

# New view on wobbling in $A \sim 130$ nuclei

1. Wobbling : transverse vs longitudinal
2. Coupling of wobbling/chiral and scissor mode

## Experiments

Orsay, CSNSM – C. Petrache, B. Lv et al.

Jyväskylä, JYFL – P. Greenlees et al.

Stockholm, KTH – B. Cederwall et al.

Vancouver, TRIUMF – C. Andreoiu et al.

Lanzhou, IMP – S. Guo et al.

Warsaw, HIL – A. Tucholski et al.

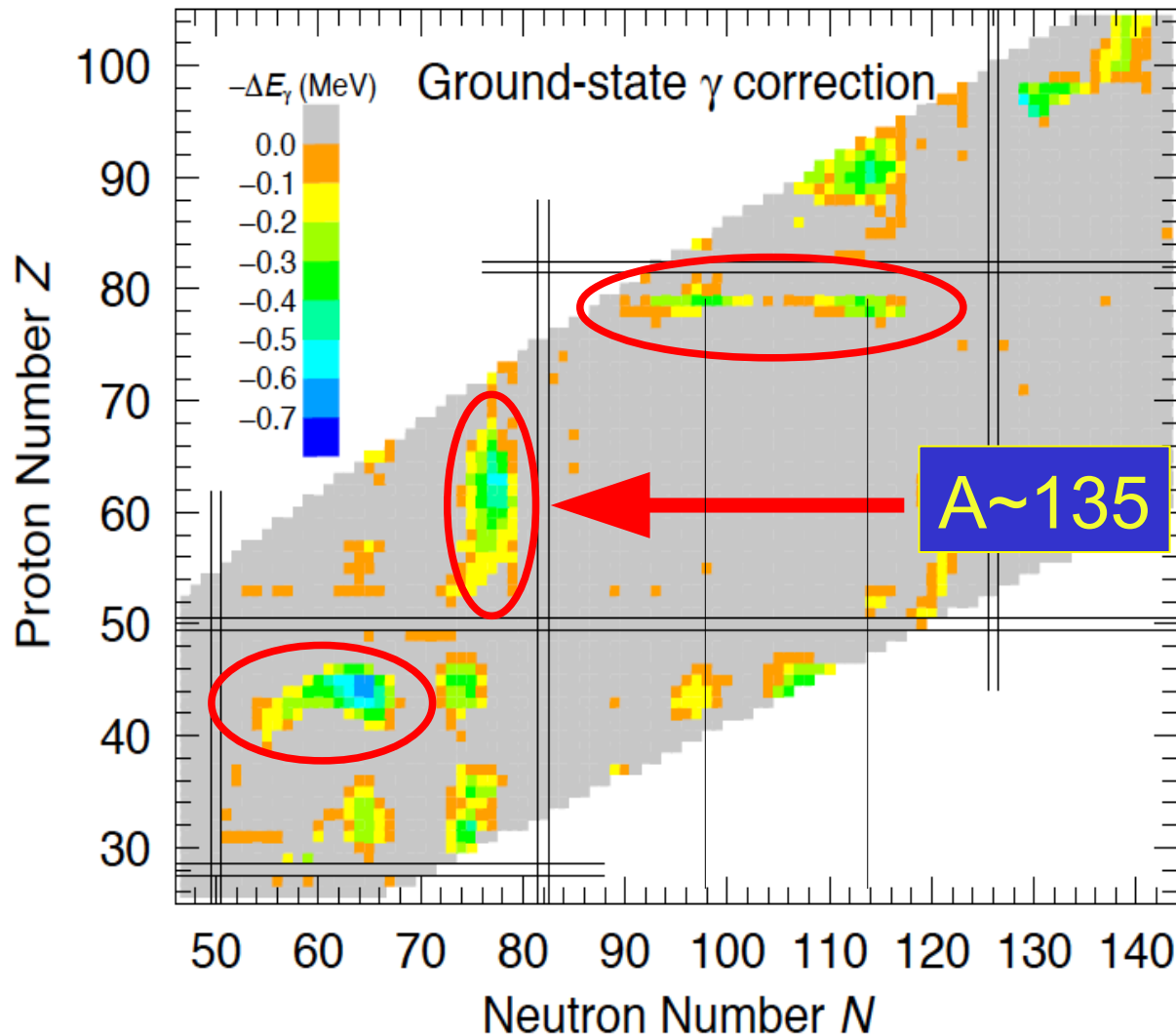
Debrecen, INR – J. Timar et al.

## Theory

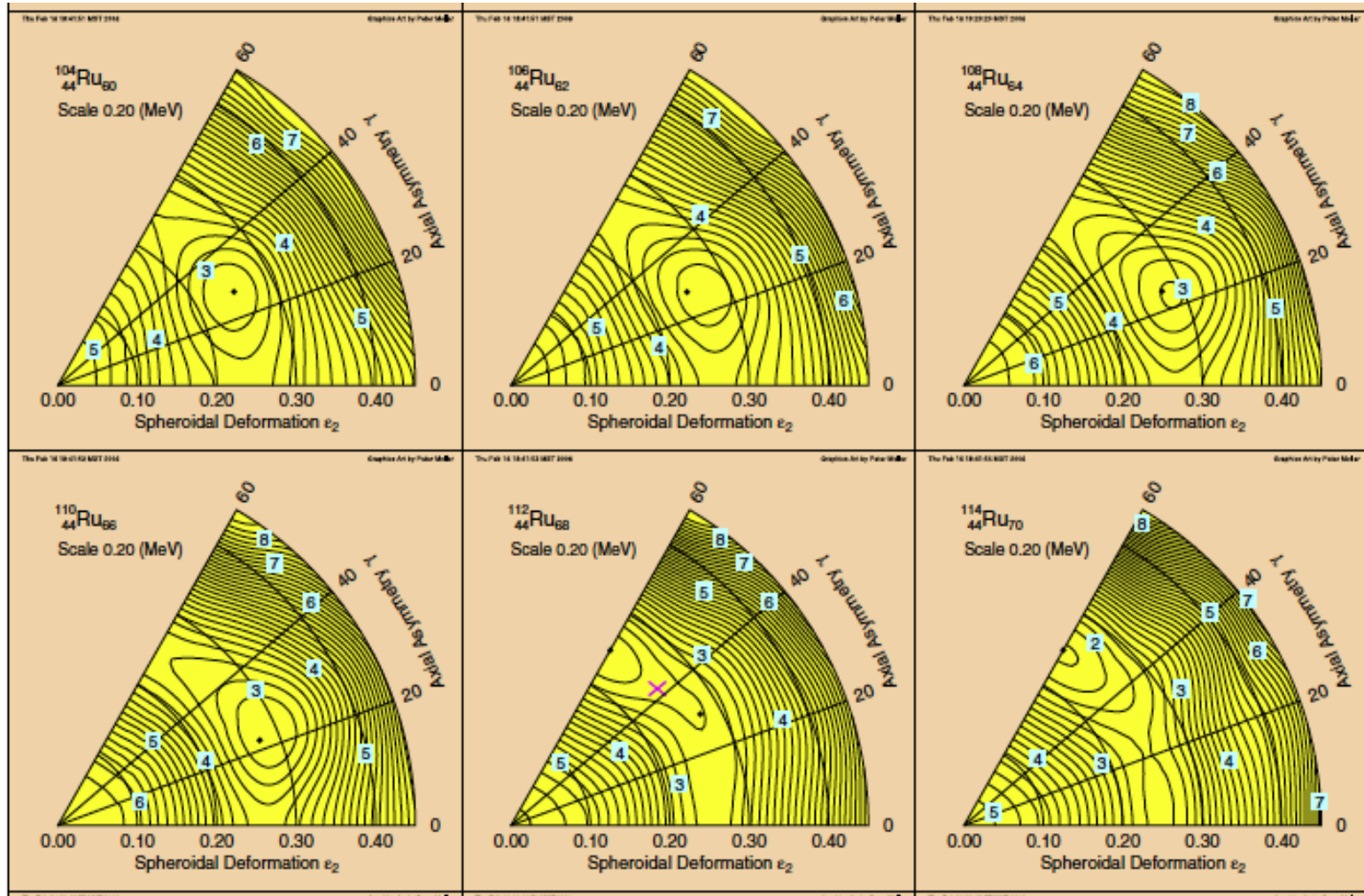
Beijing, Peking Univ. – Q. B. Chen, J. Meng

Bucharest, IFIN-HH – A. A. Raduta

# Axial asymmetry, triaxiality



# Axial asymmetry, triaxiality



# Wobbling motion in $^{135}\text{Pr}$ within a collective Hamiltonian

Q. B. Chen (陈启博),<sup>1</sup> S. Q. Zhang (张双全),<sup>1,\*</sup> and J. Meng (孟杰)<sup>1,2,3,†</sup>

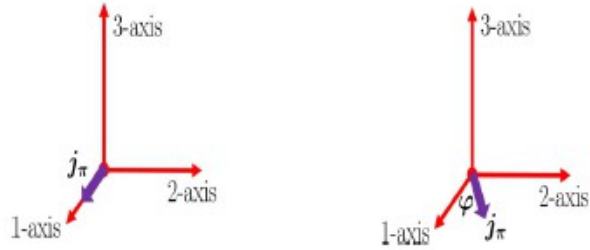


FIG. 1. Sketch of the angular momentum vector of the proton particle with respect to the principal axis frame.

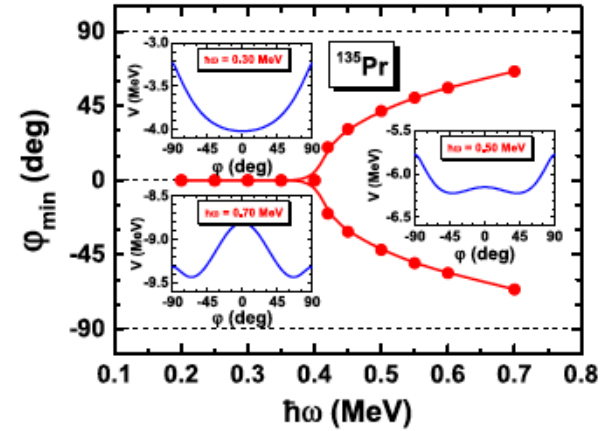


FIG. 3.  $\varphi_{\min}$ , i.e., the  $\varphi$  which minimizes the total Routhian surface, as a function of rotational frequency and the extracted collective potential  $V(\varphi)$  at  $\hbar\omega = 0.30, 0.50$ , and  $0.70$  MeV.

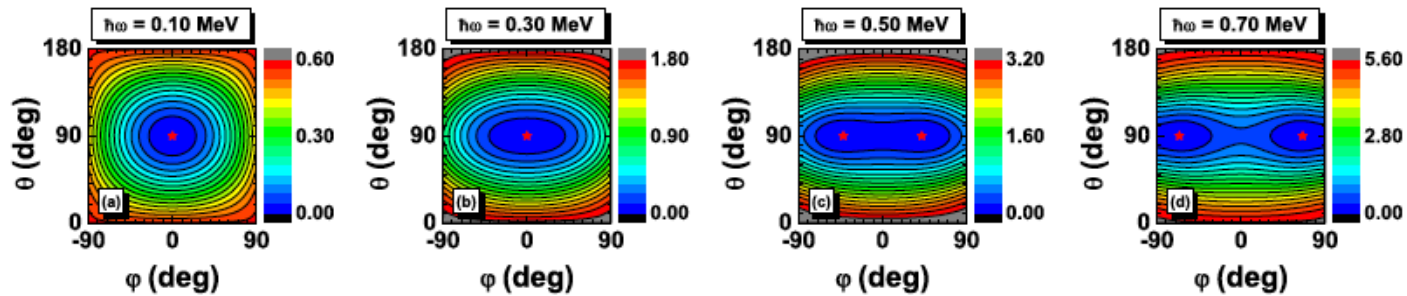
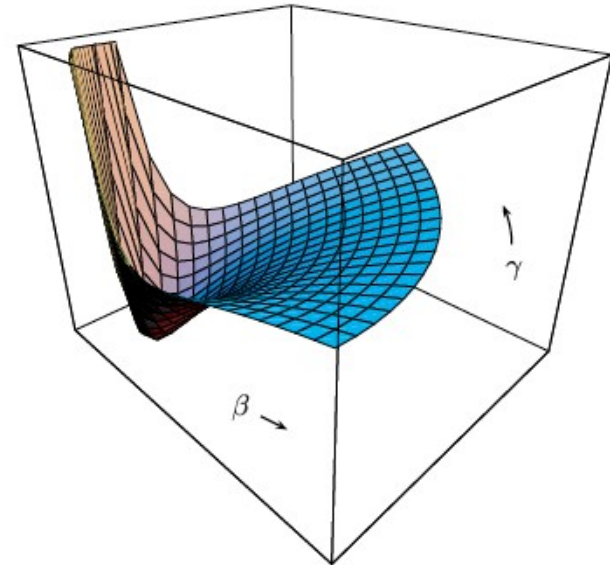
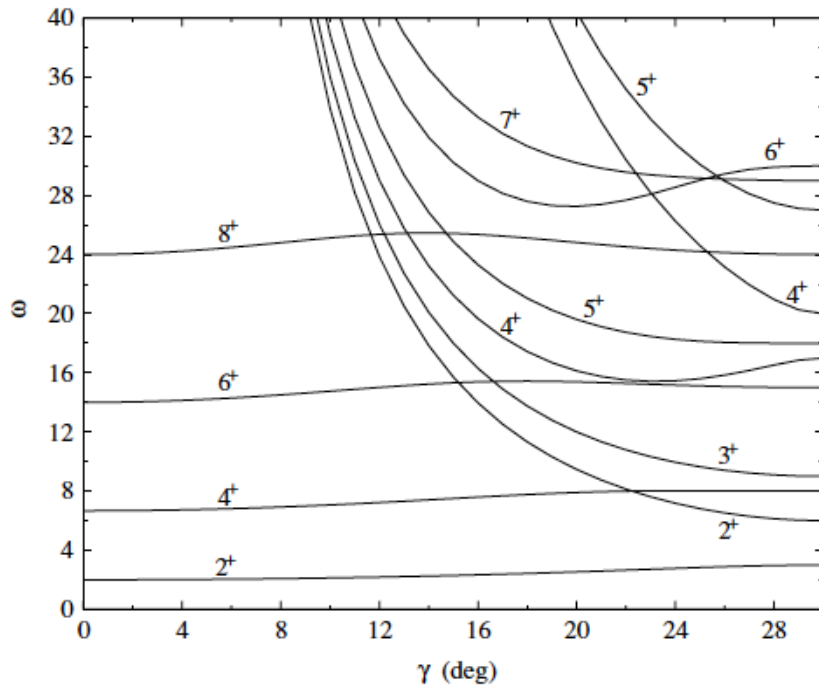


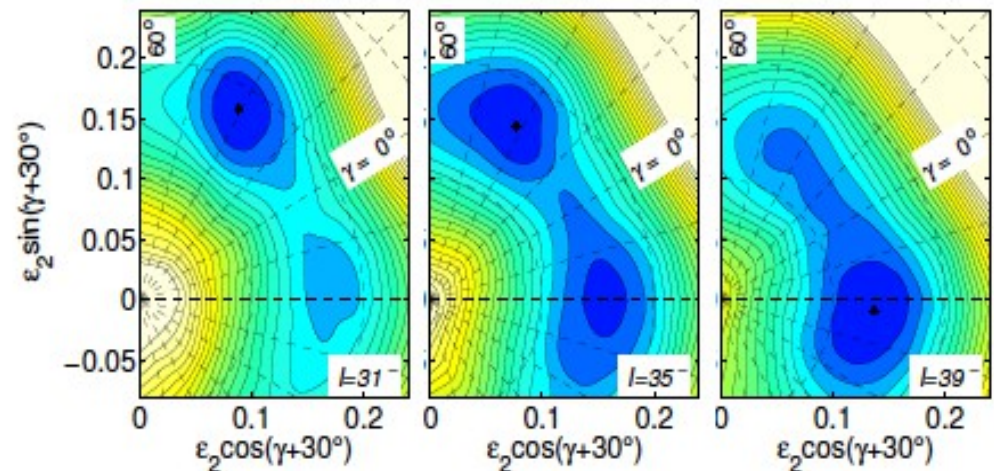
FIG. 2. Contour plots of the total Routhian surface calculation  $E'(\theta, \varphi)$  for  $^{135}\text{Pr}$  at the frequencies  $\hbar\omega = 0.10, 0.30, 0.50$ , and  $0.70$  MeV. All energies at each rotational frequency are normalized with respect to the absolute minimum.

# Triaxiality in an even-even nucleus



Fortunato, 2006

Ragnarsson, 2013



# Triaxiality in an odd-A nucleus

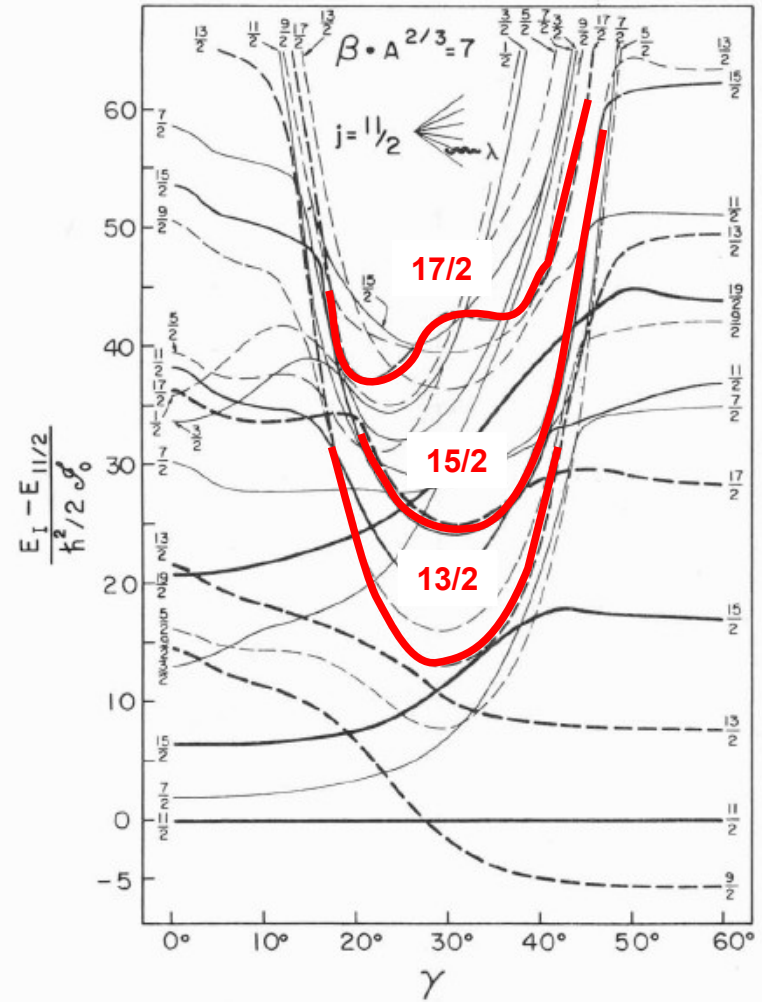
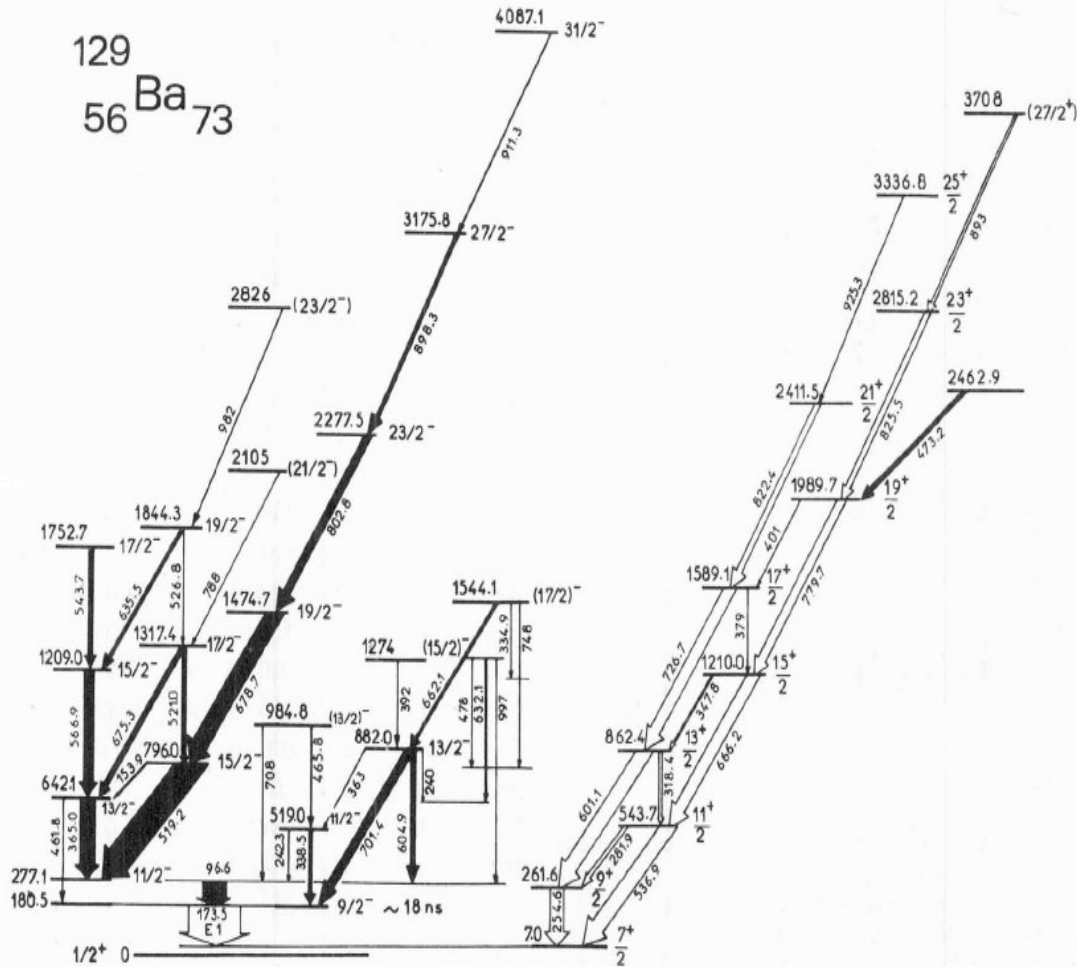
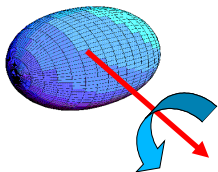


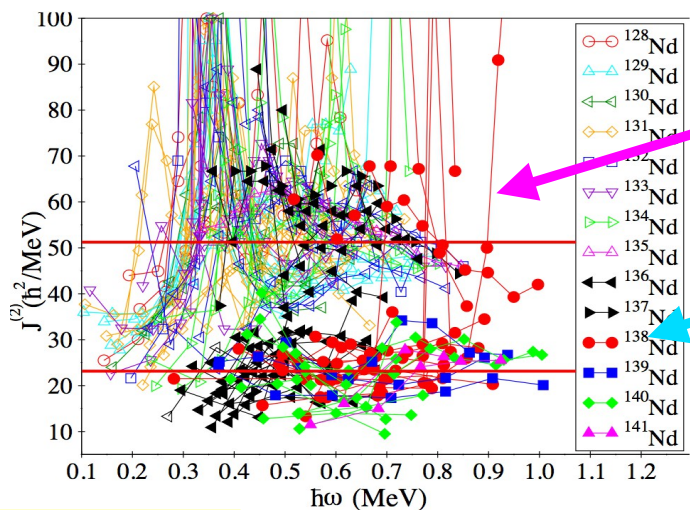
FIG. 9. — The odd-A energy spectrum as a function of  $\gamma$  for  $\beta A^{2/3} = 7$ ,  $\lambda_P = e_2$ , and  $j = 11/2$ . All states with  $E_I - E_{11/2} < 40 h^2/2 I_0$  at  $\gamma = 30^\circ$  have been plotted.

Meyer-ter-Vehn, 1978

Intermediate axis



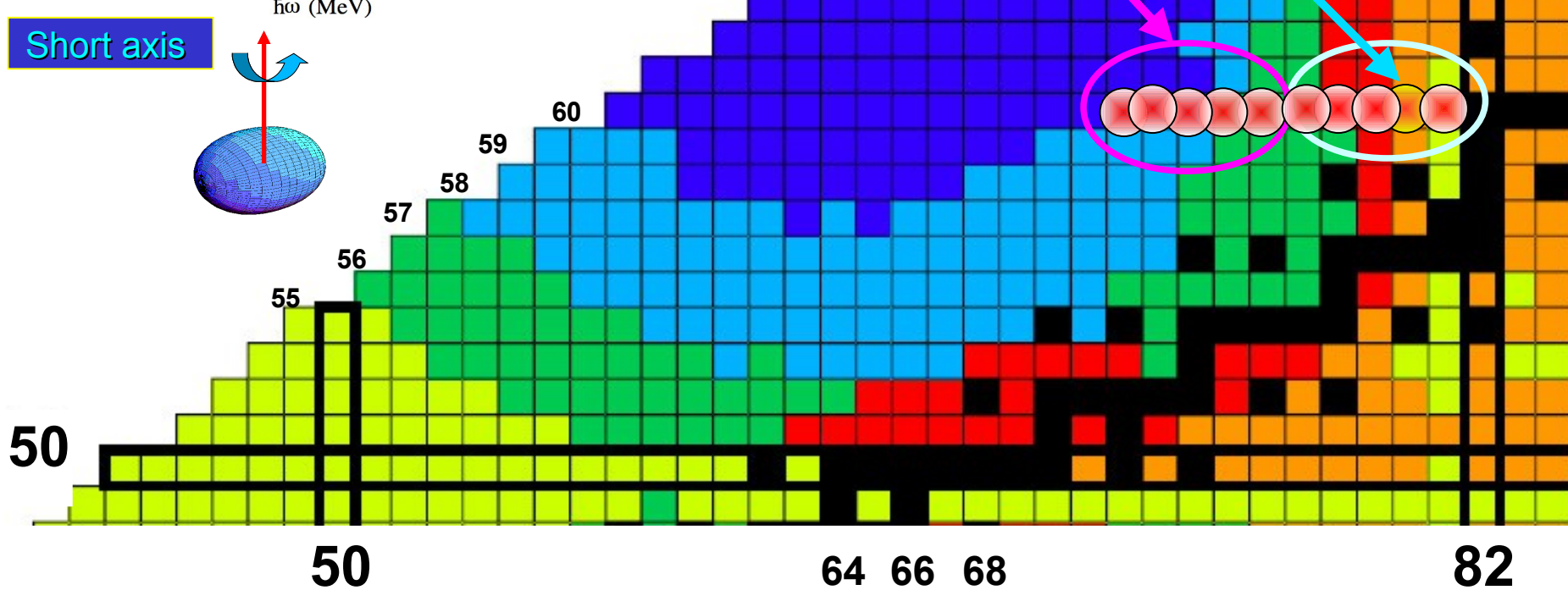
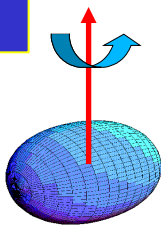
# Rotations of Nd Nuclei



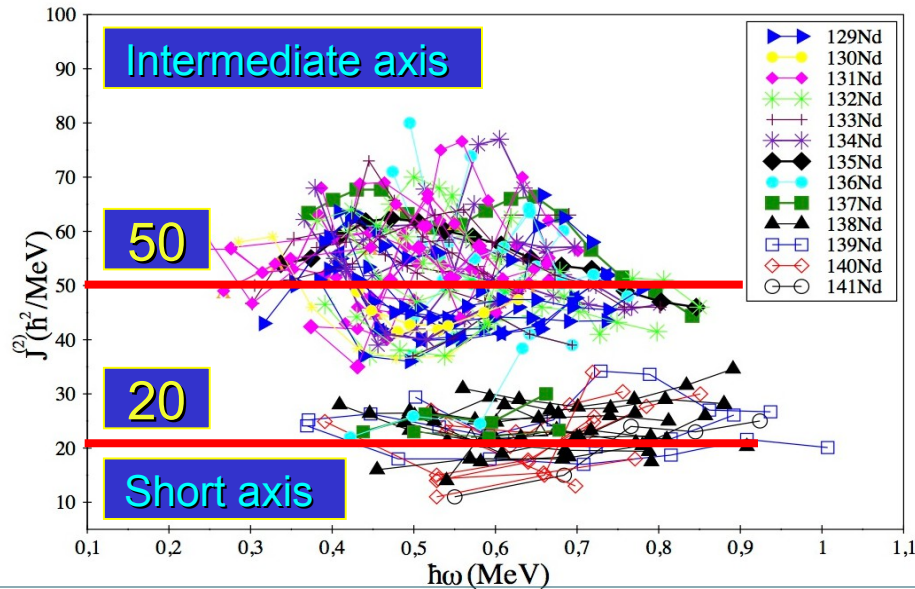
$^{133}\text{Nd}-^{137}\text{Nd}$

$^{137}\text{Nd}-^{141}\text{Nd}$

Short axis



# Mol's : rigid or hydronamical



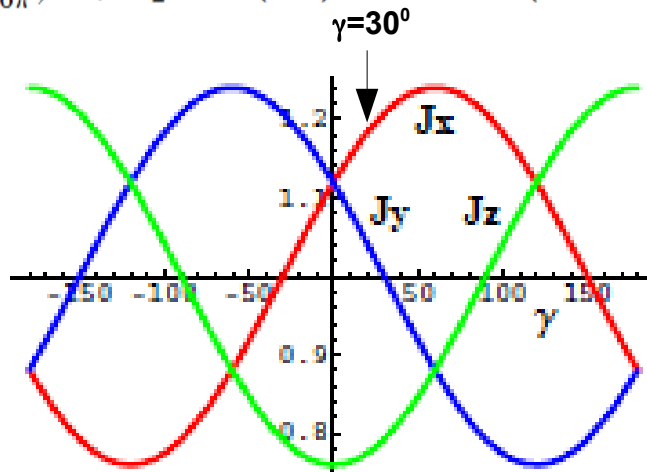
$$\mathcal{J}_k^{\text{hyd}} = \frac{4}{3} \mathcal{J}_0 \sin^2 \left( \gamma - \frac{2\pi}{3} k \right)$$

$$\frac{\mathcal{J}_i^{\text{hyd}}}{\mathcal{J}_s^{\text{hyd}}} \sim \frac{\sin^2(\gamma - 240^\circ)}{\sin^2(\gamma - 120^\circ)} = \frac{\sin^2(-30^\circ - 240^\circ)}{\sin^2(-30^\circ - 120^\circ)} = 4$$

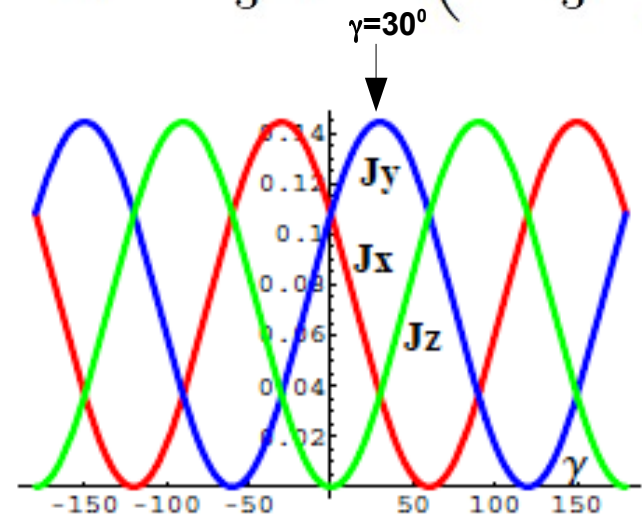
$$\mathcal{J}_k^{\text{rig}} = \frac{\mathcal{J}_0}{1 + \left(\frac{5}{16\pi}\right)^{1/2} \beta^2} \left[ 1 - \left(\frac{5}{4\pi}\right)^{1/2} \beta_2 \cos \left( \gamma - \frac{2\pi}{3} k \right) \right]$$

$$\frac{\mathcal{J}_i^{\text{rig}}}{\mathcal{J}_s^{\text{rig}}} \sim \frac{1 - \left(\frac{5}{4\pi}\right)^{1/2} 0.2 \cos(-30^\circ - 240^\circ)}{1 - \left(\frac{5}{4\pi}\right)^{1/2} 0.2 \cos(-30^\circ - 120^\circ)} = \frac{1}{1.1} = 0.9$$

$$\mathcal{J}_k^{\text{rig}} = \frac{\mathcal{J}_0}{1 + \left(\frac{5}{16\pi}\right)^{1/2} \beta_2} \left[ 1 - \left(\frac{5}{4\pi}\right)^{1/2} \beta_2 \cos \left( \gamma + \frac{2}{3} \pi k \right) \right]$$



$$\mathcal{J}_k^{\text{hyd}} = \frac{4}{3} \mathcal{J}_0 \sin^2 \left( \gamma + \frac{2}{3} \pi k \right)$$





# Wobbling mode

The quantized wobbling mode in nuclei, which is uniquely related to triaxiality of the nuclear shape, is described in the textbook by Bohr and Mottelson [1] in the case of the absence of angular momenta coming from the intrinsic motion. Though the rotation about the axis with the largest moment of inertia is energetically the cheapest, while freezing the intrinsic structure a series of rotational bands can be built by transferring some angular momentum to the two other axes. The family of such rotational bands is formulated in terms of phonon excitations, and each family member is designated by the wobbling phonon number. A family of rotational bands with wobbling excitations can be characterized by specific electromagnetic properties between them. If

$$E(I, n_{\text{wobb}}) = \frac{I(I+1)}{2\mathcal{J}_x} + \hbar\omega_{\text{wobb}} \left( n_{\text{wobb}} + \frac{1}{2} \right)$$

$$\hbar\omega_{\text{wobb}} = \hbar\omega_{\text{rot}} \sqrt{\frac{(\mathcal{J}_x - \mathcal{J}_y)(\mathcal{J}_x - \mathcal{J}_z)}{\mathcal{J}_y\mathcal{J}_z}}$$

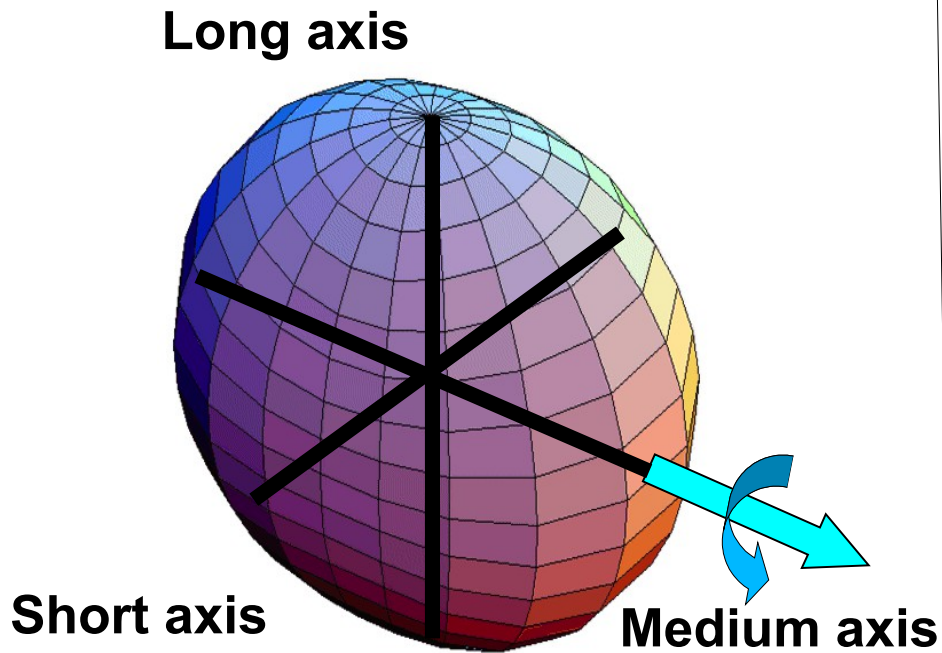
$$\hbar\omega_{\text{rot}} = \frac{I}{\mathcal{J}_x}$$

**Exotic rotational motion, in the sense that the axis of rotation does not coincide with any of the inertia axes of deformation.**

**The nuclear wobbling motion has been considered by analogy with the spinning motions of an asymmetric top (classical rigid-body), where perturbations are superimposed on the main rotation around one of the principal axes with the largest moment of inertia.**

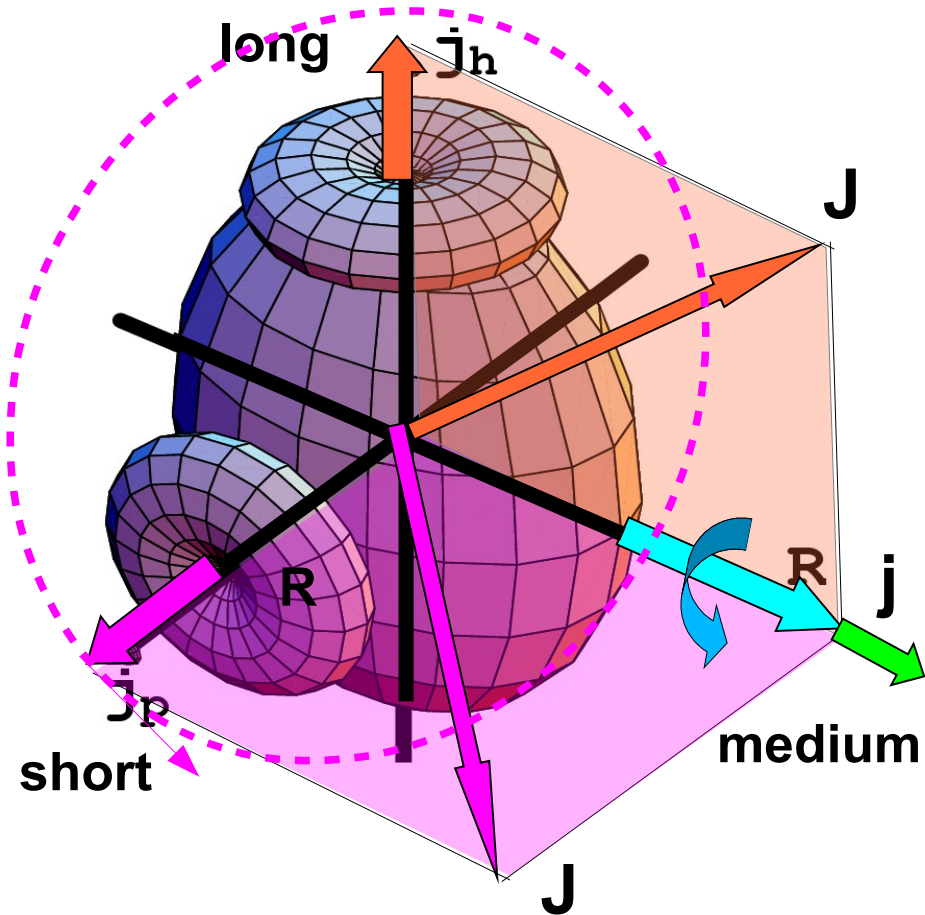
# Which rotation axis ?

Even-even nucleus



Hydro Mol

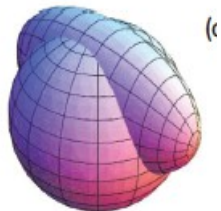
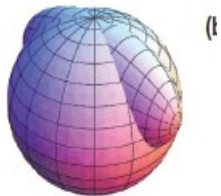
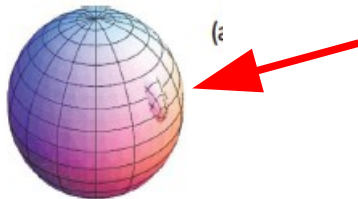
Odd-even nucleus



## Transverse

$$E = A_3(J_3 - j)^2 + A_1 J_1^2 + A_2 J_2^2$$

$$A_1 = 6A_2, A_3 = 3A_2$$



# Wobbling mode

- Old concept : discovered by Bohr and Mottelson 50 years ago, but the first example was observed experimentally at the beginning of this century.

- has the character of a harmonic vibration.

$$E(I, n_{\text{wobb}}) = \frac{I(I+1)}{2\mathcal{J}_x} + \hbar\omega_{\text{wobb}} \left( n_{\text{wobb}} + \frac{1}{2} \right)$$

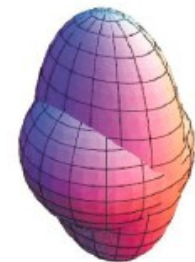
$$\hbar\omega_{\text{wobb}} = \hbar\omega_{\text{rot}} \sqrt{\frac{(\mathcal{J}_x - \mathcal{J}_y)(\mathcal{J}_x - \mathcal{J}_z)}{\mathcal{J}_y \mathcal{J}_z}} \quad \hbar\omega_{\text{rot}} = \frac{I}{\mathcal{J}_x}$$

## Simple

$$J^2 = J_1^2 + J_2^2 + J_3^2 = I(I+1)$$

$$E = A_3 J_3^2 + A_1 J_1^2 + A_2 J_2^2$$

$$A_1 = 6A_3, A_2 = 3A_3$$



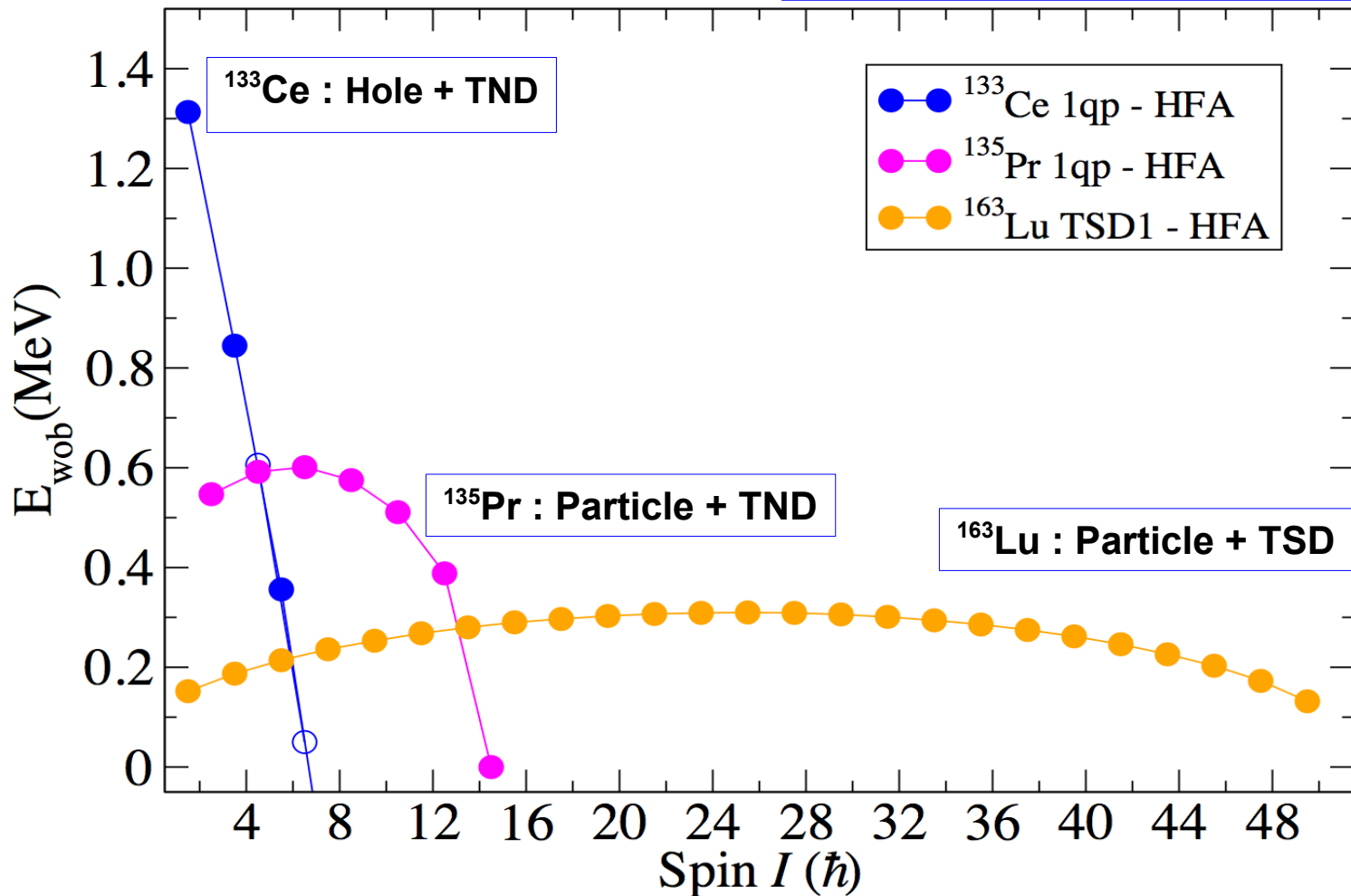
# Transverse wobbling frequency : TND versus TSD, hole versus particle

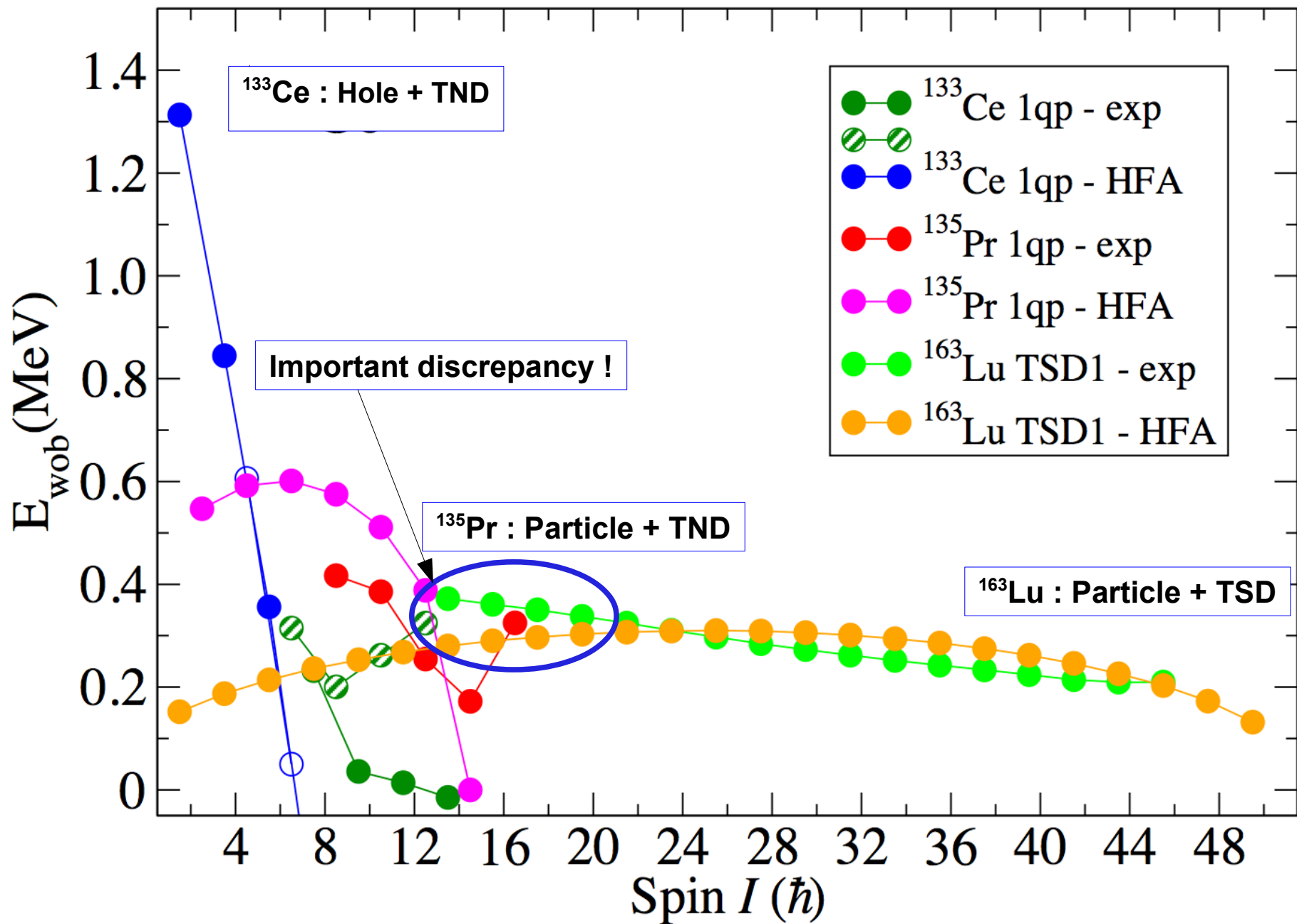
$$\hbar\omega_{wobb} = \frac{\hbar^2 j}{\mathcal{J}_3} \sqrt{\left[1 + \frac{J}{j} \left(\frac{\mathcal{J}_3}{\mathcal{J}_1} - 1\right)\right] \left[1 + \frac{J}{j} \left(\frac{\mathcal{J}_3}{\mathcal{J}_2} - 1\right)\right]}$$

$$0.116\sqrt{-0.01J^2+0.39J+1} - {}^{163}\text{Lu}$$

$$0.42\sqrt{-0.028J^2-0.34J+1} - {}^{135}\text{Pr}$$

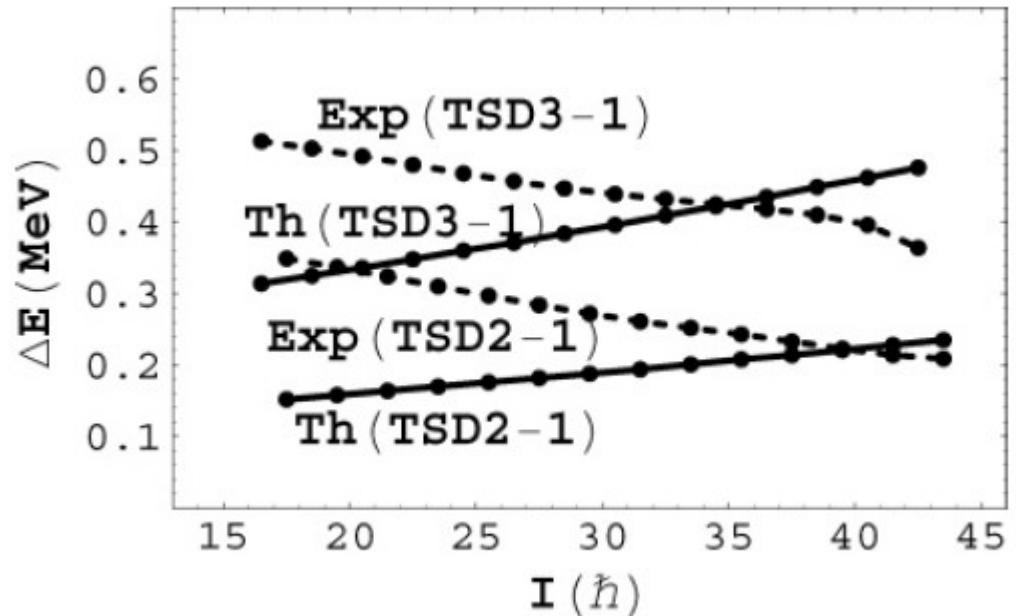
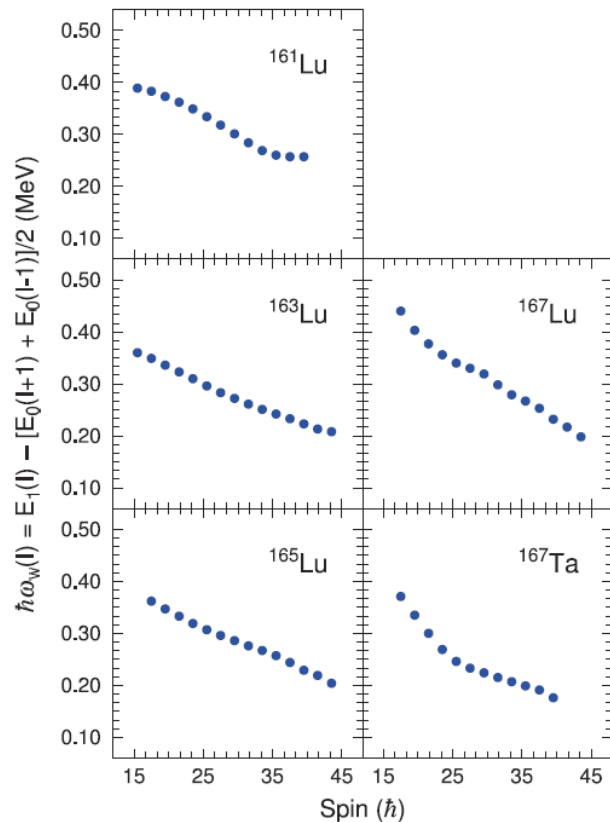
$$1.66\sqrt{0.017J^2-0.26J+1} - {}^{133}\text{Ce}$$





# Wobbling frequency in odd-even Lu nuclei not reproduced by the originally proposed longitudinal wobbling

$$E_{wobb} = E(I, n_w = 1) - \frac{E(I + 1, n_w = 0) + E(I - 1, n_w = 0)}{2}$$



How to explain the conflict ?

# Which other measurable quantities are characteristic for the wobbling mode ?

- **EM character of the connecting transitions :**

**$\Delta I=1$  transitions should be predominantly E2**

- **Transition probabilities of the connecting transitions :**

**large  $B(E2)$ , small  $B(M1)$ , staggering**

# Transverse Wobbling in $^{135}\text{Pr}$

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$^{123}\text{Sb}(^{16}\text{O},4n)$

pure M1

60% E2

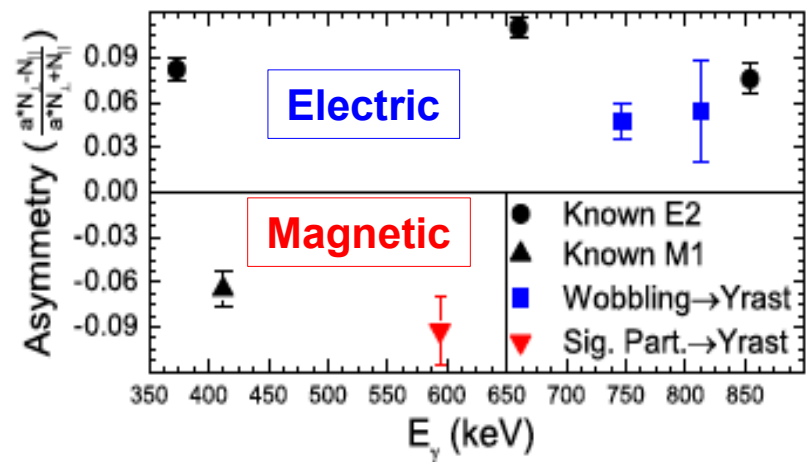
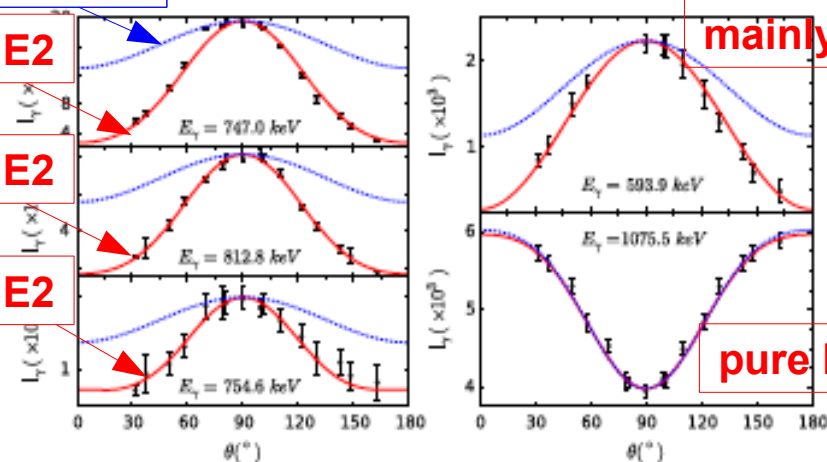
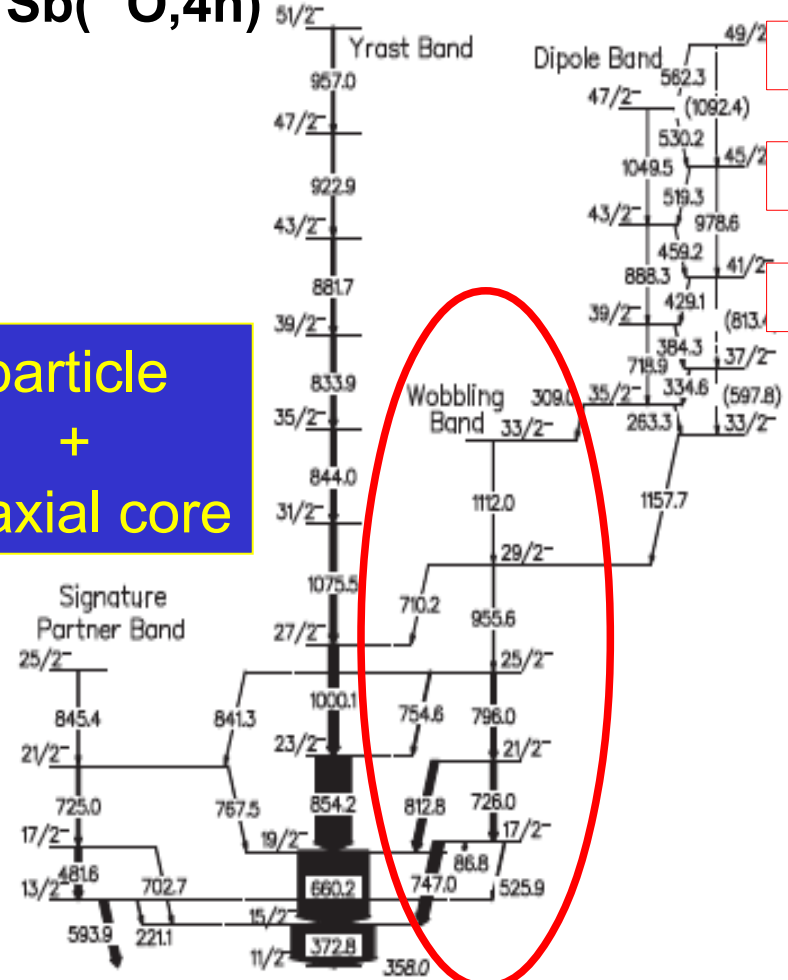
70% E2

85% E2

mainly M1

pure E2

particle  
+  
triaxial core

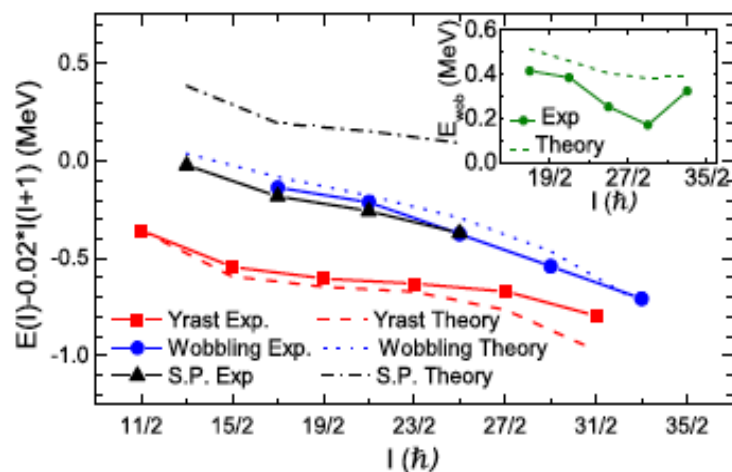
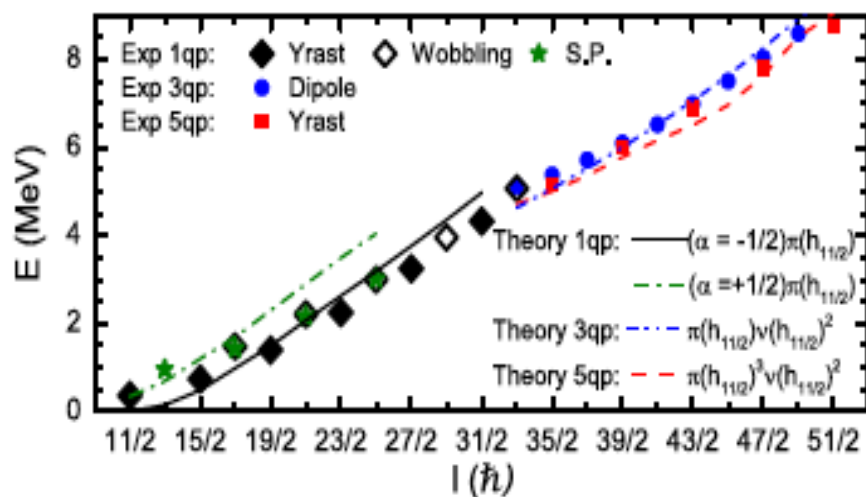




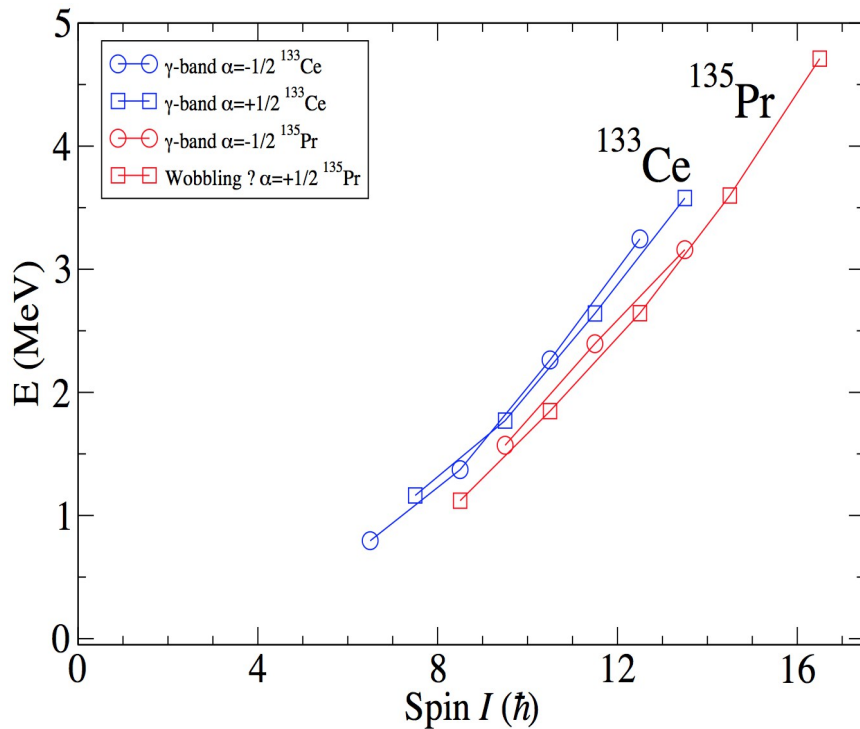
# 135Pr

TABLE I. The mixing ratios,  $\delta$ , E2 fractions, and the experimental and theoretical transition probability ratios for transitions from the  $n_{\omega} = 1$  to  $n_{\omega} = 0$  wobbling bands in  $\gamma$ . The in-band transitions were assumed to be of pure E2 character in calculations of the probability ratios. The mixing ratio of the  $\frac{25}{2}^{-} \rightarrow \frac{23}{2}^{-}$  transition has been taken as a lower limit when deriving the probability ratios for the  $\frac{29}{2}^{-} \rightarrow \frac{27}{2}^{-}$  transition. Shown at the bottom is the measured mixing ratio for the lowest Signature partner to Yrast transition.

Initial $I^{\pi}$	Final $I^{\pi}$	$E_{\gamma}$ (keV)	$\delta$	Asymmetry	E2 Fraction (%)	$\frac{B(M1_{out})}{B(E2_{in})} \left( \frac{\mu_N^2}{e^2 b^2} \right)$		$\frac{B(E2_{out})}{B(E2_{in})}$	
						Experiment	QTR	Experiment	QTR
$\frac{17}{2}^{-}$	$\frac{15}{2}^{-}$	747.0	$-1.24 \pm 0.13$	$0.047 \pm 0.012$	$60.6 \pm 5.1$	...	0.213	...	0.908
$\frac{21}{2}^{-}$	$\frac{19}{2}^{-}$	812.8	$-1.54 \pm 0.09$	$0.054 \pm 0.034$	$70.3 \pm 2.4$	$0.164 \pm 0.014$	0.107	$0.843 \pm 0.032$	0.488
$\frac{25}{2}^{-}$	$\frac{23}{2}^{-}$	754.6	$-2.38 \pm 0.37$	...	$85.0 \pm 4.0$	$0.035 \pm 0.009$	0.070	$0.500 \pm 0.025$	0.290
$\frac{29}{2}^{-}$	$\frac{27}{2}^{-}$	710.2	...	...	...	$\leq 0.016 \pm 0.004$	0.056	$\geq 0.261 \pm 0.014$	0.191
$\frac{13}{2}^{-}$	$\frac{11}{2}^{-}$	593.9	$-0.16 \pm 0.04$	$-0.092 \pm 0.023$	$2.5 \pm 1.2$	...	...	...	...

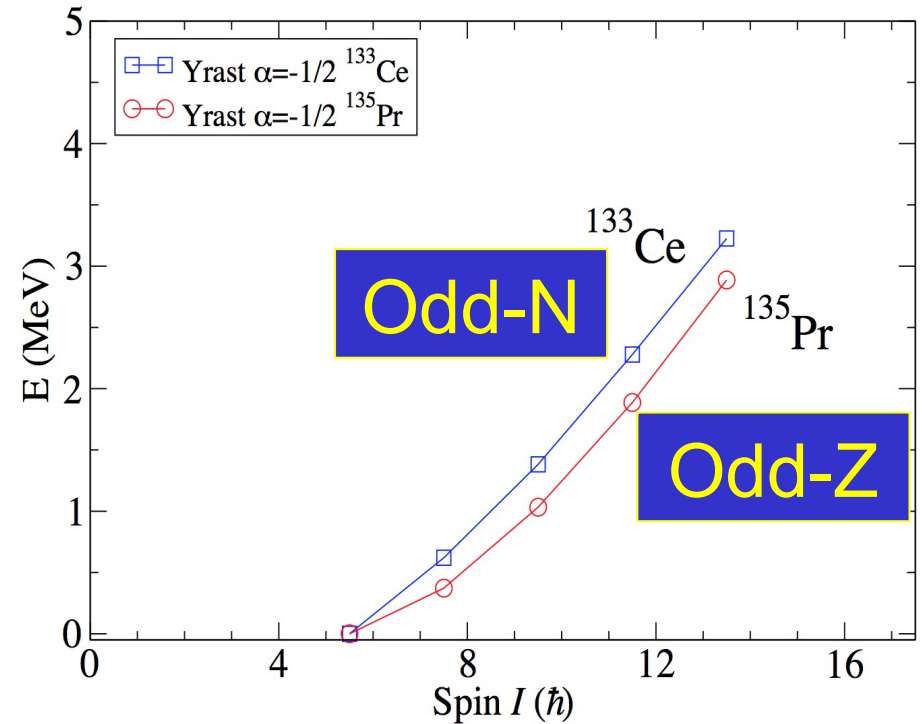


# Similarity between the $\gamma$ -bands of $^{135}\text{Pr}$ and $^{133}\text{Ce}$



The  $\gamma$ -bands do not feel the difference between 1 particle or 1 hole

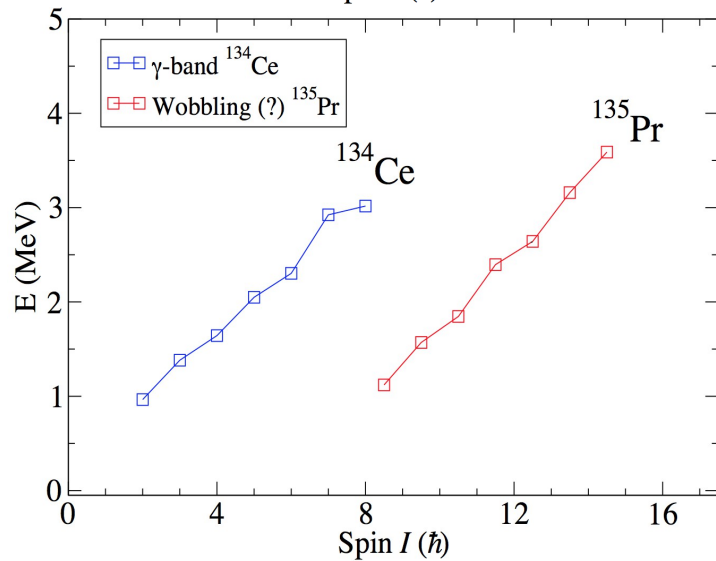
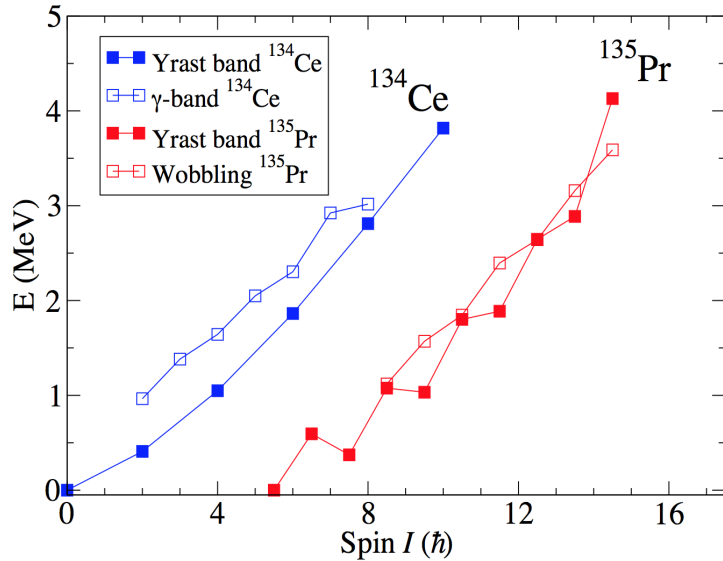
# Difference between the yrast bands of $^{135}\text{Pr}$ and $^{133}\text{Ce}$



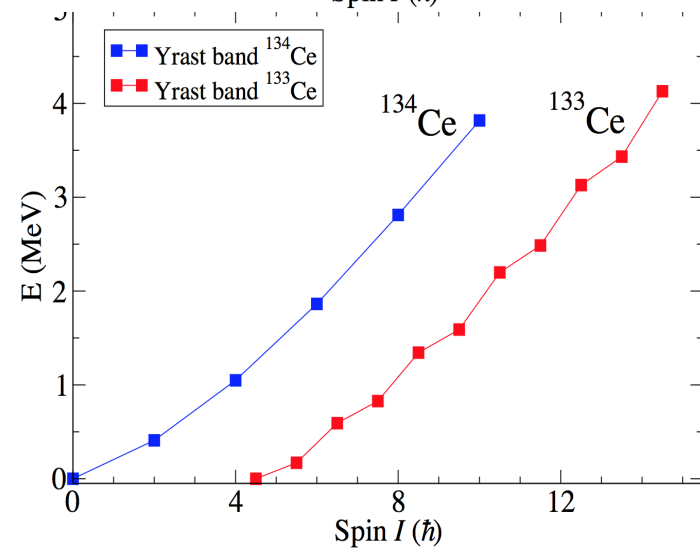
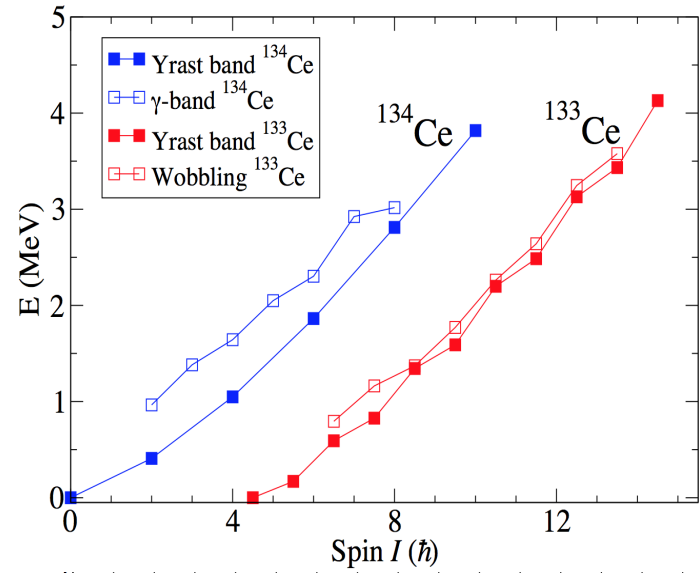
The yrast bands feel the difference between 1 particle or 1 hole

# Difference between $^{135}\text{Pr}$ and $^{133}\text{Ce}$

## Similarity between the wobbling/ $\gamma$ -bands of $^{135}\text{Pr}$ and $^{134}\text{Ce}$

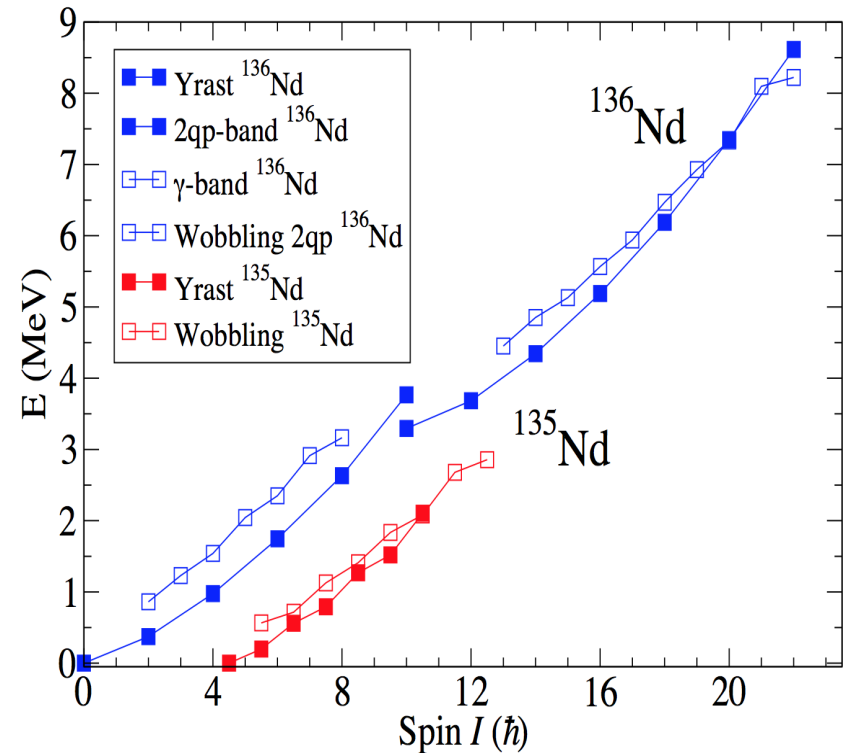
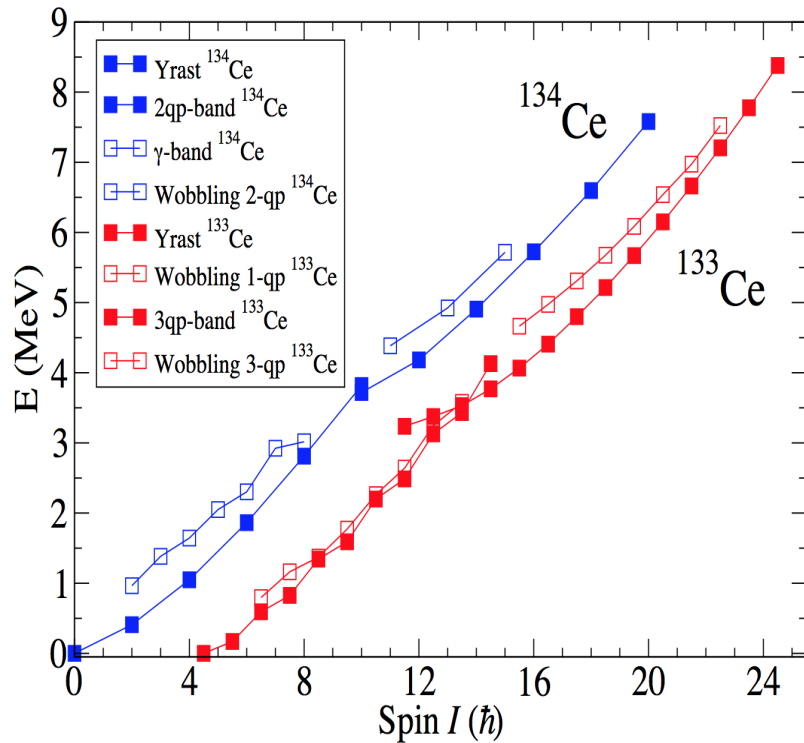


## Difference between the yrast bands of $^{133}\text{Ce}$ and $^{134}\text{Ce}$



# Wobbling 2-qp and 3-qp bands of $^{134}\text{Ce}$ and $^{133}\text{Ce}$

# Wobbling 2-qp and 3-qp bands of $^{136}\text{Nd}$ and $^{135}\text{Nd}$



No staggering in 3-qp bands  $\rightarrow$  More axial symmetric  
 Quadratic E-I behavior  $\rightarrow$  More rigid

# Difference between $^{135}\text{Pr}$ and $^{133}\text{Ce}$

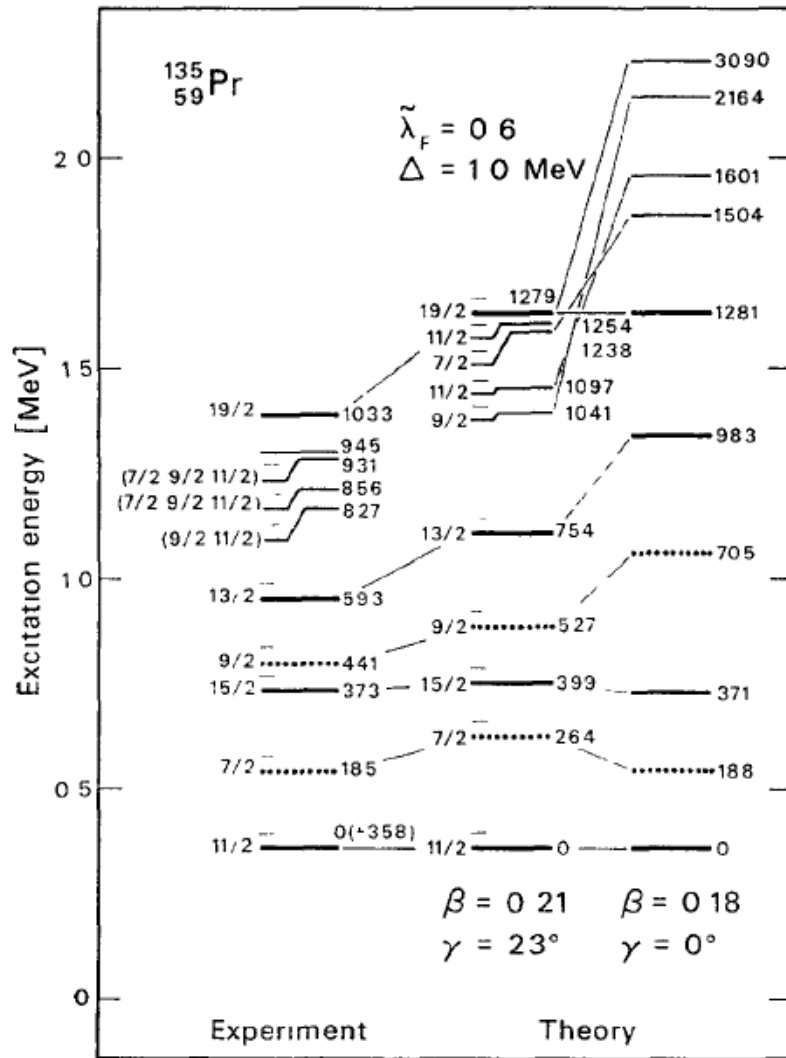


Fig. 7. Negative-parity states in  $^{135}\text{Pr}$  below the  $19/2^-$  state

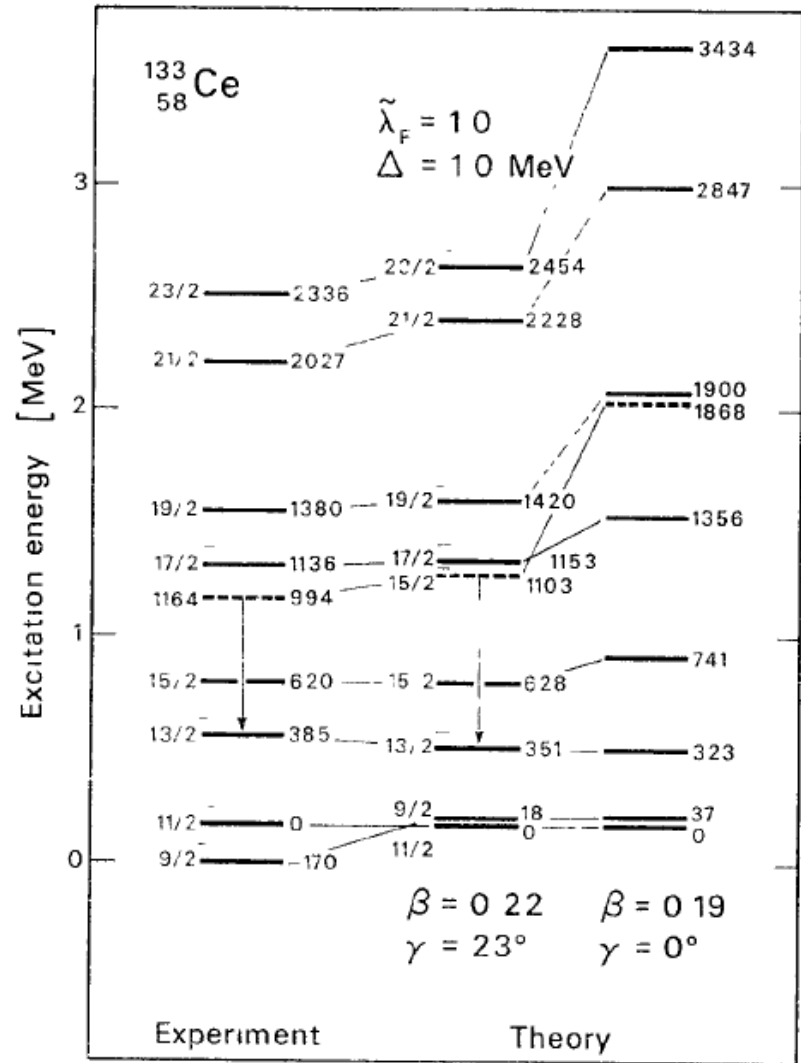
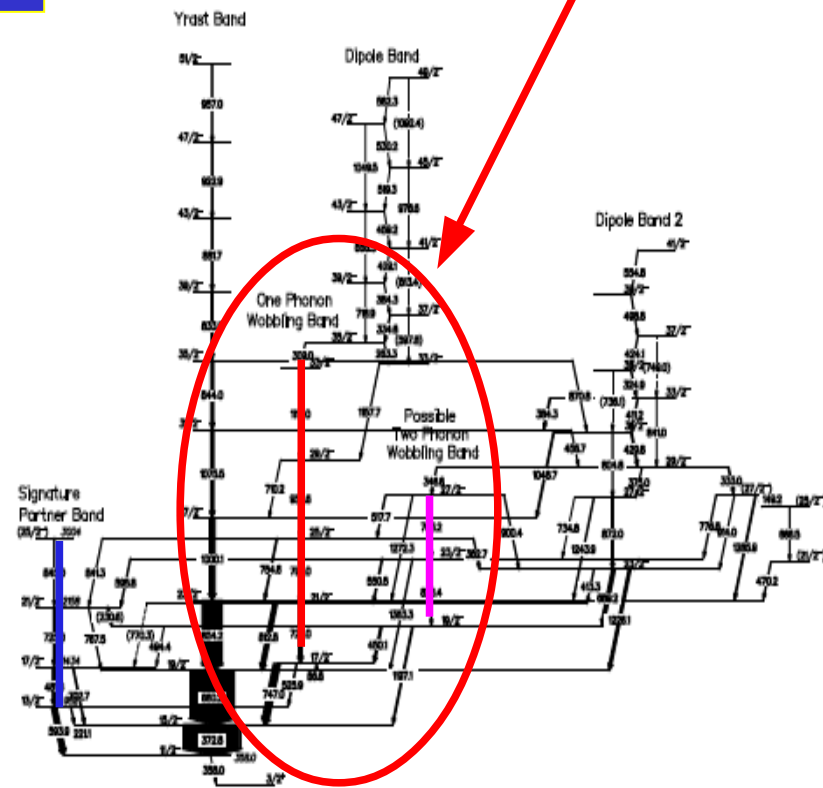
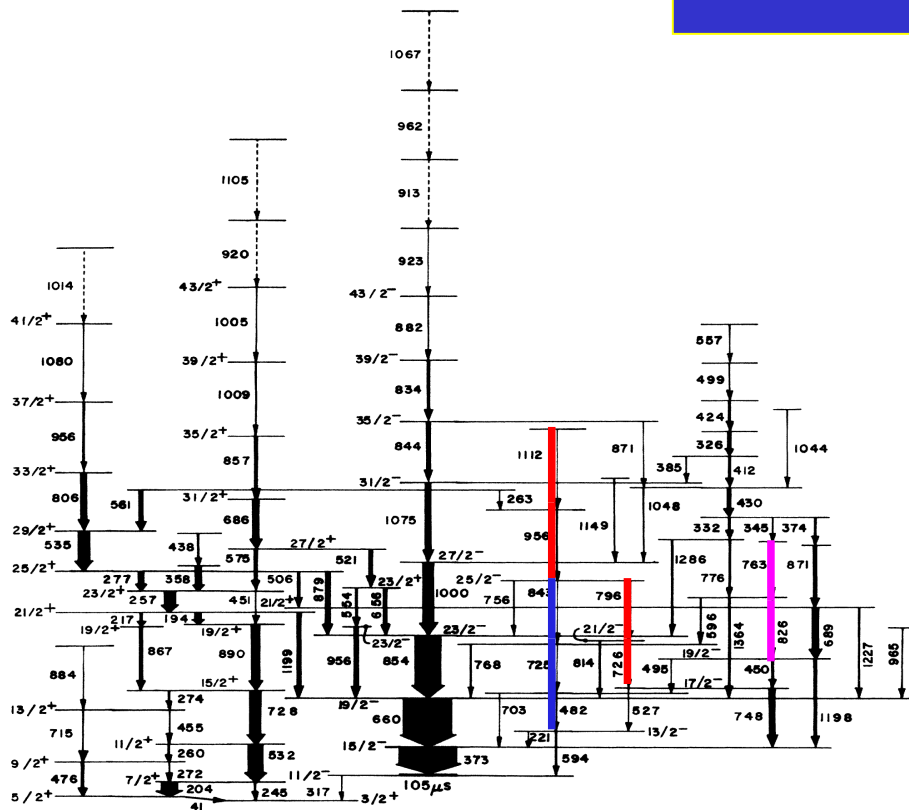


Fig. 9 Negative-parity states in  $^{133}\text{Ce}$

$^{135}\text{Pr}$

Wobbling or  $\gamma$ 2 bands ?

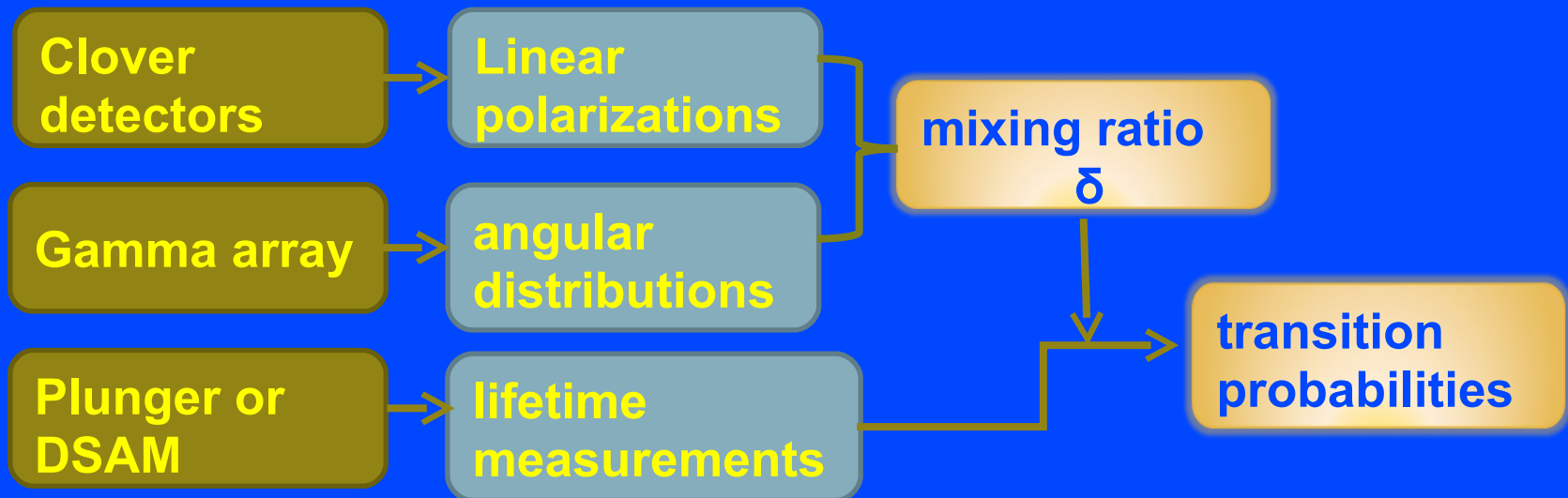


Semkow et al., PRC 34, 523 (1986)

J. Matta, PhD Thesis (2015)

# Perspectives on wobbling $\gamma$ bands ? from an experimental point of view

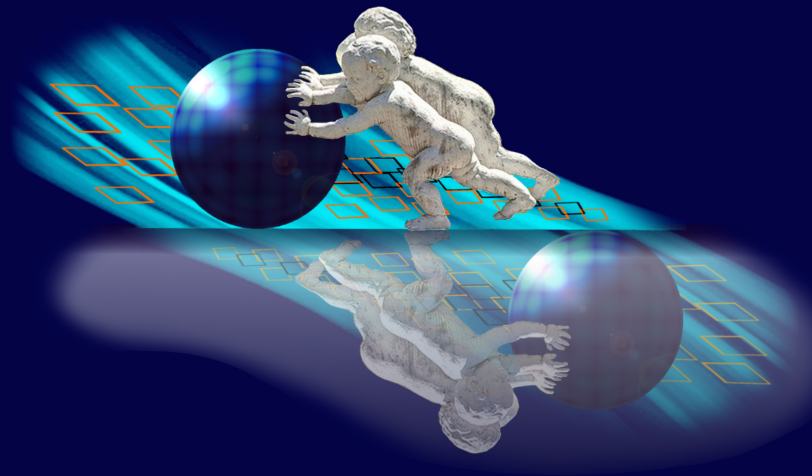
1. Precise measurement of angular distribution, polarization, and lifetimes are needed.
2. QRPA with QQ+IVLL for TND nuclei are needed.



# SSNET'17

International Workshop on  
Shapes and Symmetries in Nuclei :  
from Experiment to Theory

Gif sur Yvette, November 6-10, 2017



## Topics

Methods & instrumentation  
New facilities  
Experimental studies: deformation  
of ground & excited states  
Theoretical models:  
exotic collective excitations  
exotic symmetries

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