



Chirality and Vorticity in Heavy-Ion Collisions



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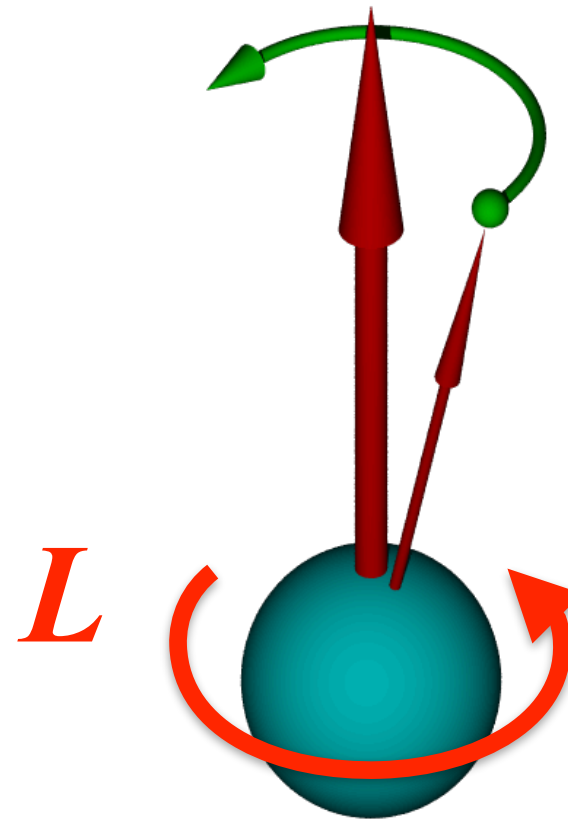
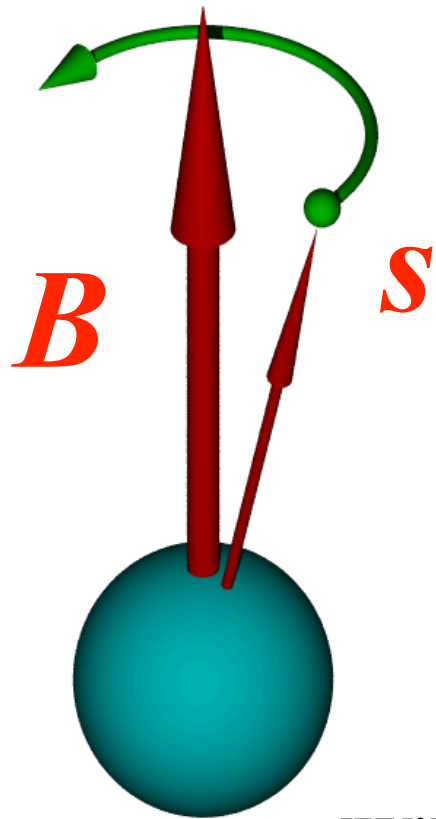
— Colloquium in RIKEN Nishina Center —

Chirality? Vorticity??



Chirality $\sim S$

Vorticity $\sim L$



[Wikipedia]

Spin and Mass



Magnetic Moment of Spin-1/2 Particles

$$\mu = \frac{q\hbar}{2m}$$

Spin effect is more suppressed by larger mass

Spin and Mass



Quark Model

Wave-function \rightarrow

$$\mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$
$$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$$

“Constituent Quark”

$$\mu_u = \frac{q_u}{2m_q} = -2\mu_d \rightarrow m_q \approx 340 \text{ MeV}$$

Spin and Mass



Quark Model

Phenomenological Mass Formula

$$M_{\text{hadron}} = \sum_i m_i + \Delta M$$

$$\Delta M = \sum \frac{4\pi\alpha_s}{9} \frac{\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{m_i m_j} |\psi(0)|^2$$

“Constituent Quark” $m_{u,d} \approx 360 \text{ MeV}$

Mass in QCD



Constituent Quark Mass from the “QCD Vacuum”

RHIC: From dreams to beams in two decades

Gordon Baym
Department of Physics, University of Illinois at Urbana-Champaign
Urbana, IL 61801, U.S.A.

This talk traces the history of RHIC over the last two decades, reviewing the scientific motivations underlying its design, and the challenges and opportunities the machine presents.

1. THE VERY EARLY DAYS

The opening of RHIC culminates a long history of fascination of nuclear and high energy physicists with discovering new physics by colliding heavy nuclei at high energy. As far back as the late 1960's the possibility of accelerating uranium ions in the CERN ISR for this purpose was contemplated [1]. The subject received “subtle stimulation” by the workshop on “Bev/nucleon collisions of heavy ions” at Bear Mountain, New York, organized by Arthur Kerman, Leon Lederman, Mal Ruderman, Joe Weneser and T.D. Lee in the fall of 1974 [1]. In retrospect, the Bear Mountain meeting was a turning point in bringing heavy ion physics to the forefront as a research tool. The driving question at the meeting was, as Lee emphasized, whether the vacuum is a medium whose properties one could change; “we should investigate,” he pointed out, “. . . phenomena by distributing high energy or high nucleon density over a relatively large volume.” If in this way one could restore broken symmetries of the vacuum, then it might be possible to create abnormal dense states of nuclear matter, as Lee and Gian-Carlo Wick speculated [2].

Vacuum
~ **Medium?**
~ **Changeable??**

Quark mass
changeable?

Mass in QCD



**So far, there is NO HIC experimental data
that can tell us anything about quark mass changes**

Color deconfinement ~ Bulk properties of media

Lattice-QCD:

QCD has only one criticality that is “chiral”!

Quark mass (chiral) ~ Excitations

Phenomenology:

Hagedorn picture works WITHOUT mass shift!

Mass in QCD

Zero-point Oscillation Energy [Peskin-Schroeder]

$$= \int \frac{d^3 p}{(2\pi)^3} \omega_{\mathbf{p}} \left(a_{\mathbf{p}}^\dagger a_{\mathbf{p}} + \frac{1}{2} [a_{\mathbf{p}}, a_{\mathbf{p}}^\dagger] \right). \quad (2.31)$$

The second term is proportional to $\delta(0)$, an infinite c-number. It is simply the sum over all modes of the zero-point energies $\omega_{\mathbf{p}}/2$, so its presence is completely expected, if somewhat disturbing. Fortunately, this infinite energy shift cannot be detected experimentally, since experiments measure only energy *differences* from the ground state of H . We will therefore ignore this infinite constant term in all of our calculations. It is possible that this energy shift of the ground state could create a problem at a deeper level in the theory; we will discuss this matter in the Epilogue.

Not true for QCD! $\omega_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$
Dynamical Quantity

Mass in QCD

Zero-Point Oscillation Energy

$$-2 \int^{\Lambda} \frac{d^3 p}{(2\pi)^3} \sqrt{p^2 + M^2}$$
$$\simeq -\frac{\Lambda^4}{8\pi^2} [2 + \xi^2 + \mathcal{O}(\xi^4)]$$

$\xi = M/\Lambda$

negative

(Some) Interaction

$$\frac{M^2}{2\lambda_{\Lambda}} = \frac{\Lambda^4}{2\hat{\lambda}_{\Lambda}} \xi^2$$

positive

**Dynamical mass
generated for $\hat{\lambda}_{\Lambda} > 2\pi^2$**

Nambu—Jona-Lasinio 1961

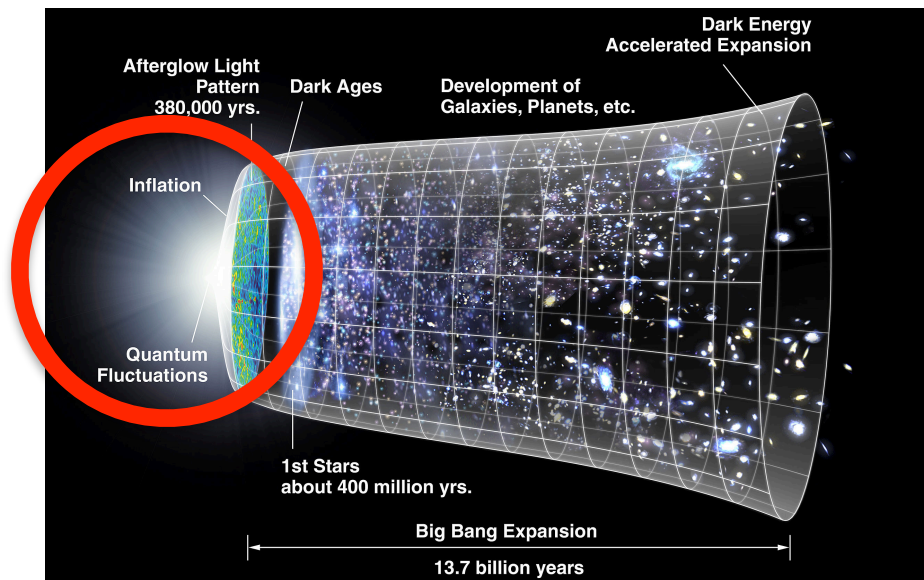
Mass in QCD

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

What is the RHS in the QCD Vacuum?

Chiral condensate, real or illusion?

How to “renormalize” the zero-point energy?

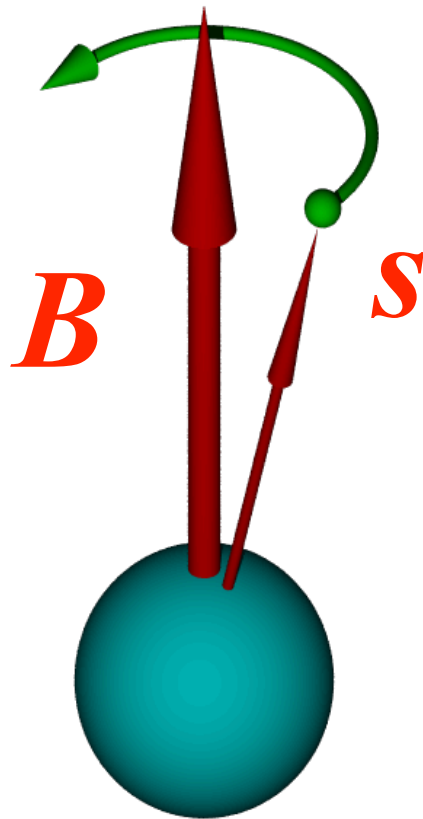


Crucial problem for HIC
Early Thermalization Puzzle

“Evolution to the quark-gluon plasma”
K.Fukushima, Rept.Prog.Phys.80 (2017)

Chirality in B

How to see the mass? **Magnetic field!**

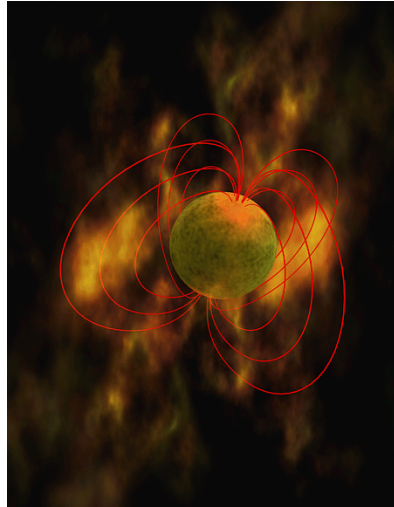


E — work given to charge carriers
(screened easily)

B — no work done to charge
(no screening)

B is everywhere in nature!

Stronger B



[Wikipedia]

Surface of the neutron star

$$\lesssim 10^{12} \text{ gauss} \sim 10^{-2} \text{ MeV}^2$$

Surface of the magnetar

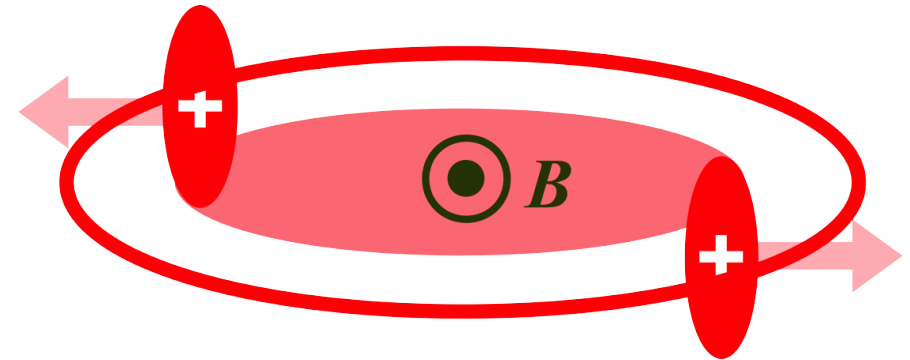
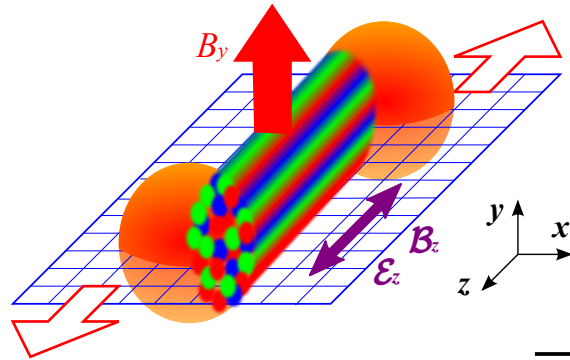
$$\lesssim 10^{15} \text{ gauss} \sim 10 \text{ MeV}^2$$

Interior of the magnetar

$$\lesssim 10^{18} \text{ gauss} \sim 10^4 \text{ MeV}^2 \sim m_{\pi}^2$$

Color superconductivity in a magnetic field
Mixture of photon and gluon (unscreened B)

Strongest B



$$\frac{eB_0}{[1 + (t/t_0)^2]^{3/2}}$$

$$eB_0 = (47.6 \text{ MeV})^2 \left(\frac{1 \text{ fm}}{b} \right)^2 Z \sinh Y$$

$$t_0 = \frac{b}{2 \sinh Y}$$

$$\lesssim 10^{20} \text{ gauss} \sim \text{GeV}^2$$

Strongest magnetic field in the (present) Universe

HIC: Old and New Ideas



Solution of the Dirac Equation for Strong External Fields*

Berndt Müller, Heinrich Peitz, Johann Rafelski, and Walter Greiner
Institut für Theoretische Physik der Universität Frankfurt, Frankfurt am Main, Germany
(Received 14 February 1972)

The 1s bound state of superheavy atoms and molecules reaches a binding energy of $-2mc^2$ at $Z \approx 169$. It is shown that the K shell is still localized in r space even beyond this critical proton number and that it has a width Γ (several keV large) which is a positron escape width for ionized K shells. The suggestion is made that this effect can be observed in the collision of very heavy ions (superheavy molecules) during the collision.

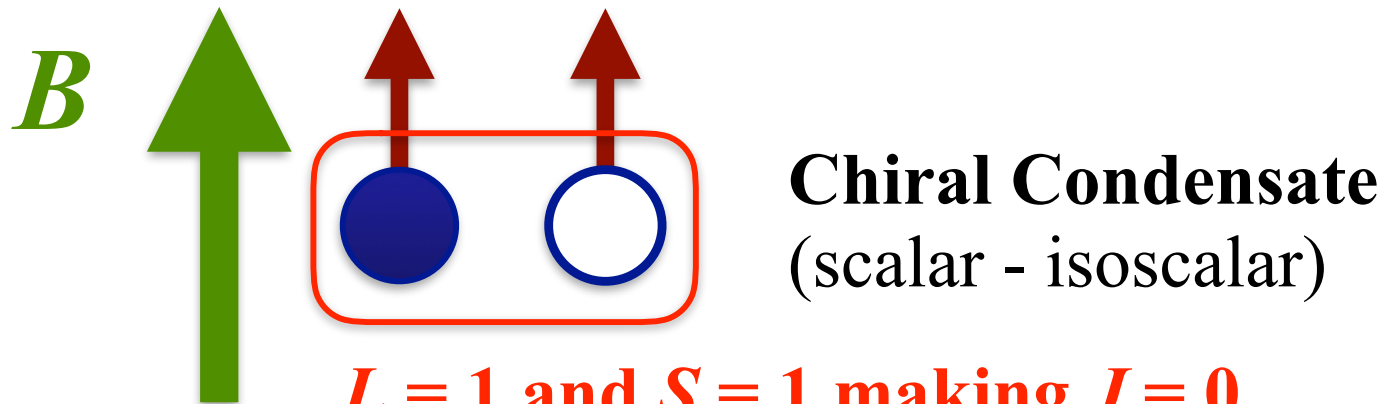
In the HIC super critical E is realized if $Z_1+Z_2 > Z_{cr}$

**Later, B effects considered too : PRL36, 517 (1976)
(Magnetic splitting predicted as an observable)**

QCD Vacuum Changed by B

Magnetic Catalysis

Gorbar, Gusynin, Miransky, Shovkovy
Klimenko, ... 1994



$L = 1$ and $S = 1$ making $J = 0$
more favored by strong B

Chiral Perturbation Theory (Shushpanov-Smilga 1997)

$$\Sigma(B) = \Sigma(0) \left(1 + \frac{\ln 2}{16\pi^2 f_\pi^2} eB + \dots \right)$$

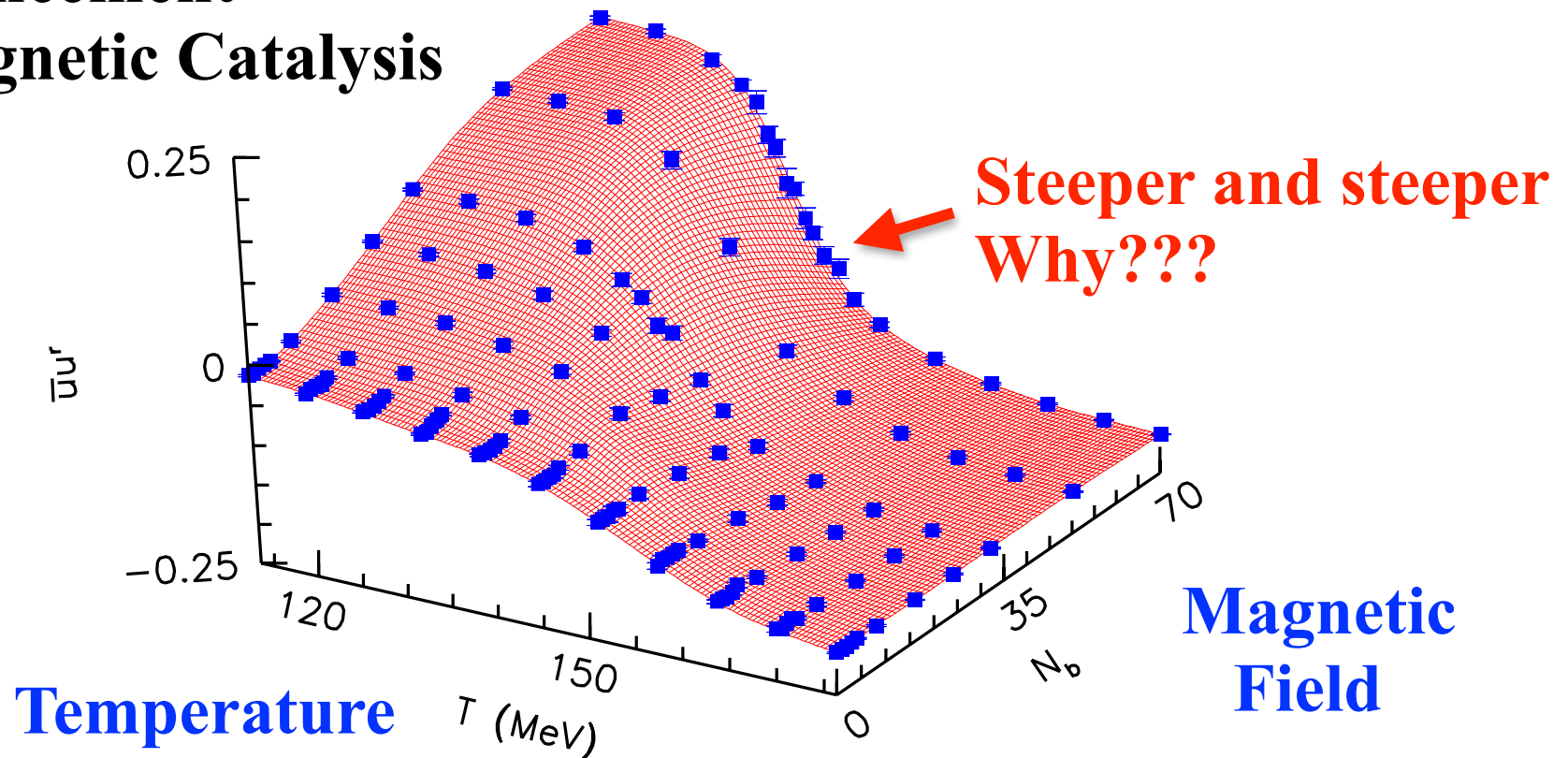
Positive coefficient

QCD Vacuum Changed by B



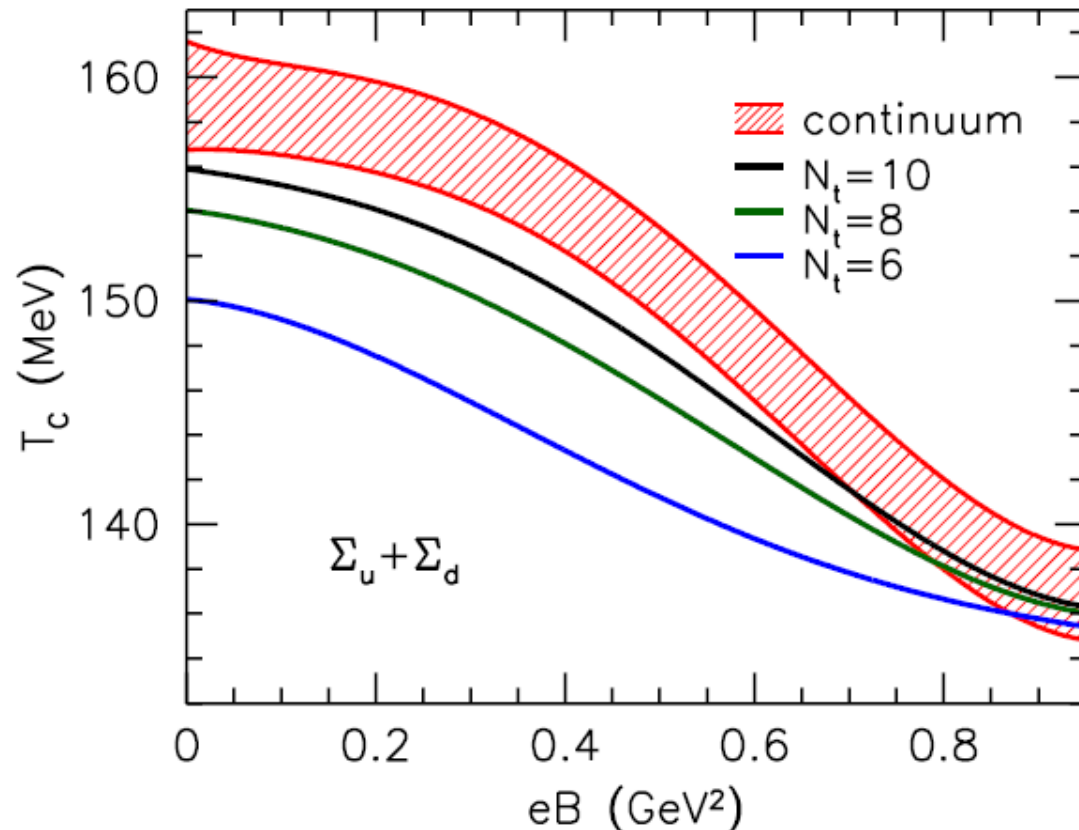
Enhancement
~ Magnetic Catalysis

Bali *et al.* 2011



We are such ignorant about QCD even today!

QCD Vacuum Changed by B



Fodor et al. 2011

Any impact to HIC?

Yes, if B survives

What happens with B ?

(B may survive with backreaction of matter)

Chemical Freezeout

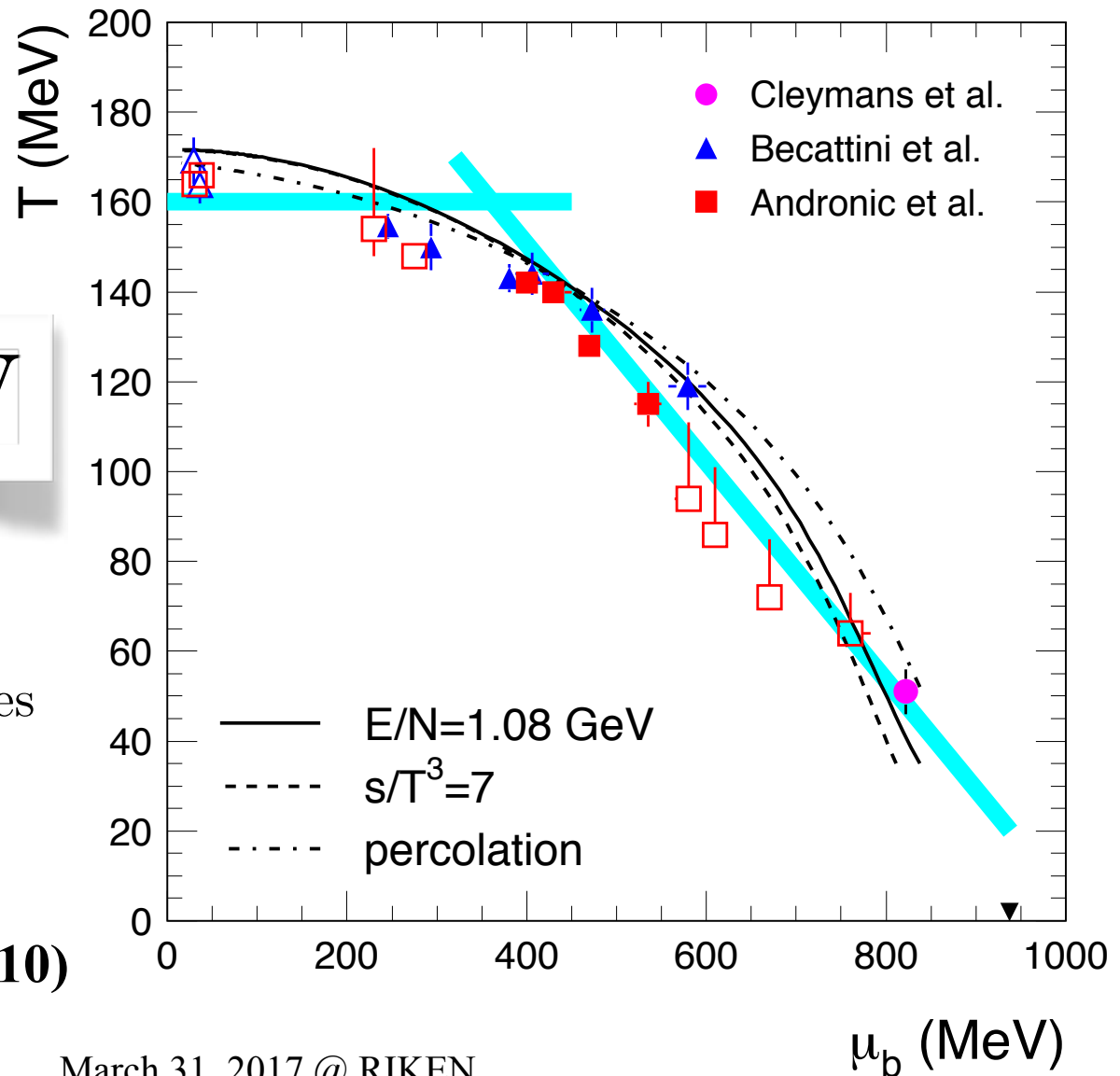
Cleymans-Redlich
PRL81, 5284 (1998)

$$E/N \sim 1 \text{ GeV}$$

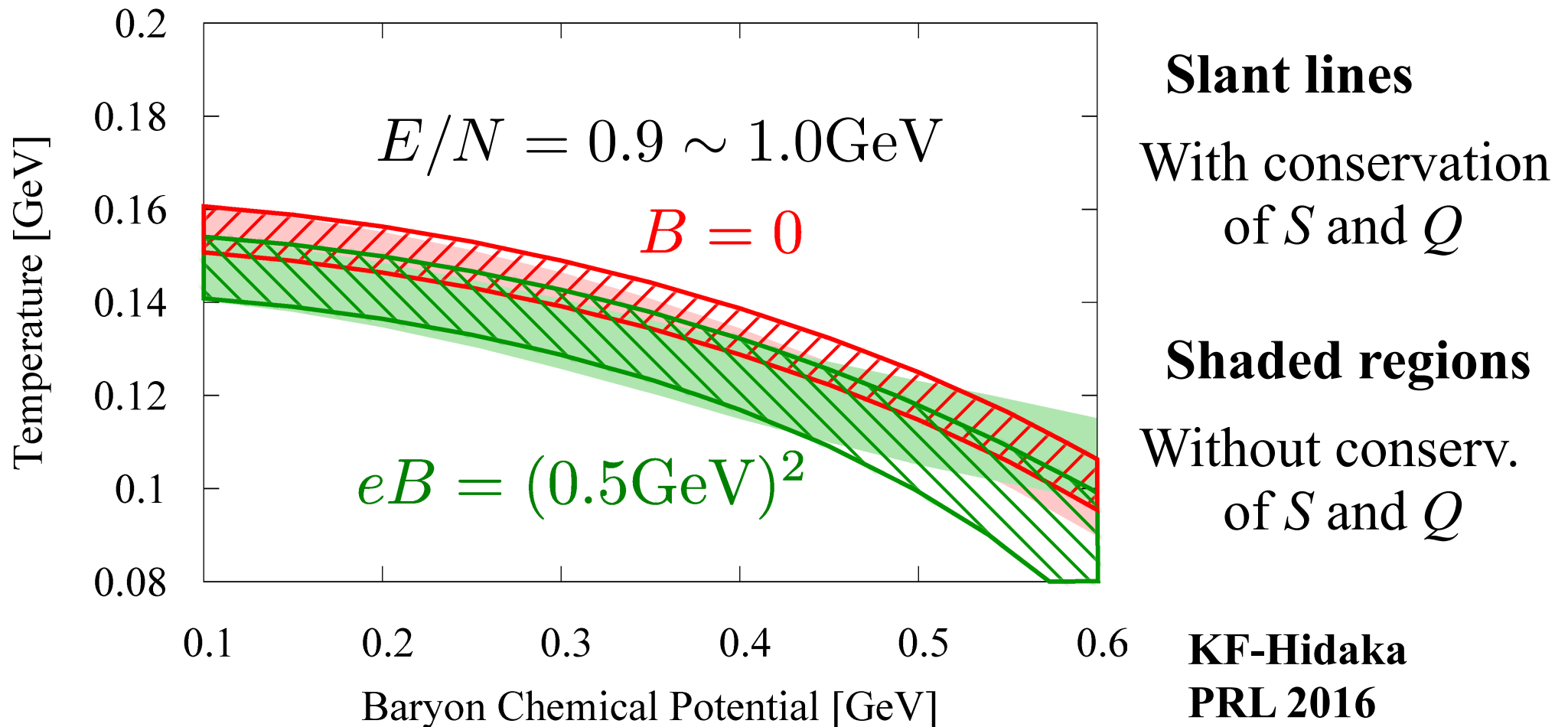
E : internal energy

N : particles + antiparticles

Andronic et al. (2010)

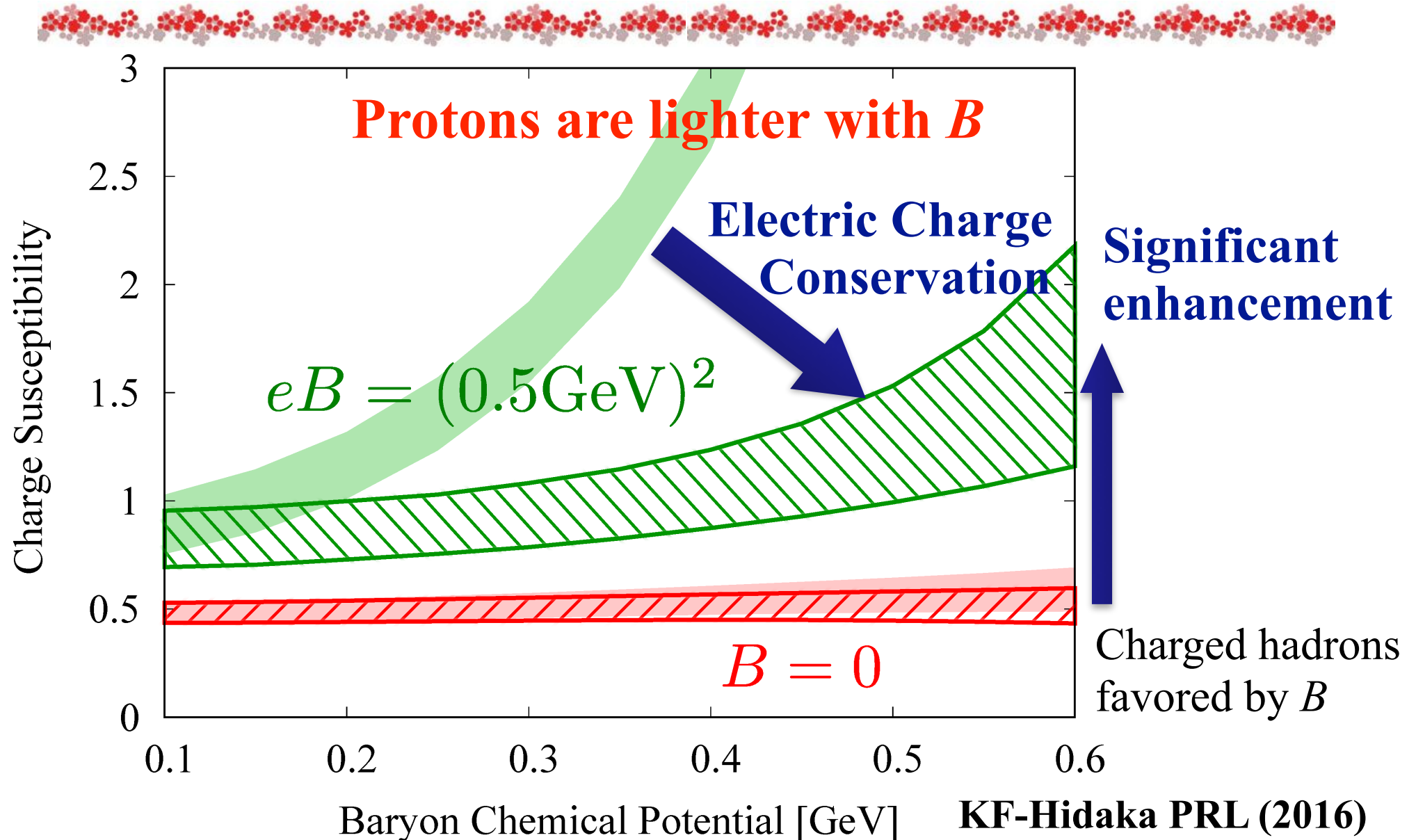


Chemical Freezeout



Inverse Magnetic Catalysis naturally reproduced

Chemical Freezeout



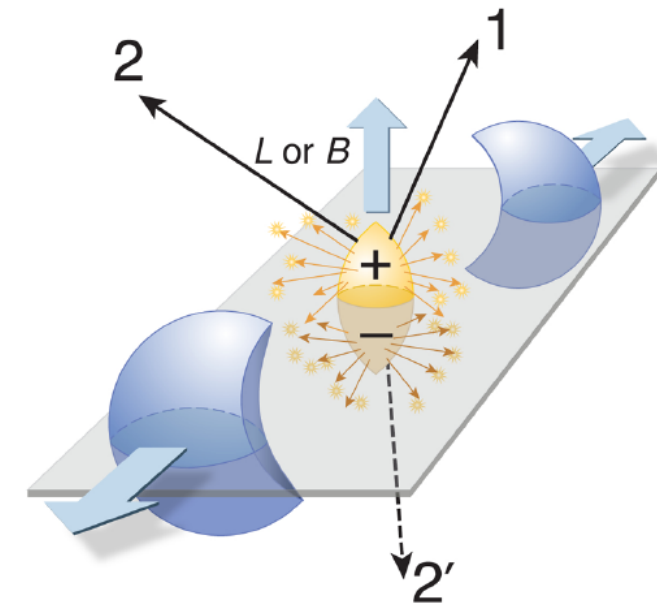
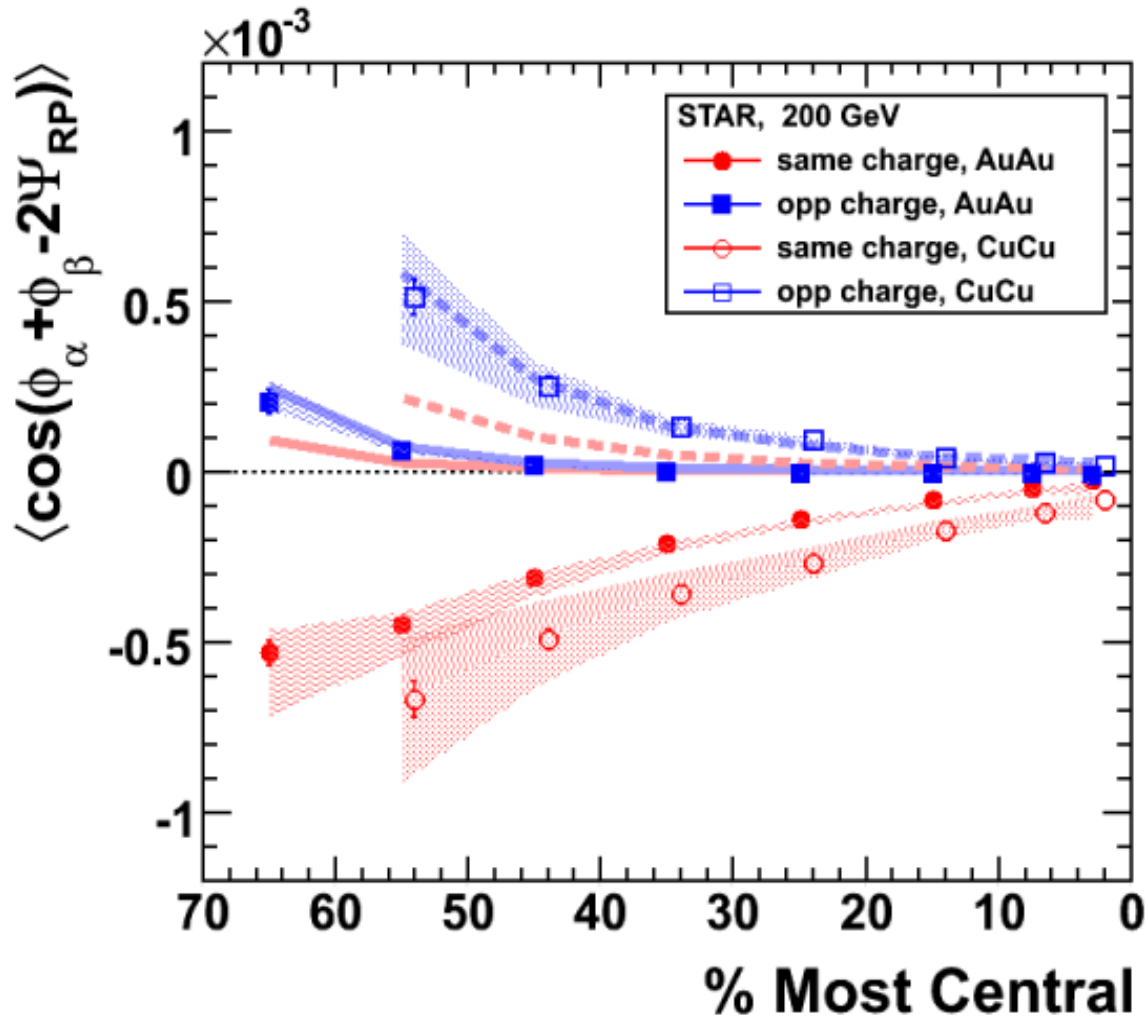
Chiral Magnetic Effect



$$\mathbf{j} = \frac{q_e^2 \mu_5}{2\pi^2} \mathbf{B}$$

‘Historical’ overview : 1209.5064 [hep-ph]

Chiral Magnetic Effect



B. Muller

STAR PRL 2009

Chiral Magnetic Effect

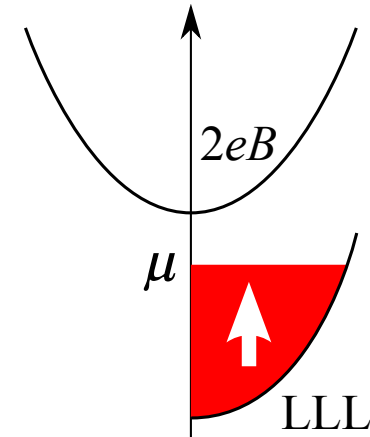
$$j_A^i = \langle \bar{\psi} \gamma^i \gamma_5 \psi \rangle = \phi_R^\dagger \sigma^i \phi_R + \phi_L^\dagger \sigma^i \phi_L$$



$$\frac{q_e B}{2\pi} \times q_e \frac{\mu q}{\pi}$$

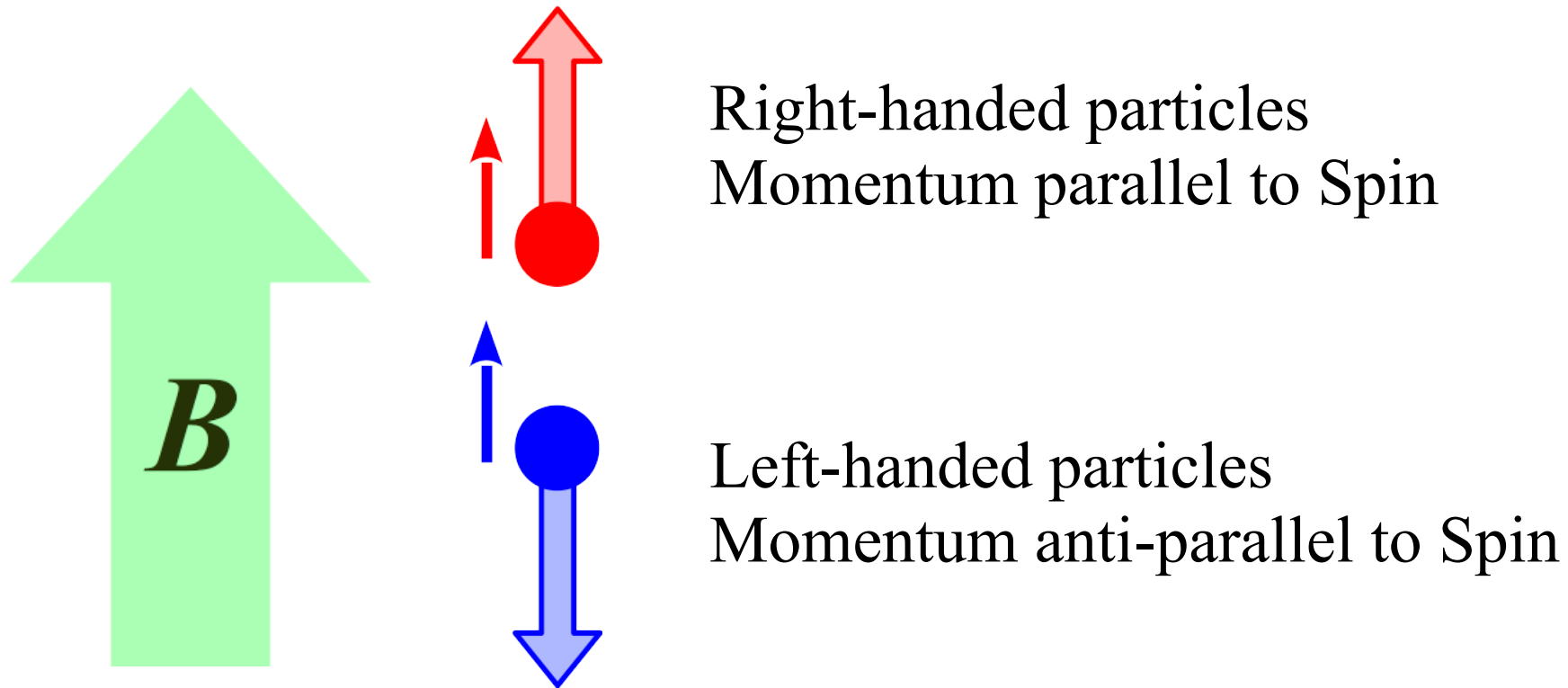
Density of states

1D charge density



Axial Current ~ Magnetization ~ B

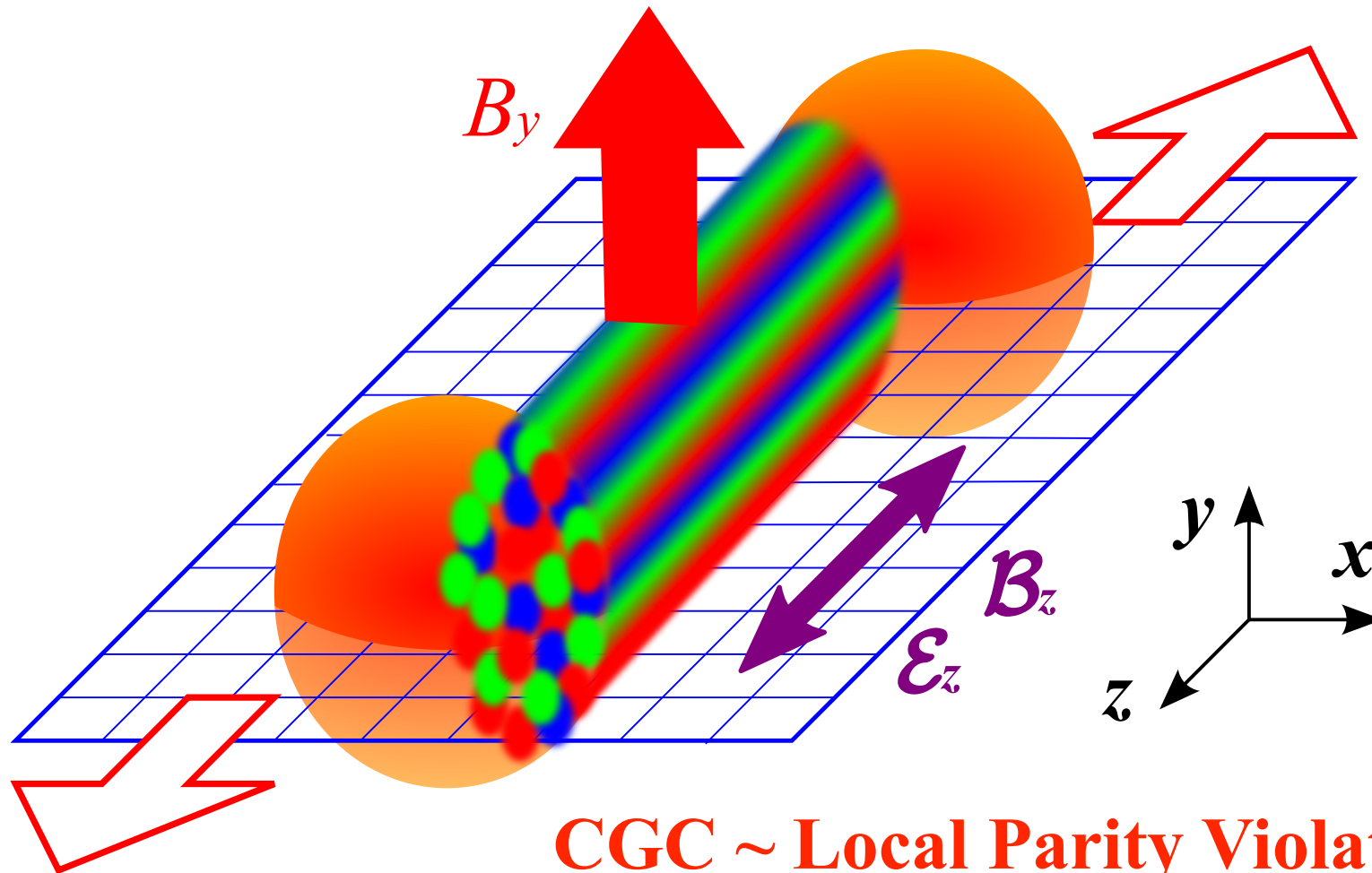
Chiral Magnetic Effect



Topological Current $\sim B$ (energy from chirality)

“Chiral Battery”

Chiral Magnetic Effect



Caveats



***B* decays very quickly**

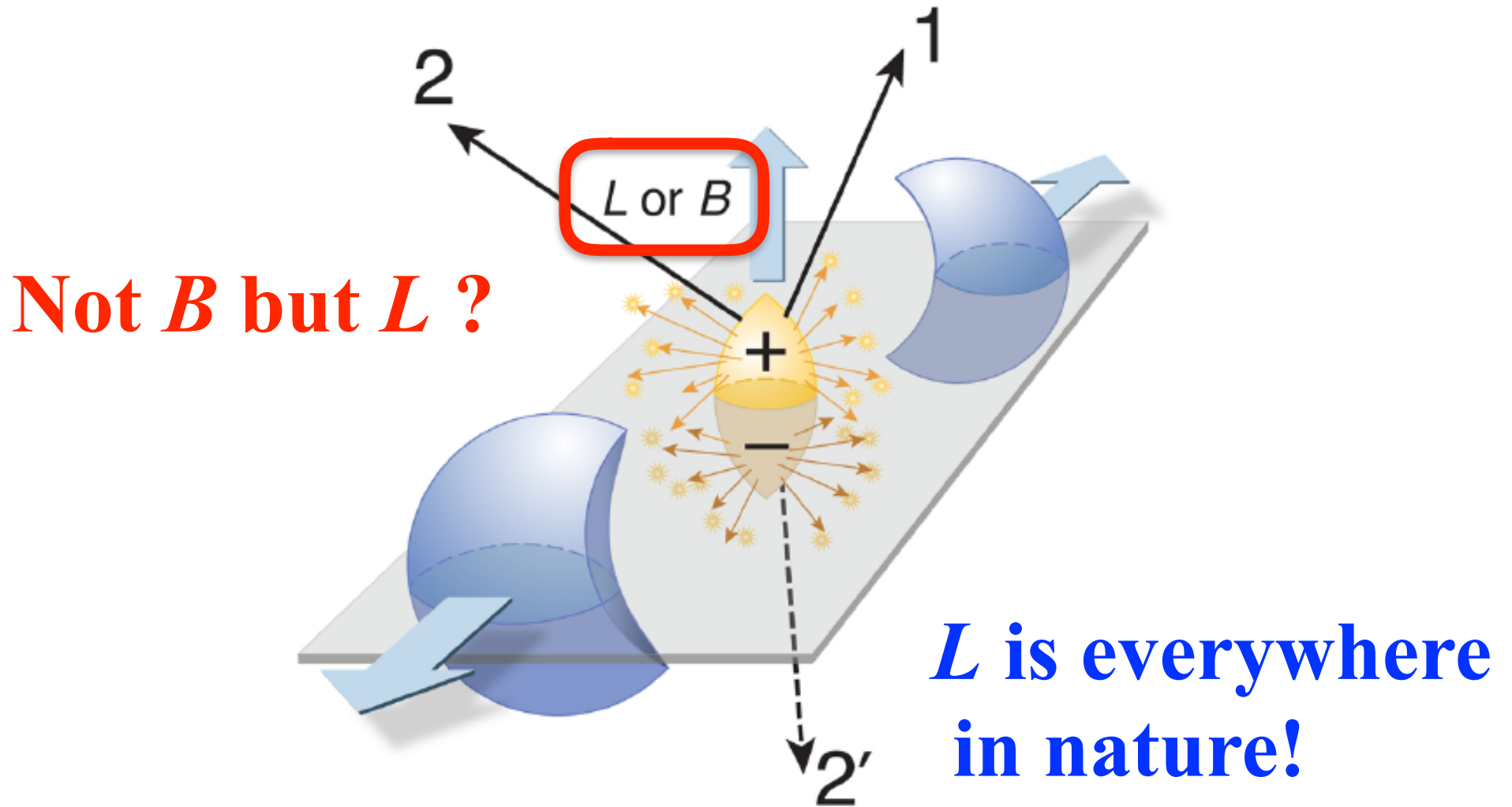
$$t_0 = \frac{b}{2 \sinh Y}$$

***B* is not an intrinsic property of matter
but created by passing spectators**

IF the electric conductivity is large, *B* could survive
Conductivity at strong *B* and finite density
(with Y. Hidaka — very hard theoretical calculation)

CGC simulation in *B* needed

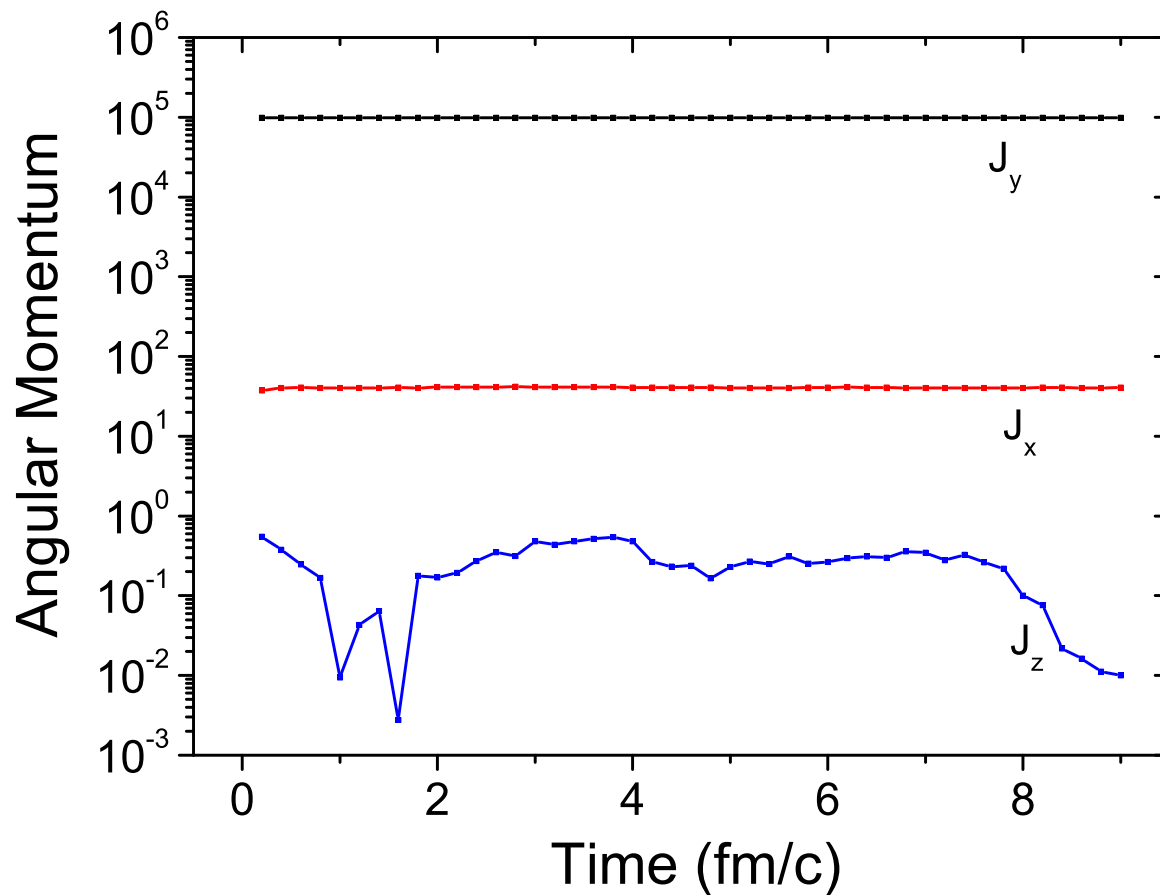
Better Alternative!?



Better Alternative!?



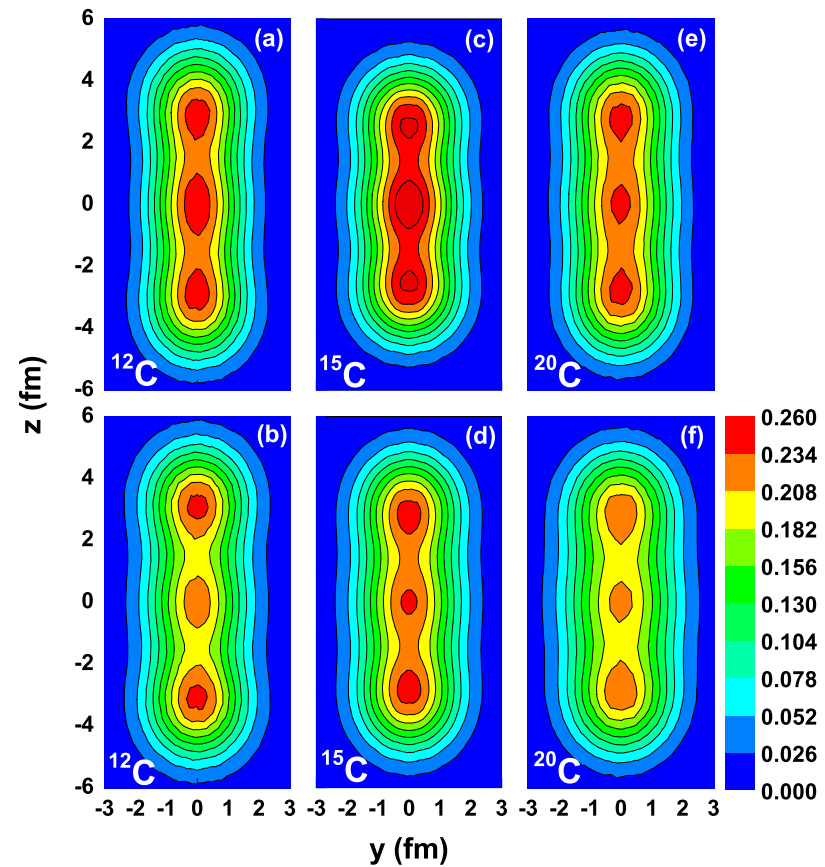
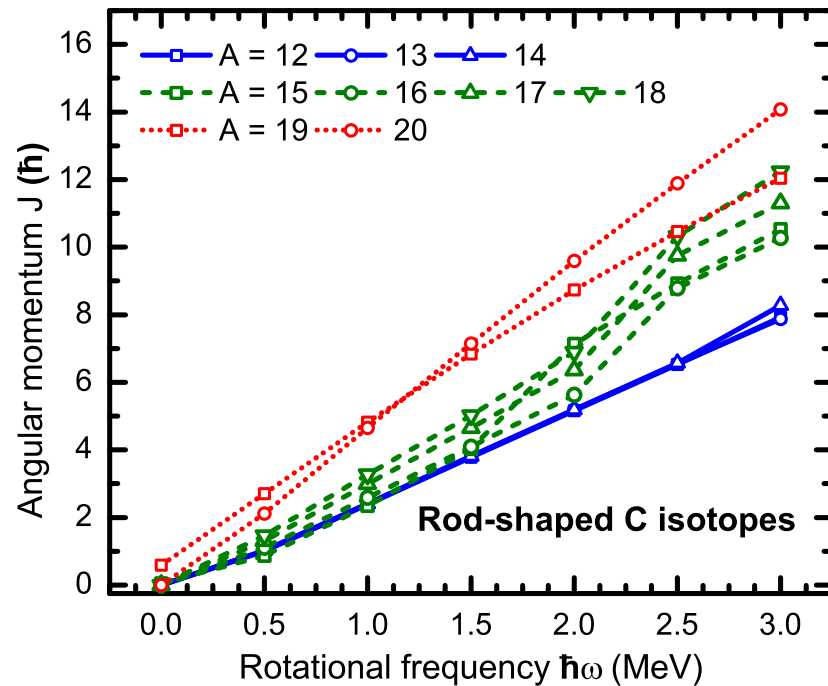
L is an intrinsic property of matter



Jiang-Lin-Liao 2016

Physics with Large L

Rod-shaped Nuclei at Extreme Spin and Isospin P.W. Zhao, N. Itagaki, J. Meng, PRL 2015



Physics with Large L



Lattice QCD in rotating frames A. Yamamoto, Y. Hirono, PRL 2013

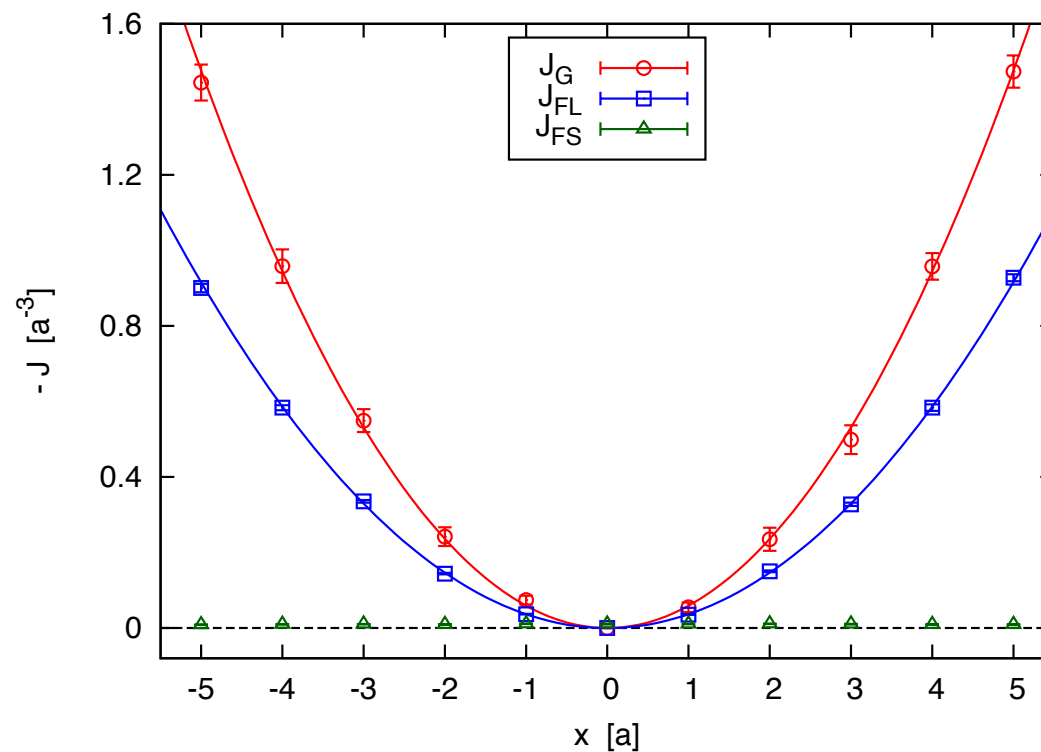


FIG. 2: Angular momentum density J along the x axis with the angular velocity $a\Omega = 0.06$. The solid curves are quadratic fitting functions.

Physics with Large L



Theoretical treatment for deformed nuclei

Cranking model $H_{\text{rot}} = H - \omega \underline{J_z}$

Chemical Potential?

J

Angular Momentum ~ Magnetic Field
(Topological Current expected)

B

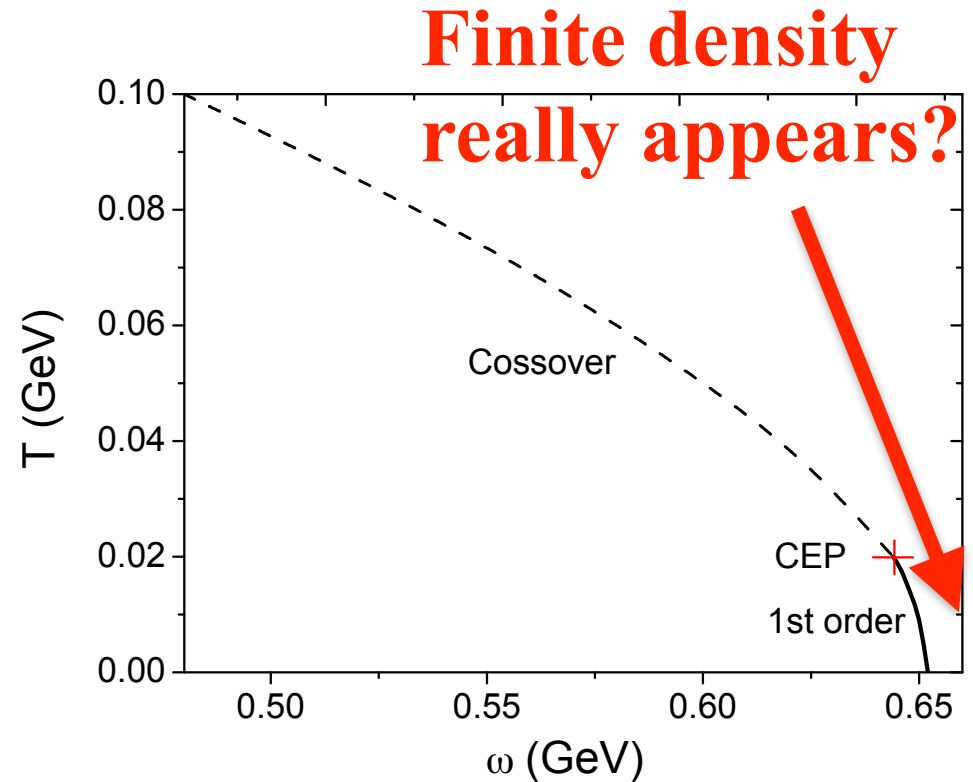
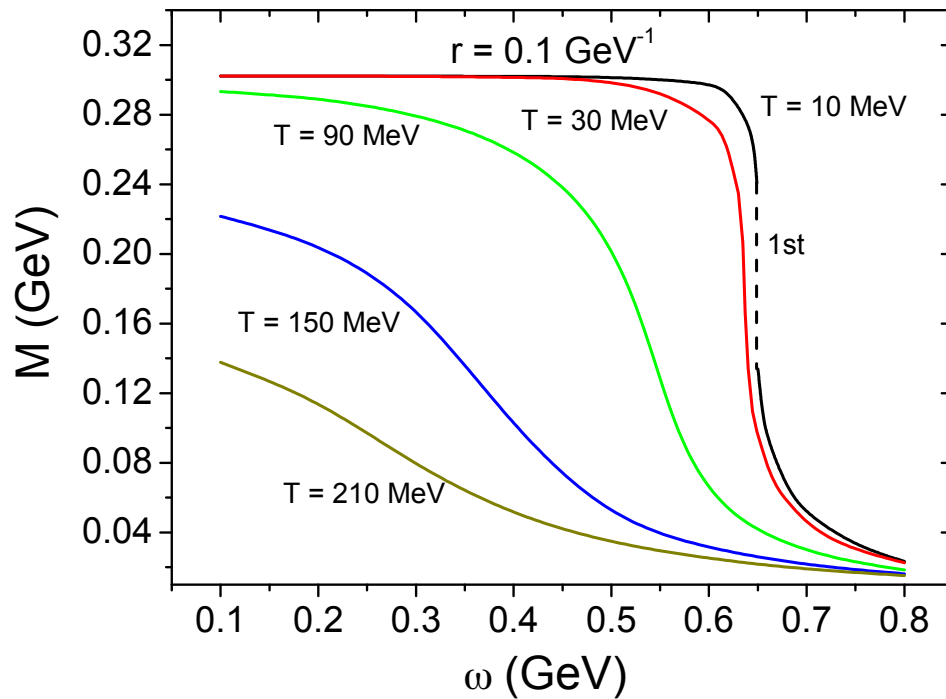
Angular Momentum ~ Finite Density
("Critical Point"???)

μ

Quantum Transition with Rotation



Jiang-Liao, PRL 2016



Rotating Quark Matter has Stronger 1st-order PT

Vilenkin's Cautions

Vilenkin (1980)

Here,

$$n_{\omega m} = (e^{\beta(\omega - m\Omega)} - 1)^{-1} \quad (23)$$

⁵Note that $n_{\omega m}$ has a peculiarity, however, it is the Bose-Einstein distribution for a rotating system, ⁵ $\tau = \tau_1 - \tau_2$, the upper and lower lines in cannot have size greater than $c\tau$ (the velocity at the boundary would exceed the velocity of light), and in a finite system the energy is quantized in such a way that ω is always greater than $m\Omega$. (There are some exceptions in which the field has exponentially growing modes. See Ref. 6.) As an example, consider an infinite cylinder of radius R rotating around its axis. Requiring that Ψ vanishes at the boundary, we find the energy levels $\omega_{nmp} = (p^2 + \mu^2 + \xi_{mn}^2 R^{-2})^{1/2}$, where ξ_{mn} is the n th root of $J_m(x)$. It can be shown (Ref. 7) that $\xi_{mn} > m$. Thus, $\omega_{nmp} > \xi_{mn} R^{-1} > m\Omega$. In the present paper we shall assume that the lowest energy modes are unimportant and thus the infinite-space solutions (17) can be used.

Vilenkin's Cautions

Vilenkin (1980)

Here,

⁵Note that $n_{\omega m}$ has a singularity at $\omega = m\Omega$. This singularity, however, is unphysical. A rotating system cannot have size greater than Ω^{-1} (otherwise the velocity at the boundary would exceed the velocity of light), and in a finite system the energy is quantized in such a way that ω is always greater than $m\Omega$. (There are some exceptions in which the field has exponentially growing modes. See Ref. 6.) As an example, consider an infinite cylinder of radius R rotating around its axis. Requiring that Ψ vanishes at the boundary, we find the energy levels $\omega_{nmp} = (p^2 + \mu^2 + \xi_{mn}^2 R^{-2})^{1/2}$, where ξ_{mn} is the n th root of $J_m(x)$. It can be shown (Ref. 7) that $\xi_{mn} > m$. Thus, $\omega_{nmp} > \xi_{mn} R^{-1} > m\Omega$. In the present paper we shall assume that the lowest energy modes are unimportant and thus the infinite-space solutions (17) can be used.

(23)

rotating
r lines in

No Liquid-Vacuum Phase Transition



$$\underbrace{\varepsilon - \omega|\ell + 1/2|}_{\text{effective chem. pot.}} \geq \frac{1}{R} \left[\underbrace{\xi_{\ell,1}}_{\text{smallest "mass" ~ Matsubara mode}} - \omega R(\ell + 1/2) \right]$$

Causality \nearrow

$$\omega R \leq 1 \geq \frac{1}{R} \left[\xi_{\ell,1} - (\ell + 1/2) \right] > 0$$

Mass Gap > Effective Chemical Potential
No “baryon density” induced by rotation alone

**Ebihara-KF-
-Memeda 2016**

Anomalous effects from coupling with...

μ
Gauge CVE

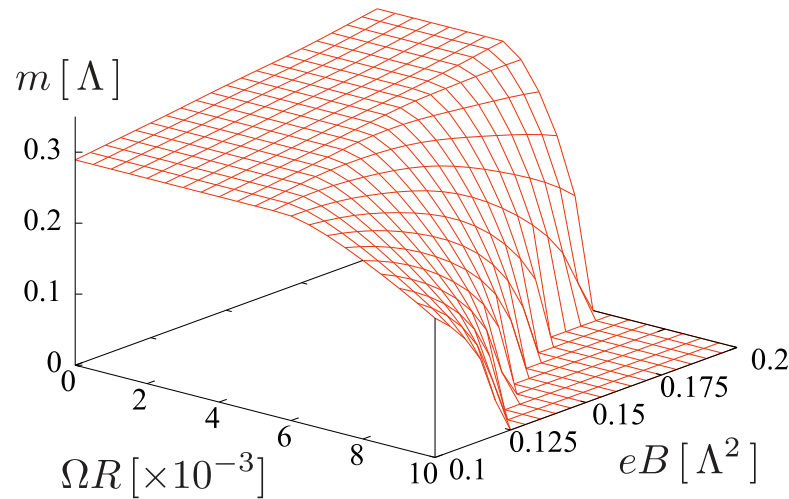
B
Chiral Pumping Effect

T
Gravity CVE

Rotation + B

Chen-KF-Huang-Mameda (2015)

Chen-KF-Huang-Mameda in progress



Finite Density really induced:

$$n = - \left. \frac{\partial \Omega}{\partial \mu} \right|_{\mu=0} = \frac{eB\omega}{4\pi^2}$$

interpreted as anomaly (**Hattori-Yin 2016**)

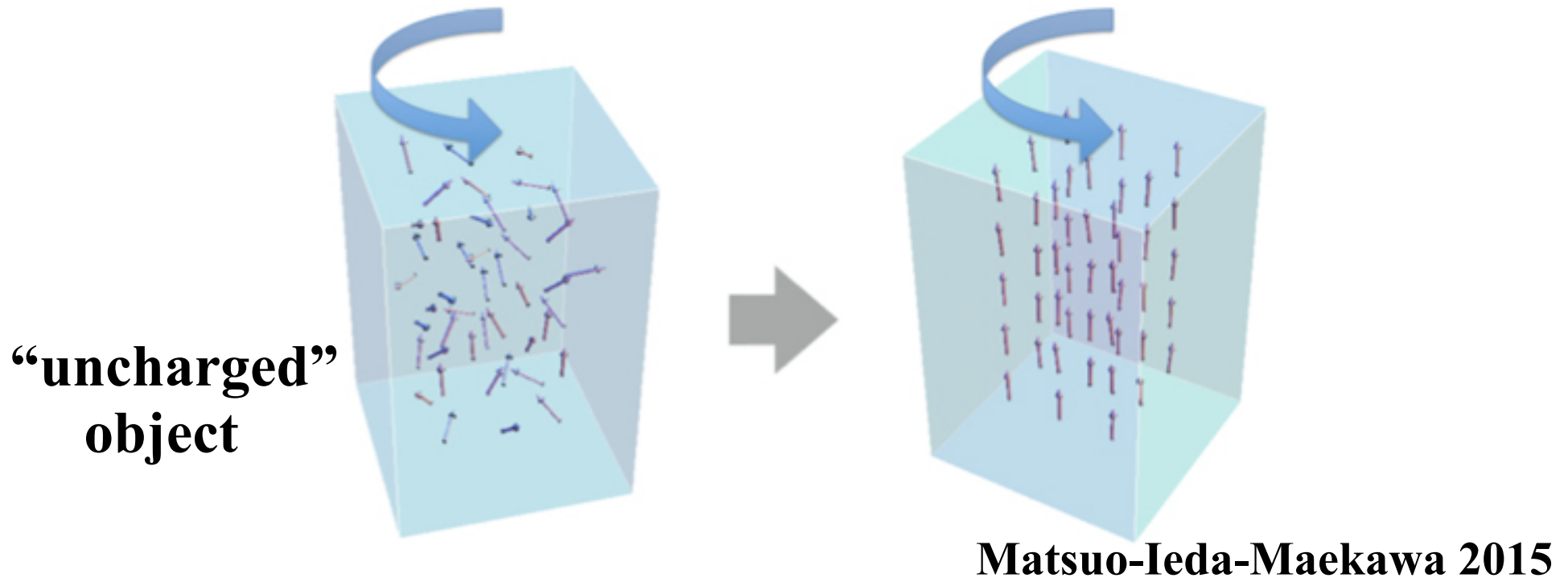
**Rotating Quark Matter (in HIC) has larger baryon density!
(detectable in the thermal model fit)**

Rotation + μ (Chiral Vortical Effect)



Barnett Effect

Magnetization



$$\text{Axial Current} \sim \text{Magnetization} \sim \mu^2 \omega$$

Rotation + B + μ (in HIC)



Rotating Quark Matter has
higher density induced by the chiral pumping
larger magnetic field induced by the Barnett effect
higher density ... larger magnetic field ...

NS

Origin of strong magnetic field (magnetar)
Equation of state (more stiff?)
Vortex structures (especially in a superfluid)

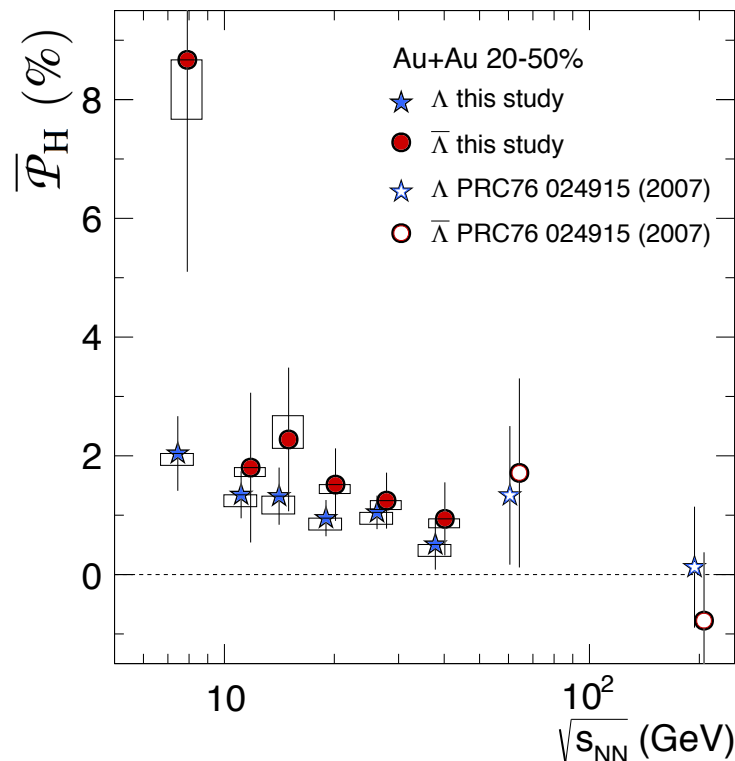
HIC

Quark matter magnetized (hadron polarized)
(STAR: Lambda “global” polarization ~ 0.02)
More chance to see a 1st-order phase transition
Finite temperature?

Rotation + B + μ (in HIC)

Global Polarization of Λ

$$P_{\text{Vortical}} = \frac{1}{2} (P_{\Lambda} + P_{\bar{\Lambda}}) \quad P_{\text{Magnetic}} = \frac{1}{2} (P_{\Lambda} - P_{\bar{\Lambda}})$$



Becattini, Csernai, Wang, ...

At high energies $L \gg B$

At low energies B larger?

STAR: 2016

Rotation + T

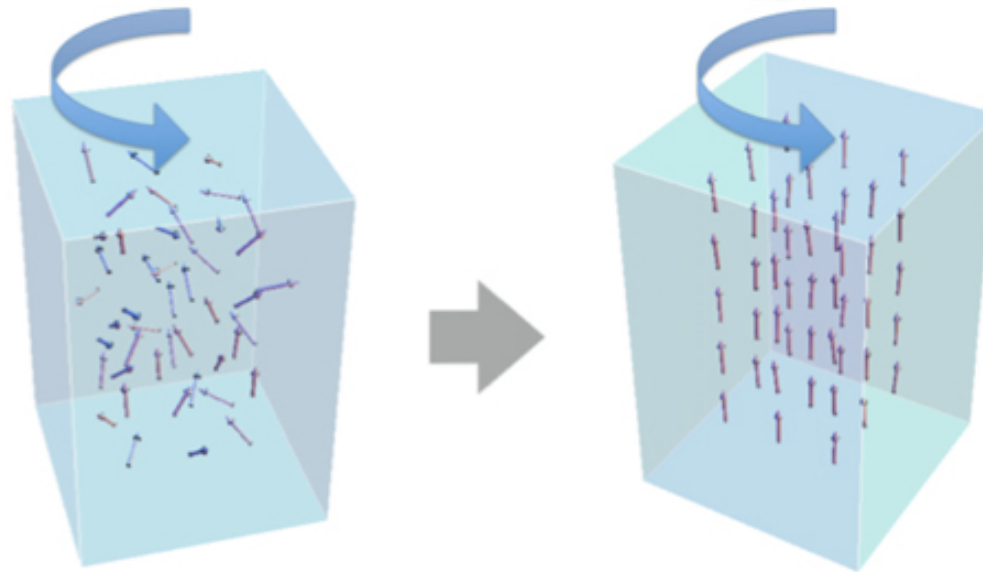


$T \gg R^{-1}$ Boundary can be neglected (Debye screening)

Vilenkin (1980)

$$\langle \mathbf{j}_A \rangle = \frac{T^2}{12} \boldsymbol{\omega}$$

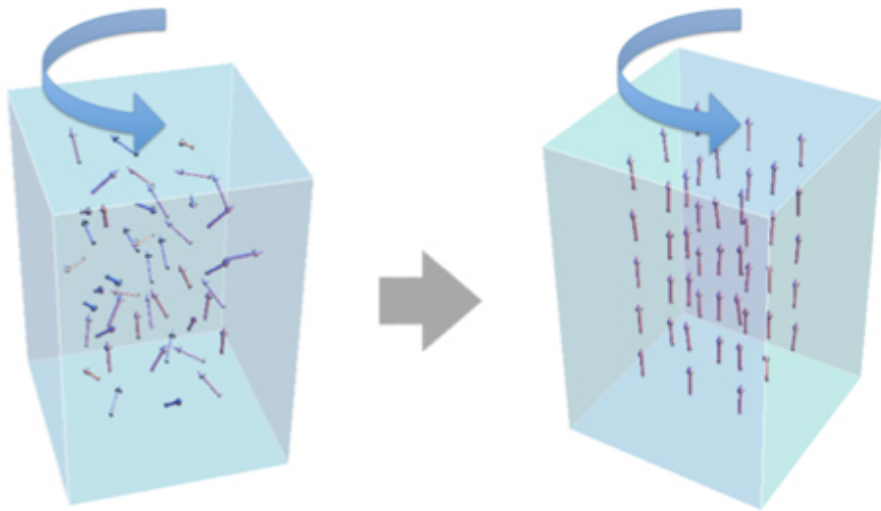
$\mu \rightarrow T$



Contradicting General Relativity?



$$X_{\mu\nu} \rightarrow X_{\mu'\nu'} = \frac{\partial x^\mu}{\partial x'^{\mu'}} \frac{\partial x^\nu}{\partial x'^{\nu'}} X_{\mu\nu}$$



$$x' = x \cos(\omega t) - y \sin(\omega t)$$

$$y' = y \cos(\omega t) + x \sin(\omega t)$$

$$j_{z'} = j_z = 0 \text{ (???)}$$

This axial current is NOT a vector!

Contradicting General Relativity?

$$\bar{\Gamma}^k_{ij} = \frac{\partial x^p}{\partial y^i} \frac{\partial x^q}{\partial y^j} \Gamma^r_{pq} \frac{\partial y^k}{\partial x^r} + \frac{\partial y^k}{\partial x^m} \frac{\partial^2 x^m}{\partial y^i \partial y^j}$$

Christoffel symbol is NOT a tensor

cf. Geodesic equation $\frac{d^2 x_\gamma}{d\lambda^2} = -\Gamma_{\gamma}^{\alpha\beta} \cdot \frac{dx_\alpha}{d\lambda} \cdot \frac{dx_\beta}{d\lambda}$

**Fictitious forces
(centrifugal, Coriolis)**

**Chiral Vortical Effect is as fictitious as Coriolis force
(or as real as Coriolis force)**

Gravitational CS Current



$$J_A^\mu = 4C_R \epsilon^{\mu\nu\rho\lambda} \Gamma_{\nu\beta}^\alpha \left(\partial_\rho \Gamma_{\alpha\lambda}^\beta + \frac{2}{3} \Gamma_{\rho\sigma}^\beta \Gamma_{\alpha\lambda}^\sigma \right)$$

$\sim \omega$ $\sim R$

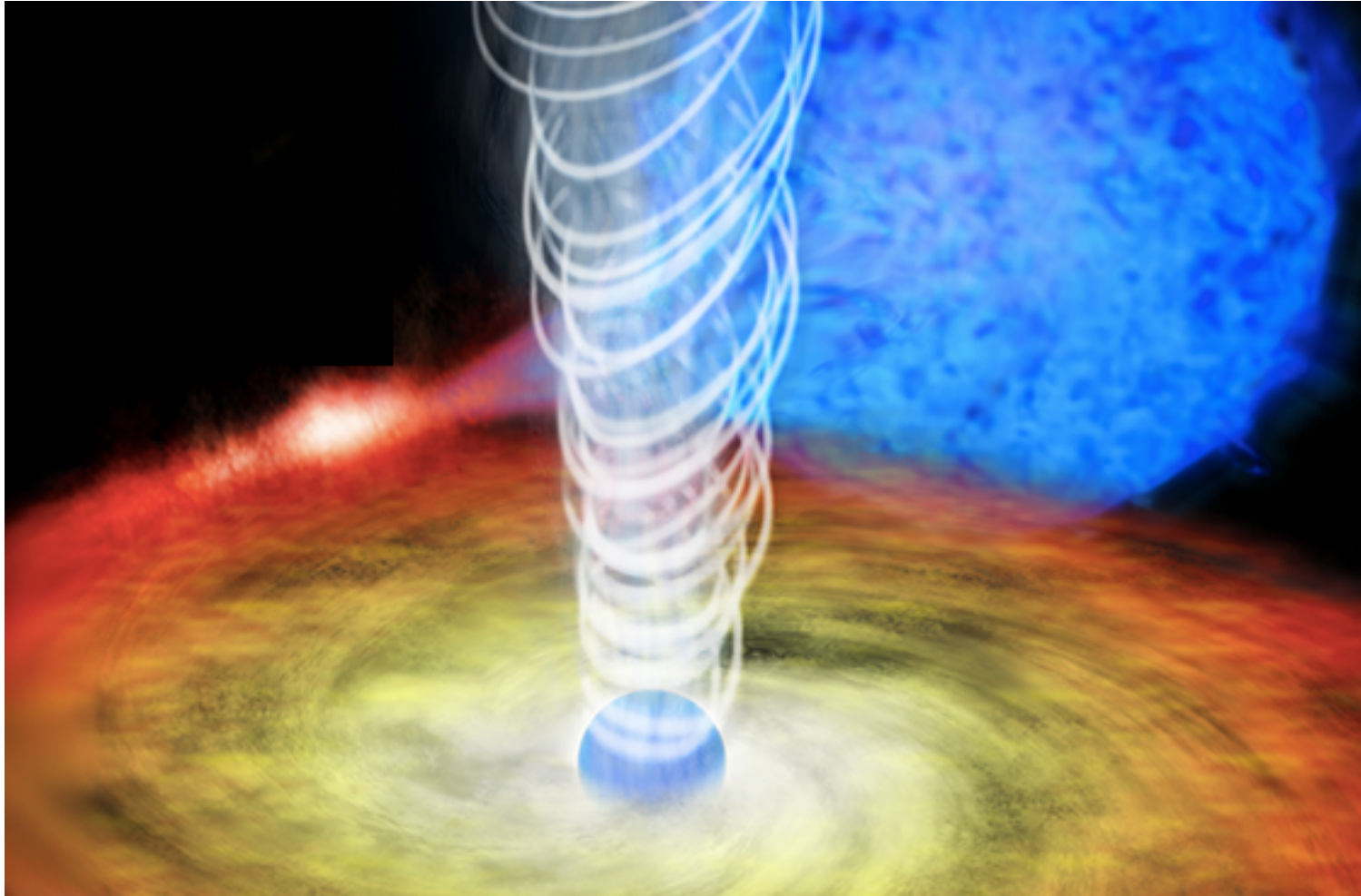
leading to the **Chiral Vortical Effect**

Fukushima-Flachi 2017

Rotation + $T \sim$ Rotation + R

→ Topological Axial Current

Gravitational CS Current



@ University of Wisconsin-Madison

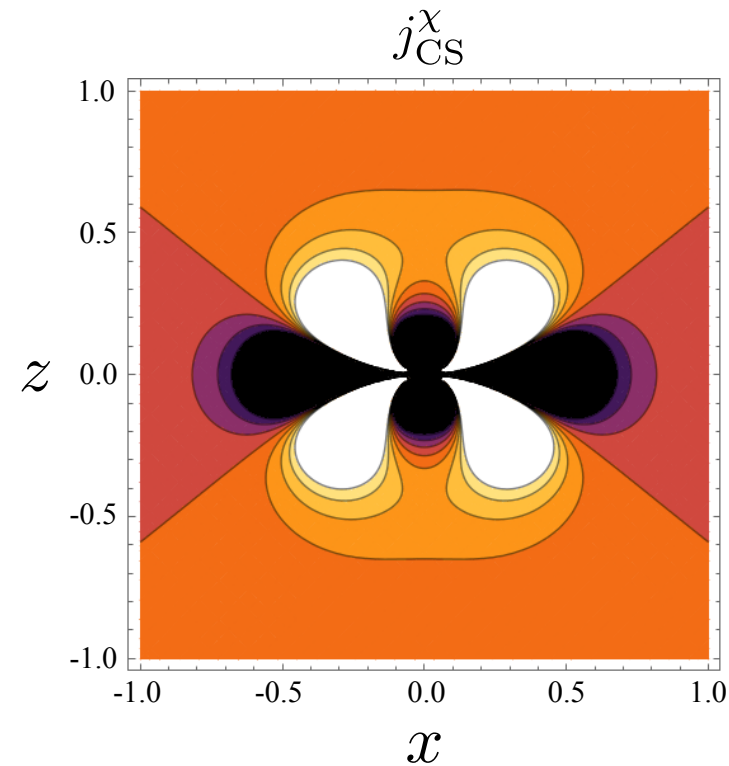
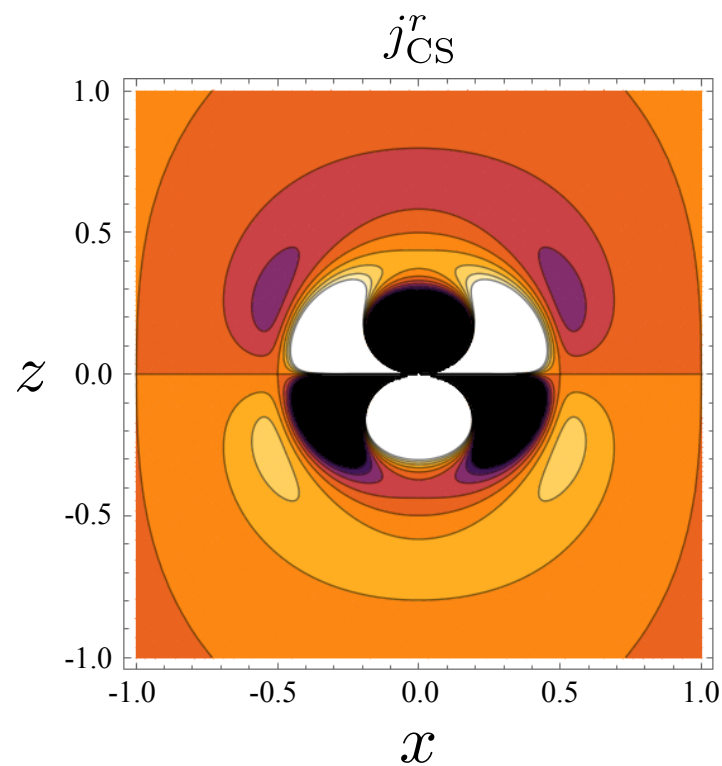
March 31, 2017 @ RIKEN

44

Gravitational CS Current



Fukushima-Flachi 2017



Neutrino Flux with Jet + Disk structures

Is this also relevant for the HIC at low energies?

Summary

Rotating quark matter

- Better experimental chance than B
 - ◆ **Beam energy scan ~ hot astrophysics simulator**
 - ◆ Lambda global polarization in HIC
 - ◆ Astrophysical jet counterpart?
- More rich structures in theory
 - ◆ Technically complicated than B
- Topologically induced density and current
 - ◆ Finite- T induced quantum anomaly

CS current and Barnett effect

- Rotation + R ~ Origin of Astrophysical jet?