Lambda-N and Sigma-N interactions from 2+1 lattice QCD

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for HAL QCD Collaboration
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Outline

- Introduction
  - Brief introduction of HAL QCD method
  - Importance of LN-SN tensor force for hypernuclei
- Effective block algorithm for various baryon-baryon channels,
  HN, Comput.Phys.Commun.**207**,91(2016)
  [arXiv:1510.00903[hep-lat]]
- Preliminary results of LN-SN potentials at nearly physical point [arXiv:1702.00734[hep-lat]]
  - LN-SN(I=1/2), central and tensor potentials
  - SN(I=3/2), central and tensor potentials
- Summary

Plan of research

QCD

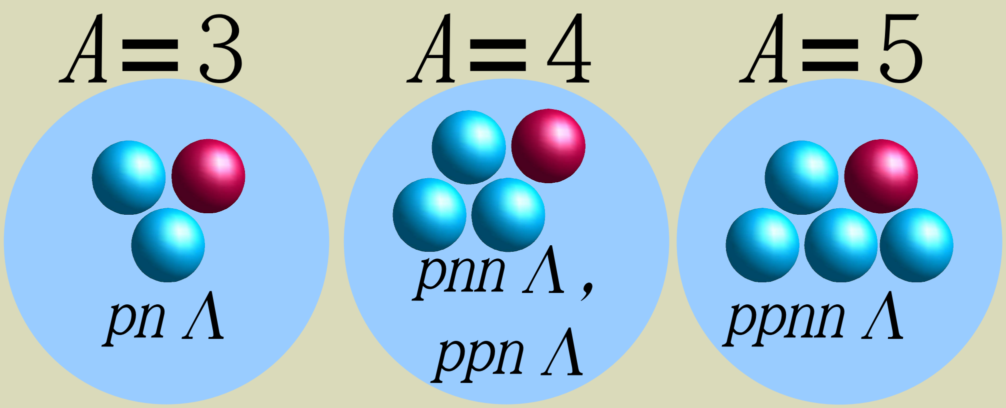
Baryon interaction

J-PARC,
JLab, GSI, MAMI, ...
YN scattering,
hypernuclei

Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

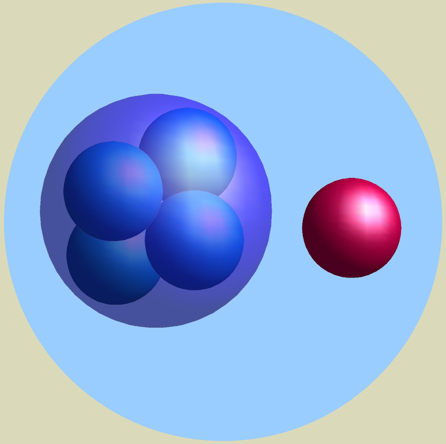
Neutron star and supernova



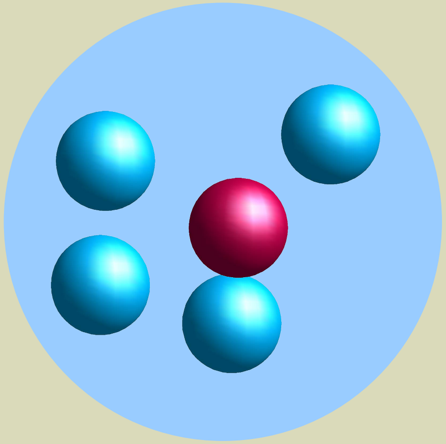
What is realistic picture of hypernuclei?

- $B(\text{total}) = B(^4\text{He}) + B_\Lambda(^5_\Lambda\text{He})$

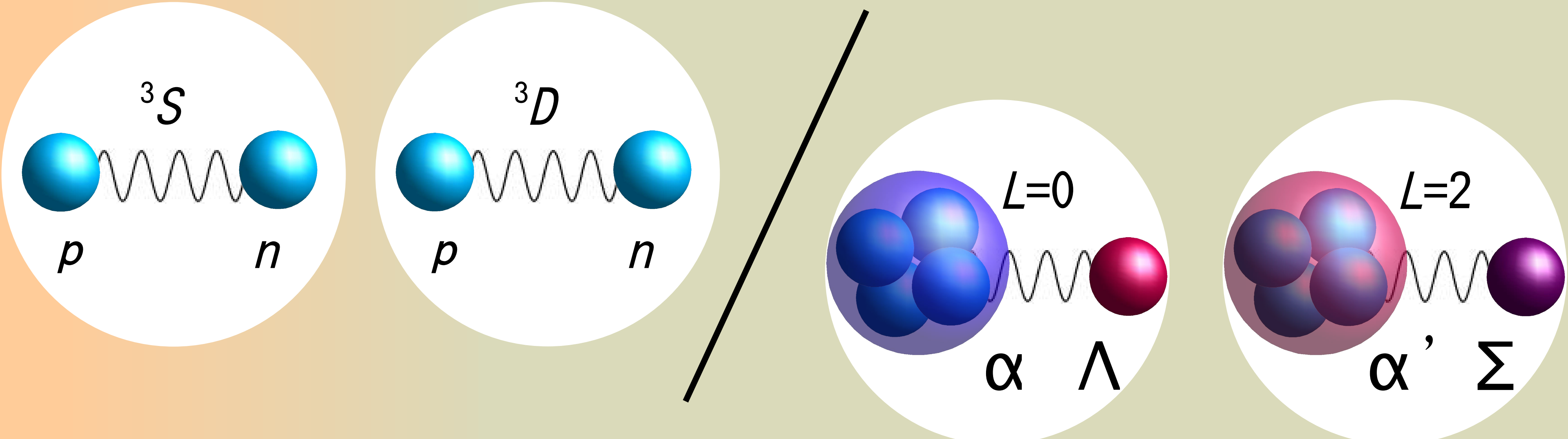
- A conventional picture:
  $B(\text{total})$
  $= B(^4\text{He}) + B_\Lambda(^5_\Lambda\text{He})$
  $= 28+3$ MeV.



- A (probably realistic) picture:
  $B(\text{total})$
  $= (B(^4\text{He}) - \Delta E_c) + (B_\Lambda(^5_\Lambda\text{He}) + \Delta E_c)$
  $= ??+??$ MeV.



Comparison between d=p+n and core+Y



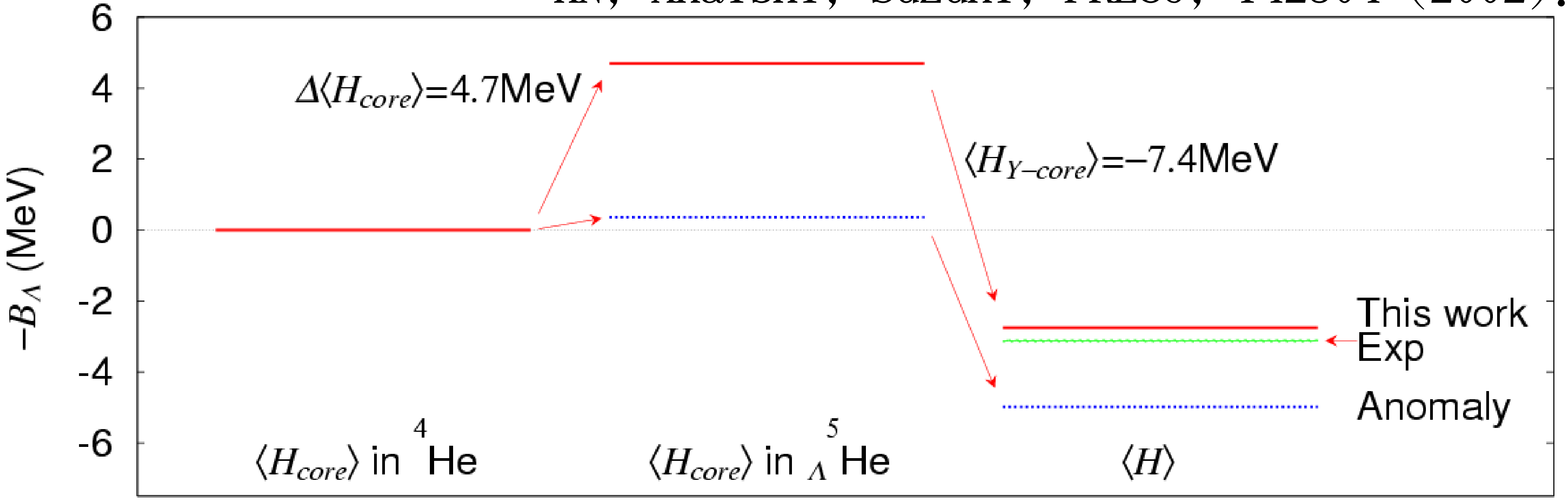
	$\langle T_S \rangle$ (MeV)	$\langle T_D \rangle$ (MeV)	$\langle V_{NN}(\text{central}) \rangle$ (MeV)	$\langle V_{NN}(\text{tensor}) \rangle$ (MeV)	$\langle V_{NN}(\text{LS}) \rangle$ (MeV)
AV8	8.57	11.31	−4.46	−16.64	−1.02
G3RS	10.84	5.64	−7.29	−11.46	0.00

	$\langle T_{Y\text{-}c} \rangle_\Lambda$	$\langle T_{Y\text{-}c} \rangle_\Sigma + \Delta \langle H_C \rangle$	$\langle V_{YN}$(のこり)$\rangle$	$2\langle V_{\Lambda N-\Sigma N}(\text{tensor}) \rangle$
$^5_\Lambda$He	9.11	3.88+4.68	−0.86	−19.51
$^4_\Lambda$H*	5.30	2.43+2.02	0.01	−10.67
$^4_\Lambda$H	7.12	2.94+2.16	−5.05	−9.22

HN, Akaishi, Suzuki, PRL89, 142504 (2002).

Rearrangement effect of ${}^{5}_{\Lambda}$He

HN, Akaishi, Suzuki, PRL89, 142504 (2002).



$$H = \sum_{i=1}^{A}\left(m_i c^2 + \frac{\boldsymbol{p}_i^2}{2\,m_i}\right) - T_{CM} + \sum_{i<j}^{A-1} v_{ij}^{(NN)} + \sum_{i=1}^{A-1} v_{iY}^{(NY)} = H_{core} + H_{Y-core} \quad ,$$

$$H_{core} = \sum_{i=1}^{A-1}\frac{\boldsymbol{p}_i^2}{2\,m_N} - \frac{\left(\sum_{i=1}^{A-1}\boldsymbol{p}_i\right)^2}{2(A-1)\,m_N} + \sum_{i<j}^{A-1} v_{ij}^{(NN)} = T_{core} + V_{NN} \quad .$$

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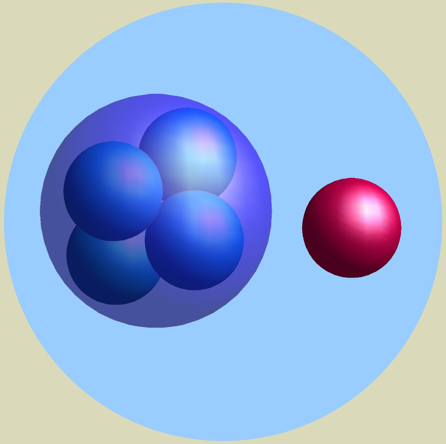

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  $B(\text{total})$
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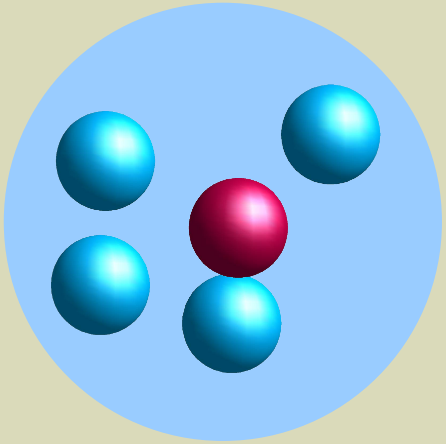
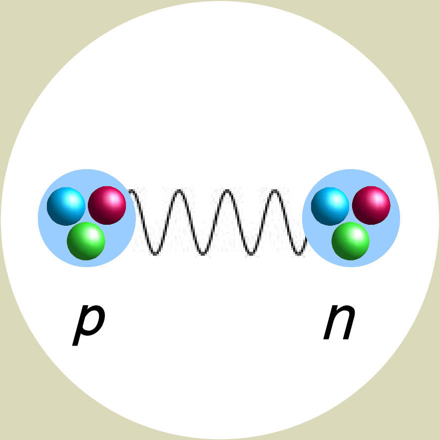
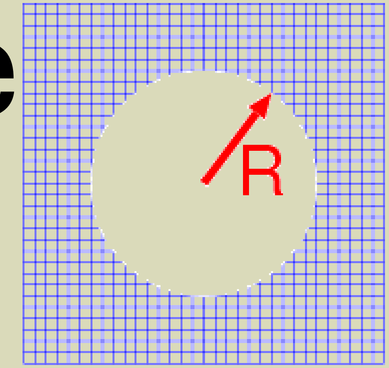
  $=$ 24+7 MeV.



Lattice QCD calculation



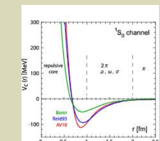
Multi-hadron on lattice



i) basic procedure:

asymptotic region

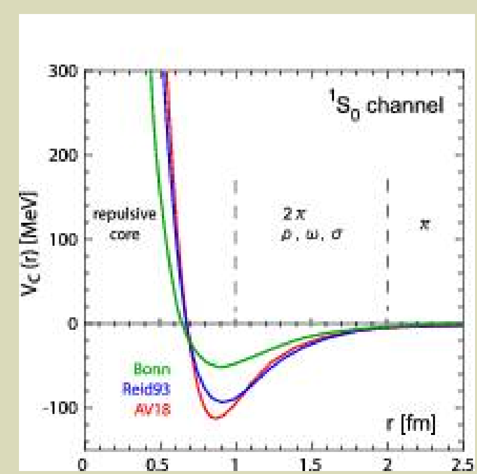
--> phase shift



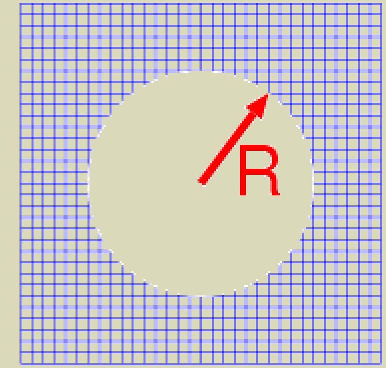
ii) HAL's procedure:

interacting region

--> potential



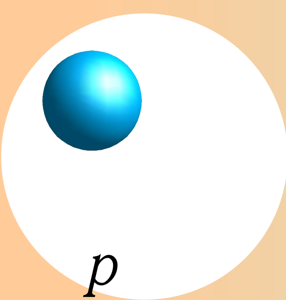
Formulation

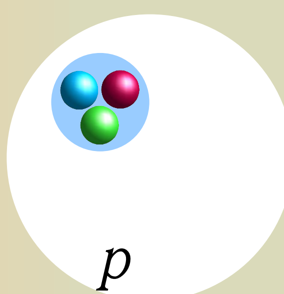


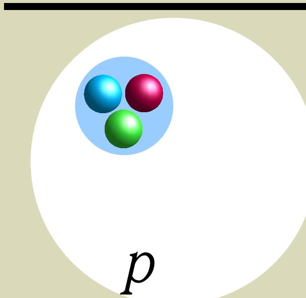
Lattice QCD simulation

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

$$\begin{aligned}\langle O(\bar{q}, q, U)\rangle &= \int dU\, d\bar{q}\, dq\, \mathrm{e}^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U) \mathrm{e}^{-S_U(U)} O(D^{-1}(U)) \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))\end{aligned}$$

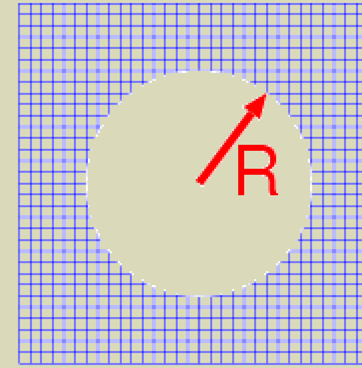


$\rightarrow \langle$ 

(t) $\overline{}$ 

$(t_0)\rangle$

Formulation



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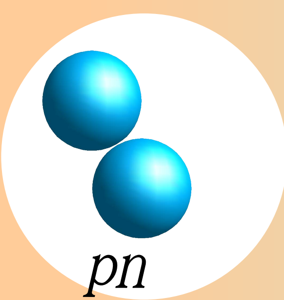
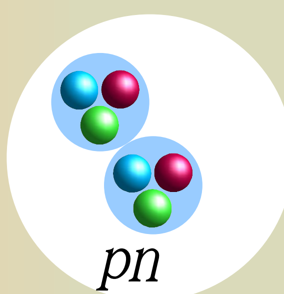
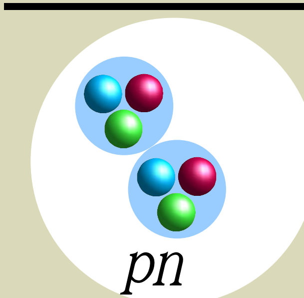
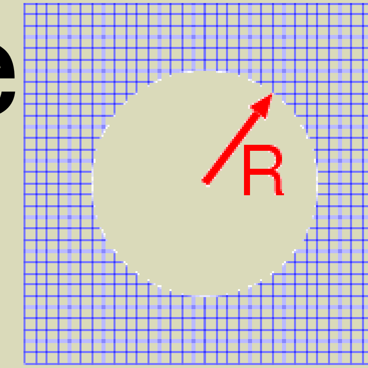


$\rightarrow \langle$ 

$(t)\ \overline{\quad}$ 

$(t_0) \rangle$

Multi-hadron on lattice



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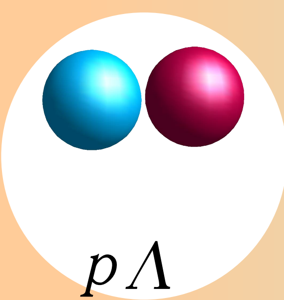
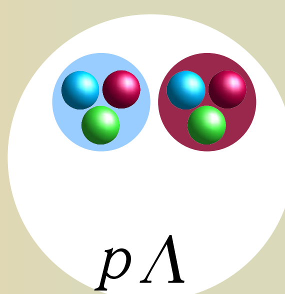
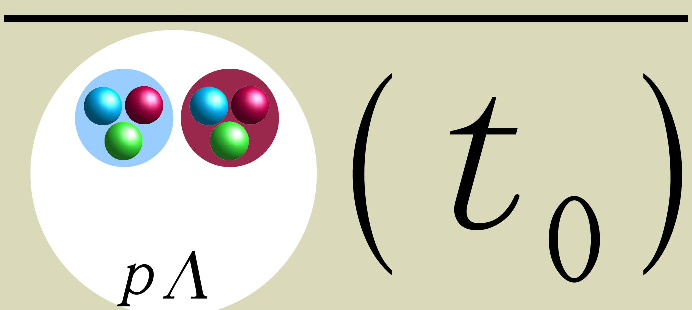


$\rightarrow \langle$ 

(t) 

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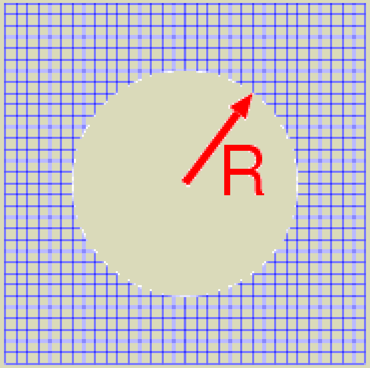

i) basic procedure:

asymptotic region

(or temporal correlation)

--> scattering energy

--> phase shift

$$E = \frac{k^2}{2\mu}$$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in Z^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re\, s > \frac{3}{2}$$

Luscher, NPB354, 531 (1991).
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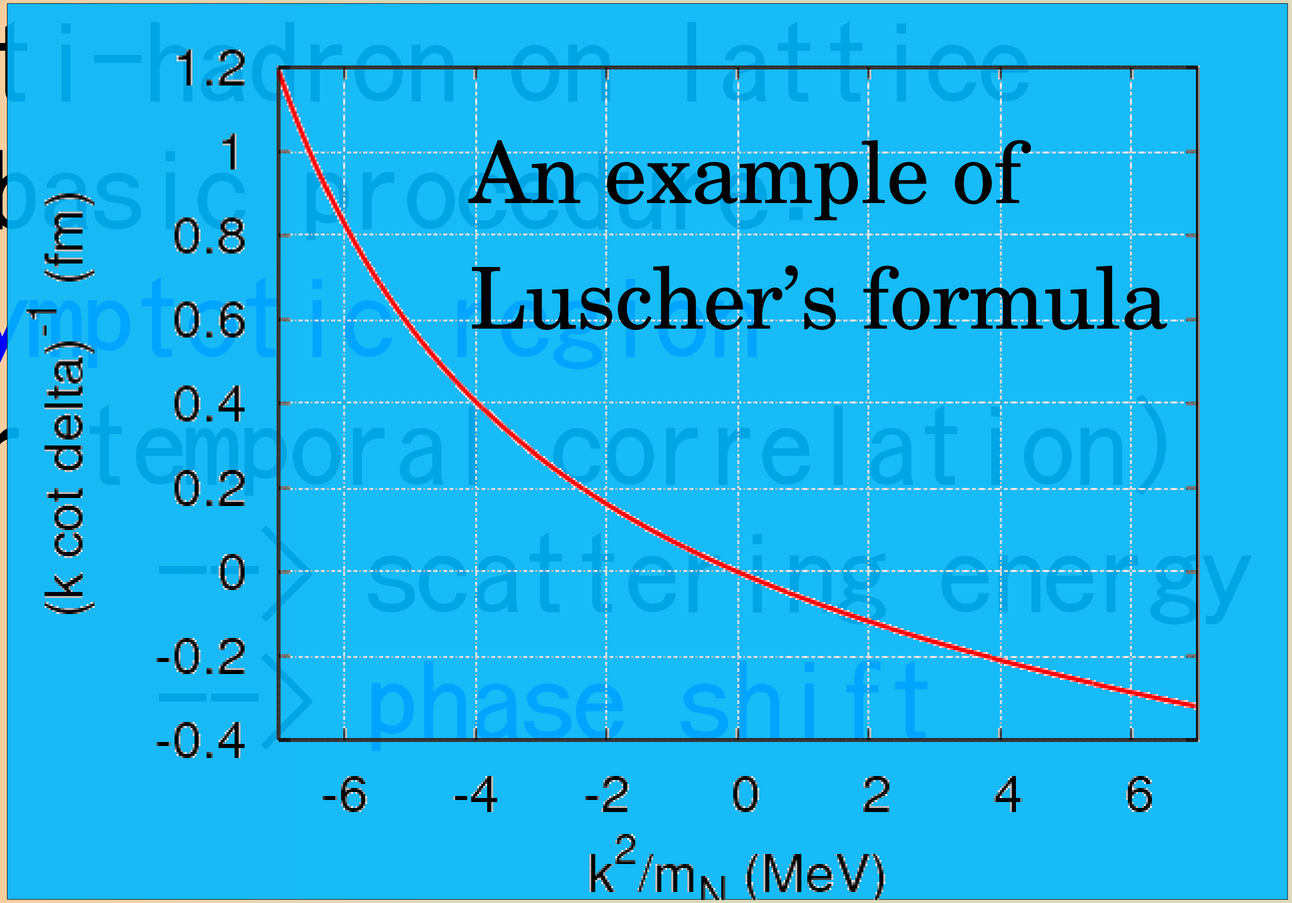

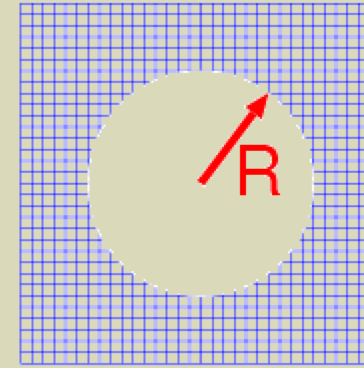

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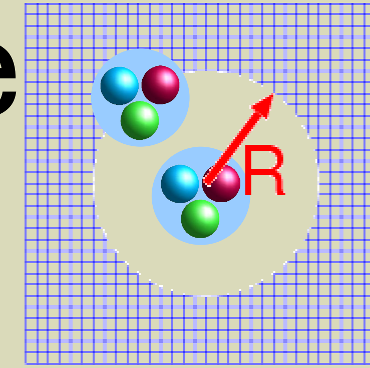

Lattice QCD simulation

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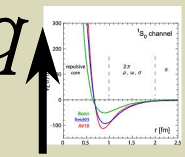

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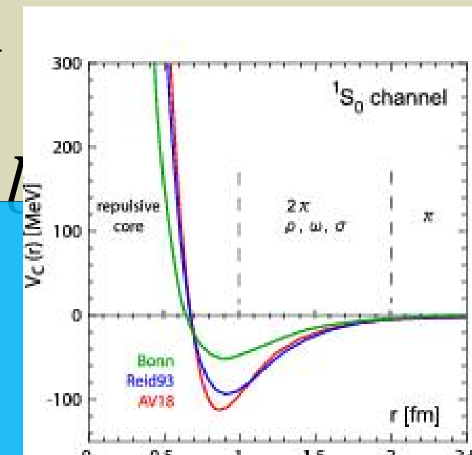
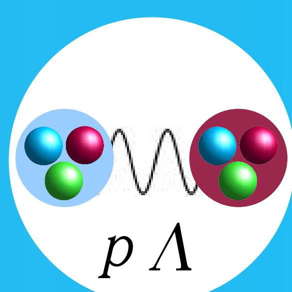
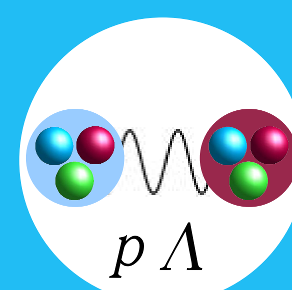


$$F^{(JM)}_{\alpha\beta}(\vec{r}, t - t_0)$$

$$\rightarrow \langle$$ 

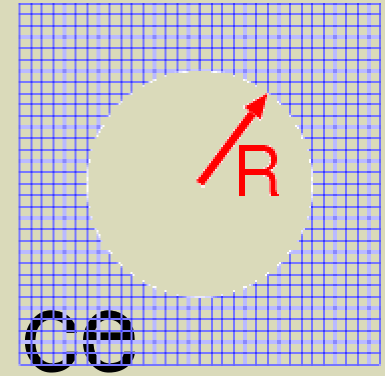
$$(\vec{r}, t)\ \overline{}$$ 

$$(t_0)\rangle$$

Calculate the scattering state

Multi-hadron on lattice

ii) HAL's procedure:



make better use of the lattice output ! (wave function)

interacting region

--> potential

Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., PTP123, 89 (2010).

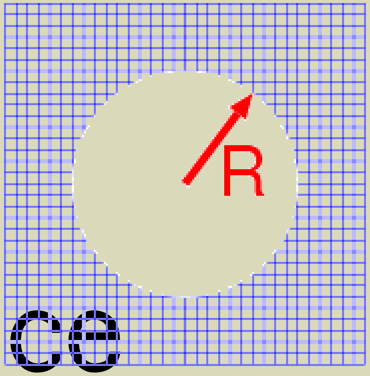
NOTE:

> Potential is not a direct experimental observable.

> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

Multi-hadron on lattice



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=>

> Phase shift

> Nuclear many-body problems

The potential is obtained at moderately large imaginary time; no single state saturation is required.

$$R_{\alpha\beta}^{(J,M)}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}_{B_3B_4}^{(J,M)}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},0) B_{2,\beta}(\vec{X},0) \right| E_n \right\rangle \mathrm{e}^{-(E_n - m_{B_1} - m_{B_2})(t-t_0)}$$
$$+O(\mathrm{e}^{-(E_{\mathrm{th}} - m_{B_1} - m_{B_2})(t-t_0)}), \quad (4)$$

where E_n ($|E_n\rangle$) is the eigen-energy (eigen-state) of the six-quark system and $A_n = \sum_{\alpha'\beta'} P_{\alpha'\beta'}^{(JM)} \langle E_n | \overline{B}_{4,\beta'} \overline{B}_{3,\alpha'} | 0 \rangle$. At moderately large $t - t_0$ where the inelastic contribution above the pion production $O(\mathrm{e}^{-(E_{\mathrm{th}}-2m_N)(t-t_0)}) = O(\mathrm{e}^{-m_\pi (t-t_0)})$ becomes exiguous, we can construct the non-local potential U through $\left(\frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu}\right) R(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r'}) R(\vec{r'})$. In lattice QCD calculations

[1] The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}$, is required for the present HAL QCD method[20], which becomes $((2\pi)^2/(2\mu L^2))^{-1} \simeq 4.6$ fm if we consider $L \sim 6$ fm and $m_N \simeq 1$ GeV. In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at $m_\pi \simeq 530$ MeV and $L \simeq 4.4$ fm.

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$$R^{(J,M)}_{\alpha\beta}(\vec{r}, t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, t) B_{2,\beta}(\vec{X}, t) \overline{\mathcal{J}^{(J,M)}_{B_3 B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, 0) B_{2,\beta}(\vec{X}, 0) \right| E_n \right\rangle \mathrm{e}^{-(E_n - m_{B_1} - m_{B_2})(t-t_0)} + O(\mathrm{e}^{-(E_{\rm th} - m_{B_1} - m_{B_2})(t-t_0)}), \quad (4)$$

where E_n ($|E_n\rangle$) is the eigen-energy (eigen-state) of the six-quark system and $A_n = \sum_{\alpha'\beta'} P^{(JM)}_{\alpha'\beta'} \langle E_n | \overline{B}_{4,\beta'} \overline{B}_{3,\alpha'} | 0 \rangle$. At moderately large $t - t_0$ where the inelastic contribution above the pion production $O(\mathrm{e}^{-(E_{\rm th} - 2m_N)(t-t_0)}) = O(\mathrm{e}^{-m_\pi (t-t_0)})$ becomes exiguous, we can construct the non-local potential U through $\left(\frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu}\right) R(\vec{r}) = \int d^3 r' \, U(\vec{r}, \vec{r'}) R(\vec{r'})$. In lattice QCD calculations

[1] The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}$, is required for the present HAL QCD method[20], which becomes $((2\pi)^2/(2\mu L^2))^{-1} \simeq 4.6$ fm if we consider $L \sim 6$ fm and $m_N \simeq 1$ GeV. In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at $m_\pi \simeq 530$ MeV and $L \simeq 4.4$ fm.

An improved recipe for NY potential:

❁ cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r}) + \int d^3r' \, U(\vec{r},\vec{r}\,') R(t,\vec{r}\,') = -\frac{\partial}{\partial t} R(t,\vec{r})$$

$$\rightarrow \frac{k^2}{2\mu} R(t,\vec{r})$$

$$U(\vec{r},\vec{r}\,') = V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}\,')$$

- A general expression of the potential:

$$V_{NY} = V_0(r) + V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y) + V_T(r) S_{12} + V_{LS}(r)(\vec{L}\cdot\vec{S}_+) + V_{ALS}(r)(\vec{L}\cdot\vec{S}_-) + O(\nabla^2)$$

Determination of baryon-baryon potentials at nearly physical point

Effective block algorithm for various baryon-baryon correlators

HN, CPC207,91(2016), arXiv:1510.00903(hep-lat)

Numerical cost (# of iterative operations) in this algorithm

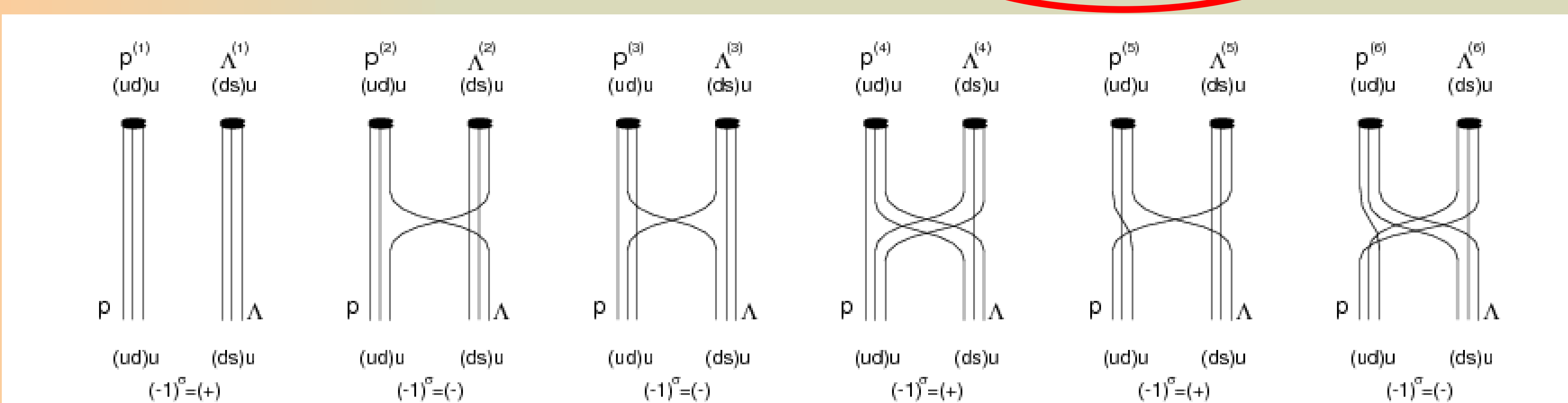
$$1+N_c{}^2+N_c{}^2N_\alpha{}^2+N_c{}^2N_\alpha{}^2+N_c{}^2N_\alpha+N_c{}^2N_\alpha=370$$

In an intermediate step:

$$(N_c!N_\alpha)^B\times N_u!N_d!N_s!\times 2^{N_\Lambda+N_{\Sigma^0}-B}=3456$$

In a naïve approach:

$$(N_c!N_\alpha)^{2B}\times N_u!N_d!N_s!=3,981,312$$



Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{aligned}
&\langle p\Lambda\overline{p\Lambda}\rangle, \quad \langle p\Lambda\overline{\Sigma^+ n}\rangle, \quad \langle p\Lambda\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^+ n\overline{p\Lambda}\rangle, \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^0 p\overline{p\Lambda}\rangle, \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{aligned} \tag{4.2}$$

$$\begin{array}{llllll}
\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, & \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, & \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, & \\
\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, & \langle p\Xi^-\overline{p\Xi^-}\rangle, & \langle p\Xi^-\overline{n\Xi^0}\rangle, & \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle,\\
\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, & \langle n\Xi^0\overline{p\Xi^-}\rangle, & \langle n\Xi^0\overline{n\Xi^0}\rangle, & \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle,\\
\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle,\\
\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, & \\
 & \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & & \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{array} \tag{4.3}$$

$$\begin{aligned}
&\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, \quad \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, \quad \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{aligned} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

Make better use of the computing resources!

HN, CPC **207**, 91(2016) [arXiv:1510.00903[hep-lat]],
(See also arXiv:1604.08346)

Almost physical point lattice QCD calculation using N_F=2+1 clover fermion + Iwasaki gauge action

- APE-Stout smearing (ρ=0.1, n_{stout}=6)
- Non-perturbatively O(a) improved Wilson Clover action at β=1.82 on $96^3 \times 96$ lattice
- $1/a$ = 2.3 GeV (a = 0.085 fm)
- Volume: $96^4 \rightarrow (8\text{fm})^4$
- m_π=145MeV, m_K=525MeV
- DDHMC(ud) and UVPHMC(s) with preconditioning
- K.-I.Ishikawa, et al., PoS LAT2015, 075; arXiv:1511.09222 [hep-lat].

- NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207configs x 4rotation x Nsrc (Nsrc=4 → 20 → 52 → 96 (2015FY+))

LN-SN potentials at nearly physical point

The methodology for coupled-channel V is based on:
Aoki, et al., Proc.Japan Acad. B87 (2011) 509.
Sasaki, et al., PTEP 2015 (2015) no.11, 113B01.
Ishii, et al., JPS meeting, March (2016).

#stat: (this/scheduled in FY2015+) < 0.05 (==>0.2) → 0.54

$\Lambda N - \Sigma N \ (I = 1/2)$

$V_C(^1S_0)$ $V_C(^3S_1 - {}^3D_1)$ $V_T(^3S_1 - {}^3D_1)$

$\Sigma N \ (I = 3/2)$

$V_C(^1S_0)$ $V_C(^3S_1 - {}^3D_1)$ $V_T(^3S_1 - {}^3D_1)$

LN-SN potentials at nearly physical point

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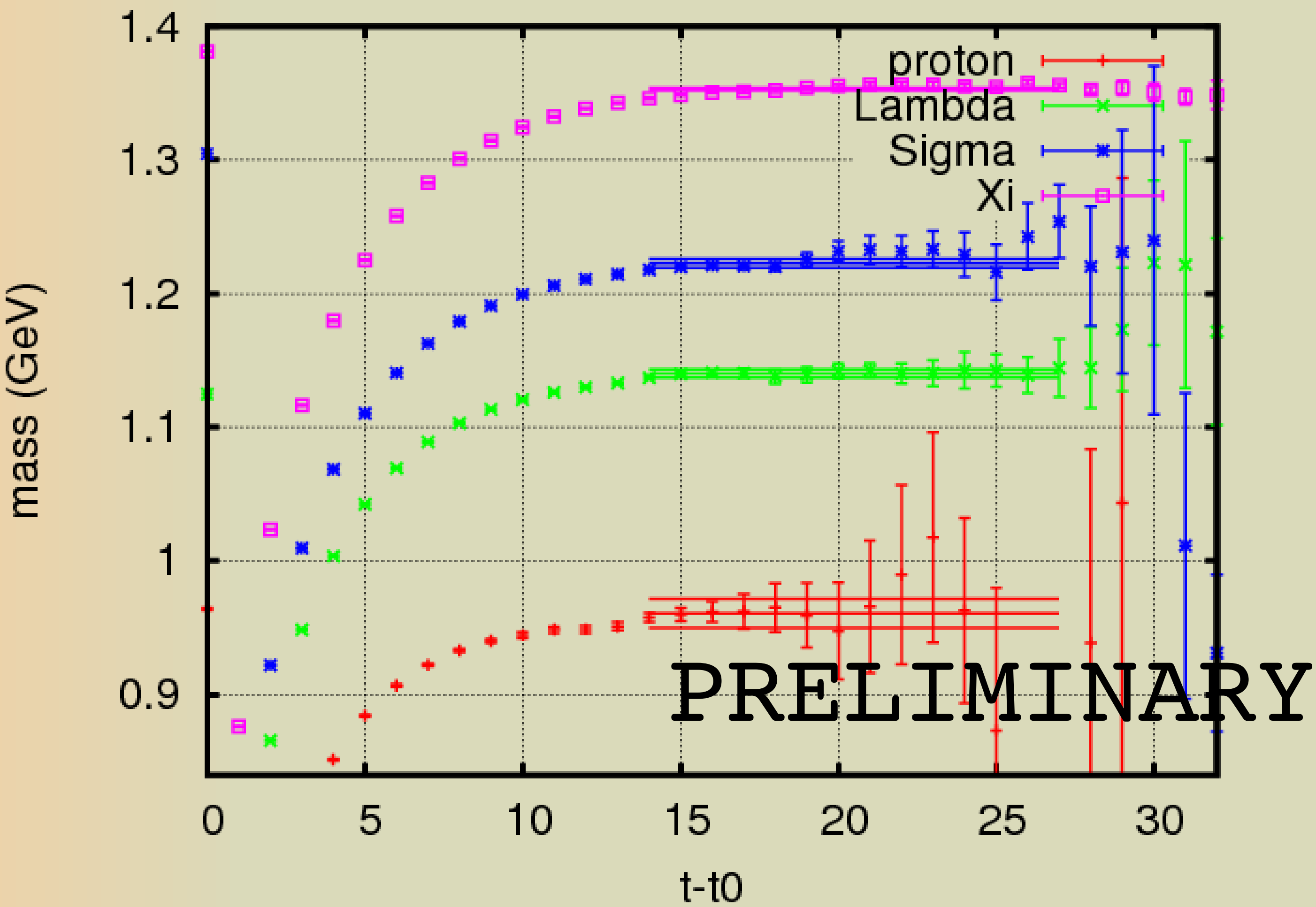
$\Lambda N - \Sigma N \; (I = 1/2)$

$V_C(^1S_0)$ $V_C(^3S_1 - {}^3D_1)$ $V_T(^3S_1 - {}^3D_1)$

$\Sigma N \; (I = 3/2)$

$V_C(^1S_0)$ $V_C(^3S_1 - {}^3D_1)$ $V_T(^3S_1 - {}^3D_1)$

Effective mass plot of the single baryon's correlation function



Potentials obtained at t-t0 = 5 to 12 will be shown.

TABLE 4

The eigenvalues of the normalization kernel in eq. (3.3) for $S=-1$ two-baryon (BB) system

$S=-1$

I	J	BB	Eigenvalues (uncoupled)	Eigenvalues (coupled)
$\frac{1}{2}$	0	$N\Lambda$	1	$0 \quad \frac{10}{9}$
		$N\Sigma$	$\frac{1}{9}$	
$\frac{1}{2}$	1	$N\Lambda$	1	$\frac{8}{9} \quad \frac{10}{9}$
		$N\Sigma$	1	
$\frac{3}{2}$	0	$N\Sigma$	$\frac{10}{9}$	
$\frac{3}{2}$	1	$N\Sigma$	$\frac{2}{9}$	

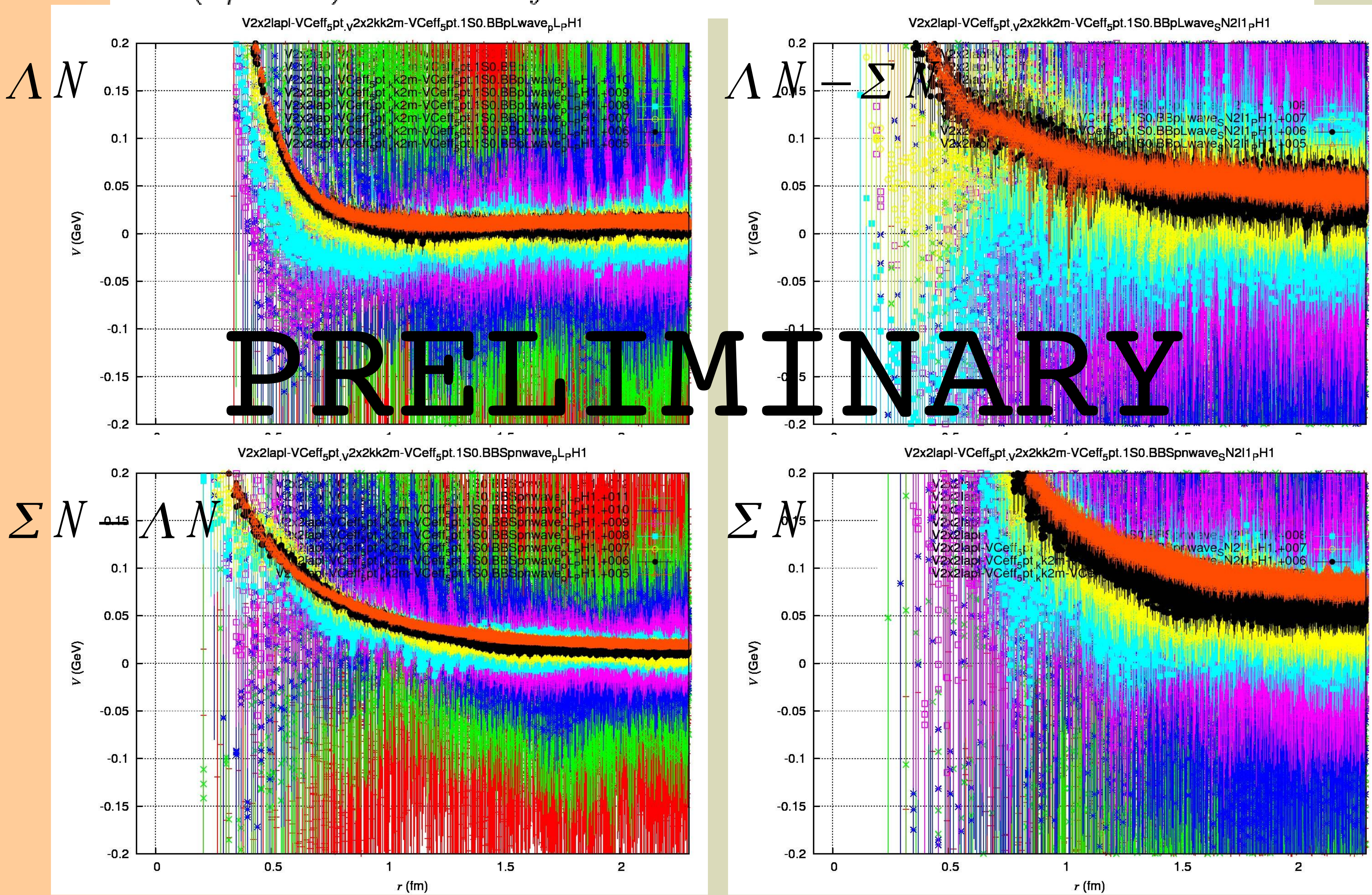
Eigenvalues of single and coupled channels are given.

Oka, Shimizu and Yazaki (1987)

Very preliminary result of LN potential at the physical point

$V_C(^1S_0)$

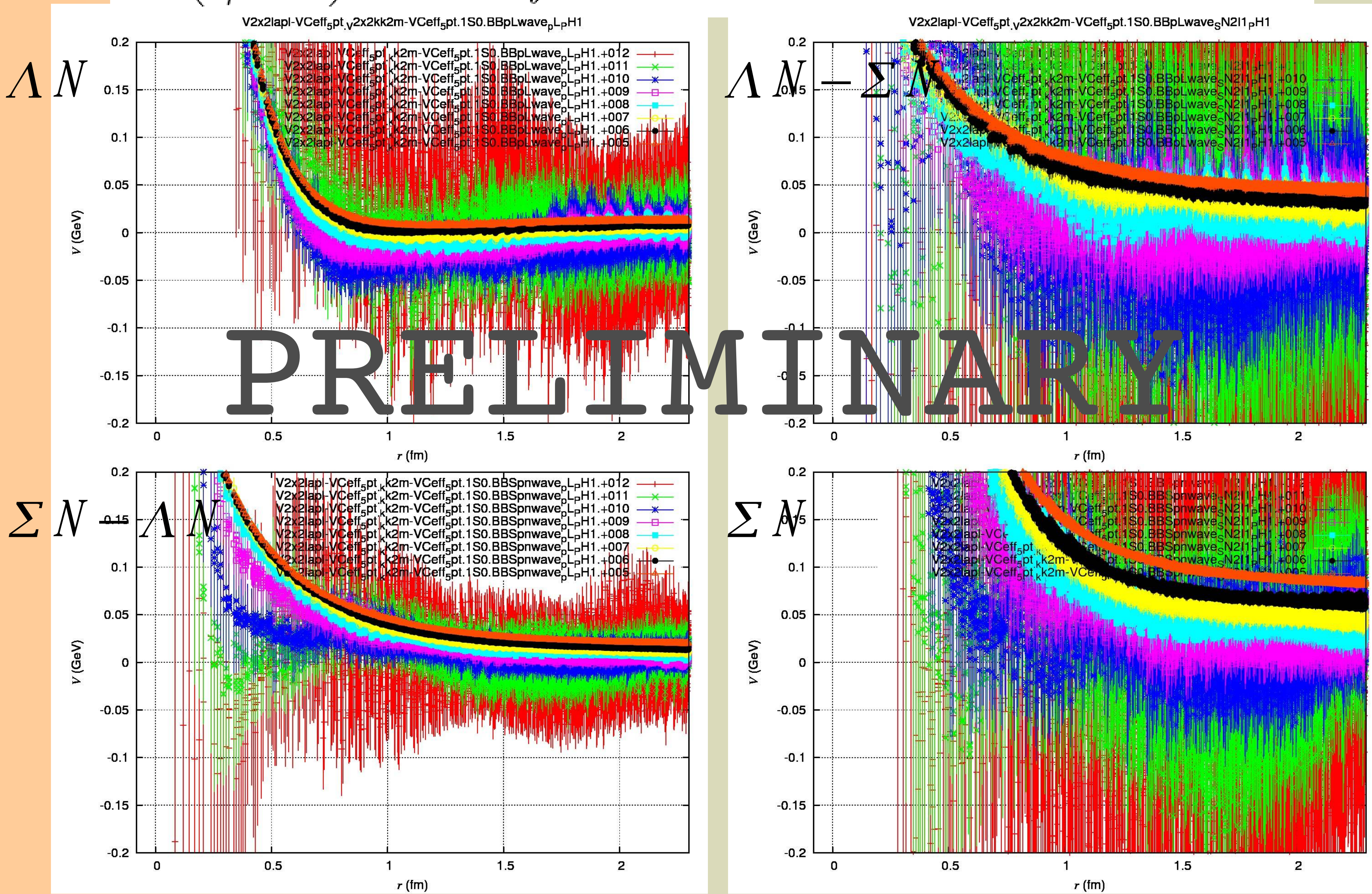
$$\left(\frac{\nabla^2}{2\mu}-\frac{\partial}{\partial t}\right)R(\vec r,t) \;=\; \int d^3r'\,U(\vec r,\vec r')R(\vec r',t)+O(k^4)=V_{\rm LO}(\vec r)R(\vec r,t)+\cdot\,(8)$$



Very preliminary result of LN potential at the physical point

$V_C(\,^1S_0)$

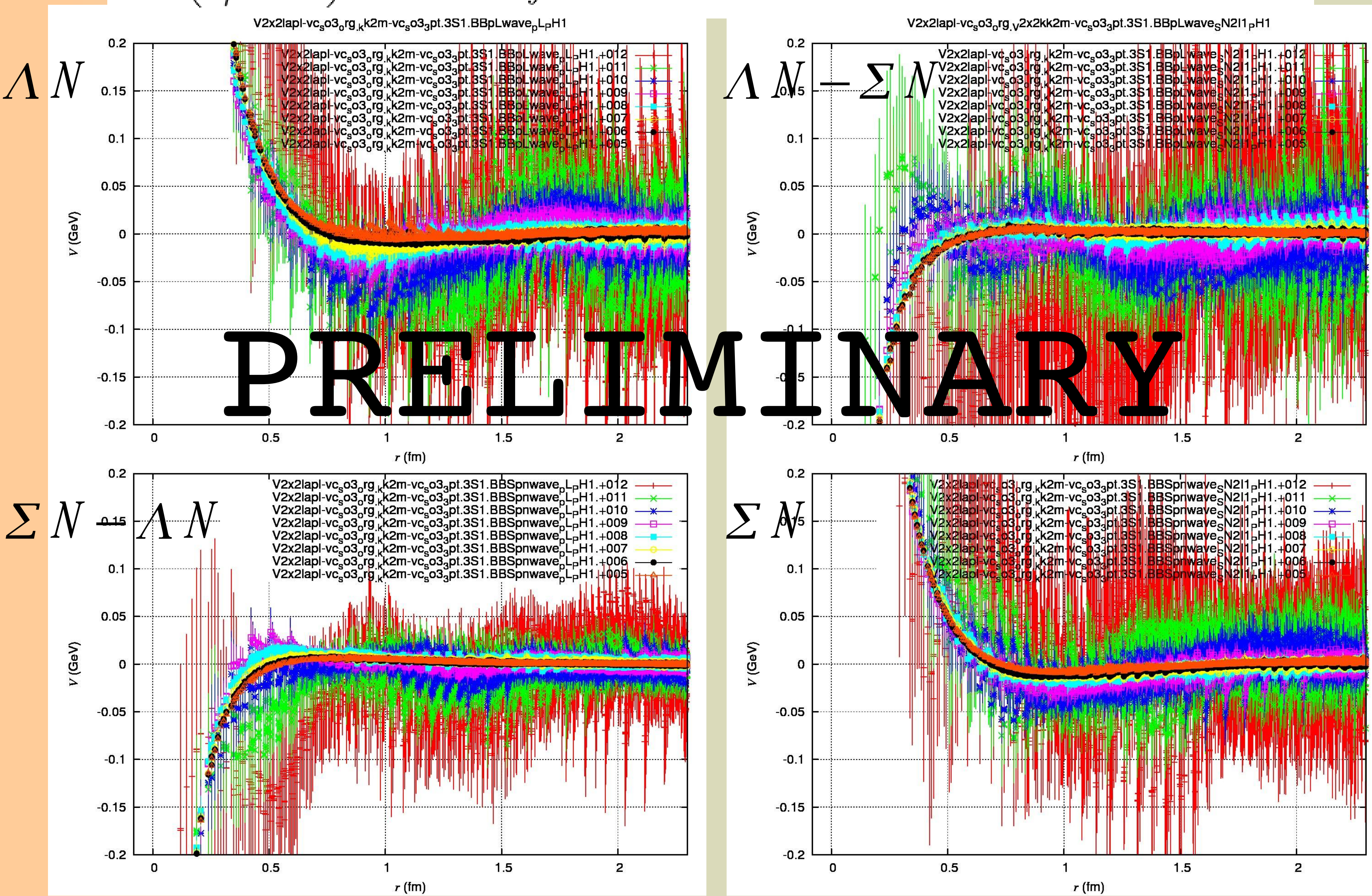
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_C({}^3S_1 - {}^3D_1)$

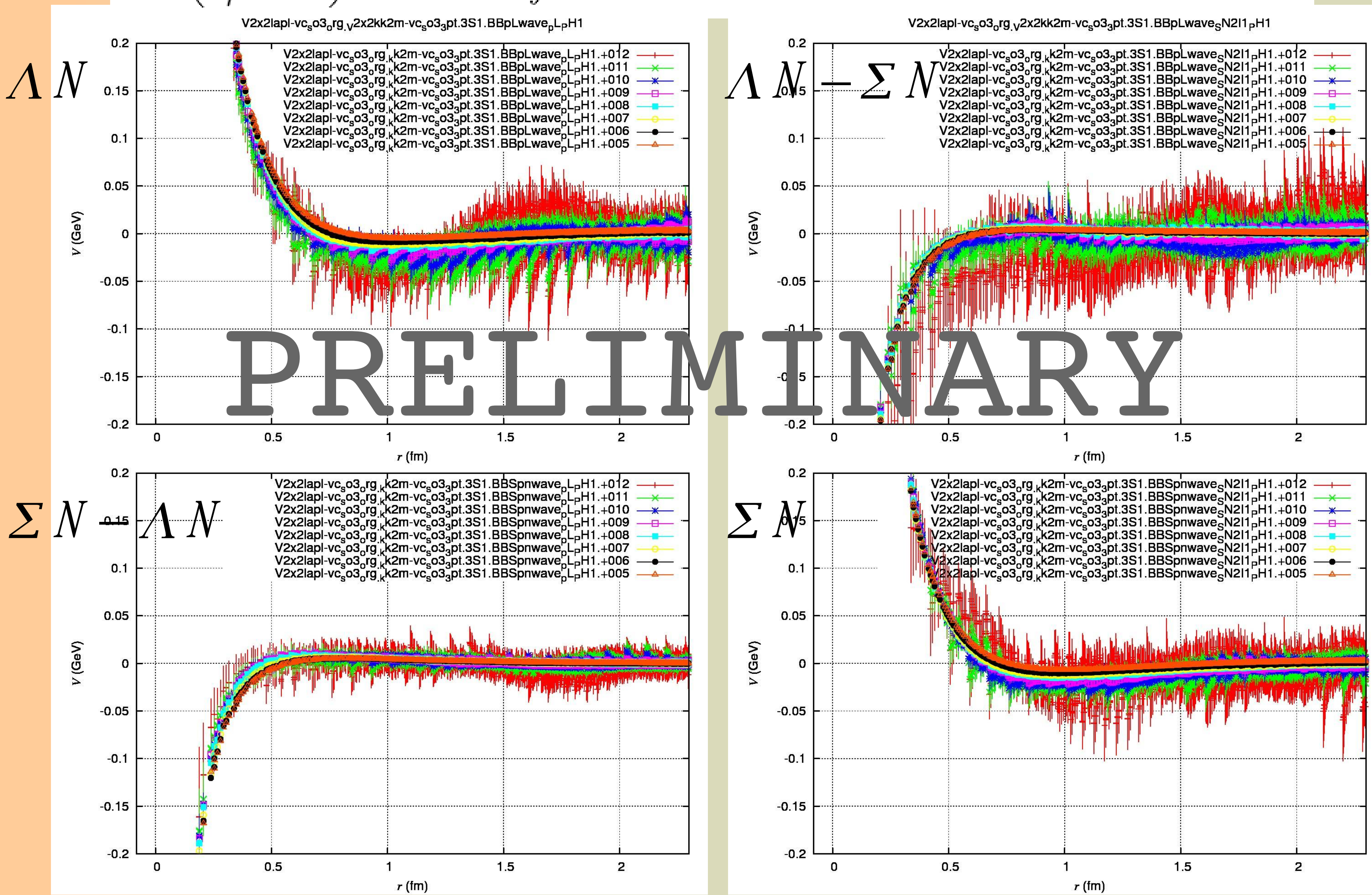
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_C({}^3S_1 - {}^3D_1)$

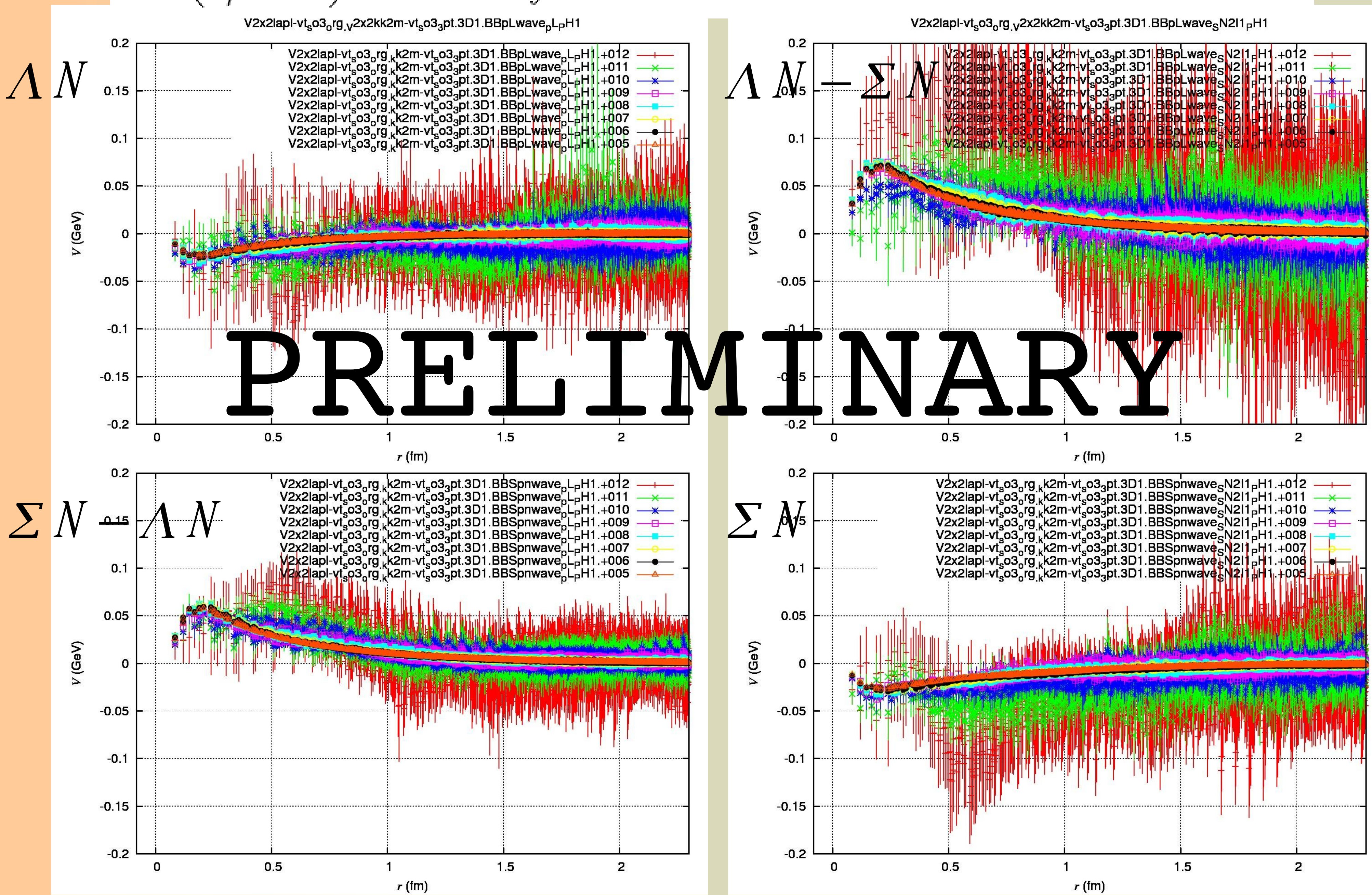
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_T({}^3S_1 - {}^3D_1)$

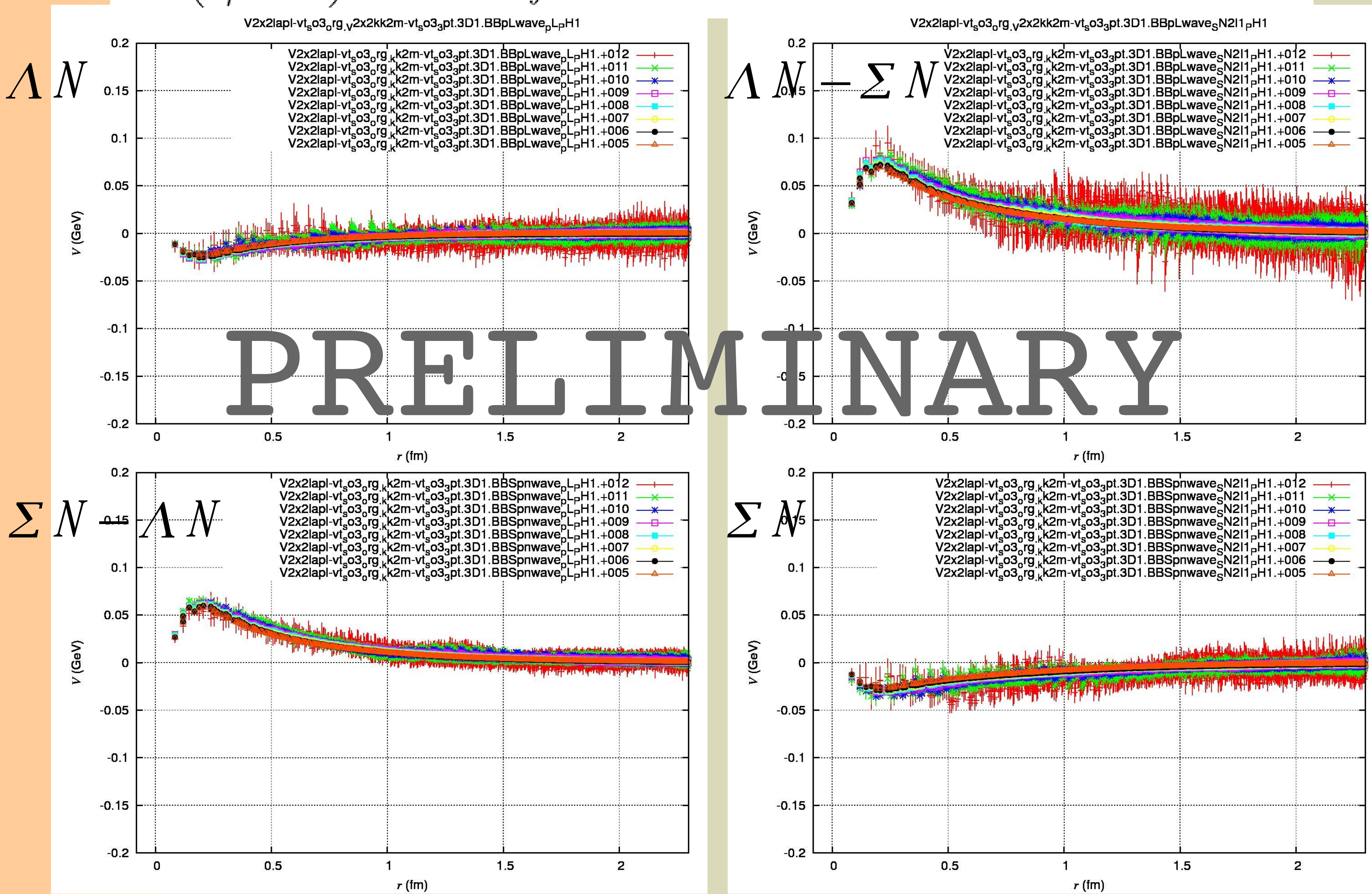
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}\,') R(\vec{r}\,',t) + O(k^4) = V_{\rm LO}(\vec{r}) R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_T(\,^3S_1 - \,^3D_1)$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r' \, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$

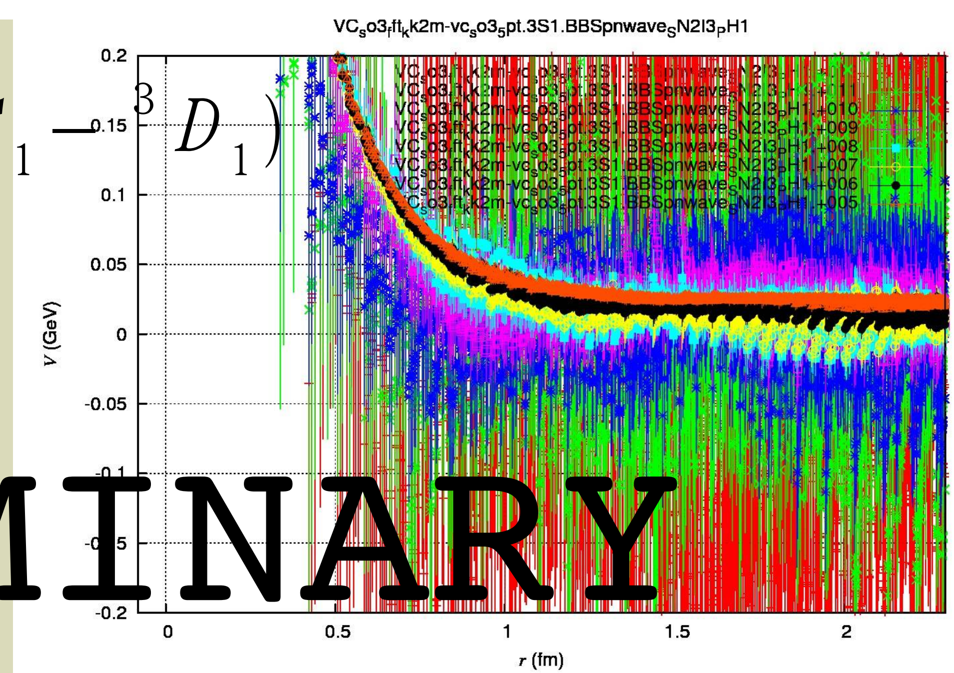


Very preliminary result of LN potential at the physical point

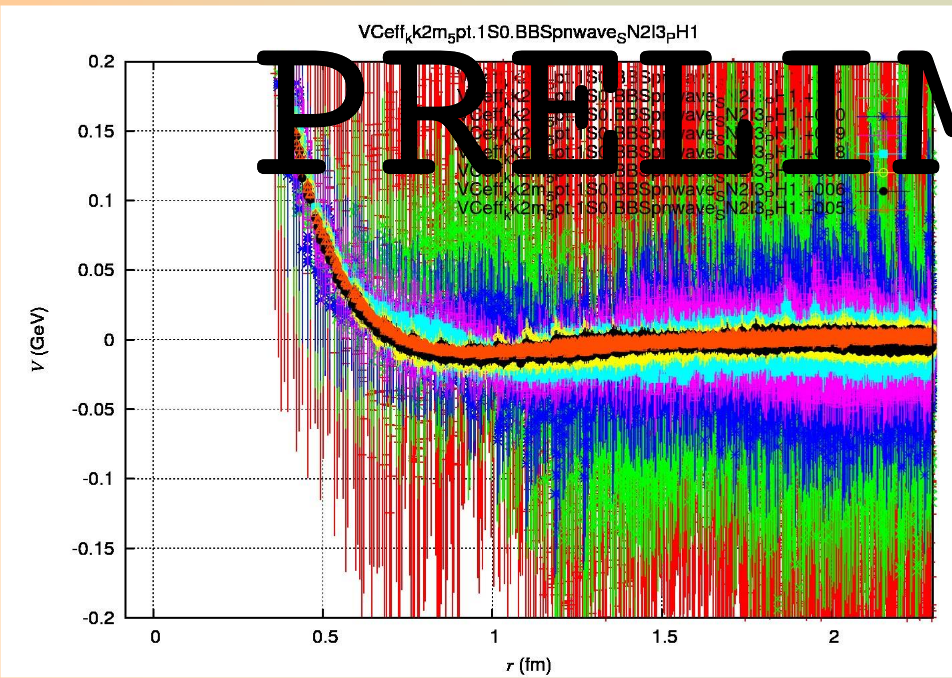
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots \quad (8)$$

$\Sigma N\,(I=3/2)$

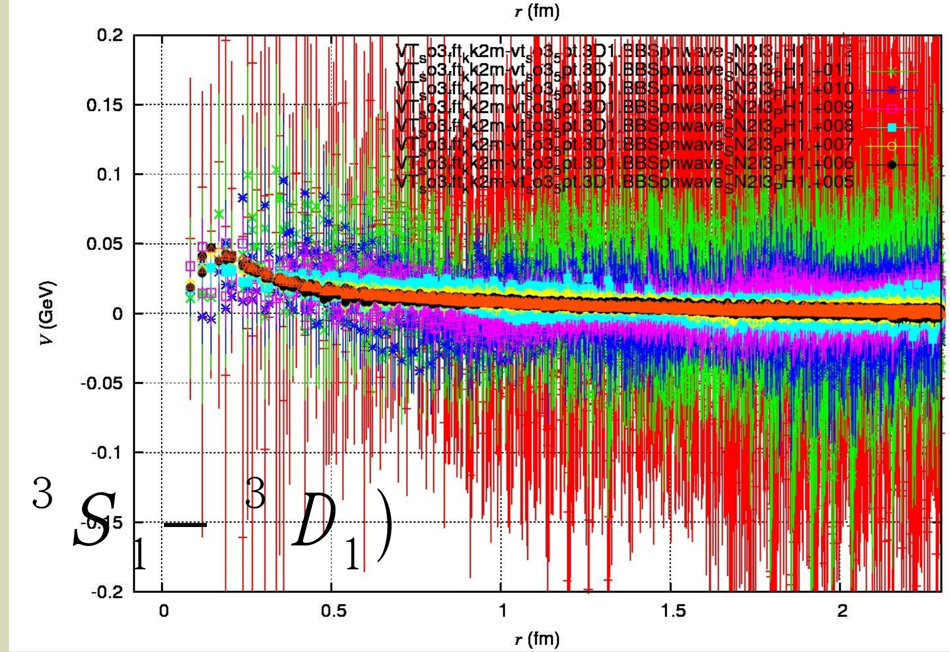
$V_C(\,^3S_1 - \,^3D_1)$



PRELIMINARY



$V_C(\,^1S_0)$



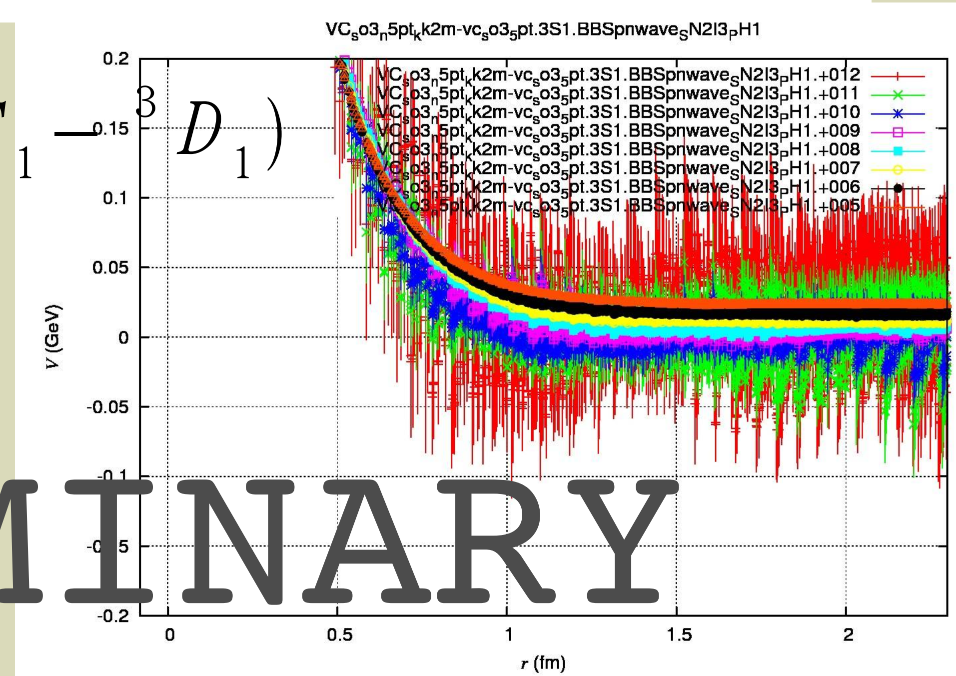
$V_T(\,^3S_1 - \,^3D_1)$

Very preliminary result of LN potential at the physical point

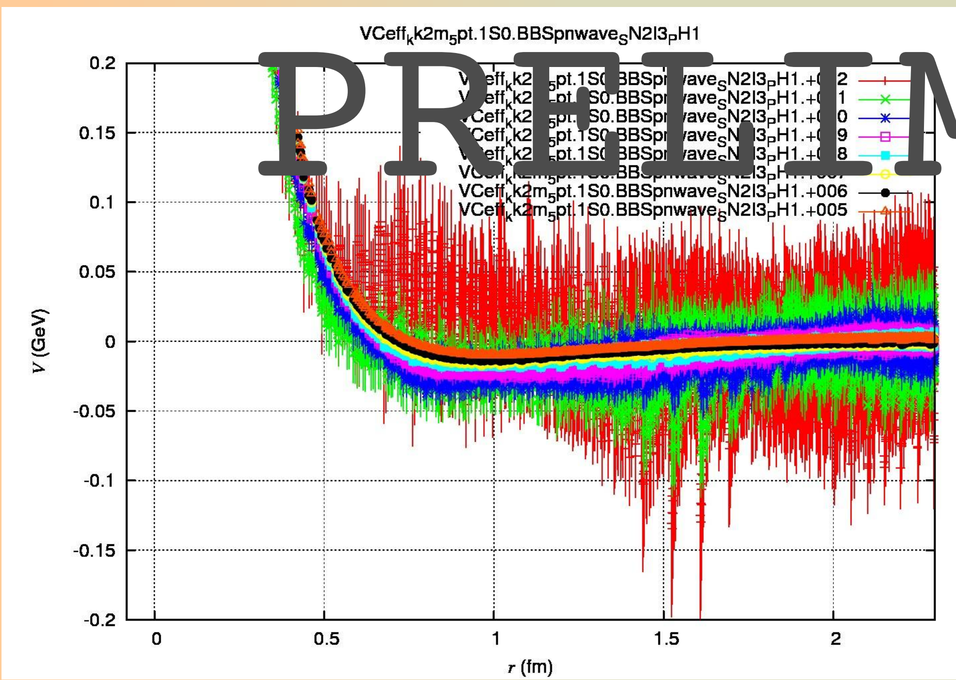
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots \quad (8)$$

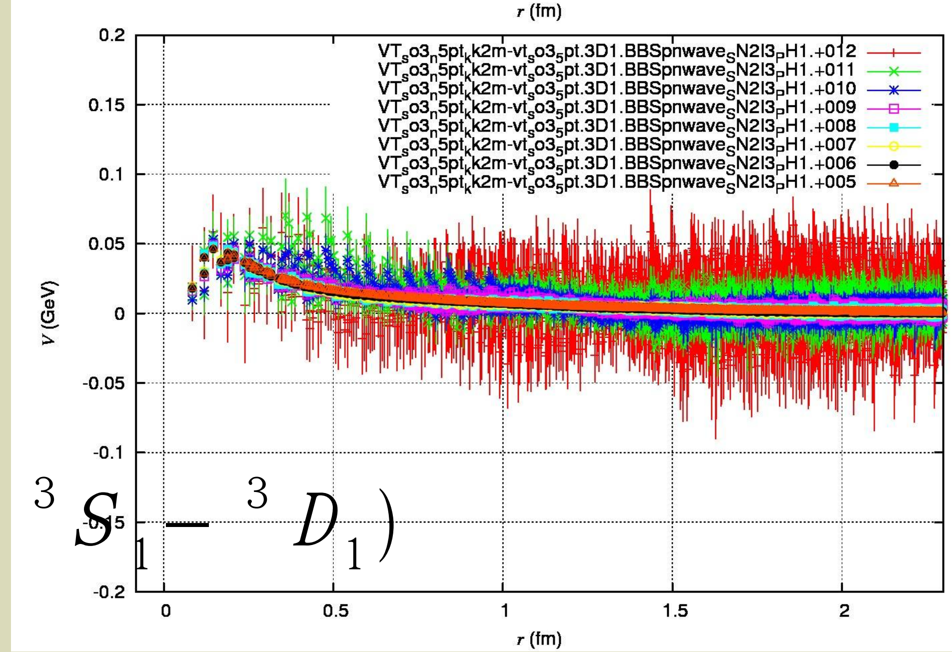
$\Sigma N\,(I=3/2)$

$V_C(^3S_1 - {}^3D_1)$



PRELIMINARY





$V_C(^1S_0)$

$V_T(^3S_1 - {}^3D_1)$

Summary

(I-1) Preliminary results of LN-SN potentials at nearly physical point. (Lambda-N, Sigma-N: central, tensor)
Statistics approaching to 0.54 (=present/scheduled)
Signals in spin-triplet are relatively going well smoothly.
We will have to increase still more statistics, particularly for spin-singlet channels
Several interesting features seem to be obtained with more high statistics.

(I-2) Effective hadron block algorithm for the various baron-baryon interaction
Paper published/available:
Comput.Phys.Commun.**207**,91(2016) [arXiv:1510.00903(hep-lat)]

Future work:
(II-1) Physical quantities including the binding energies of few-body problem of light hypernuclei with the lattice YN potentials