Lattice QCD simulations on the S=-2 baryon-baryon interaction

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for HAL QCD Collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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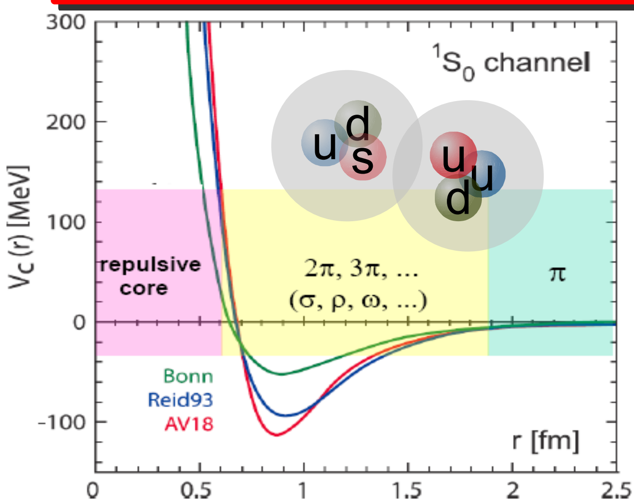
Introduction

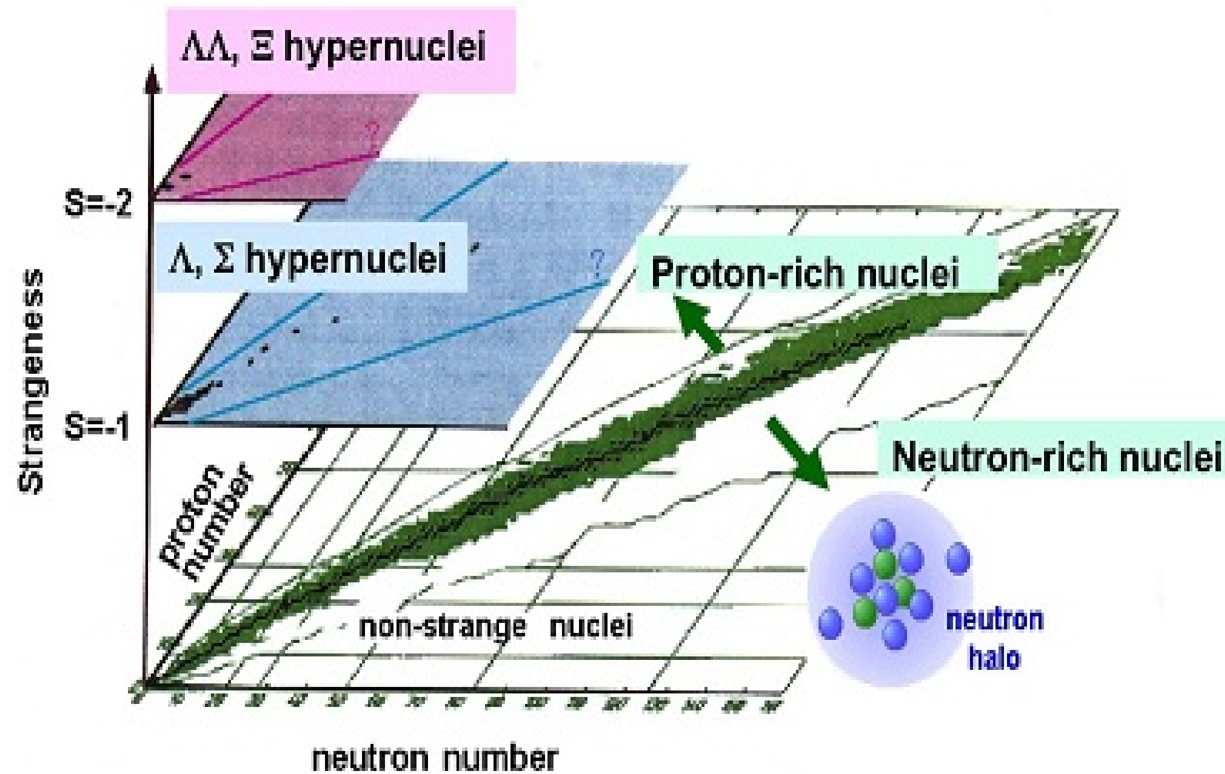
Introduction

BB interactions are crucial to investigate the nuclear phenomena

Once we obtain "realistic" nuclear potentials,
we apply them to the (hyper) nuclear structure calculations.

BB interaction (potential)





Properties of nuclear potential

- **State dependence (spin, isospin)**
- **Long range attraction**
- **Short range repulsion**

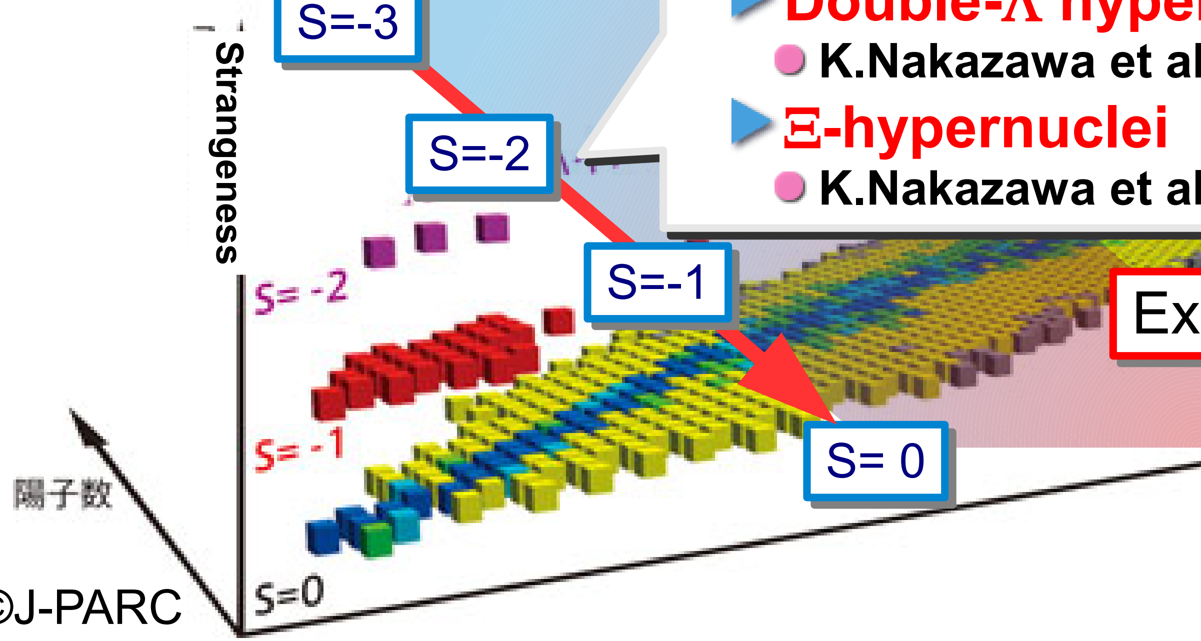
Baryon interactions from fundamental theory, QCD, are highly awaited.

Introduction

Aim : Nuclear structures and exotic states from QCD

Lattice QCD simulation

- **Advantageous for more strange quarks**
- **Signals getting worse as increasing the number of light quarks.**
- **Complementary role to experiment.**



Main topics of S=-2 multi-baryon system

- **H-dibaryon**
  - **R.L. Jaffe, PRL 38 (1977) 195**
- **Double-Λ hypernuclei**
  - **K.Nakazawa et al, KEK-E176 Collaboration**
- **Ξ-hypernuclei**
  - **K.Nakazawa et al, KEK-E373 Collaboration**

Experiment

Interests of S=-2 multi-baryon system

H-dibaryon

- The flavor singlet state with J=0 predicted by R.L. Jaffe.
  - Strongly attractive color magnetic interaction.
  - No quark Pauli principle for flavor singlet state.

Double-Λ hypernucleus

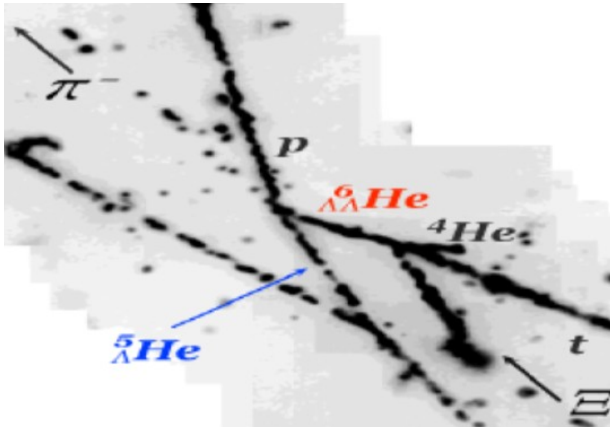
- Conclusions of the “NAGARA Event”

K.Nakazawa and KEK-E176 & E373 Collaborators

Λ–N attraction

Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV

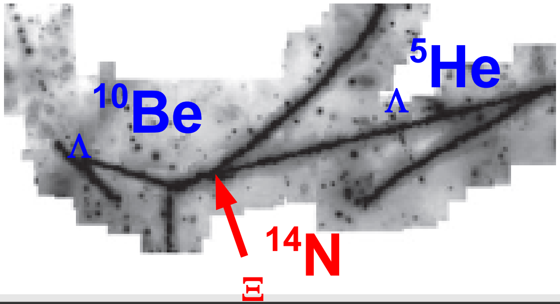


Ξ hypernucleus

- Conclusions of the “KISO Event”

K.Nakazawa and KEK-E373 Collaborators

Ξ–N attraction



HAL QCD method

QCD to hadronic interactions

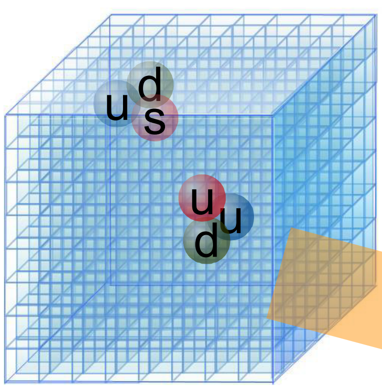
HAL QCD method enables us to derive baryon interactions directly from QCD

N. Ishii, S. Aoki and T. Hatsuda, PRL99 (2007) 022001

QCD Lagrangian

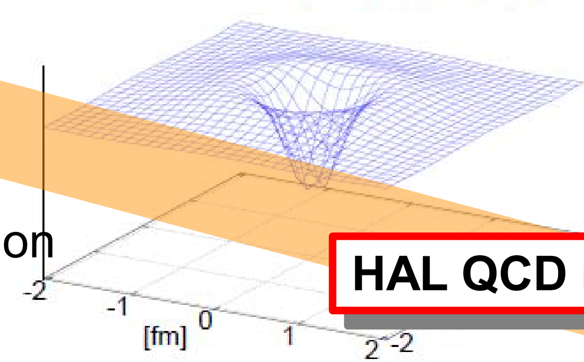
$$L_{QCD}=\bar{q}(i\gamma_\mu D^\mu - m)q + \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$$

Lattice QCD simulation



Evaluate NBS wave function on the lattice

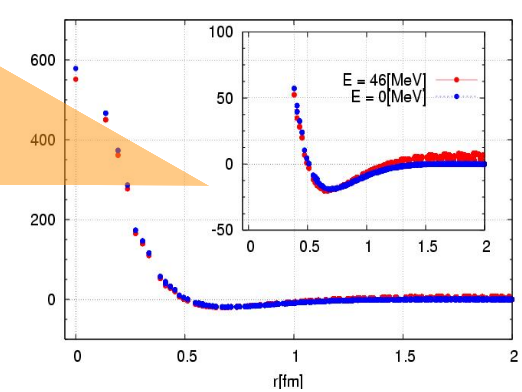
NBS wave function



HAL QCD method

Our potential is faithful to the phase shift by construction

BB interaction (potential)



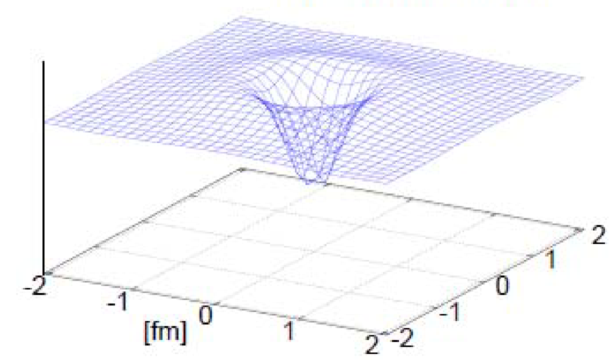
Potentials are evaluated via Schrodinger eq

Nambu-Bethe-Salpeter wave function

Definition : equal-time NBS w.f.

$$\Psi^{B_1 B_2}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0 | B_{1\alpha}(t,\vec{x}+\vec{r}) B_{2\alpha}(t,\vec{x}) | E \rangle$$

E : Total energy of the system



Local composite interpolating operators

$$B_\alpha = \epsilon^{abc}(q_a^T C \gamma_5 q_b) q_{c\alpha}$$

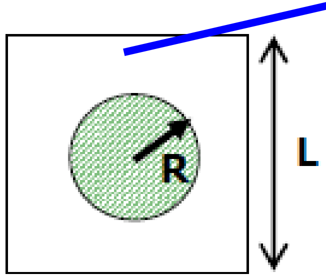
- **It satisfies the Helmholtz eq. in asymptotic region :**

$$(p^2+\nabla^2)\Psi(E,\vec{r})=0$$

- **Asymptotic form of NBS wave function**

C.-J.D.Lin et al.,NPB619 (2001) 467.

$$\Psi(E,\vec{r}) \simeq A \frac{\sin(pr+\delta(E))}{pr}$$



NBS w.f. is the same asymptotic form with quantum mechanical w. f..

(NBS wave function is characterized by phase shift)

$$S \equiv e^{i\delta}$$

N. Ishii et al Phys. Lett. B**712**(2012)437

Time-dependent Schrödinger like equation

Start with the normalized four-point correlator.

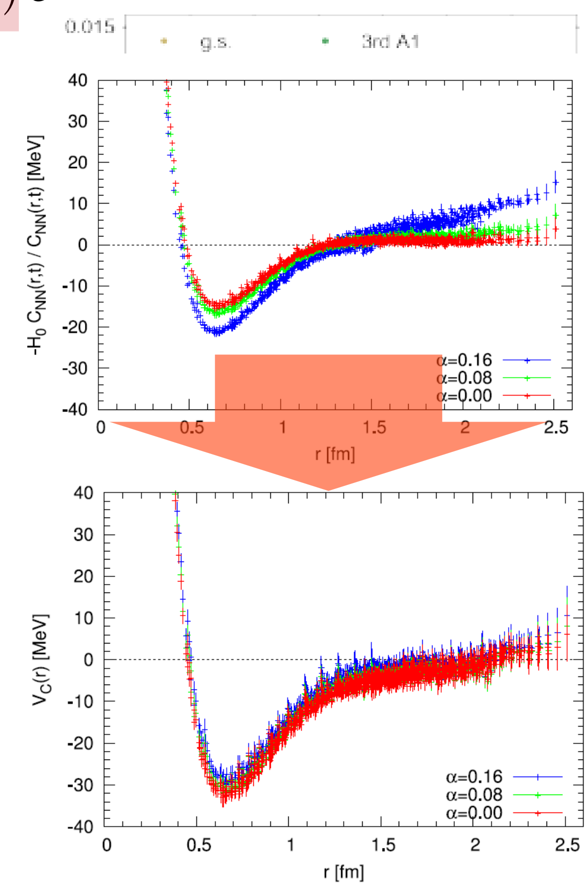
$$R_I^{B_1 B_2}(t,\vec{r}) = F^{B_1 B_2}{}_I(t,\vec{r})\, e^{(m_1+m_2)t}$$

$$= A_0 \Psi(\vec{r},E_0) e^{-(E_0-m_1-m_2)t} + A_1 \Psi(\vec{r},E_1) e^{-(E_1-m_1-m_2)t} + \cdots$$

$$\left(\frac{p_0^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_0) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_0)d^3r'$$

$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu}$$

$$\left(\frac{p_1^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_1) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_1)d^3r'$$



A single state saturation is not required!!

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1 B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,')R_I^{B_1 B_2}(t,\vec{r})d^3r'$$

Derivative (velocity) expansion of U

$$U(\vec{r},\vec{r}\,') = \left[V_C(r)+S_{12}V_T(r)\right]+\left[\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)\right]+O(\nabla^2)$$

HAL QCD method (coupled-channel)

NBS wave function

$$\Psi^\alpha(E_i,\vec{r})e^{-E_i t} = \langle 0|(B_1 B_2)^\alpha(\vec{r})|E_i\rangle$$
$$\Psi^\beta(E_i,\vec{r})e^{-E_i t} = \langle 0|(B_3 B_4)^\beta(\vec{r})|E_i\rangle$$

$$\int dr\, \tilde{\Psi}_\beta(E',\vec{r})\Psi^\gamma(E,\vec{r}) = \delta(E'-E)\delta_\beta^\gamma$$

$$R_E^{B_1 B_2}(t,\vec{r}) = A_E \Psi^{B_1 B_2}(\vec{r},E)e^{(-E+m_1+m_2)t}$$

Leading order of velocity expansion and time-derivative method.

Modified coupled-channel Schrödinger equation

$$\begin{pmatrix} (-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\alpha})R^\alpha_{E_0}(t,\vec{r}) \\ (-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta})R^\beta_{E_0}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} V^\alpha{}_\alpha(\vec{r}) & V^\alpha{}_\beta(\vec{r})\Delta^\alpha{}_\beta(t) \\ V^\beta{}_\alpha(\vec{r})\Delta^\beta{}_\alpha(t) & V^\beta{}_\beta(\vec{r}) \end{pmatrix} \begin{pmatrix} R^\alpha_{E_0}(t,\vec{r}) \\ R^\beta_{E_0}(t,\vec{r}) \end{pmatrix}$$

$$(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta})R^\beta_{E_1}(t,\vec{r}) \quad \ldots \quad (\vec{r})\Delta^\alpha{}_\beta(t) \quad V^\beta{}_\beta(\vec{r}) \quad \begin{pmatrix} R^\alpha_{E_1}(t,\vec{r}) \\ R^\beta_{E_1}(t,\vec{r}) \end{pmatrix}$$

$$\Delta^\alpha_\beta = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

S.Aoki et al [HAL QCD Collab.] Proc. Jpn. Acad., Ser. B, 87 509
K.Sasaki et al [HAL QCD Collab.] PTEP no 11 (2015) 113B01

Potential

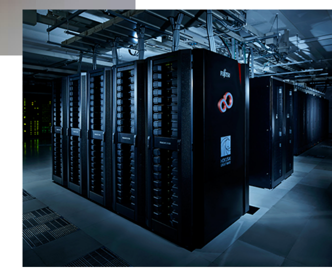
Considering two different energy eigen states

$$\begin{pmatrix} V^\alpha{}_\alpha(\vec{r}) & V^\alpha{}_\beta(\vec{r})\Delta^\alpha_\beta \\ V^\beta{}_\alpha(\vec{r})\Delta^\beta_\alpha & V^\beta{}_\beta(\vec{r}) \end{pmatrix} = \begin{pmatrix} (\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t})R^\alpha_{E0}(t,\vec{r}) & (\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t})R^\alpha_{E1}(t,\vec{r}) \\ (\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t})R^\beta_{E0}(t,\vec{r}) & (\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t})R^\beta_{E1}(t,\vec{r}) \end{pmatrix} \begin{pmatrix} R^\alpha_{E0}(t,\vec{r}) & R^\alpha_{E1}(t,\vec{r}) \\ R^\beta_{E0}(t,\vec{r}) & R^\beta_{E1}(t,\vec{r}) \end{pmatrix}^{-1}$$

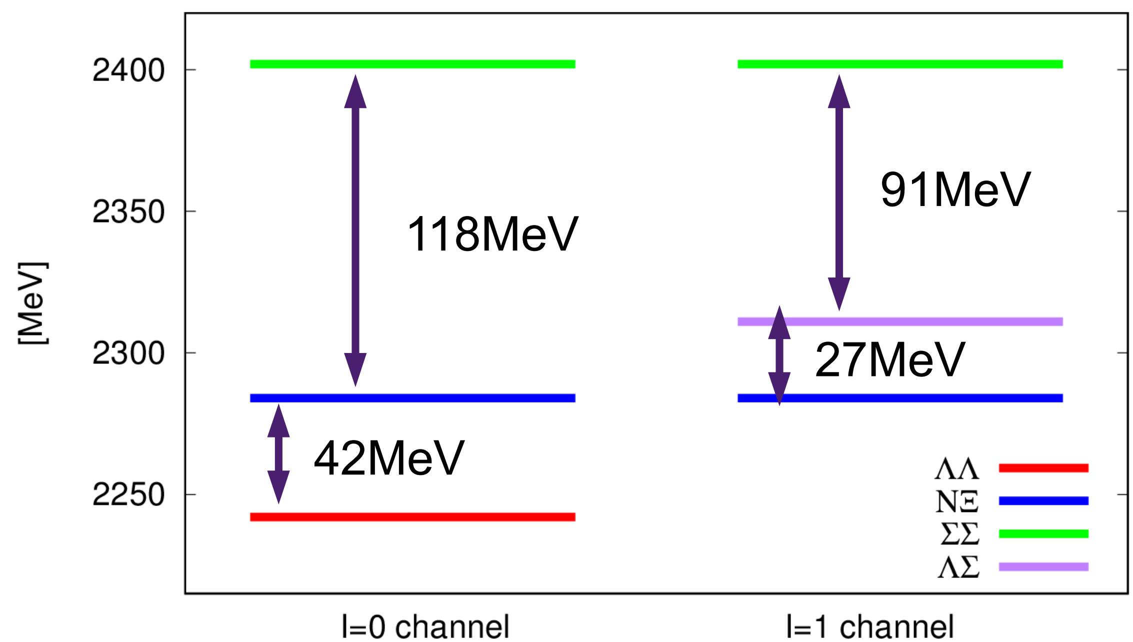
Numerical results

Numerical setup

- 2+1 flavor gauge configurations.
  - Iwasaki gauge action & O(a) improved Wilson quark action
  - a = 0.085 [fm], a^{-1} = 2.333 GeV.
  - 96^3x96 lattice, L = 8.12 [fm].
  - 414 confs x 28 sources x 4 rotations.
- Flat wall source is considered to produce S-wave B-B state.



	Mass [MeV]
π	146
K	525
m_π/m_K	0.28
N	956±12
Λ	1121±4
Σ	1201±3
Ξ	1328±3



Baryon-baryon system in S=-2 sector

Spin singlet states

Isospin	BB channels		
I=0	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$
I=1	$N\Xi$	$\Lambda\Sigma$	——
I=2	$\Sigma\Sigma$	——	——

Spin triplet states

Isospin	BB channels		
I=0	$N\Xi$	——	——
I=1	$N\Xi$	$\Lambda\Sigma$	$\Sigma\Sigma$

Relations between BB channels and SU(3) irreducible representations

$$8 \times 8 = 27 + 8_S + 1 + \overline{10} + 10 + 8_A$$

$J^P=0^+$, I=0

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

$J^P=1^+$, I=0

$$N\Xi \Leftrightarrow 8$$

$J^P=1^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ 10 \\ \overline{10} \end{pmatrix}$$

$J^P=0^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

$J^P=0^+$, I=2

$$\Sigma\Sigma \Leftrightarrow 8$$

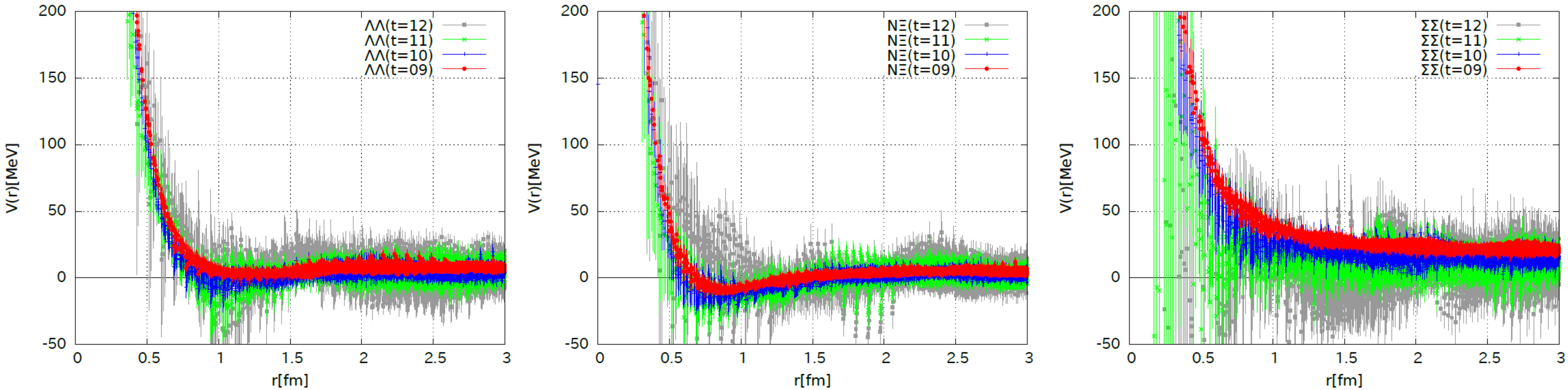
Features of flavor singlet interaction is integrated into the S=-2 $J^P=0^+$, I=0 system.

$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel

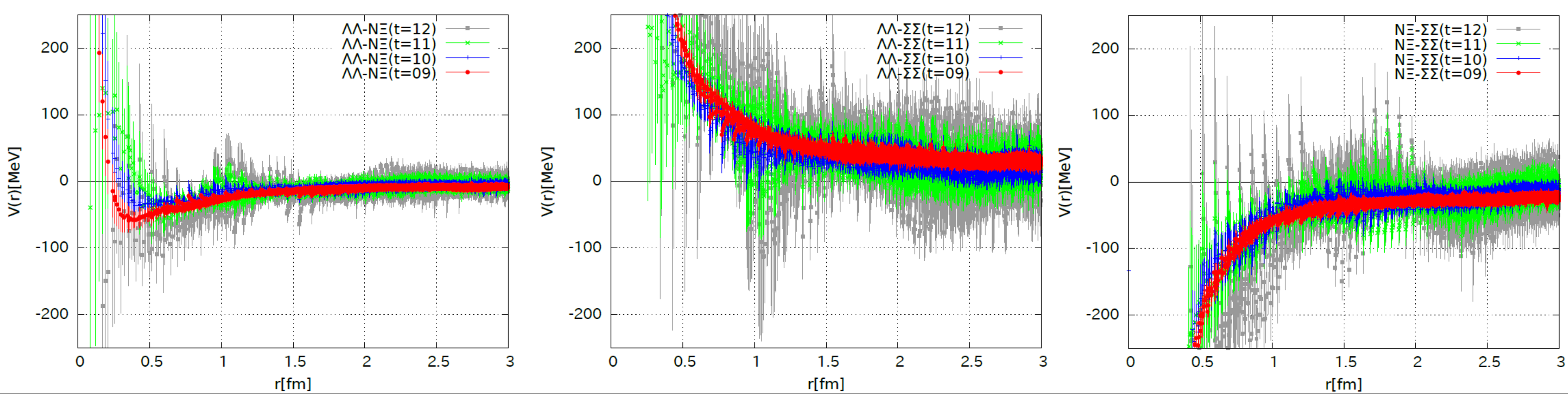
t=09
t=10
t=11
t=12

- N_f = 2+1 full QCD m_π=146MeV with L = 8.12fm

Diagonal elements



Off-diagonal elements

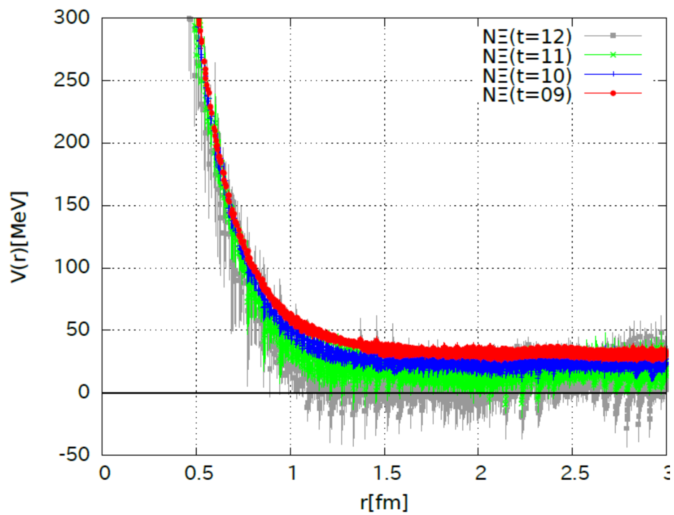


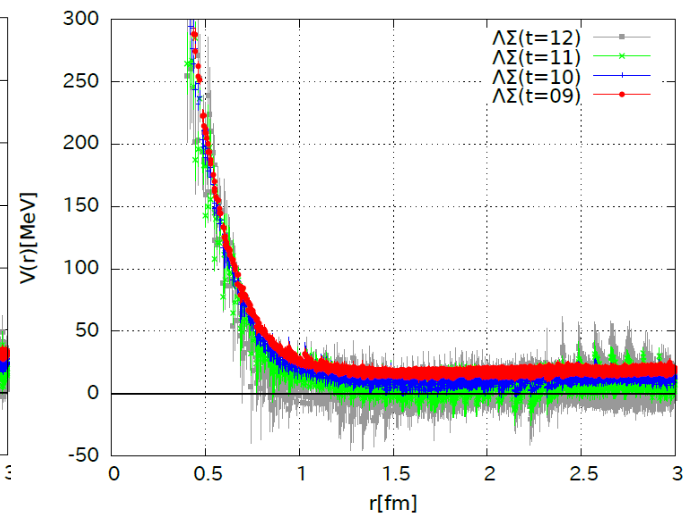
NΞ, ΛΣ (I=1) $^1S_0$ channel

t=09
t=10
t=11
t=12

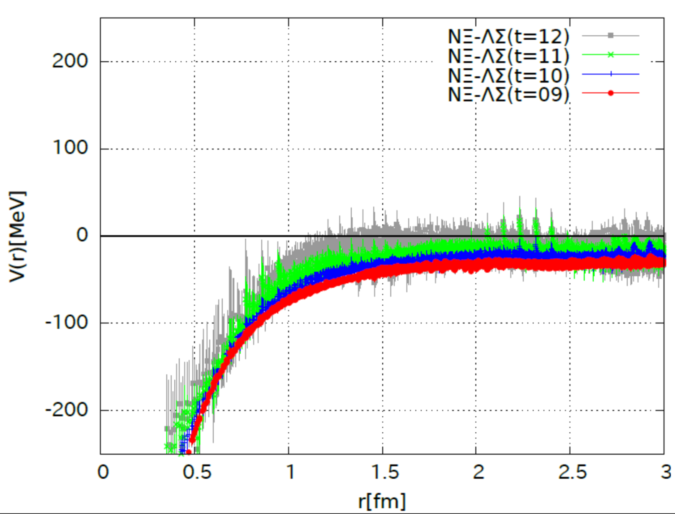
- **N_f = 2+1 full QCD m_π=146MeV with L = 8.12fm**

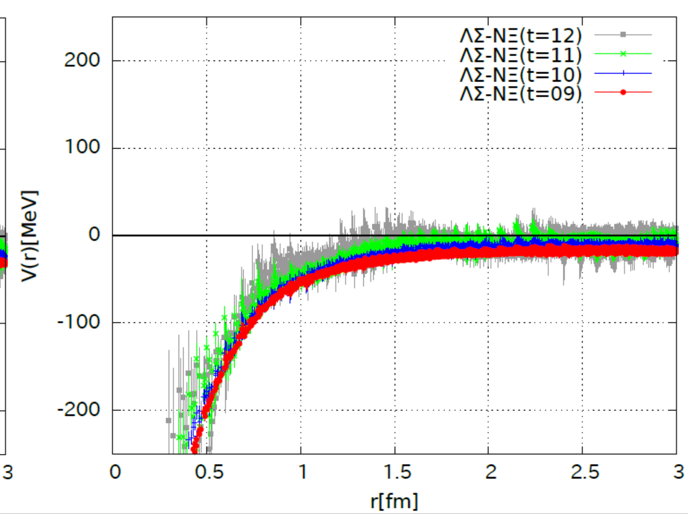
Diagonal elements





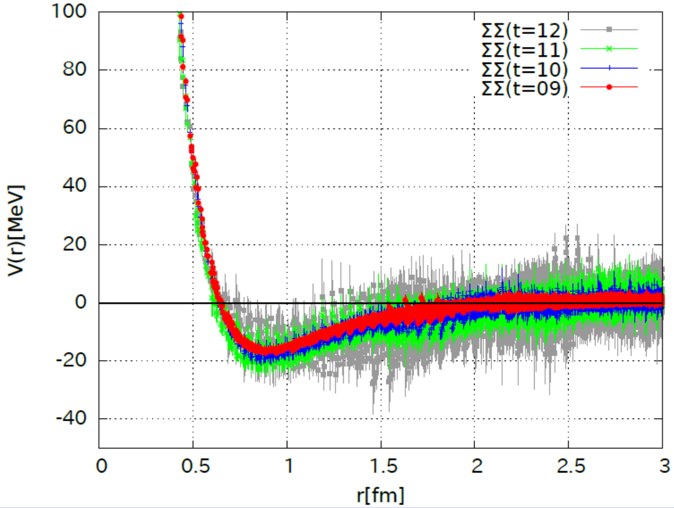
Off-diagonal elements



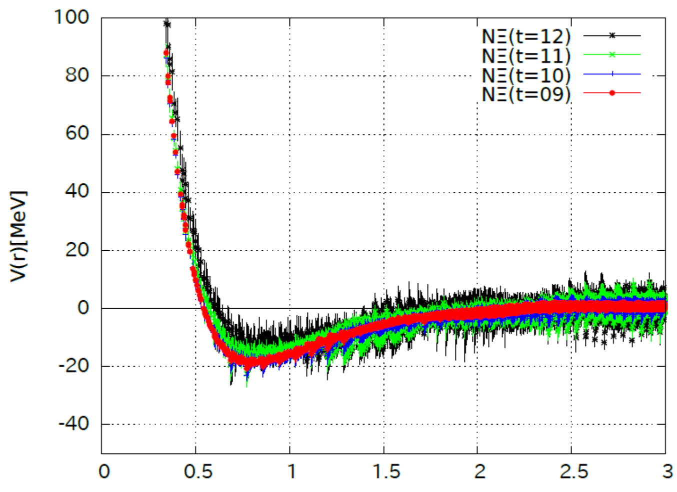


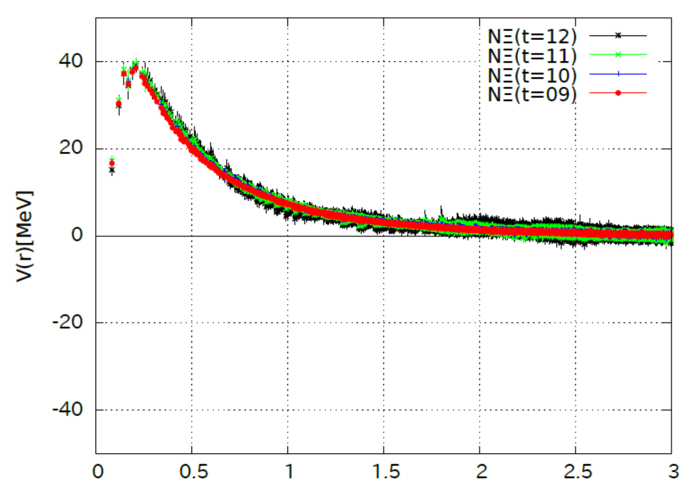
$\Sigma\Sigma$ (I=2) $^1S_0$ channel

- N_f = 2+1 full QCD m_π=146MeV with L = 8.12fm



$N\Xi$ (I=0) $^3S\text{-}D_1$ channel



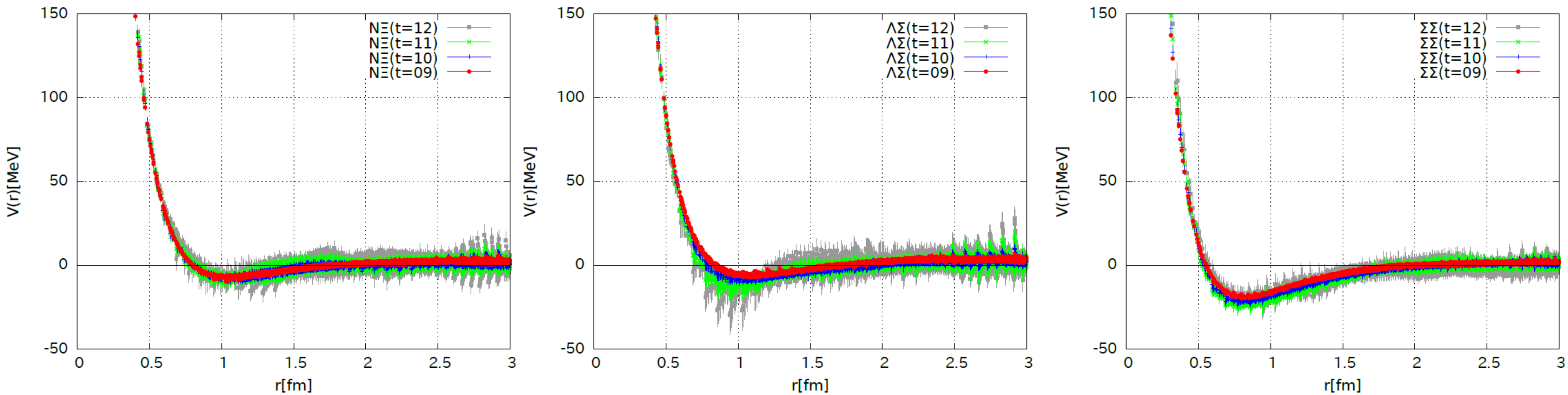


NΞ, ΛΣ, ΣΣ (I=1) ^3S-D_1 channel (central pot)

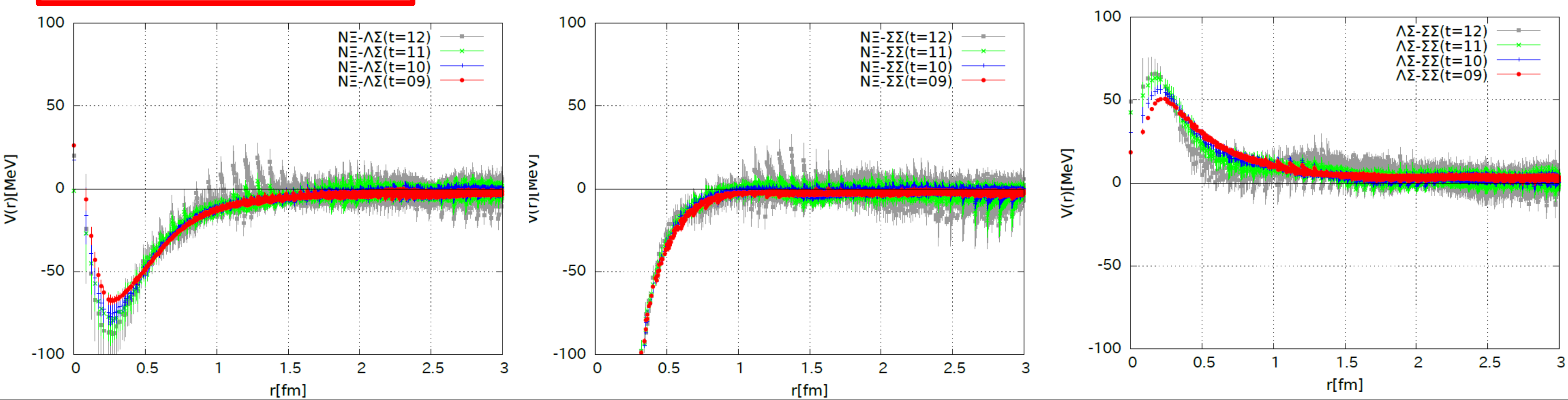
t=09
t=10
t=11
t=12

- **N_f = 2+1 full QCD mπ=146MeV with L = 8.12fm**

Diagonal elements



Off-diagonal elements

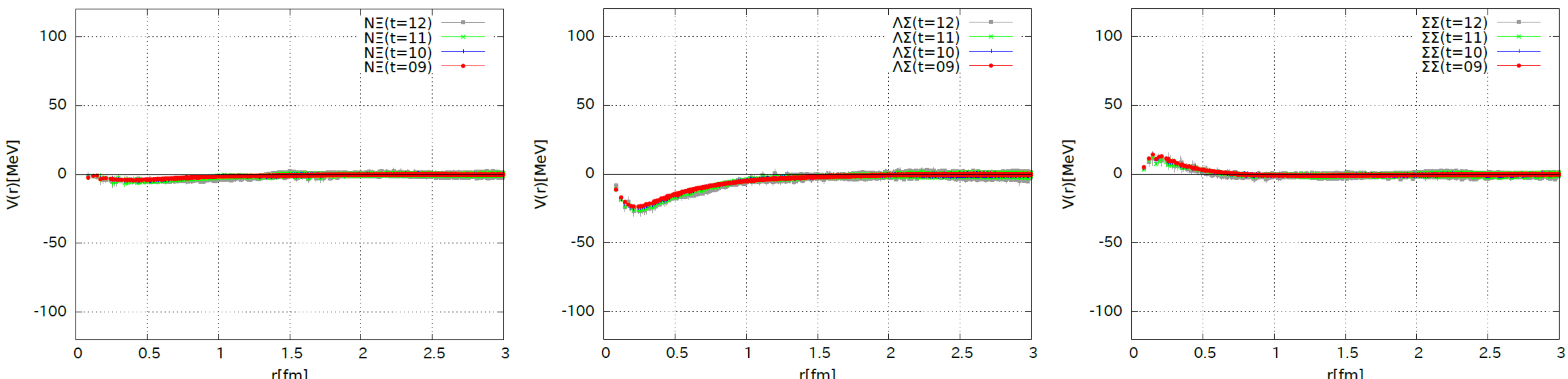


$N\Xi$, $\Lambda\Sigma$, $\Sigma\Sigma$ (I=1) $^3S\text{-}D_1$ channel (tensor pot)

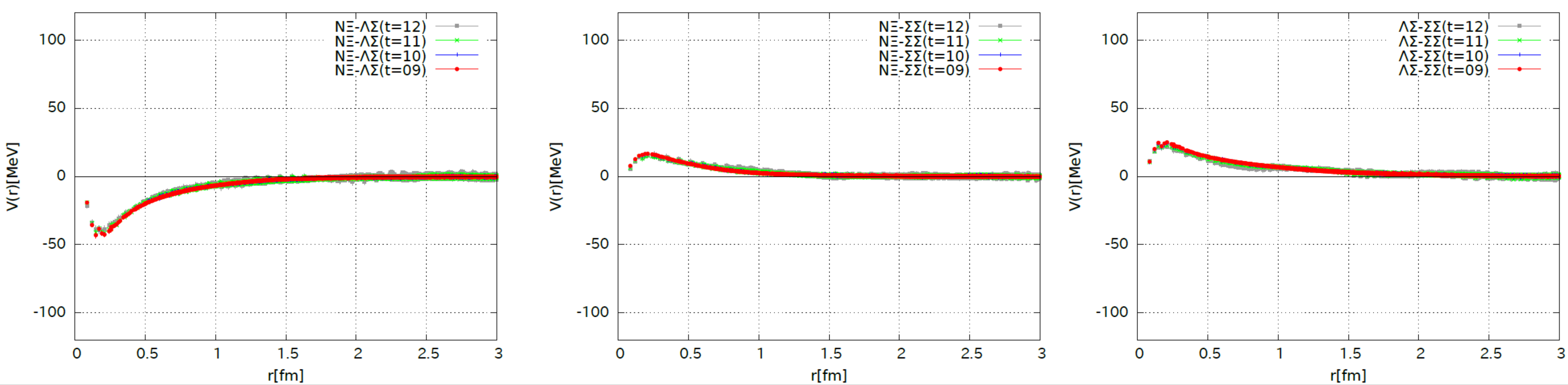
t=09
t=10
t=11
t=12

- N_f = 2+1 full QCD m_π=146MeV with L = 8.12fm

Diagonal elements



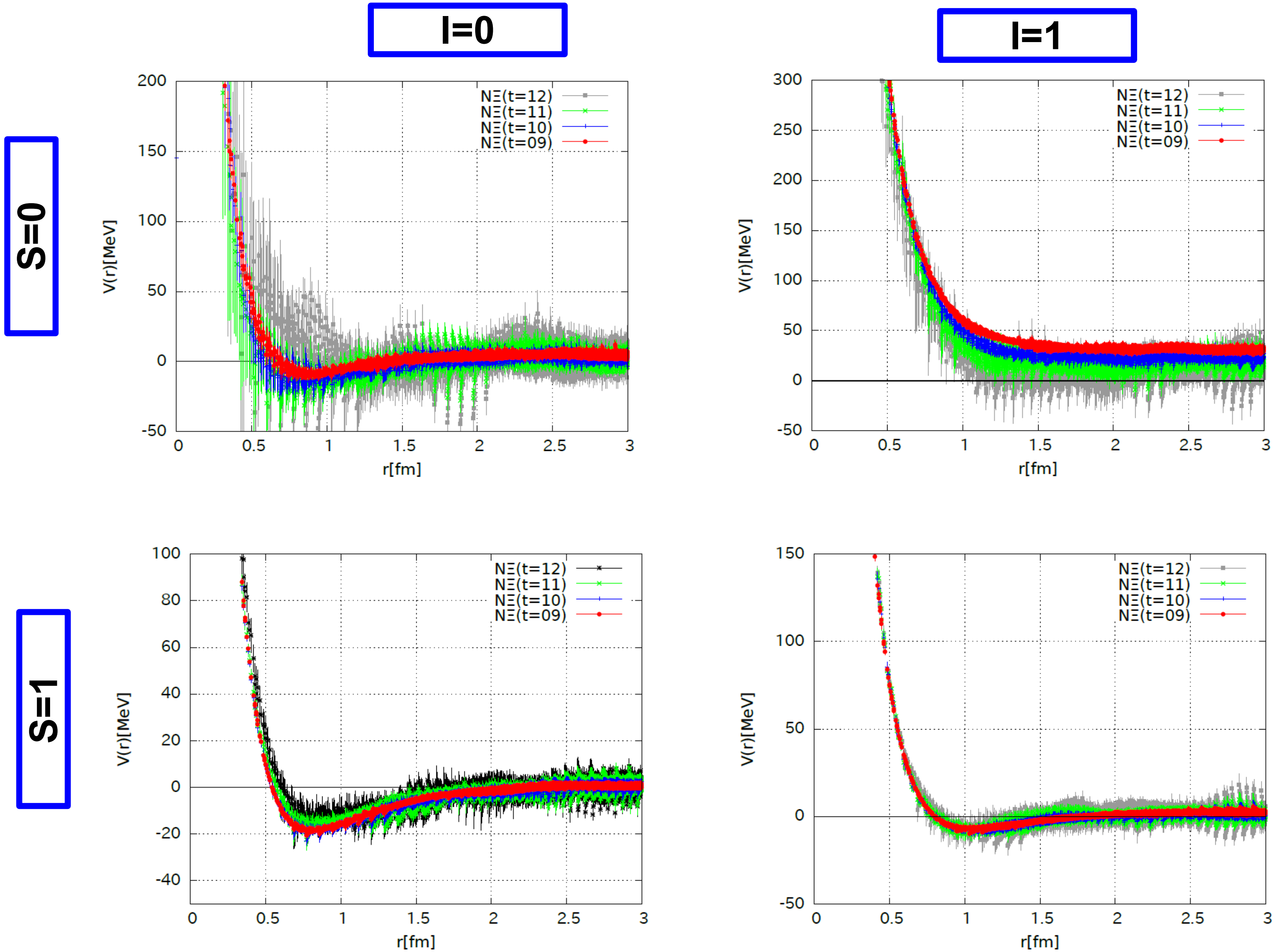
Off-diagonal elements



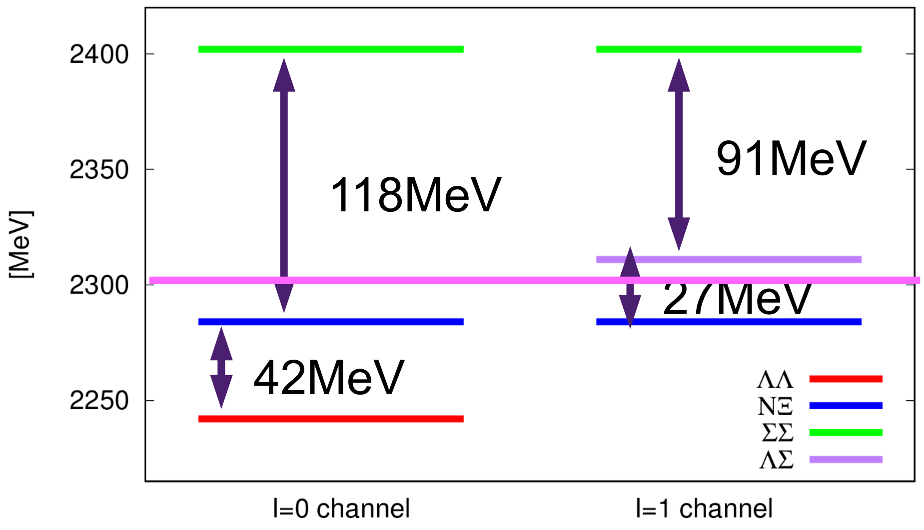
Spin and Isospin dependence of $N\Xi$ potentials

t=09
t=10
t=11
t=12

- N_f = 2+1 full QCD m_π=146MeV with L = 8.12fm



Effective ΛΛ–NΞ interactions



Effective two channel potential

Original coupled channel equation

$$\begin{pmatrix} (E^{\Lambda\Lambda}-H_0^{\Lambda\Lambda})R^{\Lambda\Lambda}(\vec{r},t) \\ (E^{\Xi N}-H_0^{\Xi N})R^{\Xi N}(\vec{r},t) \\ (E^{\Sigma\Sigma}-H_0^{\Sigma\Sigma})R^{\Sigma\Sigma}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} V^{\Lambda\Lambda}_{\Lambda\Lambda}(\vec{r}) & V^{\Lambda\Lambda}_{\Xi N}(\vec{r}) & V^{\Lambda\Lambda}_{\Sigma\Sigma}(\vec{r}) \\ V^{\Xi N}_{\Lambda\Lambda}(\vec{r}) & V^{\Xi N}_{\Xi N}(\vec{r}) & V^{\Xi N}_{\Sigma\Sigma}(\vec{r}) \\ V^{\Sigma\Sigma}_{\Lambda\Lambda}(\vec{r}) & V^{\Sigma\Sigma}_{\Xi N}(\vec{r}) & V^{\Sigma\Sigma}_{\Sigma\Sigma}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r},t) \\ R^{\Xi N}(\vec{r},t) \\ R^{\Sigma\Sigma}(\vec{r},t) \end{pmatrix}$$

Truncation of ΣΣ channel

Reduced coupled channel equation

$$\begin{pmatrix} (E^{\Lambda\Lambda}-H_0^{\Lambda\Lambda})R^{\Lambda\Lambda}(\vec{r},t) \\ (E^{\Xi N}-H_0^{\Xi N})R^{\Xi N}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} \overline{V^{\Lambda\Lambda}_{\Lambda\Lambda}}(\vec{r}) & \overline{V^{\Lambda\Lambda}_{\Xi N}}(\vec{r}) \\ \overline{V^{\Xi N}_{\Lambda\Lambda}}(\vec{r}) & \overline{V^{\Xi N}_{\Xi N}}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r},t) \\ R^{\Xi N}(\vec{r},t) \end{pmatrix}$$

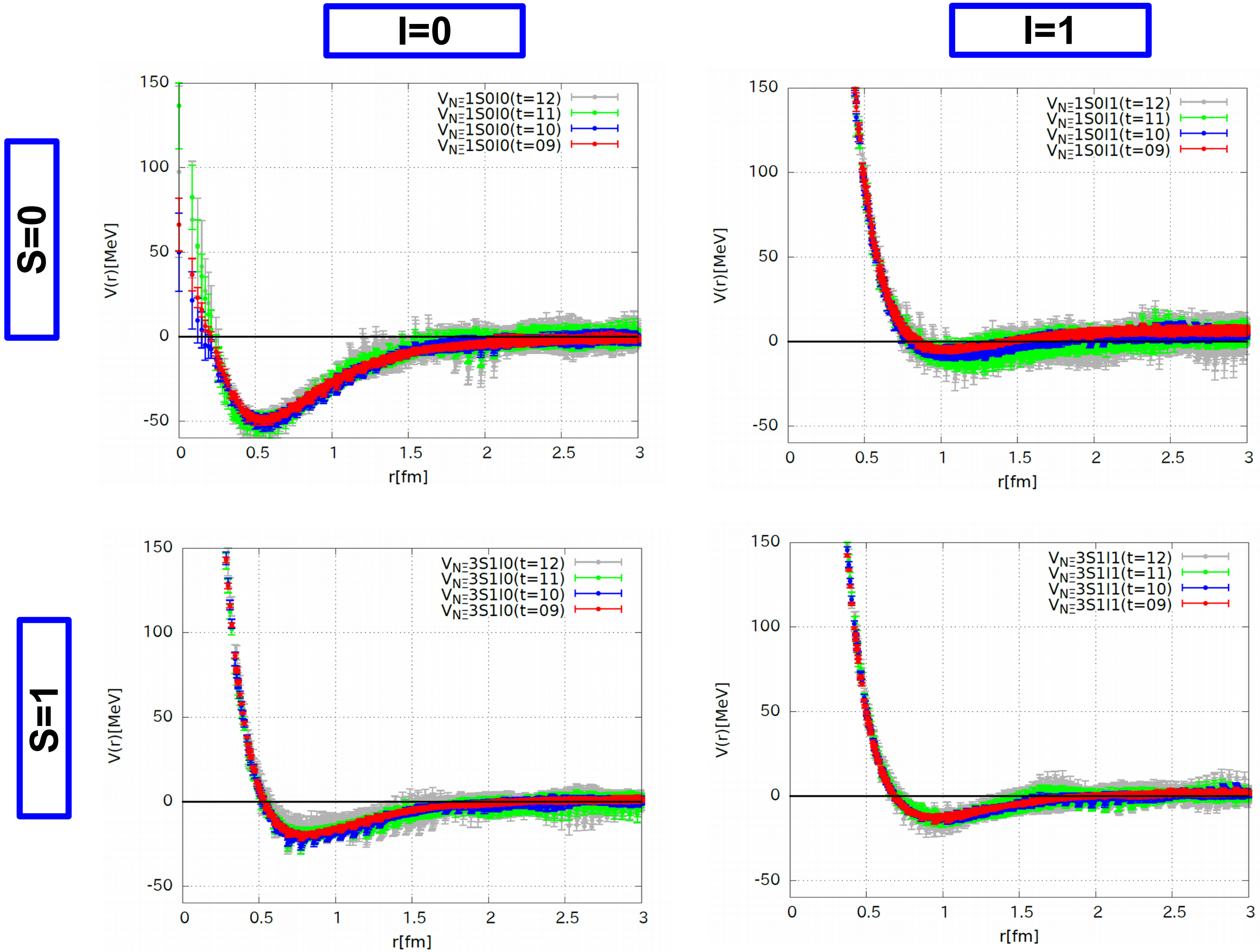
Effective ΛΛ–NΞ potential

- The same scattering phase shift would be expected in a low energy region.
- Non-locality (energy dependence, higher derivative contribution) of potential matrix could be enhanced.

Spin and Isospin dependence of $N\Xi$ potentials

t=09
t=10
t=11
t=12

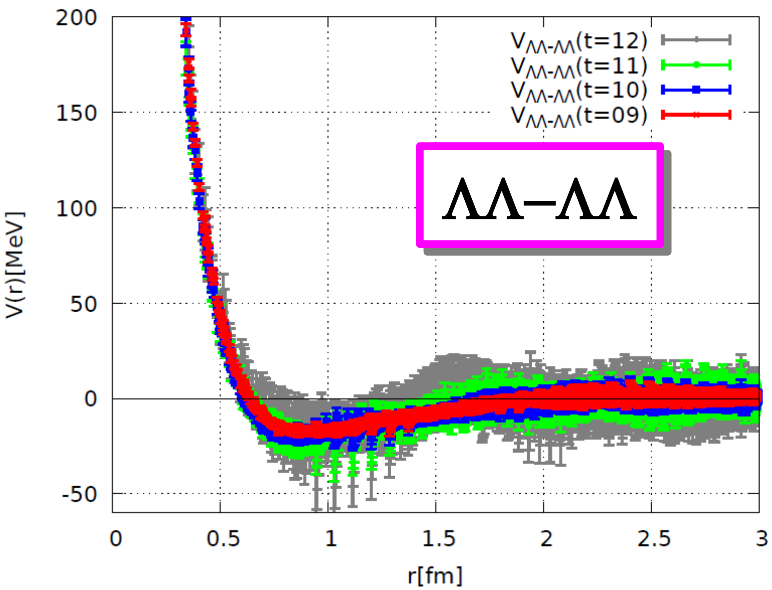
- **N_f = 2+1 full QCD m_π=146MeV with L = 8.12fm**

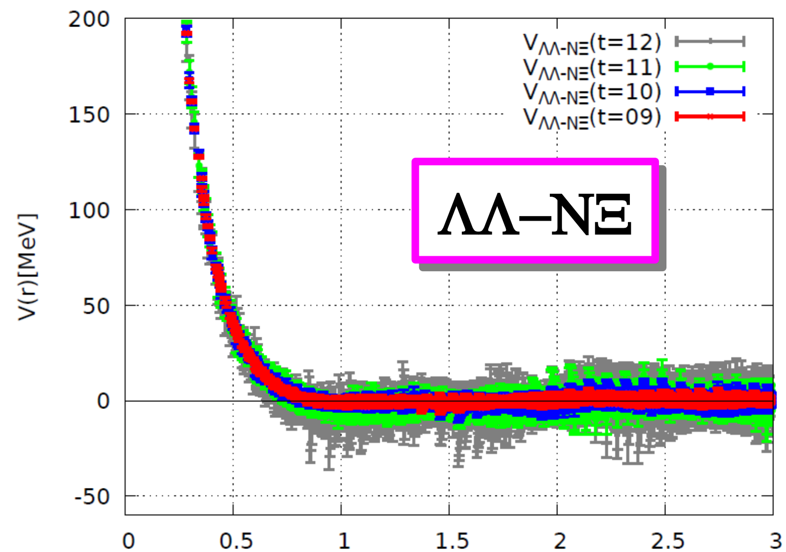


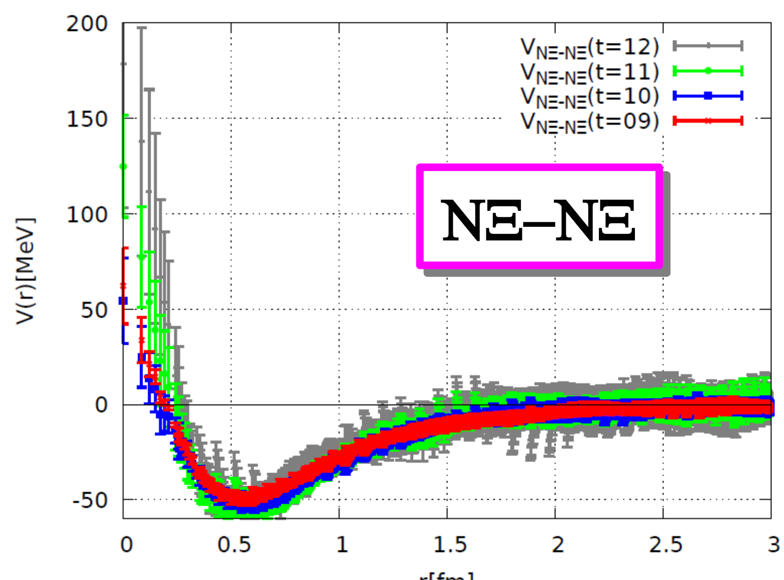
$\Lambda\Lambda$, $N\Xi$ (I=0) $^1S_0$ potential (2ch calc.)

▶ N_f = 2+1 full QCD m_π=146MeV with L = 8.12fm

Preliminary!







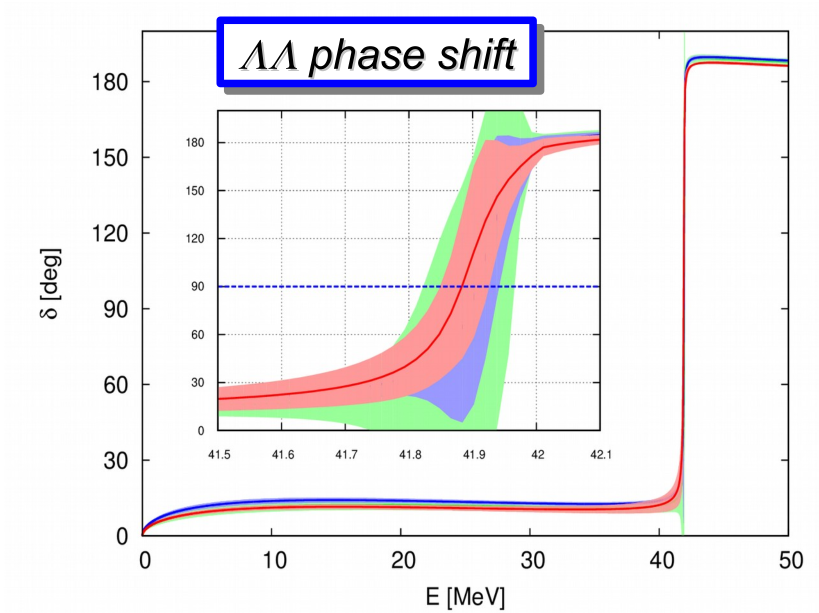
- **Potential calculated by only using $\Lambda\Lambda$ and $N\Xi$ channels.**
- **Long range part of potential is almost stable against the time slice.**
- **Short range part of $N\Xi$ potential changes as time t goes.**
- **$\Lambda\Lambda$–$N\Xi$ transition potential is quite small in r > 0.7fm region**

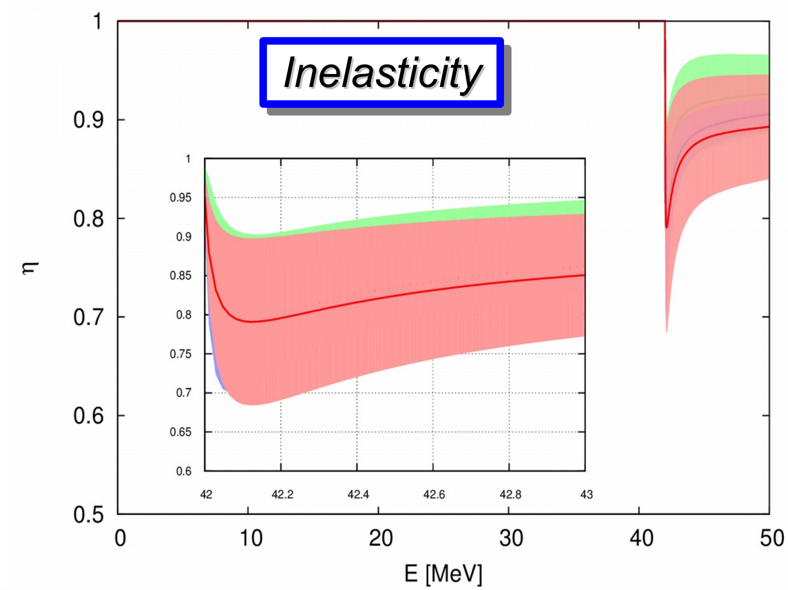
ΛΛ and NΞ phase shift and inelasticity

t=09
t=10
t=11

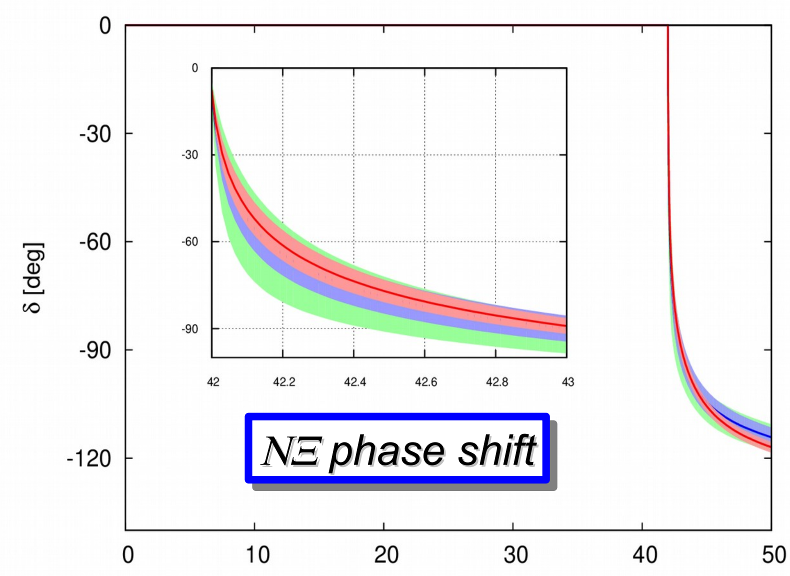
- N_f = 2+1 full QCD m_π=146MeV with L = 8.12fm

Preliminary!





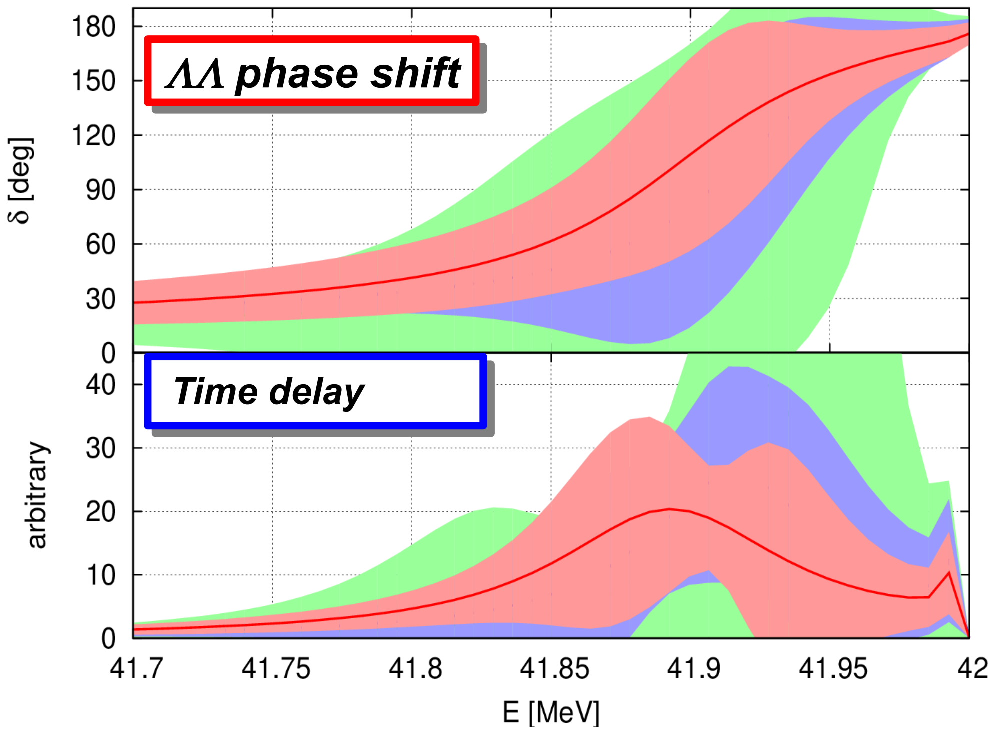
- **ΛΛ and NΞ phase shift is calculated by using 2ch effective potential.**
- **A sharp resonance is found just below the NΞ threshold.**
- **Inelasticity is small.**



Breit-Wigner mass and width

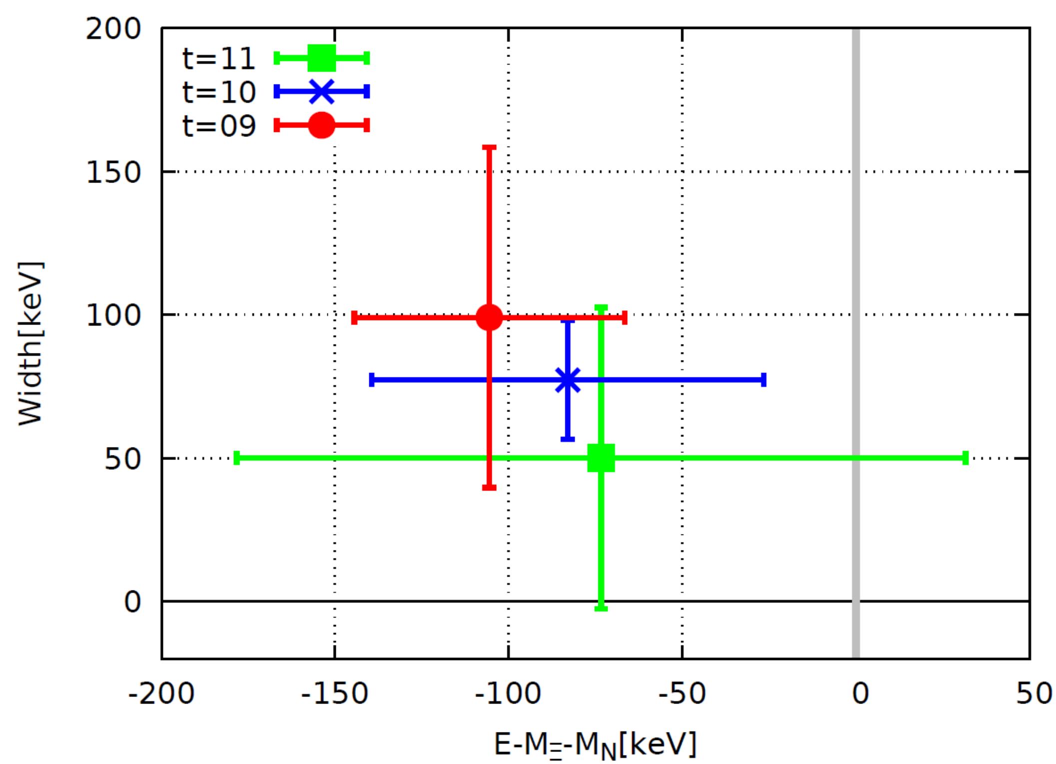
▶ $N_f = 2+1$ full QCD m_π=146MeV with L = 8.12fm

Preliminary!



- **Fitting the time delay of ΛΛ scattering by the Breit-Wigner type finction,**

Resonance enargy and width



- **In the vicinity of resonance point,**

$$\delta(E) = \delta_B - \arctan\left(\frac{\Gamma/2}{E - E_r}\right)$$

thus

$$\frac{d\,\delta(E)}{d\,E} = \frac{\Gamma/2}{(E-E_r)^2 + (\Gamma/2)^2}$$

Summary

- S=-2 BB interaction is investigated using 414confs x 28src x 4rot.
- We perform coupled-channel calculations for S=-2 BB system
- We find that the $N\Xi$ interaction largely depends on their spin and isospin.
- The 1S0 I=0 effective $N\Xi$ state is very close to the unitary limit.
- For H-dibaryon channel, we find that sharp resonance is just below the $N\Xi$ threshold.
- Resonance position from Breit-Wigner type fit tends to close to the $N\Xi$ threshold as “t” becomes larger.
    - We continue to study it by using higher statistical data.