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Nonperturbative ambiguity in double-well type matrix models

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Resurgence & Physics

Resurgence is checked to work in several theories

- beautiful!
- no ambiguity (in trans-series) $\rightarrow$ predictability

What should be done:

- Unified understanding
  - case-by-case study $\rightarrow$ classification   dim., symm., pot., op., $\cdots$
- Case of string theory   Dunne's talk
  - nonperturbative effect is more important $\sim e^{-C/g_s}$
  - vacuum structure of string theory: SSB

    nice if resurgence tells something

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DWMM
SUSY/nonSUSY
SSB of SUSY
in string theory

Bosonic DWMM: type 0B string in 1D

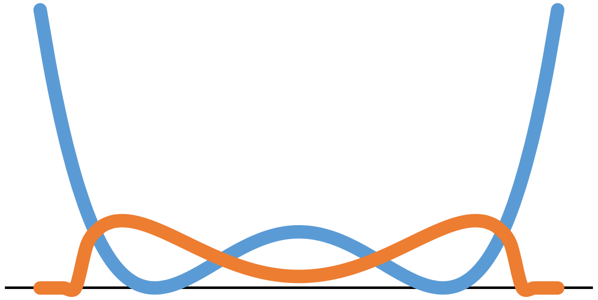
0D Hermitian MM with DW potential:

$$S = N\mathrm{tr}\left(-\frac{1}{2}\phi^2 + \frac{g}{4}\phi^4\right)$$

ϕ: $N \times N$
$g > 0$: parameter

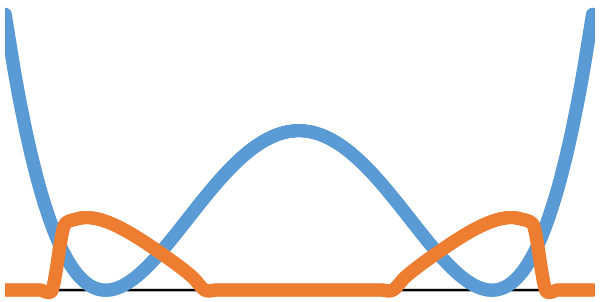
$\phi = U\mathrm{diag}(\lambda_1, \cdots, \lambda_N)U^\dagger \rightarrow$ dynamics of N eigenvalues

$N \rightarrow \infty$: two phases



$\leftarrow \; g_c = \frac{1}{4} \; \rightarrow$

3rd order PT



Klebanov, Maldacena, Seiberg '03

DSL: $N^{\frac{2}{3}}(g_c - g) \sim t \sim 1/g_s^{\frac{2}{3}} \rightarrow$ nonpert. type 0B string theory

Nonperturbative ambiguity in type 0B

$$e^{-F(t)} = \int \prod_{i=1}^{N} d\lambda_i \Delta(\lambda)^2 e^{-N\sum_i\left(-\frac{1}{2}\lambda_i^2+\frac{g}{4}\lambda_i^4\right)}$$

$$\boxed{N^{\frac{2}{3}}(g_c-g) \sim t \sim 1/g_s^{\frac{2}{3}}}$$

orthogonal polynomial $\rightarrow$ in DSL, $F(t)$ satisfies Painlevé II:

$$\partial_t^2 F = -\frac{1}{4}(f^2+t), \qquad tf = f^3 - 2\partial_t^2 f$$

large t(small g_s)-exp. pert. solution: $f_{pert} = \pm\sqrt{t} \mp \frac{1}{4}t^{-\frac{5}{2}} + \mathcal{O}\left(t^{-\frac{11}{2}}\right)$

two solutions: f_1, f_2 with same $f_{pert} \rightarrow \Delta f = f_1 - f_2$ satisfies

$$\partial_t^2 \Delta f = \left(t - \frac{3}{4}t^{-2} + \cdots\right)\Delta f \rightarrow \Delta f = Ct^{-\frac{1}{4}} e^{-\frac{2}{3}t^{\frac{3}{2}}}$$

C: undermined!

“nonperturbative ambiguity”

perturbative solution:

Kapaev '04

$$f(t) = \sqrt{\frac{2t}{3\pi^3}} \sum_{h=0}^{\infty} b_h t^{-3h}, \qquad b_h \sim \left(\frac{9}{4}\right)^h \boxed{\Gamma\left(2h - \frac{1}{2}\right)}$$

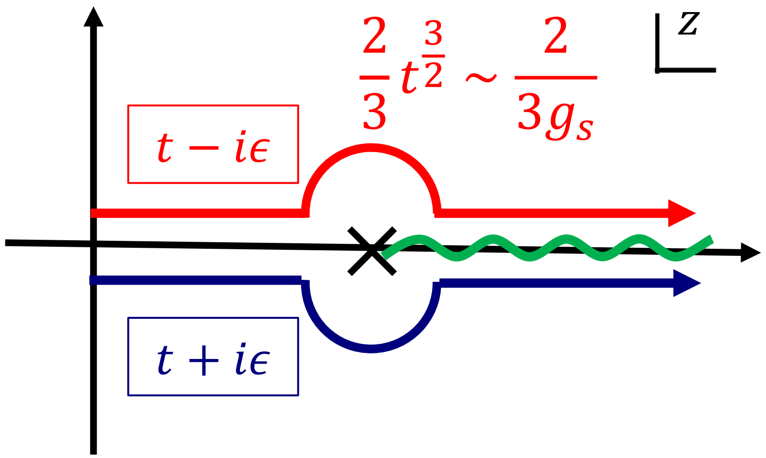
$\boxed{g_s^{2h}}$

$(2h)!$ stringy

Borel resummation: insert $1 = \frac{1}{\Gamma(2n+1)} \int_0^\infty dz\, e^{-z} z^{2n}$

$$\to f = -\frac{\sqrt{t}}{\sqrt{3}\pi} \int_0^\infty dz\, e^{-z} \left(1 - \frac{9z^2}{4t^3}\right)^{\frac{1}{2}} + \cdots$$

$$\to f_+ - f_- = i \frac{2\sqrt{t}}{\sqrt{3}\pi} K_1\left(\frac{2}{3} t^{\frac{3}{2}}\right) \sim \frac{i}{\sqrt{\pi} t^{\frac{1}{4}}} e^{-\frac{2}{3} t^{\frac{3}{2}}}$$



$$\to F_+ - F_- \sim -\frac{i}{2\sqrt{\pi} t^{\frac{3}{4}}} e^{-\frac{2}{3} t^{\frac{3}{2}}}$$

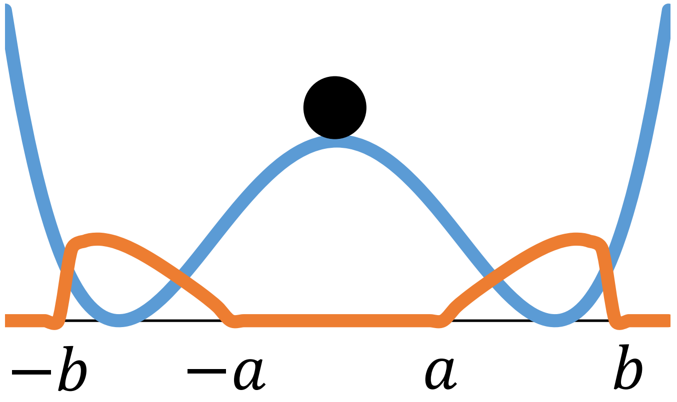
nonperturbative ambiguity consistent with Painlevé II

Instanton = isolated eigenvalue at the top

- classification of saddle pts.

$$\int_{-\infty}^{\infty} d\lambda_i = \int_0^{\infty} 2d\lambda_i = \int_a^{\infty} 2d\lambda_i + \int_0^{a} 2d\lambda_i$$

perturbative | nonperturbative



$$Z = \left(\int_a^{\infty} \prod_{i=1}^{N} 2d\lambda_i + {}_N C_1 \int_a^{\infty} \prod_{i=1}^{N-1} 2d\lambda_i \int_0^{a} 2d\lambda_N + \cdots \right) (\cdots)$$

$^\exists$ambiguity   one-instanton sector

$$\rightarrow F_{1-inst} \sim \int_{-\sqrt{t}}^{0} d\zeta \frac{\sqrt{t}}{t-\zeta^2} e^{-\frac{2}{3}(t-\zeta^2)^{\frac{3}{2}}}$$

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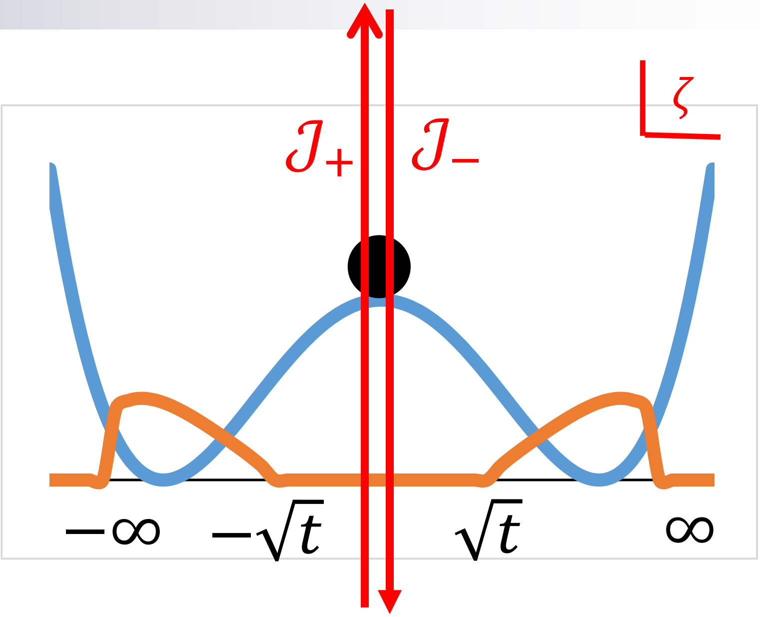
$$F_{1-inst+} - F_{1-inst-} \sim \frac{i}{2\sqrt{\pi} t^{\frac{3}{4}}} e^{-\frac{2}{3}t^{\frac{3}{2}}}$$



Kawai-T.K.-Matsuo '05

Marino '08
Chan-Irie-Yeh '10, '11
Schiappa-Vaz '13

- cancels ambiguity from perturbative series
- coefficient (Stokes constant) was proved to be universal $\rightarrow$ attributed to universality of (large order behavior of) perturbative series

SUSY DWMM: type IIA string in 2D

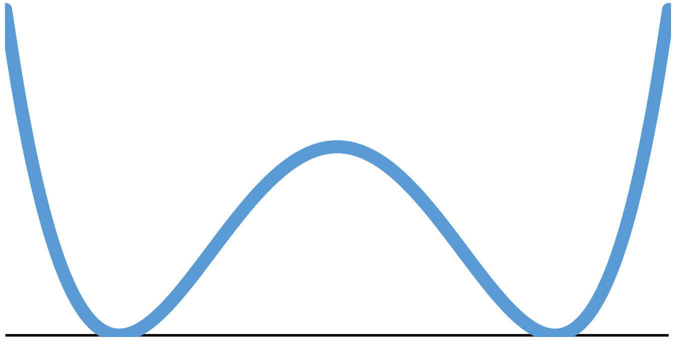
$$S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right]$$

- 0-dim. $N \times N$ MM, parameters: N, μ^2
- scalar potential: $V(\phi) = (\phi^2 - \mu^2)^2/2$
- nilpotent SUSY:

$$Q\phi = \psi, \quad Q\psi = 0, \quad Q\bar{\psi} = -iB, \quad QB = 0,$$
$$\bar{Q}\phi = -\bar{\psi}, \quad \bar{Q}\bar{\psi} = 0, \quad \bar{Q}\psi = -iB, \quad \bar{Q}B = 0,$$

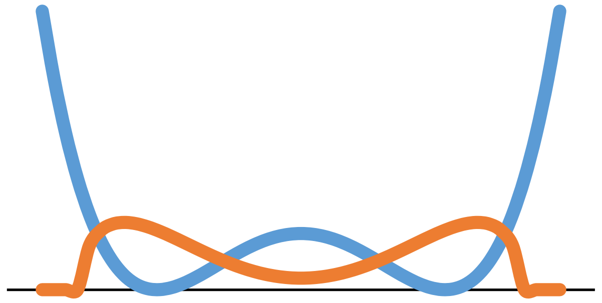
- finite N: instanton $\longrightarrow$ SUSY br.

$\longrightarrow N \to \infty$?



Large$-N$ limit & SUSY phase

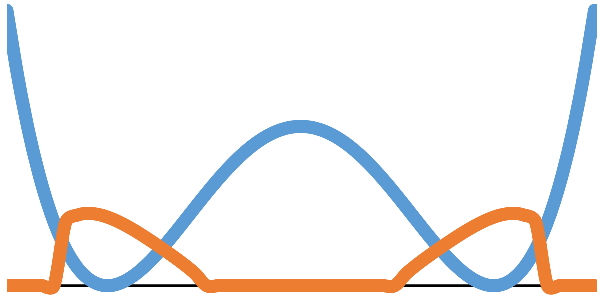
$N \to \infty$: two phases



$\leftarrow \ \mu_c^2 = 2 \ \rightarrow$

3rd order PT

SUSY/nonSUSY



one-cut phase ($\mu^2 < 2$)
no SUSY

two-cut phase ($\mu^2 > 2$)
SUSY

double scaling limit: $N \to \infty,\ \mu^2 \to 2 + 0$

: keeping perturbative SUSY (w.r.t $1/N^2$ – exp.)

tree level correlation func.$\longrightarrow$ $\boxed{N^2(\mu^2 - 2)^3 = s^3 = 1/g_s^2\text{: fixed}}$

Superstring theory side

type IIA in 2D ($\mathcal{N} = 2$ superLiouville)

Kutasov-Seiberg '90

target sp.: $(x \in S^1(R = 1), \varphi)$

WS & TS SUSY: $Q_+^2 = \bar{Q}_-^2 = \{Q_+, \bar{Q}_-\} = 0$

nilpotent!

$$q_+ = e^{-\frac{1}{2}\phi - \frac{i}{2}H - ix}, \qquad \bar{q}_- = e^{-\frac{1}{2}\bar{\phi} + \frac{i}{2}\bar{H} + i\bar{x}} \qquad (\psi_l \pm i\psi_x = \sqrt{2}e^{\mp iH})$$

physical states:

NS: $T_k \sim \exp[ikx + (1 - |k|)\varphi]$

R: $V_k^{(\epsilon)} \sim \exp[i\epsilon H/2 + ikx + (1 - |k|)\varphi]$

Agreement of tree amplitudes

T.K.-Sugino '14

MM amp. in $N \to \infty$ = type IIA tree level amp. under

$$\frac{1}{N}\operatorname{tr}\phi^{2k+1} \Leftrightarrow \text{(R+, R−)}: V^{(+)}_{k+1/2}\bar{V}^{(-)}_{-k-1/2}$$
$$\frac{1}{N}\operatorname{tr}\psi^{2k+1} \Leftrightarrow \text{(NS, R−)}: T_{-k-1/2}\bar{V}^{(-)}_{-k-1/2}$$
$$\frac{1}{N}\operatorname{tr}\bar{\psi}^{2k+1} \Leftrightarrow \text{(R+, NS)}: V^{(+)}_{k+1/2}\bar{T}_{k+1/2}$$
$$\frac{1}{N}\operatorname{tr}B \Leftrightarrow \text{(NS, NS)}: T_{-1/2}\bar{T}_{1/2}$$

$$\begin{array}{ccc} & \operatorname{tr}\phi & \\ Q \swarrow & & \searrow \bar{Q} \\ \operatorname{tr}\psi & & \operatorname{tr}\bar{\psi} \\ \bar{Q} \searrow & & \swarrow Q \\ & \operatorname{tr}B & \end{array}$$

SUSY identification

$$(Q, \bar{Q}) \longleftrightarrow (Q_+, \bar{Q}_-)$$

massless multiplet

$$\begin{array}{ccc} & V^{(+)}_{1/2}\bar{V}^{(-)}_{-1/2} & \\ Q_+ \swarrow & & \searrow \bar{Q}_- \\ T_{-1/2}\bar{V}^{(-)}_{-1/2} & & V^{(+)}_{1/2}\bar{T}_{1/2} \\ \bar{Q}_- \searrow & & \swarrow Q_+ \\ & T_{-1/2}\bar{T}_{1/2} & \end{array}$$

Note:

- Confirmed for fundamental two-pt. functions

$\left\langle \left(V_k^{(+)} \bar{V}_{-k}^{(-)} \right) \left(V_\ell^{(+)} \bar{V}_{-\ell}^{(-)} \right) \right\rangle$, $\left\langle \left(V_k^{(+)} \bar{V}_{-k}^{(-)} \right) T_{-1/2} \bar{T}_{1/2} \right\rangle$ $(k, \ell$: $^\forall$half odd integer),

$\left\langle \left(T_{-k'} \bar{V}_{-k'}^{(-)} \right) \left(V_{k'}^{(+)} \bar{T}_{k'} \right) \right\rangle$ $(k' = 1,3)$

SUSY DW MM reproduces several kinds of & infinitely many fundamental two-pt. functions at tree level

→ strong evidence that SUSY DW MM would provide nonperturbative formulation of 2D type IIA superstring theory

but so far, only at tree level ($g_s = 0 \iff N \to \infty$)

→ assuming this, discuss higher genus & nonperturbative aspect using SUSY DWMM

Spontaneous SUSY breaking

recall: DSL: $N^2(\mu^2-2)^3 = s^3 = 1/g_s^2$

nonperturbative: MM with double scaling limit $g_s \neq 0$

SUSY breaking order parameter:

$$\left\langle \frac{1}{N}\mathrm{tr}\,B \right\rangle_{MM} = \left\langle \int d^2z\;T_{-1/2}(z)\bar{T}_{1/2}(\bar{z}) \right\rangle_{IIA}$$

$$iQ(\mathrm{tr}\,\bar{\psi}) = i\bar{Q}(\mathrm{tr}\,\psi) = \mathrm{tr}\,B$$
$$Q_+\left(V_{1/2}^{(+)}\bar{T}_{1/2}\right) = \bar{Q}_-\left(T_{-1/2}\bar{V}_{-1/2}^{(-)}\right) = T_{-1/2}T_{1/2}$$

orthogonal polynomial:

$$N^{\frac{4}{3}}\left\langle \frac{1}{N}\mathrm{tr}\,B\right\rangle = 0 - i\,\frac{1}{8\pi}\,g_s^{\frac{2}{3}}\,e^{-\frac{4}{3g_s}} + \mathcal{O}\left(e^{-\frac{8}{3g_s}}\right)$$

$$\rightarrow F = \frac{1}{16\pi}\,g_s\,e^{-\frac{4}{3g_s}} + \mathcal{O}\left(e^{-\frac{8}{3g_s}}\right)$$

Endres-T.K.-Sugino-Suzuki ’13

<u>Note</u>

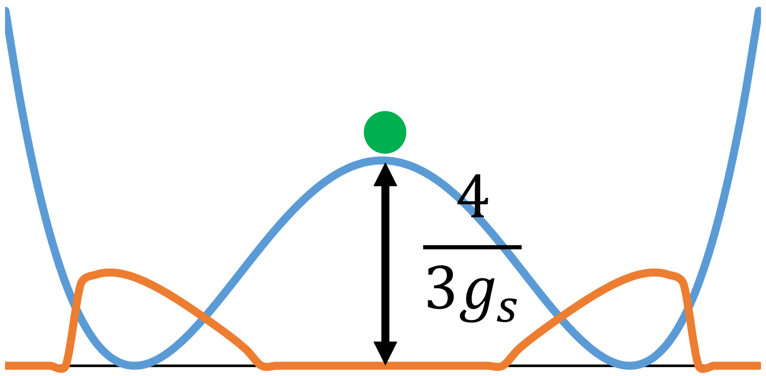
- in all order in g_s – exp., order parameter = 0, i.e. SUSY is preserved, but gets broken spontaneously & nonperturbatively

first example!

- due to MM instanton

would be D-brane in IIA Hanada et. al. ’04

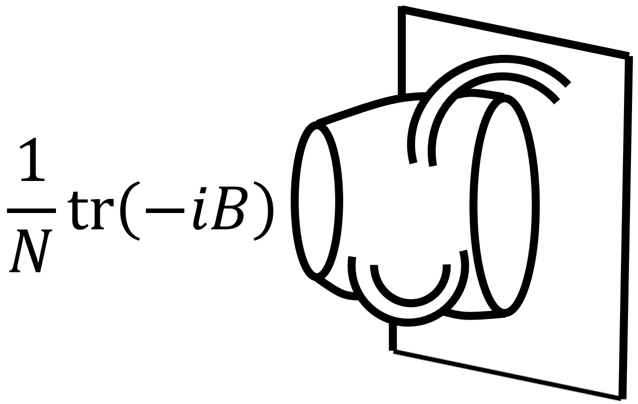
suggesting interesting possibility



- full order result in one-inst. sector:

$$N^{\frac{4}{3}}\left\langle\frac{1}{N}\mathrm{tr}(-iB)\right\rangle\Bigg|_{1-\mathrm{inst.}} = Ai'\left(g_s^{-2/3}\right)^2 - g_s^{-\frac{2}{3}}Ai\left(g_s^{-2/3}\right)^2$$

i.e. arbitrary # of holes & handles



Complete trans-series

Nishigaki-Sugino '14

$$F(s) = \int_s^\infty (t-s) q(t)^2 dt$$

$\boxed{s^3 = 1/g_s^2}$

$q(t)$ satisfies Painlevé II $q''(s) = sq(s) + 2q(s)^3$

with b.c.: $q(s) \to Ai(s)$ $(s \to \infty)$: unique!! Hastings-McLeod '80

i.e. $\partial_s^2 F(s) = q(s)^2$

trans-series is completely fixed and explicit:

$$F_{1-\mathrm{inst}}(s) \sim \frac{e^{-\frac{4}{3}s^{3/2}}}{16\pi s^{3/2}}\left(1 - \frac{35}{24 s^{3/2}} + \frac{3745}{1152 s^3} - \frac{805805}{82944 s^{9/2}} + \cdots\right)$$

$$F_{2-\mathrm{inst}}(s) \sim \frac{1}{2}\left(\frac{e^{-\frac{4}{3}s^{3/2}}}{16\pi s^{3/2}}\right)^2\left(1 - \frac{35}{12 s^{3/2}} + \frac{619}{72 s^3} - \frac{592117}{20736 s^{9/2}} + \cdots\right), \cdots$$

Comments

- trans-series of SUSY order parameter is in general completely zero or at most alternating $\rightarrow$ no ambiguity, as it should be!

  $f(t)_{0B} = q(-t)_{IIA}$: on/off the Stokes line $\rightarrow$ positive/alternating
- In general, it seems difficult to find nonperturbative object (saddle pt.) which triggers SUSY breaking only from such a well-behaved perturbative series

  cf. Dunne's & Fujimori's talk
- nonperturbative object can be probed by other “bad” operators whose perturbative series grows (non-Borel summable)

  $\rightarrow$ identify nonperturbative object, but difficult to compute
- in our case, Nicolai mapping is available to overcome these issues

Non-SUSY operators

$$S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right]$$

- $B \sim -i(\phi^2 - \mu^2)$: Q & $\bar{Q}$ – exact
- Nicolai mapping: $M = \phi^2 - \mu^2 \rightarrow S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iBM\right]$: GMM, $M \sim B$
- basic observable: $\frac{1}{N}\mathrm{tr}\,\phi^k$

even k: SUSY, $\frac{1}{N}\mathrm{tr}\,M^{k/2} \rightarrow$ order parameter as $\frac{1}{N}\mathrm{tr}\,B^{k/2}$, perturbatively zero.

odd k: non-SUSY, "$\frac{1}{N}\mathrm{tr}\,M^{k/2}$" $\rightarrow$ eigenvalue integration possible in GMM

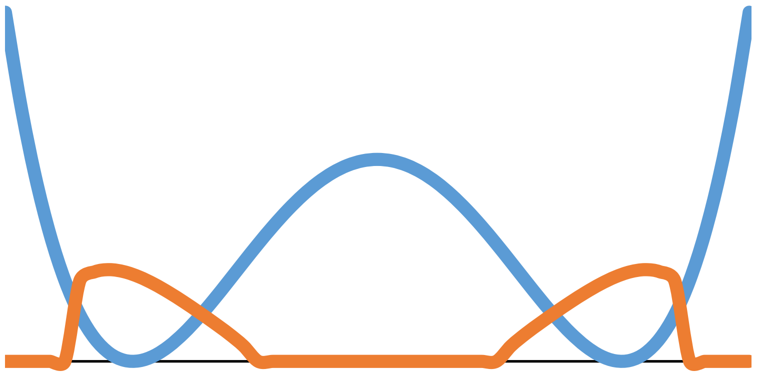
Eigenvalue dynamics

$$U\phi U^\dagger = \mathrm{diag}(\lambda_1,\cdots,\lambda_N)$$

$$Z = \int \prod_i (2\lambda_i d\lambda_i) \prod_{i>j} (\lambda_i - \lambda_j)^2 \prod_{i>j} (\lambda_i + \lambda_j)^2 e^{-\frac{N}{2}\sum_i (\lambda_i^2 - \mu^2)^2}$$

N particles w/ log repulsive force from λ_j & $-\lambda_j$ in DW pot.

filling fraction (ν_+, ν_-) sector:



$$Z = \sum_{\nu_+ N = 0}^{N} {}_N C_{\nu_+ N}\, Z^{(\nu_+,\nu_-)}$$

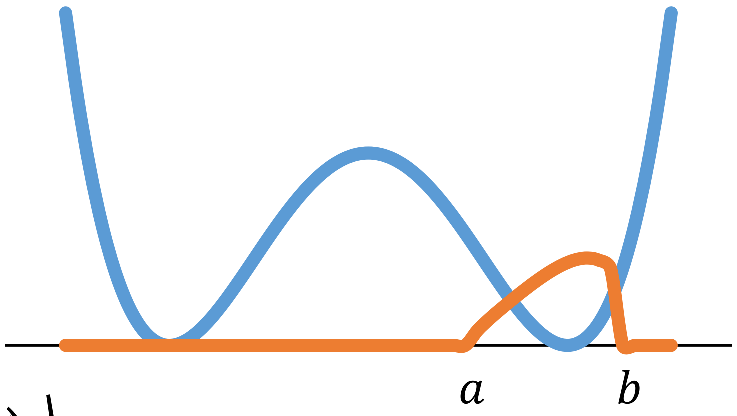
$$Z^{(\nu_+,\nu_-)} = \left(\prod_{i=1}^{\nu_+ N} \int_{\mathbb{R}_+} 2\lambda_i d\lambda_i + \prod_{i=\nu_+ N+1}^{N} \int_{\mathbb{R}_-} 2\lambda_i d\lambda_i \right) (\cdots)$$

n-pt. func. of odd ops. in (ν_+, ν_-) sector $\propto (\nu_+ - \nu_-)^n \to (1,0)$ sector

Non-SUSY corr. func. via SUSY ones

corr. func. of non-SUSY ops. ← corr. func. of SUSY ops.

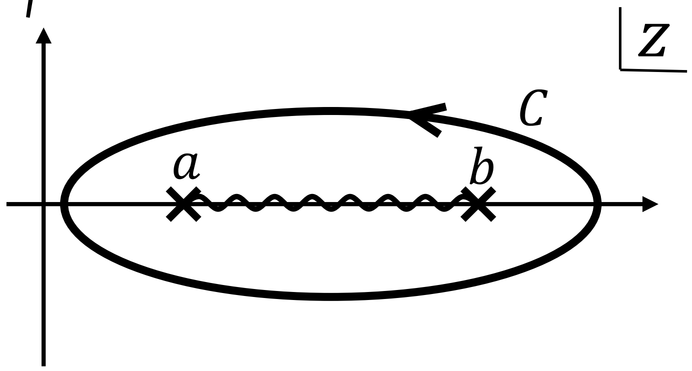
ϕ^2-resolvent: $R_2(z^2) \equiv \left\langle \frac{1}{N} \mathrm{tr} \frac{1}{z^2 - \phi^2} \right\rangle = \left\langle \frac{1}{N} \sum_i \frac{1}{z^2 - \lambda_i^2} \right\rangle$



singularity on $z \in \mathbb{C}$: cut $[a, b]$ $(a^2 = \mu^2 - 2, b^2 = \mu^2 + 2)$

$$\frac{1}{\pi i} \oint_C dz\, z f(z)\, R_2(z^2) = \frac{1}{2\pi i} \oint_C dz\, f(z) \left\langle \frac{1}{N} \sum_i \left(\frac{1}{z - \lambda_i} + \cancel{\frac{1}{z + \lambda_i}} \right) \right\rangle$$

$$= \left\langle \frac{1}{N} \sum_i f(\lambda_i) \right\rangle = \left\langle \frac{1}{N} \mathrm{tr}\, f(\phi) \right\rangle$$: arbitrary one-pt. function



key: Nicolai mapping $R_2(z^2) = \left\langle \frac{1}{N} \mathrm{tr} \frac{1}{z^2 - \mu^2 - M} \right\rangle_{GMM}$

One-point functions at arbitrary genus

$$N^{\frac{2}{3}(k+2)}\left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k+1}\right\rangle = \frac{\Gamma\left(k+\frac{3}{2}\right)}{2\pi^{\frac{3}{2}}}$$

$$\boxed{s^{-3}=g_s^2}$$

$$\times\left[\sum_{h=0}^{\left[\frac{1}{3}(k+2)\right]}\left(-\frac{1}{12}\right)^h \frac{1}{h!\,(k+2-3h)!}s^{k+2-3h}\ln s\right.$$

$$\left.+(-1)^{k+1}\sum_{h=\left[\frac{1}{3}(k+2)\right]+1}^{\infty}\left(\frac{1}{12}\right)^h\frac{(3h-k-3)!}{h!}s^{k+2-3h}\right]$$

lower genus contribution:
log singularity
alternating

higher genus contribution:
no log singularity
non-Borel summable

- double scaling limit works!!
- higher genus: positive power series $\sum_h c_h g_s^{2h}$ with $c_h \sim (2h)!\,(4/3)^{-2h}$: stringy

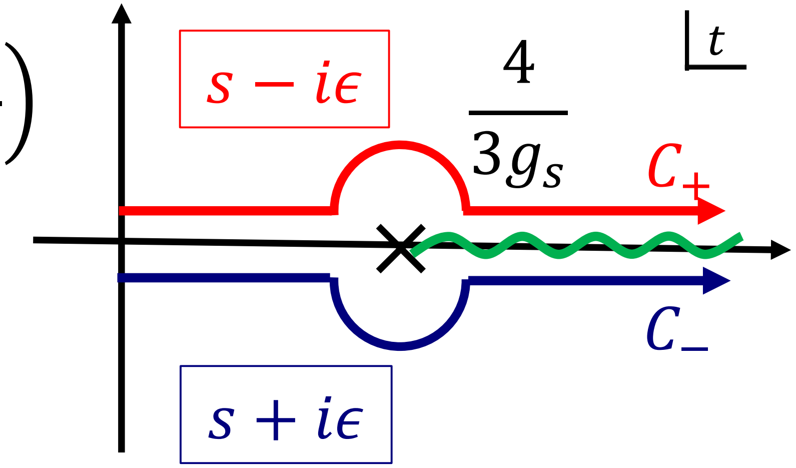
Resurgence: 0-instanton sector

$$N^{\frac{2}{3}(k+2)}\left\langle \frac{1}{N}\mathrm{tr}\,\phi^{2k+1}\right\rangle = \frac{\Gamma\left(k+\frac{3}{2}\right)}{2\pi^{\frac{3}{2}}}(-1)^{k+1}\sum_{h=\left[\frac{1}{3}(k+2)\right]+1}^{\infty}\frac{1}{h!}\left(\frac{1}{12}\right)^{h}\Gamma(3h-k-2)\,s^{k+2-3h}$$

+finite

insert $1=\frac{1}{\Gamma(2h+1)}\int_0^\infty dt\, e^{-t}t^{2h} \to \int_0^\infty dt\left(1-\frac{t^2}{t_0^2}\right)^{k+\frac{5}{2}}e^{-t}$ $\quad t_0=\frac{4}{3g_s}$

$(s-i\epsilon$ case$)-(s+i\epsilon$ case$)=(\mathrm{const.})\,g_s^{\frac{k}{3}+\frac{2}{3}}K_{k+3}\left(\frac{4}{3g_s}\right)$



$$= i(-1)^{k+1}\frac{\Gamma\left(k+\frac{3}{2}\right)}{2^{k+\frac{7}{2}}\pi}\, g_s^{\frac{k}{3}+\frac{7}{6}}\, e^{-\frac{4}{3g_s}}+\cdots$$

pure imaginary, D-brane like

1-instanton sector: instanton calculation

- eigenvalue distribution

$$\rho_2(x^2) \equiv \left\langle \frac{1}{N} \mathrm{tr}\delta(x^2-\phi^2) \right\rangle^{(1,0)} \to \left\langle \frac{1}{N} \mathrm{tr} f(\phi) \right\rangle^{(1,0)} = \int_0^\infty dx\, \rho_2(x) 2x f(x)$$

$$\rho_2(x^2) \equiv \left\langle \frac{1}{N} \mathrm{tr}\delta(\mu^2 - x^2 - M) \right\rangle_{GMM} = \rho_G(\mu^2 - x^2)$$

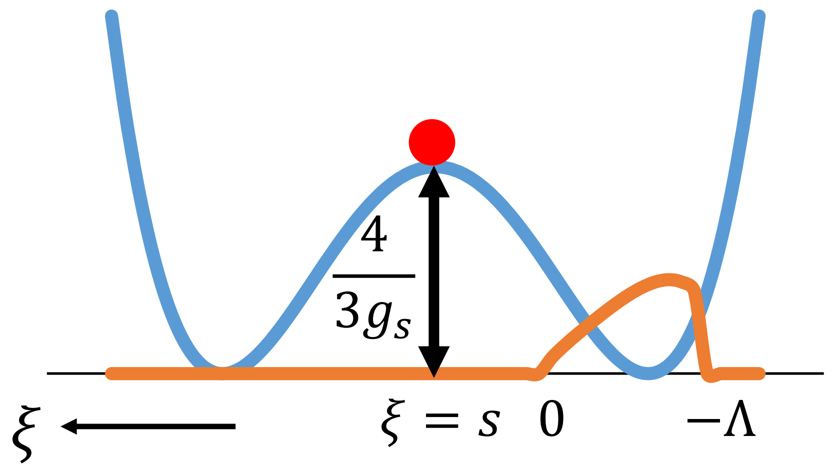
$$\left(x = 2 + N^{-\frac{2}{3}}\xi\right)$$

RMT: DSL=soft edge scaling limit $N^{\frac{1}{3}}\rho_G(x) \to K_{Ai}(\xi,\xi) = Ai'(\xi)^2 - \xi Ai(\xi)^2$

$$N^{\frac{2}{3}(k+2)} \left\langle \frac{1}{N} \mathrm{tr}\phi^{2k+1} \right\rangle = \int_{-\Lambda}^{0} d\xi K_{Ai}(\xi,\xi) \underbrace{(s-\xi)^{k+\frac{1}{2}}}_{\text{from operator}}$$

$$+ \boxed{\int_0^s d\xi K_{Ai}(\xi,\xi)(s-\xi)^{k+\frac{1}{2}}} + \cdots$$

one-instanton contribution



saddle pt.: $\xi_* = s + \frac{1}{2}\left(k+\frac{1}{2}\right)s^{-\frac{1}{2}} + \cdots$:outside $[0,s]$ $\boxed{s = g_s^{-2/3} \gg 1}$

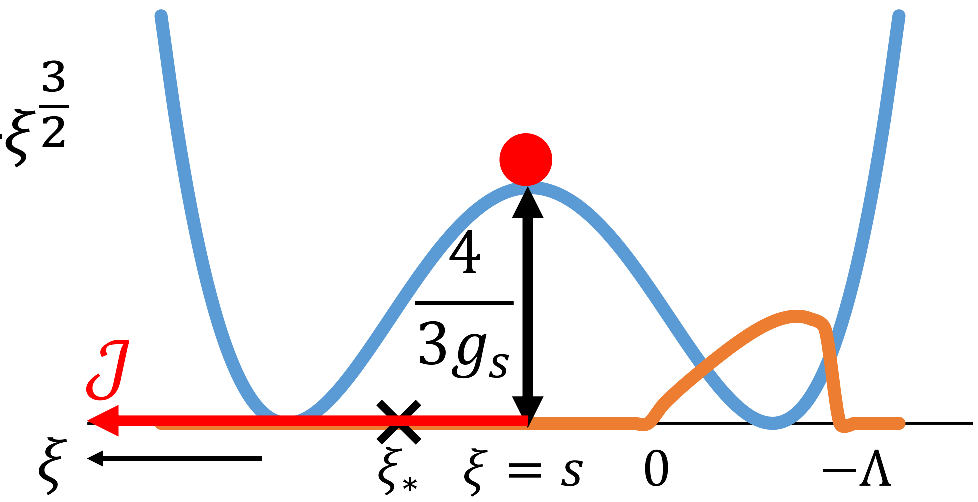
$$\rightarrow \left(k+\frac{1}{2}\right)\ln(s-\xi_*) = \left(k+\frac{1}{2}\right)\ln\left(-\frac{1}{2}\left(k+\frac{1}{2}\right)s^{-\frac{1}{2}}\right) \ni \pm\left(k+\frac{1}{2}\right)\pi i$$

operator value @saddle causes ambiguity

around $\xi \sim \xi_* \sim s \gg 1$ is relevant

$$K_{Ai}(\xi,\xi) = Ai'(\xi)^2 - \xi Ai(\xi)^2 \sim \frac{1}{8\pi\xi}e^{-\frac{4}{3}\xi^{\frac{3}{2}}}$$

$$\int_{\mathcal{J}} d\xi \frac{1}{8\pi\xi}e^{-\frac{4}{3}\xi^{\frac{3}{2}}}(s-\xi)^{k+\frac{1}{2}}$$



$$= i(-1)^k \frac{1}{2^{k+3}\sqrt{\pi}}\left(k+\frac{1}{2}\right)^{k+1} e^{-\left(k+\frac{1}{2}\right)} s^{-\frac{k}{2}-\frac{7}{4}} e^{-\frac{4}{3}s^{\frac{3}{2}}}$$

Final Issue

0-instanton ambiguity + 1-instanton ambiguity:

$$i(-1)^{k+1}\frac{\Gamma\left(k+\frac{3}{2}\right)}{2^{k+\frac{7}{2}}\pi}s^{-\frac{k}{2}-\frac{7}{4}}e^{-\frac{4}{3}s^{\frac{3}{2}}}$$

$$-i(-1)^{k+1}\frac{1}{2^{k+3}\sqrt{\pi}}\left(k+\frac{1}{2}\right)^{k+1}e^{-\left(k+\frac{1}{2}\right)}s^{-\frac{k}{2}-\frac{7}{4}}e^{-\frac{4}{3}s^{\frac{3}{2}}}$$

$$\int_{\mathcal{J}}d\xi\,\frac{1}{8\pi\xi}e^{-\frac{4}{3}\xi^{\frac{3}{2}}}(s-\xi)^{k+\frac{1}{2}}$$

$$\sim e^{-V(\xi_*)}\int_{-\infty}^{\infty}d\tilde{\xi}\;e^{-\frac{4}{2k+1}e^{\pm 2\pi i}\tilde{\xi}^2}$$

cancel only if $k \gg 1$??

$\rightarrow$ HELP!!

Summary

- bosonic DWMM = type0B in 1D
  - Painlevé II → perturbative series: non-Borel summable
    → nonpert. ambiguity (universal)
  - nonperturbative saddle = isolated eigenvalue at top
    → ambiguity from $\mathcal{J}_\pm$ (universal)

cancel

analytic conti.

- SUSY DWMM = type IIA in 2D
  - SUSY operator: perturbatively $= 0$, but $\neq 0$ nonperturbatively
    Painlevé II → trans-series: alternating
  - non-SUSY operator: probing instanton, still calculable
    via Nicolai mapping & resolvent, cancellation incomplete?
- classification: meaning of cut (conti. spectrum)? When?