

Hyperon Stars at Zero and Finite Temperatures with the Variational Method

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Outline

1 : Introduction

2 : Variational Method for Hyperon Equation of State

3 : Application to Compact Stars

4 : Summary

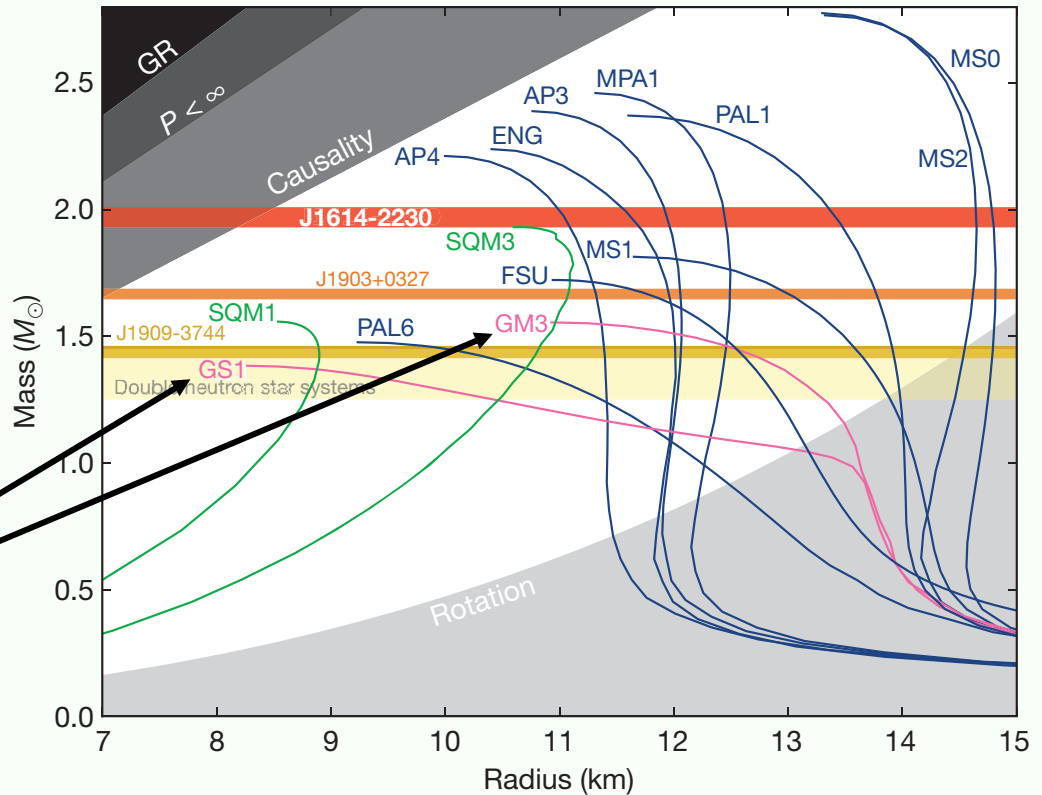
1. Introduction

*Neutron star structure is governed by
the nuclear equation of state (EOS) at zero temperature.*

HYPERON PUZZLE

- EOS becomes softer due to hyperon mixing.
- Maximum mass tends to be lower than the observational data.

*Nuclear EOS with
Hyperons*



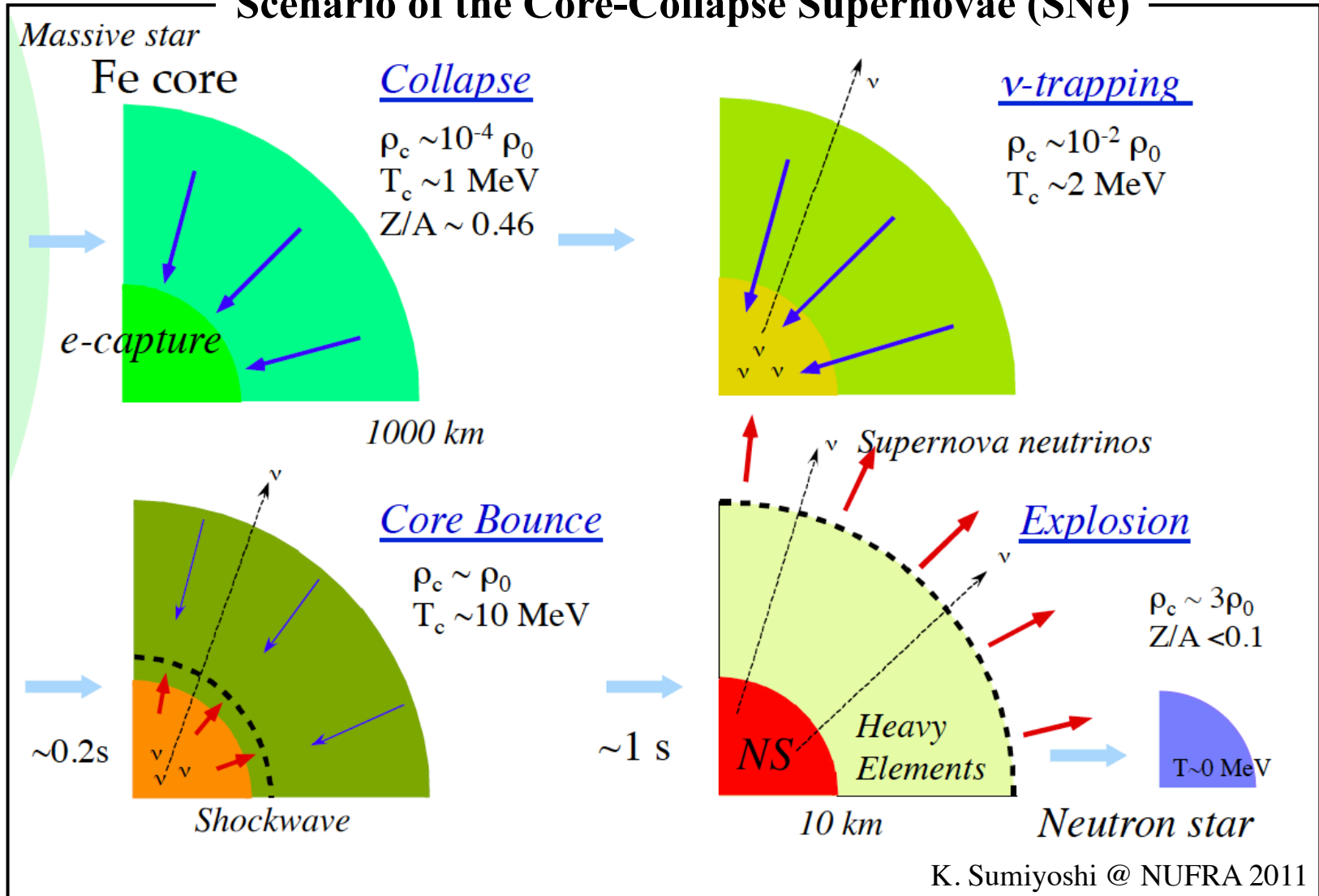
P. B. Demorest *et al.*, NATURE 467 (2010)

The hyperon mixing in neutron stars has been studied with various nuclear theories.

Nuclear EOS and Core-Collapse Supernovae

Nuclear EOS at finite temperature is one of the crucial ingredients
for the numerical simulations of *Core-Collapse Supernovae*.

Scenario of the Core-Collapse Supernovae (SNe)

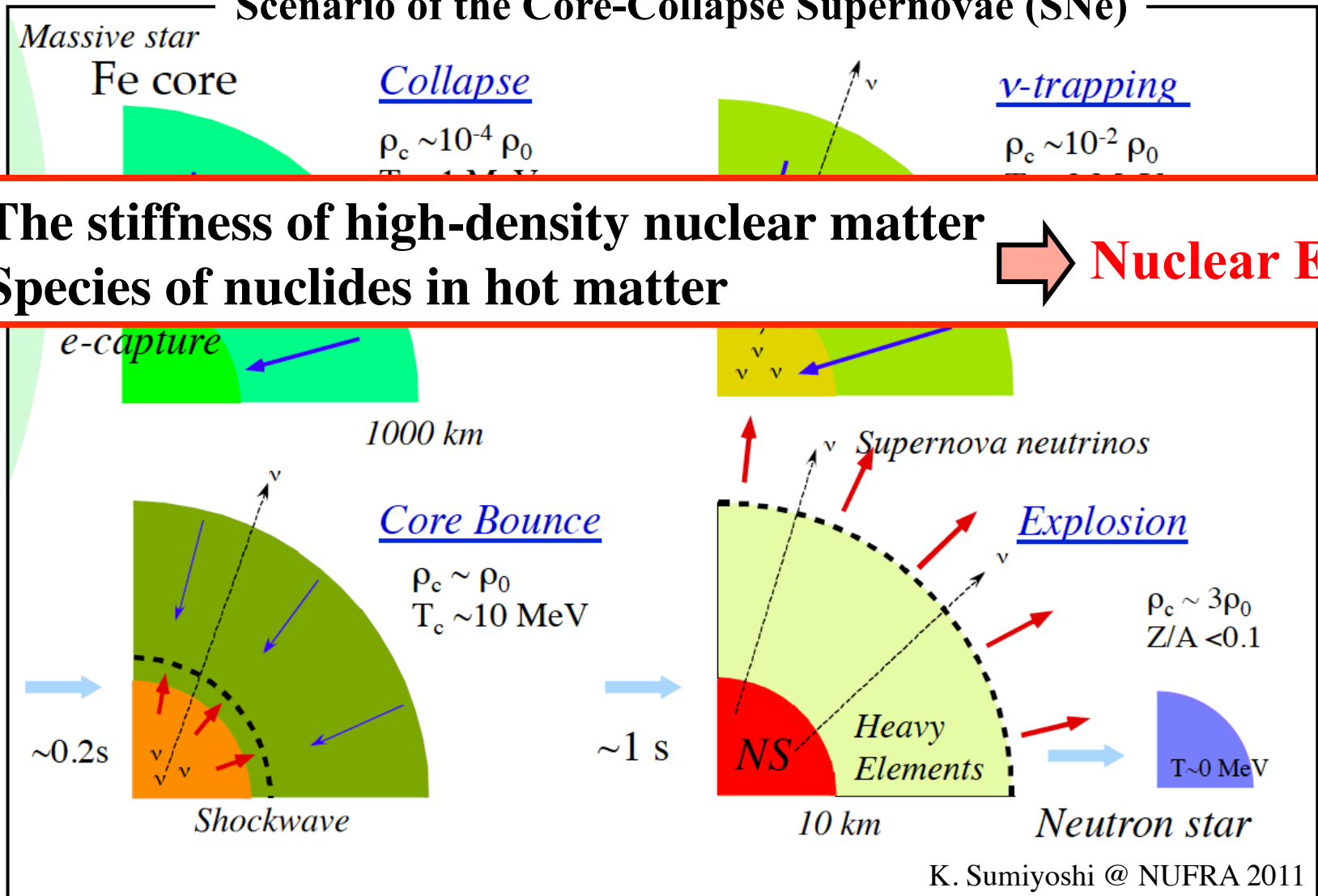


Nuclear EOS and Core-Collapse Supernovae

Nuclear EOS at finite temperature is one of the crucial ingredients
for the numerical simulations of *Core-Collapse Supernovae*.

Scenario of the Core-Collapse Supernovae (SNe)

- The stiffness of high-density nuclear matter
 - Species of nuclides in hot matter
- ➡ **Nuclear EOS**



Current status of SN-EOS with hyperons

Nuclear Interaction	n_{sat} (fm^{-3})	BE/A (MeV)	K (MeV)	Q ($\frac{\text{MeV}}{\text{fm}^3}$)	J (MeV)	L (MeV)	type of int.	used in
SKa	0.155	16.0	263	-300	32.9	74.6	Skyrme	H&W
LS180	0.155	16.0	180	-451	28.6	73.8	Skyrme	LS180
LS220	0.155	16.0	220	-411	28.6	73.8	Skyrme	LS220, LS220 Λ , LS220 π
LS375	0.155	16.0	375	176	28.6	73.8	Skyrme	LS375
TMA	0.147	16.0	318	-572	30.7	90.1	RMF	HS(TMA)
NL3	0.148	16.2	272	203	37.3	118.2	RMF	SHT, HS(NL3)
FSUgold	0.148	16.3	230	-524	32.6	60.5	RMF	SHO(FSU1.7), HS(FSUgold)
FSUgold2.1	0.148	16.3	230	-524	32.6	60.5	RMF	SHO(FSU2.1)
IUFSU	0.155	16.4	231	-290	31.3	47.2	RMF	HS(IUFSU)
DD2	0.149	16.0	243	169	31.7	55.0	RMF	HS(DD2), BHBA, BHBA ϕ
SFH ϕ	0.158	16.2	245	-468	31.6	47.1	RMF	SFH ϕ
SFH χ	0.160	16.2	239	-457	28.7	23.2	RMF	SFH χ
TM1	0.145	16.3	281	-285	36.9	110.8	RMF	STOS, FYSS, HS(TM1), STOSA, STOSY, STOSY π , STOS π , STOS π Q, STOSQ, STOSB139, STOSB145, STOSB155, STOSB162, STOSB165

Hyperon EOS

There is no SN-EOSs based on the microscopic many-body theory.

- Shen EOS with Λ , Σ , Ξ [$M_{\text{max}} = 1.67 M_{\odot}$] (C. Ishizuka et al., JPG 35 (2008) 085201)
- Shen EOS with Λ [$M_{\text{max}} = 1.75 M_{\odot}$] (H. Shen et al., APJS 197 (2011) 20)
- LS EOS with Λ [$M_{\text{max}} = 1.91 M_{\odot}$] (M. Oertel et al., PRC 85 (2012) 055806)
- HS EOS with Λ [$M_{\text{max}} = 2.11 M_{\odot}$] (S. Banik et al., APJS 214 (2014) 22)

Nuclear EOS table with the variational method

<http://www.np.phys.waseda.ac.jp/EOS/>

Equation of state for nuclear matter with the variational method

Equation of state (EOS) based on the variational many-body theory with realistic nuclear forces is provided. For uniform matter, the EOS is constructed with the cluster variational method starting from the Argonne v18 two-body nuclear potential and the Urbana IX three-body nuclear potential. Non-uniform nuclear matter is treated in the Thomas-Fermi approximation. Alpha particle mixing is also taken into account. See Togashi et al., Nucl. Phys. A 961 (2017) 78 for details. This EOS table is open for general use in any studies for nuclear physics and astrophysics, provided that our paper is referred to in your publication.

User's Guide (read me first)

[guide.pdf](#)

EOS tables

[eos.zip](#)

Contact

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Table A.1: Ranges of temperature T , proton fraction Y_p , and baryon mass density ρ_B in the table of the variational EOS. At the top of the last column, "+1" represents the case at $T = 0$ MeV.

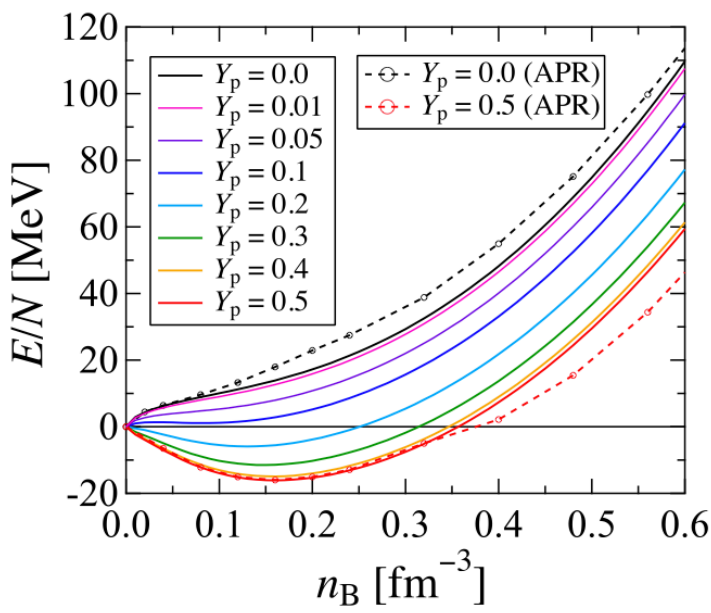
Parameter	Minimum	Maximum	Mesh	Number
$\log_{10}(T)$ [MeV]	-1.00	2.60	0.04	91 + 1
Y_p	0	0.65	0.01	66
$\log_{10}(\rho_B)$ [g/cm ³]	5.1	16.0	0.10	110

(HT *et al.*, NPA961 (2017) 78)

Nuclear EOS table with the variational method

<http://www.np.phys.waseda.ac.jp/EOS/>

Equation of state for nuclear matter with the variational method



Uniform EOS: AV18 + UIX

theory with realistic nuclear forces is provided. For uniform
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temperature T , proton fraction Y_p , and baryon mass density ρ_B in
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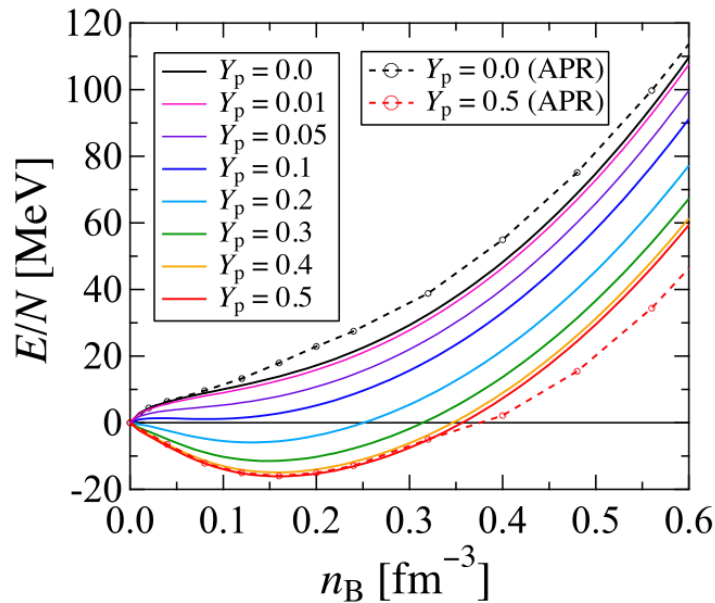
• Hajime Togashi
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(HT *et al.*, NPA961 (2017) 78)

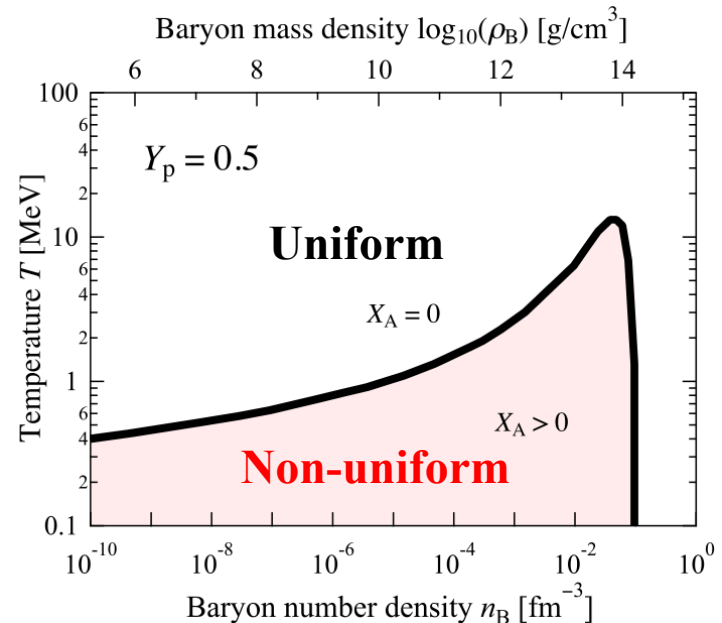
Nuclear EOS table with the variational method

<http://www.np.phys.waseda.ac.jp/EOS/>

Equation of state for nuclear matter with the variational method



Uniform EOS: AV18 + UIX



Non-uniform EOS: Thomas-Fermi method

Parameter	Minimum	Maximum	Mesh	Number
$\log_{10}(T)$ [MeV]	-1.00	2.60	0.04	91 + 1
Y_p	0	0.65	0.01	66
$\log_{10}(\rho_B)$ [g/cm^3]	5.1	16.0	0.10	110

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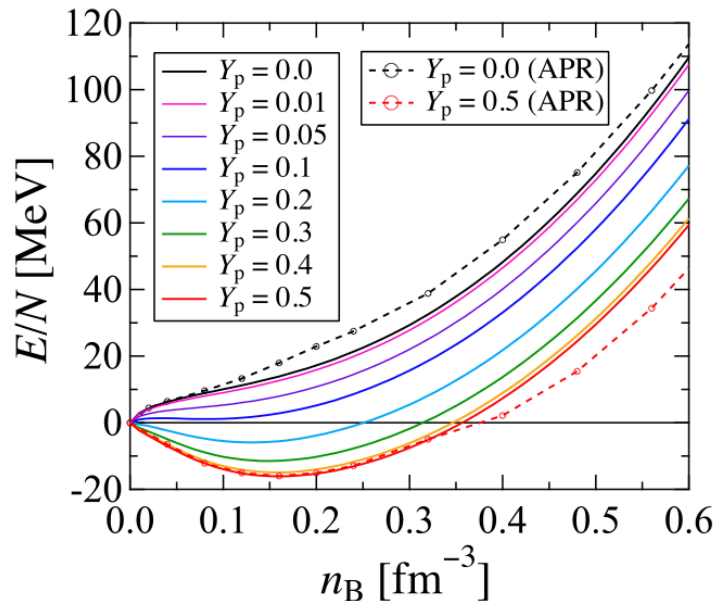
2-1 Hirosawa, Wako, Saitama 351-0198, Japan

(HT *et al.*, NPA961 (2017) 78)

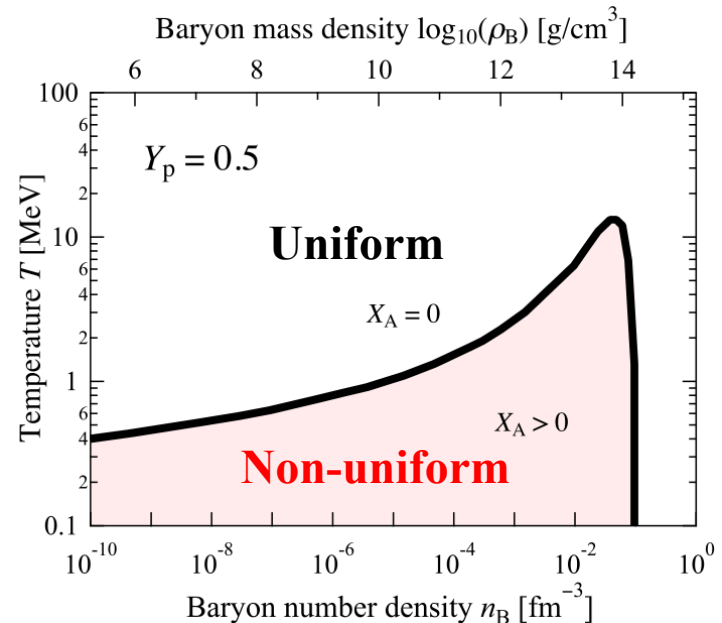
Nuclear EOS table with the variational method

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Equation of state for nuclear matter with the variational method



Uniform EOS: AV18 + UIX



Non-uniform EOS: Thomas-Fermi method

*We aim to extend the microscopic SN-EOS table to consider **Λ hyperon mixing**.*

2. Variational Method for Hyperon EOS

Hamiltonian of Λ hyperon matter

$$H = H_N - \frac{\hbar^2}{2m_\Lambda} \sum_i \nabla_i^2 + \sum_{i < j} [V_{ij}^{\Lambda N} + V_{ij}^{\Lambda \Lambda} + \bar{v}_{ij}^{\text{TBF}}]$$

H_N : Nuclear Hamiltonian (AV18+UIX)

Two-Body Central Potentials

$V_{ij}^{\Lambda N}$: Λ -Nucleon (N) potential (E. Hiyama et al., PRC 74 (2006) 054312)
- Constructed so as to reproduce the experimental binding energies of
light Λ hypernuclei with the Gaussian expansion method.

$V_{ij}^{\Lambda \Lambda}$: Λ - Λ potential (E. Hiyama et al., PRC 66 (2002) 024007)
- the experimental double- Λ binding energy from ${}^6_{\Lambda\Lambda}\text{He}$ (NAGARA event)

$\bar{v}_{ij}^{\text{TBF}}$: Effective potential based on **Three-Baryon Force (TBF)**
for ΛNN , $\Lambda\Lambda N$, $\Lambda\Lambda\Lambda$ systems

(Y. Yamamoto et al., PRC 90 (2014) 045805, HT et al., PRC 93 (2016) 035808)

Energy of nuclear matter with Λ hyperons

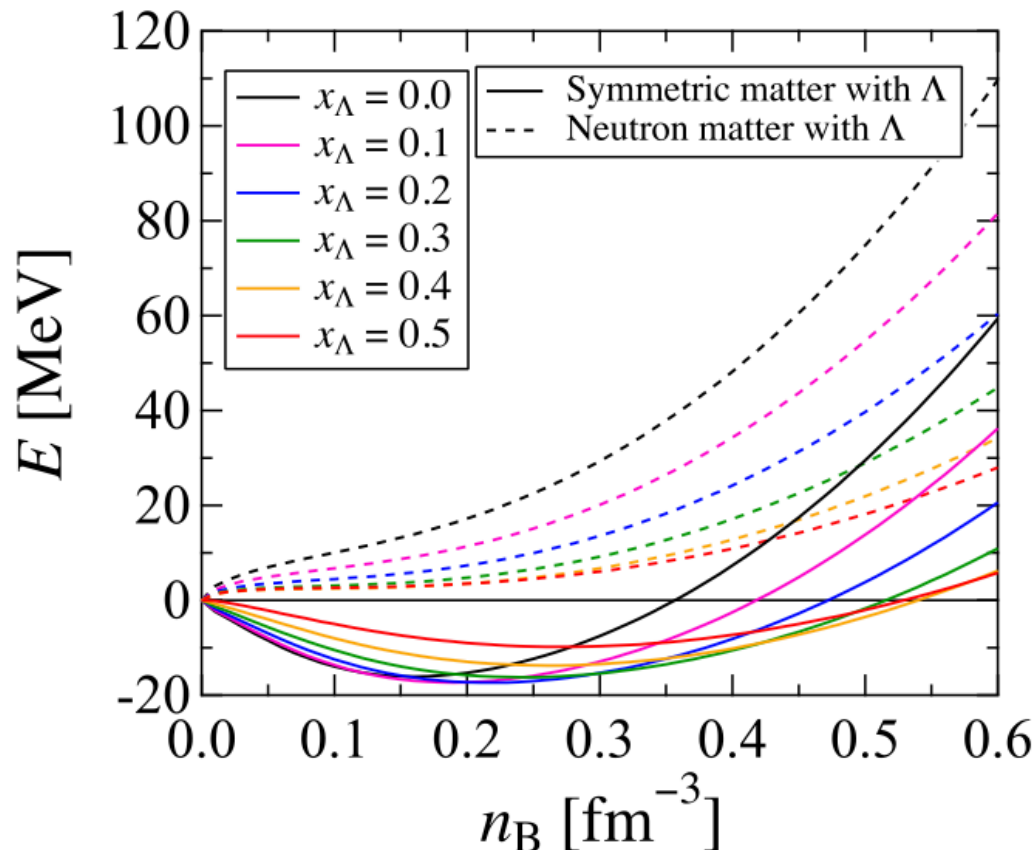
Jastrow wave function

$$\Psi = \text{Sym} \left[\prod_{i < j} f_{ij} \right] \Phi_F$$

f_{ij} : Correlation function (Variational function)

Φ_F : Fermi-gas wave function

$E(n_B, x_p, x_\Lambda)$: The expectation value of H in the two-body cluster approximation



- Baryon number density

$$n_B = n_p + n_n + n_\Lambda$$

- Particle fraction

$$x_i = n_i / n_B \quad (i = p, n, \Lambda)$$

(HT *et al.*, PRC 93 (2016) 035808)

Free energy at finite temperature

We follow the prescription proposed by Schmidt and Pandharipande.

(K. E. Schmidt et al., Phys. Lett. 87B(1979) 11, A. Mukherjee et al., PRC 75(2007) 035802)

Free energy per baryon $F(n_B, x_p, x_\Lambda, T)$

$$F = U_0 - TS_0$$

- **Approximate Internal Energy : U_0**

$$E[f_{ij}, n_{T=0i}(k)] \longrightarrow U_0[f_{ij}, n_{Ti}(k)]$$

**Occupation probability
at $T = 0$ MeV**

$$n_{T=0i}(k) = \Theta(k_{Fi} - k)$$

Average occupation probability

$$n_{Ti}(k) = \left\{ 1 + \exp \left[\frac{\varepsilon_i(k) - \mu_{0i}}{k_B T} \right] \right\}^{-1}$$

$$\varepsilon_i(k) = \hbar^2 k^2 / (2m_i^*)$$

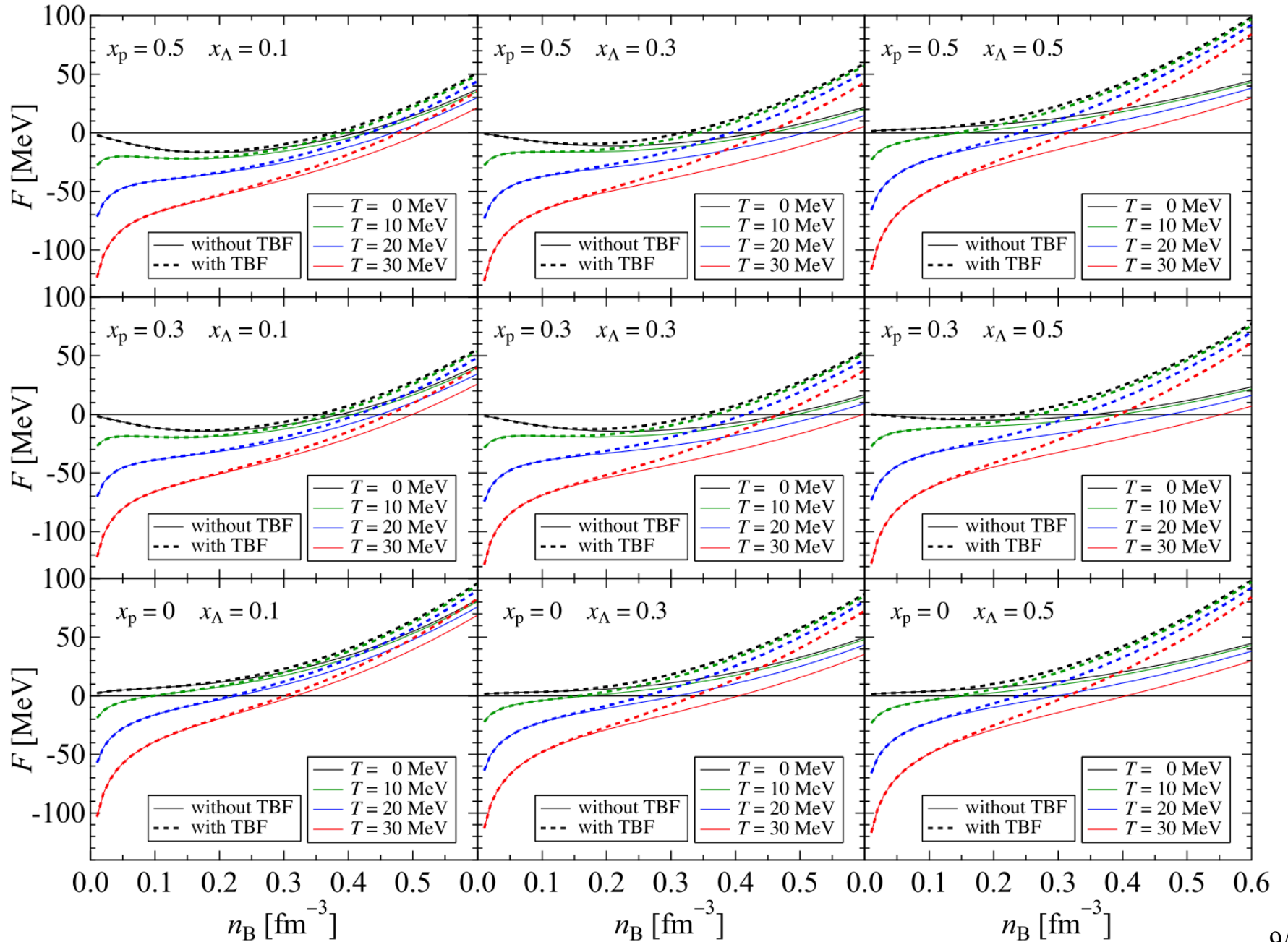
m_i^* : Effective mass ($i = p, n, \Lambda$)

- **Approximate Entropy : S_0**

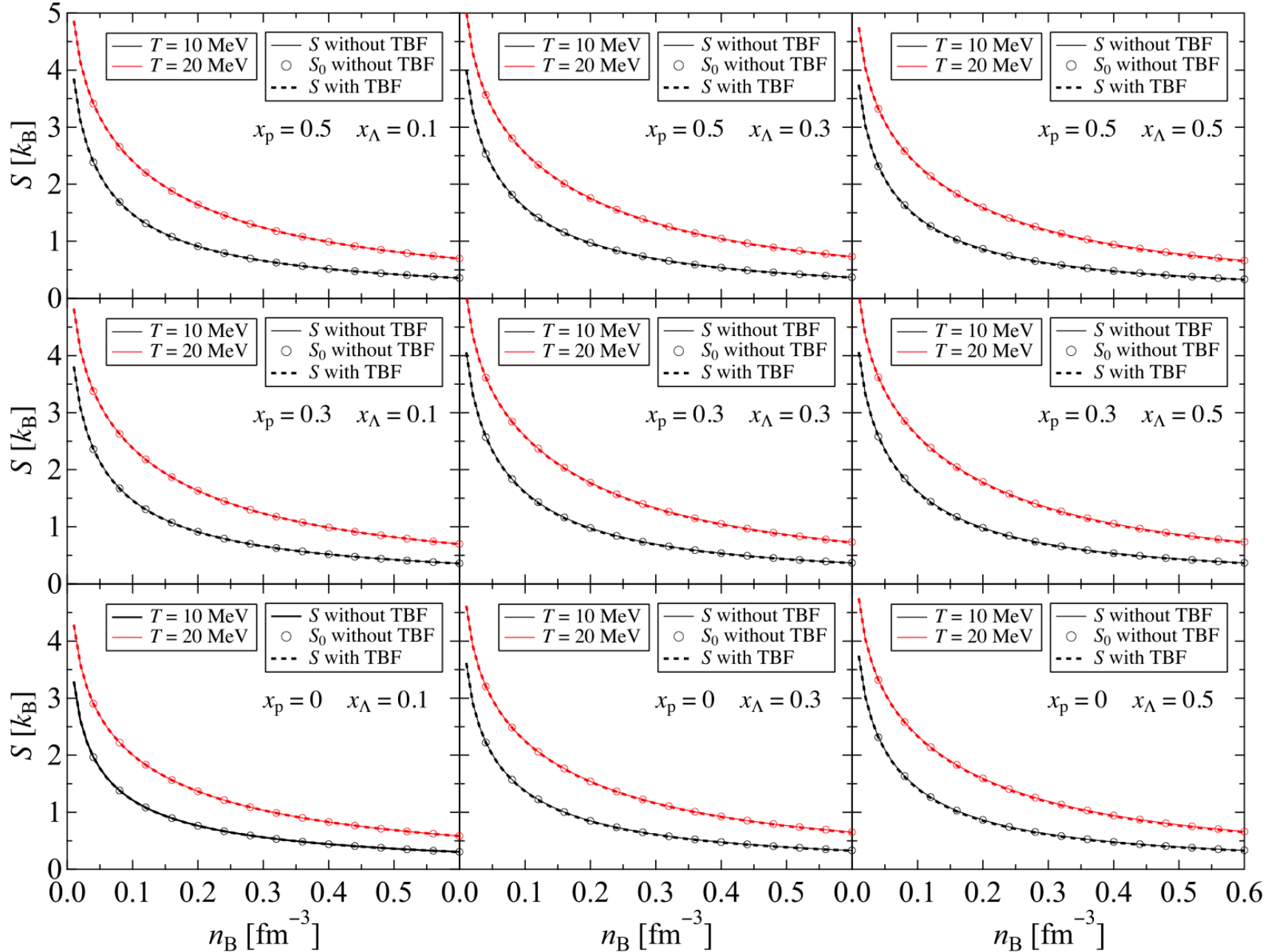
Expressed with $n_{Ti}(k)$ as in the case of the non-interacting quasi-baryon system

Free energy F is minimized with respect to m_i^ .*

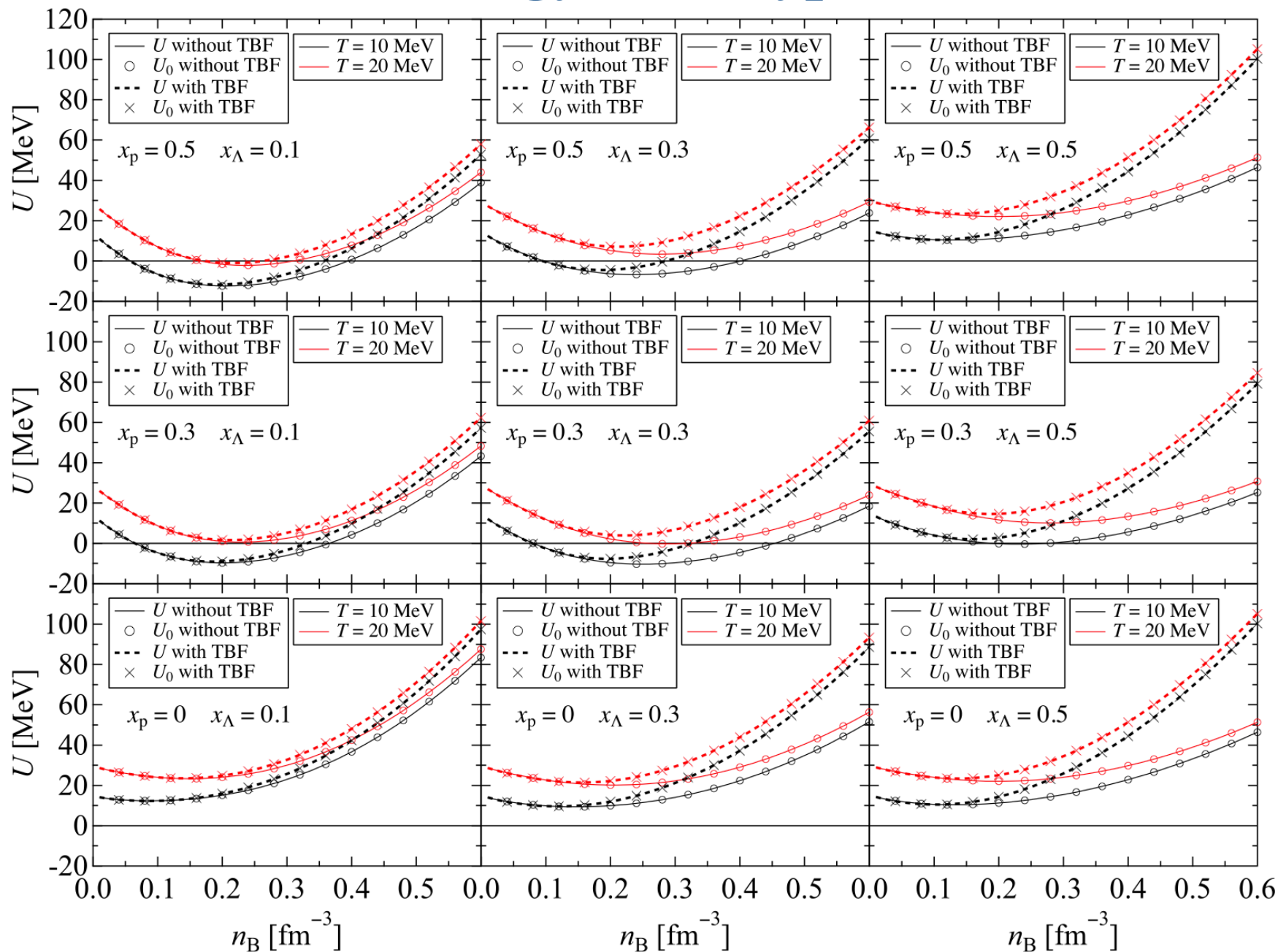
Free Energy of Λ Hyperon Matter



Entropy of Λ Hyperon Matter



Internal Energy of Λ Hyperon Matter



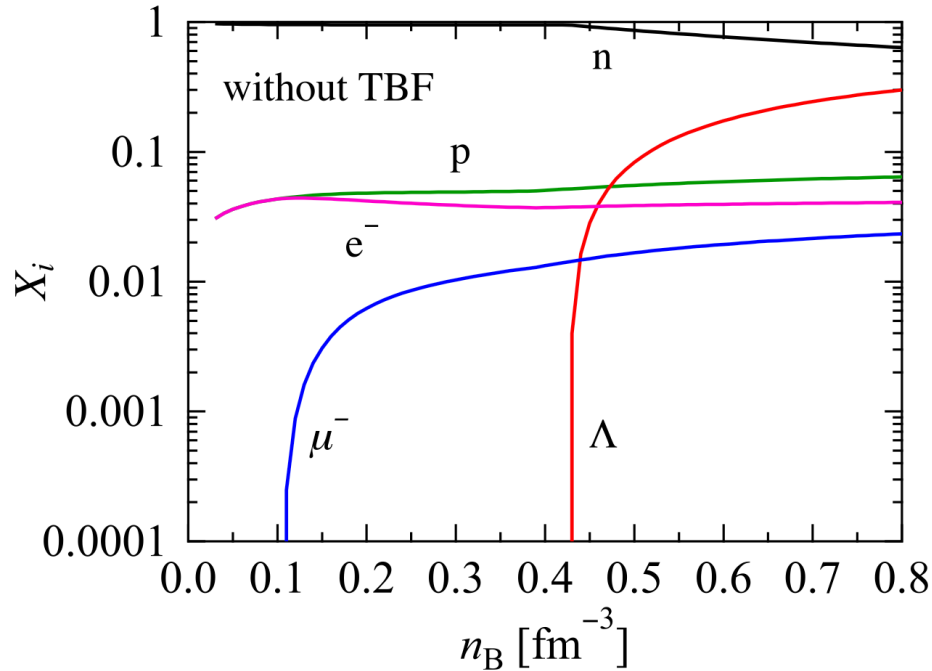
3. Application to Compact Stars

*The obtained EOS is applied to
the calculations of cold and hot neutron star structures.*

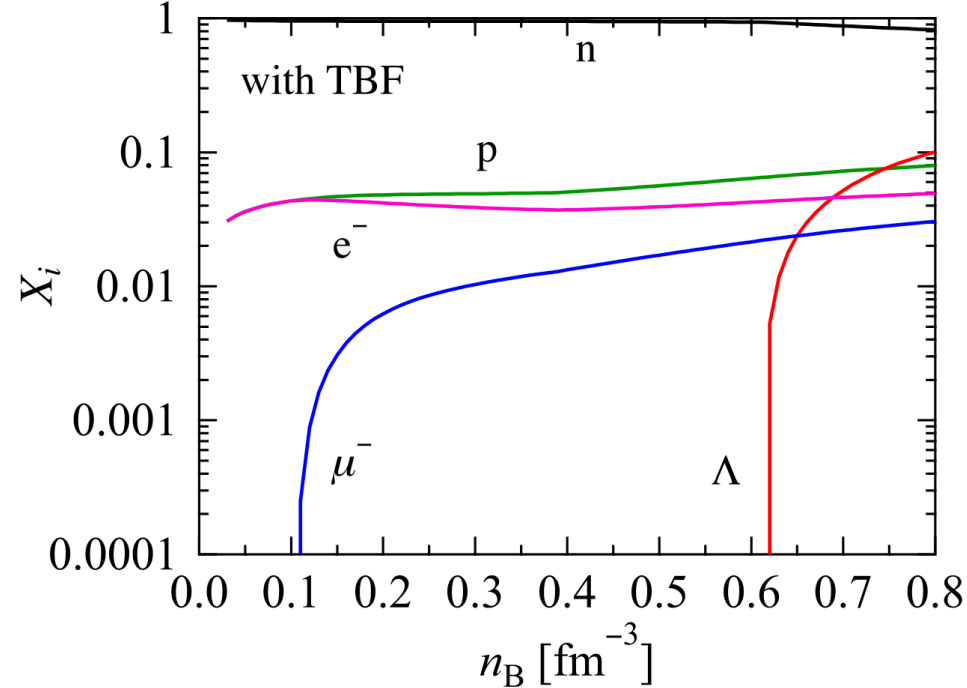
- 1. Cold Neutron Star (CNS) matter** (*Time after bounce t_{pb} \sim some minutes*)
 - $T = 0$ MeV
 - Charge neutral and β -stable mixtures of n, p, Λ , e^- , and μ^-
- 2. Proto Neutron Star (PNS) matter** (*$t_{\text{pb}} \sim 10 - 30s$*)
 - Isentropic matter (The entropy per baryon $S \sim 1-2$)
 - Charge neutral and β -stable mixtures of n, p, Λ , e^- , e^+ , and γ
- 3. Supernova (SN) matter** (*$t_{\text{pb}} \sim 0.5 - 1s$*)
 - Isentropic matter (The entropy per baryon $S \sim 1-2$)
 - Charge neutral and β -stable mixtures of n, p, Λ , e^- , e^+ , ν_e , $\bar{\nu}_e$, and γ
 - Fixed lepton number fraction $Y_l = Y_e + Y_{\nu_e} \sim 0.3 - 0.4$

Particle Fraction of CNS matter

- 1. Cold Neutron Star (CNS) matter** (*Time after bounce t_{pb} \sim some minutes*)
- $T = 0$ MeV
 - Charge neutral and β -stable mixtures of n , p , Λ , e^- , and μ^-

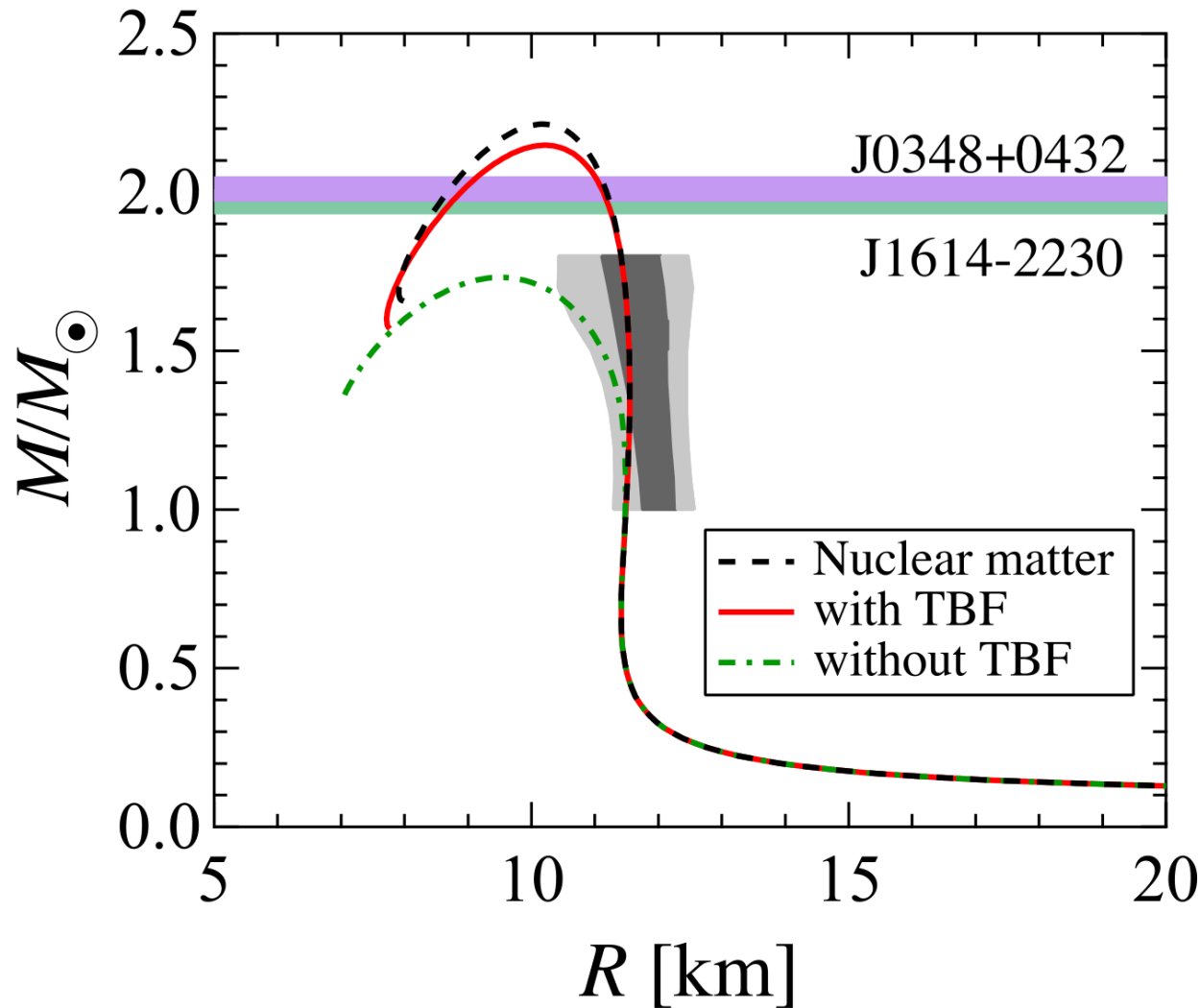


Particle fractions of CNS matter **without** hyperon three-baryon forces



Particle fractions of CNS matter **with** hyperon three-baryon forces

Mass-Radius relation of CNS



J0348+0432: Science 340 (2013) 1233232

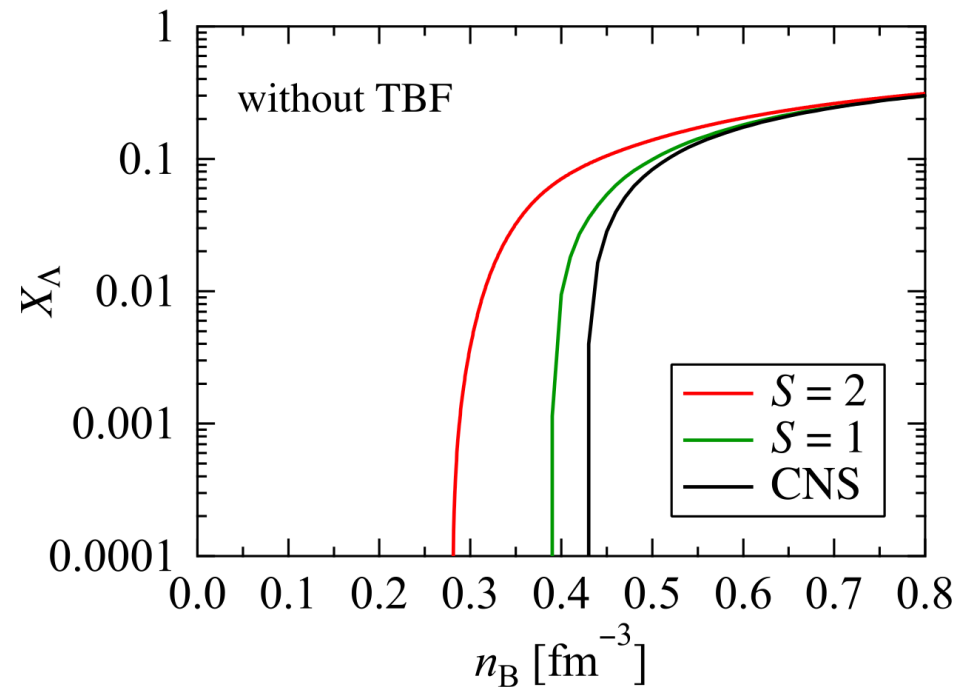
J1614-2230: Nature 467 (2010) 1081

Shaded region is the observationally suggested region by Steiner et al.
(Astrophys. J. 722 (2010) 33)

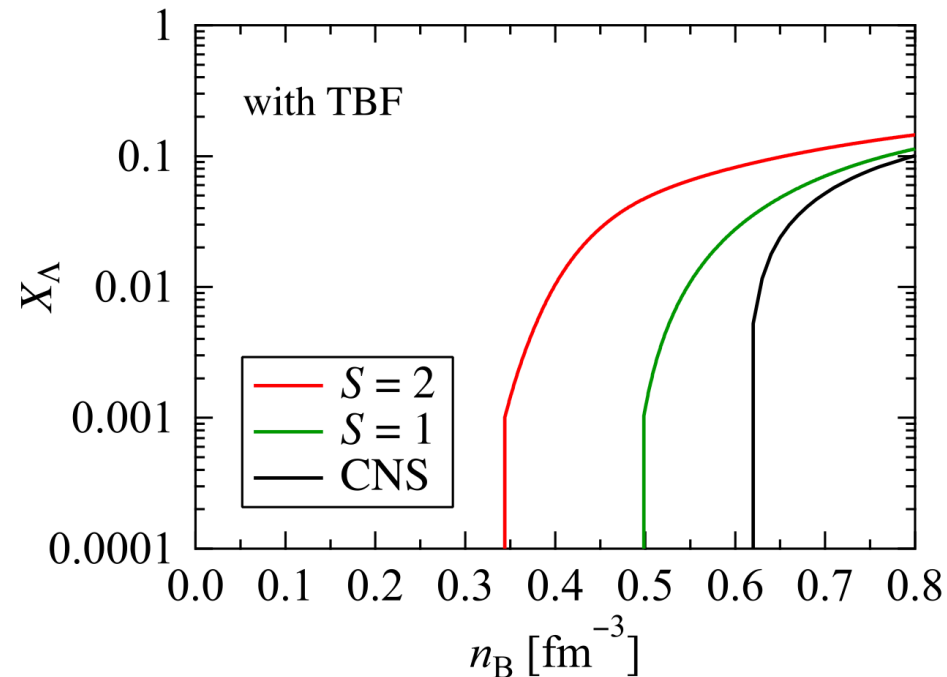
Λ hyperon Fraction of PNS matter

2. Proto Neutron Star (PNS) matter ($t_{\text{pb}} \sim 10 - 30s$)

- Isentropic matter (The entropy per baryon $S \sim 1-2$)
- Charge neutral and β -stable mixtures of n , p , Λ , e^- , e^+ , and γ



Λ fractions of PNS matter **without** hyperon three-baryon forces

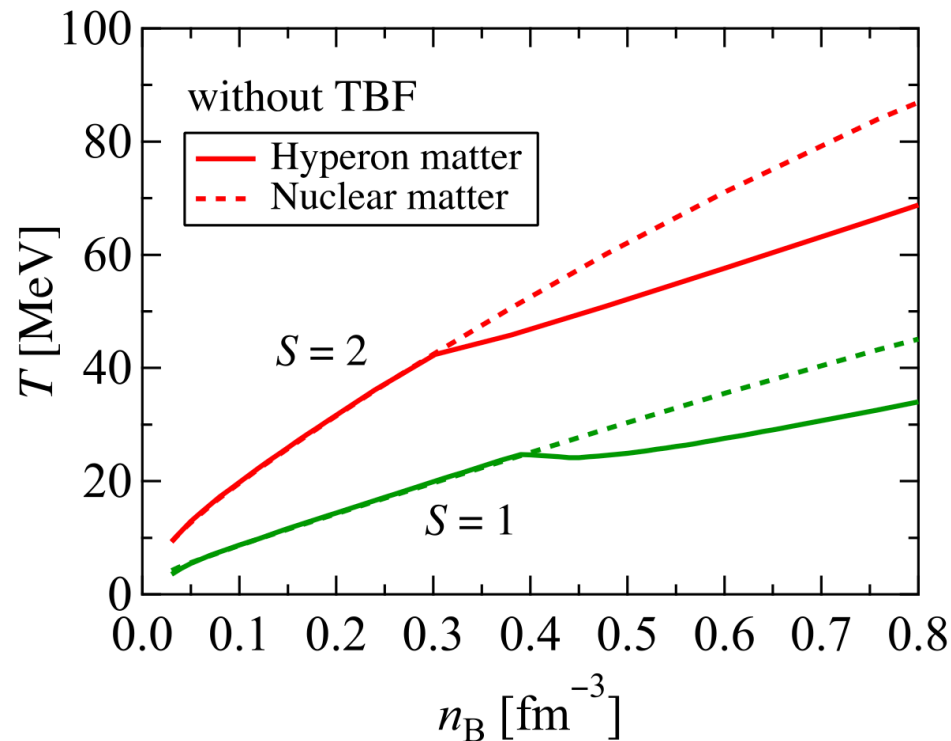


Λ fractions of PNS matter **with** hyperon three-baryon forces

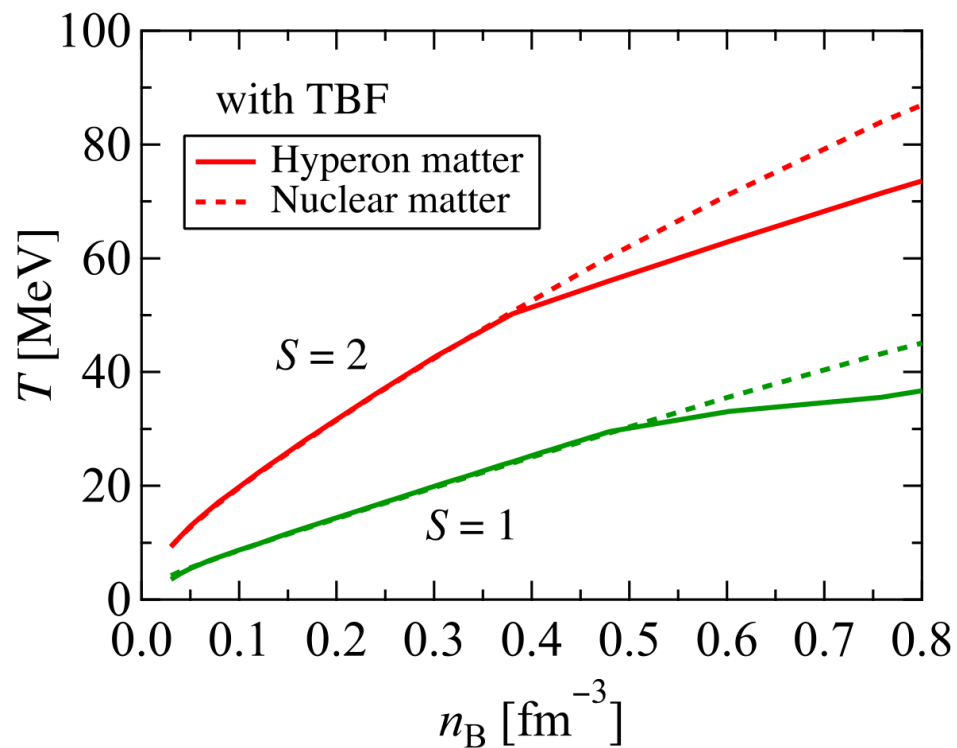
Temperature of PNS matter

2. Proto Neutron Star (PNS) matter ($t_{\text{pb}} \sim 10 - 30s$)

- Isentropic matter (The entropy per baryon $S \sim 1-2$)
- Charge neutral and β -stable mixtures of n , p , Λ , e^- , e^+ , and γ



Temperature of PNS matter **without**
hyperon three-baryon forces

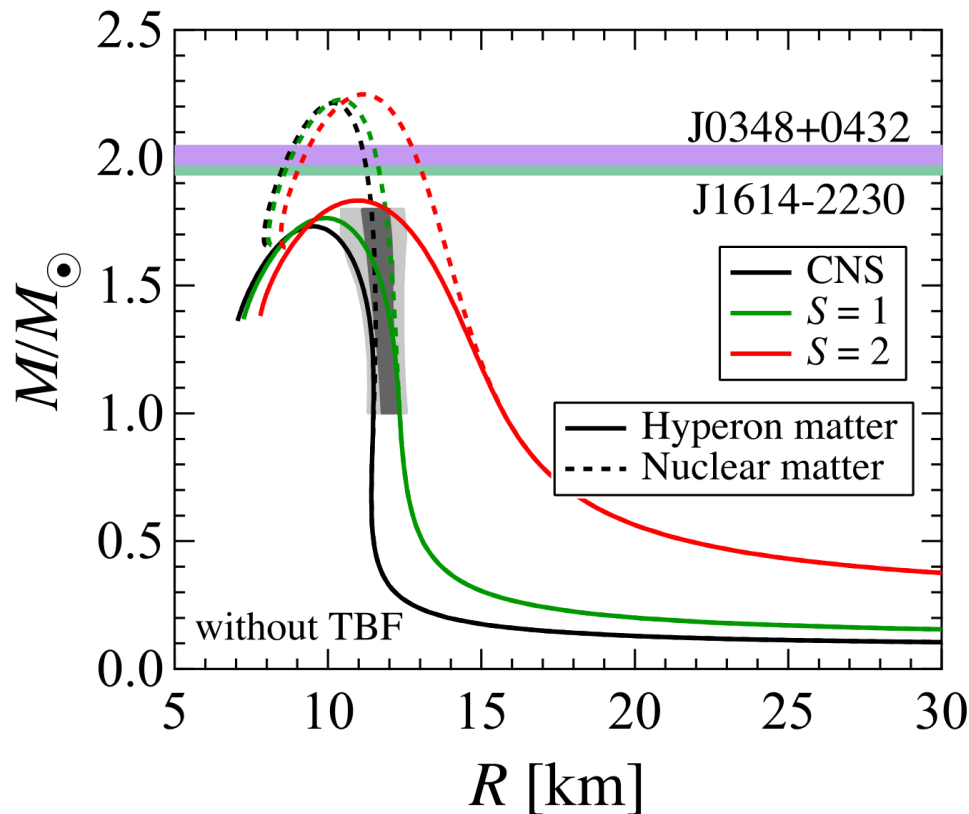


Temperature of PNS matter **with**
hyperon three-baryon forces

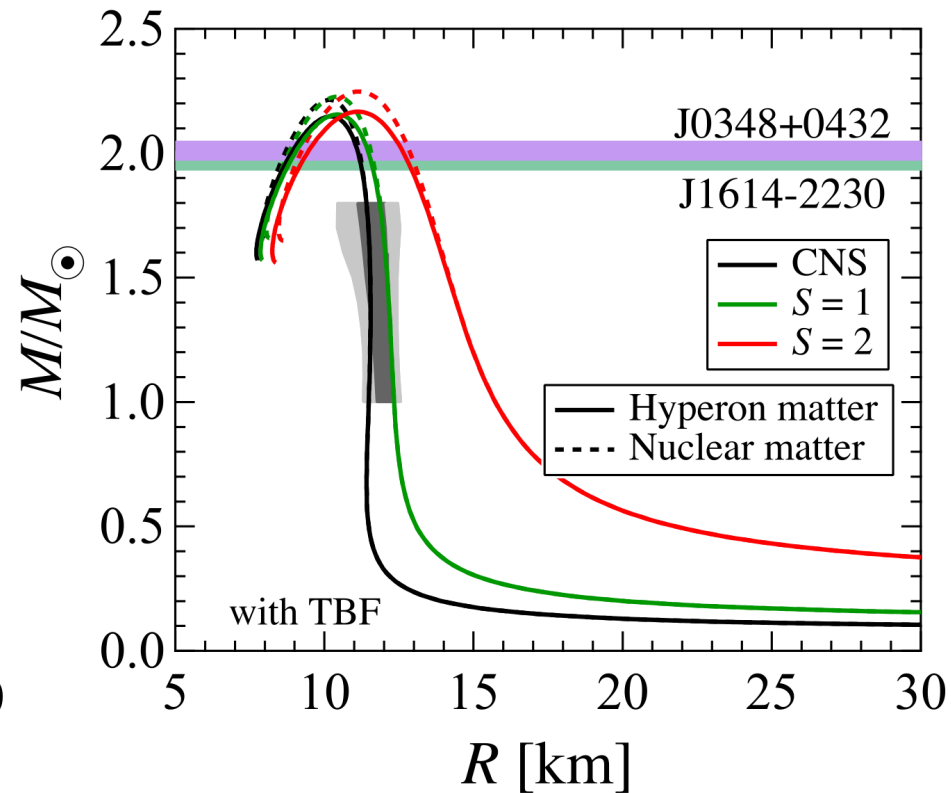
Mass-Radius relation of PNS

2. Proto Neutron Star (PNS) matter ($t_{\text{pb}} \sim 10 - 30\text{s}$)

- Isentropic matter (The entropy per baryon $S \sim 1-2$)
- Charge neutral and β -stable mixtures of n , p , Λ , e^- , e^+ , and γ



Mass-radius relations of PNS **without** hyperon three-baryon forces

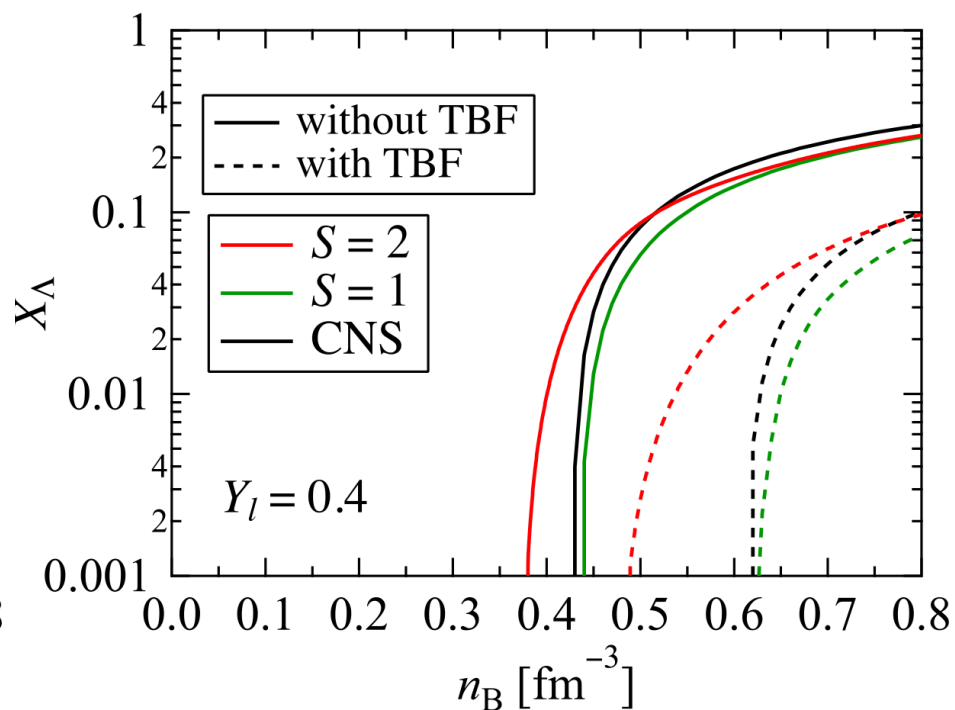
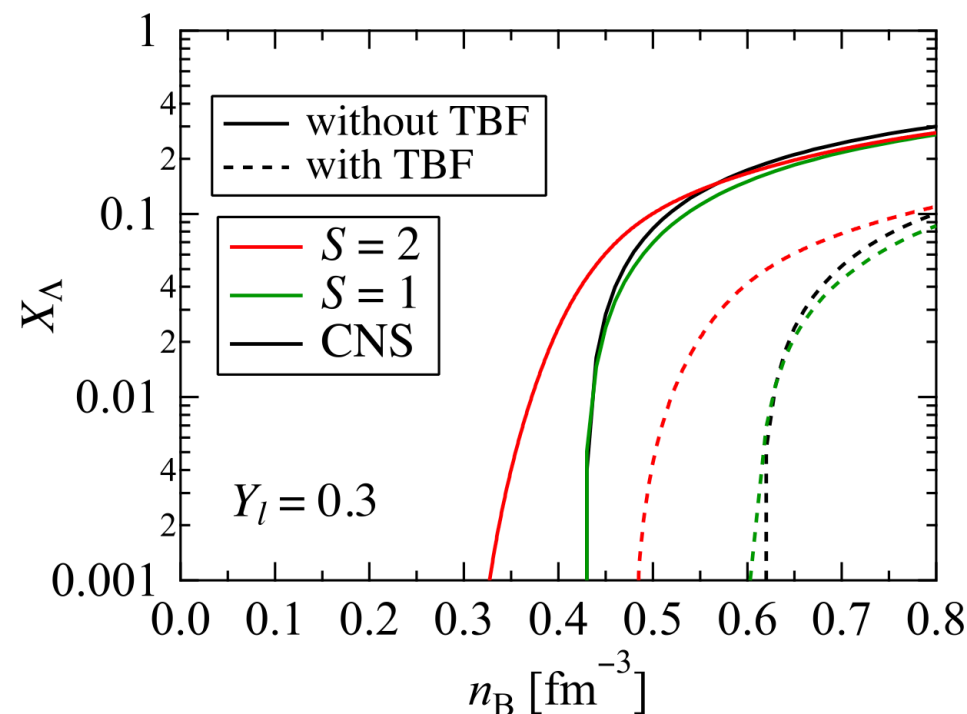


Mass-radius relations of PNS **with** hyperon three-baryon forces

Λ hyperon Fraction of SN matter

3. Supernova (SN) matter ($t_{\text{pb}} \sim 0.5 - 1\text{s}$)

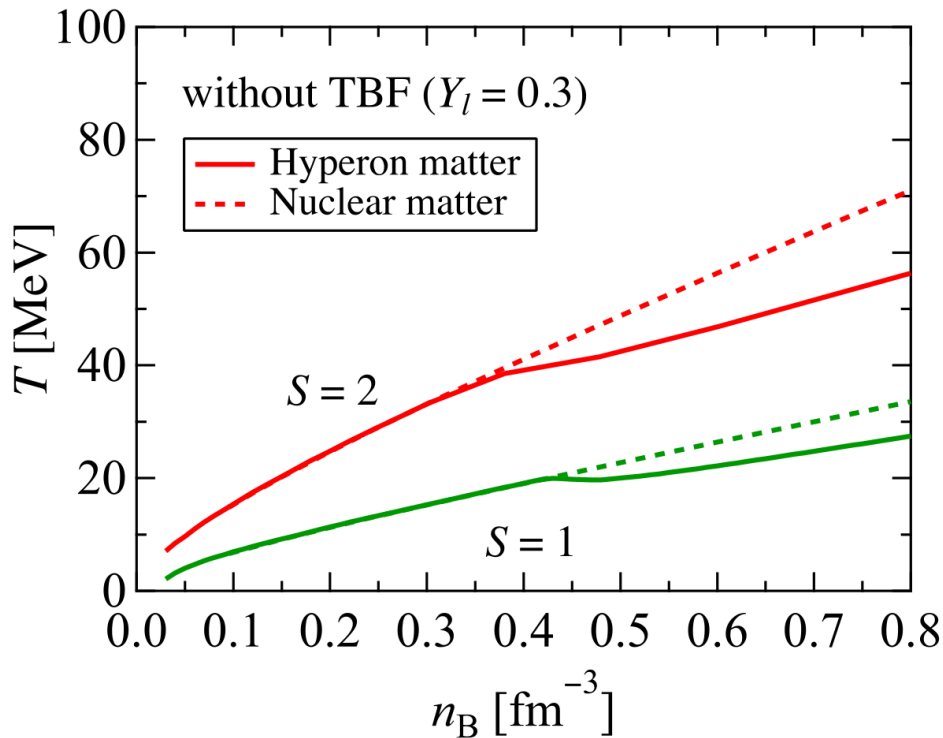
- Isentropic matter (The entropy per baryon $S \sim 1-2$)
- Charge neutral and β -stable mixtures of n , p , Λ , e^- , e^+ , ν_e , $\bar{\nu}_e$, and γ
- Fixed lepton number fraction $Y_l = Y_e + Y_{\nu_e} \sim 0.3 - 0.4$



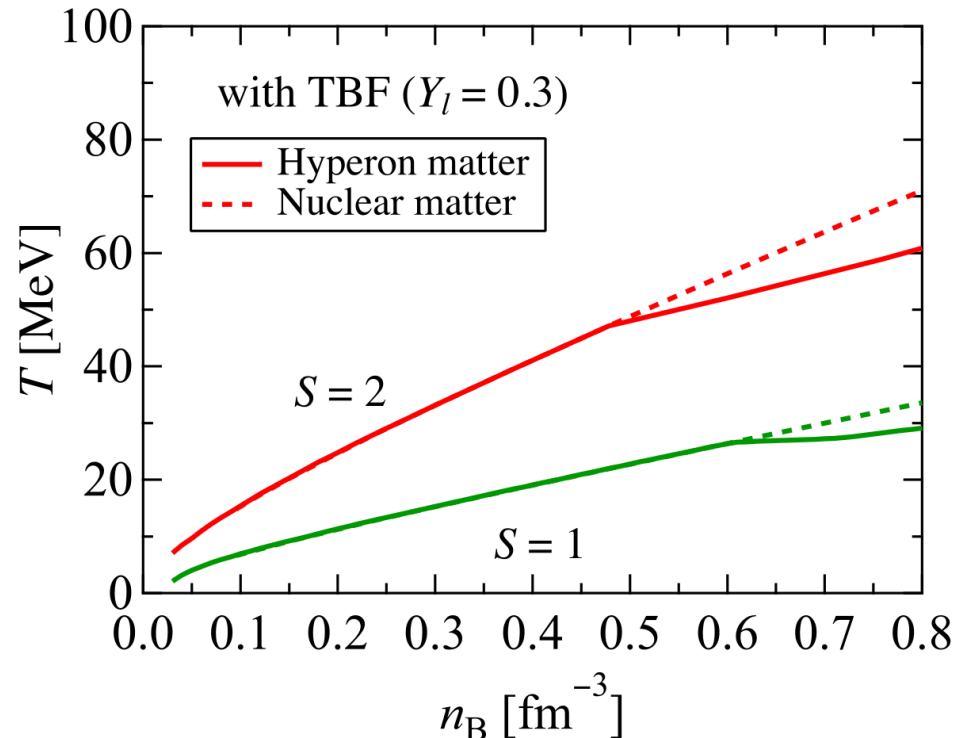
Temperature of SN matter

3. Supernova (SN) matter ($t_{\text{pb}} \sim 0.5 - 1\text{s}$)

- Isentropic matter (The entropy per baryon $S \sim 1-2$)
- Charge neutral and β -stable mixtures of n , p , Λ , e^- , e^+ , ν_e , $\bar{\nu}_e$, and γ
- Fixed lepton number fraction $Y_l = Y_e + Y_{\nu_e} \sim 0.3 - 0.4$



Temperature of SN matter **without** hyperon three-baryon forces

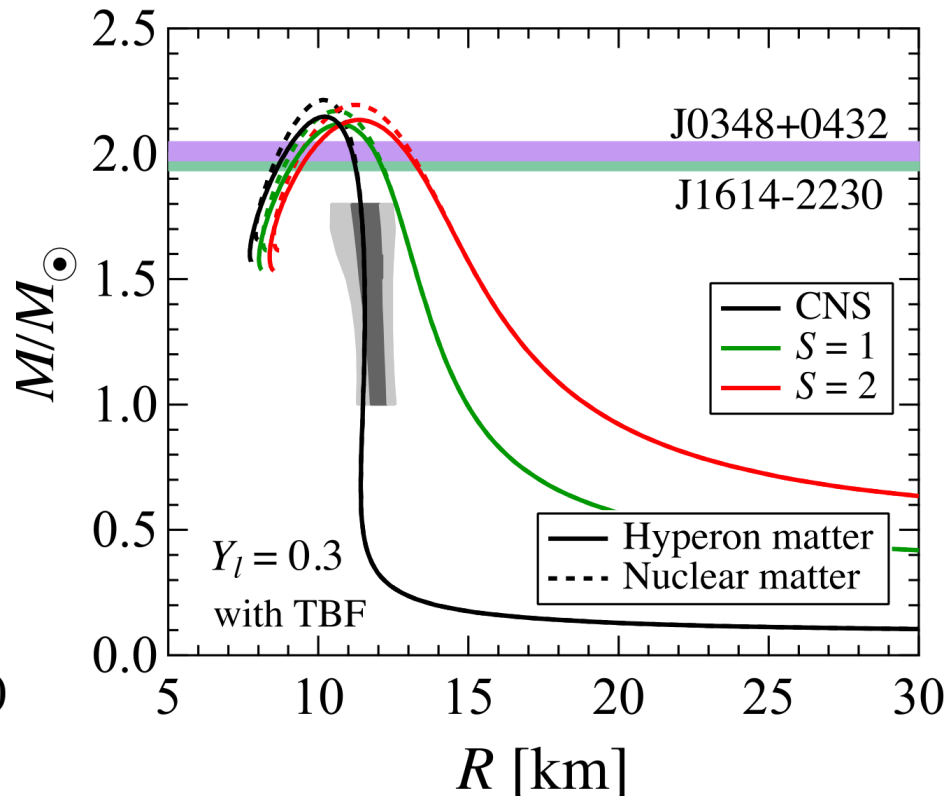
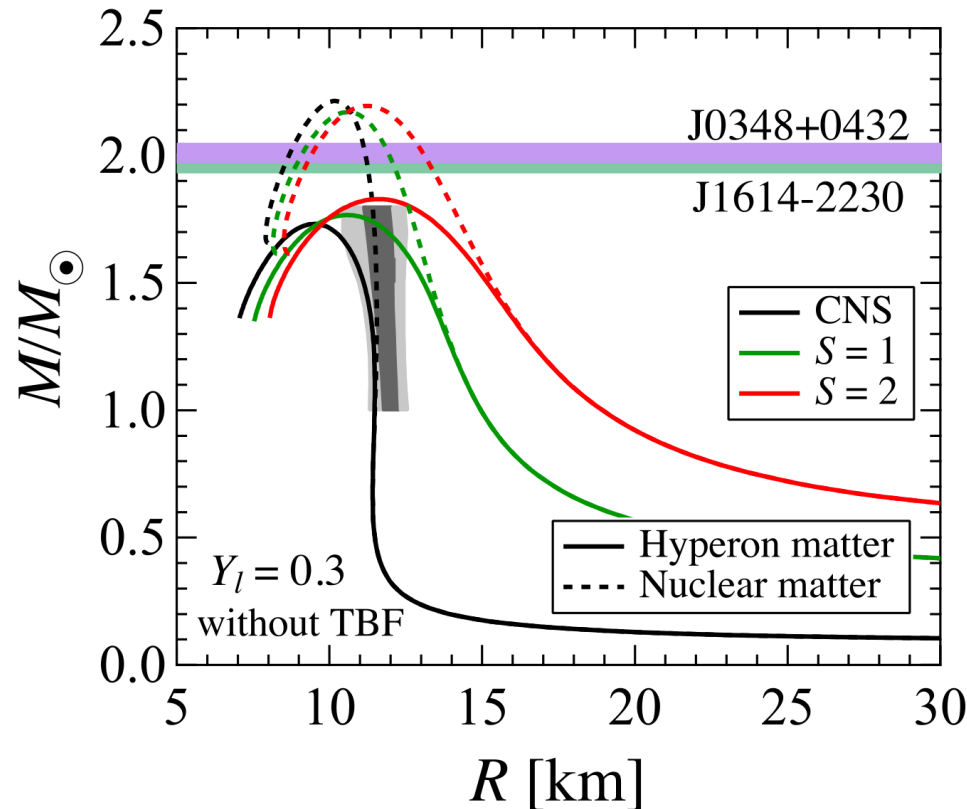


Temperature of SN matter **with** hyperon three-baryon forces

Mass-Radius relation of SN core

3. Supernova (SN) matter ($t_{\text{pb}} \sim 0.5 - 1s$)

- Isentropic matter (The entropy per baryon $S \sim 1-2$)
- Charge neutral and β -stable mixtures of $n, p, \Lambda, e^-, e^+, \nu_e, \bar{\nu}_e$, and γ
- Fixed lepton number fraction $Y_l = Y_e + Y_{\nu_e} \sim 0.3 - 0.4$



Summary

We construct the EOS for nuclear matter including Λ hyperons at zero and finite temperatures by the variational method.

Variational method for hyperon matter

- The obtained thermodynamic quantities are **reasonable**.

Application of the EOS to compact stars

- The effect of Λ hyperons on neutrino-free matter becomes larger **at higher entropies**.
- Trapped neutrino reduces Λ mixing in SN matter.

Does the three-baryon force play key role to understand the inner structure of compact stars?