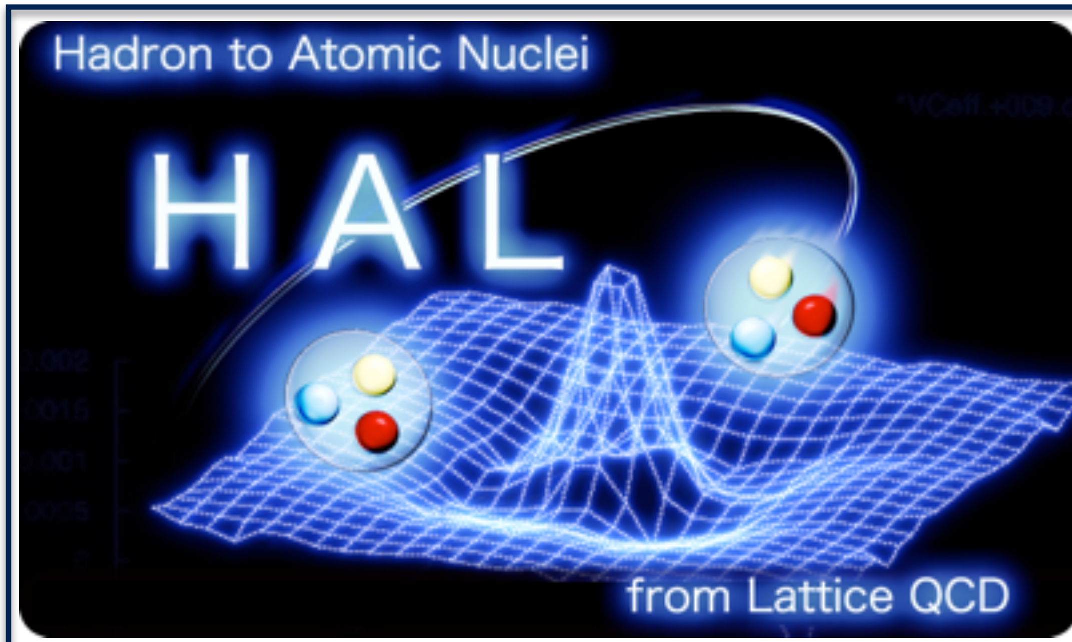
ΛcN interaction from lattice QCD and Λc-nuclei

Takaya Miyamoto

(Yukawa Institute for Theoretical Physics, Kyoto University)

for **HAL QCD Collaboration**

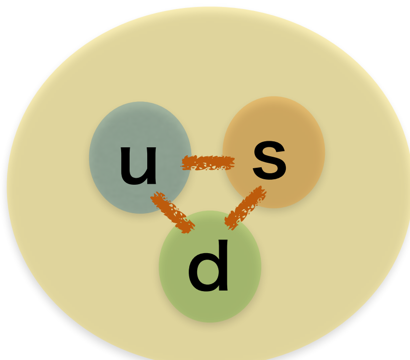


S. Aoki, K. Sasaki, D. Kawai	(YITP Kyoto Univ.)
T. Doi, T. Hatsuda, T. Iritani	(RIKEN Nishina)
T. Inoue	(Nihon Univ.)
N. Ishii, Y. Ikeda, K. Murano	(RCNP Osaka Univ.)
H. Nemura	(Univ. Tsukuba)
S. Gongyo	(Univ. of Tours)
F. Etminan	(Univ. of Birjand)

Introduction

u, d, s quark physics

Chiral SU(3) symmetry

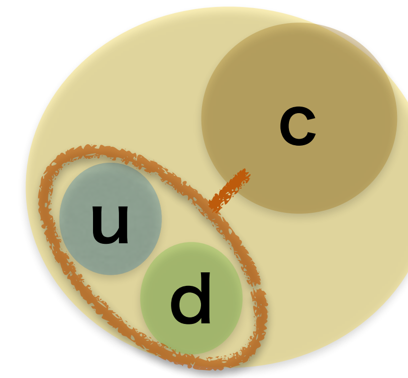


Λ, Σ, Σ^* baryon

Not only the quark mass but also the symmetry changes

charm quark physics

Heavy quark (HQ) symmetry

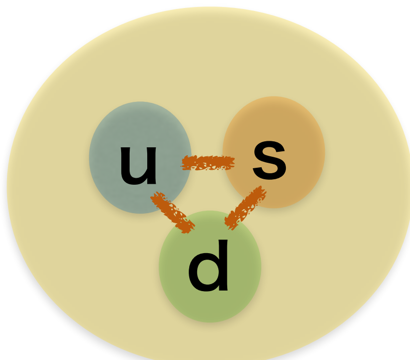


Λ_c, Σ_c, Σ_c^* baryon

Introduction

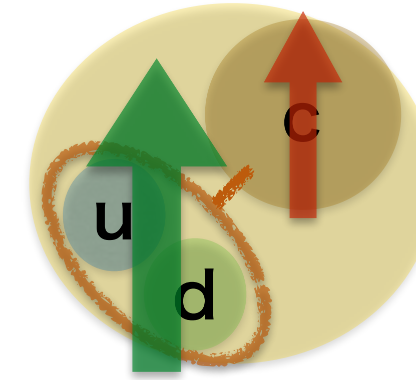
u, d, s quark physics

Chiral SU(3) symmetry

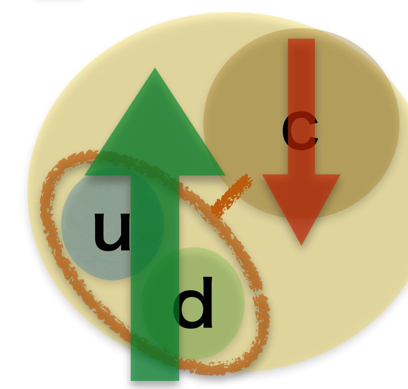


charm quark physics

Heavy quark (HQ) symmetry



Σc*

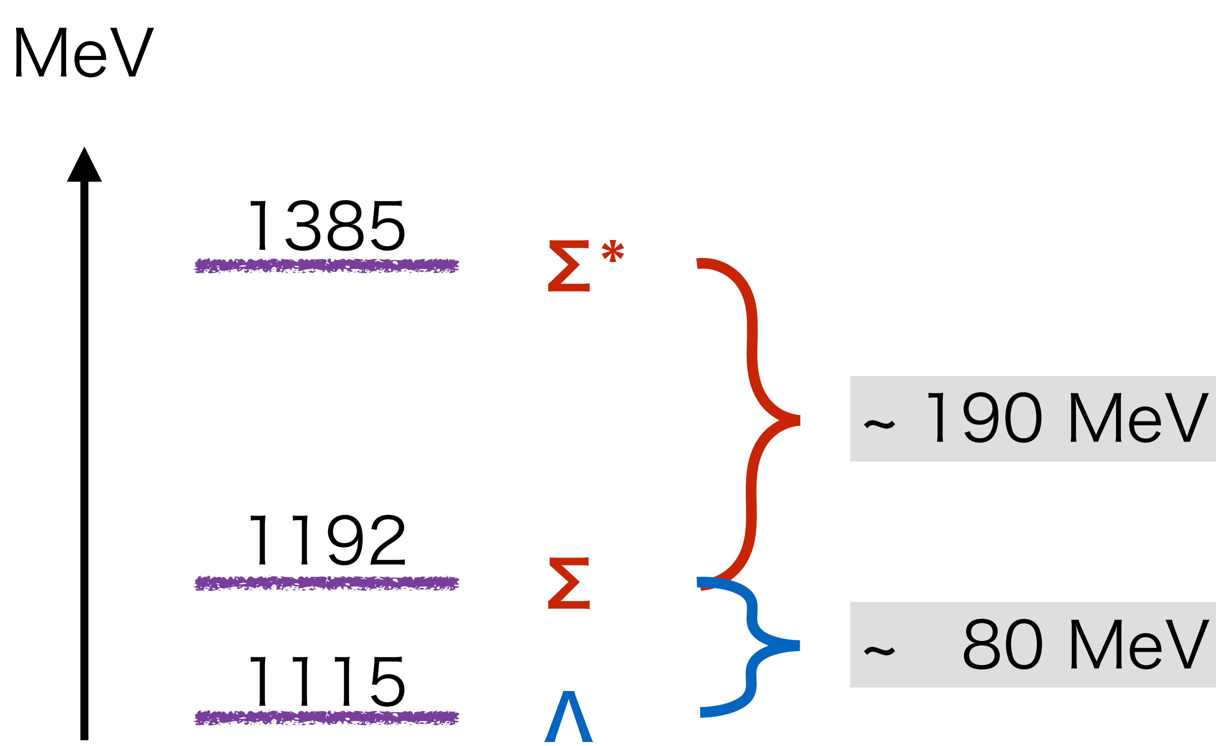


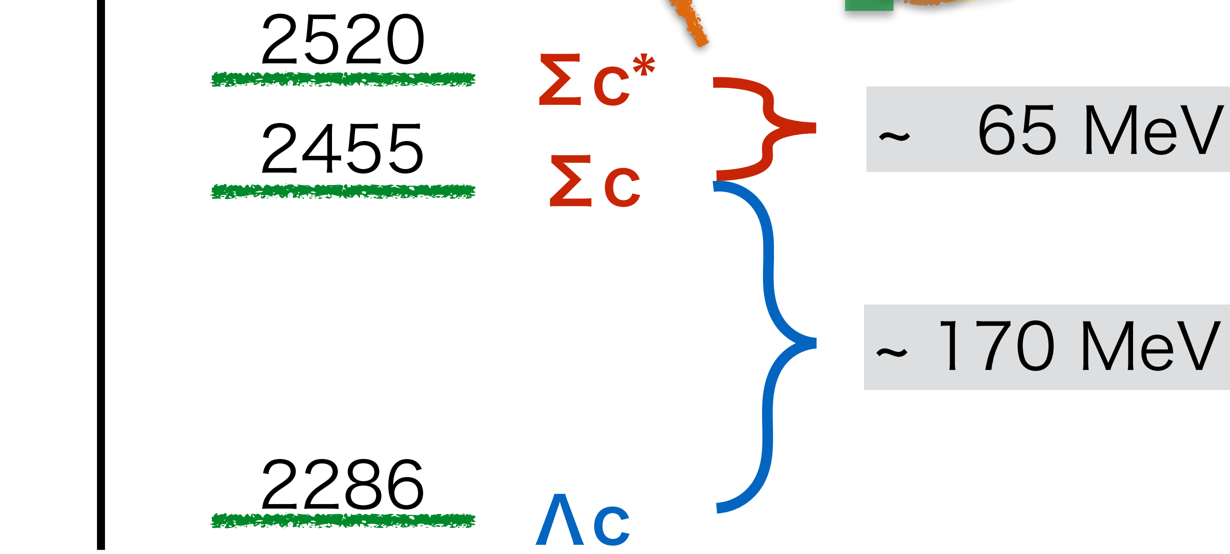
Σc

<u>HQ spin symmetry</u>

heavy quark spin is conserved

→ $(m_{\Sigma^*}-m_{\Sigma}) > (m_{\Sigma c^*}-m_{\Sigma c})$

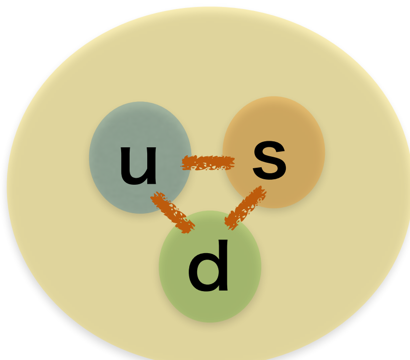




Introduction

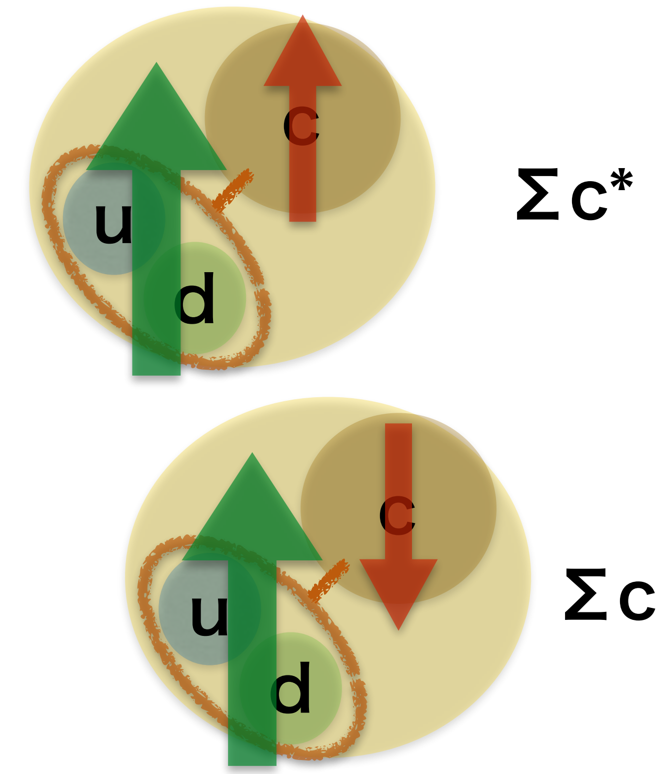
u, d, s quark physics

Chiral SU(3) symmetry

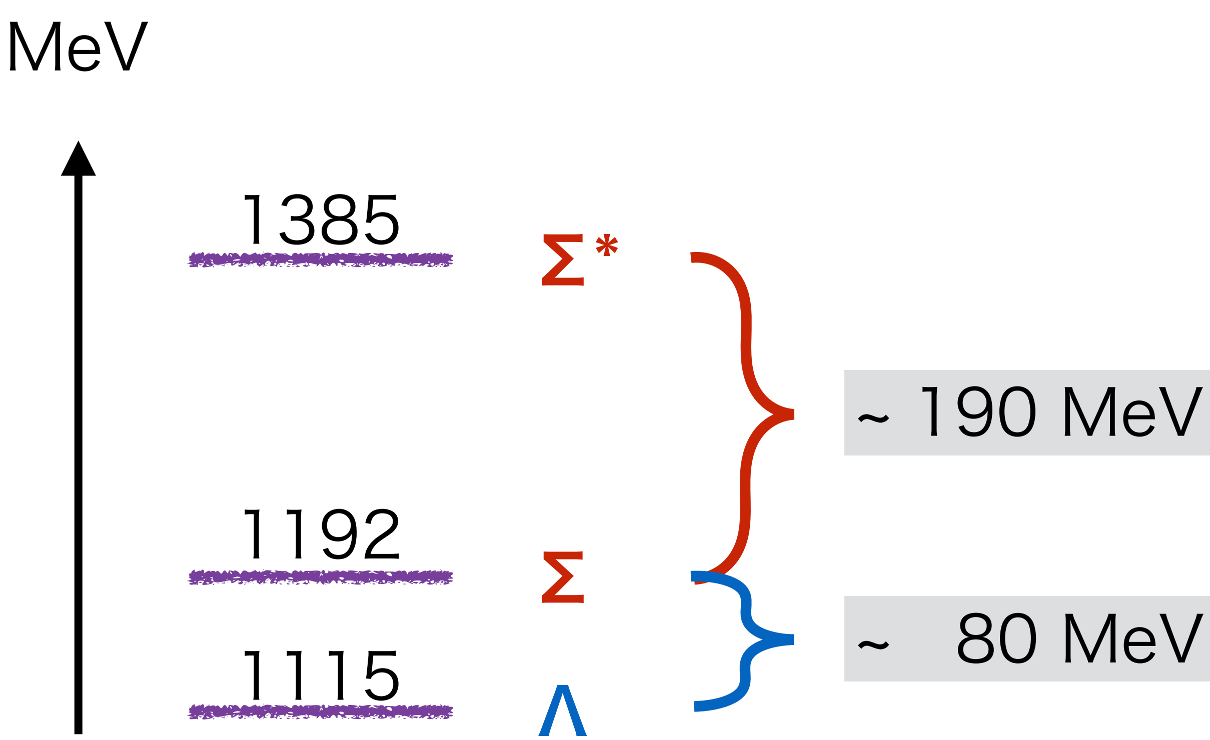


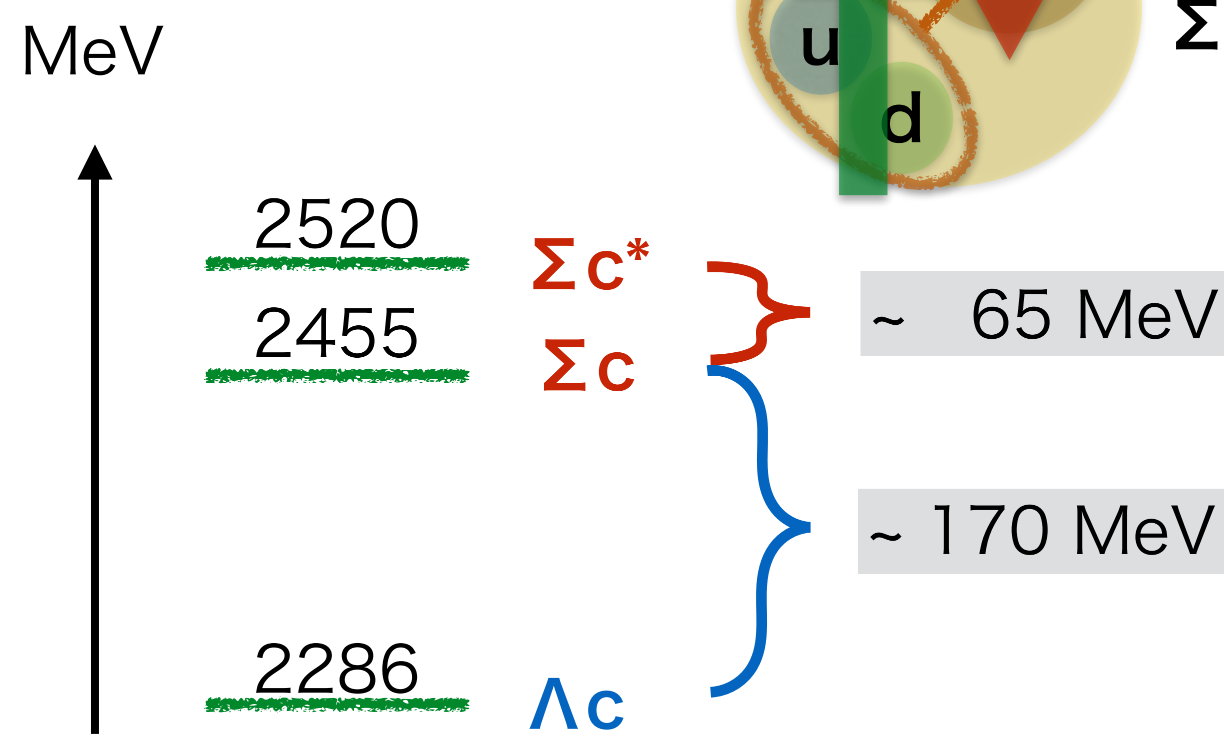
charm quark physics

Heavy quark (HQ) symmetry



How does HQ spin symmetry affect charm hadron interactions ?

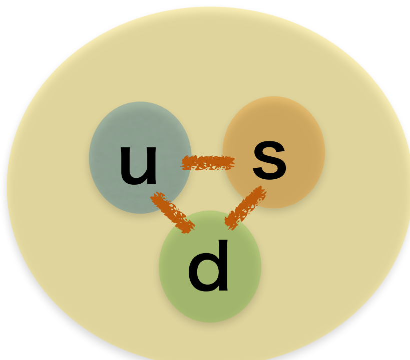




Introduction

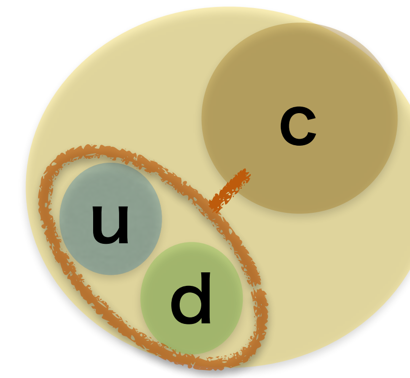
u, d, s quark physics

Chiral SU(3) symmetry



charm quark physics

Heavy quark (HQ) symmetry



How does HQ spin symmetry affect charm hadron interactions ?

<u>e.g. There are no ΛN bound states</u>

OBEP model based on
<u>the heavy quark effective theory</u>

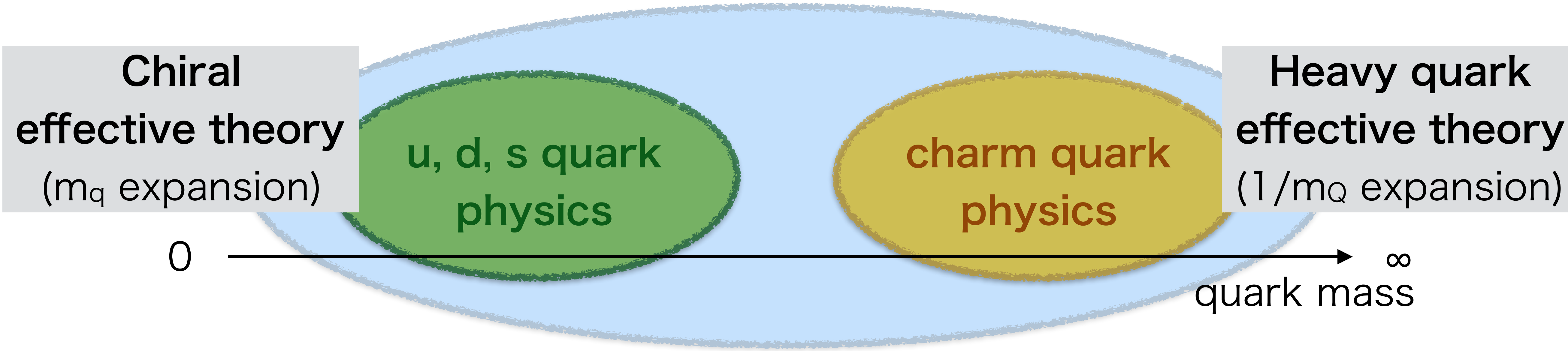
Y. R. Liu, M. Oka, Phys. Rev. D85, 014015 (2012).

The possibility of **bound ΛcN** was claimed

Introduction

● How does HQ spin symmetry affect charm hadron interactions ?

- We would like to investigate the difference of u,d,s quark physics and c quark physics
- Because of the different symmetry, it is difficult to construct the EFT includes two sectors



Lattice QCD: Ab-initio calculation of QCD

We extract interactions by using HAL QCD method

● Dbar-N system：Y. Ikeda（previous talk）

● Λc-N　system：T. M

Outline

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys., 123 (2010).
S. Aoki *et al*, [HAL QCD Collaboration], PTEP., 01A105 (2012).

- **Nambu-Bethe-Salpeter (NBS) wave functions**

$$\psi^{(W_n)}_{\Lambda_c N}(\vec{r})\, e^{-W_n t} = \sum_{\vec{x}} \langle 0 | \Lambda_c(\vec{r}+\vec{x},t) N(\vec{x},t) | \Lambda_c N, W_n \rangle$$

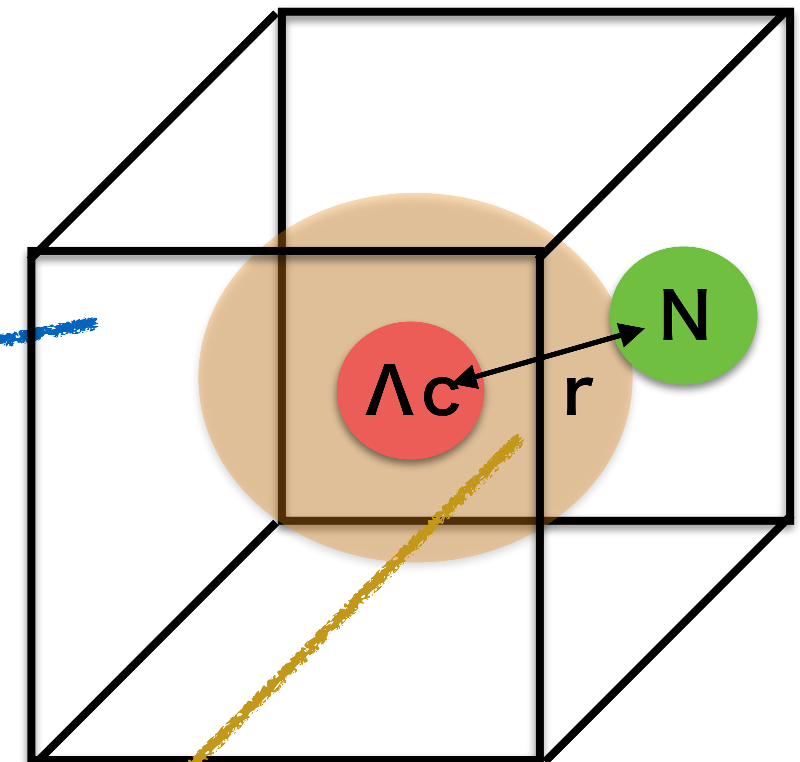
$$W_n \equiv \sqrt{k_n^2 + m_{\Lambda_c}^2} + \sqrt{k_n^2 + m_N^2}$$

At large r

$$(k_n^2 + \nabla^2)\psi^{(W_n)}(\vec{r}) = 0$$

$$\psi^{(W_n)}(|\vec{r}|) \propto \frac{\sin(k_n r - l\pi/2 + \delta_l(k_n))}{k_n r}$$

outside interactions



interacting region

Define the energy-independent non-local potentials through the **Schrödinger-type equation**
(non-relativistic approx. is not necessary)

$$(E_n - H_0)\,\psi^{(W_n)}_{\Lambda_c N}(\vec{r}) = \int d^3r'\, U_{\Lambda_c N}(\vec{r},\vec{r}\,')\psi^{(W_n)}_{\Lambda_c N}(\vec{r}\,')$$

- Potentials **faithful to phase shift** by construction
- **All 2PI contributions are included** in potentials
- Potentials are energy-independent until a new channel opens

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys., 123 (2010).
S. Aoki *et al*, [HAL QCD Collaboration], PTEP., 01A105 (2012).

- **To extract "energy-independent" potentials, we employ time-dependent HAL QCD method**

N. Ishii et al [HAL QCD Coll.], PLB712 (2012) 437.

$$R_{\Lambda_c N}(\vec{r},t) \equiv \frac{G_{\Lambda_c N}(\vec{r},t)}{e^{-m_{\Lambda_c}t}e^{-m_N t}}$$

Normalized 4pt-correlation function (**R-correlator**)

$$(E_0 - H_0)\,\psi_0(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}\,')\psi_0(\vec{r}\,')$$

$$(E_1 - H_0)\,\psi_1(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}\,')\psi_1(\vec{r}\,')$$

$$(E_2 - H_0)\,\psi_2(\vec{r}) = \int d^3r' U(\vec{r},\vec{r}\,')\psi_2(\vec{r}\,')$$

All equations are combined into one t-dep. eq.

$$\boxed{\left(-\frac{\partial}{\partial t} + \left[\frac{1+\delta^2}{8\mu}\right]\frac{\partial^2}{\partial t^2} - H_0\right) R_{\Lambda_c N}(\vec{r},t) = \int d^3r' U_{\Lambda_c N}(\vec{r},\vec{r}\,')R_{\Lambda_c N}(\vec{r}\,',t)}$$

$$\mu \equiv \frac{m_{\Lambda_c} m_N}{m_{\Lambda_c} + m_N} \qquad \delta \equiv \frac{m_{\Lambda_c} - m_N}{m_{\Lambda_c} + m_N}$$

Within the approximation up to O(k^2)

- **Non-local potentials —> local potentials**

Derivative (velocity) expansion

$$U(\vec{r},\vec{r}\,') = V(\vec{r},\vec{\nabla})\,\delta^3(\vec{r}-\vec{r}\,')$$

LO term

$$V(\vec{r},\vec{\nabla}) = V_0(\vec{r}) + V_\sigma(\vec{r})\,(\vec{\sigma}_1\cdot\vec{\sigma}_2) + V_T(\vec{r})S_{12} + \mathcal{O}(\vec{\nabla})$$

In the low energy state,
LO term of the potentials is significant.

Outline

(1) Motivation

(2) HAL QCD method

(3) Simulation setup

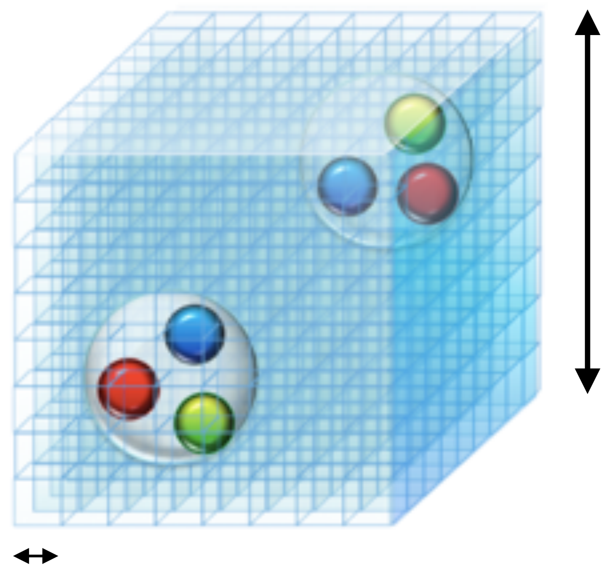
(4) Numerical results of $\Lambda_c N$ and ΛN interactions

(5) Folding potential analysis for Λ_c-nuclei

(6) Summary and conclusion

Lattice QCD setup

Nf=2+1 full QCD configurations generated by PACS-CS Coll



2.9 fm

0.0907 fm

PACS-CS Collaboration:

S. Aoki, et al., Phys. Rev. D79 (2009) 034503

- Iwasaki gauge action
- O(a) improved Wilson-clover quark action
- a ~ 0.09 fm, L ~ 3 fm (32^3 x 64)

m_π ~ 700, 570, 410 MeV

For charm quark, we use

Relativistic Heavy Quark (RHQ) action

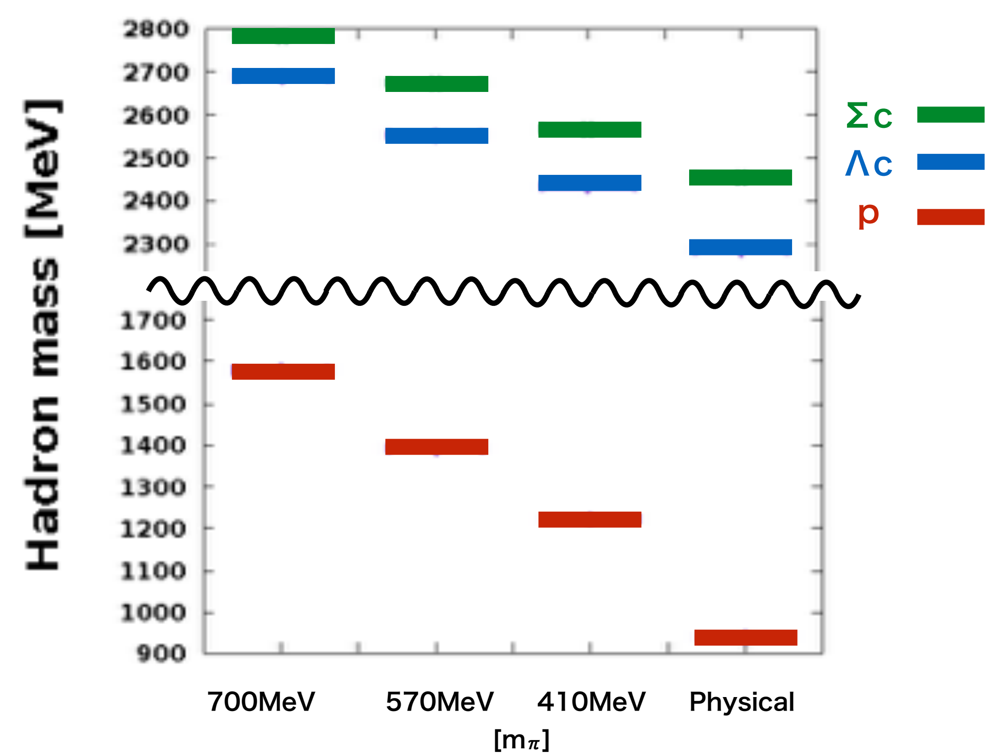
to remove the leading O($(ma)^n$) discretization error

For more details, see,

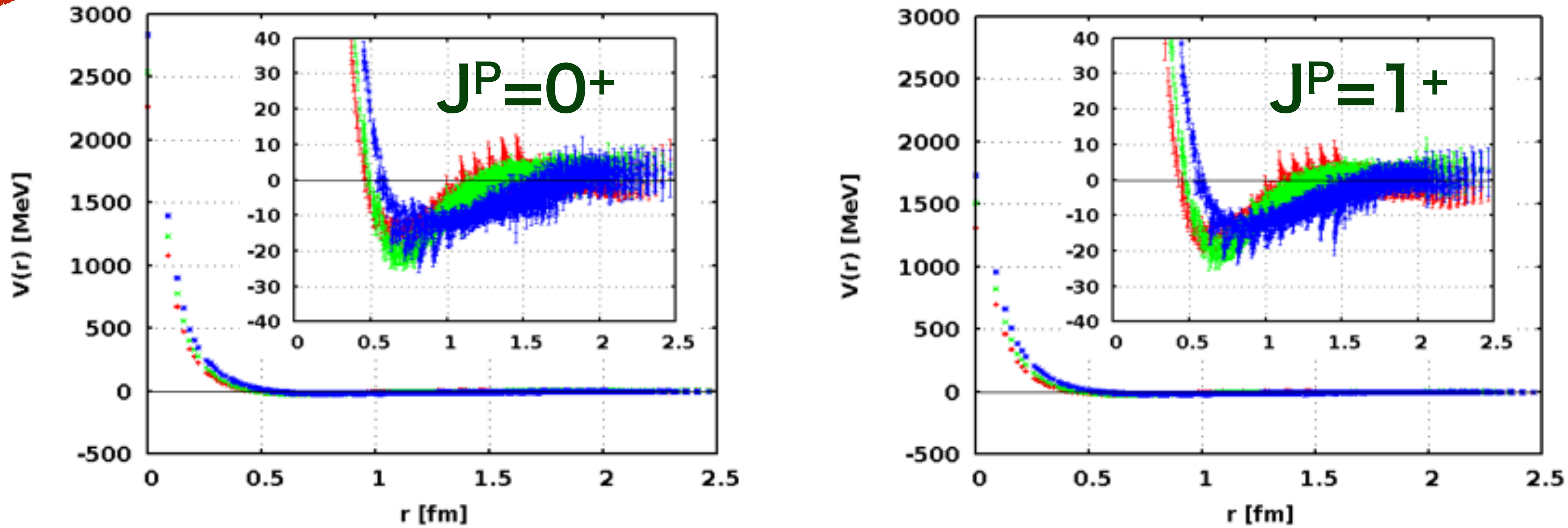
Namekawa, et al.,

Phys. Rev. D84 (2011) 074505

BG/Q @ KEK



S-wave Λc-N effective central potentials



Λc-N potential = repulsive core + attractive pocket

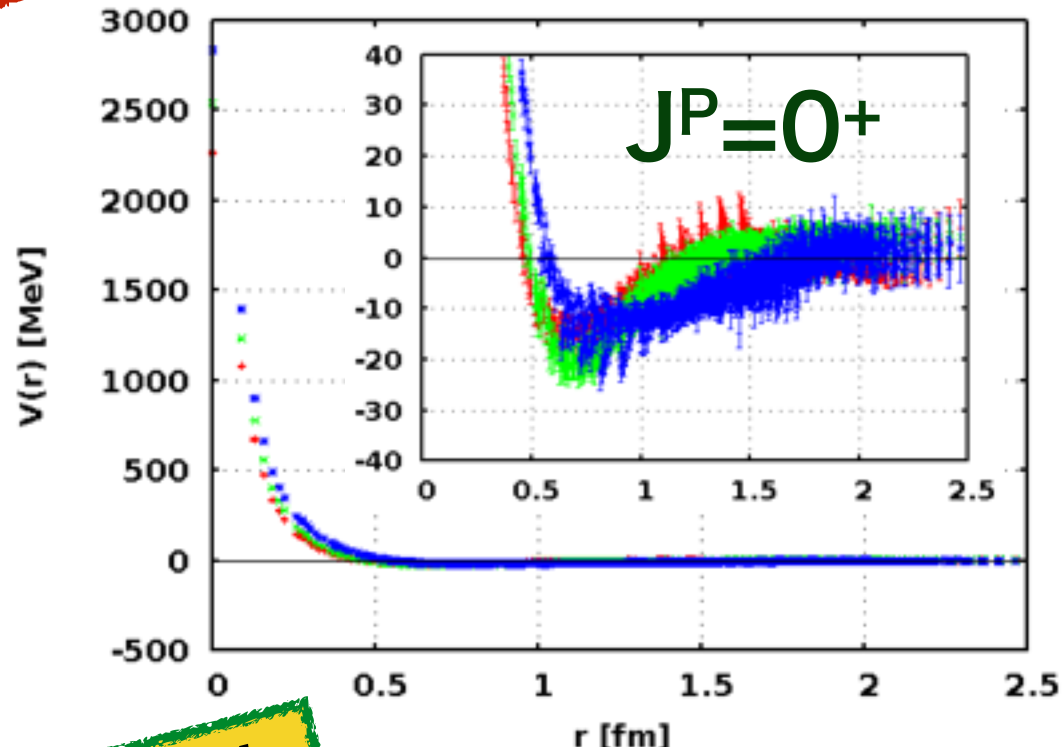
As m_q decreasing,

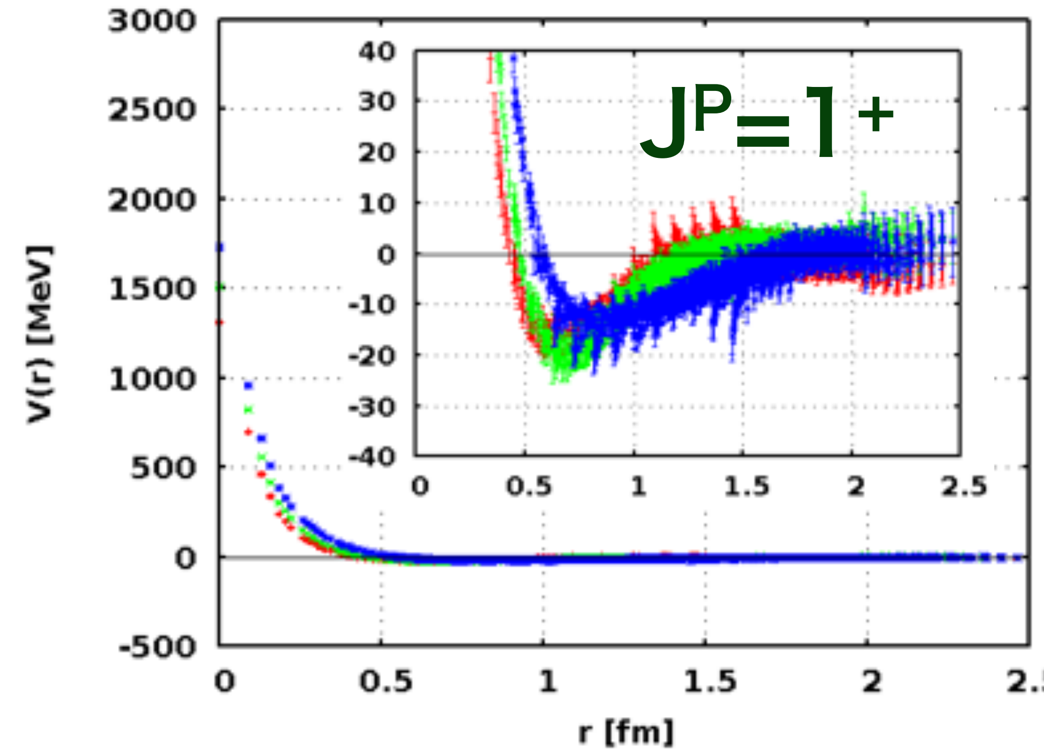
- repulsive core becomes larger
- attractive pocket shifts outward

Results

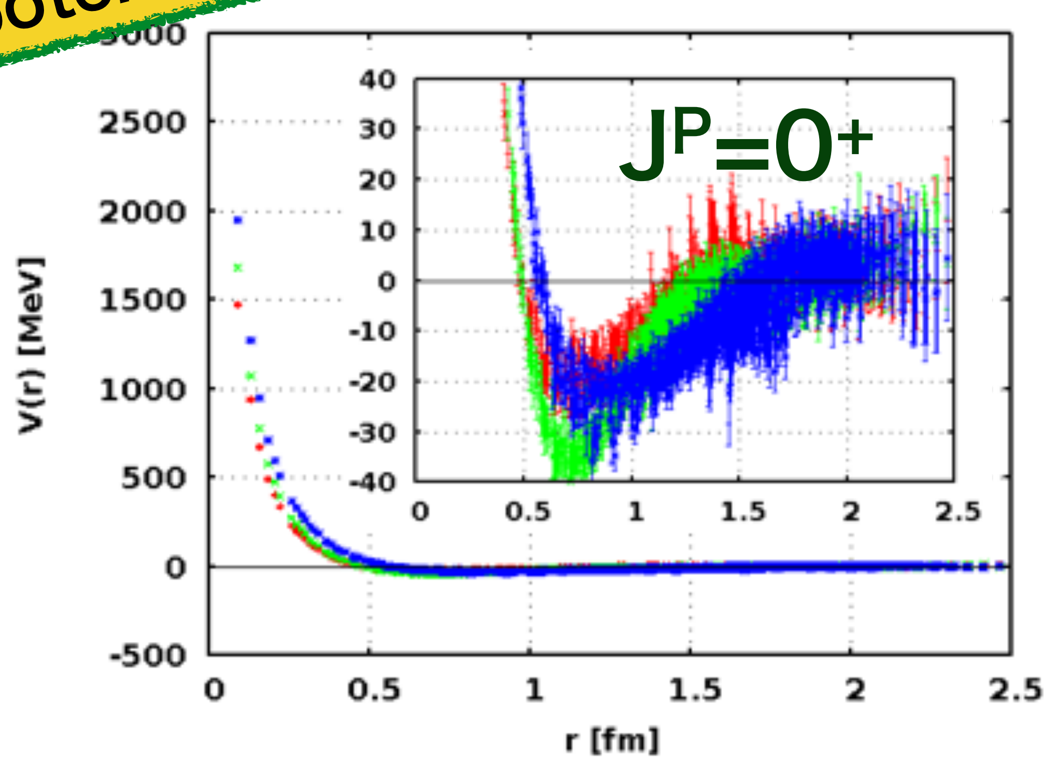
S-wave Λ(c)-N effective central potentials

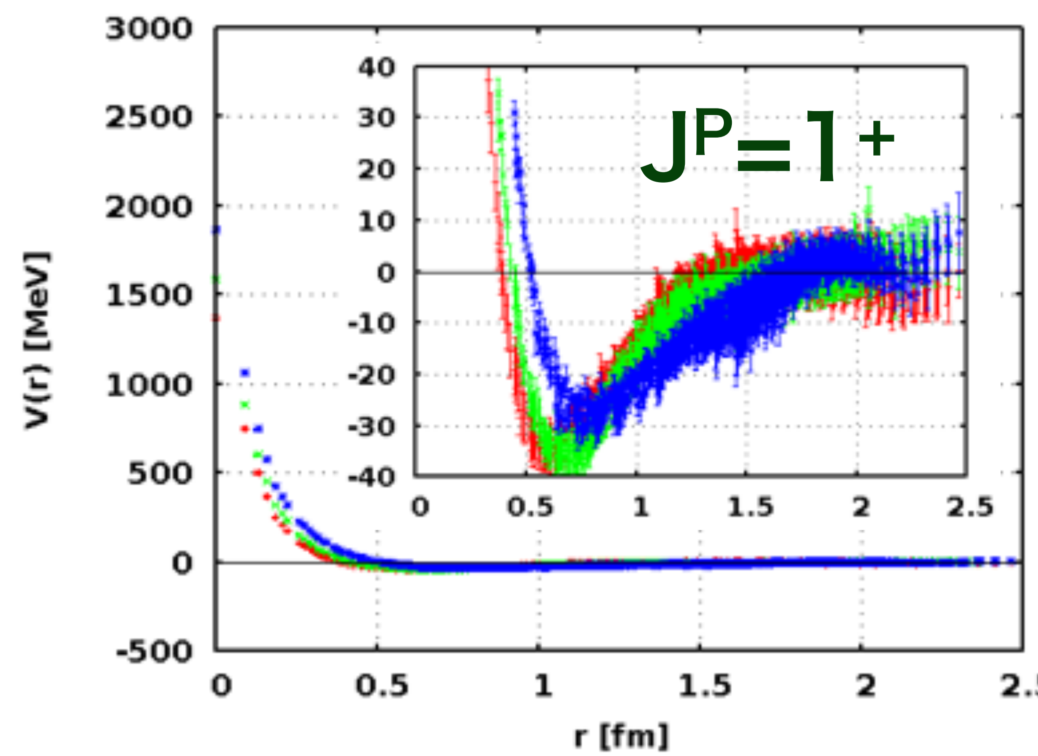
Mpi ~ 700 MeV
Mpi ~ 570 MeV
Mpi ~ 410 MeV



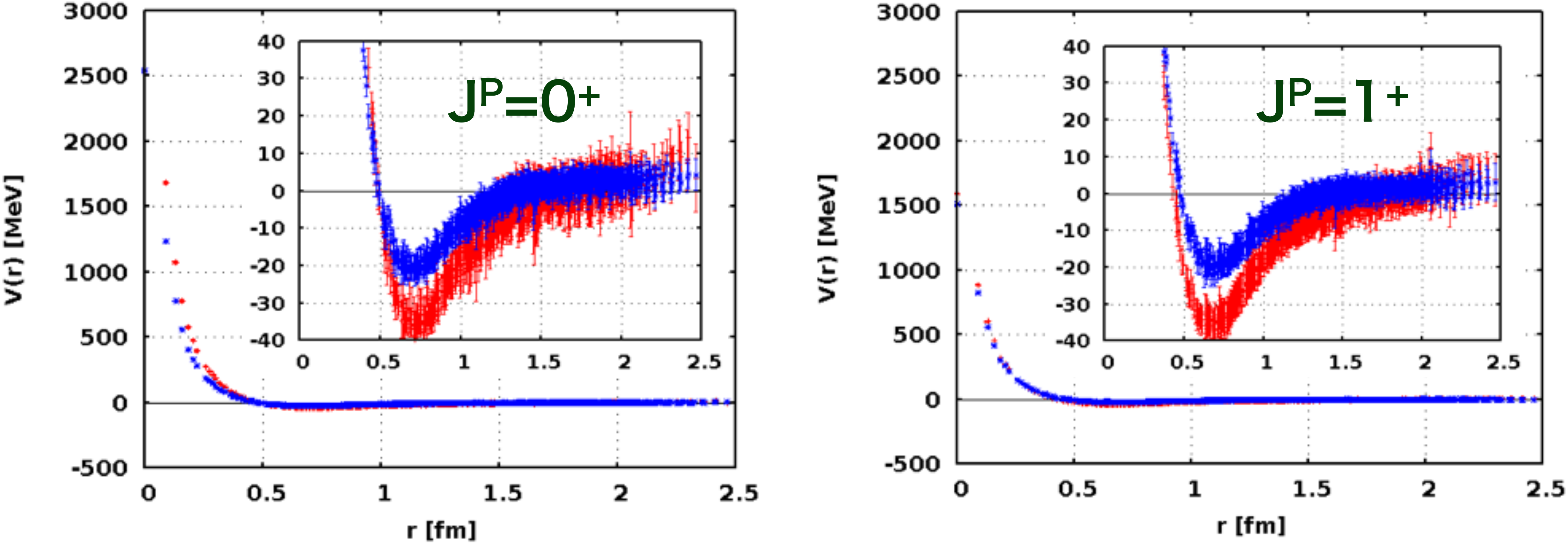


ΛN potentials





Comparison of ΛN potential and ΛcN potential (m_π = 570 MeV)

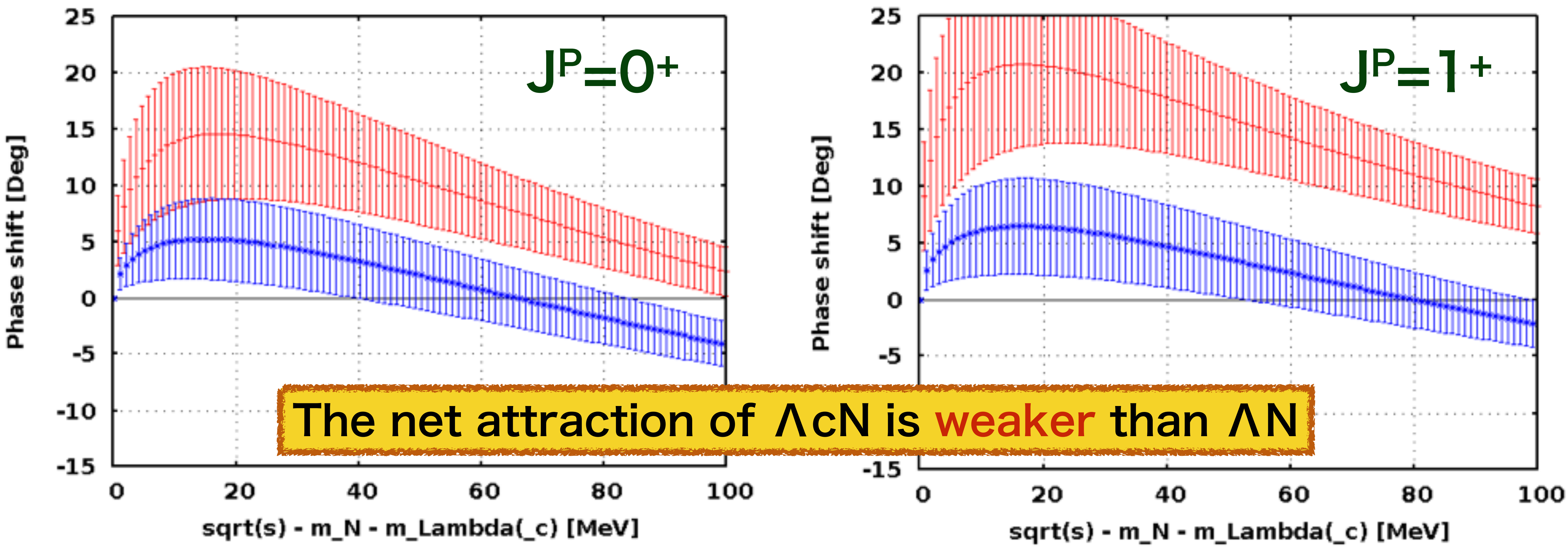


Both attractive pocket and repulsive core of ΛcN potentials are **smaller** than those of ΛN potentials

Comparison of ΛN potential and ΛcN potential (m_π = 570 MeV)

Lambda N
Lambda_c N

Phase shifts of Λ(c)N scattering



The net attraction of ΛcN is weaker than ΛN

Scattering length:

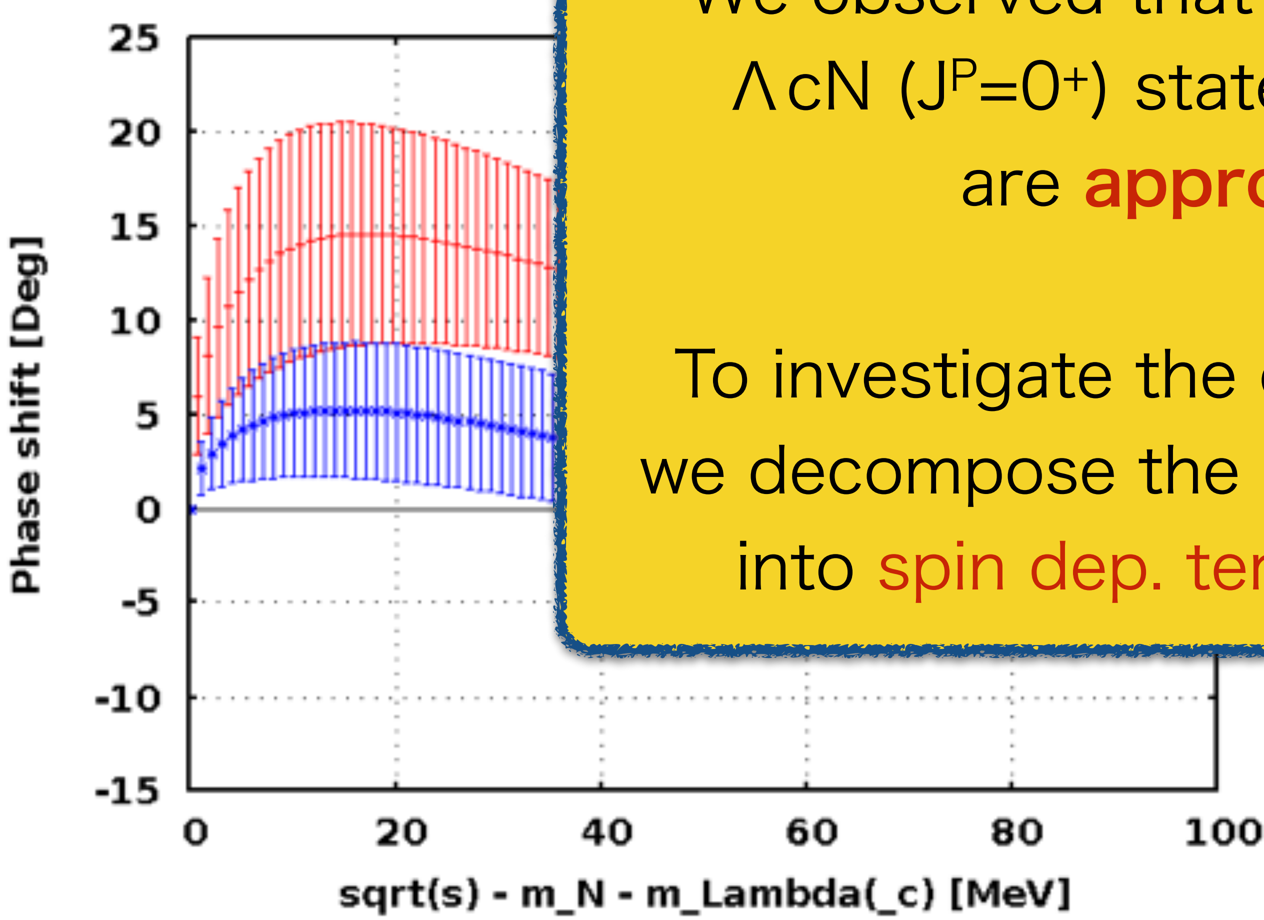
$$a = \lim_{k\to 0} \frac{\tan\delta(k)}{k}$$

$J^P=0^+$: $a_{\Lambda N}$ = 0.56(30) fm, $a_{\Lambda cN}$ = 0.18(12) fm

$J^P=1^+$: $a_{\Lambda N}$ = 0.87(45) fm, $a_{\Lambda cN}$ = 0.21(15) fm

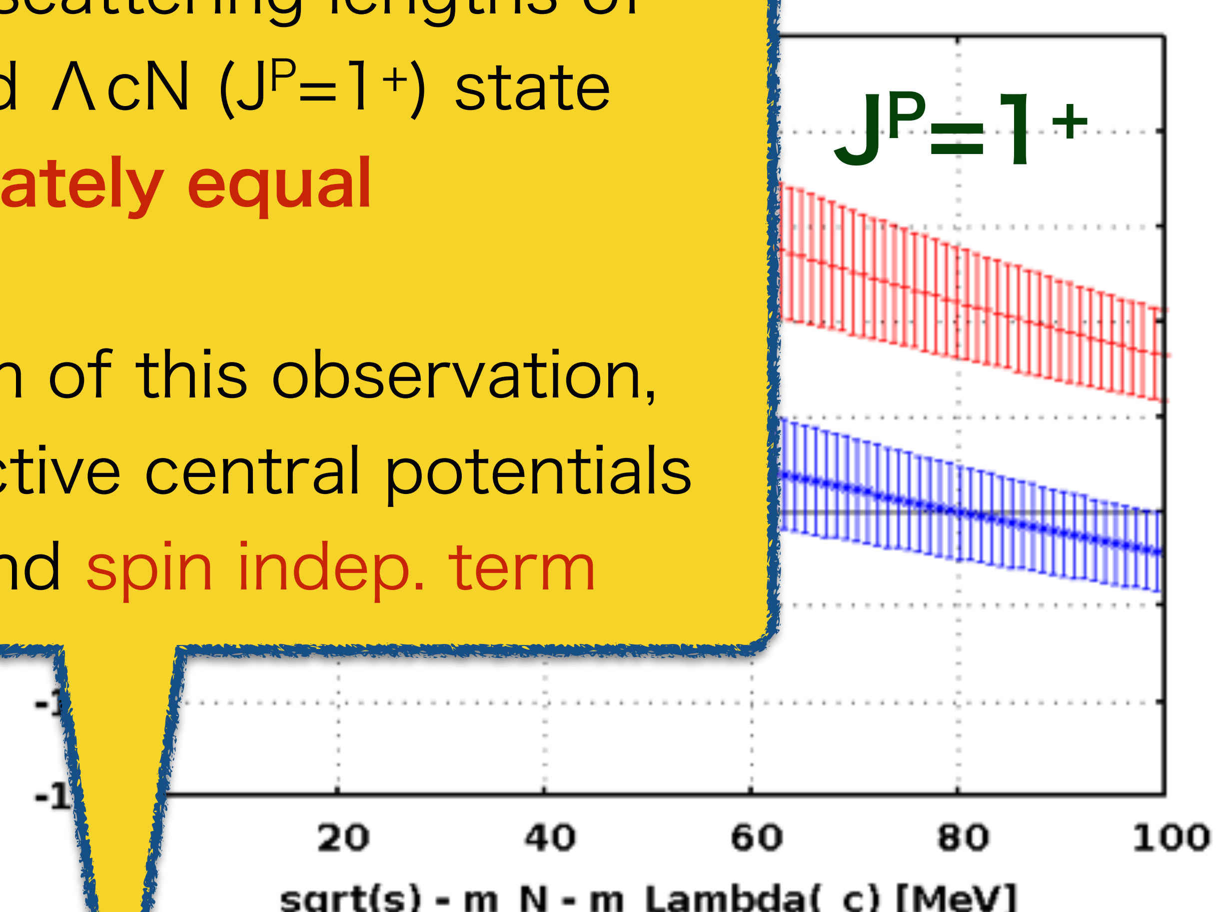
Comparison of ΛN potential and ΛcN potential (m_π = 570 MeV)

Lambda N
Lambda_c N



We observed that the scattering lengths of ΛcN ($J^P=0^+$) state and ΛcN ($J^P=1^+$) state are **approximately equal**

To investigate the origin of this observation, we decompose the effective central potentials into spin dep. term and spin indep. term



Scattering length:

$$a = \lim_{k\to 0} \frac{\tan\delta(k)}{k}$$

$a_{\Lambda N}$ = 0.56(30) fm
$a_{\Lambda cN}$ = 0.18(12) fm

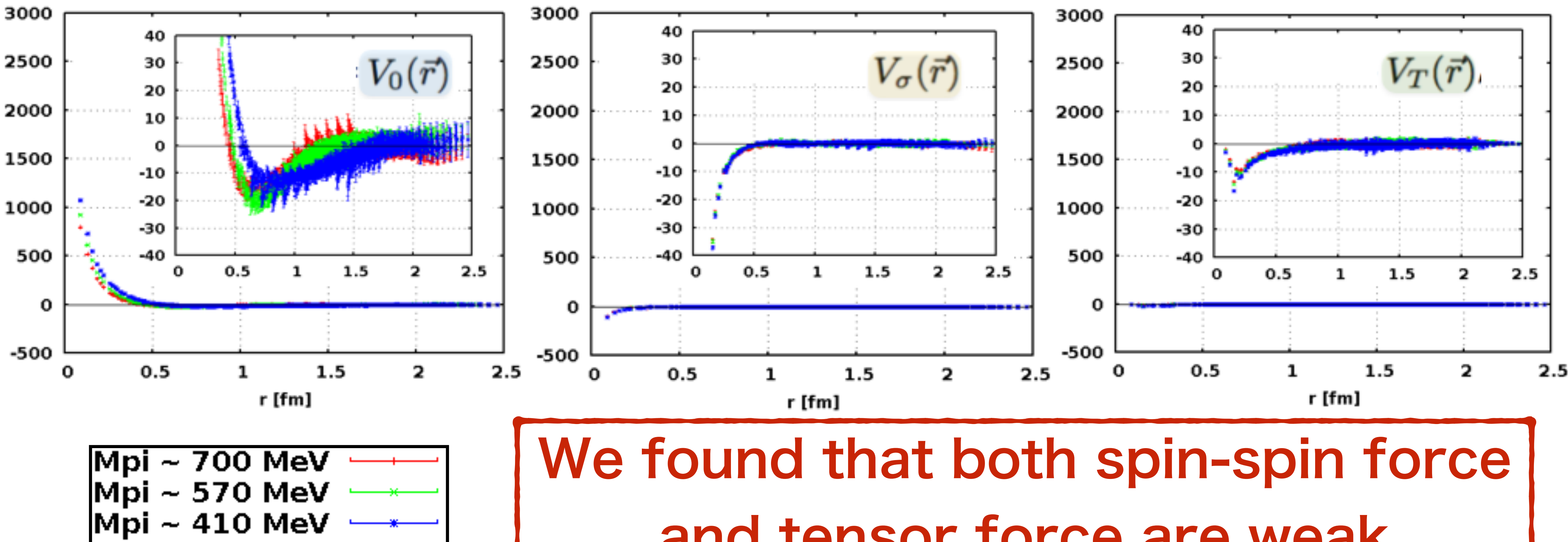
$a_{\Lambda N}$ = 0.87(45) fm
$a_{\Lambda cN}$ = 0.21(15) fm

Effective central potential of ΛcN

$$V^{eff}_{\Lambda_c N}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r}) S_{12} + \cdots$$

These three leading terms (in velocity expansion)
can be obtained from NBS wave functions in $J^P=0^+$ and 1^+

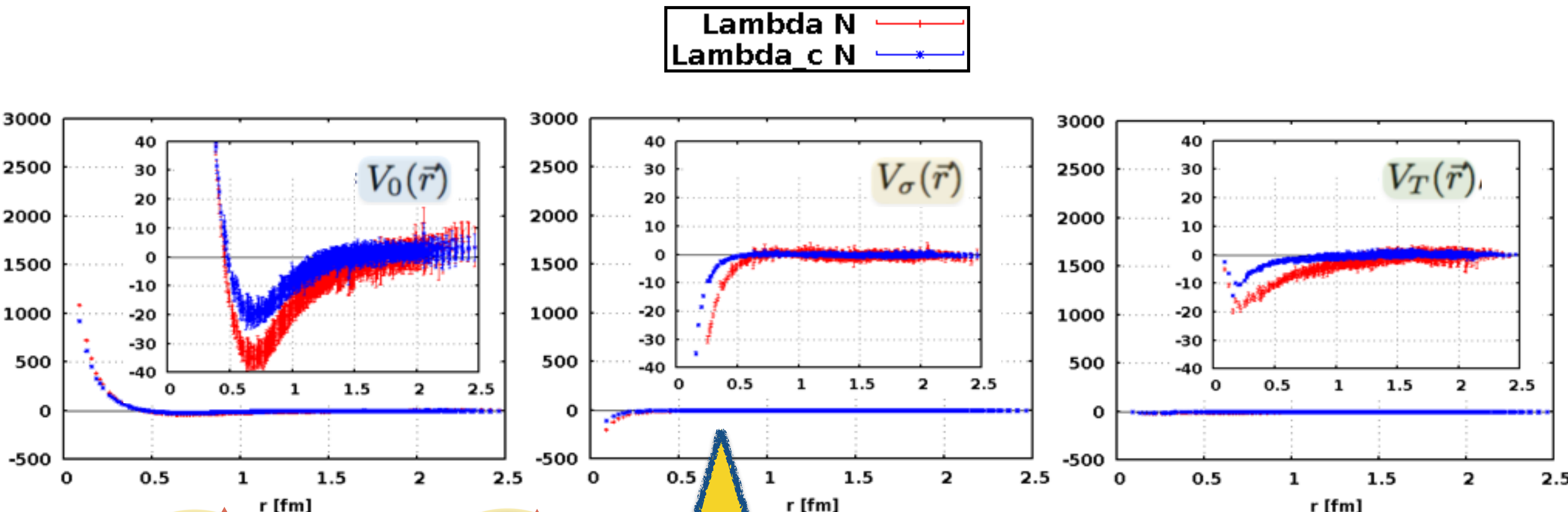
Each potential can be applied to nuclear calculations

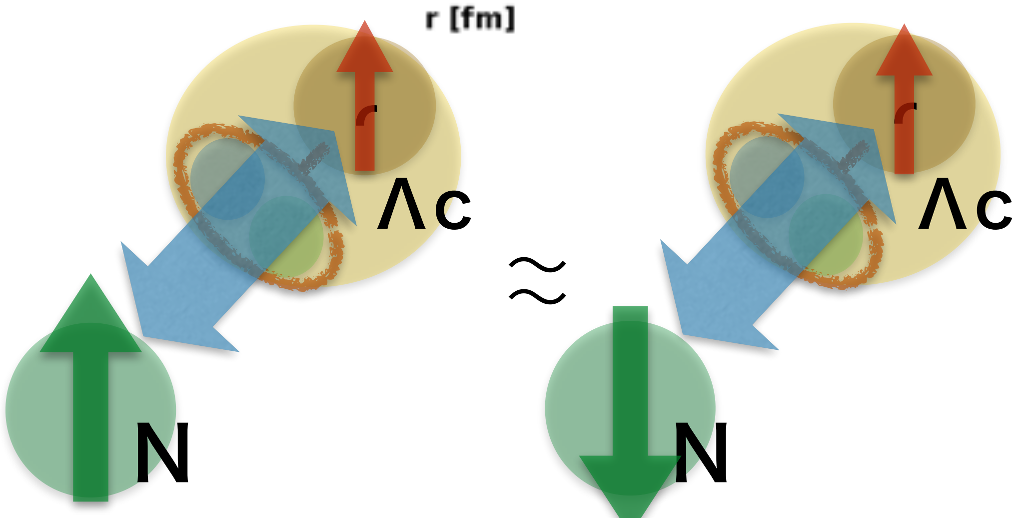


We found that both spin-spin force and tensor force are weak

Comparison of ΛN potential and ΛcN potential (m_π = 570 MeV)

$$V^{eff}_{\Lambda_{(c)}N}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r})S_{12} + \cdots$$

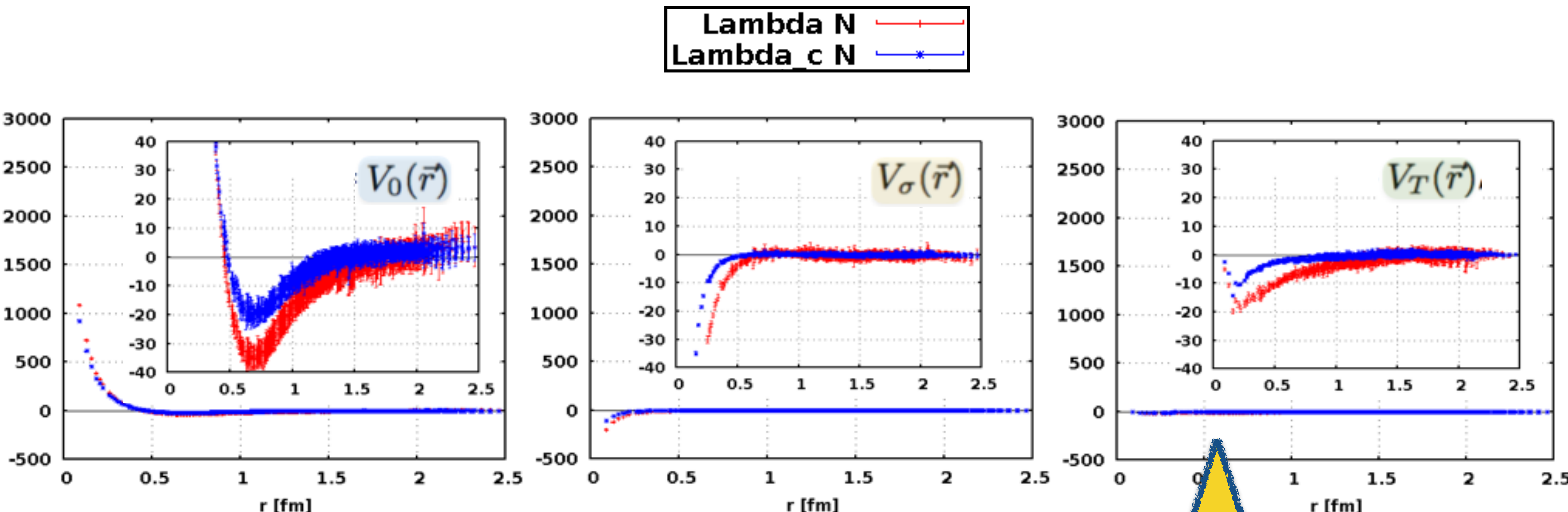


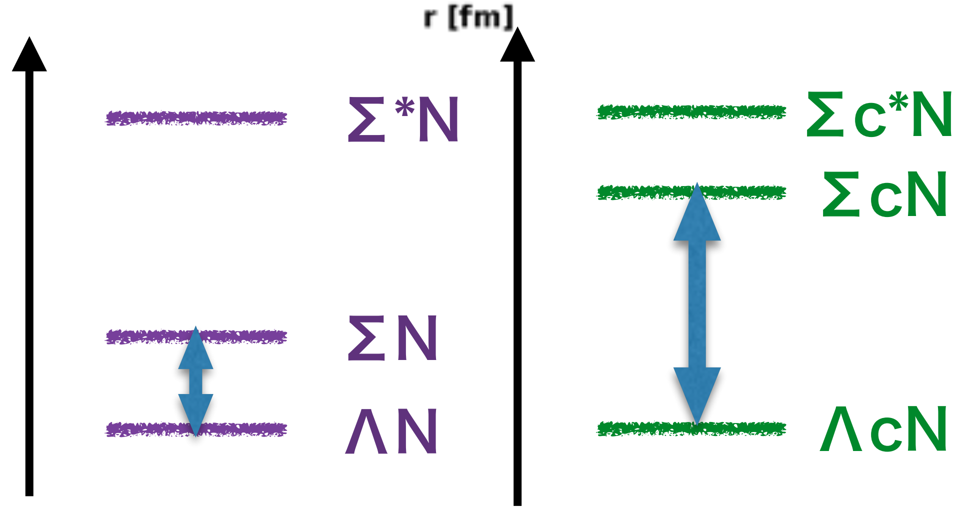


Weak spin-spin force of ΛcN
—> It can be explained from
the HQ spin symmetry

Comparison of ΛN potential and ΛcN potential
(m_π = 570 MeV)

$$V^{eff}_{\Lambda_{(c)}N}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r})S_{12} + \cdots$$

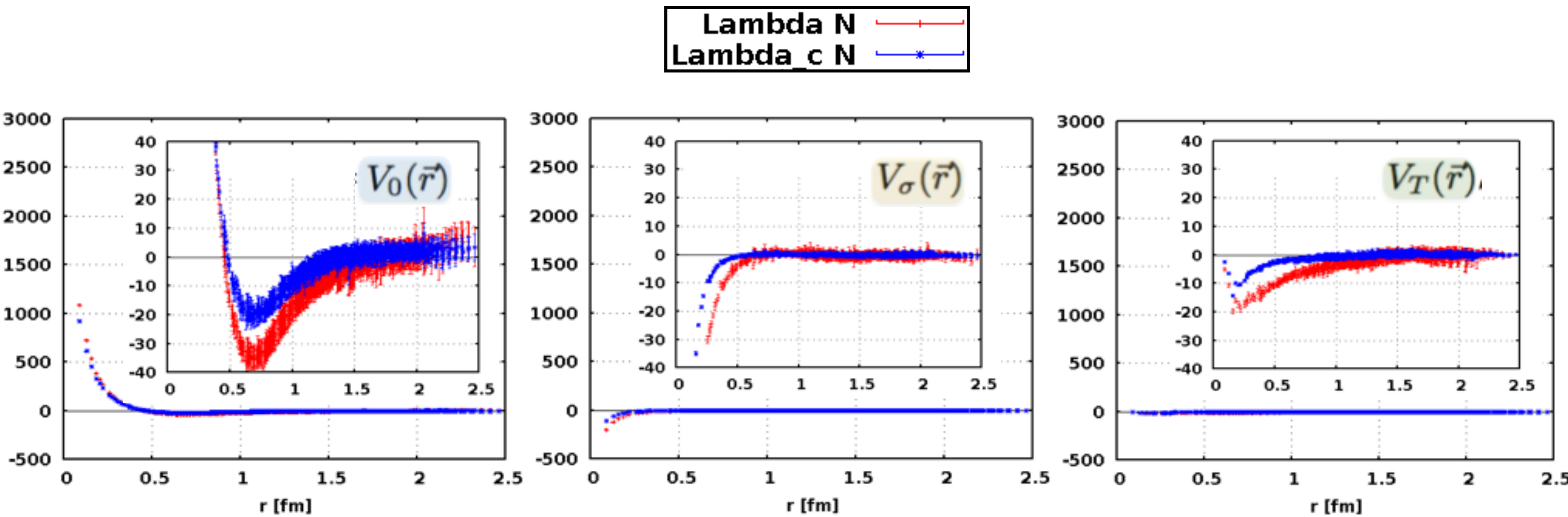




Weak tensor force of ΛcN
—> It can be explained from
large difference of ΛcN-ΣcN threshold

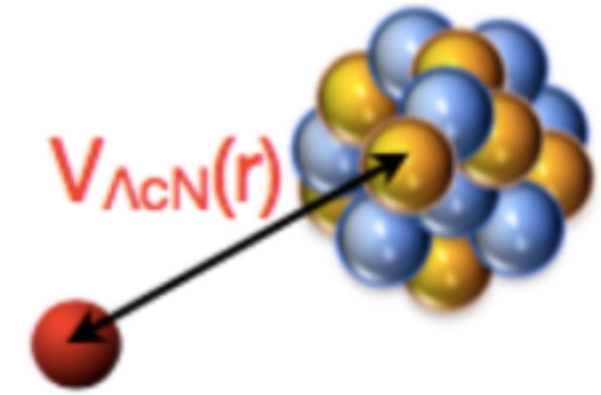
Comparison of ΛN potential and ΛcN potential
(m_π = 570 MeV)

$$V^{eff}_{\Lambda_{(c)}N}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})\,(\vec{\sigma}_1\cdot\vec{\sigma}_2) + V_T(\vec{r})S_{12} + \cdots$$



Since the s-wave ΛcN interactions are attraction,
the Λc could be bound into heavy nuclei

Λc-nuclei



(Single-) folding potential

$$V_F(\boldsymbol{r}) = \int d^3r' \underline{\rho_A(\boldsymbol{r}')} V_{\Lambda_c N}(\boldsymbol{r} - \boldsymbol{r}')$$

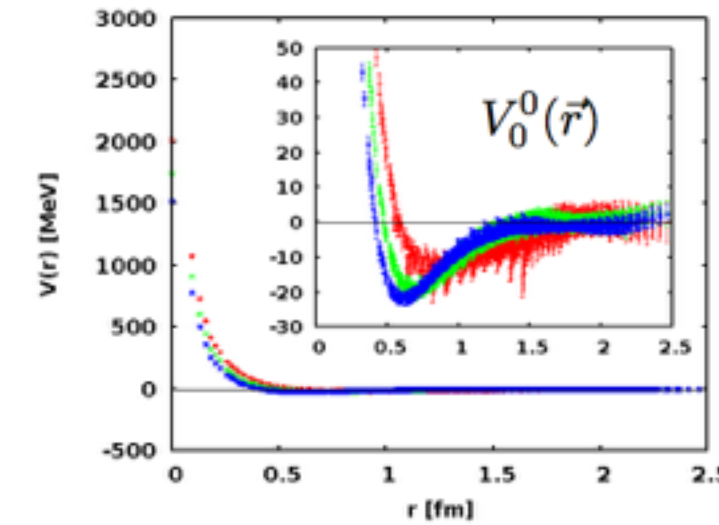
density distributions
for nuclear matter

$$\rho_A(r) = \rho_0 \left[1 + \exp\left(\frac{r - c}{a} \right) \right]^{-1}$$

two-parameter
Fermi (FM) form

$$\left(\int d^3r \; \rho_A(r) = A \right)$$

HAL QCD potential



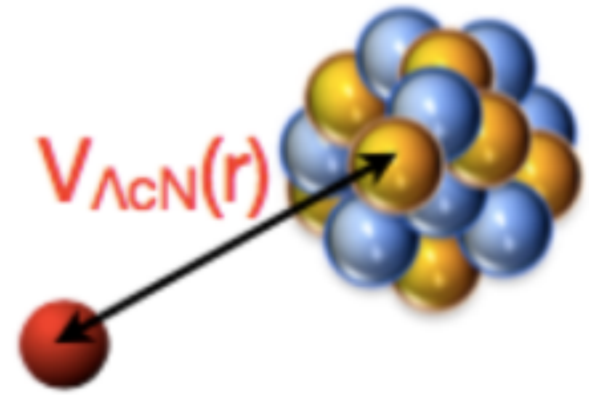
Since the spin dep. force is weak, we use only the spin indep. force for the folding potentials.

・Parameters for several stable nuclei

Nucleus	^{12}C	^{28}Si	^{40}Ca	^{58}Ni	^{90}Zr	^{208}Pb
ρ_0 (fm^{-3})	0.207	0.175	0.169	0.172	0.165	0.150
c (fm)	2.1545	3.15	3.60	4.094	4.90	6.80
α (fm)	0.425	0.475	0.523	0.54	0.515	0.515

Ref: M. El-Azab Farid a, M.A. Hassanain, Nuclear Physics A 678 (2000) 39–75

Λc-nuclei

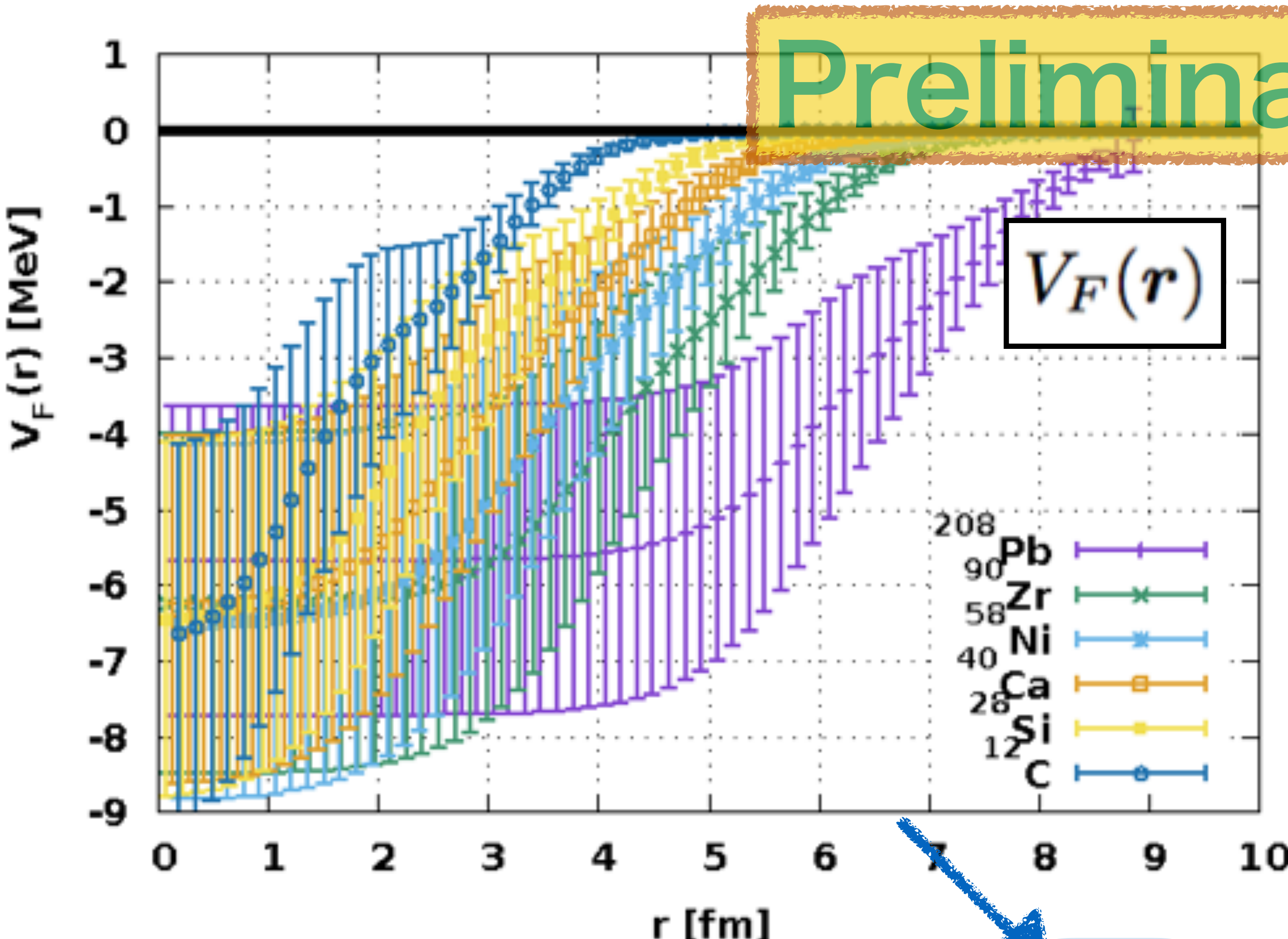


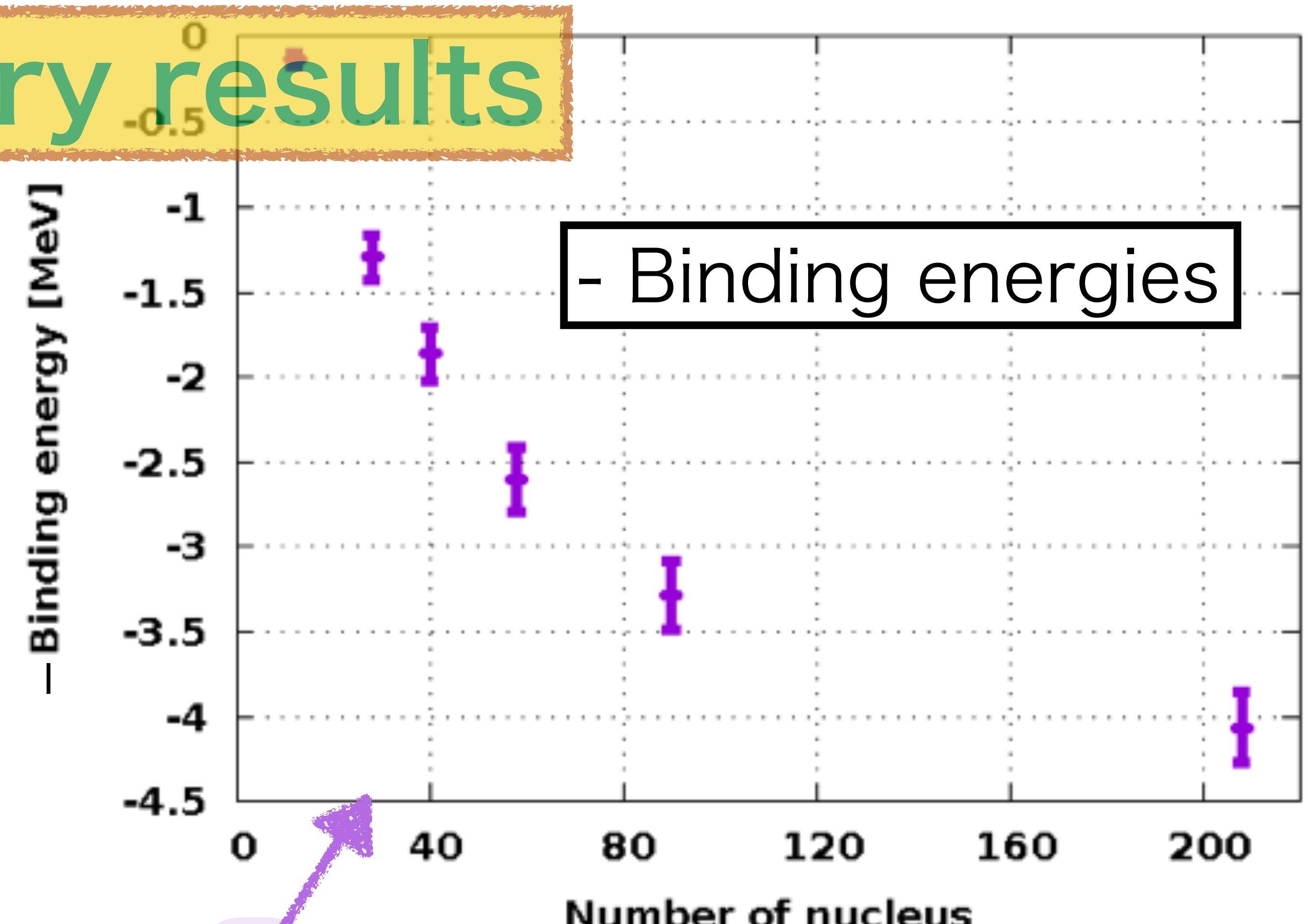
(Single-) folding potential

$$V_F(\boldsymbol{r}) = \int d^3r' \rho_A(\boldsymbol{r}') V_{\Lambda_c N}(\boldsymbol{r} - \boldsymbol{r}')$$

HAL QCD potential
with m_π = 570 MeV

Preliminary results





$$\left(H_0 + V_F(\boldsymbol{r})\right)\psi(\boldsymbol{r}) = E\,\psi(\boldsymbol{r})$$

In order to calculate binding energies, we solve the Schrodinger equation with physical mass of Λc and nucleus.

Conclusion

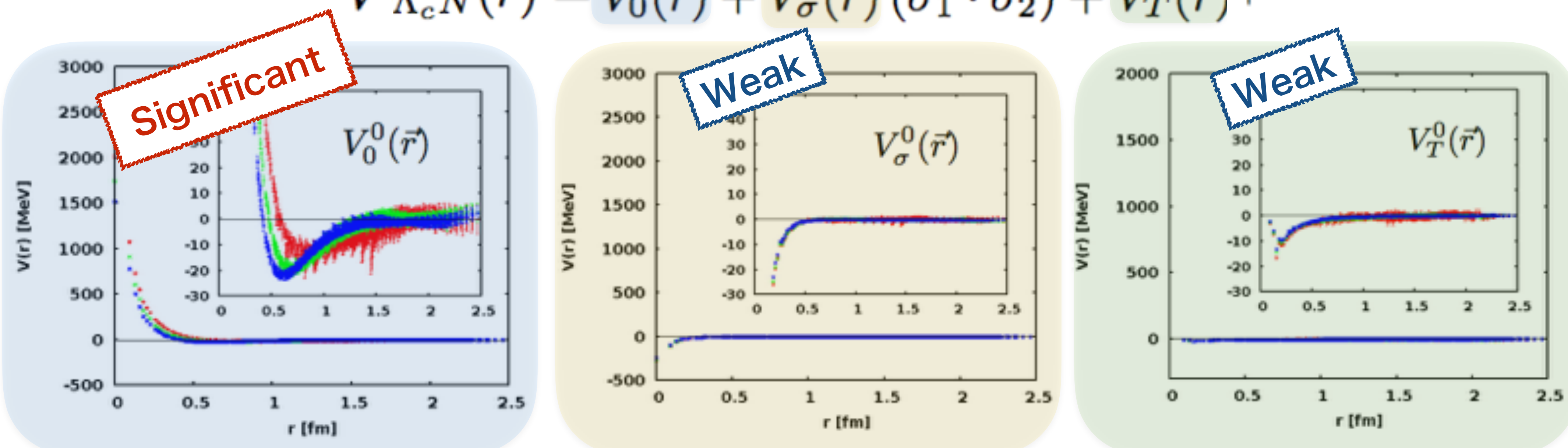
- We investigate ΛcN interactions by the HAL QCD method

Our results show that ΛcN interactions are **attractive**

The feature of ΛcN potentials is **spin-independent**

→ The spin-spin force and the tensor force are almost negligible

$$V_{\Lambda_c N}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r}) + \cdots$$



The analysis of the folding potentials with LQCD potentials shows that Λc could be bound with heavy nuclei.

Prospects:

— We will investigate Λc - light nuclei by using few/many-body calculation (GEM, AMD, ...).

— Physical point calculation is the next step

Backup

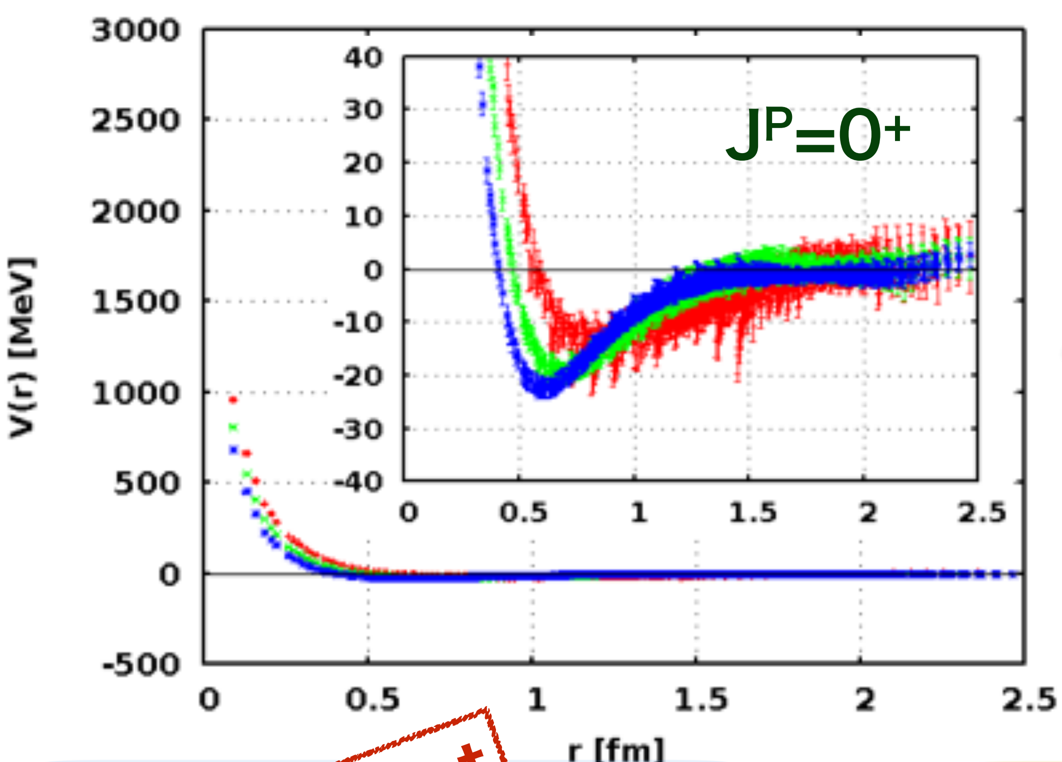
Short summary

$\Lambda_c N$ single-channel potential

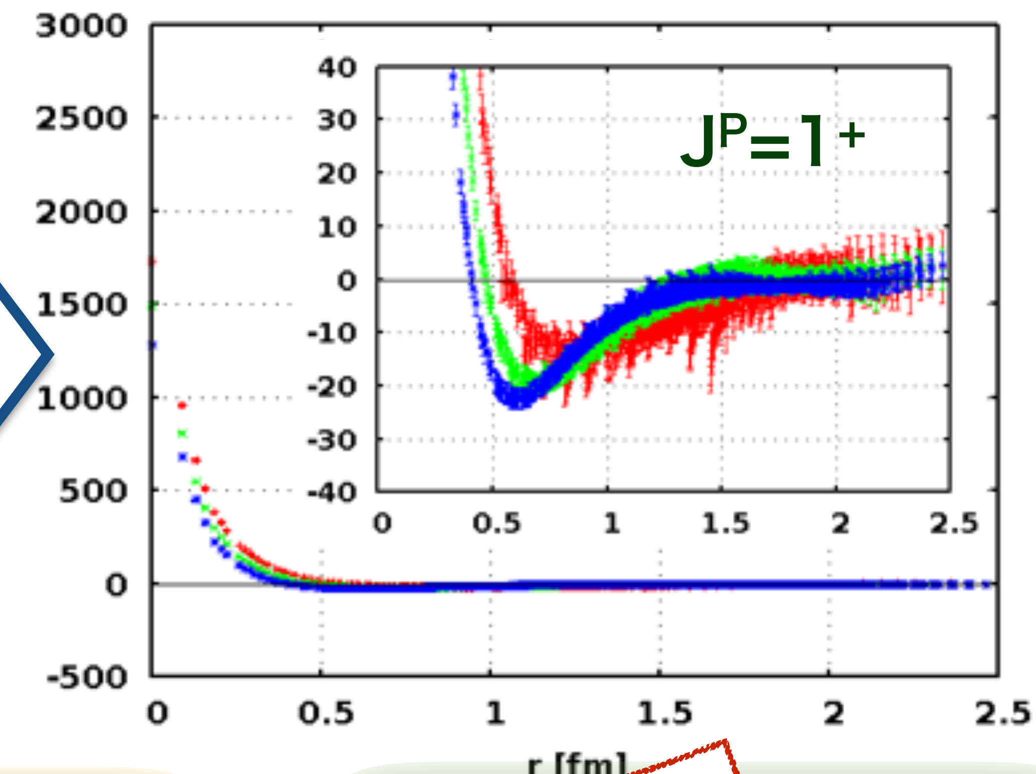
Mpi ~ 410 MeV
Mpi ~ 570 MeV
Mpi ~ 700 MeV

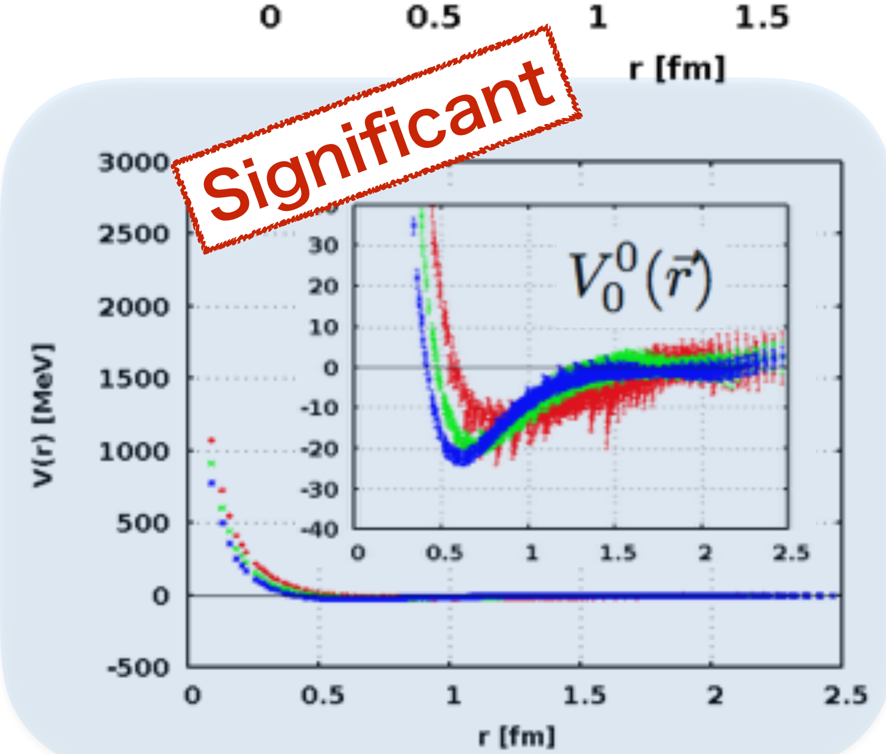
$$V^{(J^P=0^+)}_{\Lambda_c N}(\vec{r}) = V_0^0(\vec{r}) - 3V_\sigma^0(\vec{r})$$

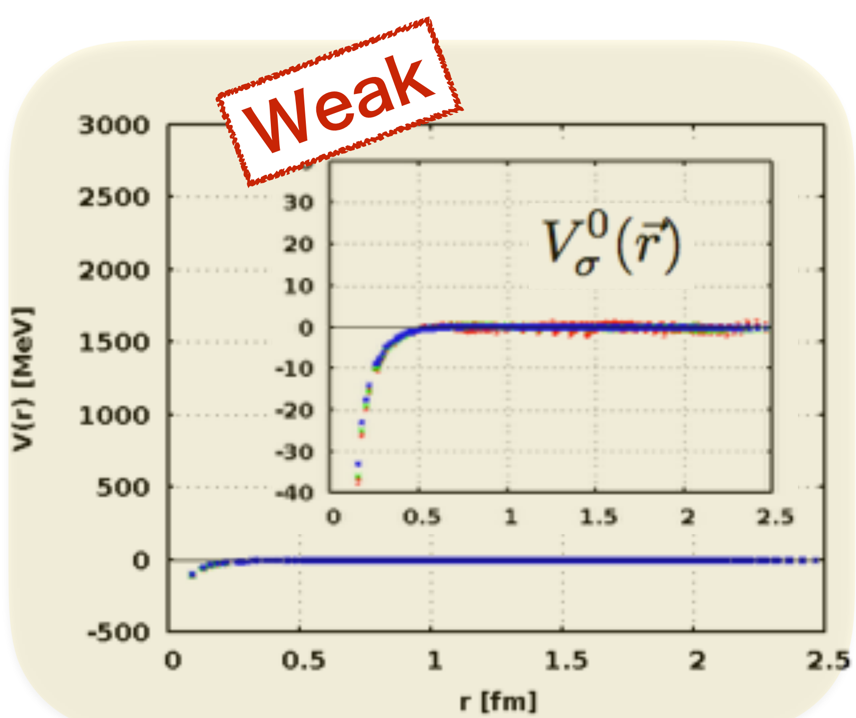
$$V^{(J^P=1^+)}_{\Lambda_c N}(\vec{r}) = V_0^0(\vec{r}) + V_\sigma^0(\vec{r}) + V_T^0(\vec{r})S_{12}$$

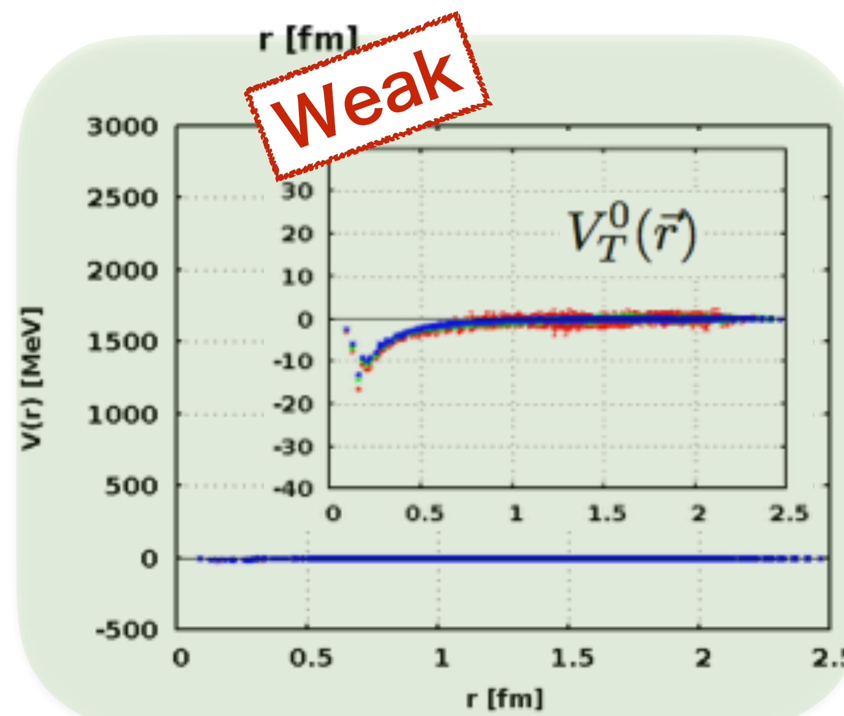


Almost same

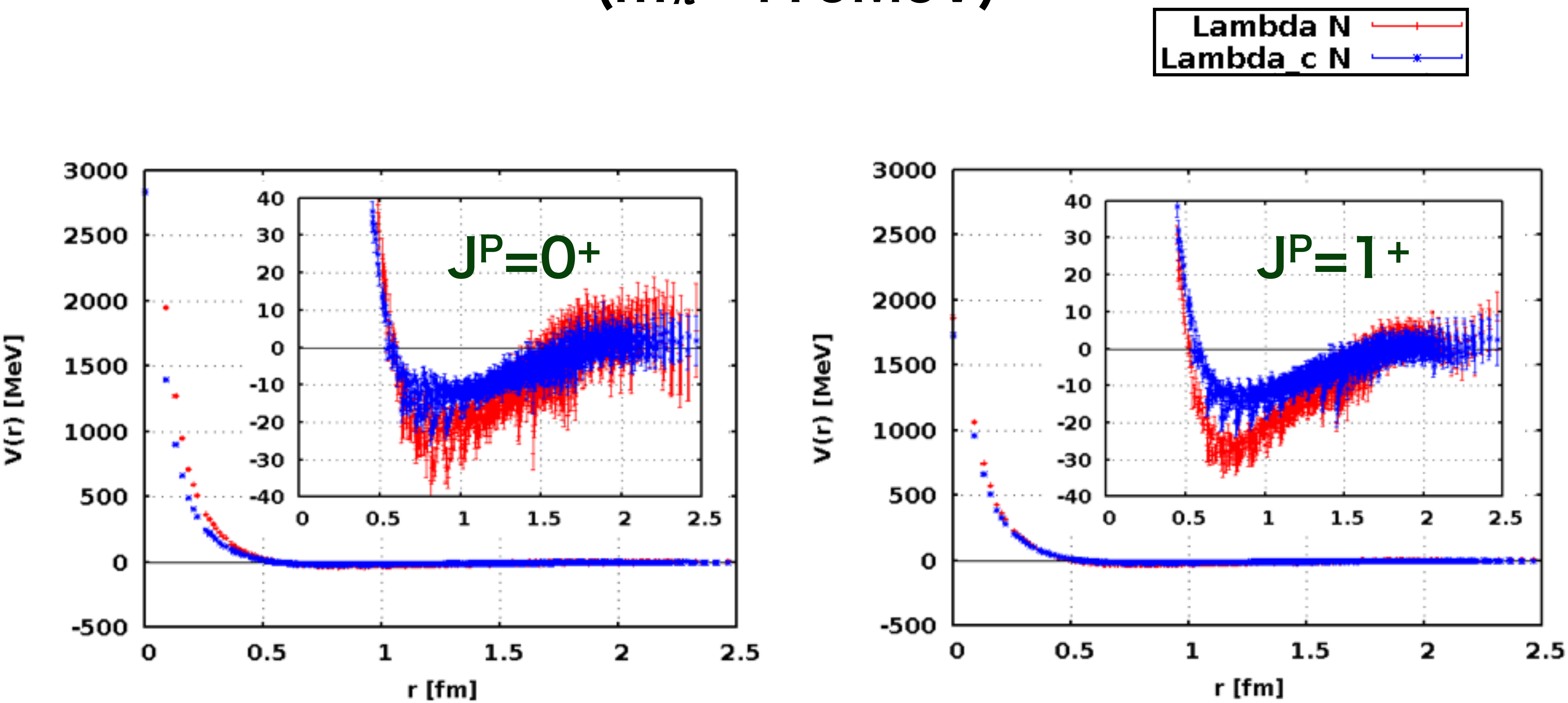








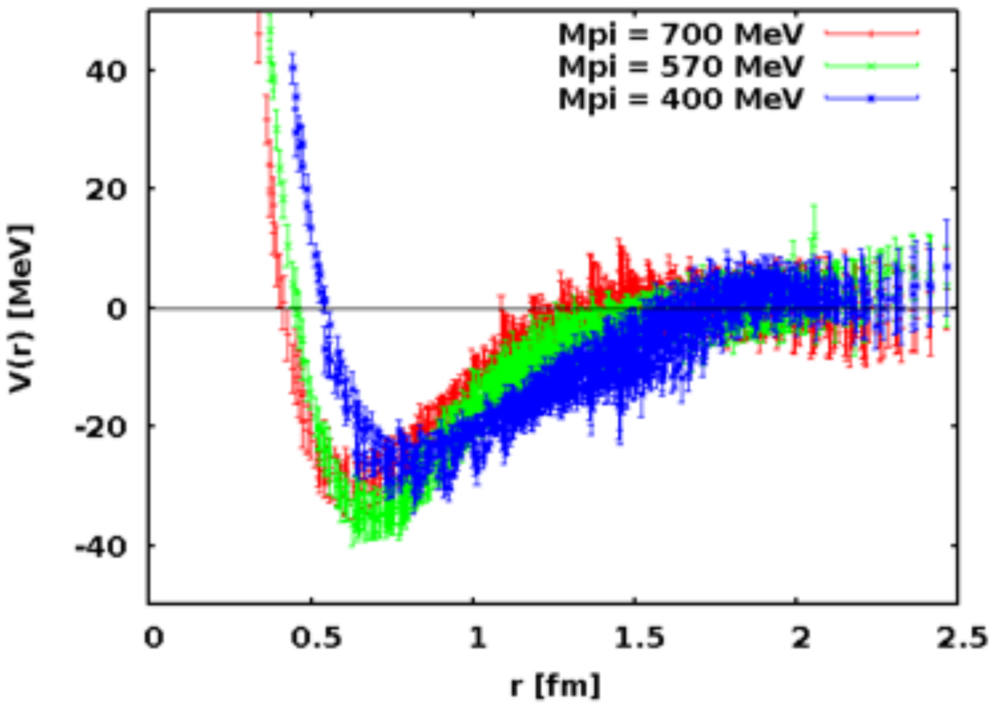
Comparison of ΛN potential and ΛcN potential (m_π=410MeV)



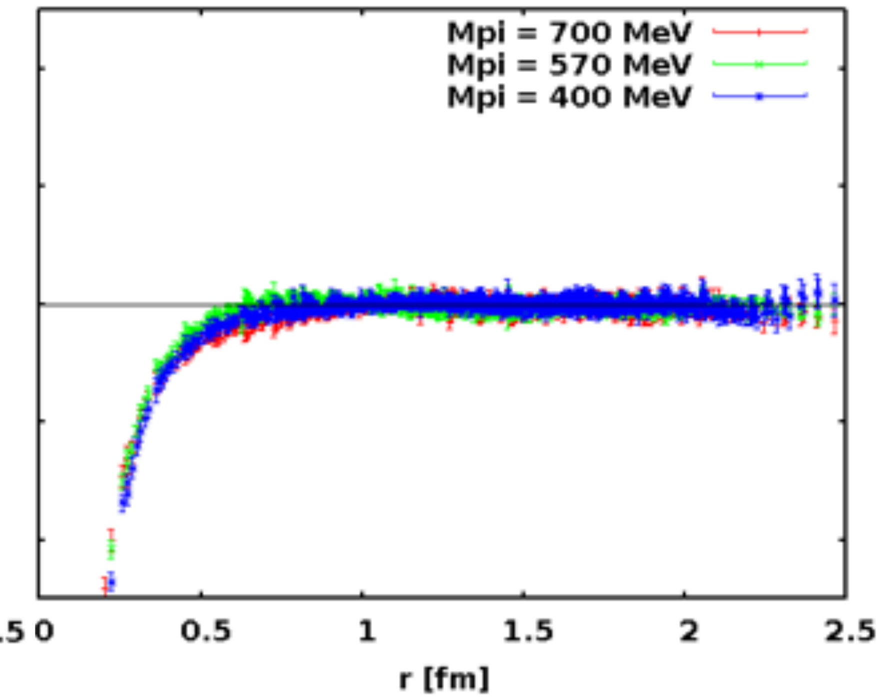
ΛcN potentials (both $J^P=0^+$ and $J^P=1^+$) are weaker than ΛN potentials

ens1:t=13, ens2:t=12, ens3:t=10

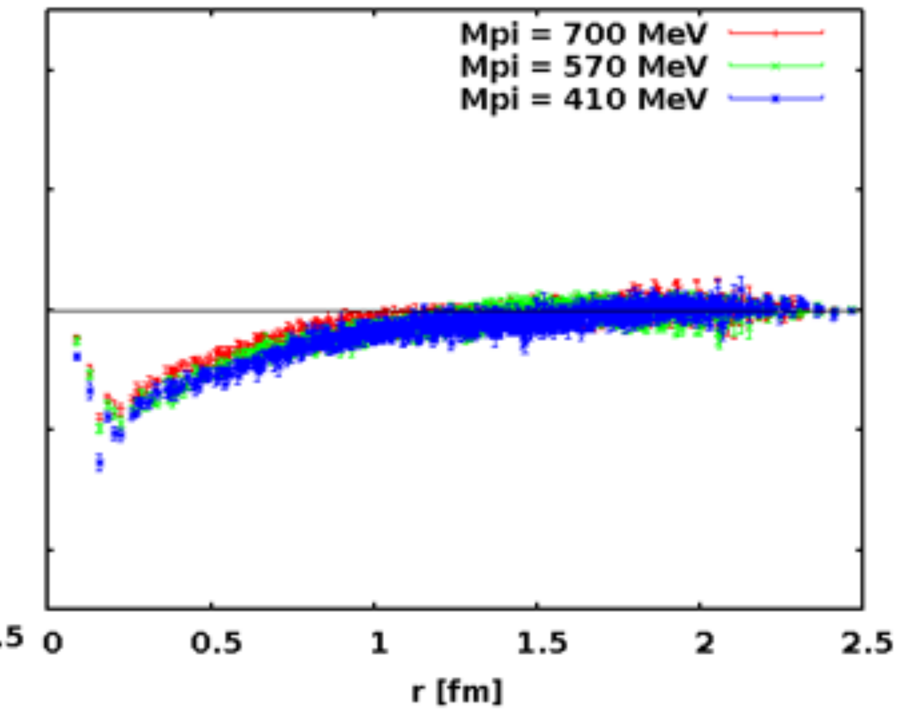
ΛN spin indep.



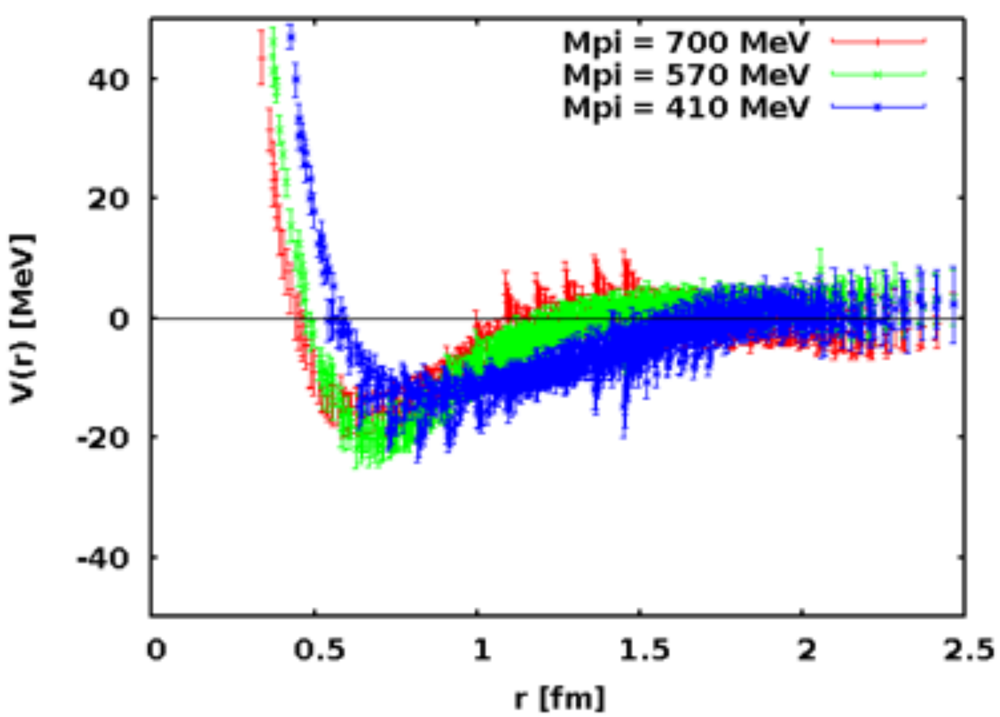
ΛN spin dep.



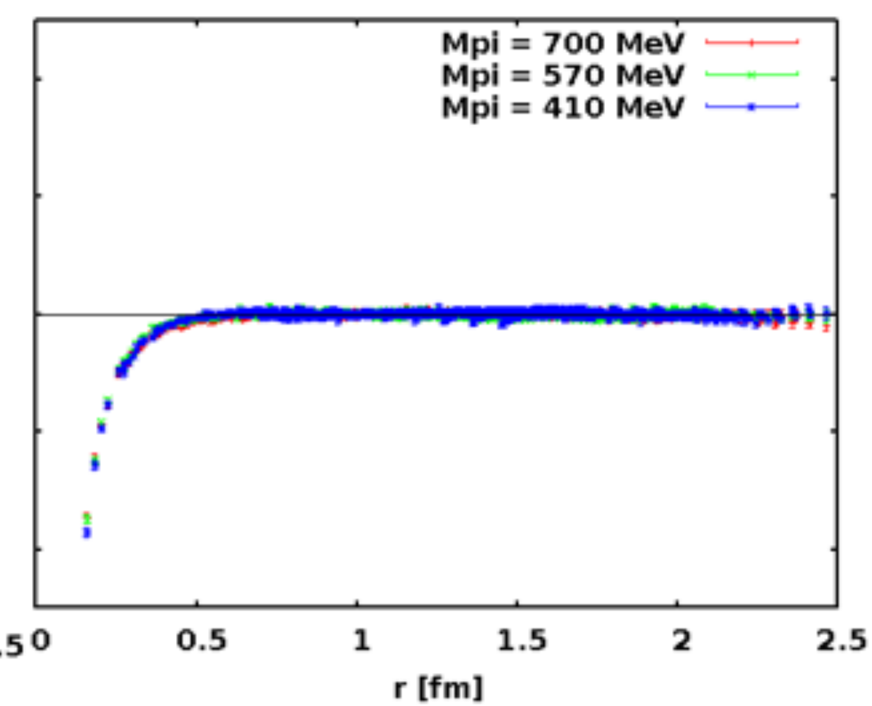
ΛN tensor



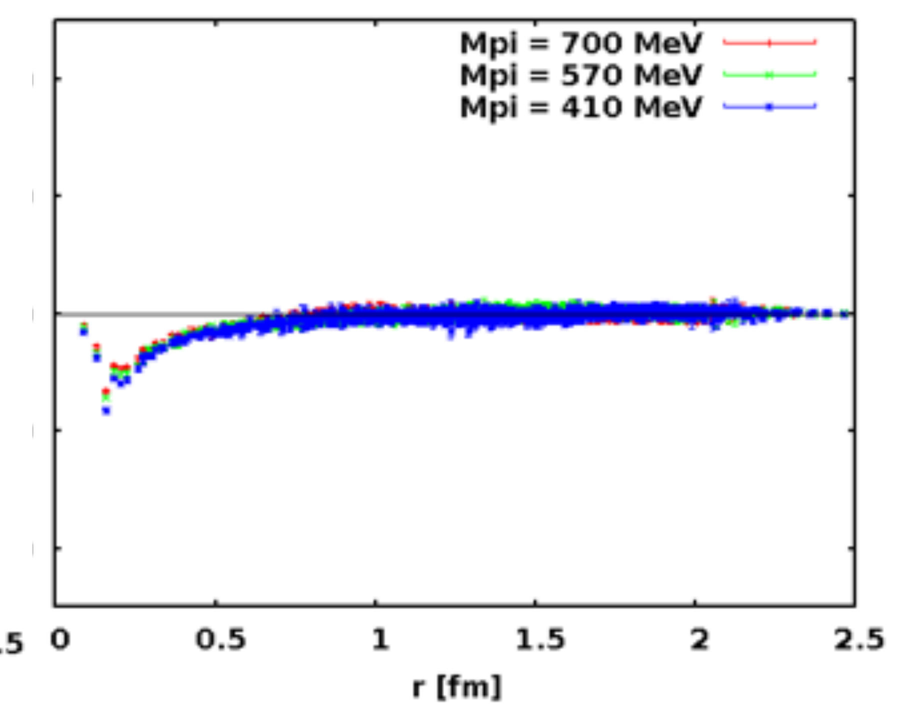
ΛcN spin indep.



ΛcN spin dep.

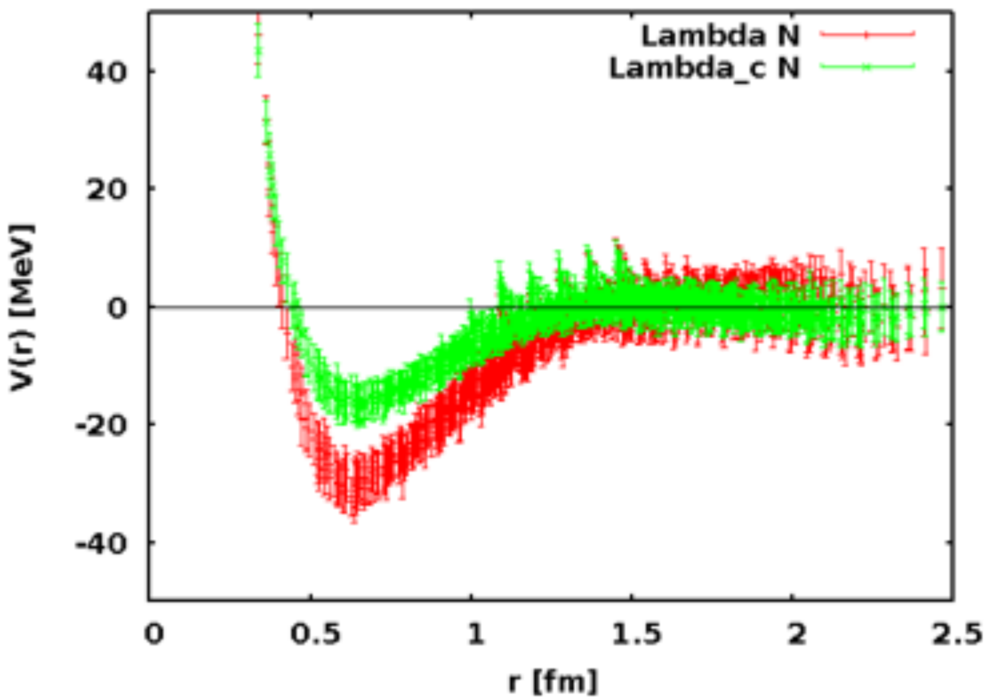


ΛcN tensor

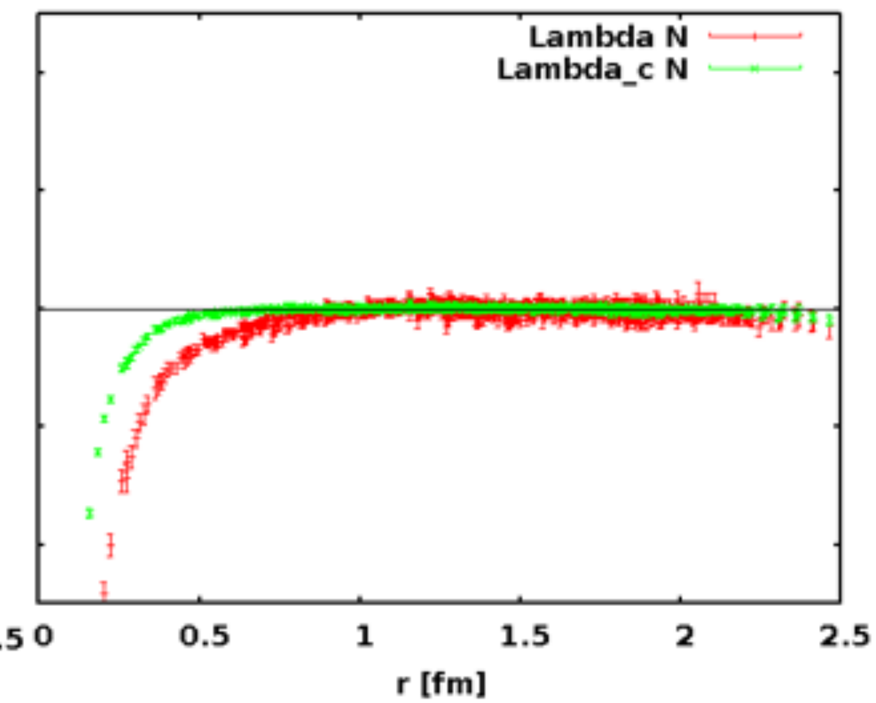


ens1:t=13, ens2:t=12, ens3:t=10

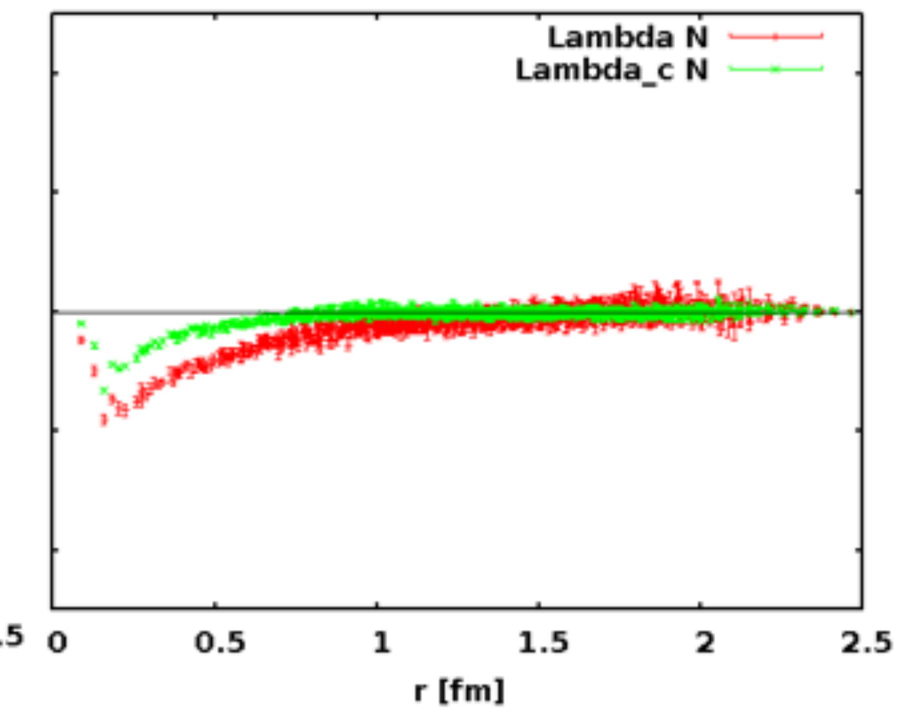
spin indep. (m_π=700MeV)



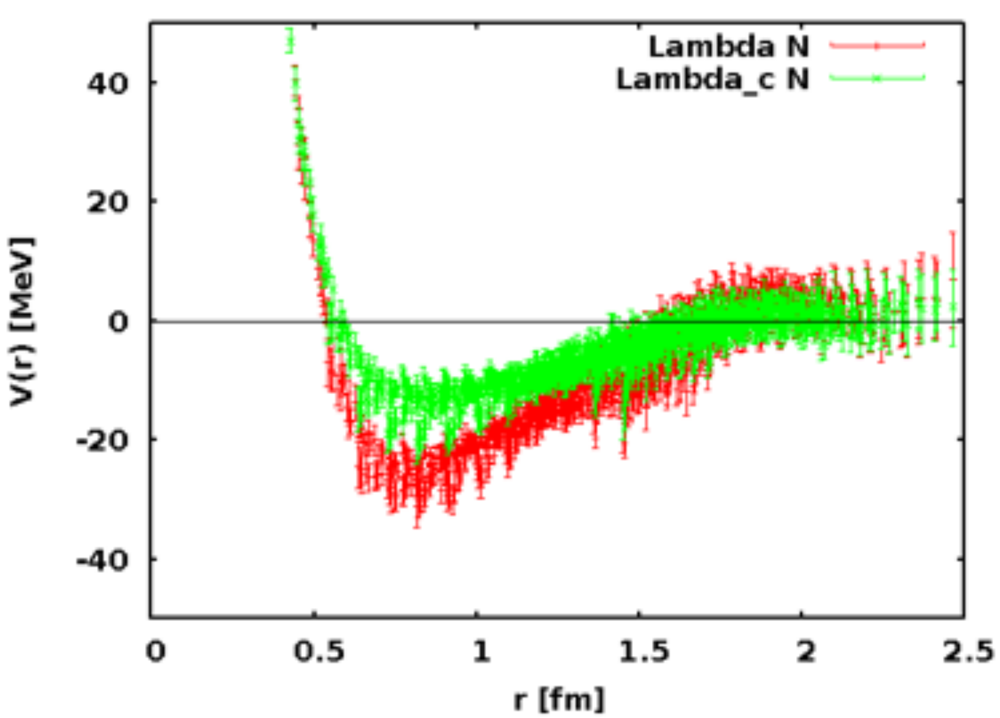
spin dep. (m_π=700MeV)



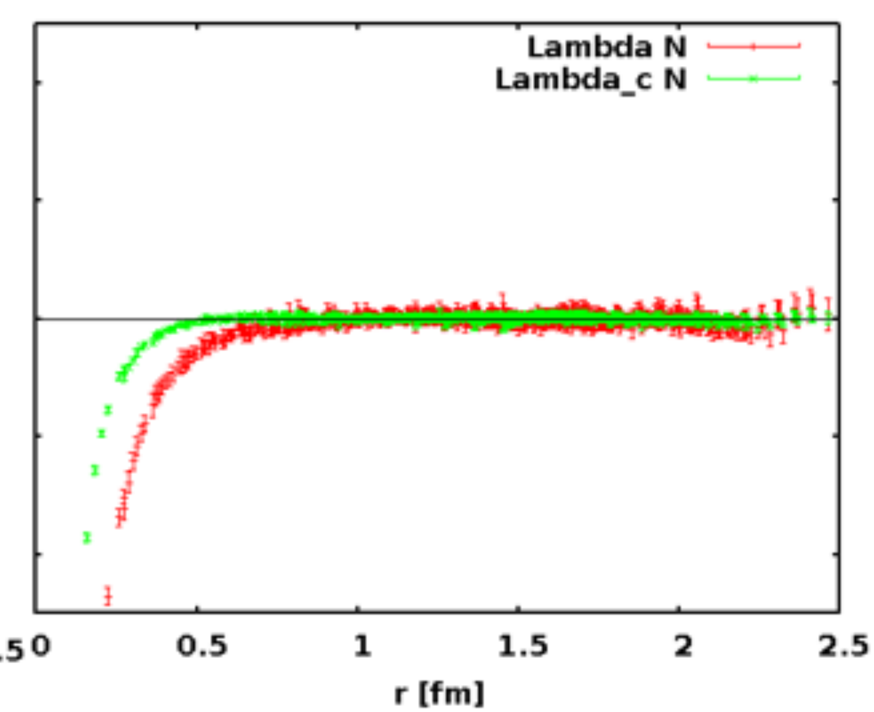
tensor (m_π=700MeV)



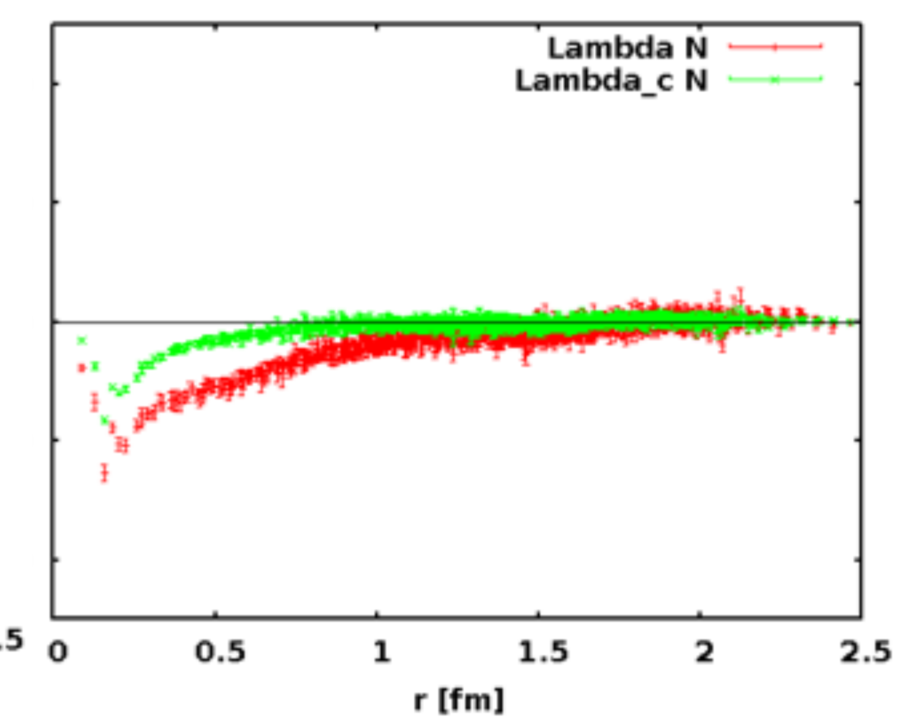
spin indep. (m_π=410MeV)



spin dep. (m_π=410MeV)

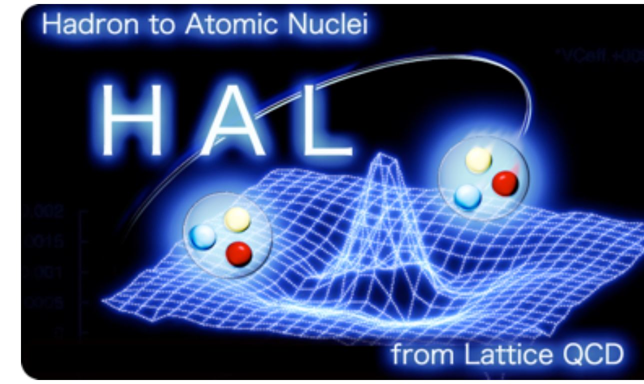


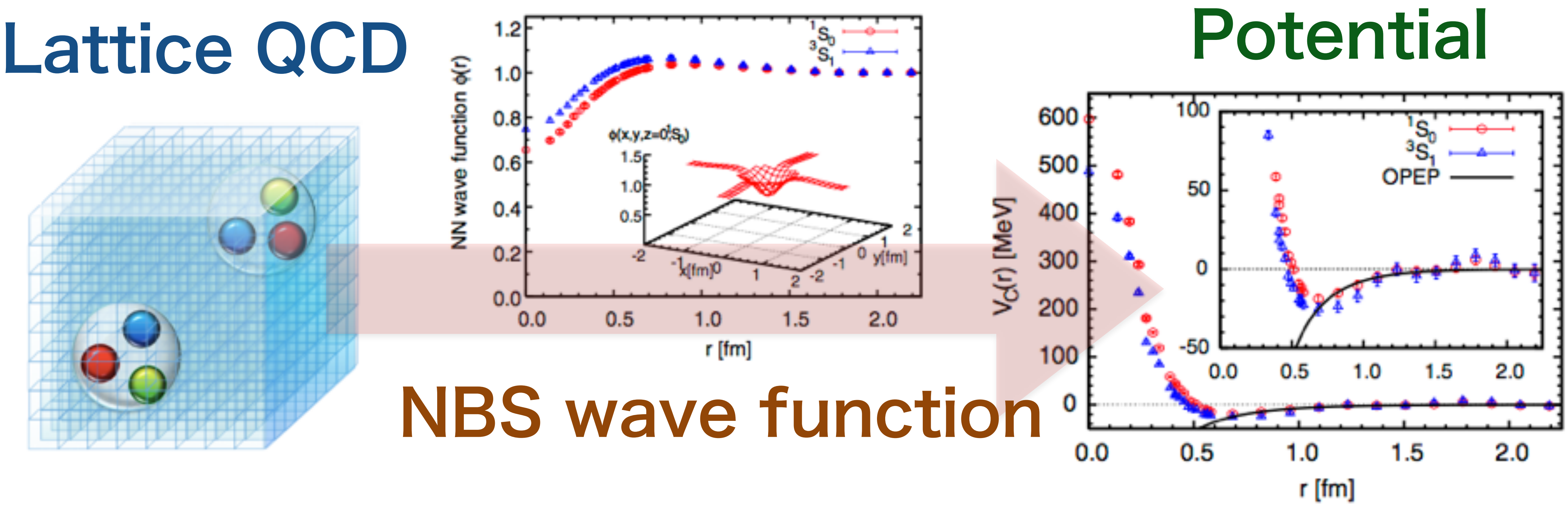
tensor (m_π=410MeV)



Hadron force from lattice QCD

N. Ishii, S. Aoki, T. Hatsuda, Phys.Rev.Lett. 99, 022001 (2007)





- Octet Baryon Interactions
- Decaplet Baryon Interactions
- Meson Interactions, Meson-Baryon Interactions
- Three-body forces
- **Charmed Baryon Interactions** <- **This work**

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys., 123 (2010).
S. Aoki *et al*, [HAL QCD Collaboration], PTEP., 01A105 (2012).

- **Derivative (velocity) expansion:** (Non-local potentials —> Local potentials)

$$U(\vec{r},\vec{r}') = V(\vec{r},\vec{\nabla})\ \delta^3(\vec{r}-\vec{r}')$$

LO term

$$V(\vec{r},\vec{\nabla}) = V_0(\vec{r}) + V_\sigma(\vec{r})\,(\vec{\sigma}_1\cdot\vec{\sigma}_2) + V_T(\vec{r})S_{12} + \mathcal{O}(\vec{\nabla})$$

$$V_X(\vec{r}) = V_X^0(\vec{r}) + V_X^\tau(\vec{r})\,(\vec{\tau}_1\cdot\vec{\tau}_2) \qquad X = 0,\sigma,T,\cdots$$

In the low energy state, **LO term of the potential** is significant.

$$(E_n - H_0)\,\psi_{\Lambda_c N}^{(W_n)}(\vec{r}) = V_{\Lambda_c N}(\vec{r})\ \psi_{\Lambda_c N}^{(W_n)}(\vec{r})$$

$$V_{\Lambda_c N}(\vec{r}) = \frac{(E_n - H_0)\,\psi_{\Lambda_c N}^{(W_n)}(\vec{r})}{\psi_{\Lambda_c N}^{(W_n)}(\vec{r})}$$

We can construct local potentials by using derivative expansion.

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys., 123 (2010).
S. Aoki *et al*, [HAL QCD Collaboration], PTEP., 01A105 (2012).

- **Derivative (velocity) expansion:** (Non-local potentials —> Local potentials)

$$U(\vec{r},\vec{r}') = V(\vec{r},\vec{\nabla})\,\delta^3(\vec{r}-\vec{r}')$$

LO term

$$V(\vec{r},\vec{\nabla}) = V_0(\vec{r}) + V_\sigma(\vec{r})\,(\vec{\sigma}_1\cdot\vec{\sigma}_2) + V_T(\vec{r})S_{12} + \mathcal{O}(\vec{\nabla})$$

$$V_X(\vec{r}) = V_X^0(\vec{r}) + \cancel{V_X^\tau(\vec{r})\,(\vec{\tau}_1\cdot\vec{\tau}_2)} \qquad X = 0,\sigma,T,\cdots$$

The case of ΛcN potentials, the LO term of derivative expansion is following form.

$$(\vec{\tau}_1\cdot\vec{\tau}_2)\,\psi_{\Lambda_c N}(\vec{r}) = 0$$

$$(\vec{\sigma}_1\cdot\vec{\sigma}_2)\,\psi_{\Lambda_c N}^{(J^P=0^+)}(\vec{r}) = -3\,\psi_{\Lambda_c N}^{(J^P=0^+)}(\vec{r})$$

$$(\vec{\sigma}_1\cdot\vec{\sigma}_2)\,\psi_{\Lambda_c N}^{(J^P=1^+)}(\vec{r}) = +1\,\psi_{\Lambda_c N}^{(J^P=1^+)}(\vec{r})$$

- $J^P=0^+$ state,

$$V_{\Lambda_c N}^{(J^P=0^+)}(\vec{r},\vec{\nabla}) = V_0^0(\vec{r}) - 3V_\sigma^0(\vec{r}) + \mathcal{O}(\vec{\nabla})$$

- $J^P=1^+$ state,

$$V_{\Lambda_c N}^{(J^P=1^+)}(\vec{r},\vec{\nabla}) = V_0^0(\vec{r}) + V_\sigma^0(\vec{r}) + V_T^0(\vec{r})S_{12} + \mathcal{O}(\vec{\nabla})$$

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys., 123 (2010).
S. Aoki *et al*, [HAL QCD Collaboration], PTEP., 01A105 (2012).

- How to extract the NBS wave functions on the lattice
  - Baryon 4pt-correlation function

$$
\begin{aligned}
\underline{G_{\Lambda_c N}(\vec{r}, t-t_0)} &= \sum_{\vec{x}} \langle 0|\Lambda_c(\vec{r}+\vec{x}, t) N(\vec{x}, t) \overline{\mathcal{J}_{\Lambda_c N}}(t_0)|0\rangle \\
&= \sum_n A_n \, \psi^{(W_n)}_{\Lambda_c N}(\vec{r}) \, e^{-W_n(t-t_0)} + \cdots \\
&\xrightarrow{t\to\infty} A_0 \, \underline{\psi^{(W_0)}_{\Lambda_c N}}(\vec{r}) \, e^{-W_0(t-t_0)} + \mathcal{O}\left(e^{-W_1(t-t_0)}\right)
\end{aligned}
$$

Ground state of the NBS wave functions.

$$
\mathcal{J}_{\Lambda_c N}(t_0) = \Lambda_c^{\mathrm{smear}}(t_0) N^{\mathrm{smear}}(t_0)
$$

$$
A_n = \langle \Lambda_c N, W_n | \overline{\mathcal{J}_{\Lambda_c N}}(t_0)|0\rangle
$$

For the operator smearing,
we employ the wall-source.
(Total momentum to be zero.)

We choose the following operator.

$$
N = \epsilon_{abc}\left[u_a^T C\gamma_5 d_b\right] q_c, \qquad q = \begin{pmatrix} u \\ d \end{pmatrix}
$$

$$
\Lambda_c = \frac{\epsilon_{abc}\left[c_a^T C\gamma_5 d_b\right] u_c + \epsilon_{abc}\left[u_a^T C\gamma_5 c_b\right] d_c - 2\epsilon_{abc}\left[d_a^T C\gamma_5 u_b\right] c_c}{\sqrt{6}}
$$

HAL QCD method

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys., 123 (2010).
S. Aoki *et al*, [HAL QCD Collaboration], PTEP., 01A105 (2012).

$$G_{\Lambda_c N}(\vec{r}, t-t_0) = \sum_{\vec{x}} \langle 0|\Lambda_c(\vec{r}+\vec{x}, t) N(\vec{x}, t) \overline{\mathcal{J}_{\Lambda_c N}}(t_0)|0\rangle$$

$$\xrightarrow{t\to\infty} A_0\, \psi^{(W_0)}_{\Lambda_c N}(\vec{r})\, e^{-W_0(t-t_0)} + \mathcal{O}\left(e^{-W_1(t-t_0)}\right)$$

Difficulty of ground state saturation

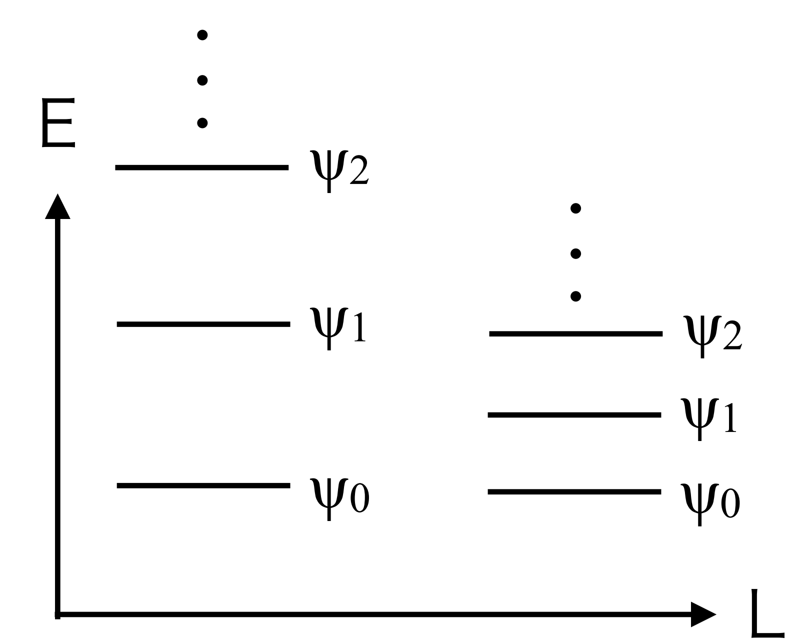
(1) Bad S/N at large t $\quad S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathrm{t}]$

In general, bad S/N occurs at large t-t_0 separation.

(2) Small ΔE at large volume

At large volume, excited state contaminations remain even at a large t due to small ΔE.

$$\Delta E \simeq \frac{1}{m_N}\frac{(2\pi)^2}{L^2}$$



To avoid this problem, we use time-dependent HAL QCD method.

time-dependent HAL QCD method

N. Ishii et al [HAL QCD Coll.], PLB712 (2012) 437.

- Normalized 4pt-correlation function (**R-correlator**)

$$R_{\Lambda_c N}(\vec{r}, t) \equiv \frac{G_{\Lambda_c N}(\vec{r}, t)}{e^{-m_{\Lambda_c} t} e^{-m_N t}}$$

$$= \sum_n A_n \, \psi^{(W_n)}_{\Lambda_c N}(\vec{r}) \, e^{-\Delta W_n t} + \cdots$$

$$\Delta W_n \equiv \sqrt{k_n^2 + m_{\Lambda_c}^2} + \sqrt{k_n^2 + m_N^2} - (m_{\Lambda_c} + m_N)$$

$$E_n = (\Delta W_n) + \frac{1+\delta^2}{8\mu} (\Delta W_n)^2 + \mathcal{O}\left[(\Delta W_n)^3\right]$$

$$\mu \equiv \frac{m_{\Lambda_c} m_N}{m_{\Lambda_c} + m_N} \qquad \delta \equiv \frac{m_{\Lambda_c} - m_N}{m_{\Lambda_c} + m_N}$$

$$(E_0 - H_0)\, \psi_0(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}\,')\psi_0(\vec{r}\,')$$

$$(E_1 - H_0)\, \psi_1(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}\,')\psi_1(\vec{r}\,')$$

$$(E_2 - H_0)\, \psi_2(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}\,')\psi_2(\vec{r}\,')$$

$$\ddots$$

The ground state saturation is not necessary.

We can obtain the signals at small t as long as the contributions of inelastic scatterings are negligible.

Within the approximation up to $(\Delta W)^2$.

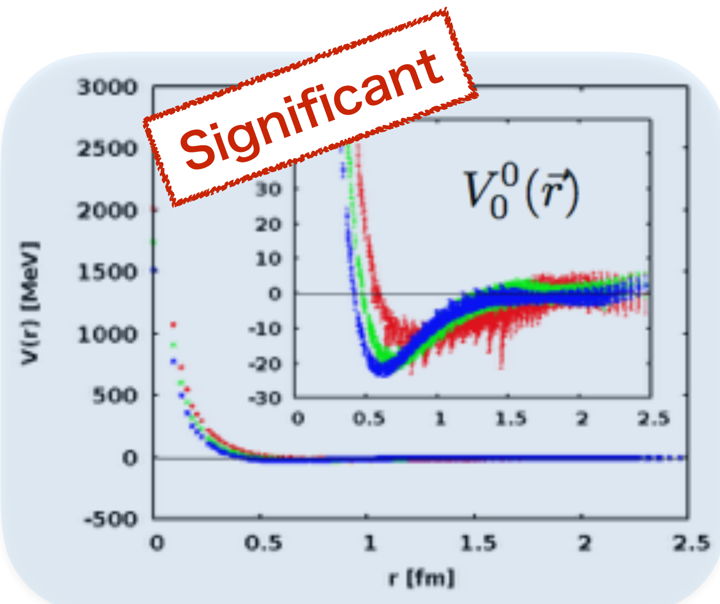
$$\boxed{\left(-\frac{\partial}{\partial t} + \left[\frac{1+\delta^2}{8\mu}\right] \frac{\partial^2}{\partial t^2} - H_0\right) R_{\Lambda_c N}(\vec{r}, t) = \int d^3 r' U_{\Lambda_c N}(\vec{r}, \vec{r}\,') R_{\Lambda_c N}(\vec{r}\,', t)}$$

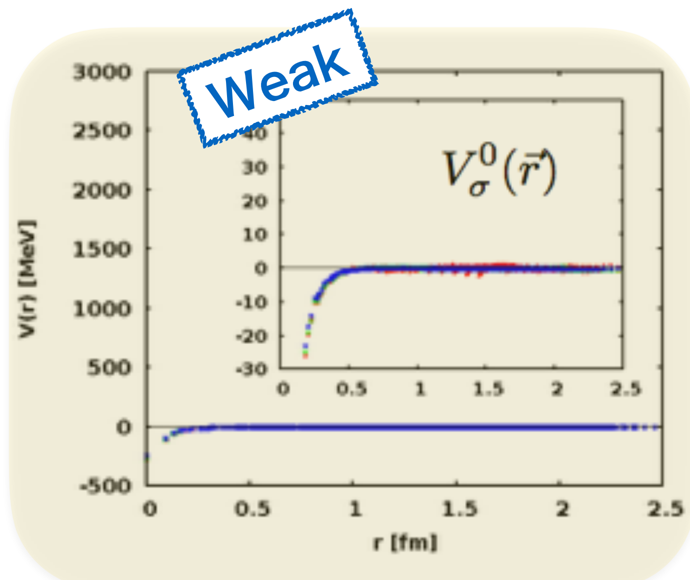
Summary of ΛcN interaction

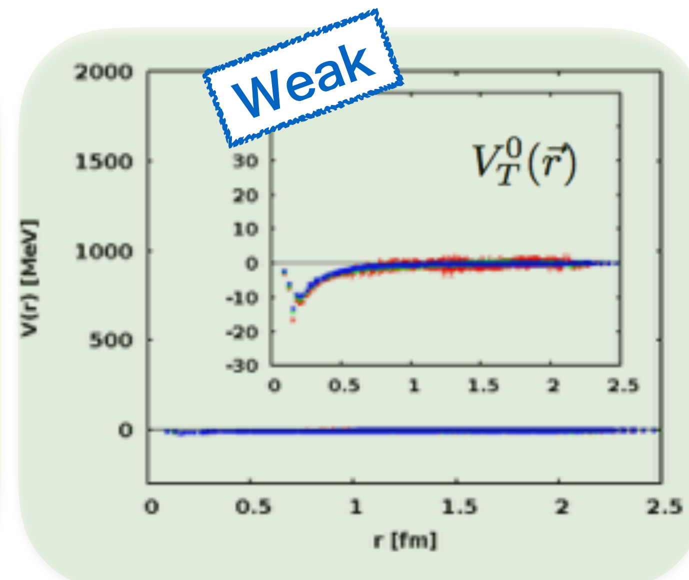
- Although ΛcN effective central potentials are attractive, the strength is weaker than that of ΛN potentials
- The effective central potentials for ΛcN ($J^P=0^+$) state and ΛcN ($J^P=1^+$) state are almost same

- We found the spin-spin force and tensor force of ΛcN potentials are almost negligible

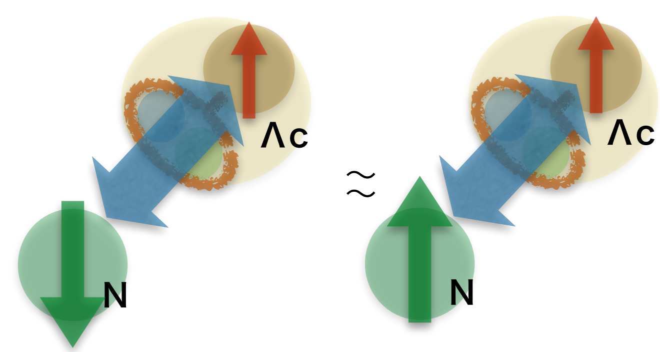
$$V_{\Lambda_c N}(\vec{r}) = V_0(\vec{r}) + V_\sigma(\vec{r})\,(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T(\vec{r}) S_{12} + \cdots$$

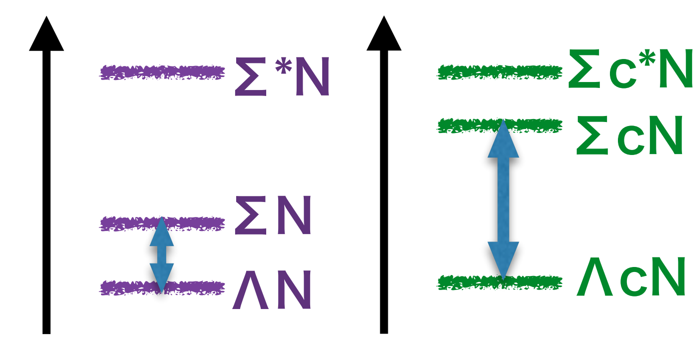






- Weak spin-spin force
  -> **Consequence of HQ spin sym.**
- Weak tensor force
  -> Because the coupling became smaller
  due to spreading of ΛcN-ΣcN threshold





Our results suggest that ΛcN interactions are spin-independent and Λc-nuclei spectrum could be simple