Clusters in ^{12}C and $^{13}{}_{\Lambda}C$

Yasuro Funaki (Beihang U.)

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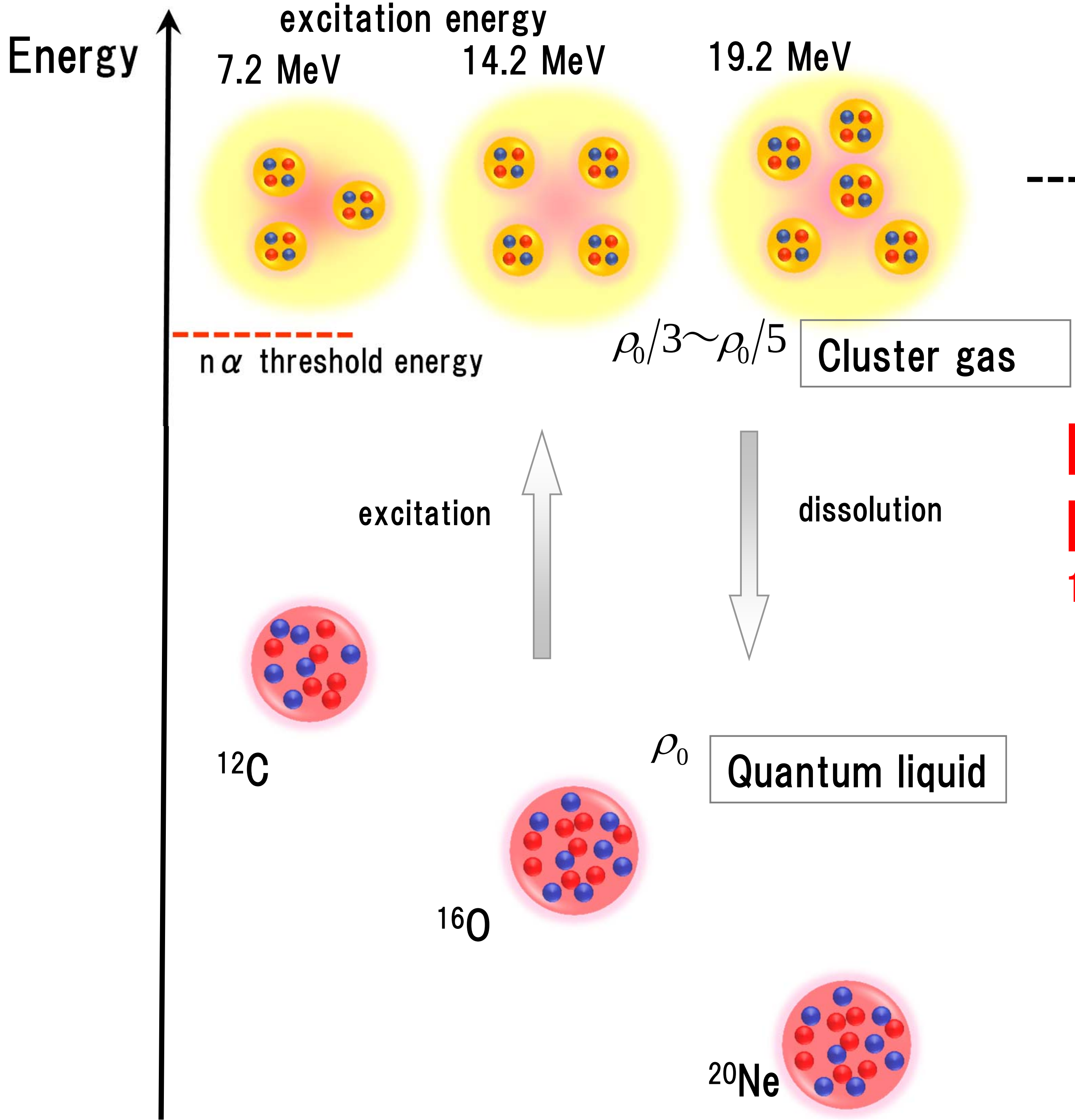
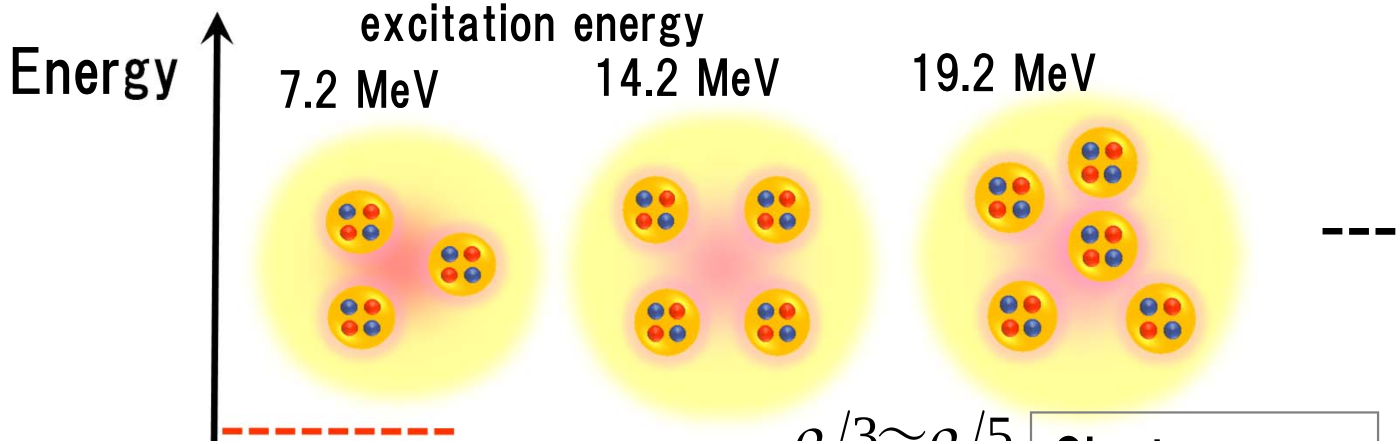
``gas phase'' in finite nuclei



Infinite nuclear matter
(low density)
$< \rho_0/5$
Crust of proto-neutron star？

Investigation in heavier nuclei than ^{12}C

“gas phase” in finite nuclei



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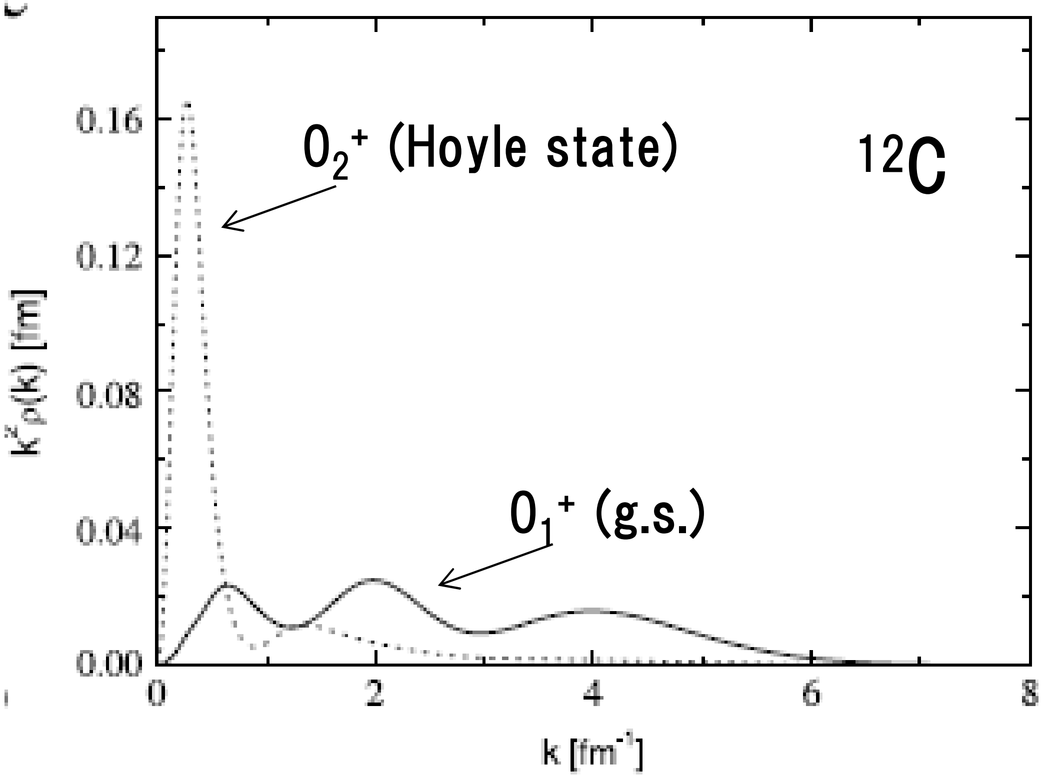
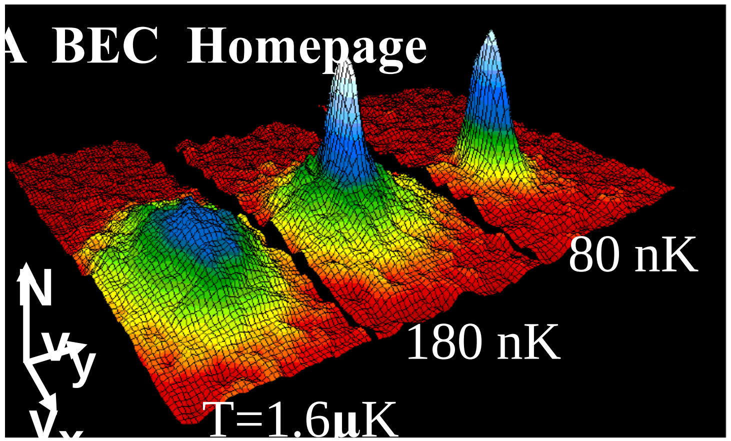
Crust of proto-neutron star？

T. Yamada et al., EPJA **26**, 185(2005).

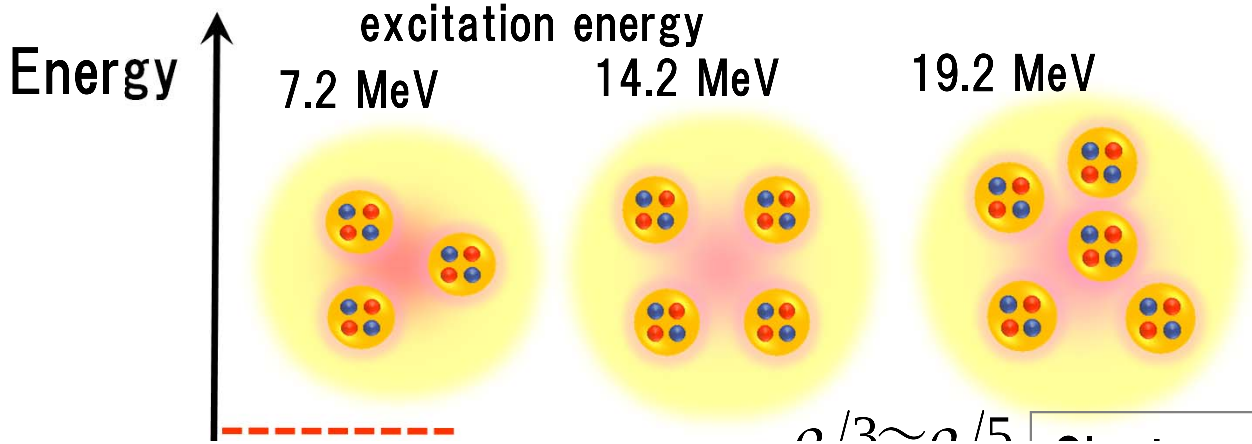
Momentum distributions of the alpha particles



BEC in the atomic world



``gas phase'' in finite nuclei

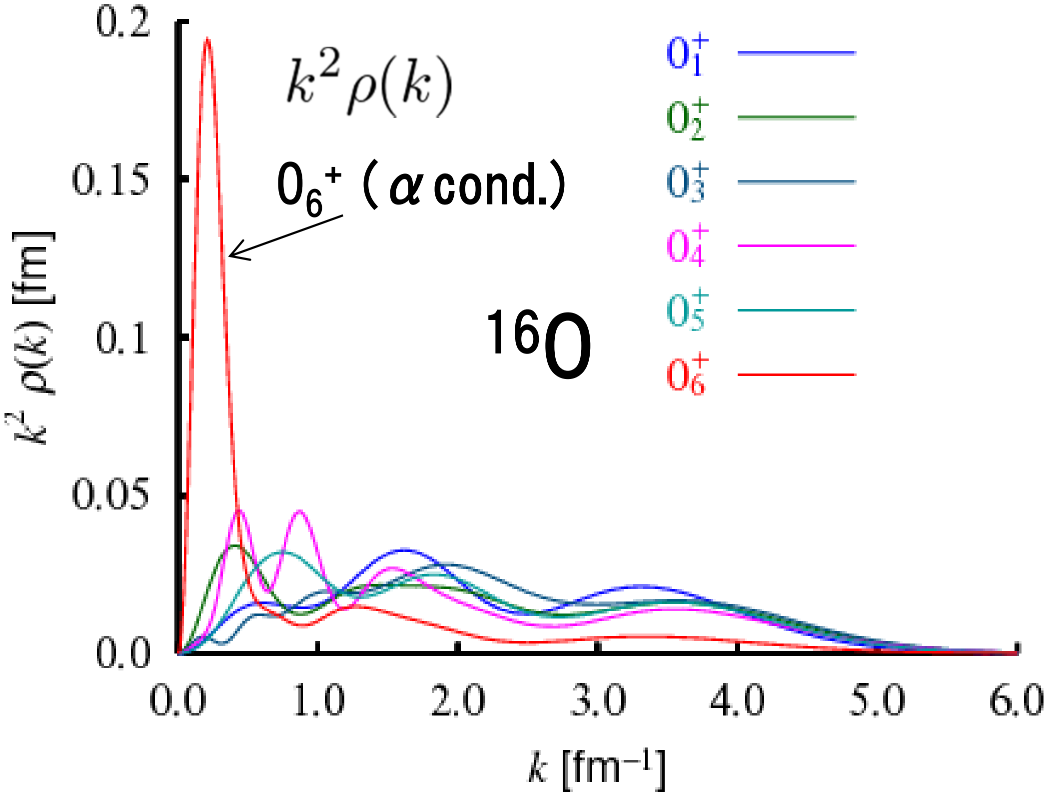


Infinite nuclear matter (low density) $< \rho_0/5$

Crust of proto-neutron star？

Y. F. et al., PRL **101**, 082502(2008).

Momentum distributions of the alpha particles



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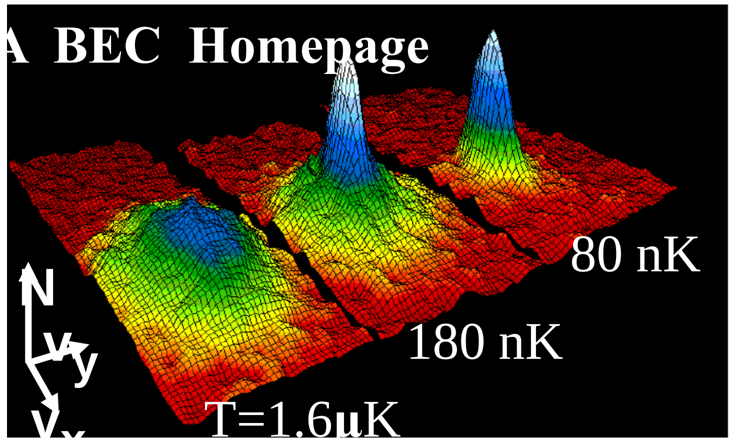
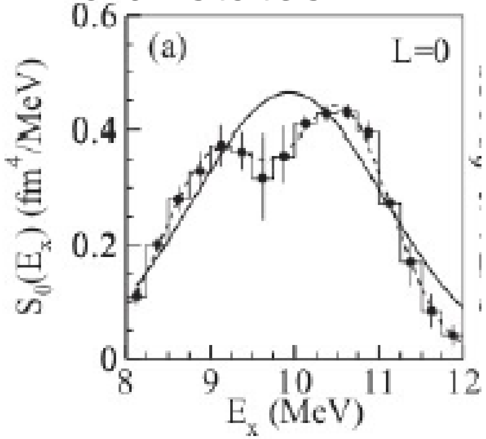


0^+ states and rotational states in ^{12}C

Broad 0^+ state
at10.3 MeV

- AMD: linear-chain-like: (0_4^+)
- OCM+CSM: family of αcond. (0_3^+)
  + linear-chain-like (0_4^+)
  by Kurokawa and Kato
- Kamimura CSM:
  exactly confirmed two states
  $(0_3^+, 0_4^+)$
- Two 0^+ states

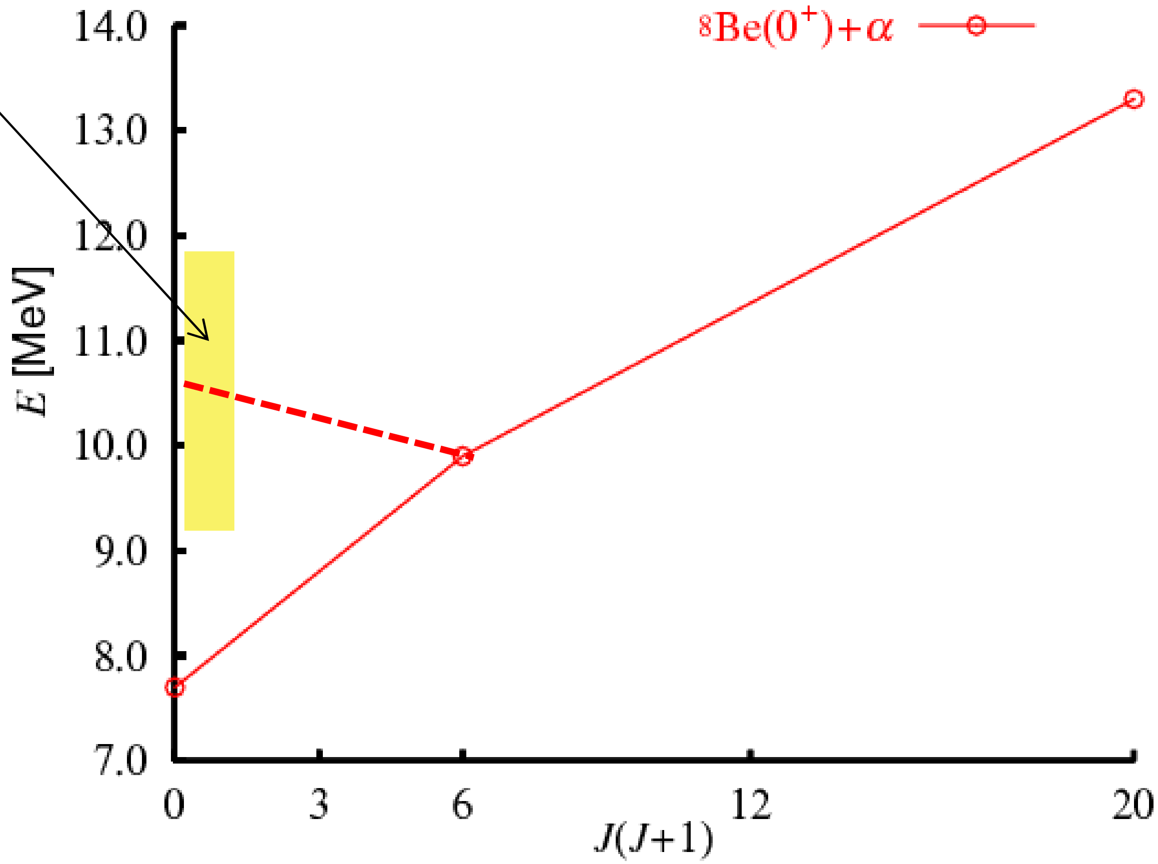


Result of MDA

Ex=9.04±0.09 MeV
Γ =1.45±0.18 MeV

Ex=10.56±0.06 MeV
Γ = 1.42±0.08 MeV

M. Itoh al., PRC 84, 054308 (2011).



4^+ : *M. Freer et al., PRC 83, 034314 (2011)*

2^+ : *M. Itoh et al., NPA 738, 268 (2004)*

M. Freer et al., PRC 80, 041303 (2009)

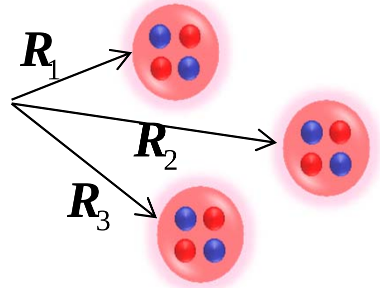
W. R. Zimerman et al., PRC 84, 027304 (2011)

So-called ``THSR'' wave function

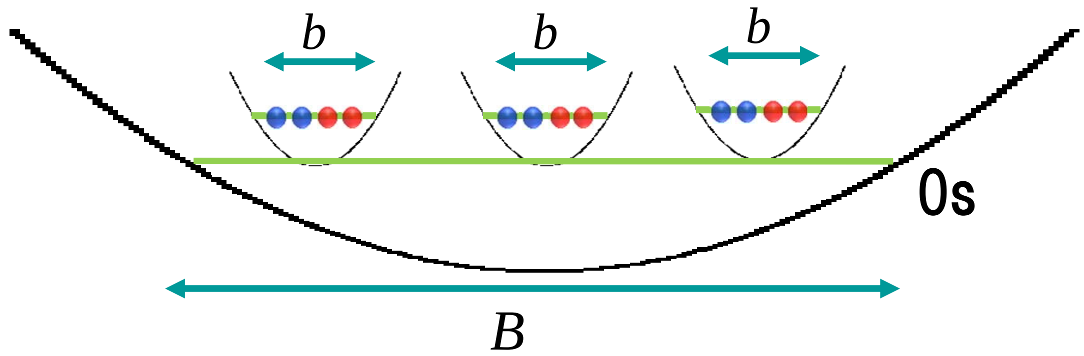
A. Tohsaki, H. Horiuchi, P. Schuck, and Ropke, PRL 87, 192501(2001)

$$\Phi_{\alpha}^{\text{con.}}(\boldsymbol{\beta},b)=\exp\left(-2\sum_{k}^{x=y,z}\frac{(\boldsymbol{R})_k^{\;2}}{b^2+2\beta_k^2}\right)\phi_\alpha\left(\boldsymbol{r_1},\boldsymbol{r_2},\boldsymbol{r_3},\boldsymbol{r_4}:b\right)$$

$$\Phi_{^{12}\text{C}}(\boldsymbol{\beta},b)=\Psi_G^{-1}\left(\frac{\boldsymbol{r}_1+\cdots+\boldsymbol{r}_{12}}{12}\right)\mathcal{A}\left\{\Phi_{\alpha}^{\text{con.}}(\boldsymbol{\beta},b)\Phi_{\alpha}^{\text{con.}}(\boldsymbol{\beta},b)\Phi_{\alpha}^{\text{con.}}(\boldsymbol{\beta},b)\right\}$$



$\sim \mathcal{A}\Big\{$



$\Big\}$

$$B_k^2=b^2+2\beta_k^2\ (k=x,y,z)$$

Total center-of-mass w. f. to be eliminated.

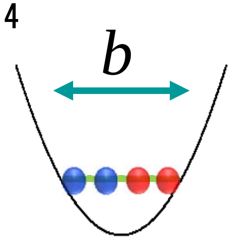
$$\Psi_G\left(\frac{\boldsymbol{r}_1+\cdots+\boldsymbol{r}_{12}}{12}\right)$$

Internal w.f. of α particle

$$\phi_\alpha\left(\boldsymbol{r_1},\boldsymbol{r_2},\boldsymbol{r_3},\boldsymbol{r_4}:b\right)\propto\exp\left(-\frac{1}{8b^2}\sum_{k<l}(\boldsymbol{r}_k-\boldsymbol{r}_l)^2\right)$$

Nicely approximated by $(0s)^4$

b=1.35 fm: fixed



Two limits

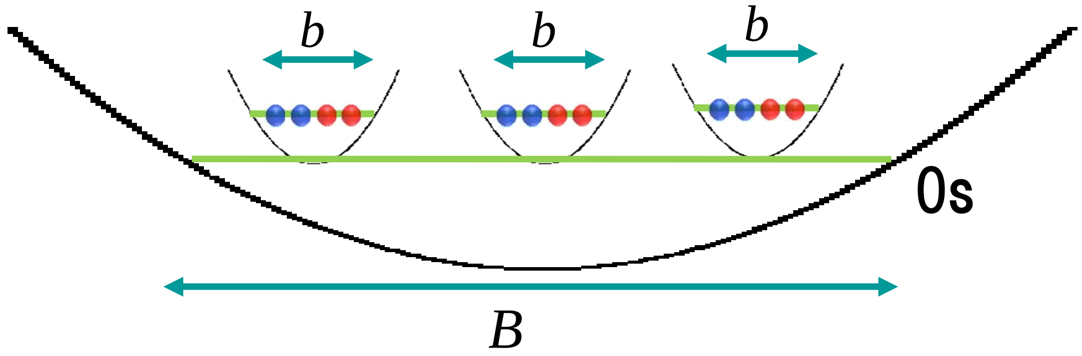
B =b: Shell model w.f.

B >>b: Gas of independent α-particles

Ext.-``THSR'' w.f. and the way of calculating energy spectra

$$\Phi_{^{12}\mathrm{C}}(\boldsymbol{\beta},b)=\Psi_G^{-1}\left(\frac{\boldsymbol{r}_1+\cdots+\boldsymbol{r}_{12}}{12}\right)\mathcal{A}\left\{\Phi_\alpha^{\mathrm{con.}}(\boldsymbol{\beta},b)\Phi_\alpha^{\mathrm{con.}}(\boldsymbol{\beta},b)\Phi_\alpha^{\mathrm{con.}}(\boldsymbol{\beta},b)\right\}$$

$\sim \mathcal{A}\ \{$



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Hill-Wheeler eq. or GCM(generator coordinate method)

$$\sum_{\boldsymbol{\beta}''}\left\langle \hat{P}^J_{MK}\Phi_{^{12}\mathrm{C}}(\boldsymbol{\beta},b)\middle|\hat{H}-E\middle|\hat{P}^J_{MK}\Phi_{^{12}\mathrm{C}}(\boldsymbol{\beta}',b)\right\rangle f\left(\boldsymbol{\beta}'\right)=0$$

$\hat{P}^J_{MK}$: Angular momentum projection operator

Hamiltonian (NN force: Volkov No.2 force)

$$\hat{H}=-\frac{\hbar^2}{2m}\sum_{i=1}^{12}\nabla_i^2-T_G+\sum_{i<j}^{12}(V_{ij}^{(N)}+V_{ij}^{(C)})$$

$\boldsymbol{\beta}_i=\left(\beta_{ix}=\beta_{iy},\beta_{iz}\right)$

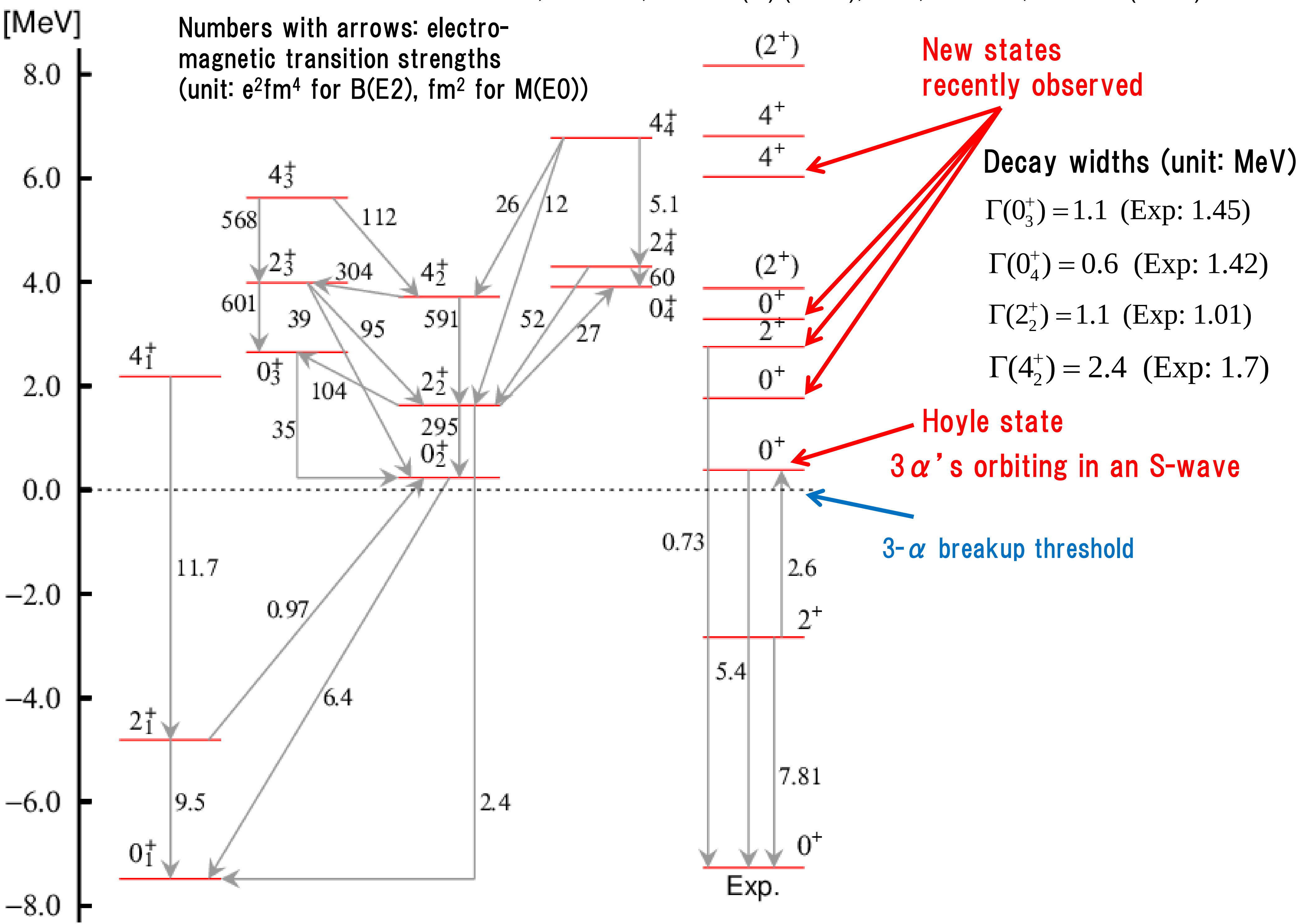
With (axially symmetric) deformation

Spurious continuum components are effectively eliminated by r^2 constraint method.

See Y. F. et al., PTP **115,** 115 (2006).

Results (of ^{12}C)

Y. F. et al., Progress in Particle and Nuclear Physics **82**, 78-132 (2015).
Y. F., PRC **92**, 021302(R) (2015); Y. F., PRC **94**, 024344 (2016)



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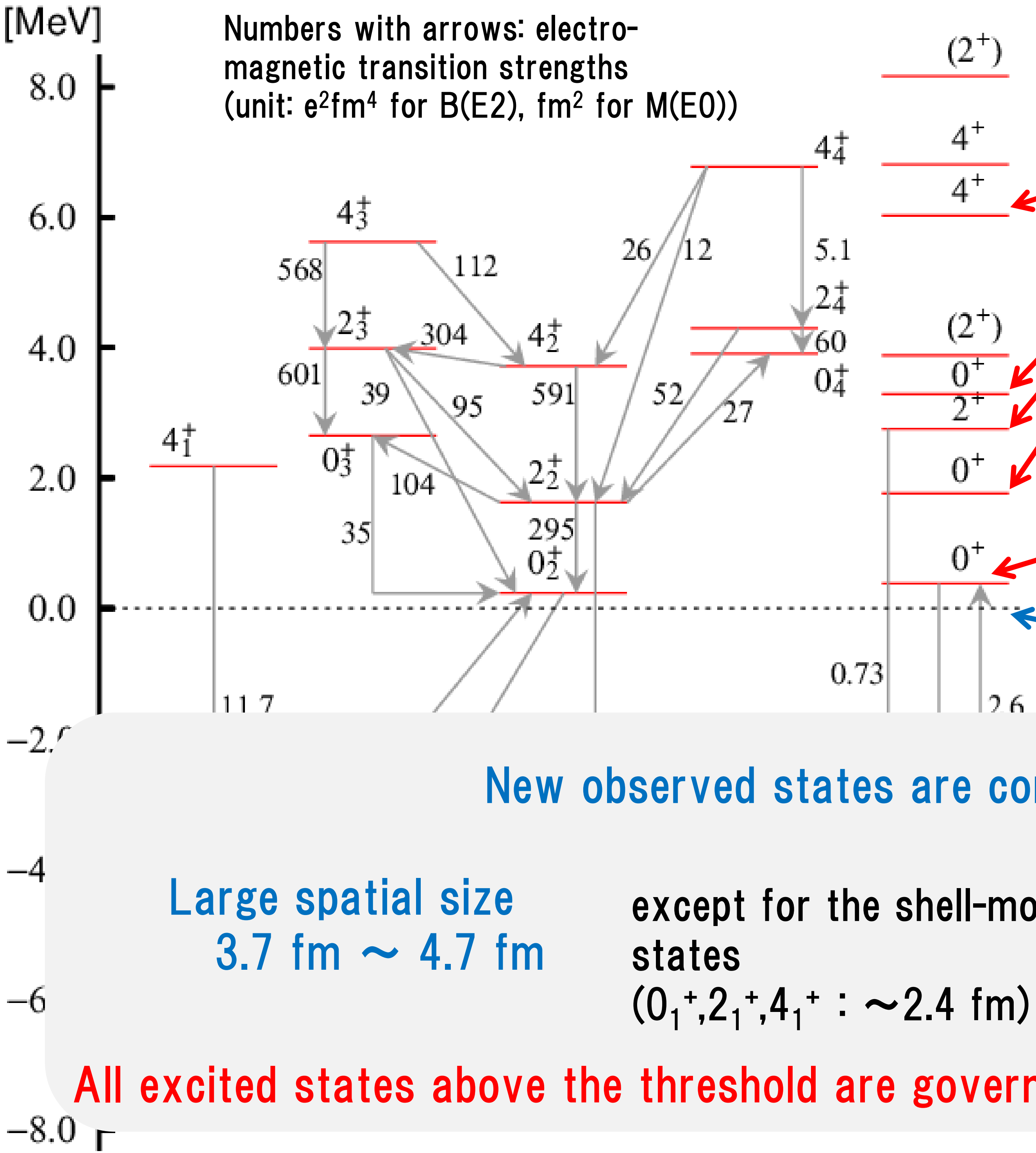
Numbers with arrows: electro-magnetic transition strengths (unit: e^2fm^4 for B(E2), fm^2 for M(E0))



New states recently observed

Decay widths (unit: MeV)

$\Gamma(0_3^+) = 1.1$ (Exp: 1.45)

$\Gamma(0_4^+) = 0.6$ (Exp: 1.42)

$\Gamma(2_2^+) = 1.1$ (Exp: 1.01)

$\Gamma(4_2^+) = 2.4$ (Exp: 1.7)

Hoyle state

3α's orbiting in an S-wave

3-α breakup threshold

New observed states are consistently reproduced.

Large spatial size
3.7 fm ∼ 4.7 fm

except for the shell-model-like states
($0_1^+, 2_1^+, 4_1^+$: ∼2.4 fm)

All excited states above the threshold are governed by cluster dynamics

Results (of ^{12}C)

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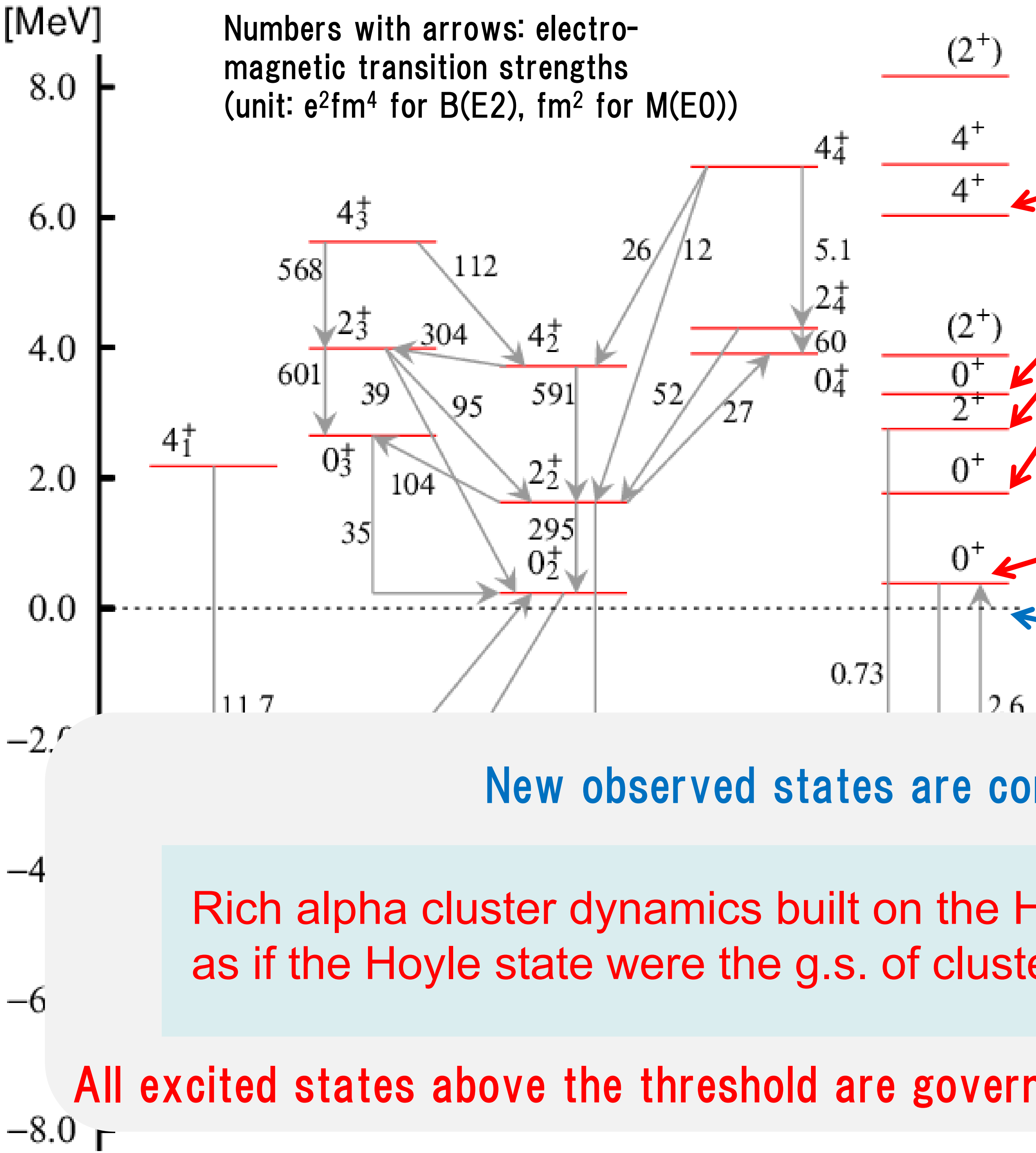

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Hoyle state

3α's orbiting in an S-wave

3-α breakup threshold

New observed states are consistently reproduced.

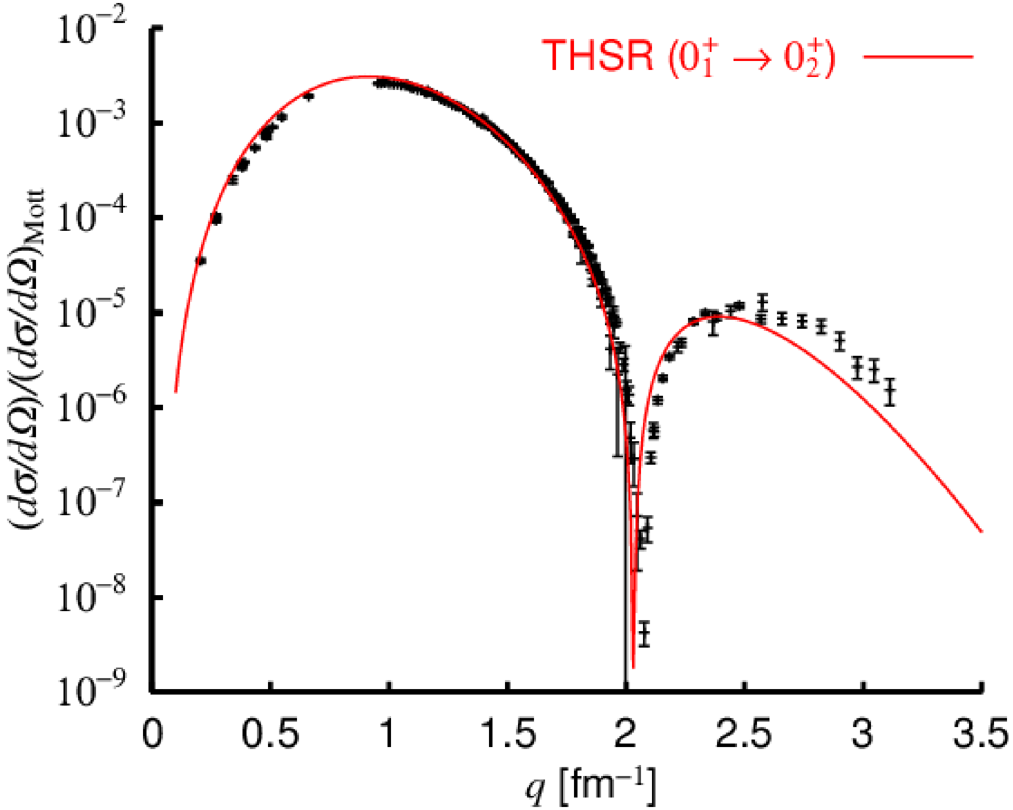
Rich alpha cluster dynamics built on the Hoyle state, as if the Hoyle state were the g.s. of cluster excitations

All excited states above the threshold are governed by cluster dynamics

Comparison with exp. data

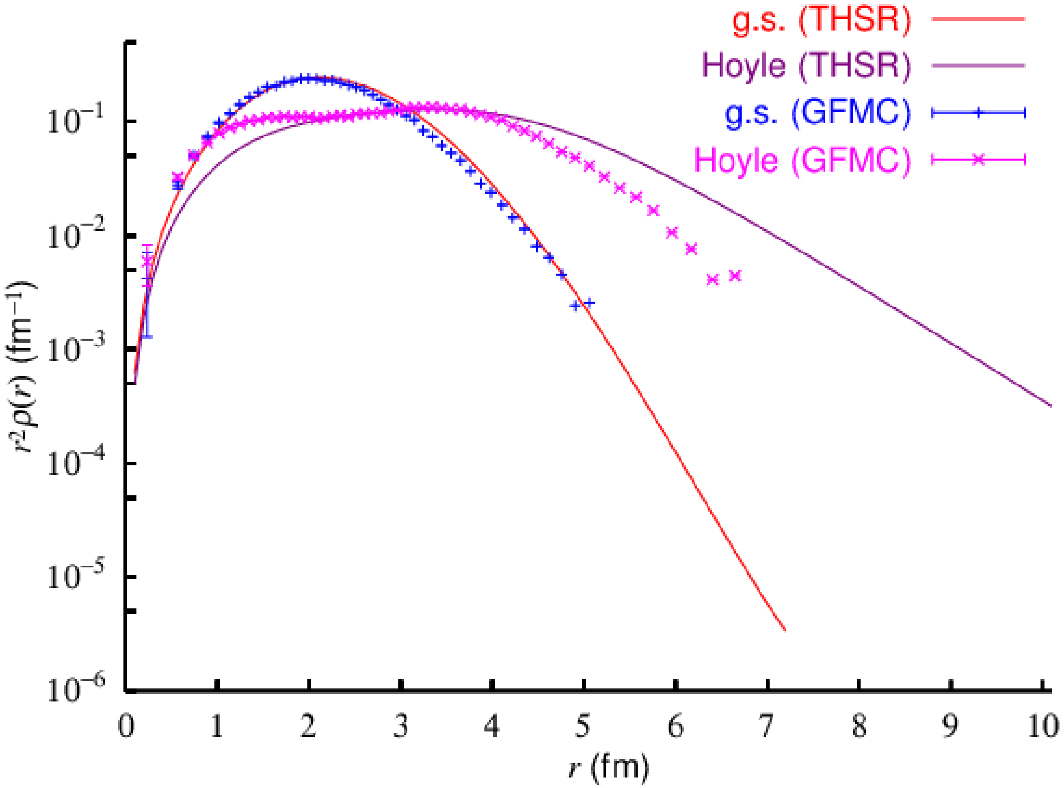
Comparison with ab-initio calc. (GFMC (data from Wringa))

Inelastic electron scattering ($0_1^+ \rightarrow 0_2^+$)



Very nicely reproduced by THSR w.f.

One-body density distribution



THSR w.f.: R_{rms}=3.7 fm

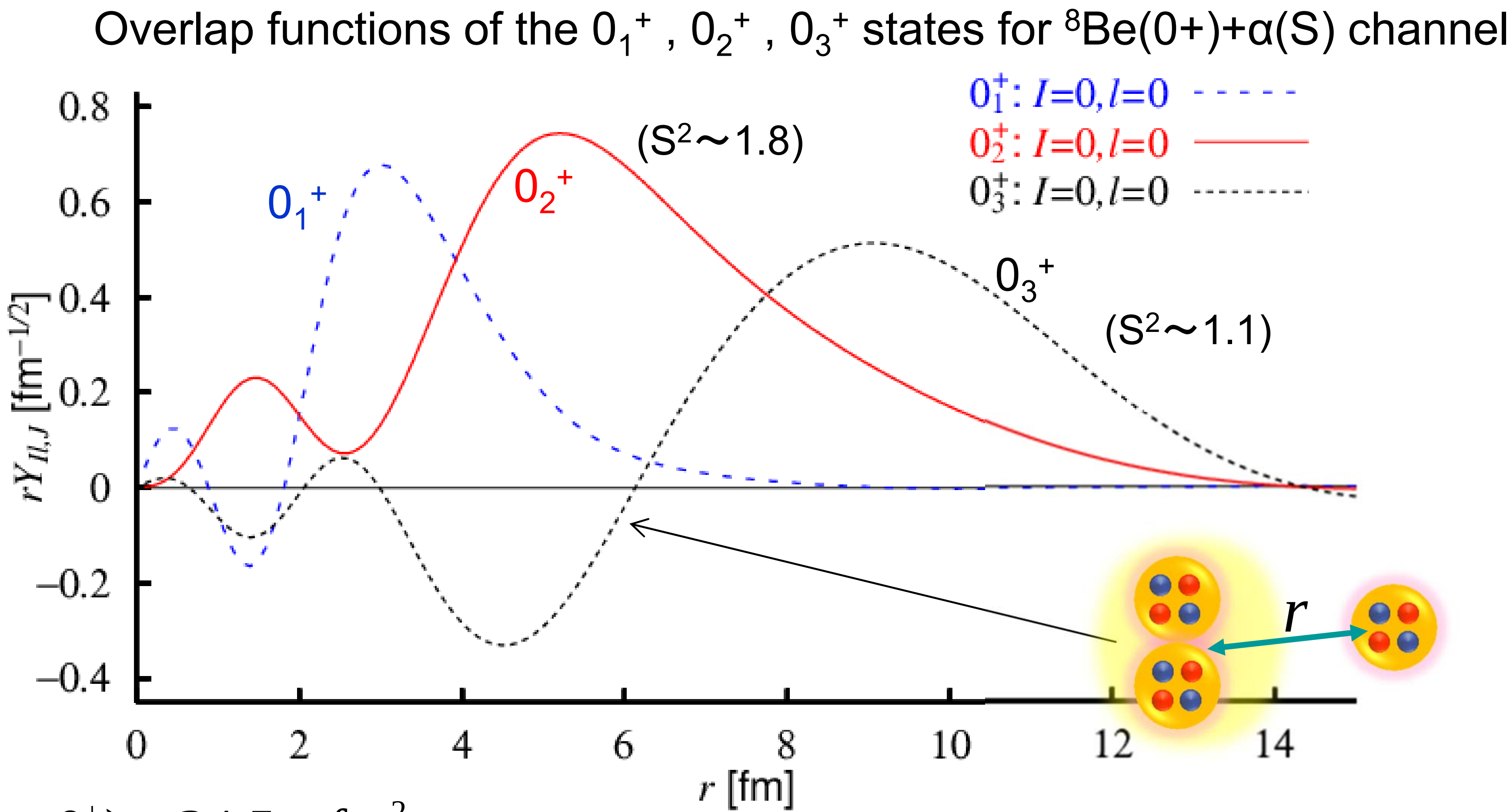
GFMC: R_{rms}=3.0 – 3.5 fm

For Hoyle state

AV18+IL7: 10.4 MeV

Exp: 7.65 MeV

0_3^+ state: higher nodal excitation of the Hoyle state



$$M(E0;\ 0_3^+ \to 0_2^+) = 34.5\ e\ \mathrm{fm}^2$$

Very large monopole transition strength
between the 0_2^+ and 0_3^+ states c.f. $M(E0;\ 0_2^+ \to 0_1^+) = 6.4\ e\ \mathrm{fm}^2$

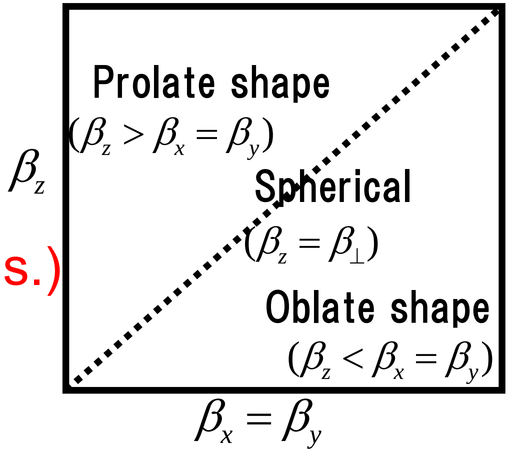
0_1^+ state: 2 nodes
0_2^+ state: 3 nodal oscillation (nodes disappear due to the dissolution of ^{8}Be core)
0_3^+ state: 4 nodes (higher nodal structure)

Squared overlap with single THSR config. for 0_2^+, 0_3^+ , 0_4^+ states of ^{12}C

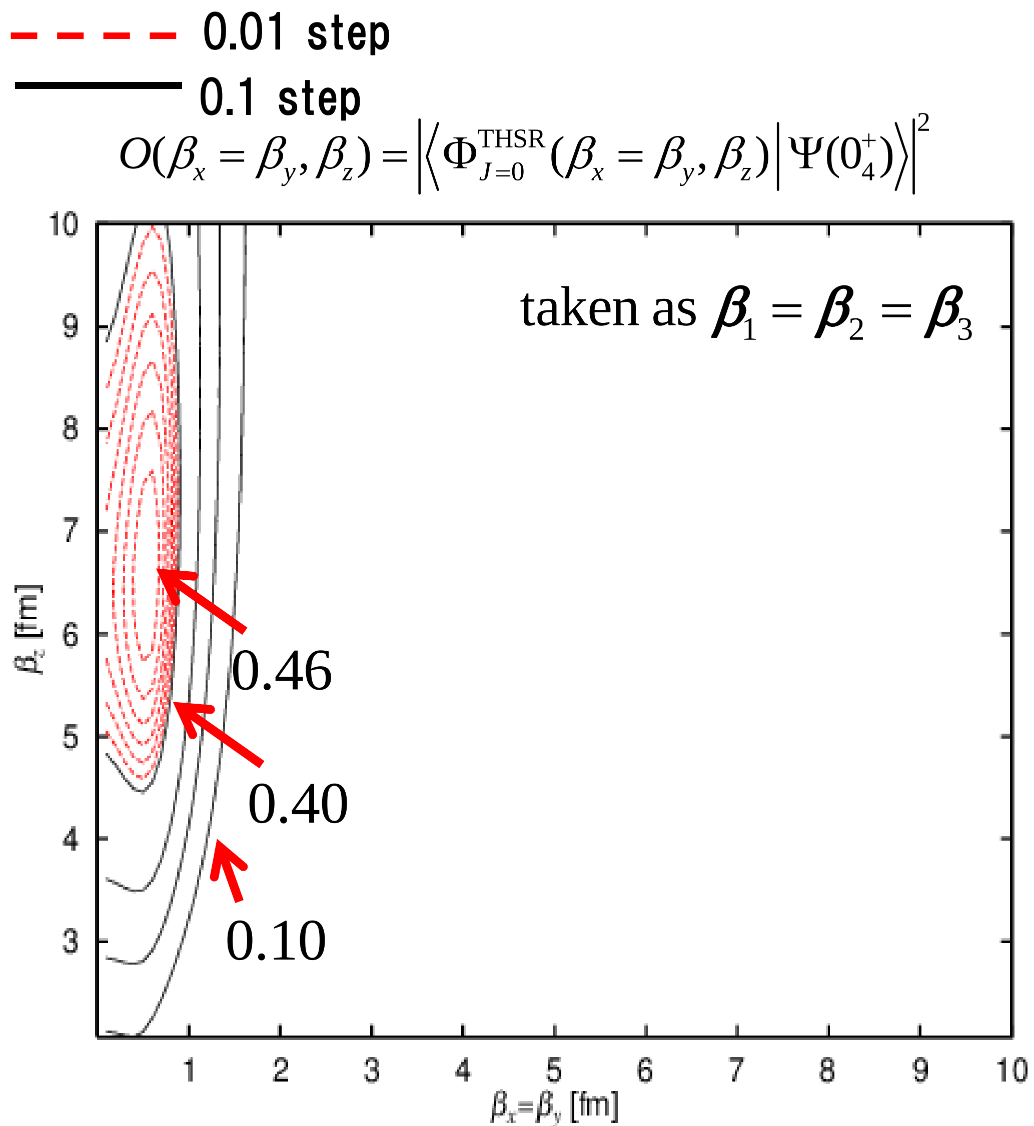
For the 0_4^+ state

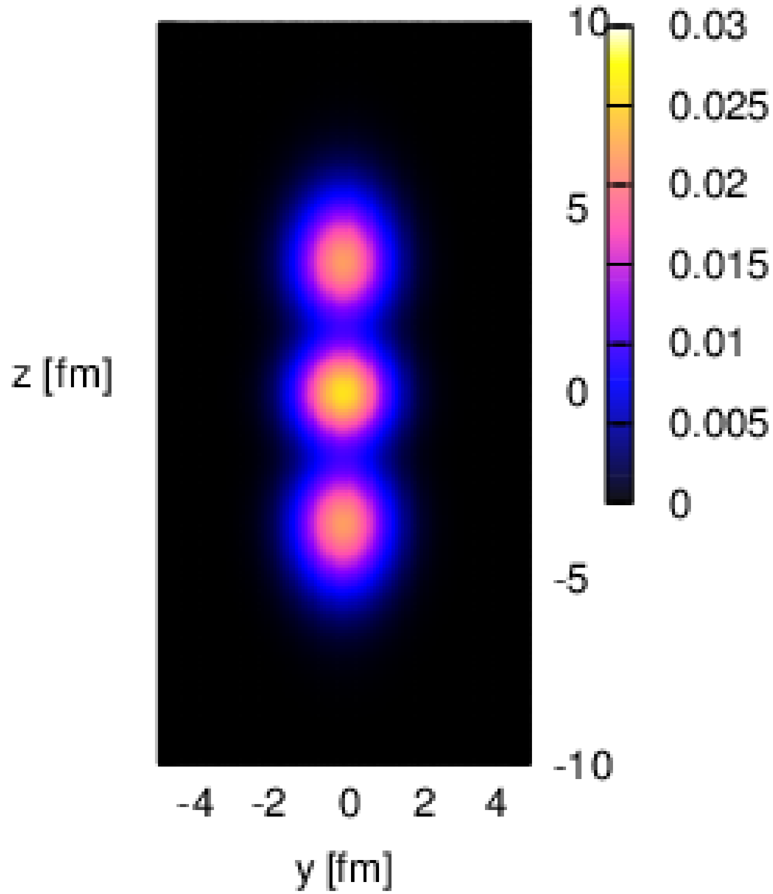
Clear linear-chain structure (l.h.s.)

Clear linear-chain structure + other config.(benging motion?) (r.h.s.)



$$O(\beta_x = \beta_y, \beta_z) = \left| \left\langle \Phi_{J=0}^{\rm THSR}(\beta_x = \beta_y, \beta_z) \middle| \Psi(0_4^+) \right\rangle \right|^2$$





Y. F., PRC **94**, 024344 (2016)

Hoyle state (0_2^+): coherent gas of 3 alphas

0_3^+: higher nodal state of ^{8}Be+α (family of the Hoyle)
strongly connected by monopole transition with Hoyle

0_4^+: dominantly linear-chain of 3 alphas

What happens when Λ is added ??

New thresholds appear below the 3α threshold.
(new species of clusters)

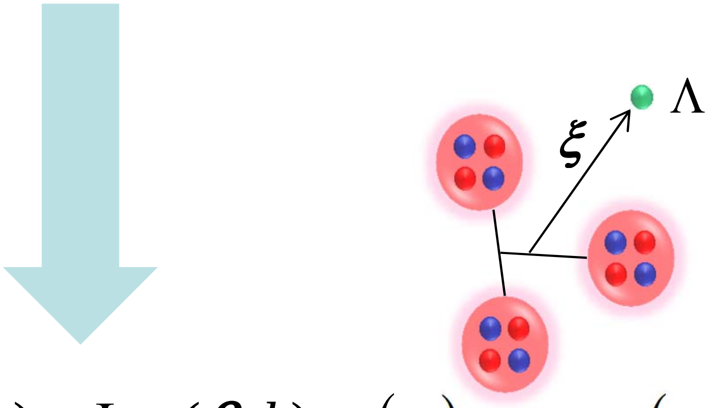
$$B_\Lambda({}^5_\Lambda\mathrm{He}) = 3.12\ \mathrm{MeV}$$
$$B_\Lambda({}^9_\Lambda\mathrm{Be}) = 6.71\ \mathrm{MeV}$$

First study of multi-cluster + Λ dynamics.
(3α+Λ in ^{12}C is the typical example)

Λ does not disturb the core structure.
(out of antisymmetrization of nucleons)

Hyper-Ext.-``THSR'' w.f. and the way of calculating energy spectra for Λ hypernuclei

$$\Phi_{^{12}\mathrm{C}}(\boldsymbol{\beta},b)=\Psi_G^{-1}\left(\frac{\boldsymbol{r}_1+\cdots+\boldsymbol{r}_{12}}{12}\right)\mathcal{A}\left\{\Phi_\alpha^{\mathrm{con.}}(\boldsymbol{\beta}_1,b)\Phi_\alpha^{\mathrm{con.}}(\boldsymbol{\beta}_2,b)\Phi_\alpha^{\mathrm{con.}}(\boldsymbol{\beta}_3,b)\right\}$$



$$\Phi_{^{12}\mathrm{C}}^{\mathrm{H\text{-}THSR}}(\boldsymbol{\beta},\boldsymbol{\kappa},b)=\Phi_{^{12}\mathrm{C}}(\boldsymbol{\beta},b)\varphi_\Lambda(\boldsymbol{\kappa})\qquad \varphi_\Lambda(\boldsymbol{\kappa})=\exp\left(-\mu_\Lambda\sum_{k=x,y,z}\frac{\xi^2}{2b^2+\kappa_k^2}\right)$$

Hill-Wheeler eq. or GCM(generator coordinate method)

$$\sum_{\boldsymbol{\beta}'',\boldsymbol{\kappa}'}\left\langle \hat{P}_{MK}^J\Phi_{^{12}\mathrm{C}}^{\mathrm{H\text{-}THSR}}(\boldsymbol{\beta},\boldsymbol{\kappa},b)\middle|\hat{H}-E\middle|\hat{P}_{MK}^J\Phi_{^{12}\mathrm{C}}^{\mathrm{H\text{-}THSR}}(\boldsymbol{\beta}',\boldsymbol{\kappa}',b)\right\rangle f(\boldsymbol{\beta}',\boldsymbol{\kappa}')=0$$

$\boldsymbol{\kappa}=(\kappa_x=\kappa_y,\kappa_z)$

With (axially symmetric) deformation

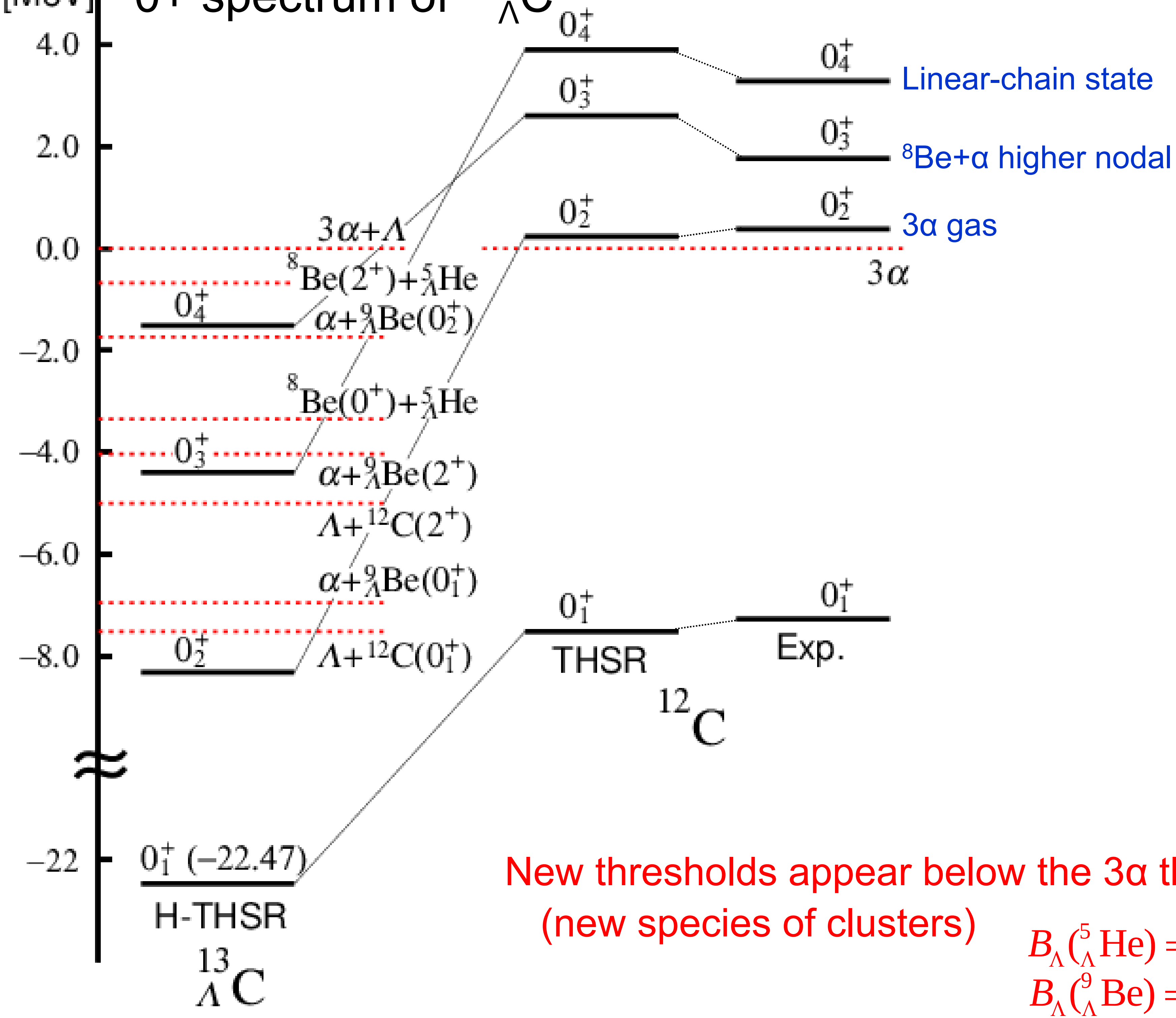
Hamiltonian (NN force: Volkov No.2 force)+ΛN interaction

ESC04a interaction

kf=1.0734

$$\hat{H}=\sum_{i=1}^{13}T_i-T_G+\sum_{i<j}^{12}(V_{ij}^{(NN)}+V_{ij}^{(C)})+\sum_{i=1}^{12}V_i^{(\Lambda N)}$$

0+ spectrum of $^{13}_{\Lambda}$C



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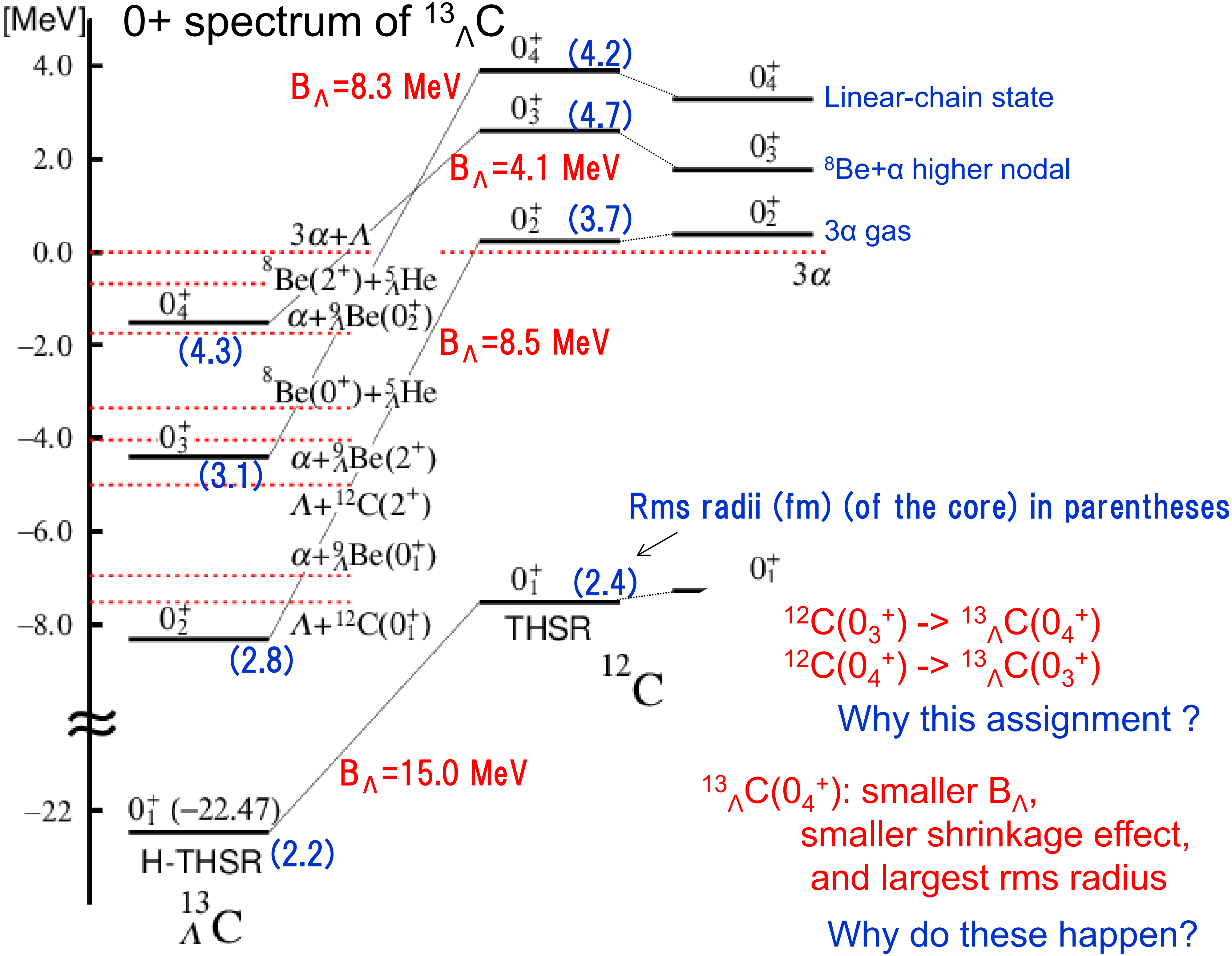
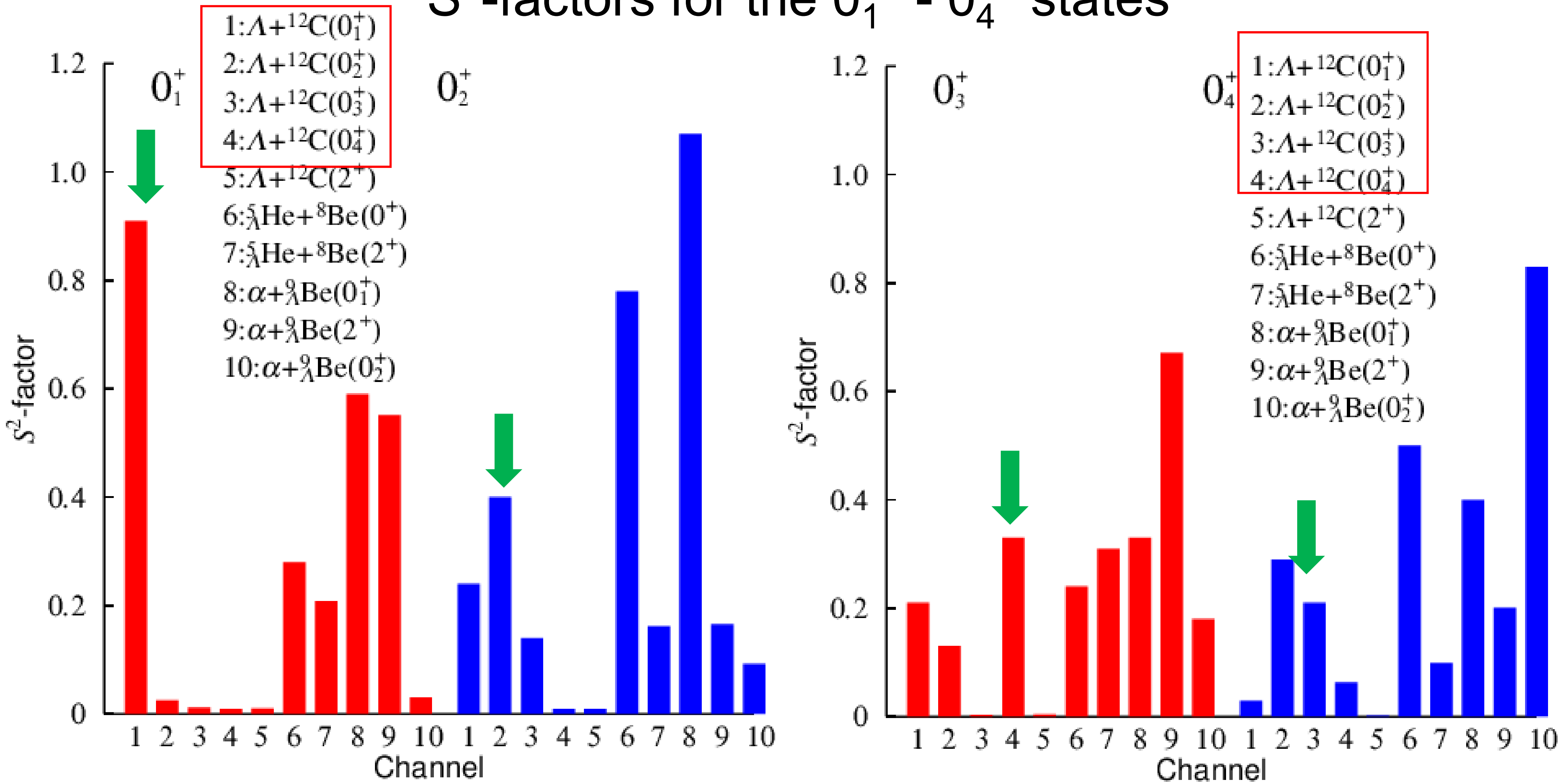


[MeV]
0+ spectrum of ¹³ΛC
4.0
2.0
0.0
−2.0
−4.0
−6.0
−8.0
−22
B_Λ=8.3 MeV
B_Λ=4.1 MeV
B_Λ=8.5 MeV
B_Λ=15.0 MeV
3α+Λ
⁸Be(2⁺)+⁵ΛHe
α+⁹ΛBe(0₂⁺)
⁸Be(0⁺)+⁵ΛHe
α+⁹ΛBe(2⁺)
Λ+¹²C(2⁺)
α+⁹ΛBe(0₁⁺)
Λ+¹²C(0₁⁺)
0₄⁺ (4.3)
0₃⁺ (3.1)
0₂⁺ (2.8)
0₁⁺ (−22.47)
H-THSR (2.2)
¹³ΛC
0₄⁺ (4.2)
0₃⁺ (4.7)
0₂⁺ (3.7)
0₁⁺ (2.4)
THSR
¹²C
0₄⁺ Linear-chain state
0₃⁺ ⁸Be+α higher nodal
0₂⁺ 3α gas
3α
0₁⁺
Rms radii (fm) (of the core) in parentheses
¹²C(0₃⁺) -> ¹³ΛC(0₄⁺)
¹²C(0₄⁺) -> ¹³ΛC(0₃⁺)
Why this assignment ?
¹³ΛC(0₄⁺): smaller B_Λ,
smaller shrinkage effect,
and largest rms radius
Why do these happen?

Correspondence between spectra in ^{12}C and $^{13}{}_{\Lambda}C$

$^{13}{}_{\Lambda}C(0_1^+)$: $^{12}C(0_1^+)$ +Λ
$^{13}{}_{\Lambda}C(0_2^+)$: $^{12}C(0_2^+)$ +Λ
$^{13}{}_{\Lambda}C(0_3^+)$: $^{12}C(0_4^+)$ +Λ
$^{13}{}_{\Lambda}C(0_4^+)$: $^{12}C(0_3^+)$ +Λ, $^{12}C(0_2^+)$ +Λ

S^2-factors for the 0_1^+ - 0_4^+ states



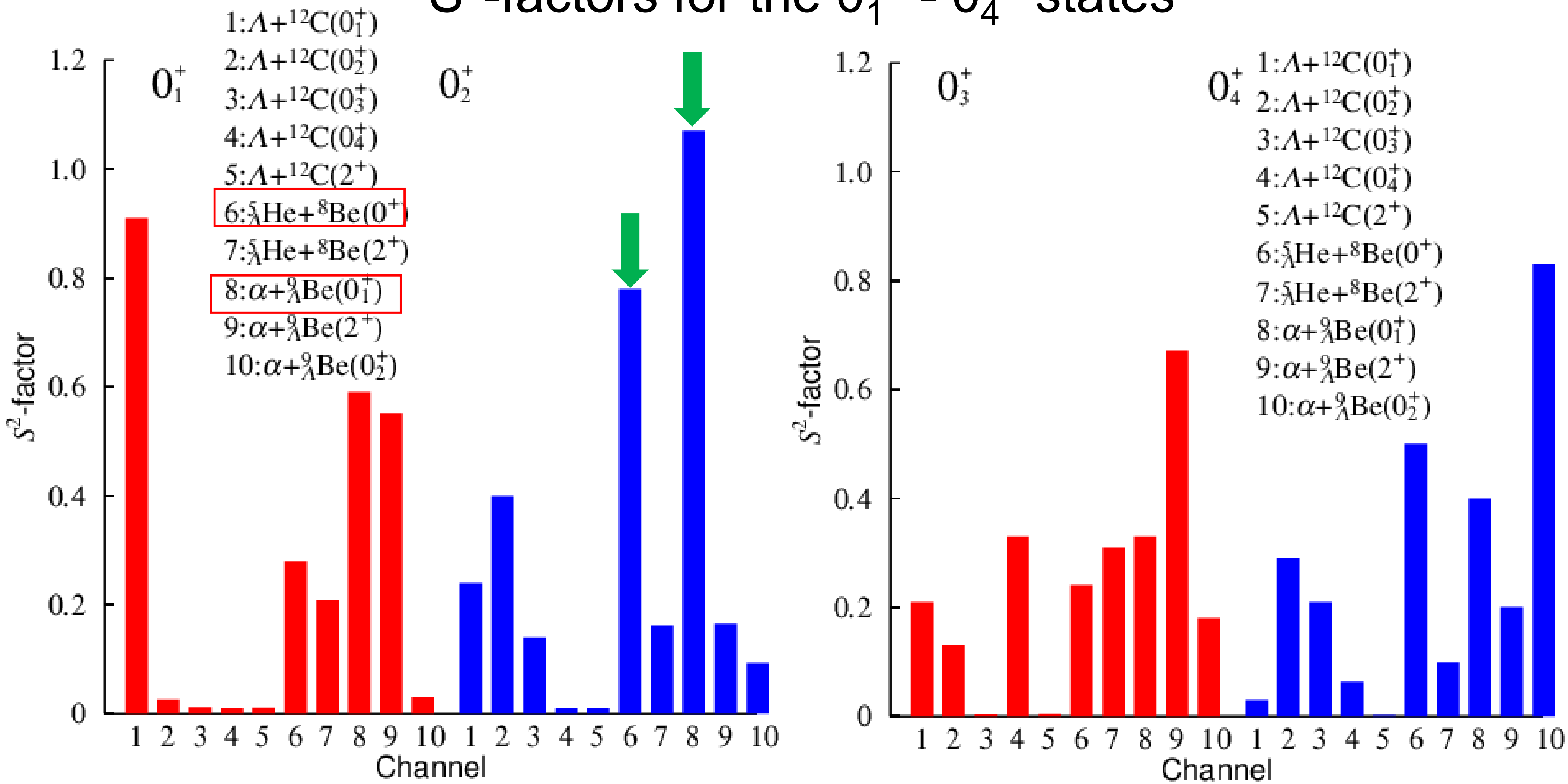
Why differences of B_Λ and amount of shrinkage happen.

$^{12}C(0_2^+)$ R=3.7 fm $\rightarrow$ $^{13}{}_\Lambda C(0_2^+)$ R=2.8 fm $(3.7/2.8)^3=2.4$ $B_\Lambda=8.5$ MeV

$^{12}C(0_3^+)$ R=4.7 fm $\rightarrow$ $^{13}{}_\Lambda C(0_4^+)$ R=4.3 fm $(4.7/4.3)^3=1.3$ $B_\Lambda=4.1$ MeV

$^{12}C(0_4^+)$ R=4.2 fm $\rightarrow$ $^{13}{}_\Lambda C(0_3^+)$ R=3.1 fm $(4.2/3.1)^3=2.5$ $B_\Lambda=8.3$ MeV

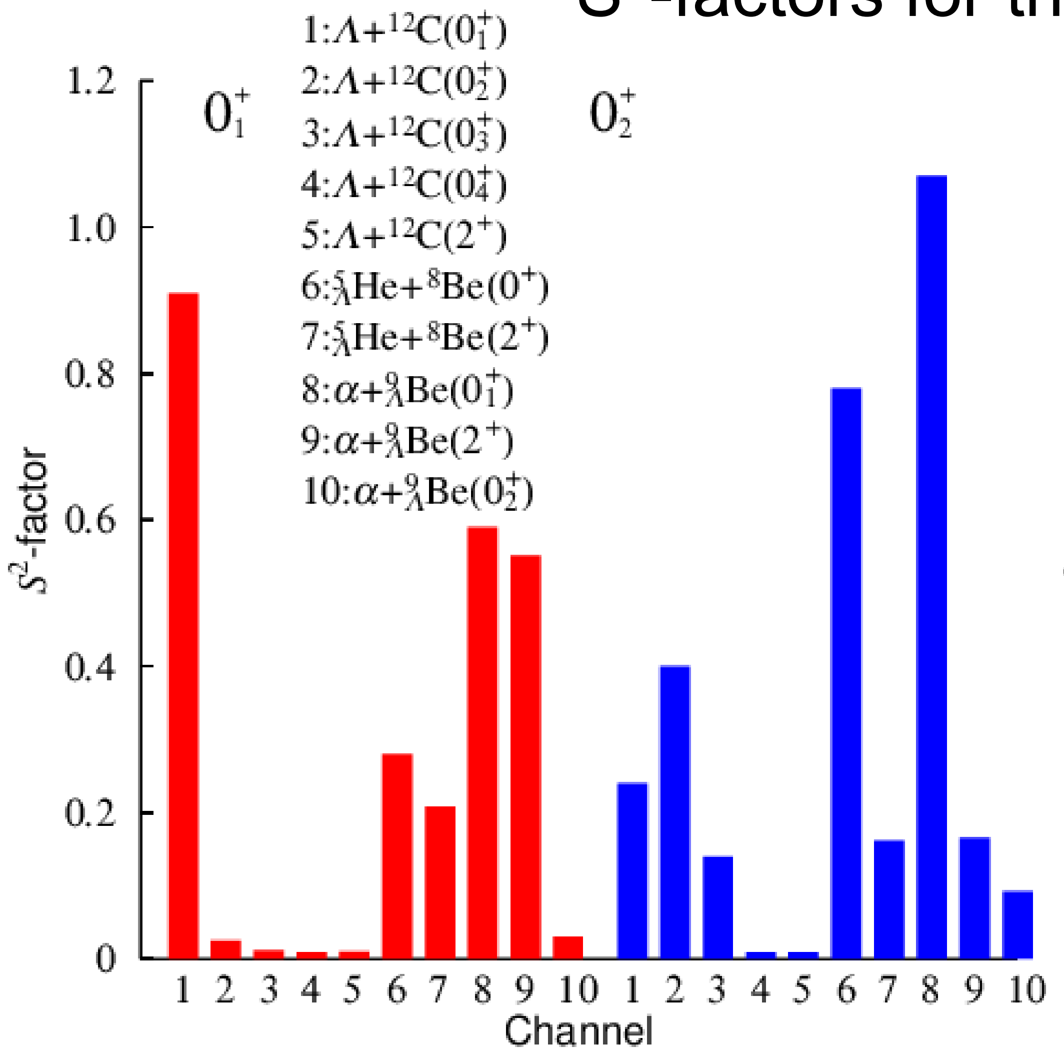
S^2-factors for the 0_1^+ - 0_4^+ states

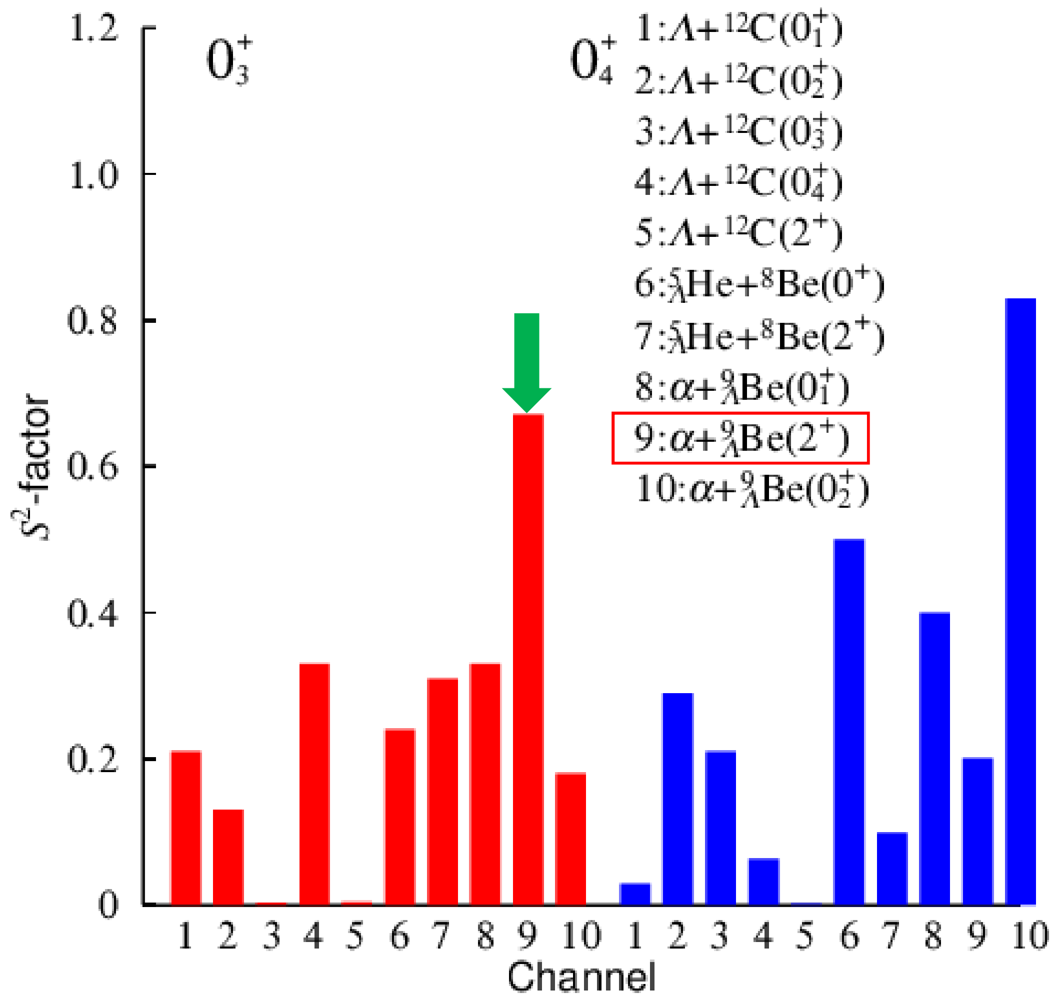


Why differences of B_Λ and amount of shrinkage happen.

$^{12}C(0_2^+)$	R=3.7 fm	$^{13}_{\Lambda}C(0_2^+)$	R=2.8 fm	$(3.7/2.8)^3=2.4$	B_Λ=8.5 MeV
$^{12}C(0_3^+)$	R=4.7 fm	$^{13}_{\Lambda}C(0_4^+)$	R=4.3 fm	$(4.7/4.3)^3=1.3$	B_Λ=4.1 MeV
$^{12}C(0_4^+)$	R=4.2 fm	$^{13}_{\Lambda}C(0_3^+)$	R=3.1 fm	$(4.2/3.1)^3=2.5$	B_Λ=8.3 MeV

S^2-factors for the 0_1^+ - 0_4^+ states





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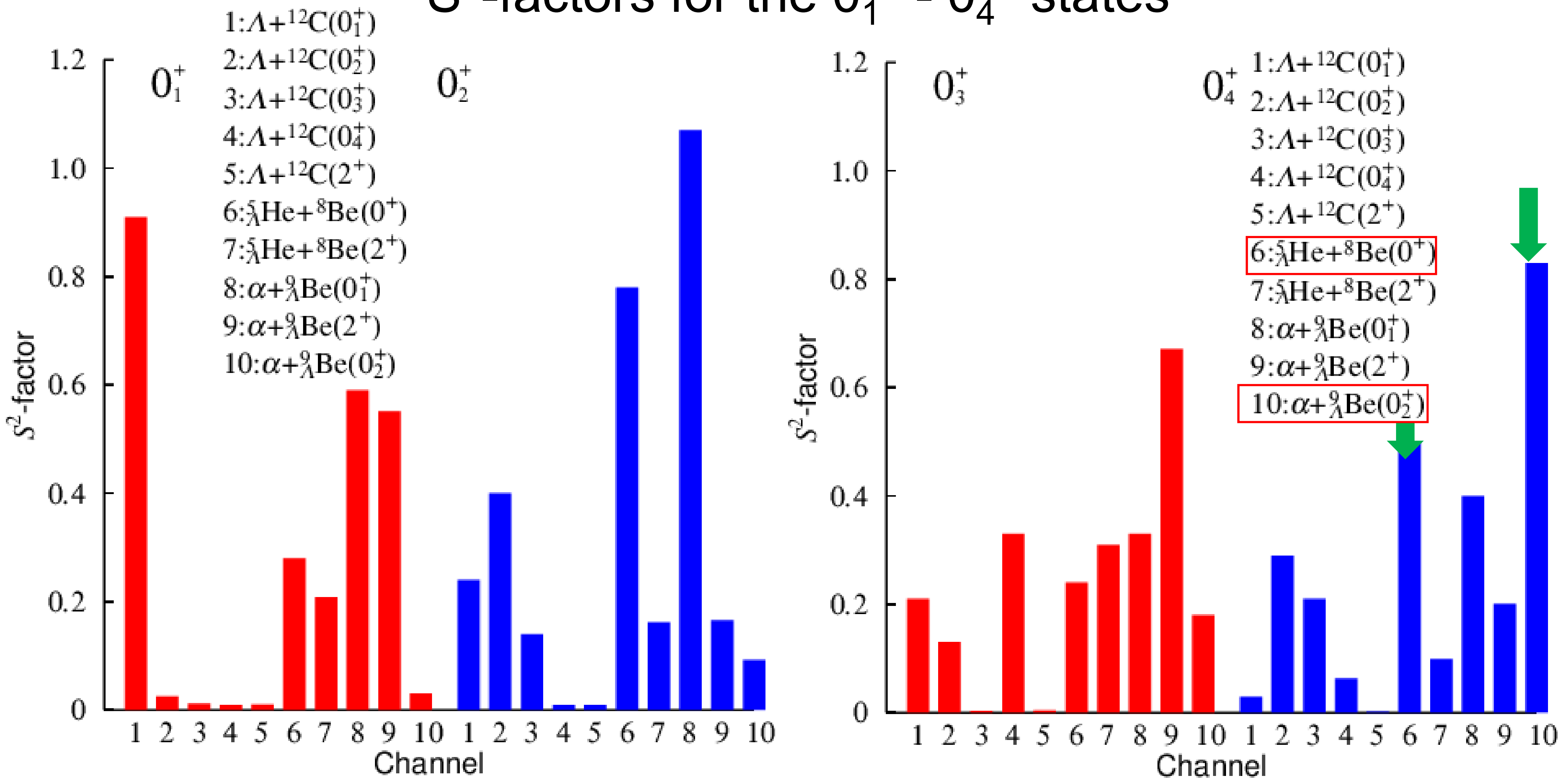
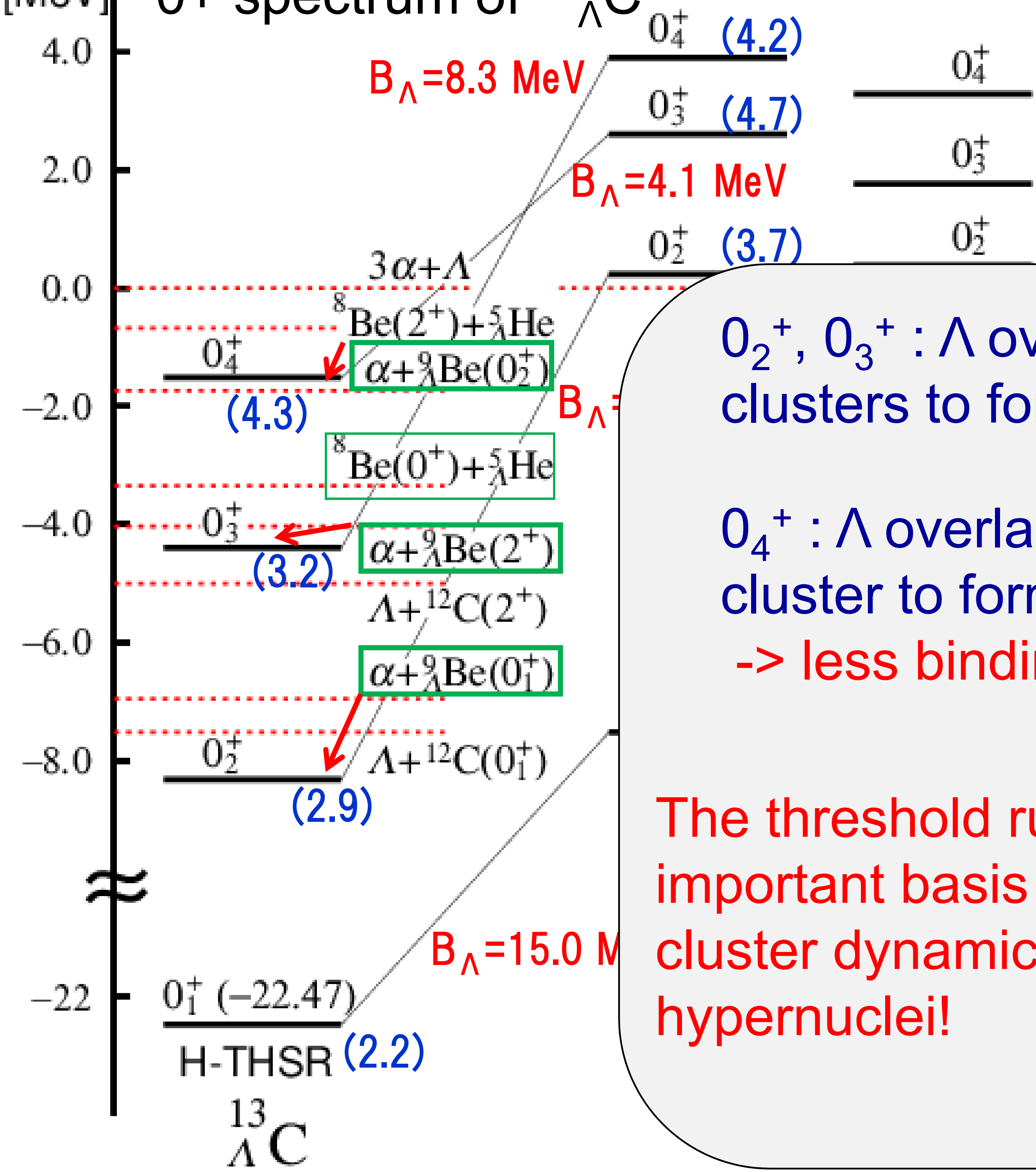
$^{12}C(0_3^+)$ R=4.7 fm $^{13}{}_\Lambda C(0_4^+)$ R=4.3 fm $(4.7/4.3)^3=1.3$ $B_\Lambda=4.1$ MeV

$^{12}C(0_4^+)$ R=4.2 fm $^{13}{}_\Lambda C(0_3^+)$ R=3.1 fm $(4.2/3.1)^3=2.5$ $B_\Lambda=8.3$ MeV

S^2-factors for the 0_1^+ - 0_4^+ states



0+ spectrum of $^{13}_{\Lambda}C$



$^{9}_{\Lambda}Be(0_2^+)$: $^{5}_{\Lambda}He+\alpha$ quasi-stable state

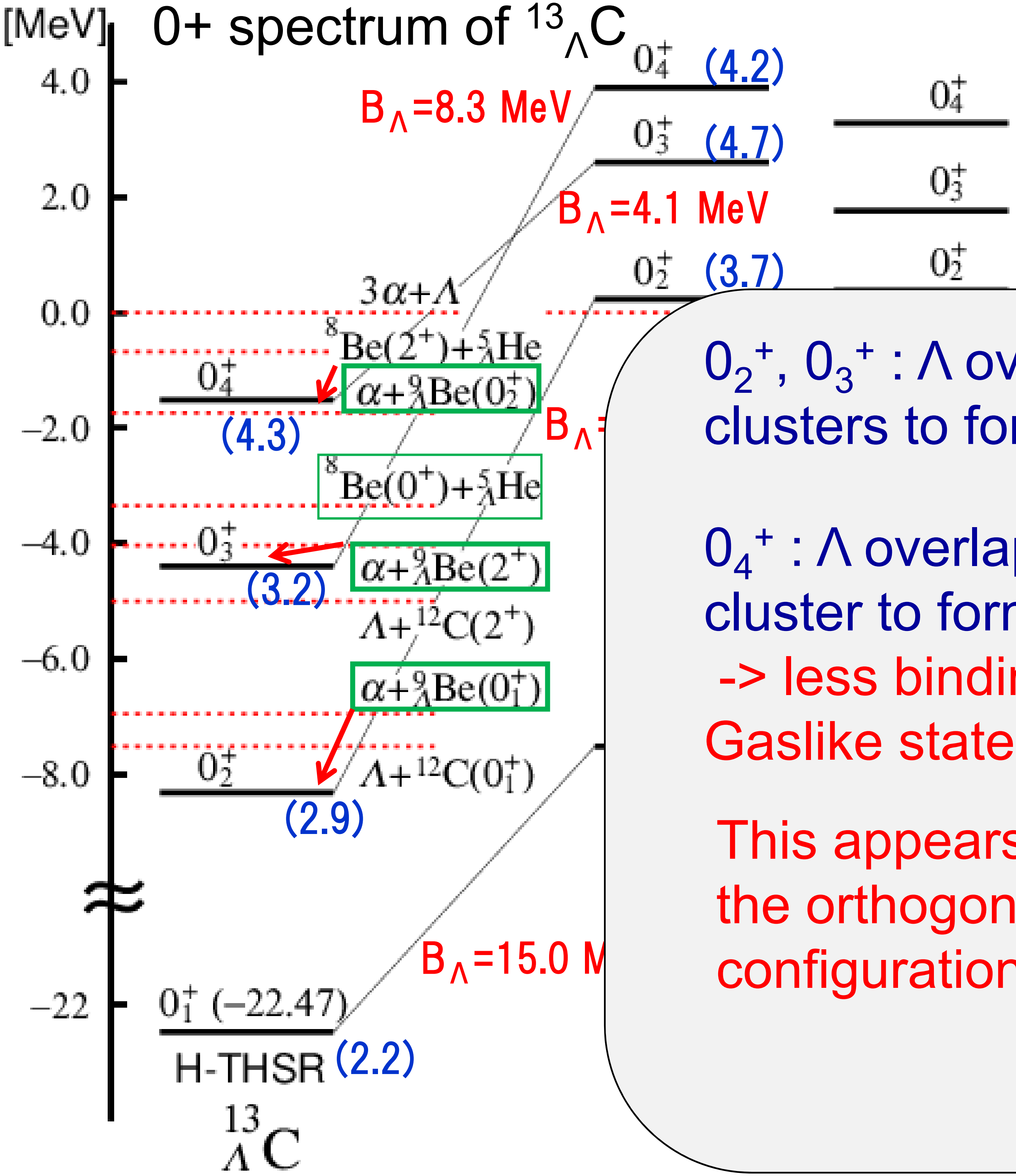
0_2^+, 0_3^+ : Λ overlaps with 2α clusters to form $^{9}_{\Lambda}Be$

0_4^+ : Λ overlaps with only an α cluster to form $^{5}_{\Lambda}He+\alpha$

-> less binding and shrinkage

The threshold rule is still an important basis to understand cluster dynamics even in hypernuclei!

0+ spectrum of $^{13}{}_{\Lambda}C$



$^{9}{}_{\Lambda}Be(0_2{}^+)$: $^{5}{}_{\Lambda}He+\alpha$ quasi-stable state

$0_2{}^+$, $0_3{}^+$: Λ overlaps with 2α clusters to form $^{9}{}_{\Lambda}Be$

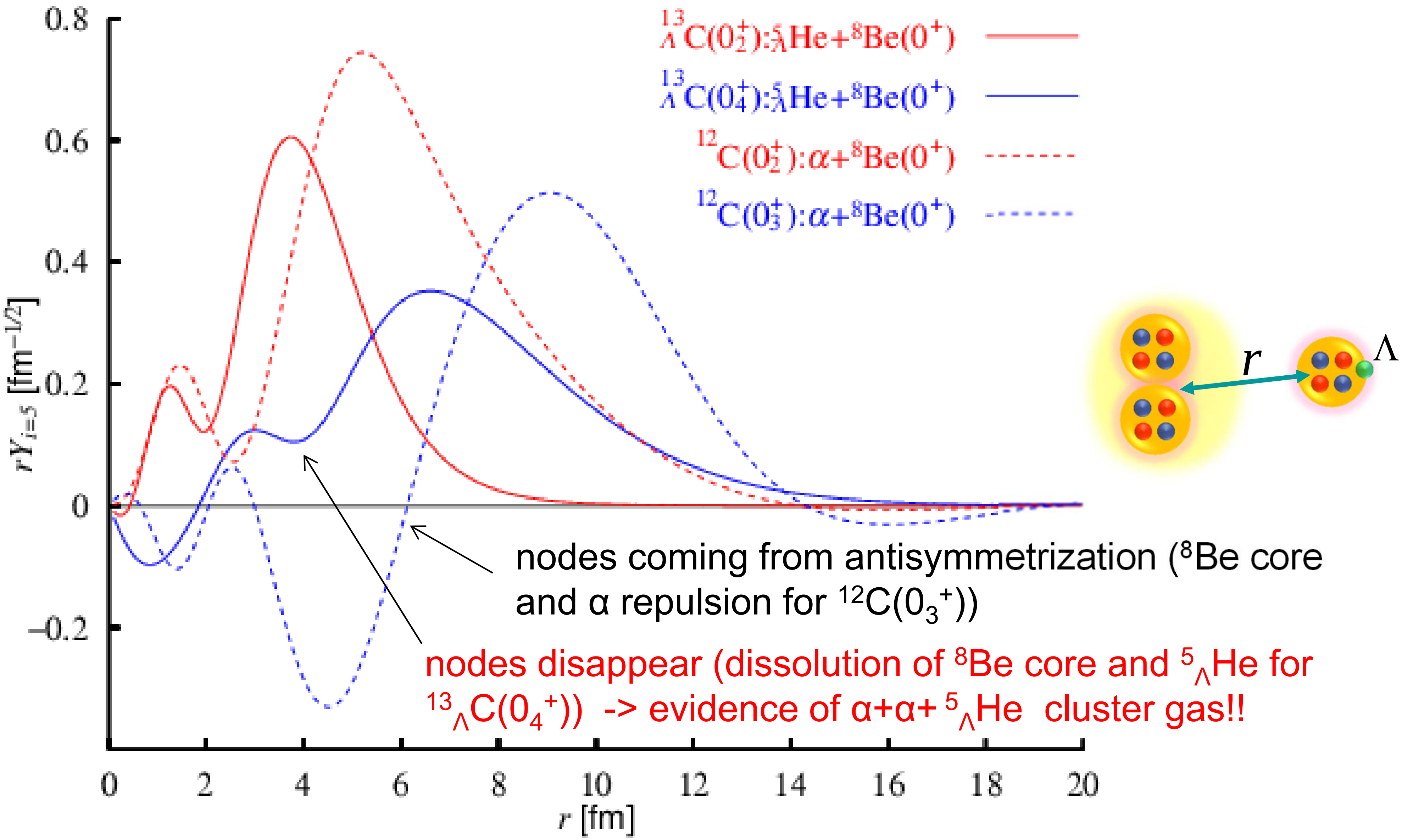
$0_4{}^+$: Λ overlaps with only an α cluster to form $^{5}{}_{\Lambda}He+\alpha$

-> less binding and shrinkage

Gaslike state of $\alpha+\alpha+{}^{5}{}_{\Lambda}He$

This appears because of the orthogonality to the lower configurations.

Reduced width amplitudes (overlap amplitude) of $^{8}Be+^{5}_{\Lambda}He$ channel ($^{13}_{\Lambda}C(0_2^+, 0_4^+)$) and $^{8}Be+^{4}He$ channel ($^{12}C(0_2^+, 0_3^+)$)

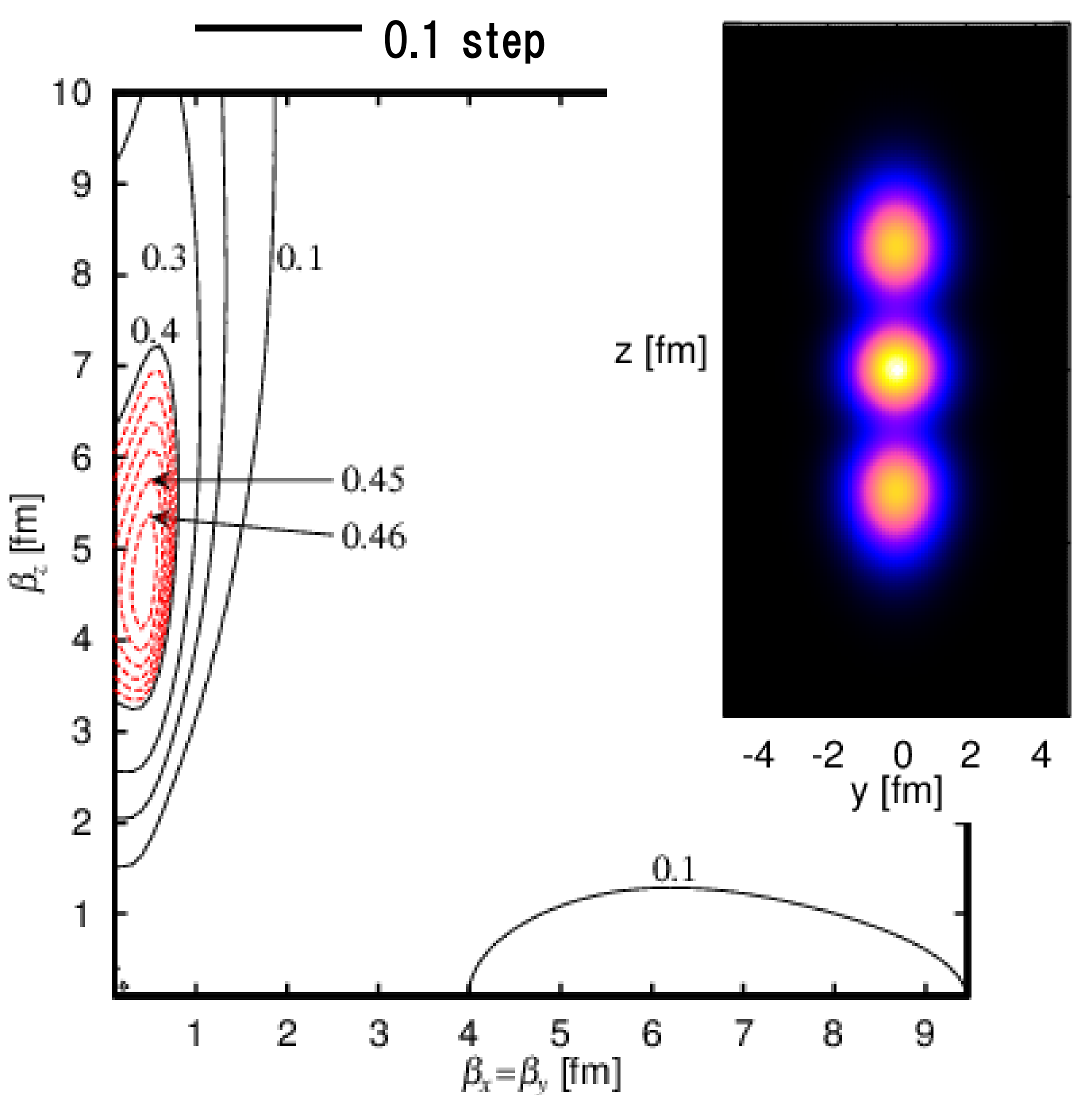


Overlap surface between the 0_3^+ state and single config. of H-THSR

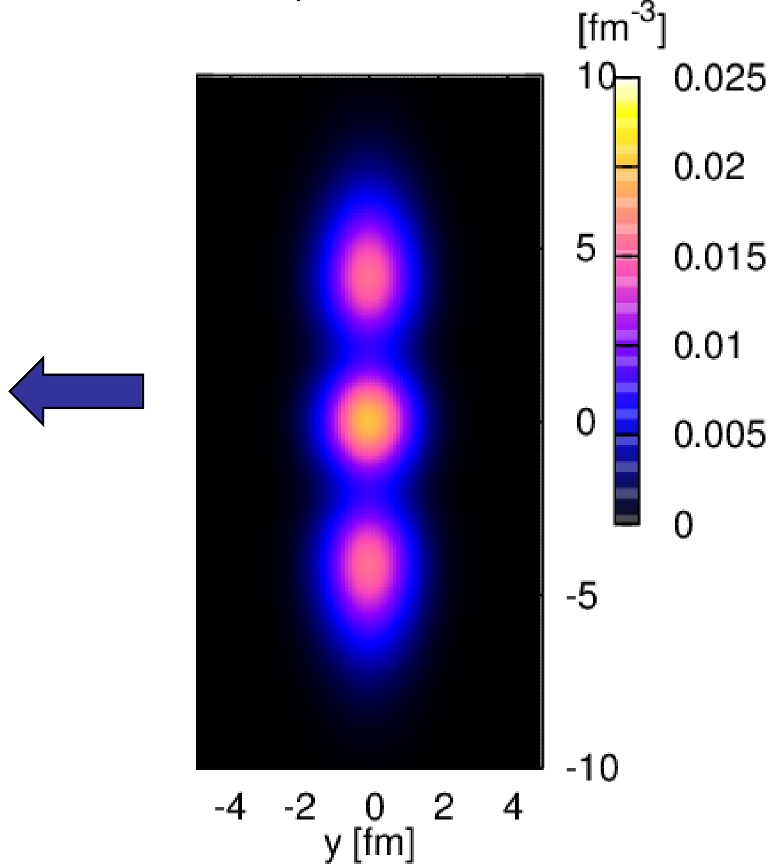
$$O(\beta_\perp,\beta_z,\kappa_\perp,\kappa_z)=\left|\sum_{\beta'_\perp,\beta'_z,\kappa'_\perp,\kappa'_z}\left\langle\Phi_{J=0}^{\text{H-THSR}}(\beta_\perp,\beta_z,\kappa_\perp,\kappa_z)\middle|\Phi_{J=0}^{\text{H-THSR}}(\beta'_\perp,\beta'_z,\kappa_\perp,\kappa_z)\right\rangle f_\lambda(\beta'_\perp,\beta'_z,\kappa'_\perp,\kappa'_z)\right|^2$$

0_3^+

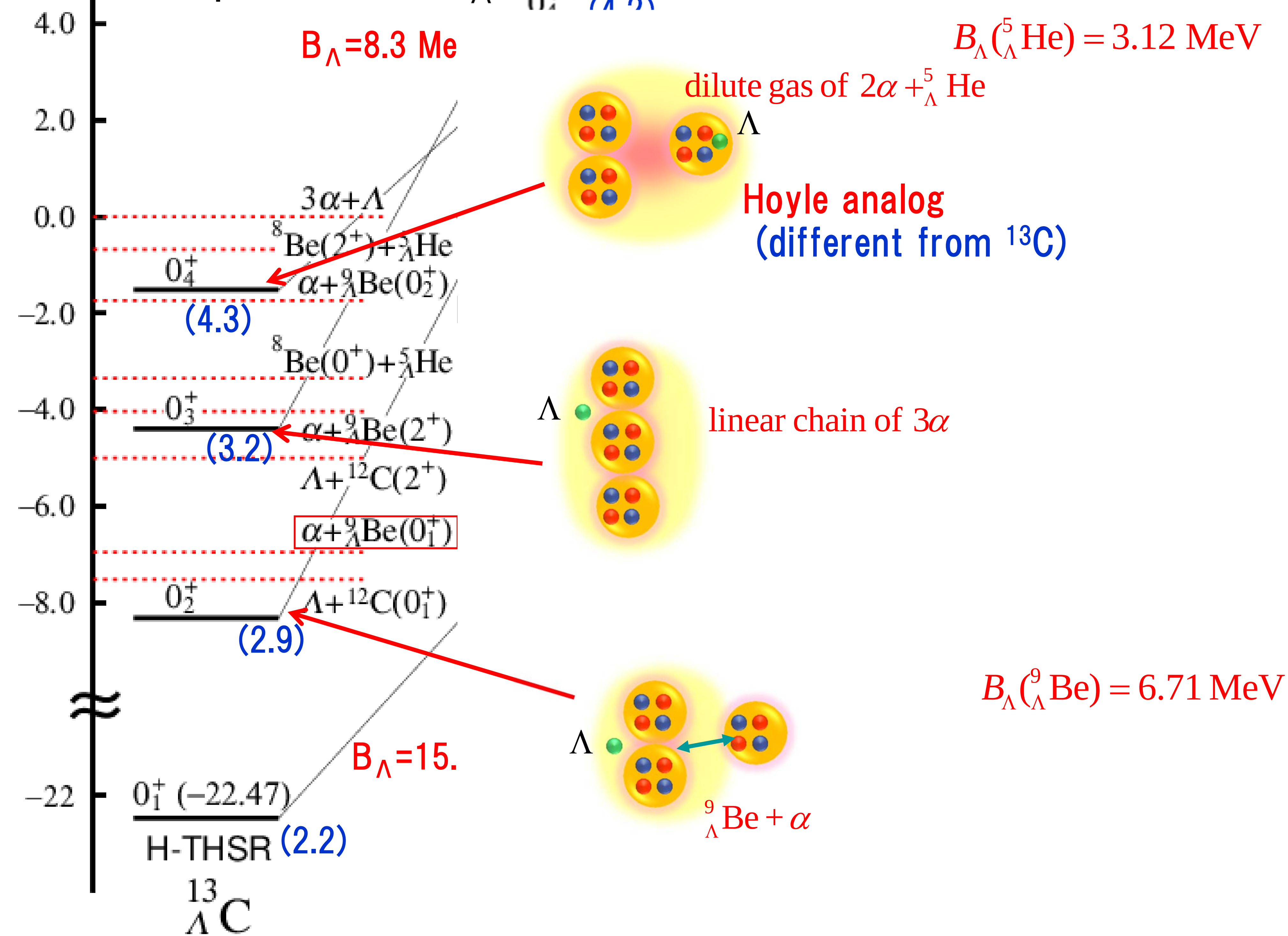
Rrms=3.1 fm



$^{12}C(0_4^+)$ Rrms=4.2 fm



0+ spectrum of $^{13}_{\Lambda}C$



Summary

Hoyle state (0_2^+): coherent gas of 3 alphas

0_3^+: higher nodal state of ^{8}Be+α (family of the Hoyle)
strongly connected by monopole transition with Hoyle

0_4^+: dominantly linear-chain of 3 alphas

The Hoyle + Λ gives $^9_\Lambda$Be+α structure.

^{12}C(0_3^+) + Λ gives dilute gas of 2α+$^5_\Lambda$He structure (Hoyle analog state)

⇨ Indicating that ^{12}C(0_3^+) is a family of the Hoyle state

Linear chain state survives and appears as the 0_3^+ state of $^{13}_\Lambda$C

Thank you for your attention

and thanks to my collaborators for the $^{13}{}_{\Lambda}C$ study,

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