r-z_{m})_{\sigma}^{2}\right] \chi_{m} \quad \chi_{m} = a_{m} \chi_{\uparrow} + b_{m} \chi_{\downarrow} \end{split}$$

### Theoretical framework: HyperAMD

### Procedure of the calculation

Variational Calculation $\frac{dX_i}{dt} = \frac{\kappa}{\hbar} \frac{\partial H^{\pm}}{\partial X_i^*}$  $\kappa < 0$ • Imaginary time development method $\frac{dX_i}{dt} = \frac{\kappa}{\hbar} \frac{\partial H^{\pm}}{\partial X_i^*}$  $\kappa < 0$ • Variational parameters:  $X_i = Z_i, z_i, \alpha_i, \beta_i, a_i, b_i, v_i, c_i$ 



# Actual calculation of HyperAMD

Energy variation with constraint on nuclear quadrupole deformation

Ex.) <sup>8</sup>Be



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Energy variation with constraint on nuclear quadrupole deformation



### For hypernuclei



#### Theoretical Framework: HyperAMD M. Isaka, et al., PRC83(2011) 044323 M. Isaka, et al., PRC83(2011) 054304

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**Angular Momentum Projection** 

$$\left|\Phi_{K}^{s};JM\right\rangle = \int d\Omega D_{MK}^{J^{*}}(\Omega) R(\Omega) \Phi^{s+}$$

#### Generator Coordinate Method(GCM)

•Superposition of the w.f. with different configuration •Diagonalization of  $H^{J\pm}_{sK,s'K'}$  and  $N^{J\pm}_{sK,s'K'}$ 

$$H_{sK,s'K'}^{J\pm} = \left\langle \Phi_{K}^{s}; J^{\pm}M \left| \hat{H} \right| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$
$$\left| \Psi^{J\pm M} \right\rangle = \sum_{sK} g_{sK} \left| \Phi_{K}^{s}; J^{\pm}M \right| \Phi_{K'}^{s'}; J^{\pm}M \right\rangle$$

# **1. Change of cluster structure**

# How does a $\Lambda$ modify 2 $\alpha$ clustering in Be ?

Examples:  ${}^{10}_{\Lambda}Be$ ,  ${}^{11}_{\Lambda}Be$  and  ${}^{12}_{\Lambda}Be$ 



- Degrees of structure changes
- Dependence of structure changes on core structure

<u>M. Isaka</u> and M. Kimura, PRC**92**, 044326 (2015) H. Homma, <u>M. Isaka</u> and M. Kimura, PRC**91**, 014314(2015)



Strength of LS force is weakened by 5% (strengthen by 17%) in <sup>9</sup>Be (<sup>10</sup>Be) to reproduce observed  $Ex(1/2^+)$  ( $Ex(0^+_2)$ )



Degrees of  $2\alpha$  clustering is dependent on states



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# Corresponding states in Be hypernuclei



#### How is structure change?

# Change of $2\alpha$ clustering



- $2\alpha$  distance is largely reduced in well developed cluster states
- Difference of  $2\alpha$  clustering still remains despite of large reduction



 $B_{\Lambda}$  : energy gain from the core states

 $B_{\Lambda}$  (energy gain) is smaller in well-developed cluster states

### Energy components

#### Ex. <sup>11</sup> Be



#### **Energy components**

	<sup>11</sup> <sub>Λ</sub> Be				<sup>10</sup> Be	
$J^{\pi}$	Е	E <sub>N</sub>	$T_{\Lambda}$	$V_{\Lambda N}$	$J^{\pi}$	Е
1/2 <sup>+</sup> 1	-77.8	-67.1	7.3	-18.0	0 <sup>+</sup> 1	-67.2
1/2 <sup>+</sup> 2	-68.4	-60.6	6.4	-14.2	0 <sup>+</sup> 2	-60.9

- Main difference of  ${\sf B}_\Lambda$  comes from that of  $\Lambda {\sf N}$  potential energy  ${\sf V}_{\Lambda {\sf N}}$
- Same trend is found in  ${}^{10}_{\Lambda}$ Be

### $\blacklozenge \Lambda$ binding energy and degrees of 2 $\alpha$ clustering

Intrinsic energy difference:  $b_{\Lambda}(\beta) = E_{core}(\beta) - E_{hvp}(\beta)$ 12.0 -50 10.0  $^{10}$ Be ( E [MeV] b<sub>A</sub> [MeV 8.0 -60 6.0  $^{11}_{\Lambda} Be(+)$  $^{11}_{\Lambda}$ Be (+) -70 050.00.0quadrupole deformation  $\beta$ 0 5 1.0 () ()quadrupole deformation  $\beta$ 

Large  $\beta$  corresponds to well-developed 2  $\alpha$  clustering

Energy gain  $b_{\Lambda}$  decreases as  $\beta$  increases mainly due to decreasing overlap Same trend is found in  ${}^{10}_{\Lambda}Be$ 

### $\blacklozenge \Lambda$ binding energy and degrees of 2 $\alpha$ clustering



Energy gain  $b_{\Lambda}$  decreases as  $\beta$  increases mainly due to decreasing overlap Same trend is found in  ${}^{10}{}_{\Lambda}$ Be

## Structure change in ${}^{10}_{\Lambda}$ Be and ${}^{11}_{\Lambda}$ Be



- $\Lambda$  causes dynamical reduction of  $2\alpha$  clustering
- But, the difference of the  $2\alpha$  clustering still remains in hypernuclei

 $\rightarrow$  Change of excitation spectra due to the difference of  $B_{\Lambda}$ 

### Similar phenomena in n-rich hypernucleus ${}^{12}_{\Lambda}$ Be

- ◆ Parity inverted ground state of n-rich <sup>11</sup><sub>4</sub>Be<sub>7</sub>
  - The <sup>11</sup>Be ground state is 1/2<sup>+</sup>, while ordinary nuclei have a 1/2<sup>-</sup> state as the ground state

→ Vanishing of the magic number N=8



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- $2\alpha$  clusters with 3 surrounding neutrons occupying molecular orbits (MO)
- MO explains parity inversion, but difficult to see difference of 2 $\alpha$  clustering

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### Results: Parity reversion of ${}^{12}_{\Lambda}$ Be

♦ Ground state of <sup>12</sup><sup>∧</sup> Be



# Results: Parity reversion of ${}^{12}_{\Lambda}$ Be

- ♦ Ground state of <sup>12</sup> ABe
  - The parity reversion of the  ${}^{12}{}_{\Lambda}$ Be g.s. occurs by the  $\Lambda$  hyperon



# Deformation and $\Lambda$ binding energy



Λ slightly reduces deformations, but the deformation is still different
 Λ hyperon coupled to the 1/2<sup>-</sup> state is more deeply bound than that coupled to the 1/2<sup>+</sup> state

– Due to the difference of the deformation between the  $1/2^-\,$  and  $1/2^+\,$  states

Parity reversion occur reflecting the difference in  $2\alpha$  clustering of <sup>11</sup>Be

could be an evidence for difference of clustering in <sup>11</sup>Be

• To study structure change by  $\Lambda$ , we applied an extended version of AMD to Be hypernuclei.

### ${}^{10}{}_{\Lambda}\text{Be}$ and ${}^{11}{}_{\Lambda}\text{Be:}$ changes of various 2 $\alpha$ cluster structure

- $\Lambda$  largely reduces radii of well-developed cluster states
- Difference of deg. of 2 $\alpha$  clustering still remains even in Be hypernuclei

### ${}^{12}{}_{\Lambda}$ Be: Exotic cluster structure

– The abnormal parity of  $^{11}Be$  ground state is reverted in  $^{12}{}_{\Lambda}Be$ 

# **2.** Effects of core structure on $\mathbf{B}_{\Lambda}$ values

# Many-body force effects appear in $B_{\Lambda}$ ?

M.Isaka, Y. Yamamoto and Th.A. Rijken, PRC**94,** 044310(2016) M. Isaka, Y. Yamamoto and Th.A. Rijken, PRC in print

### **Individual problems**

1)  $B_{\Lambda}$  values and density dependence of the interaction

#### Today's talk: importance of core description

2) Many-body force effects

Is it possible to describe mass dependence of observed  $B_{\Lambda}$ ?

What is essential to reproduce it?



# Toward heavier and exotic $\Lambda$ hypernuclei

### **Experiments at J-PARC and JLab etc.**

#### • Hypernuclear chart will be extended to heavier regions



Different structure affects  $B_{\Lambda}$  values through dens. dep. of interaction?

# $B_{\Lambda}$ as a function of mass number A

#### Observed data of binding energy of $\Lambda$ (B<sub> $\Lambda$ </sub>) (9 $\leq$ A $\leq$ 51)



#### "Mass dependence of $B_{\Lambda}$ " and "Density dependence of $\Lambda N$ ( $\Lambda NN$ ) interaction"

Bertini *et al.*, NPA**83**,306(1979), Davis, Juric , *et al.*, NPB**52**(1973), Davis, NPA**547**,369(1992);NPA**754**,3c(2005), Ajimura *et. al.*, NPA**639**(1998)93c, Pile *et al.*, PRL**66**,2585(1991), Hotchi *et al.*, PRC**64**, 044302(2001), Hashimoto and Tamura, PPNP**57**,564(2006), Tang, *et. al.*, PRC**90**,034320(2014).

# Purpose of this study

# Purpose

• To reveal many-body force effects on  $B_{\Lambda}$  values based on the interaction model ESC proposed by Nijmegen group



### Results: $B_{\Lambda}$ as a function of mass number A

#### HyperAMD w/ ESC08c

$$\langle \rho \rangle = \int dr^3 \rho_N(\mathbf{r}) \rho_\Lambda(\mathbf{r})$$
$$k_F = (1+\alpha) \left(\frac{3\pi^2 \langle \rho \rangle}{2}\right)^{1/3}$$

Small parameter  $\alpha$  is chosen so as to



 $-B_{\Lambda}^{\text{calc}}$ 

-8.1

-8.0

-8.1

-8.9

-9.1

-10.4

-11.2

 $\langle \rho \rangle$ 

0.072

0.060

0.072

0.077

0.075

0.081

0.083

в

0.50

0.87

0.45

0.57

0.58

0.50

0.39

%Li

 $^{9}_{\Lambda}$ Be

 $^{9}_{\Lambda}B$ 

 $^{10}_{\Lambda}$ Be

 $^{10}_{\Lambda}\mathrm{B}$ 

 ${}^{\hat{1}1}_{\Lambda}{}^{\mathrm{B}}_{\mathrm{B}}$ 

 $\gamma$  $2^{\circ}$ 

 $1^{\circ}$ 

 $2^{\circ}$ 

 $1^{\circ}$ 

 $1^{\circ}$ 

 $29^{\circ}$ 

 $48^{\circ}$ 

 $k_F$ 

1.01

0.95

1.01

1.04

1.03

1.05

1.06

 $-B^{exp}_{\Lambda}$ 

 $-8.50 \pm 0.12$ 

 $-6.71 \pm 0.04$ 

 $-8.29 \pm 0.18$ 

 $-9.11 \pm 0.22$  $-8.55 \pm 0.18$ 

 $-8.89 \pm 0.12$ 

 $-10.24 \pm 0.05$  $-11.37 \pm 0.06$ 

 $-11.38 \pm 0.02$ 

HyperAMD w/ ESC08c successfully reproduces B<sub>A</sub> in wide mass regions



### Description of the core nuclei

#### • "Full calculation" vs. "Spherical calculation"



"Full calculation": all of the intrinsic w.f. are used in the GCM calc.



**Deformation of the ground states is essential to reproduce**  $B_A$ 

# "Description of the core structure"

Ex.: <sup>11</sup>B More sophisticated treatment: GCM calc. on ( $\beta$ ,  $\gamma$ ) plane 10г <sup>11</sup>**B** NEG **B(E2)** e<sup>2</sup>fm<sup>4</sup> T. Suhara and Y. Kanada-En'yo, PTP123,303(2010) 8 β const.  $β-\gamma$  const. Exp. [MeV] 60° -45  $7/2^{-}$ 6  $1.9 \pm 0.4$ **Present calc.** 6 Δ (AMD)  $-3/2\overline{2}$ -60  $5/2^{-}$ 4 6 16  $14 \pm 3$ -75 1/2-2 0° 0.2 0.4 0.6 0.8 1.0  $3/2_{1}$ β <sup>12</sup> <sub>Δ</sub> B (β-γ) <sup>12</sup><sup>A</sup> B (Spherical) <sup>12</sup> <sub>A</sub>B (EXP) **B**<sub>A</sub> = 9.7 MeV  $B_{\Lambda} = 11.2 \text{ MeV}$  $B_{\Lambda} = 11.4 \pm 0.02 \text{ MeV}$  $(k_r = 1.06 \text{ fm}^{-1})$  $(k_{r} = 1.15 \text{ fm}^{-1})$ 

# Summary and future plans

### Summary

ullet To study structure change by  $\Lambda$  in cluster states of Be hypernuclei

### ${}^{\bf 10}{}_{\Lambda} Be$ and ${}^{\bf 11}{}_{\Lambda} Be$ : changes of various 2 $\alpha$ cluster structure

- $\Lambda$  largely reduces radii of well-developed cluster states
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### ${}^{12}{}_{\Lambda}$ Be: Exotic cluster structure

– The abnormal parity of  $^{11}Be$  ground state is reverted in  $^{12}{}_{\Lambda}Be$ 

### $\bullet$ Systematics of $\mathsf{B}_\Lambda$ and density dependence of the $\Lambda N$ interaction

– Proper description of the core nuclei is important for  ${\rm B}_{\Lambda}$  values

### Future plans

- Structure study of hypernuclei
  - $\alpha$  clustering of both single and double  $\Lambda$  hypernuclei

### Based on structure calculation

CSB effects in medium-heavy hypernuclei