From Low-Energy QCD to Nuclear Physics
and
Energy Density Functionals

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PART I : Emergence of phases and structures in QCD

- From quarks to nuclei and dense baryonic matter
- Chiral symmetry and interface with nuclear physics
- Constraints from massive neutron stars

PART II : Chiral EFT and nuclear energy density functionals

- Nuclear chiral effective field theory update
- Skyrme phenomenology - state of the art
- Energy density functional from nuclear chiral dynamics

PART I

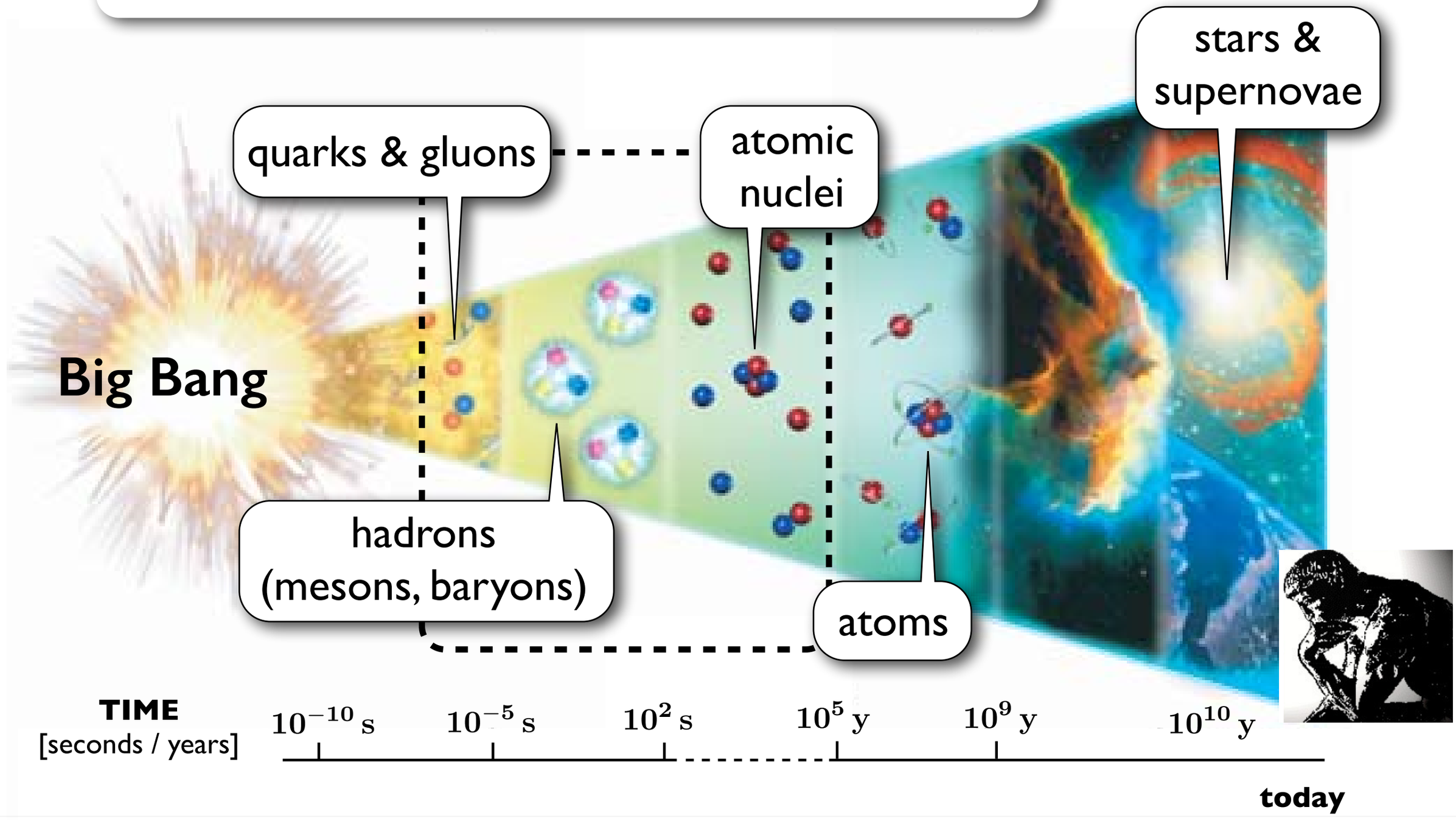
Overview:

PHASES of **QCD**
and interface with
Nuclear Physics

Keywords

- **Chiral symmetry : QCD interface with nuclear physics**
- **Nuclear chiral thermodynamics**
- **Dense baryonic matter and constraints from neutron stars**

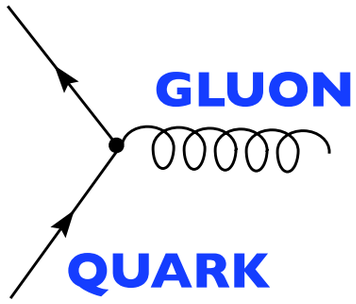
1. From **Quarks** and **Gluons** to **Nuclei** and **Neutron Stars**

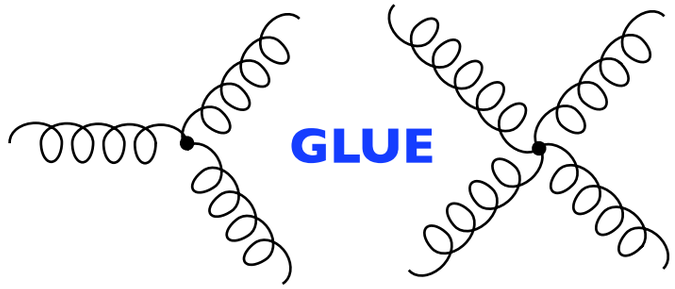


Framework:

QUANTUM CHROMO DYNAMICS

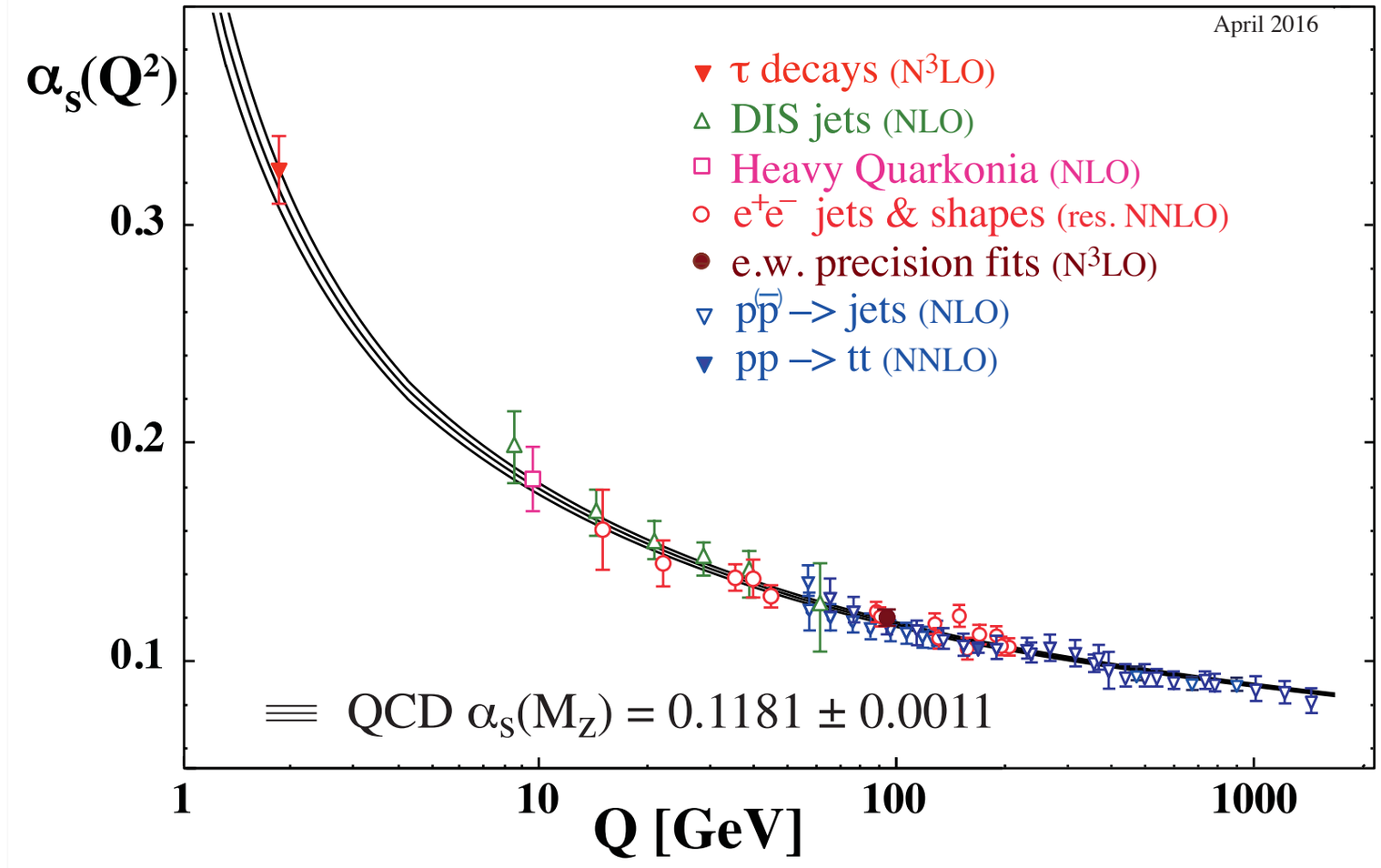
$$\mathcal{L}_{\mathbf{QCD}} = \bar{\psi}\left(i\gamma_\mu \mathcal{D}^\mu - \mathbf{m}\right)\psi - \frac{1}{4}\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}$$





Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

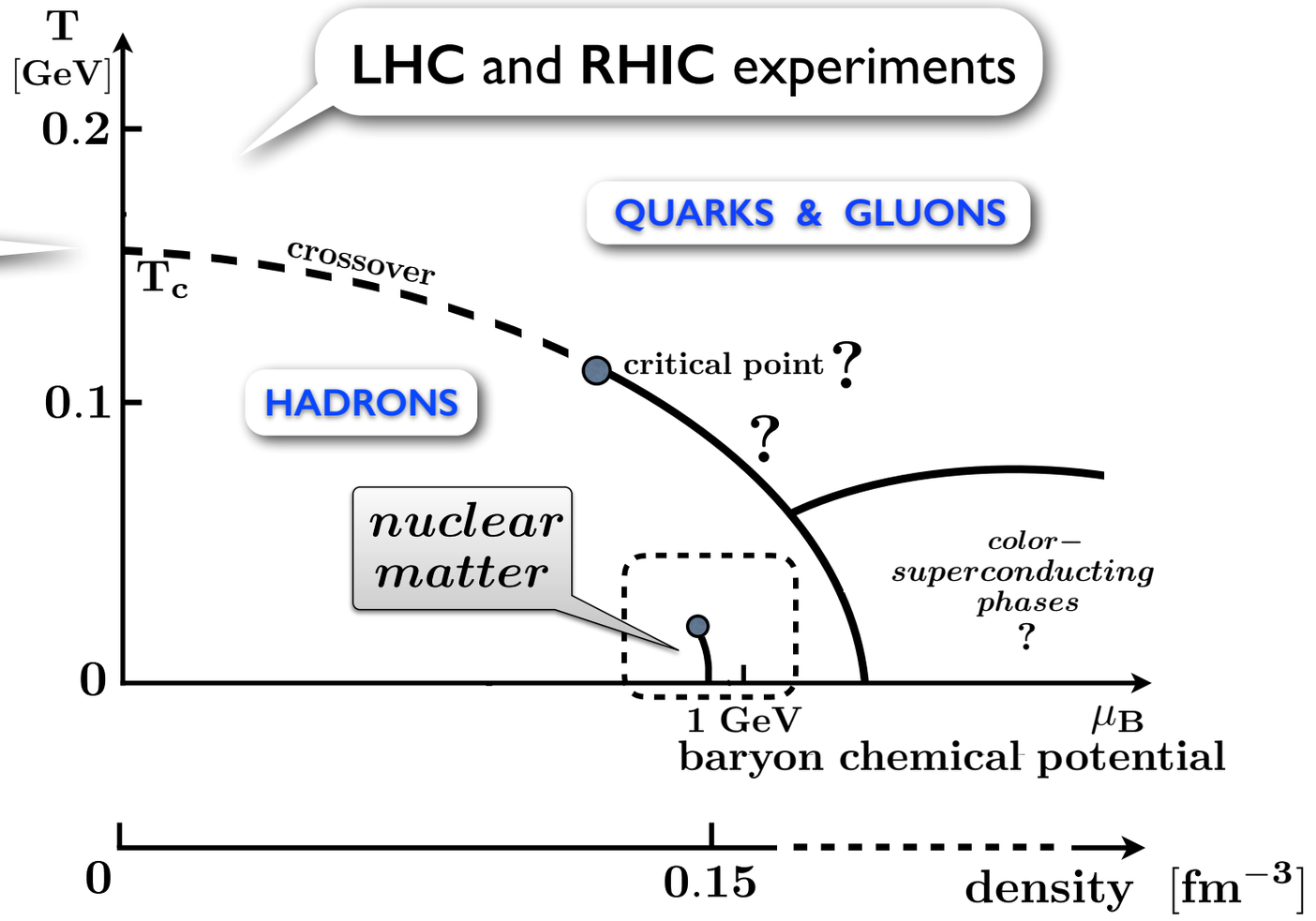
Particle Data Group — Chin. Phys. **C40** (2016) 100001



PHASES and STRUCTURES of QCD

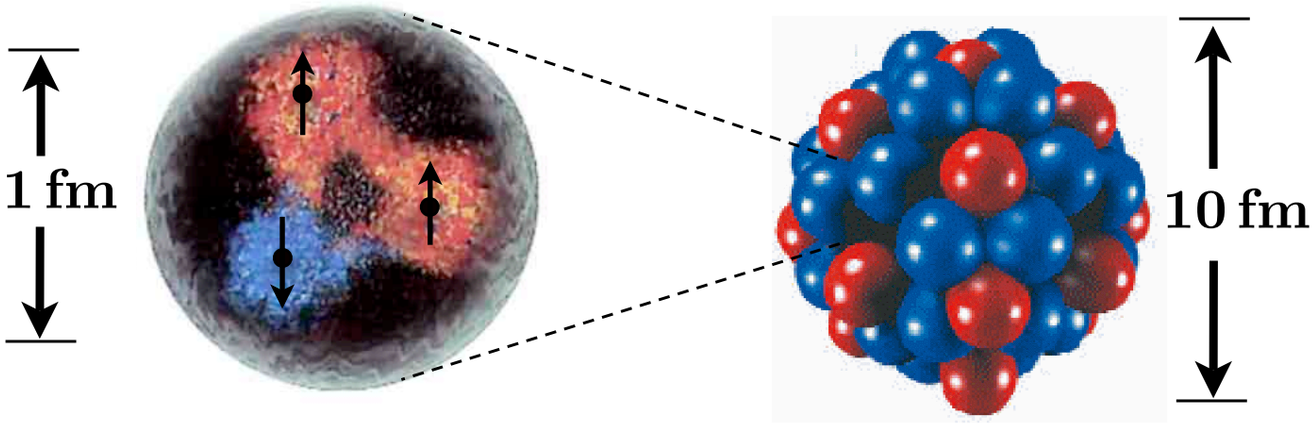
LHC and RHIC experiments

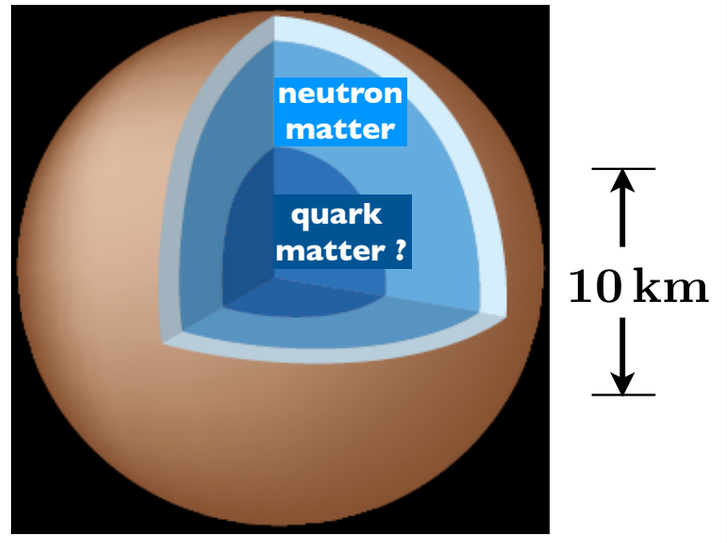
LATTICE QCD



nucleons & nuclei

neutron stars



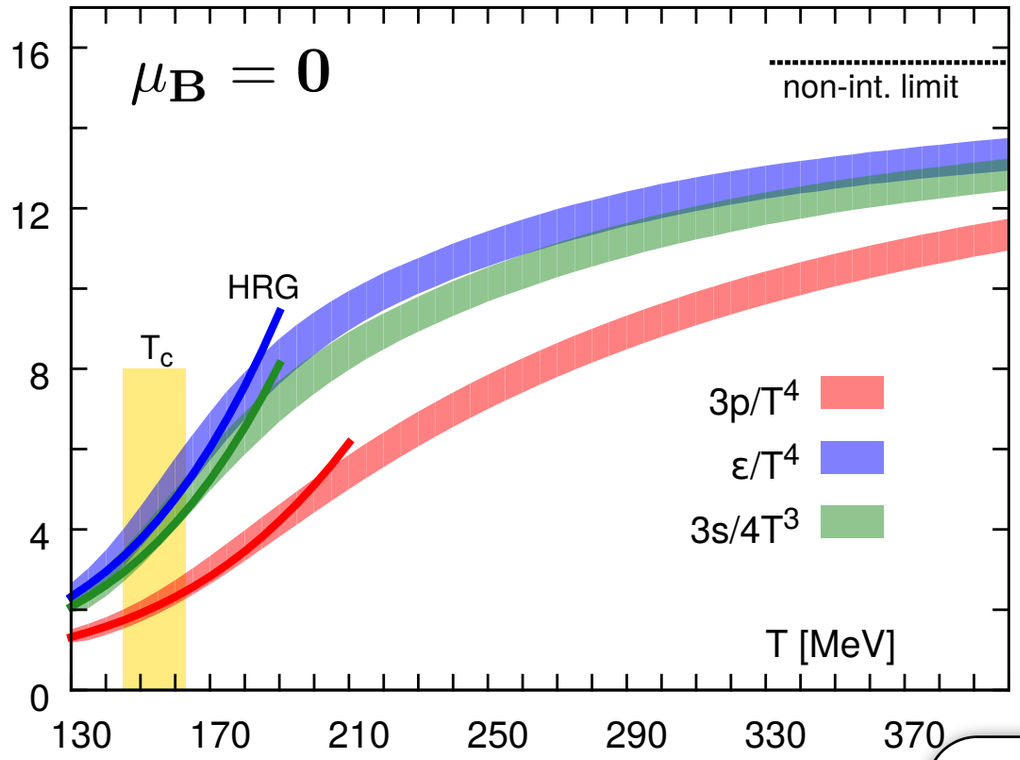


QCD THERMODYNAMICS on the LATTICE

Deconfinement transition

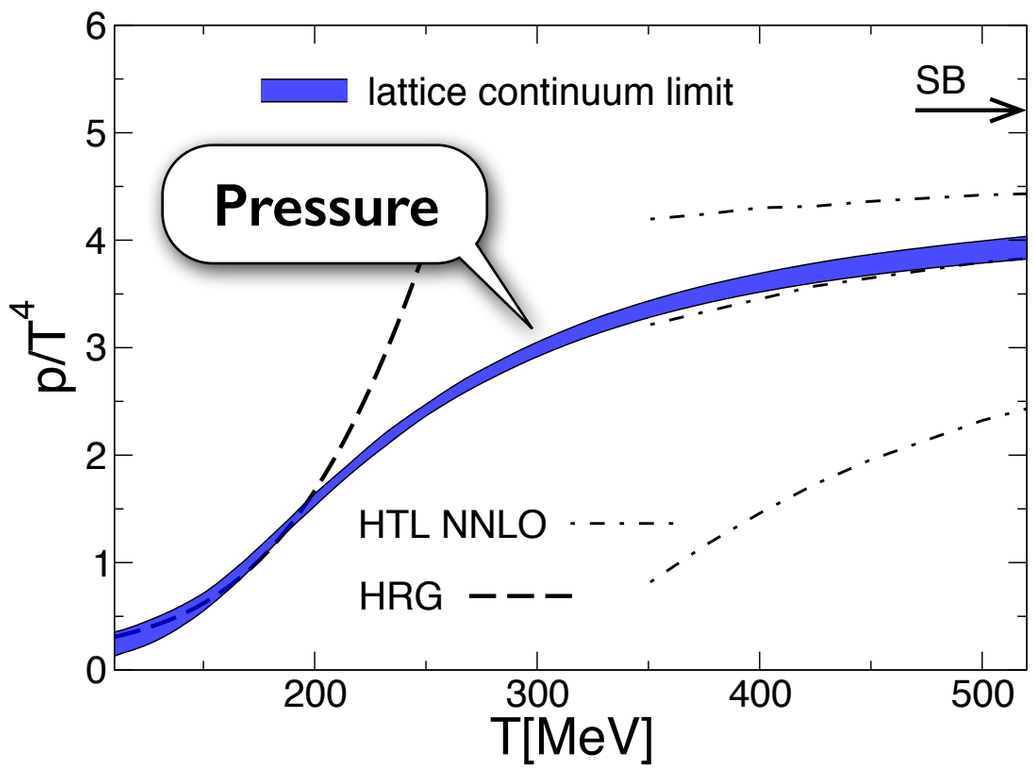
crossover
(not a phase transition)

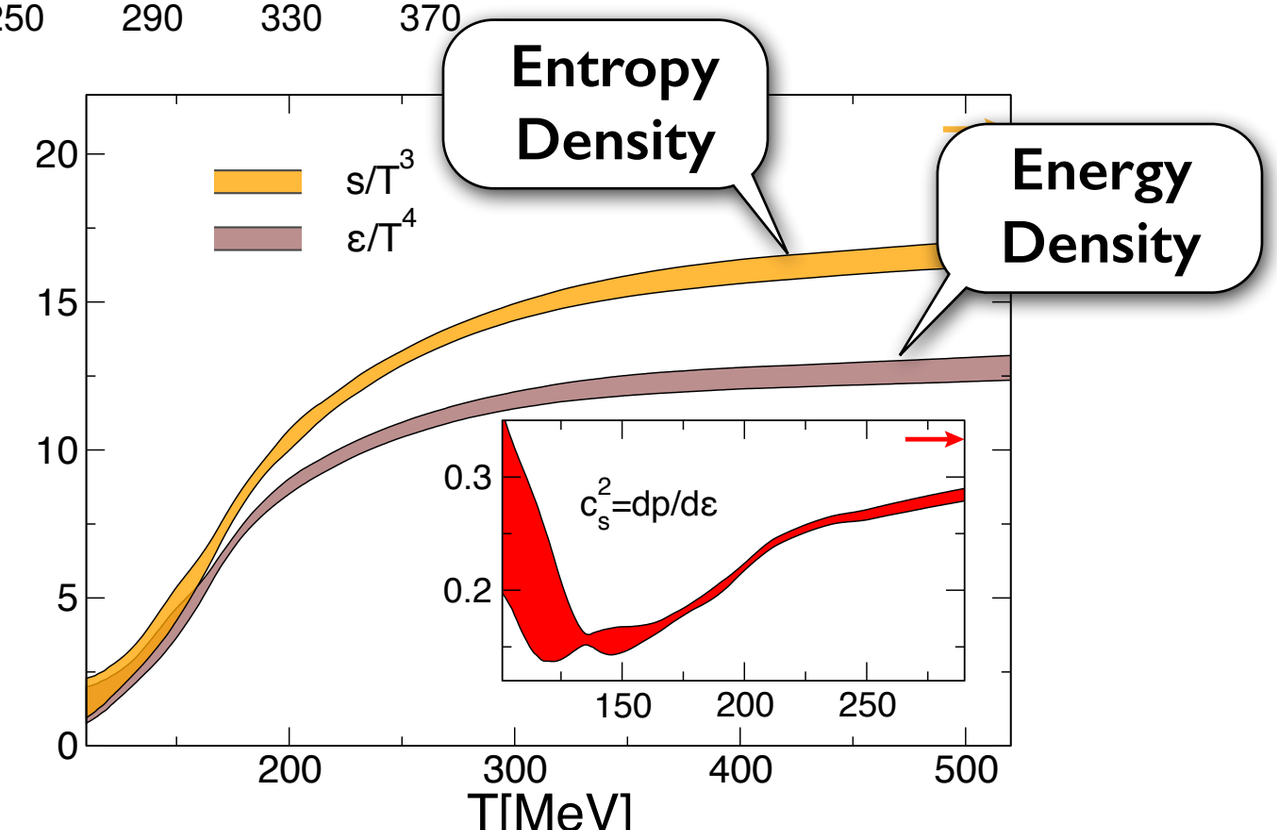
S. Borsanyi et al.
Phys. Lett. B 738 (2014) 187



A. Bazavov et al. (HotQCD collab.)
Phys. Rev. D90 (2014) 094503

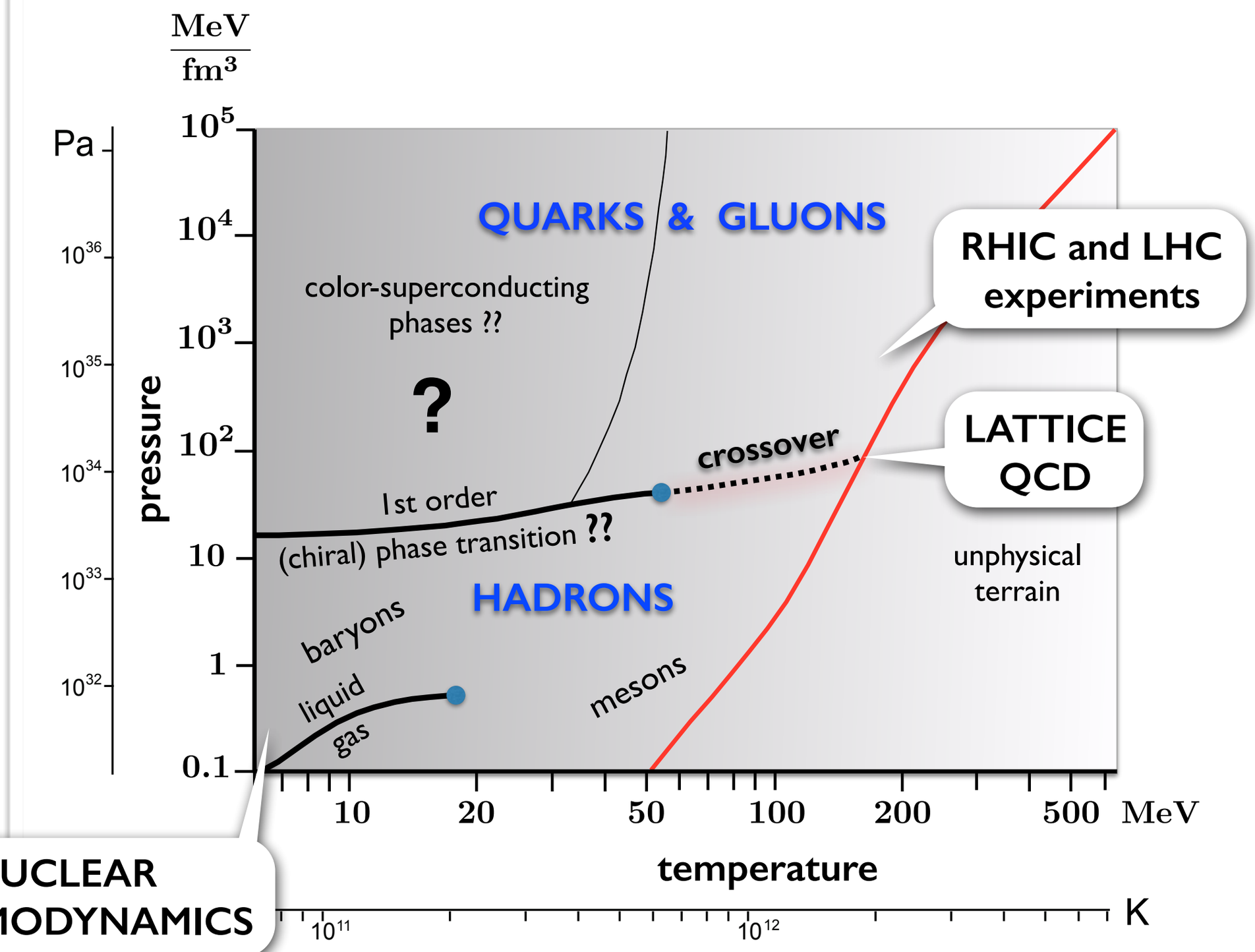
transition temperature
$T_c \simeq$
$150 - 160$ MeV



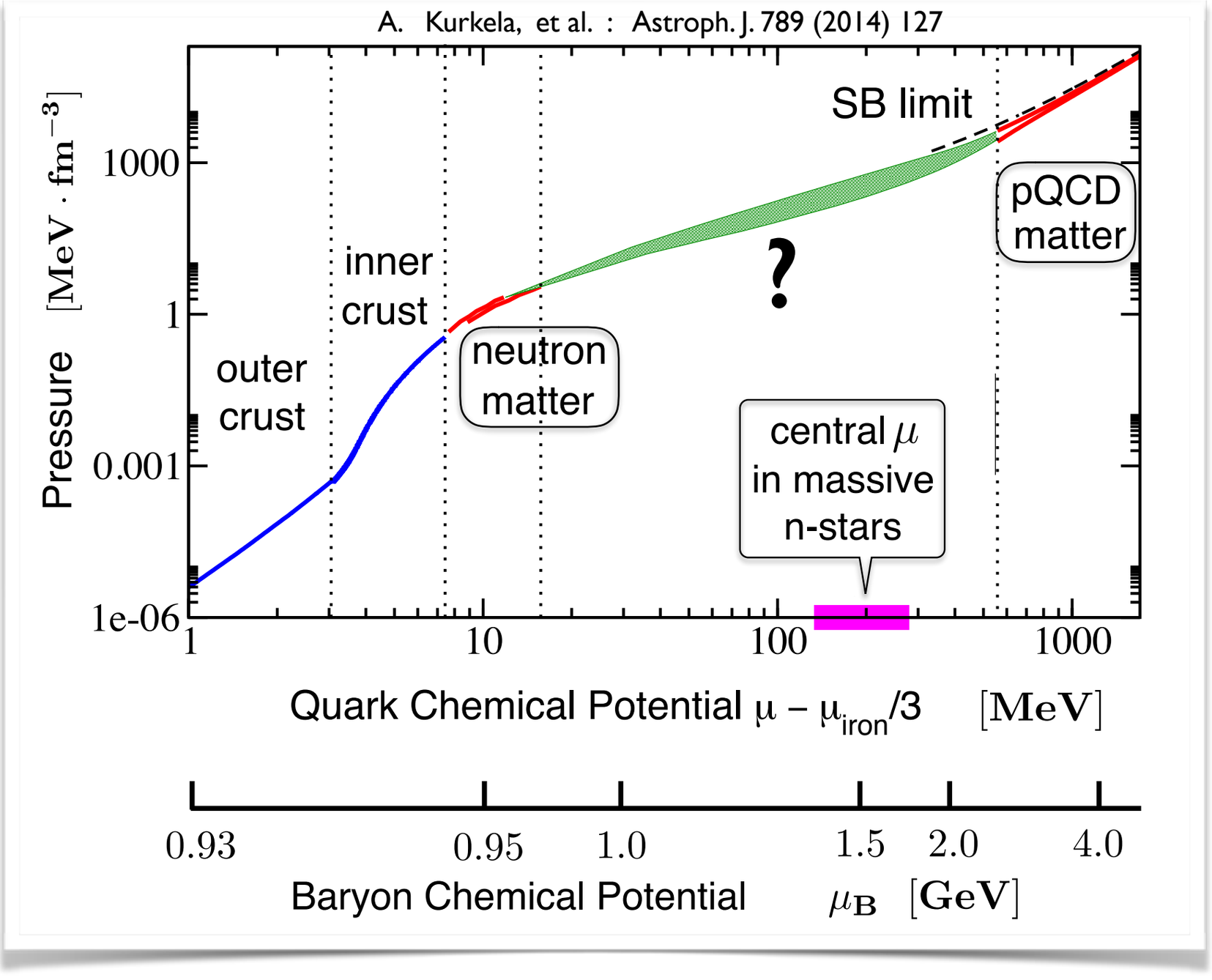


PHASES and **STRUCTURES** of **QCD**

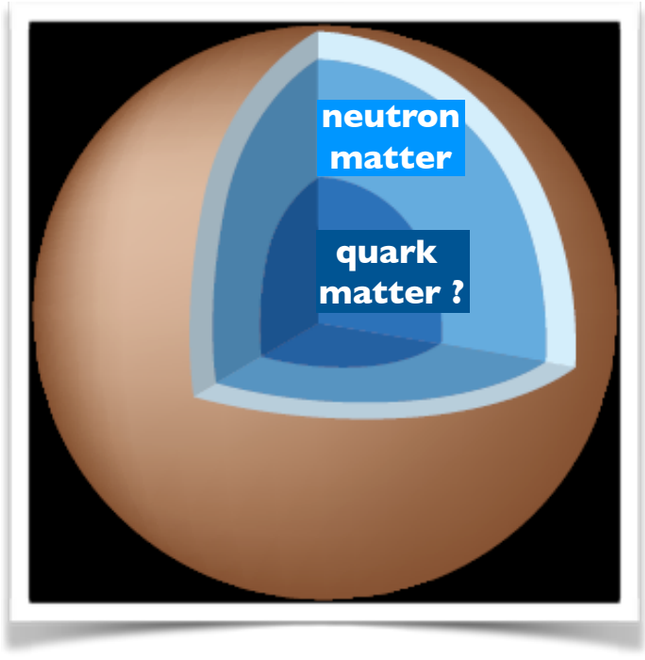
- facts and visions -



EQUATION of STATE
COLD and DENSE BARYONIC MATTER



NEUTRON STARS and beyond

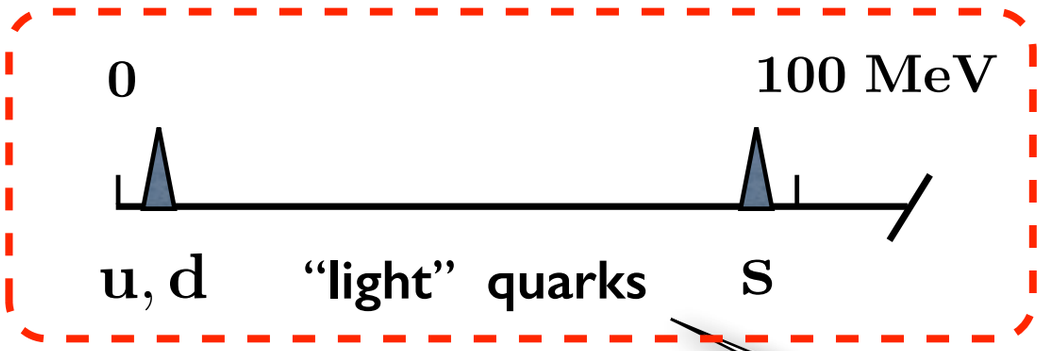


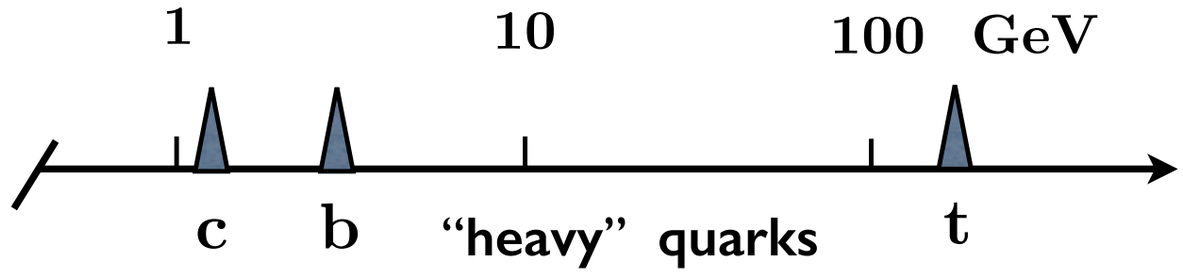
2.

The **Hadronic** Phase: Low-Energy QCD and Spontaneously Broken **Chiral Symmetry**

Hierarchy of **QUARK MASSES** in **QCD**

- Separation of Scales -





$$m_u = 2.2 \pm 0.5 \text{ MeV}$$

$$m_d = 4.7 \pm 0.5 \text{ MeV}$$

$$m_s = 96^{+8}_{-4} \text{ MeV}$$

$$(\mu \simeq 2 \text{ GeV})$$

PDG 2016

CHIRAL EFFECTIVE FIELD THEORY
of pions & nucleons

LOW-ENERGY QCD

Confinement
of quarks & gluons in hadrons

Chiral Symmetry
spontaneously broken (QCD dynamics)
explicitly broken (non-zero quark masses)

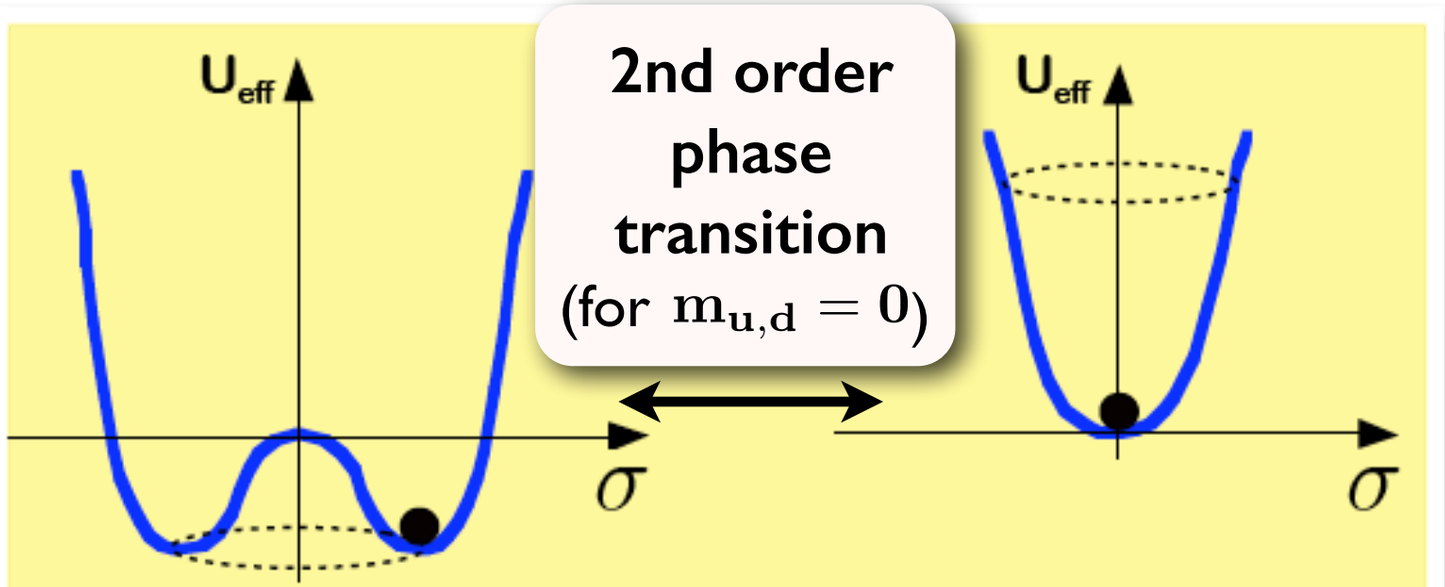
Low-Energy QCD : **CHIRAL SYMMETRY**

- **QCD** with (almost) **MASSLESS u-** and **d-QUARKS**

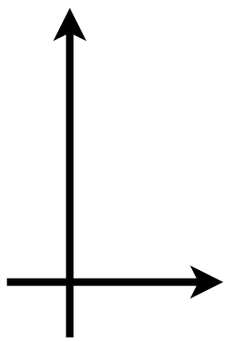
$$\mathbf{SU}(2)_R \times \mathbf{SU}(2)_L$$

$$\psi = (\mathbf{u}, \mathbf{d})^{\mathsf{T}}$$

- **Realizations of CHIRAL SYMMETRY:**



pseudoscalar
isovector
$$\pi^a \leftrightarrow \bar{\psi}\, i\gamma_5\, \tau^a \psi$$



scalar
isoscalar
$$\sigma \leftrightarrow \bar{\psi}\, \psi$$

invariant:
$$\sigma^2 + \boldsymbol{\pi}^2 = f_\pi^2$$

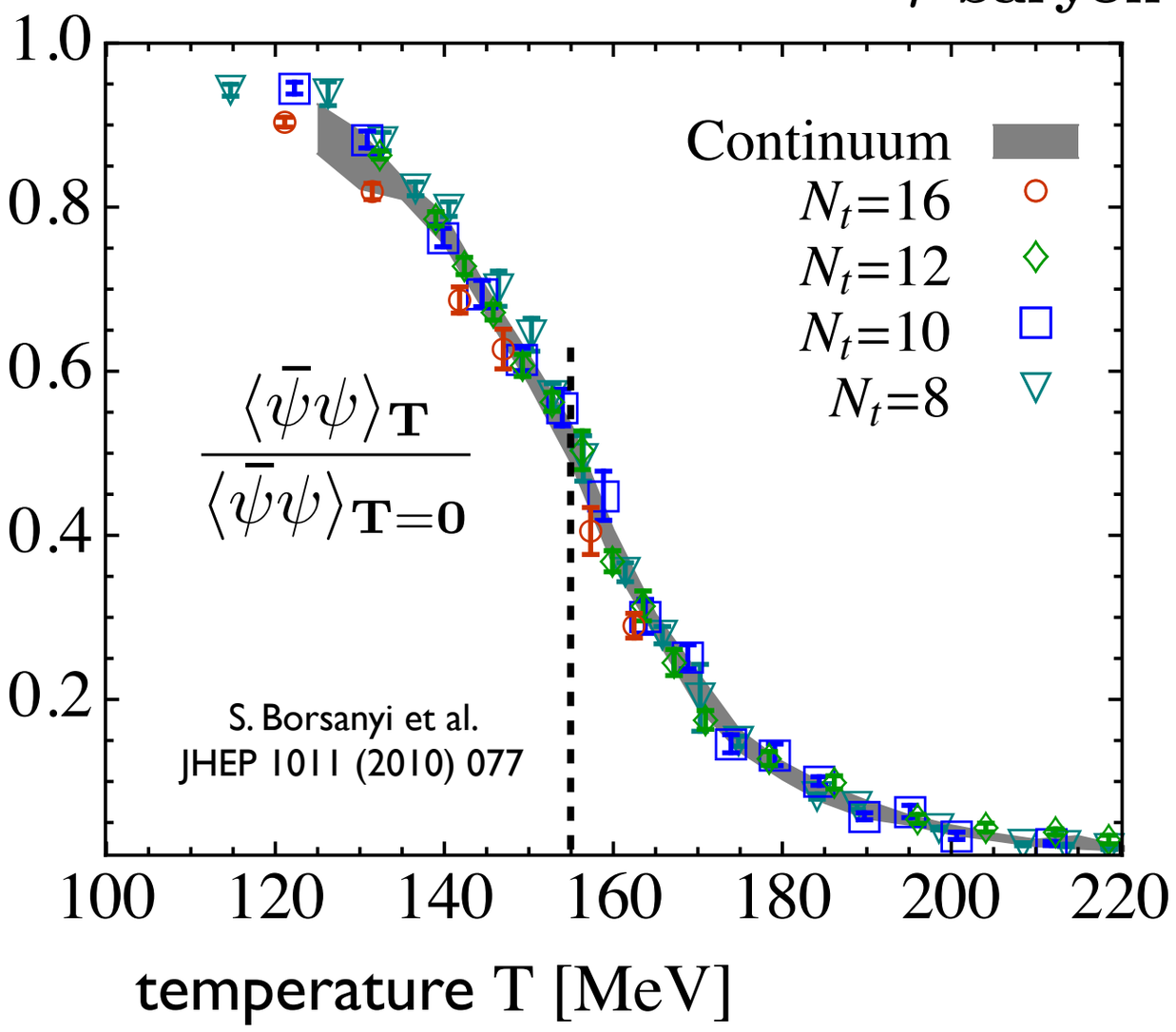
Nambu-Goldstone
$$\langle \bar{\psi}\psi \rangle \neq 0$$
at low energy / low temperature

chiral (quark) condensate

Wigner-Weyl
$$\langle \bar{\psi}\psi \rangle = 0$$
at high energy / high temperature

LATTICE QCD THERMODYNAMICS: **CHIRAL** and **DECONFINEMENT** TRANSITIONS

$$\mu_{\mathrm{baryon}} = 0$$



Quark Condensate

Order Parameter
of
spontaneously broken
Chiral Symmetry

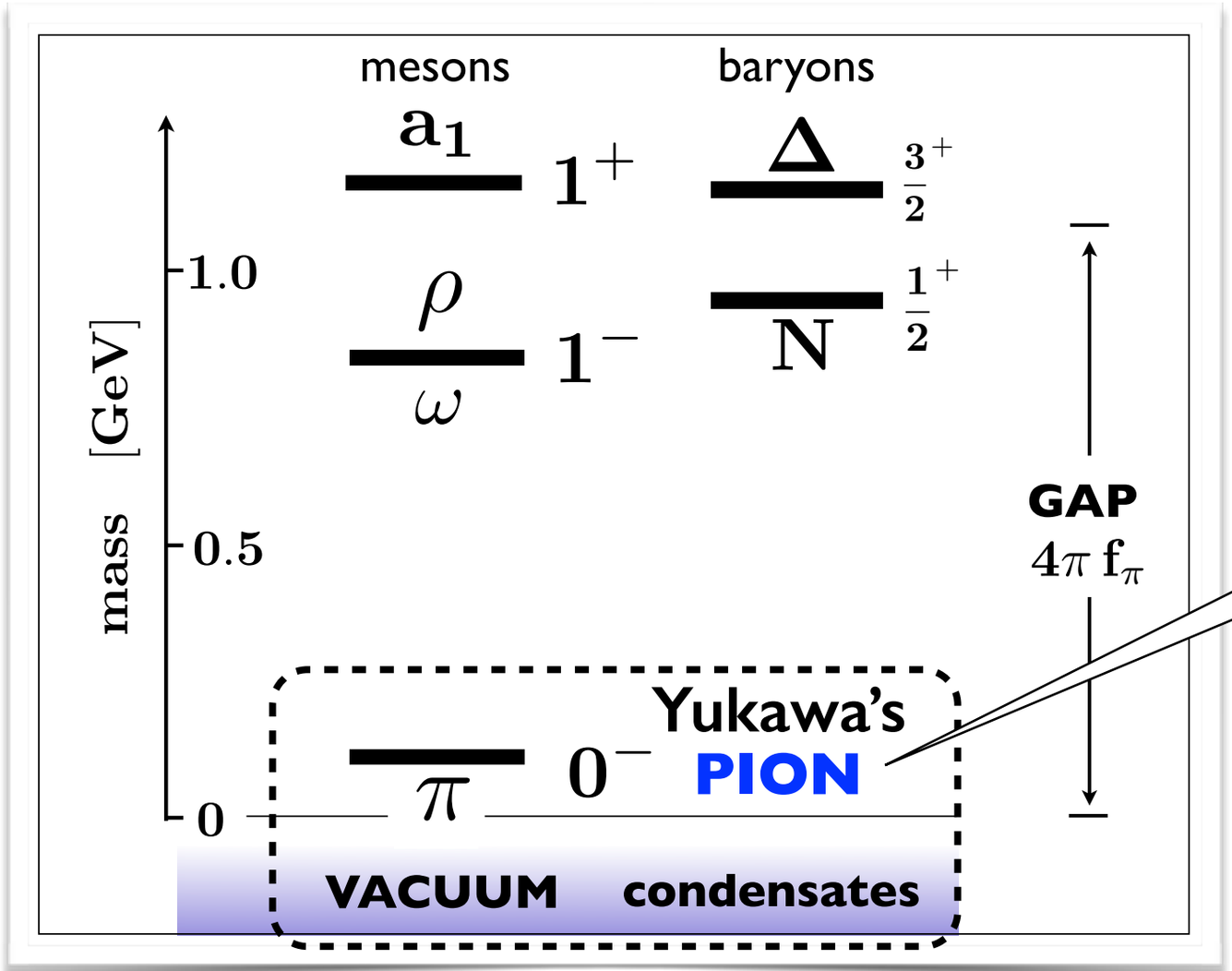
Crossover
transition temperature

$$T_c \simeq 155\,\mathrm{MeV}$$

CHIRAL and **DECONFINEMENT**
crossover transitions appear to be closely **connected**

Spontaneously Broken CHIRAL SYMMETRY

- Nambu-Goldstone Realization -



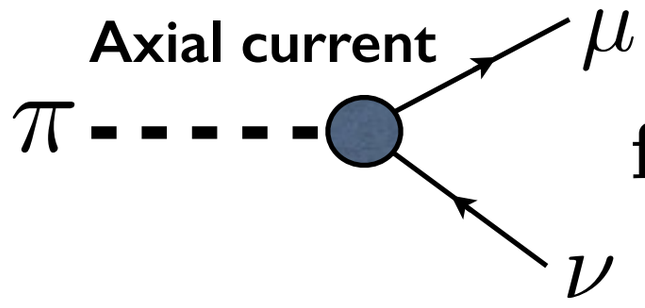
Triplet of NAMBU - GOLDSTONE BOSONS:

$\pi^+,\ \pi^0,\ \pi^-$

Characteristic Symmetry Breaking SCALE:

$4\pi\, f_\pi \sim 1\ \mathrm{GeV}$

- **PION DECAY CONSTANT**



$$f_\pi = 92.2 \pm 0.2\ \mathrm{MeV}$$

PIONS and **NUCLEI** in the context of **LOW-ENERGY QCD**

- **CONFINEMENT** of quarks and gluons in hadrons
- Spontaneously broken **CHIRAL SYMMETRY**
- **LOW-ENERGY QCD**:

  Effective **F**ield **T**heory of **weakly** interacting

  Nambu-Goldstone Bosons (PIONS)

  representing QCD at (energy and momentum) scales

  $$\mathbf{Q < 1\,GeV}$$

Recent reviews:
J.W. Holt, N. Kaiser, W.W.: Prog. Part. Nucl. Phys. 73 (2013) 35
J.W. Holt, M. Rho, W.W.: Physics Reports 621 (2016) 2
M. Drews, W.W.: Prog. Part. Nucl. Phys. 93 (2017) 69

CHIRAL EFFECTIVE FIELD THEORY

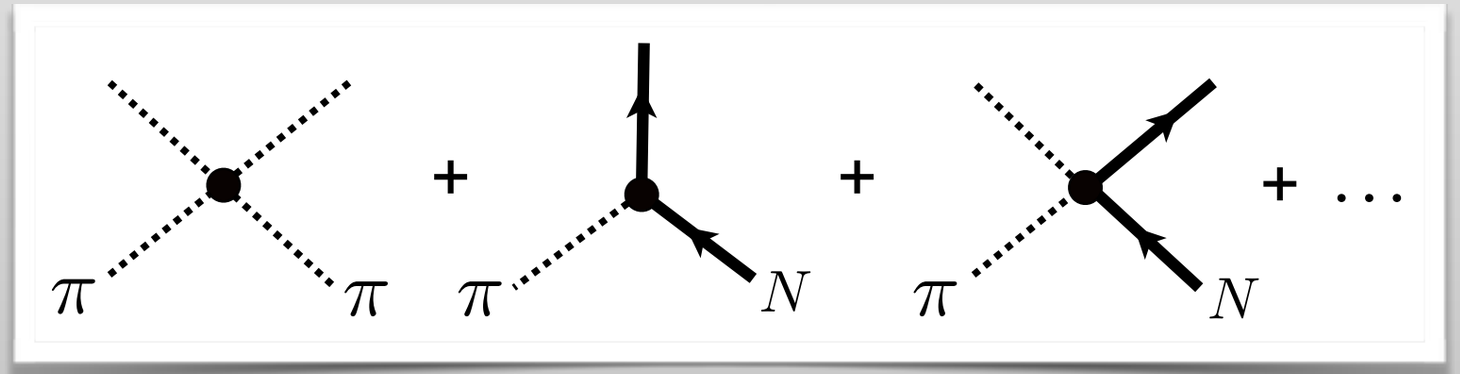
Interface of **QCD** and **Nuclear Physics**

- Interacting systems of
  PIONS (light / fast) and **NUCLEONS** (heavy / slow):

$$\mathcal{L}_{eff} = \mathcal{L}_{\pi}(U, \partial U) + \mathcal{L}_{N}(\Psi_N, U, ...)$$

$$U(x) = exp[i\tau_a \pi_a(x)/f_\pi]$$

- Construction of Effective Lagrangian: **Symmetries**



short distance dynamics: contact terms

- **Nambu-Goldstone Pions** : derivative couplings $\propto \partial^\mu \pi(x)$

3.

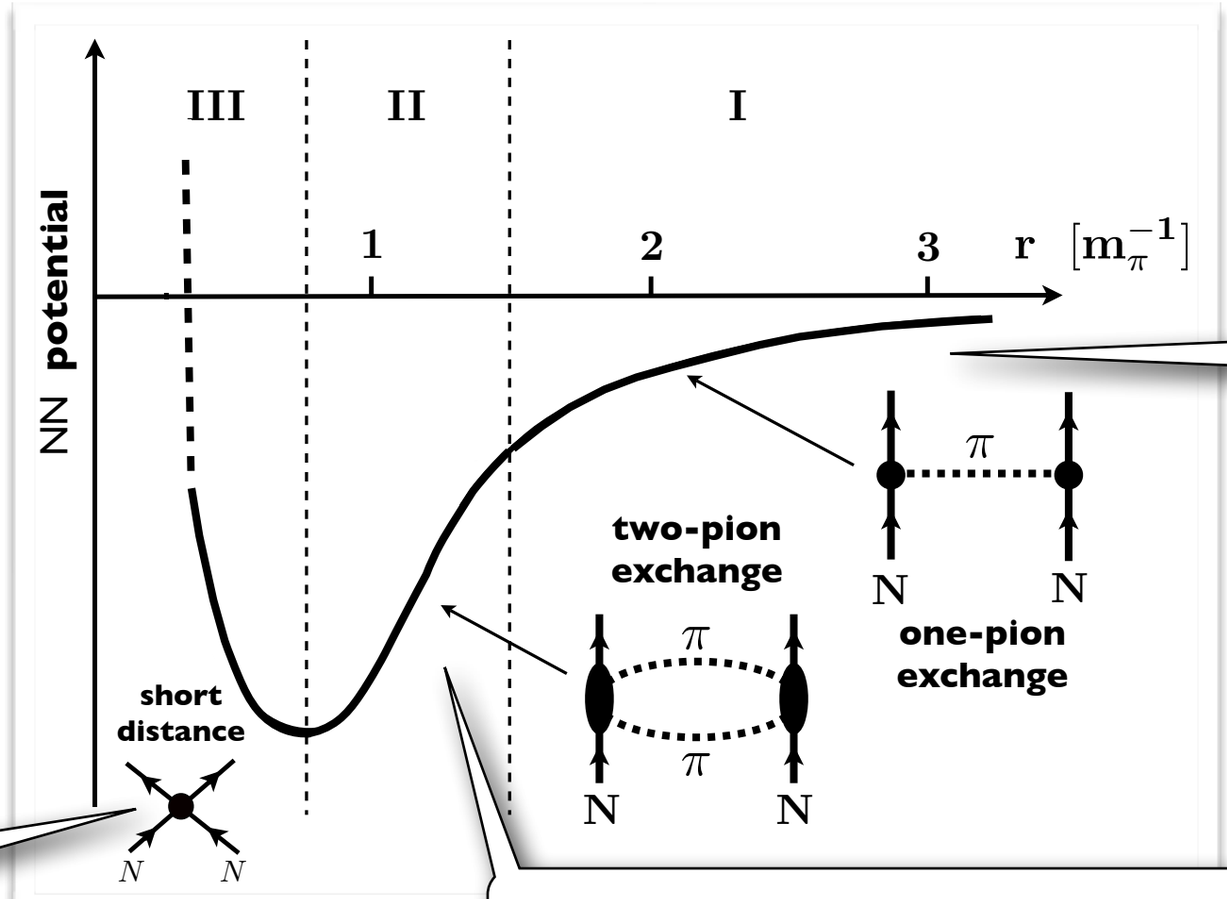
NUCLEAR FORCES

from

CHIRAL EFFECTIVE FIELD THEORY

NUCLEAR FORCES

Inward-bound Hierarchy of SCALES



I
long-range
(Yukawa 1935)

Early history:
M. Taketani,
S. Nakamura,
M. Sasaki;
Prog. Theor. Phys.
6 (1951) 581

III
contact terms
set of coupling parameters

II
intermediate range:
two-pion exchange

- Today's approach: **CHIRAL EFFECTIVE FIELD THEORY**

NUCLEAR FORCES from LATTICE QCD

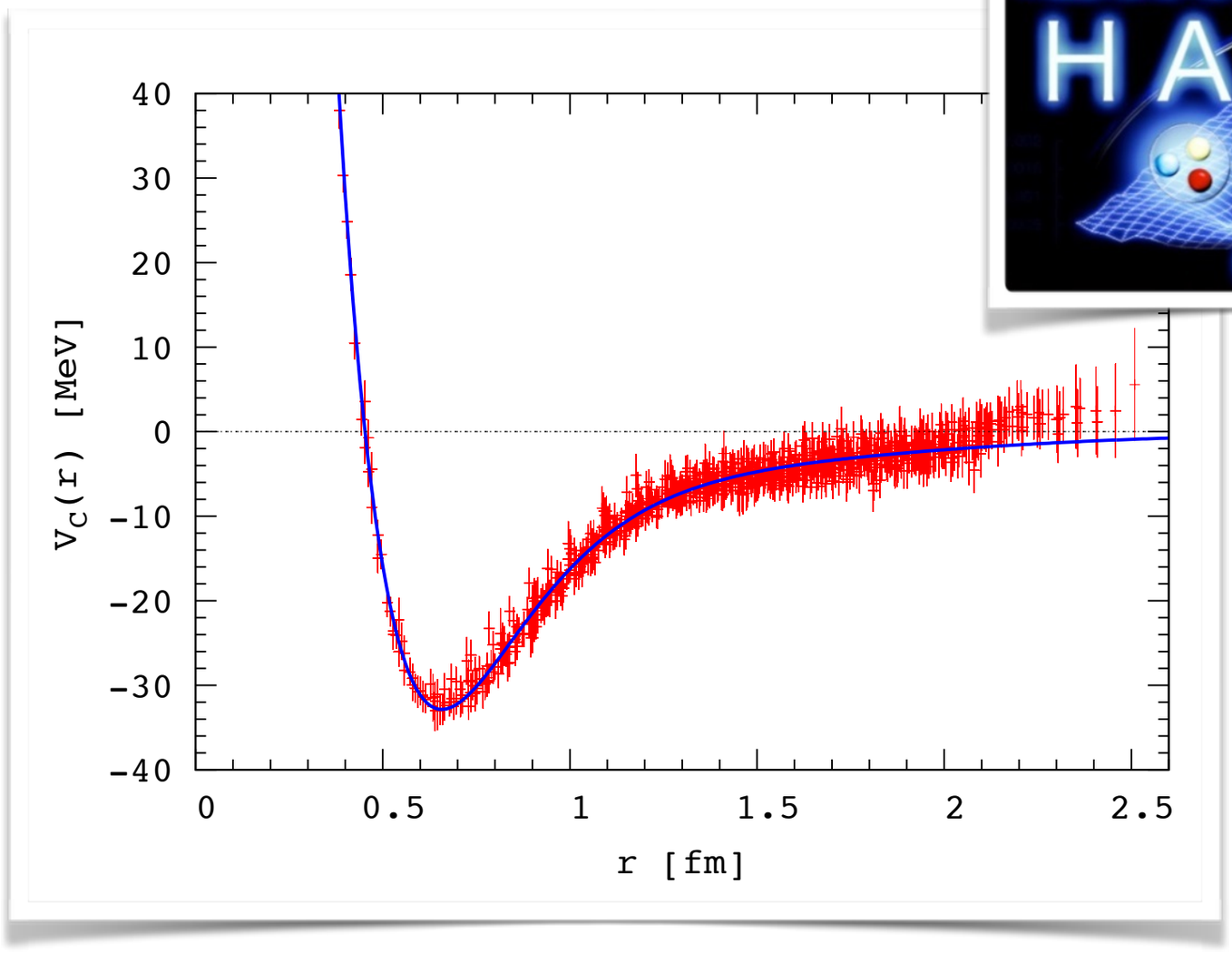
NN Central Potential (S = 0, I = 1)
deduced from LQCD two-nucleon (6-quark) correlation function

previously:
unphysically large
u- and d-quark
masses

… but :
stable 0.5 Fermi
repulsive core

S. Aoki, T. Hatsuda, N. Ishii
Prog. Theor. Phys. 123 (2010) 89

S. Aoki
Eur. Phys. J. A49 (2013) 81

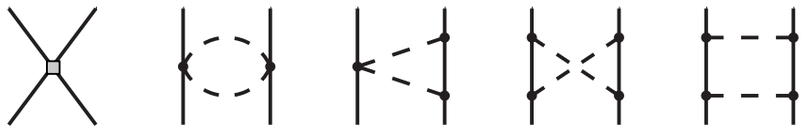
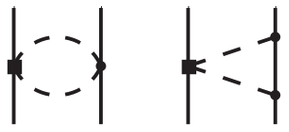
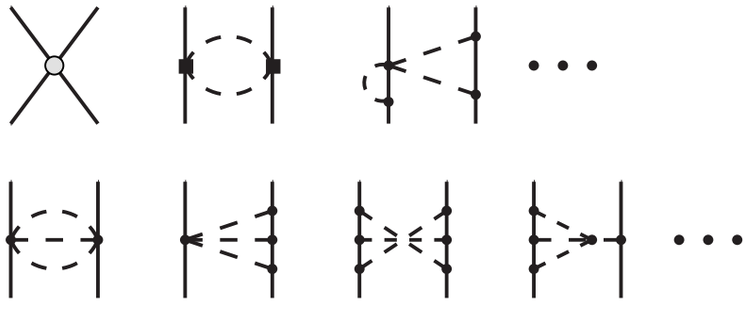


Hadrons to Atomic nuclei
HAL
from Lattice QCD

now:
physical
quark
masses

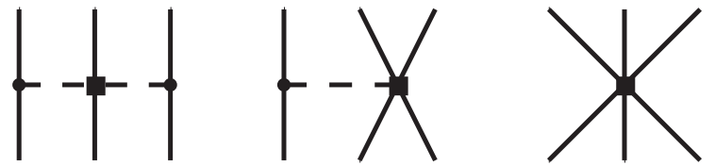
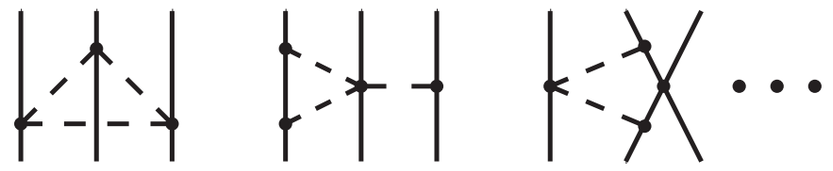
NUCLEAR INTERACTIONS from CHIRAL EFFECTIVE FIELD THEORY

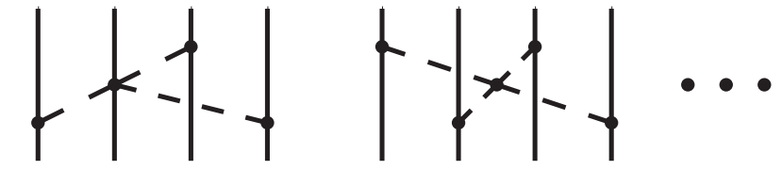
Weinberg Bedaque & van Kolck Bernard, Epelbaum, Kaiser, Meißner; …

	NN interaction
LO	
NLO	
N^2LO	
N^3LO	

- Systematically organized hierarchy in powers of $\frac{Q}{\Lambda}$

(Q: momentum, energy, pion mass)

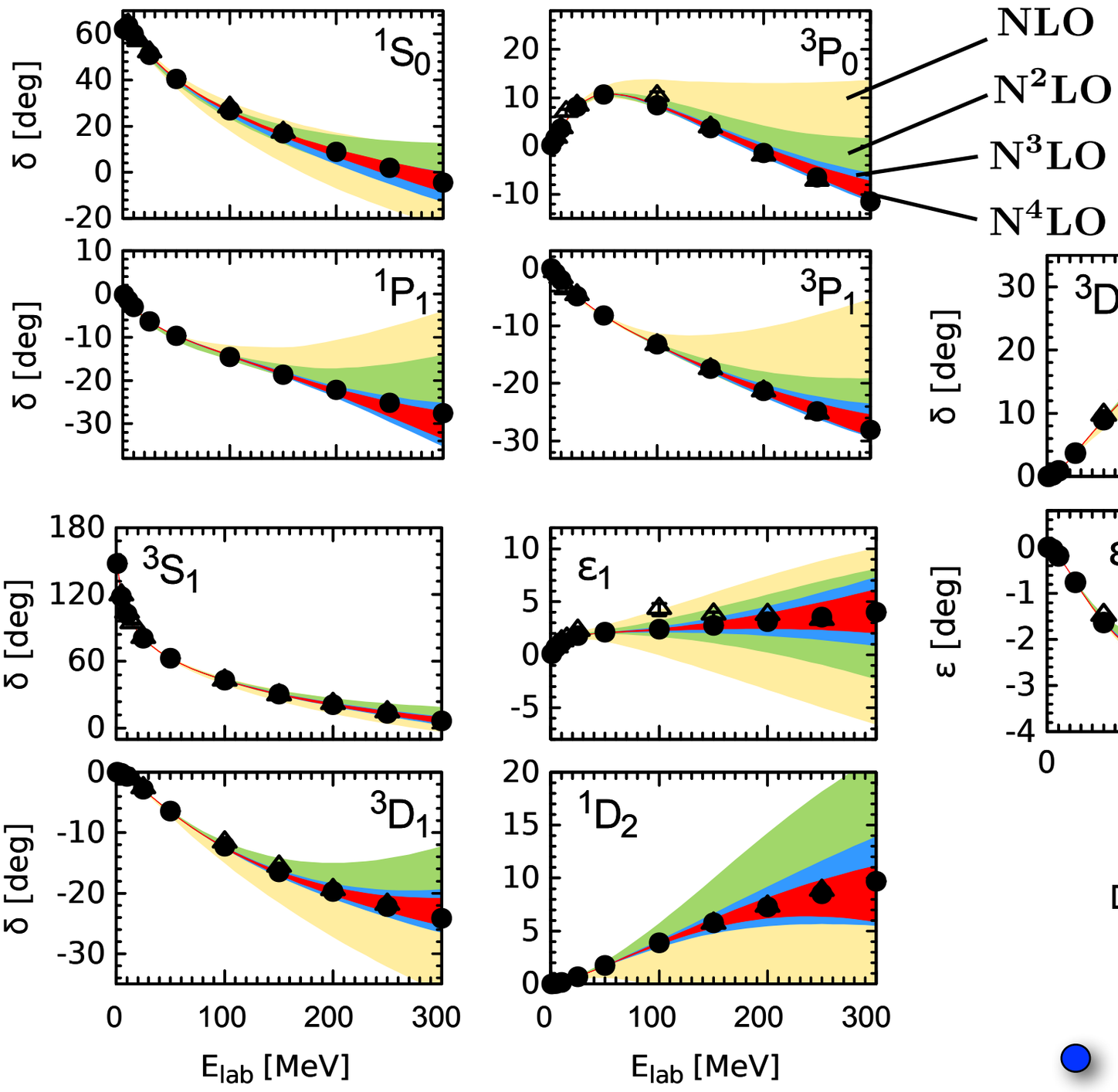
3N force	
N^2LO	
N^3LO	

4N force	
N^3LO	

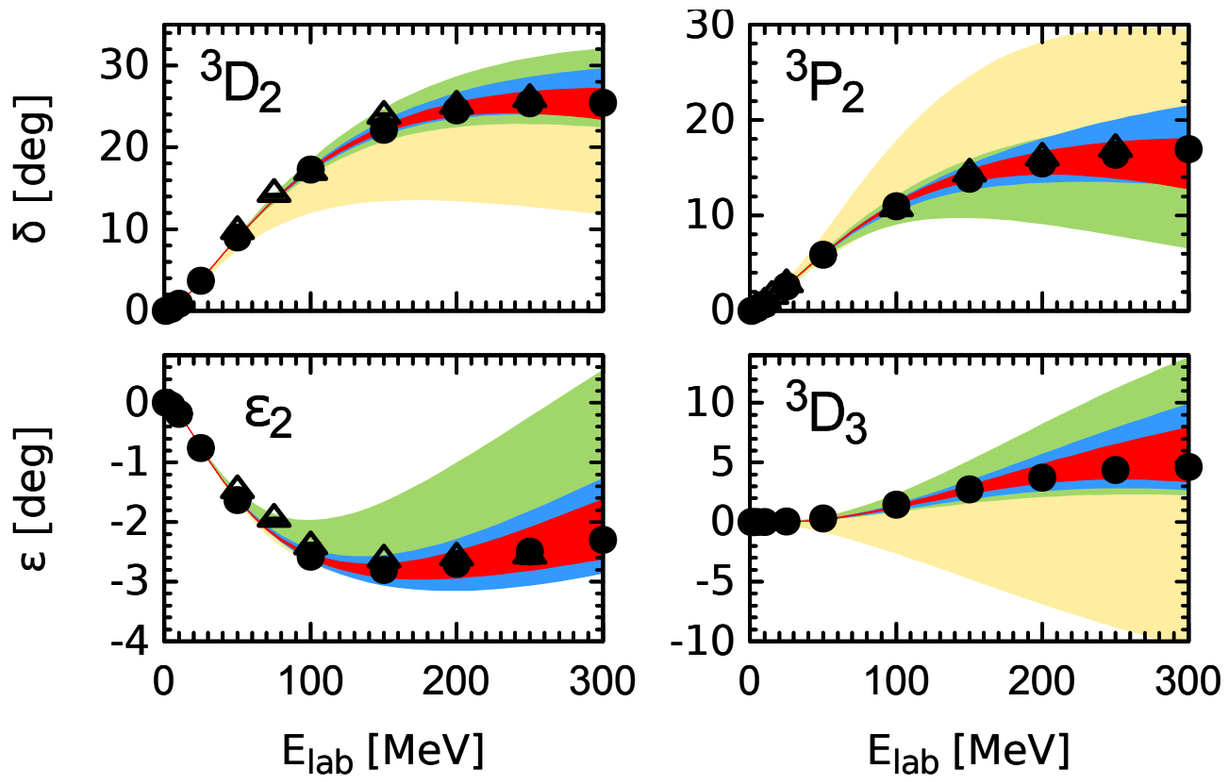
- State-of-the-art: N^4LO plus convergence tests at N^5LO

NUCLEON - NUCLEON SCATTERING

Phase Shifts



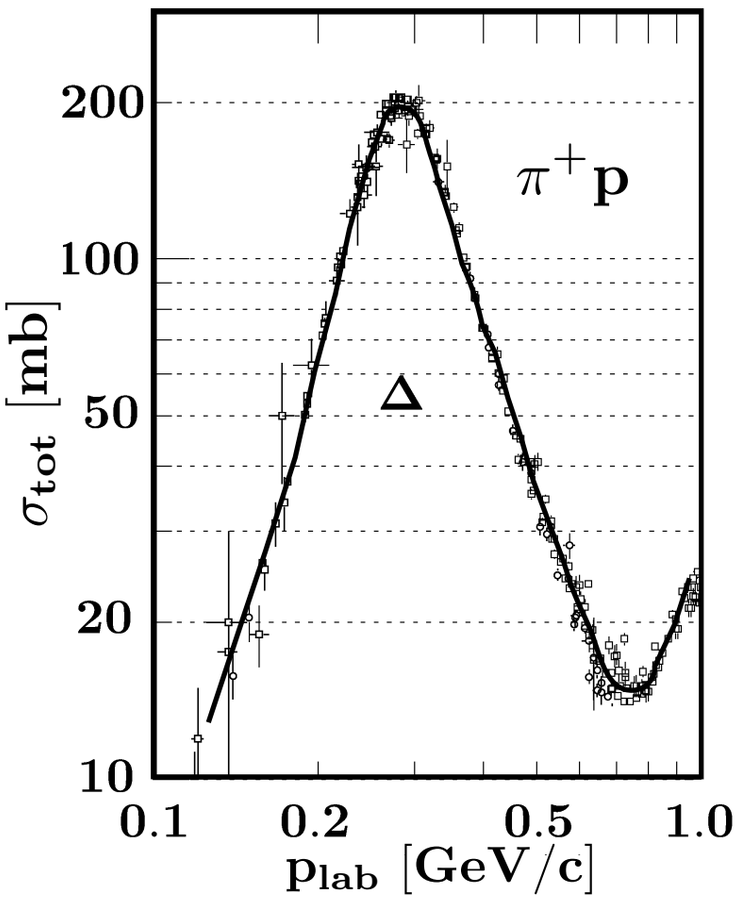
E. Epelbaum, H. Krebs, U.-G. Meißner
Phys. Rev. Lett. 115 (2015) 122301



D.R. Entem, N. Kaiser, R. Machleidt, Y. Nosyk
Phys. Rev. C91 (2015) 014002
Phys. Rev. C92 (2015) 064001

- convergence tests at N^5LO

Explicit $\Delta(1230)$ DEGREES of FREEDOM



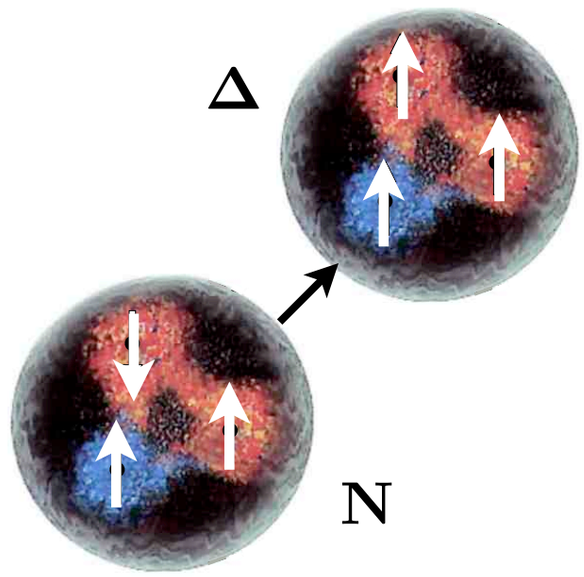
- **Large spin-isospin polarizability** of the **nucleon**

  Dominance of $\Delta(1230)$ in pion-nucleon scattering

$$\beta_\Delta = \frac{g_A^2}{f_\pi^2 (M_\Delta - M_N)} \sim 5\,\mathrm{fm}^3$$

$$M_\Delta - M_N \simeq 2\, m_\pi << 4\pi\, f_\pi$$

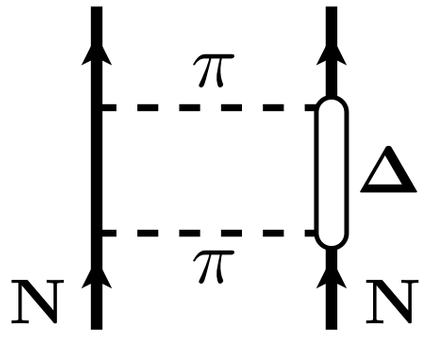
(small scale)



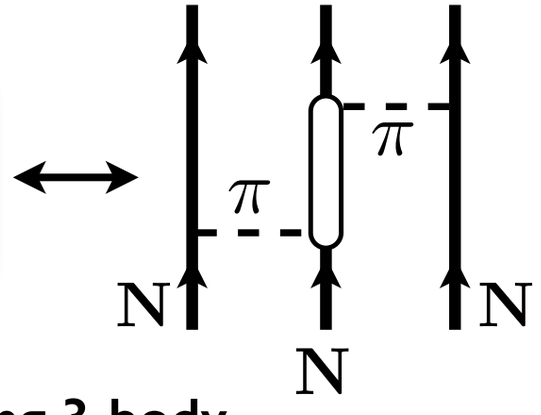
- **Pionic Van der Waals -** type intermediate range central potential

N. Kaiser, S. Gerstendörfer, W.W., NP A637 (1998) 395

N. Kaiser, S. Fritsch, W.W., NP A750 (2005) 259



$$V_c(r) = -\frac{9\, g_A^2}{32\pi^2 f_\pi^2}\, \beta_\Delta\, \frac{e^{-2m_\pi r}}{r^6}\, P(m_\pi r)$$



J. Fujita, H. Miyazawa (1957)

Pieper, Pandharipande, Wiringa, Carlson (2001)

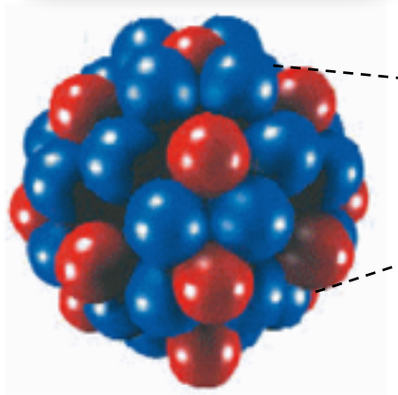
strong 3-body interaction

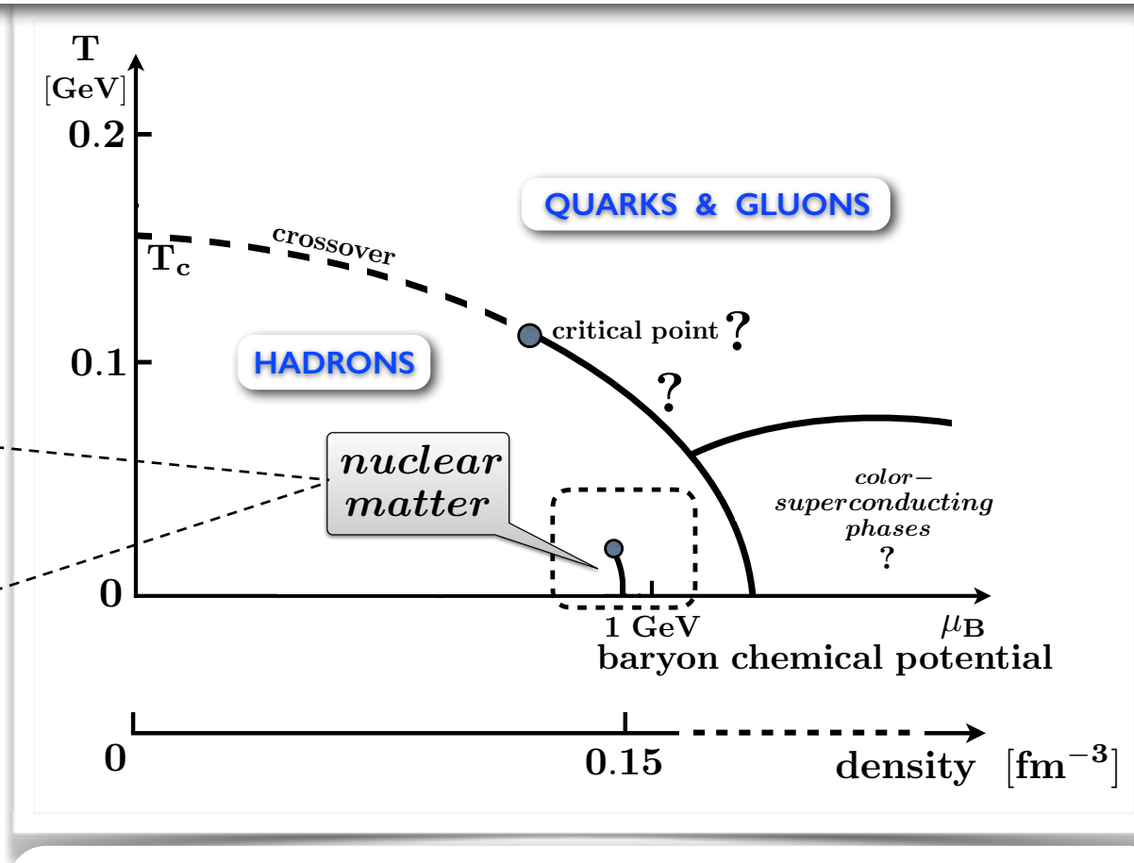
4.

NUCLEAR CHIRAL THERMODYNAMICS

NUCLEAR MATTER and QCD PHASES

bulk properties of nuclei





Scales in N = Z nuclear matter:

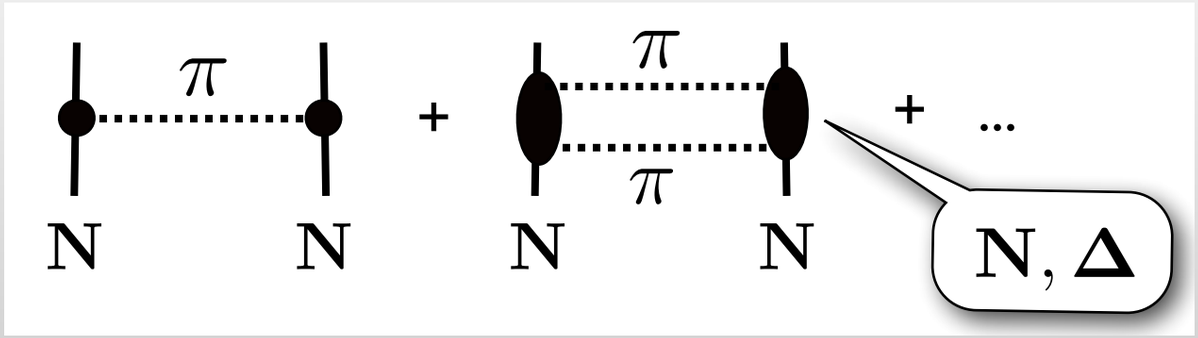
- momentum scale: **Fermi momentum** $\mathbf{k_F \simeq 1.4\ fm^{-1} \sim 2m_\pi}$
- NN distance: $\mathbf{d_{NN} \simeq 1.8\ fm \simeq 1.3\ m_\pi^{-1}}$
- compression modulus: $\mathbf{K = (260 \pm 30)\ MeV \sim 2m_\pi}$
- energy per nucleon: $\mathbf{E/A \simeq -16\ MeV}$

CHIRAL DYNAMICS and the NUCLEAR MANY-BODY PROBLEM

N. Kaiser, S. Fritsch, W.W. (2002 - 2005)

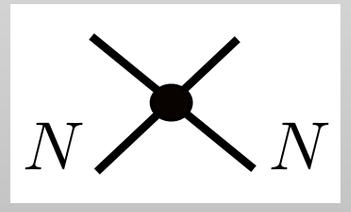
- **Small scales:** energy, momentum, $\mathbf{m_\pi},\ \mathbf{k_F} << 4\pi \mathbf{f_\pi} \sim 1\,\mathbf{GeV}$
- **PIONS** (light) and **NUCLEONS** (heavy) as **explicit degrees of freedom**
- **IN-MEDIUM CHIRAL PERTURBATION THEORY**

Pion exchange processes in the presence of a filled **Fermi sea**



in-medium

Short-distance dynamics:



contact interactions

IN-MEDIUM CHIRAL PERTURBATION THEORY

- "Medium insertion" in the nucleon propagator:

$$(\gamma_\mu \mathbf{p}^\mu + \mathbf{M_N}) \left[\frac{\mathbf{i}}{\mathbf{p^2} - \mathbf{M_N^2} + \mathbf{i}\varepsilon} - \mathbf{2}\pi\, \delta(\mathbf{p^2} - \mathbf{M_N^2})\, \theta(\mathbf{p^0})\, \theta(\mathbf{k_F} - |\vec{\mathbf{p}}|) \right]$$

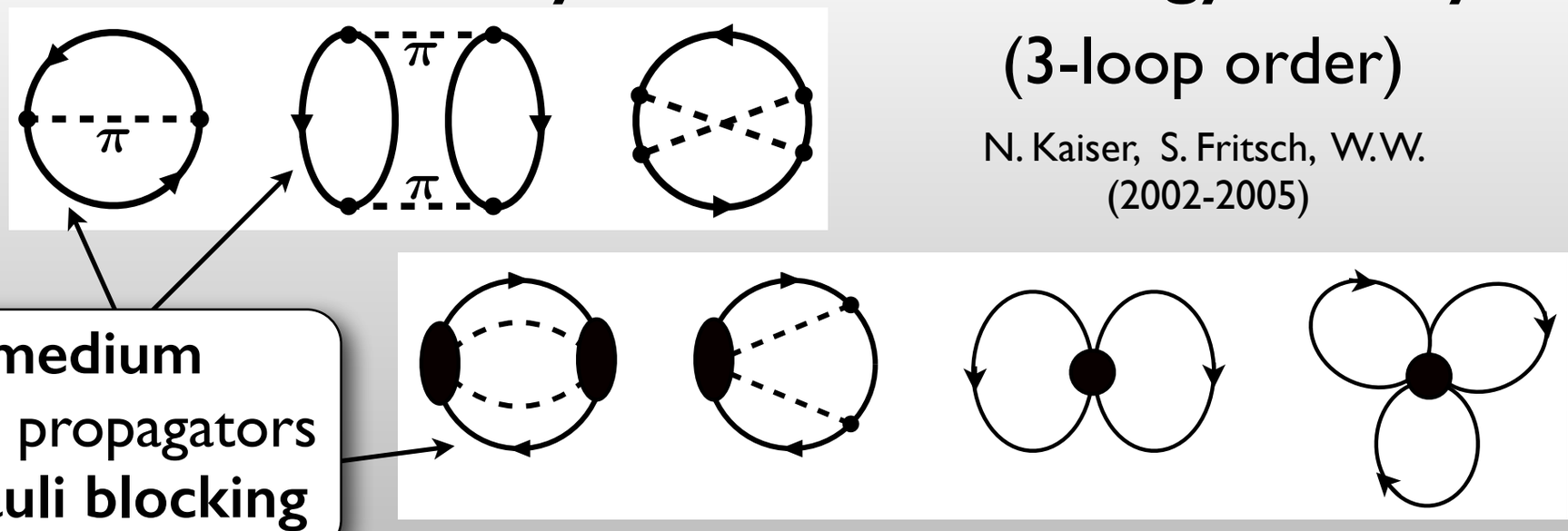
Loop expansion of (In-Medium) **C**hiral Perturbation **T**heory

$\longleftrightarrow$ Expansion of **ENERGY DENSITY** $\mathcal{E}(\mathbf{k_F})$ in **powers of Fermi momentum** [modulo functions $\mathbf{f_n}(\mathbf{k_F}/\mathbf{m}_\pi)$]

Nuclear thermodynamics: free energy density

(3-loop order)

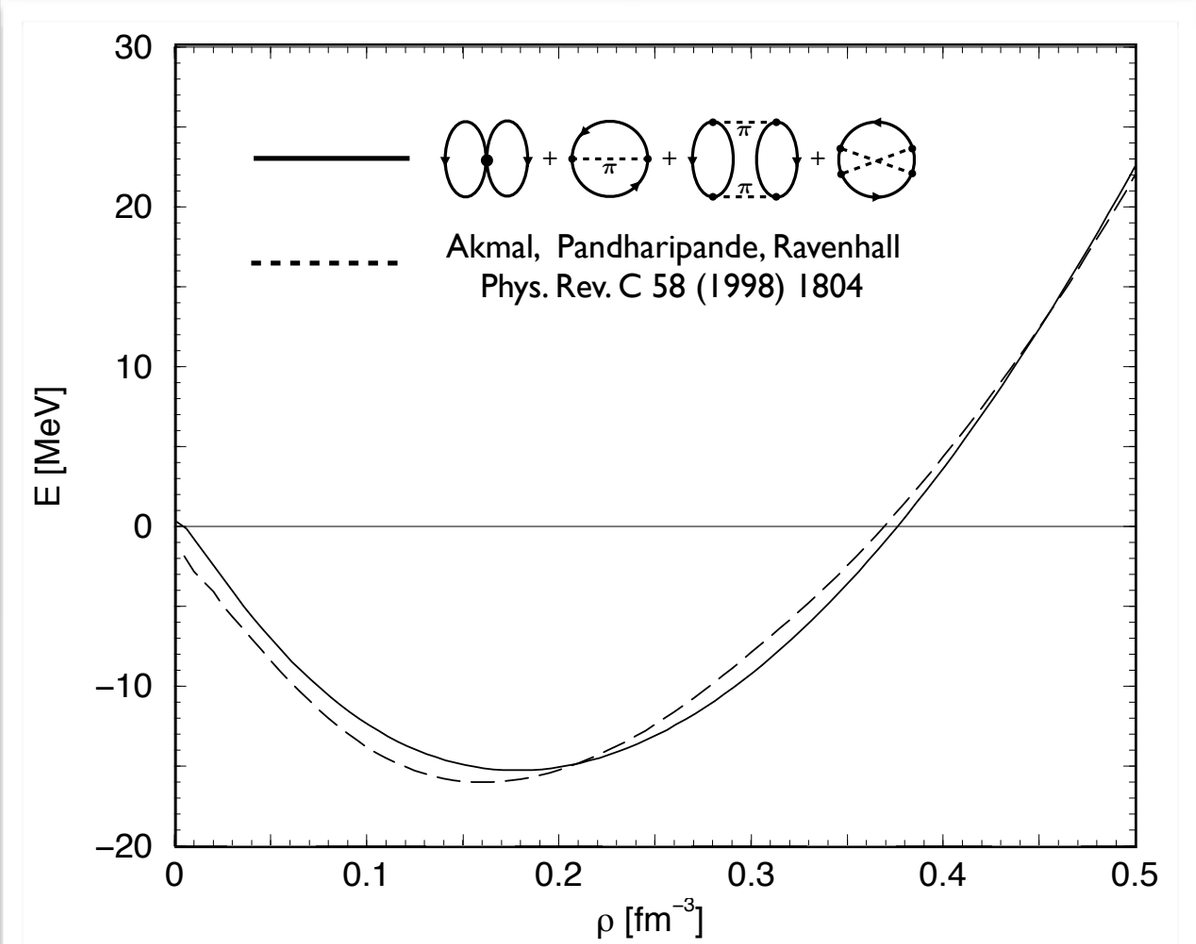
N. Kaiser, S. Fritsch, W.W. (2002-2005)



Preparations

Chiral pion-nucleon dynamics in symmetric nuclear matter

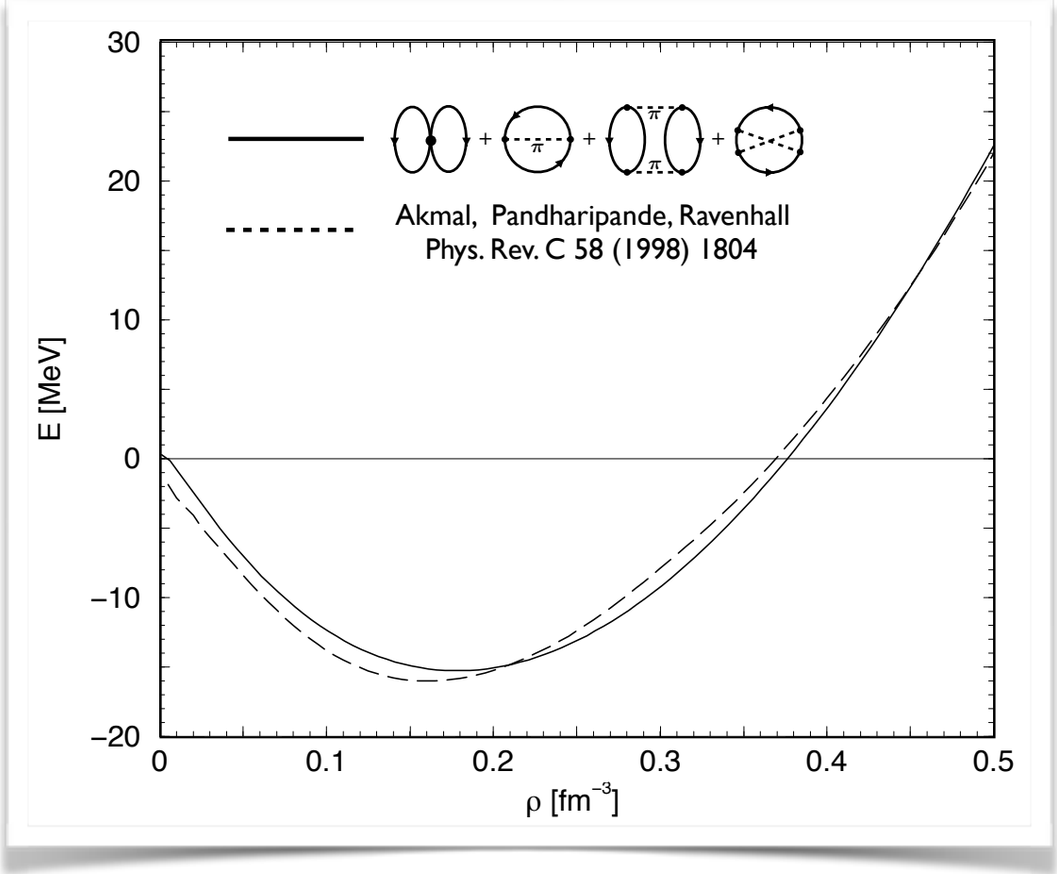
- One-pion exchange (Fock term)
- Iterated pion-exchange (2nd order tensor force) + exchange terms
- Short-distance contact interaction



N. Kaiser, S. Fritsch, W.W.: Nucl. Phys. A697 (2002) 255

… closely resembles sophisticated variational many-body calculation (APR) up to high densities $(\rho \sim 5\rho_0)$

Two-parameter description of N = Z nuclear matter



$$E = \frac{3\,k_F^2}{10\,M_N} - \alpha\frac{k_F^3}{M_N^2} + \beta\frac{k_F^4}{M_N^3}$$

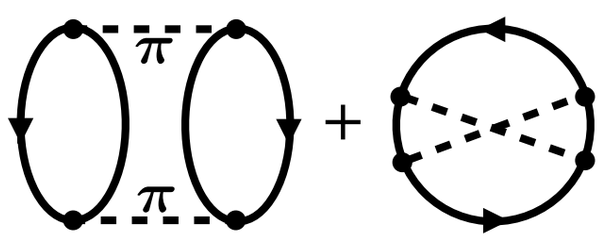
- **Fit to:** $E_0 = -16\ \mathrm{MeV}$

$$\rho_0 = \frac{2\,k_F^3(0)}{3\pi^2} = 0.16\ \mathrm{fm}^{-3}$$

$$\alpha = 5.27 \qquad \beta = 12.22$$

Prediction: **compression modulus** $K = 240\,\mathrm{MeV}$

Important role of iterated pion exchange (2nd order tensor force) in combination with PAULI principle — chiral limit $(m_\pi \to 0)$:

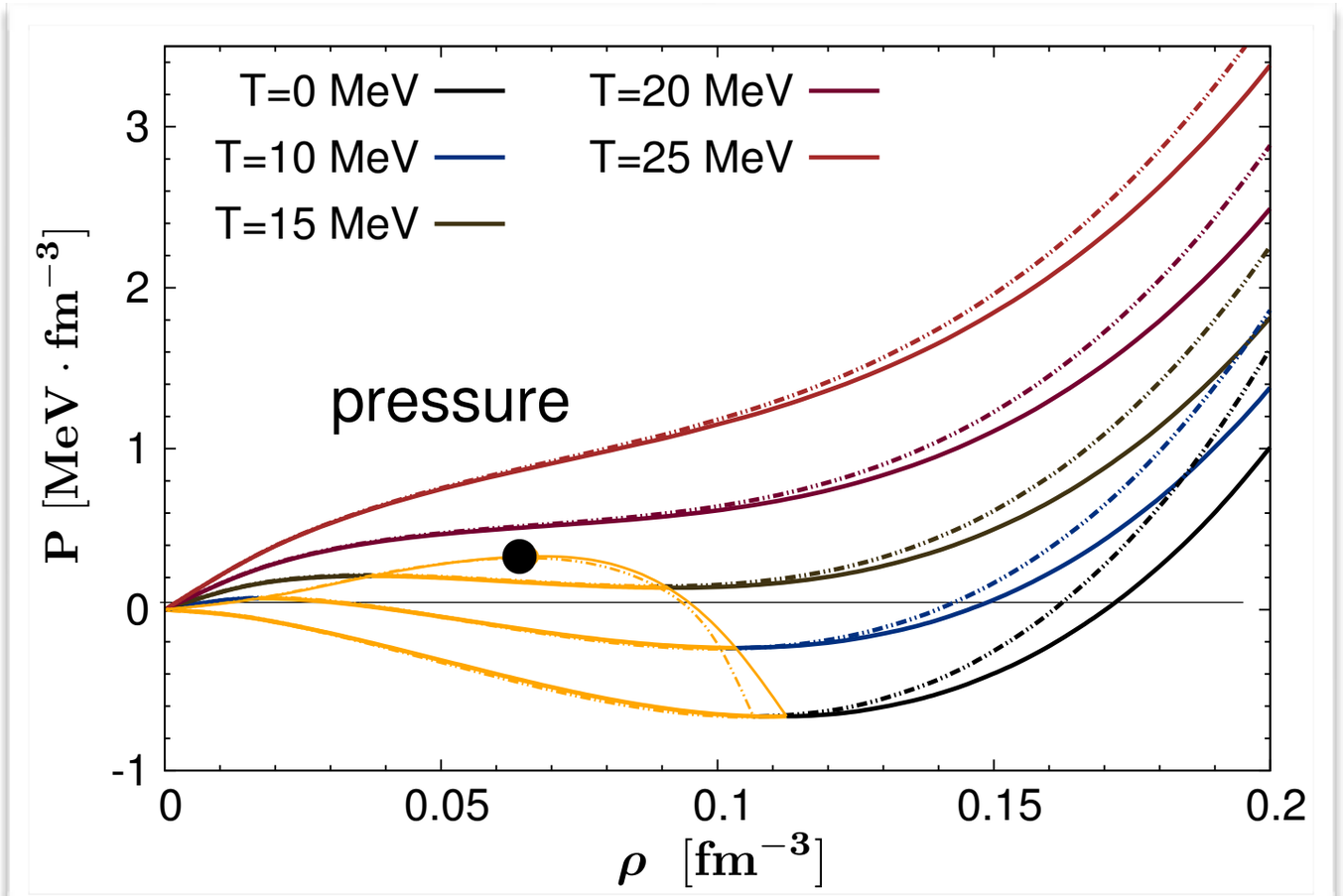


$$\beta = \frac{3}{70}\left(\frac{g_A\,M_N}{4\pi\,f_\pi}\right)^4 (4\pi^2 + 237 - 24\ln 2) = 12.47 \ \ !$$

NUCLEAR THERMODYNAMICS from CHIRAL EFT

- **Symmetric nuclear matter : liquid-gas phase diagram**

C. Wellenhofer, J.W. Holt, N. Kaiser, W.W. Phys. Rev. C89 (2014) 064009 C92 (2015) 015801



Critical temperature of liquid-gas first-order transition :

$$T_c = 17.4 \text{ MeV}$$

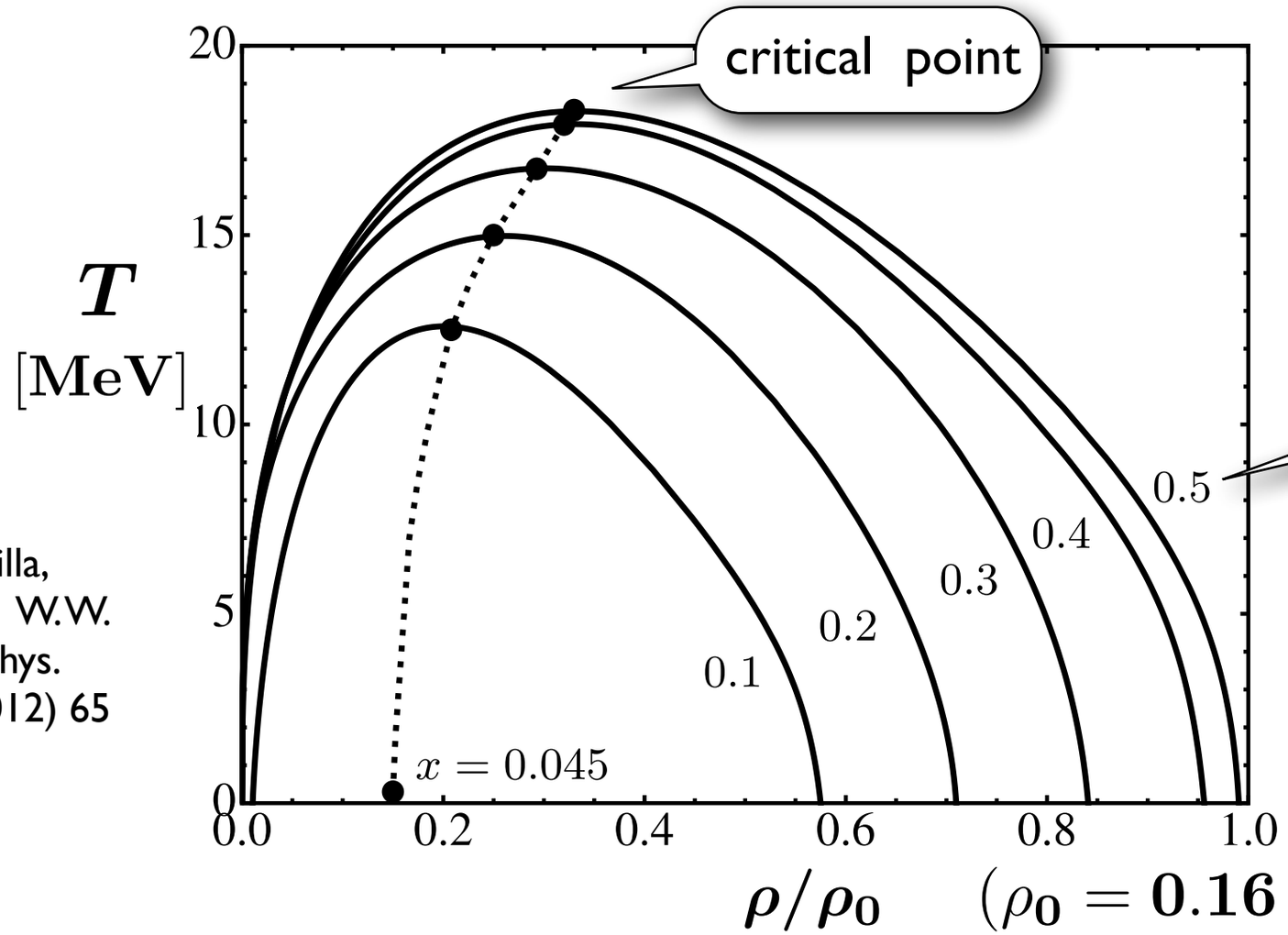
- **N3LO chiral NN interactions + N2LO 3-body forces**
- **Kohn-Luttinger-Ward many-body perturbation theory (2nd order)**

- **Empirical position of liquid-gas critical point :** J. B. Elliot et al. : Phys. Rev. C87 (2013) 054622

$$T_c = 17.9 \pm 0.4 \ MeV \qquad P_c = 0.31 \pm 0.07 \ MeV \ fm^{-3} \qquad \rho_c = 0.06 \pm 0.01 \ fm^{-3}$$

PHASE DIAGRAM of ASYMMETRIC NUCLEAR MATTER

- Trajectory of **CRITICAL POINT of Liquid - Gas transition** for **asymmetric matter** as function of proton fraction Z / A



M. Drews, T. Hell, B. Klein, W.W.
Phys. Rev. D 88 (2013) 096011

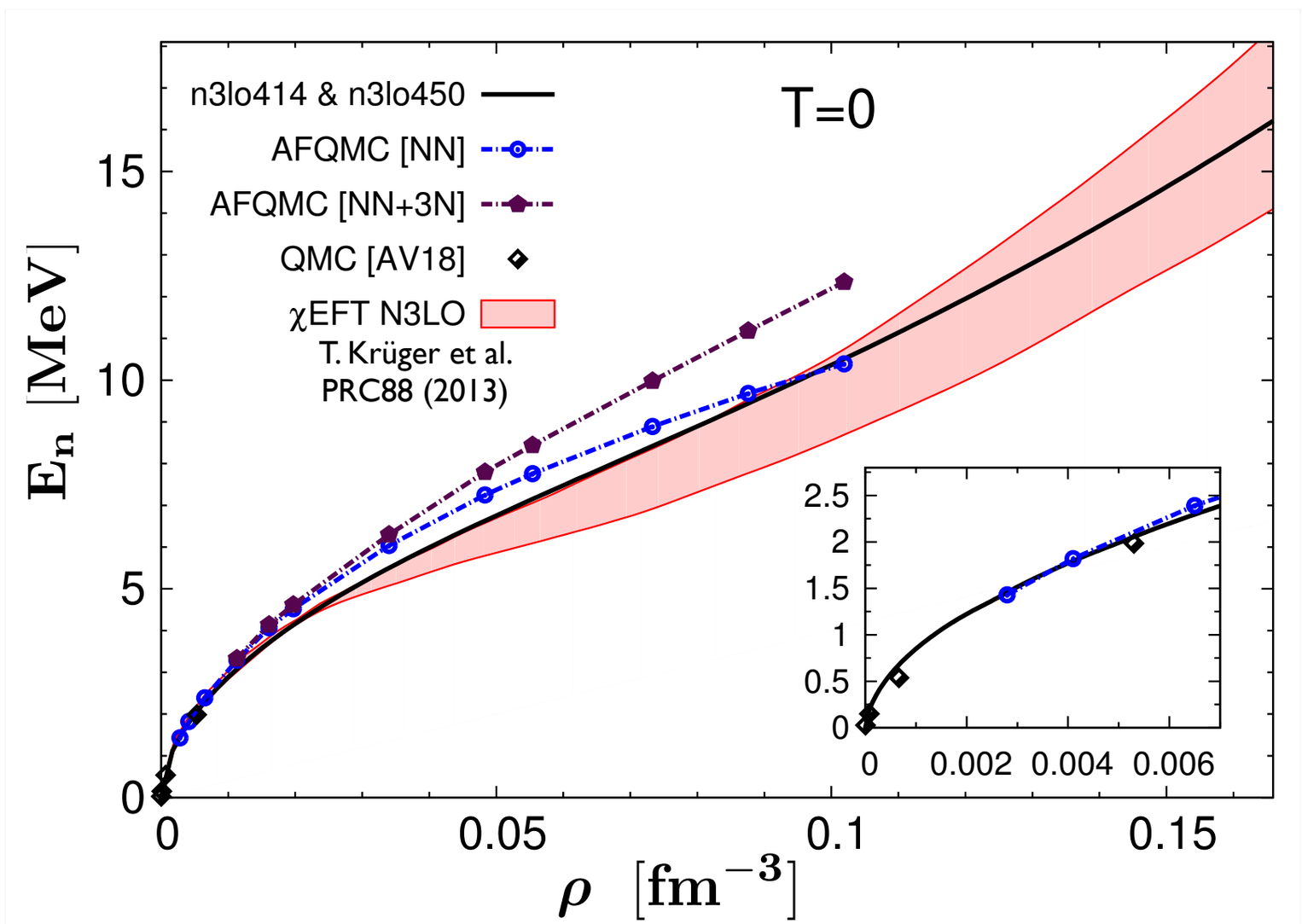
M. Drews, W.W.
Phys. Lett. B 738 (2014) 187
Phys. Rev. C 91 (2015) 035802

S. Fiorilla,
N. Kaiser, W.W.
Nucl. Phys.
A 880 (2012) 65

. . . including
non-perturbative effects
Functional
Renormalization Group

. . . determined almost entirely by
isospin dependent (one- and **two-**) **pion** exchange dynamics

NEUTRON MATTER from CHIRAL EFT

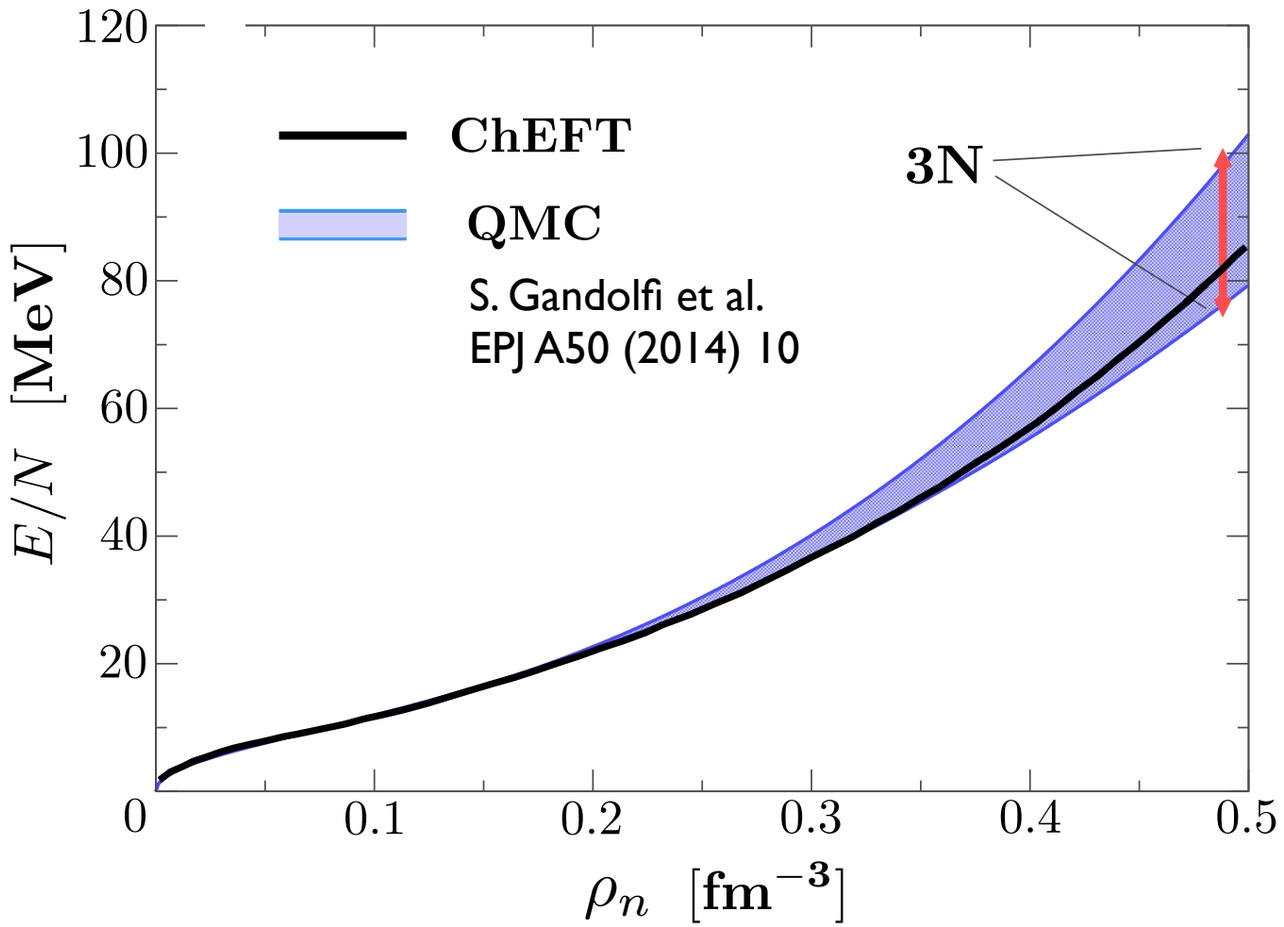


C. Wellenhofer, J.W. Holt, N. Kaiser, W.W. Phys. Rev. C89 (2014) 064009 C92 (2015) 015801

- N3LO chiral NN interactions + N2LO 3-body forces
- Many-body perturbation theory (2nd order)

COLD NEUTRON MATTER

- **In-medium Chiral Effective Field Theory** (3-loop) with resummation of short distance contact terms (large **nn scattering length**, $\mathrm{a_s = 19\ fm}$)



- **Neutron matter** behaves almost (but not quite) like **unitary Fermi gas**
- Bertsch parameter

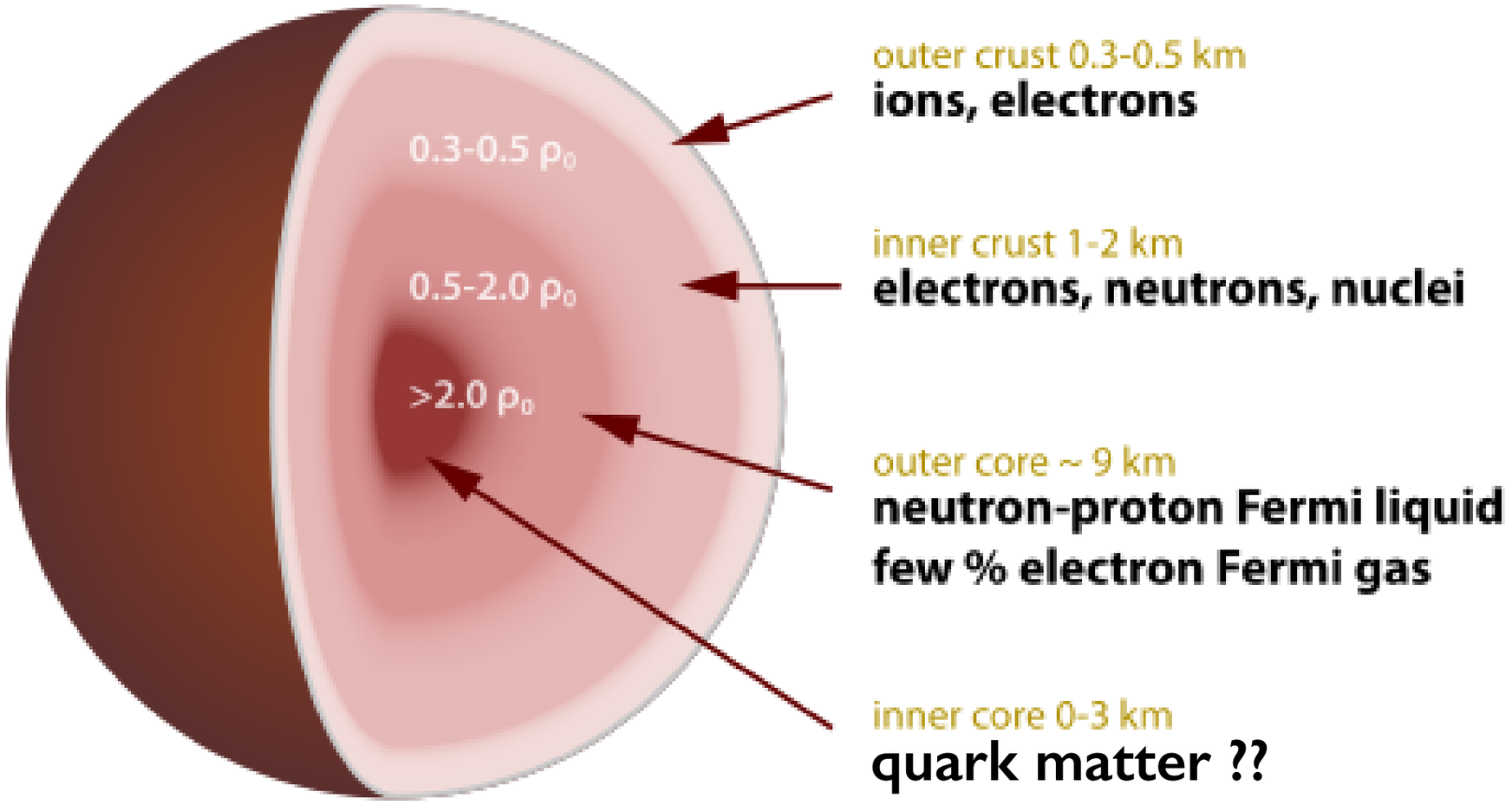
$$\xi = \frac{\bar{\mathrm{E}}}{\mathrm{E_{Fermi\,gas}}} \simeq 0.5$$

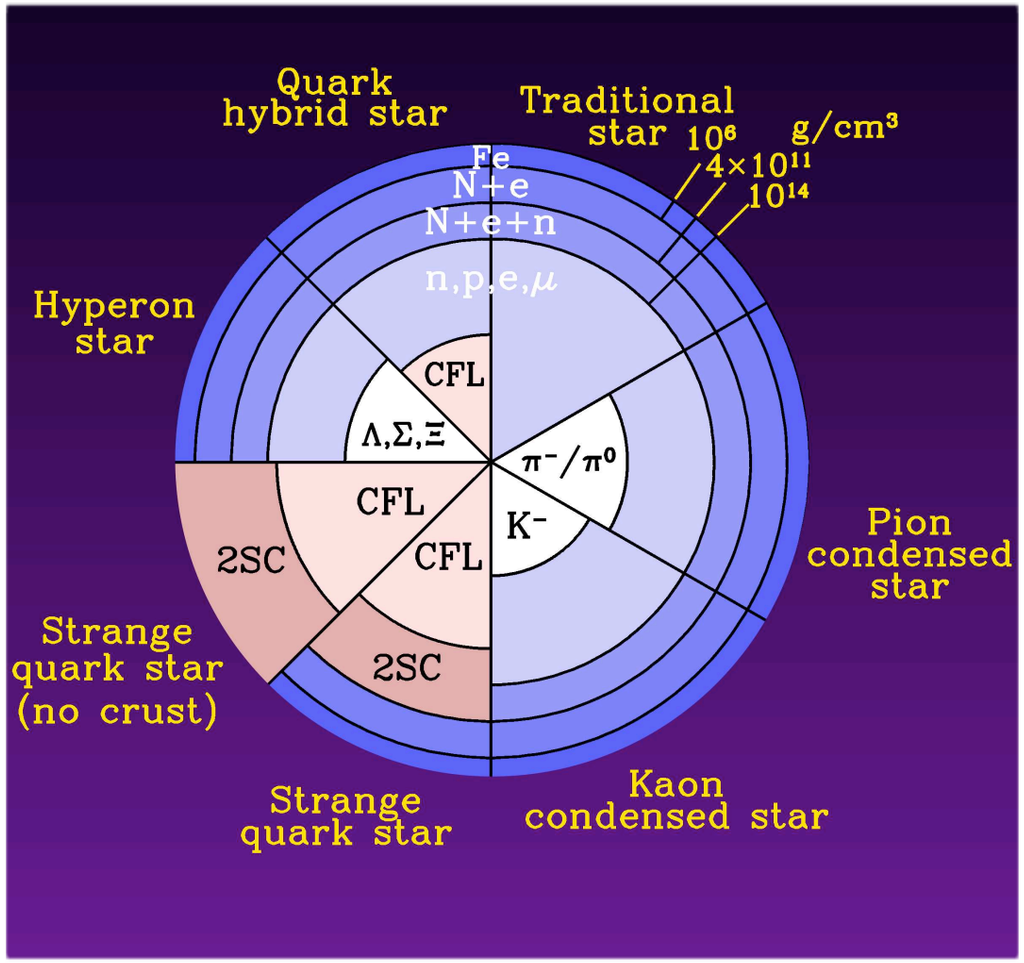
J.W. Holt, N. Kaiser, W.W.
Phys. Rev. C 87 (2013) 014338

M. Drews, W.W.
Phys. Lett. B 738 (2014) 187

- agreement with advanced many-body calculations
  (e.g. recent **Q**uantum **M**onte **C**arlo computations)

5.
Constraints
from
NEUTRON STARS

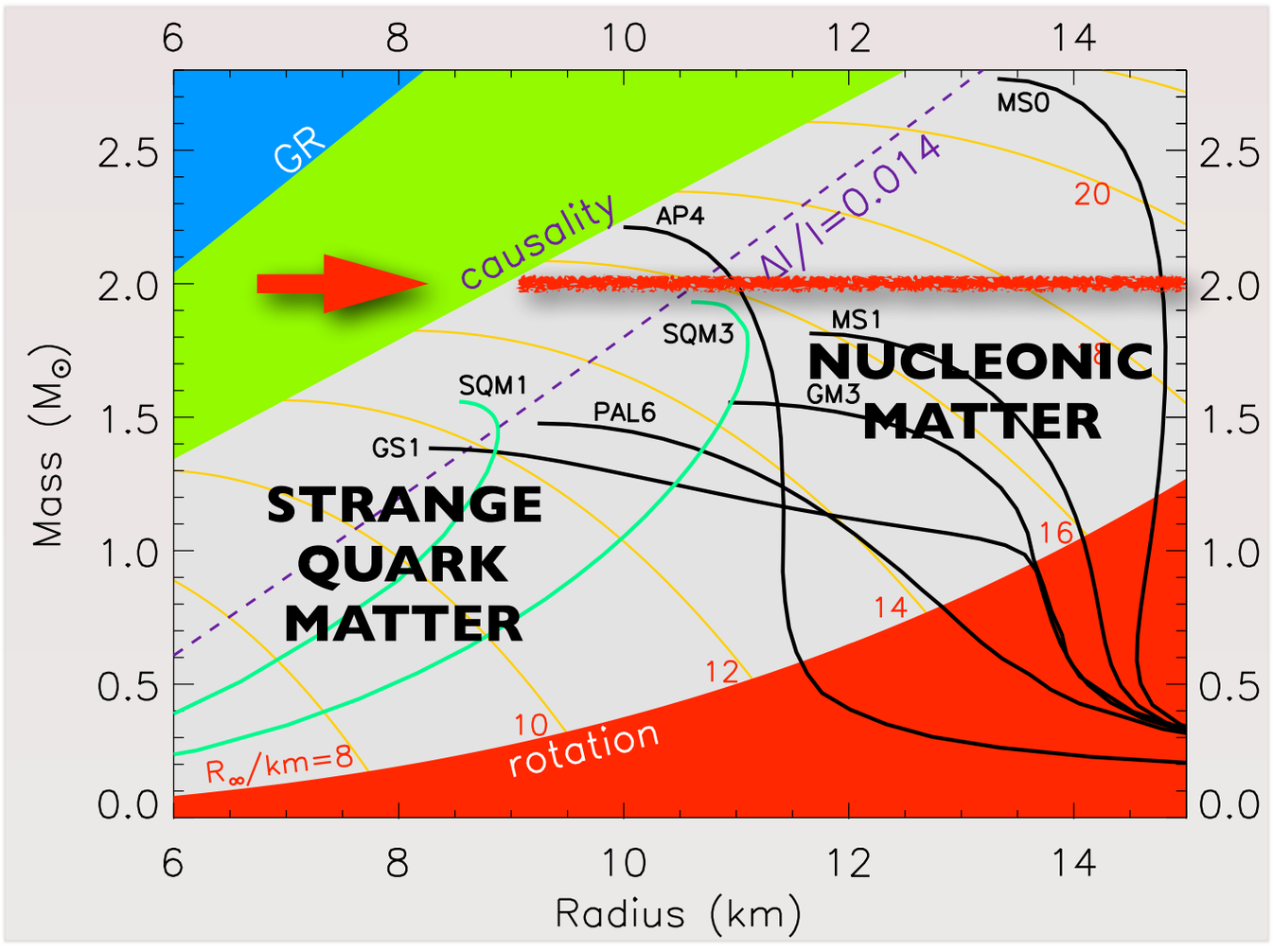




NEUTRON STARS and the EQUATION OF STATE of DENSE BARYONIC MATTER

J. Lattimer, M. Prakash: Phys. Reports 442 (2007) 109
Phys. Reports 621 (2016) 127

Mass-Radius Relation



Neutron Star Scenarios

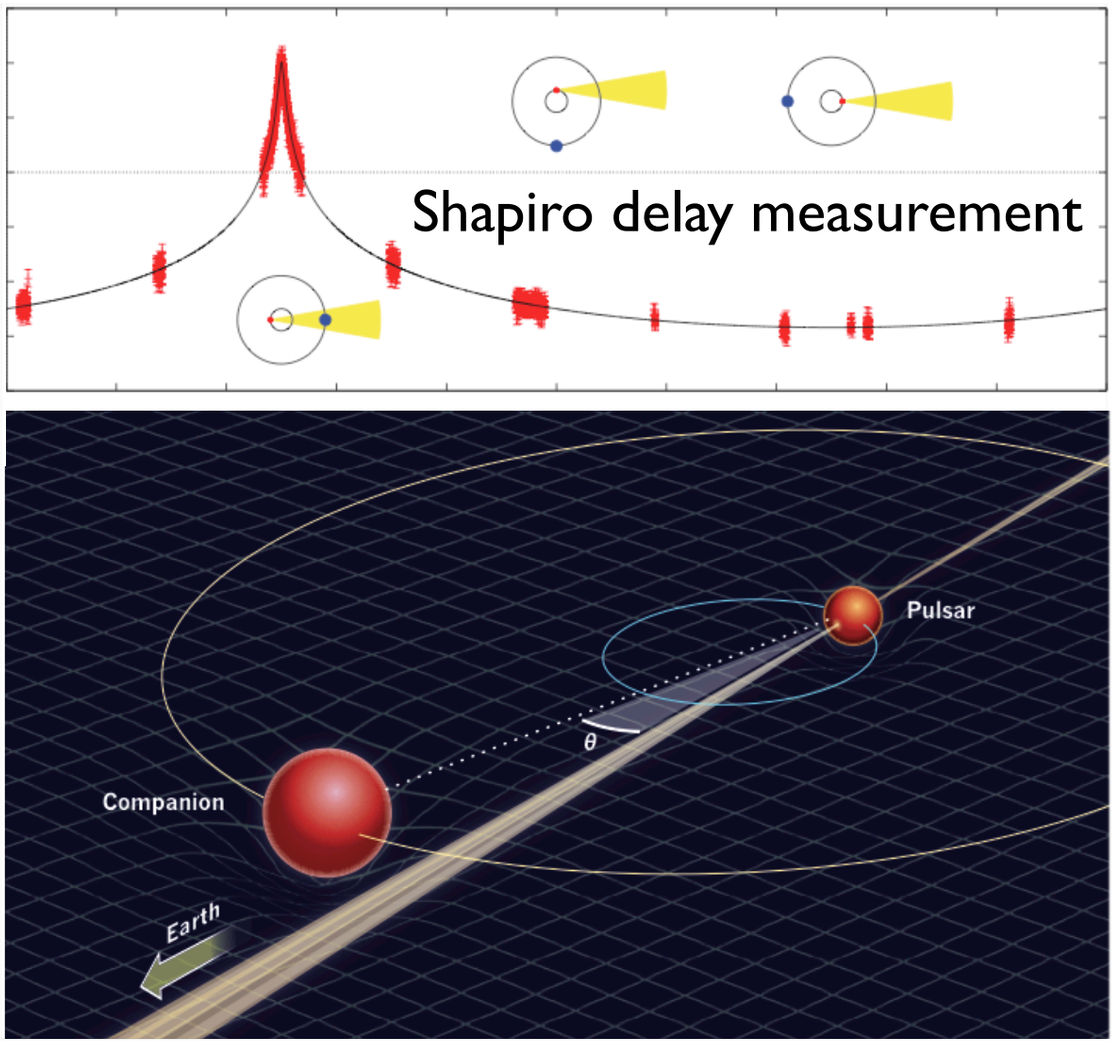
Tolman-Oppenheimer-Volkov Equations

$$\frac{dP}{dr} = -\frac{G}{c^2}\frac{(M + 4\pi P r^3)(\mathcal{E} + P)}{r(r - GM/c^2)}$$

$$\frac{dM}{dr} = 4\pi r^2 \frac{\mathcal{E}}{c^2}$$

Constraints from massive NEUTRON STARS

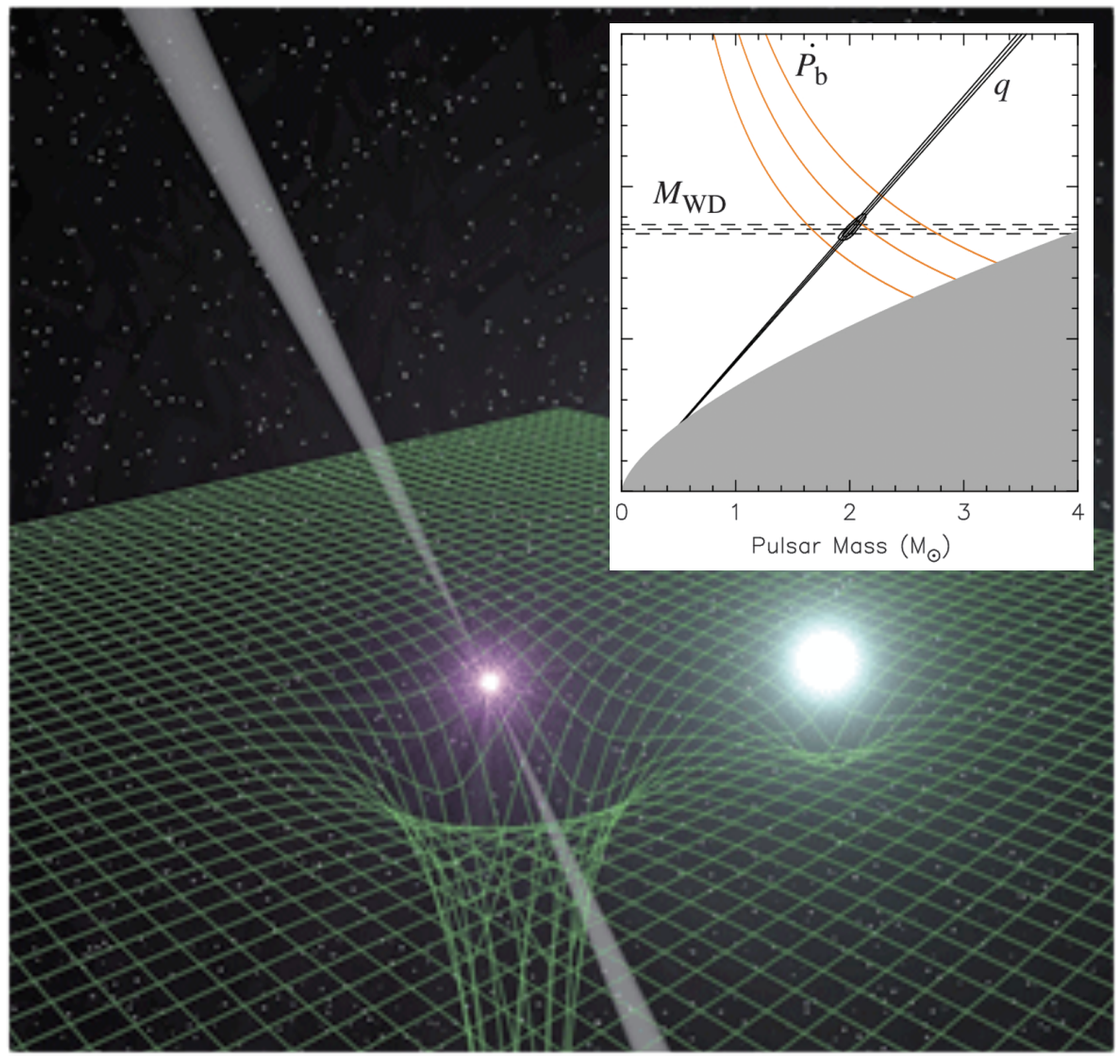
P.B. Demorest et al.
Nature 467 (2010) 1081



PSR J1614+2230

$$\mathbf{M = 1.97 \pm 0.04\ M_\odot}$$

J. Antoniadis et al.
Science 340 (2013) 6131

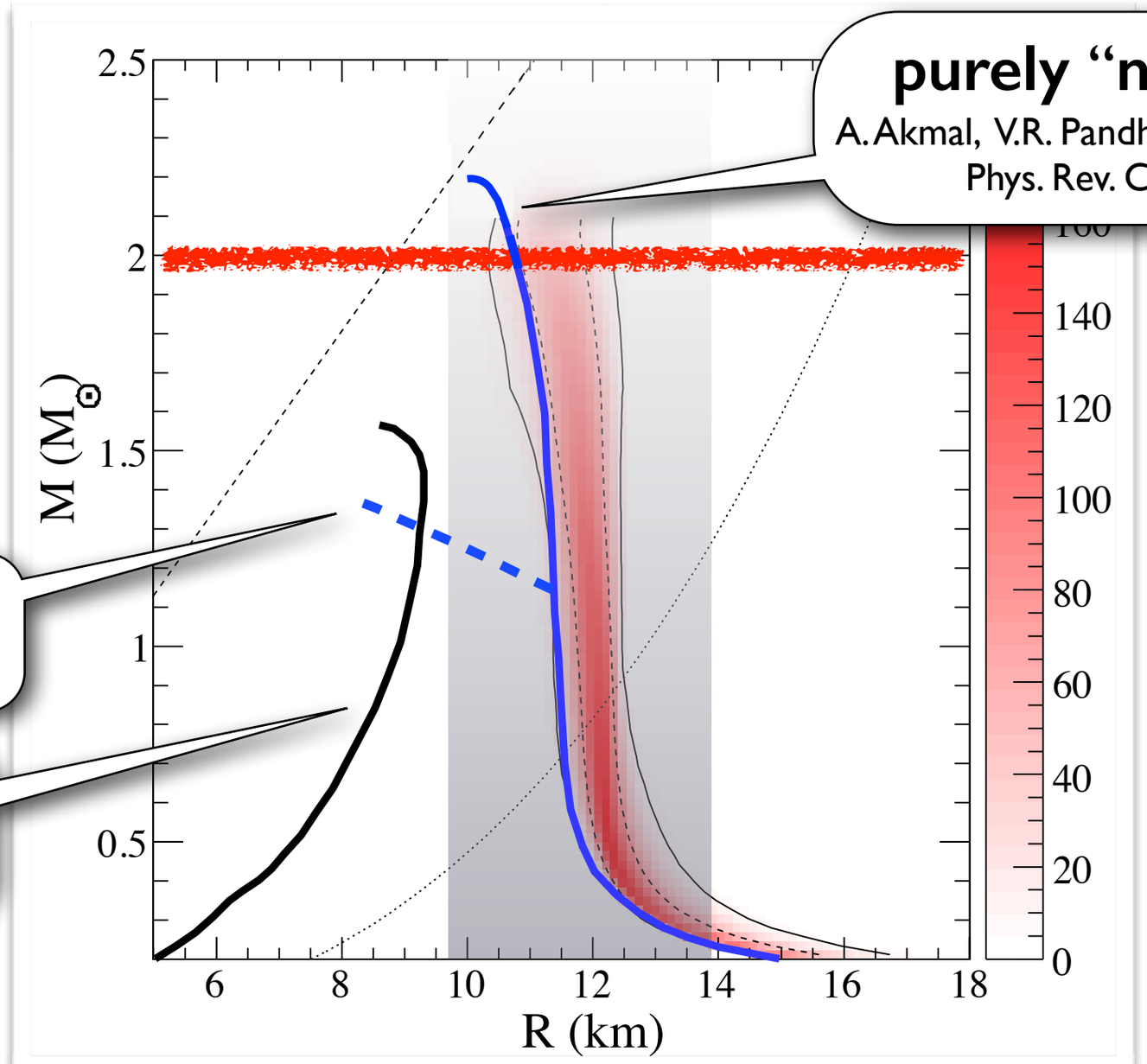


PSR J0348+0432

$$\mathbf{M = 2.01 \pm 0.04\ M_\odot}$$

CONSTRAINTS from NEUTRON STARS

Mass-Radius Relation



K. Hebeler, J. Lattimer, Ch. Pethick, A. Schwenk: Phys. Rev. Lett. 105 (2010) 161102

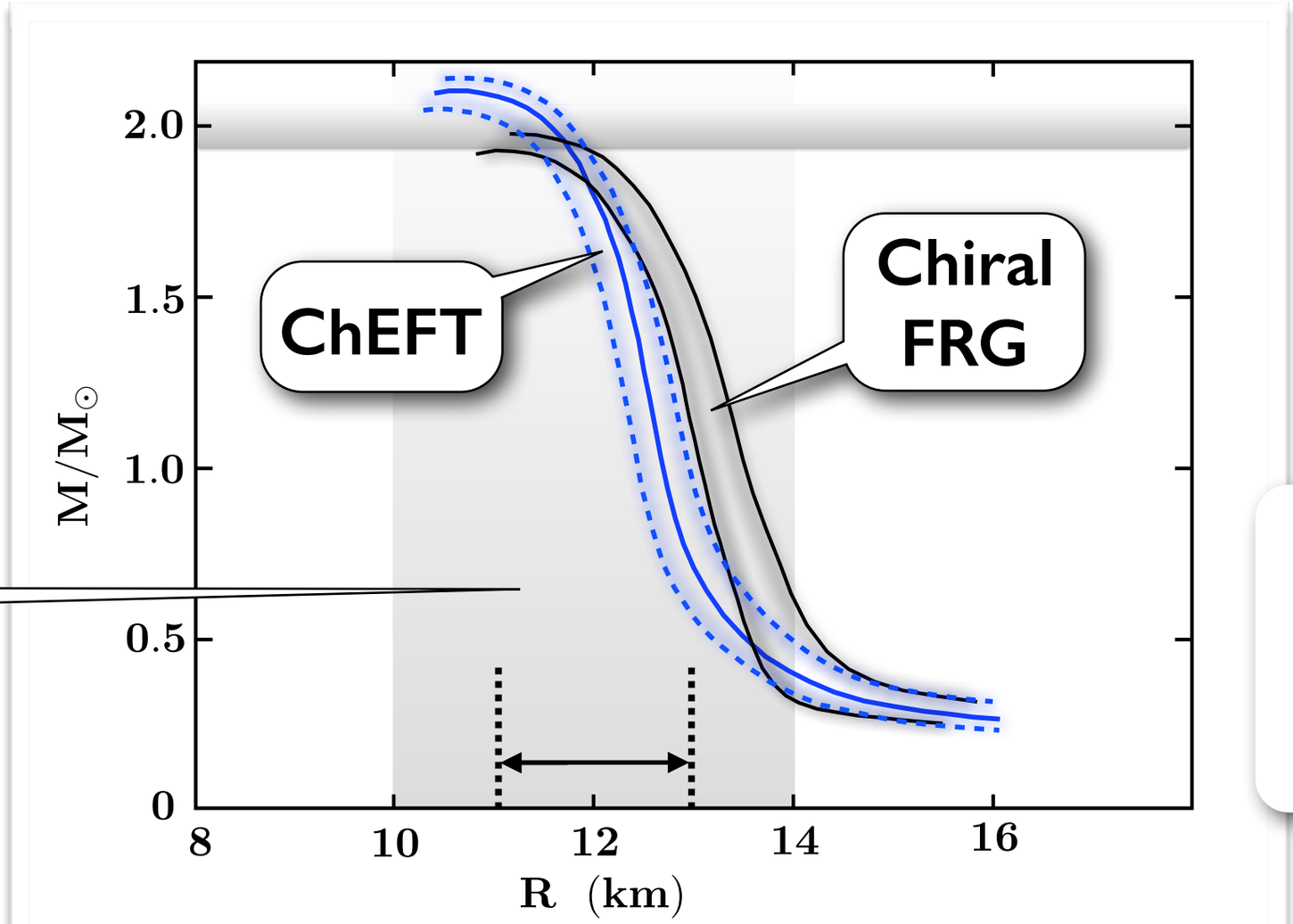
A.W. Steiner, J. Lattimer, E.F. Brown Astroph. J. 722 (2010) 33

- "**Exotic**" equations of state ruled out ?

NEUTRON STAR MATTER
from **Chiral EFT** and **FRG**

- **Symmetry energy range: 30 - 35 MeV**
- **Crust: SLy EoS**

T. Hell, W.W.
Phys. Rev.
C90 (2014) 045801



A.W. Steiner,
J.M. Lattimer, E.F. Brown
EPJ A52 (2016) 18

M. Drews, W.W.

Phys. Rev.
C91 (2015) 035802

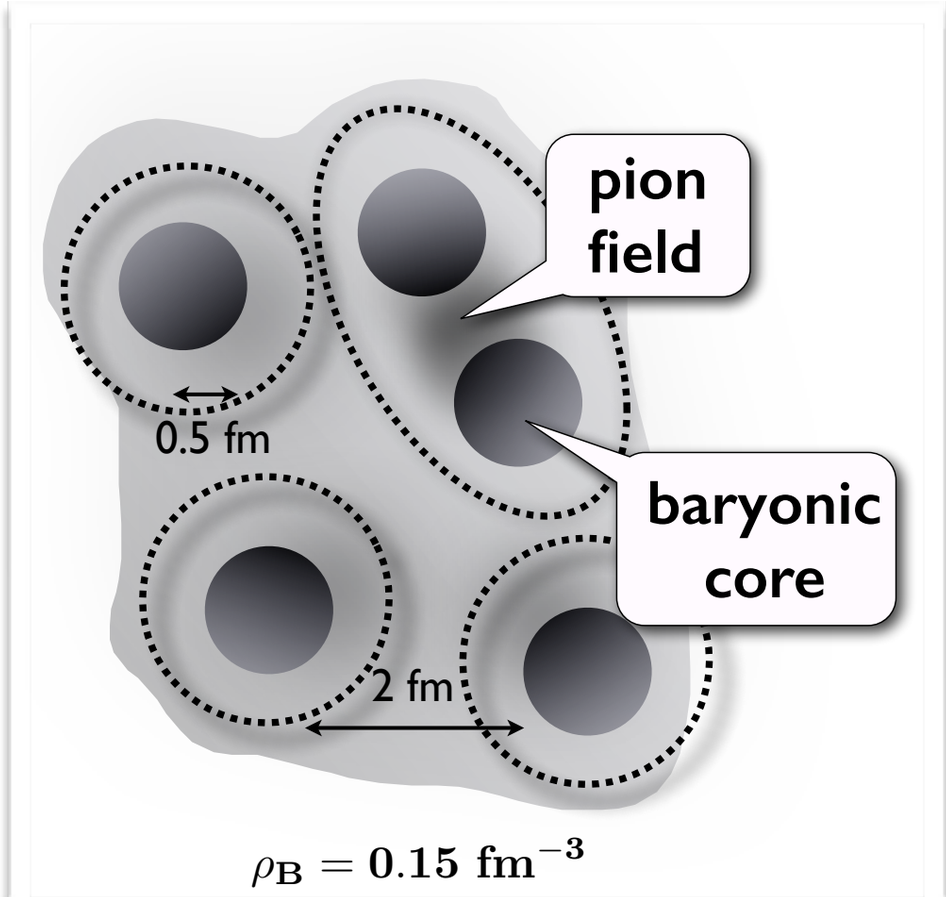
Prog. Part. Nucl. Phys.
93 (2017) 69

- **Central core density**

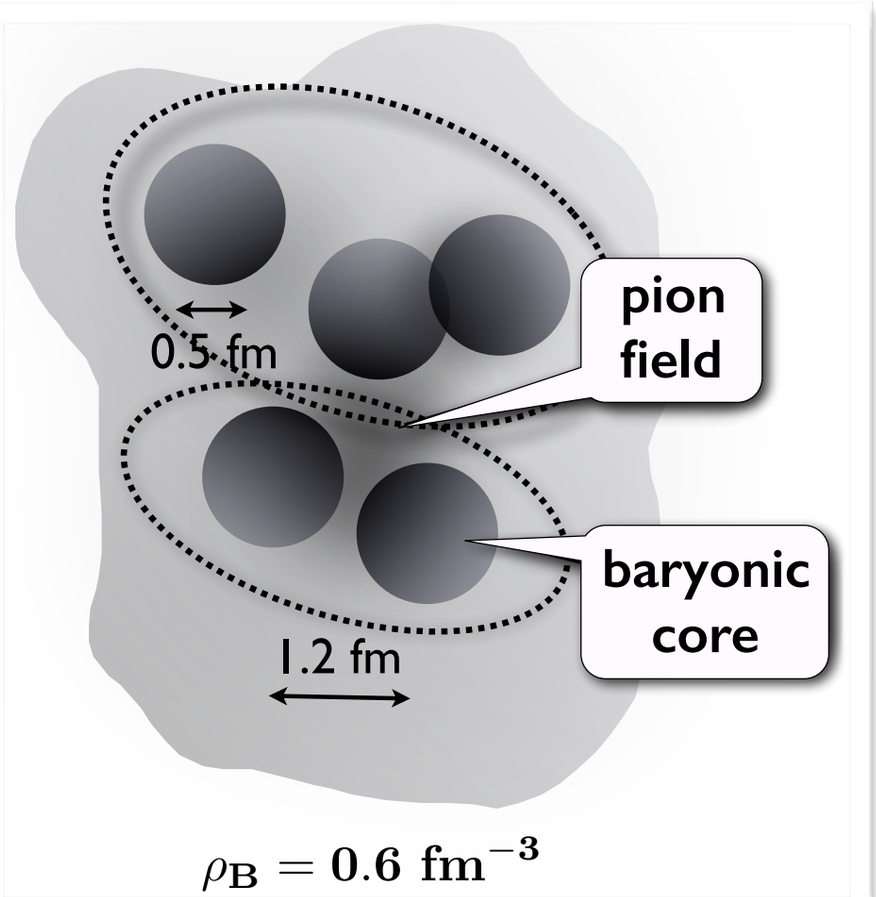
$$\rho_c \lesssim 5\,\rho_0$$

- **Chiral many-body dynamics using "conventional" (pion & nucleon) degrees of freedom is consistent with neutron star constraints**

Densities and Scales in Compressed Baryonic Matter



normal nuclear matter: dilute



neutron star core matter:
compressed but not superdense

- Quark cores of nucleons $(\mathrm{R} \sim \frac{1}{2}\ \mathrm{fm})$

begin to touch and overlap (percolate) at baryon densities

$$\rho_B \gtrsim 6\,\rho_0$$

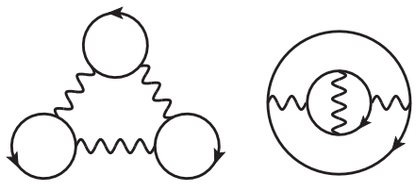
PART II

Chiral Effective Field Theory

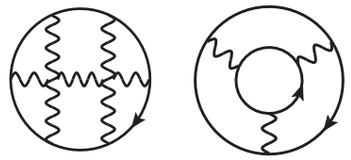
and

Nuclear Energy Density Functional

- **Nuclear chiral many-body dynamics : latest update**
- **Nuclear Skyrme phenomenology : state-of-the-art**
- **From nuclear chiral dynamics to energy density functionals**

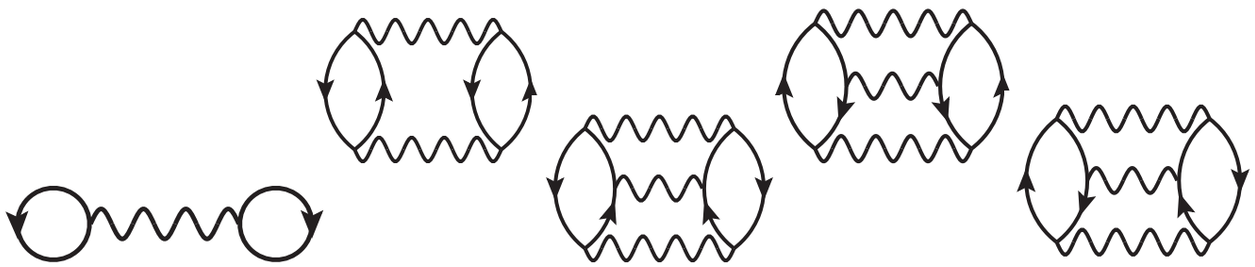


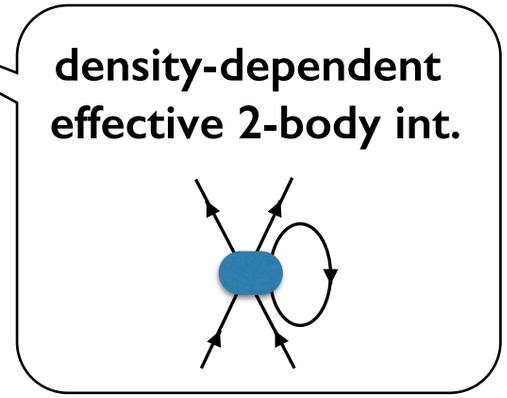
6. NUCLEAR MATTER from CHIRAL EFT 3rd Order Perturbation Theory



J.W. Holt, N. Kaiser : Phys. Rev. C95 (2017) 034326

- Convergence issues and uncertainty estimates
- N3LO chiral NN interactions
- N2LO 3-body forces
- Self-consistent single particle energies (2nd order)



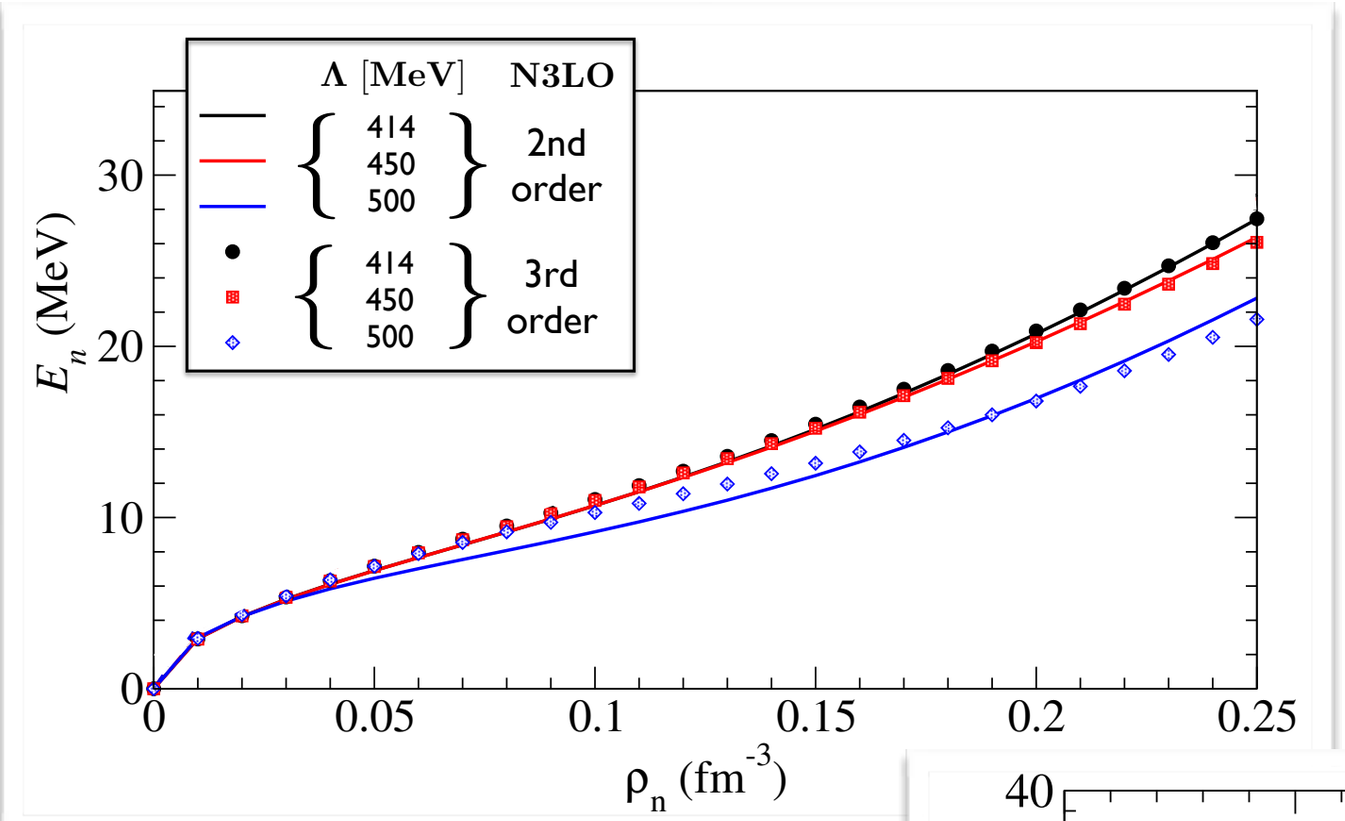


+ complete sets of exchange terms

Λ	$E^{(1)}$	$E^{(2)}$	$E^{(3)}_{pp}$	$E^{(3)}_{hh}$	$E^{(3)}_{ph}$	$E = E_{kin} + \sum_n E^{(n)}$
450	-30.1	-9.7	0.8	-0.3	-0.3	-17.7
500	-25.9	-13.2	1.0	-0.2	-1.5	-13.1

Contributions to the energy per nucleon [MeV] up to 3rd order

$$\rho = \rho_0 = 0.16\,\mathrm{fm}^{-3}$$



NEUTRON MATTER from CHIRAL EFT

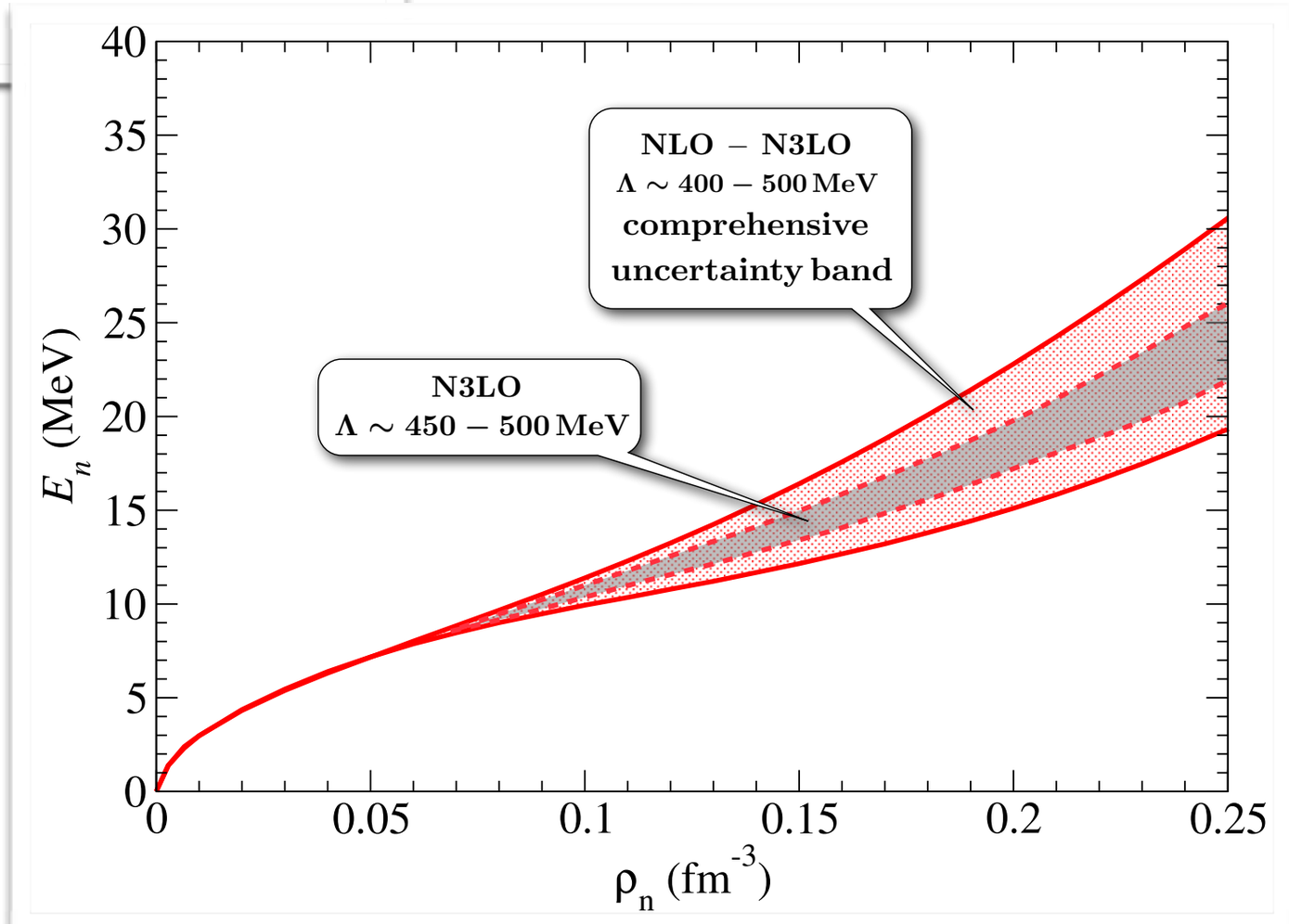
3rd Order Perturbation Theory

- **Convergence of many-body perturbation theory**

$$\rho < 2\rho_0$$

- **Uncertainties primarily from Chiral EFT expansion**

J.W. Holt, N. Kaiser
Phys. Rev. C95 (2017) 034326



ISOSPIN-ASYMMETRIC MATTER and SYMMETRY ENERGY

- **Energy per particle :** $E(\rho,\eta) = E(\rho,\eta=0) + S(\rho)\,\eta^2 + \ldots \qquad \eta = \dfrac{\rho_n - \rho_p}{\rho}$

- **Symmetry energy :**

$$S(\rho) = E_n(\rho) - E(\rho)$$

neutron matter

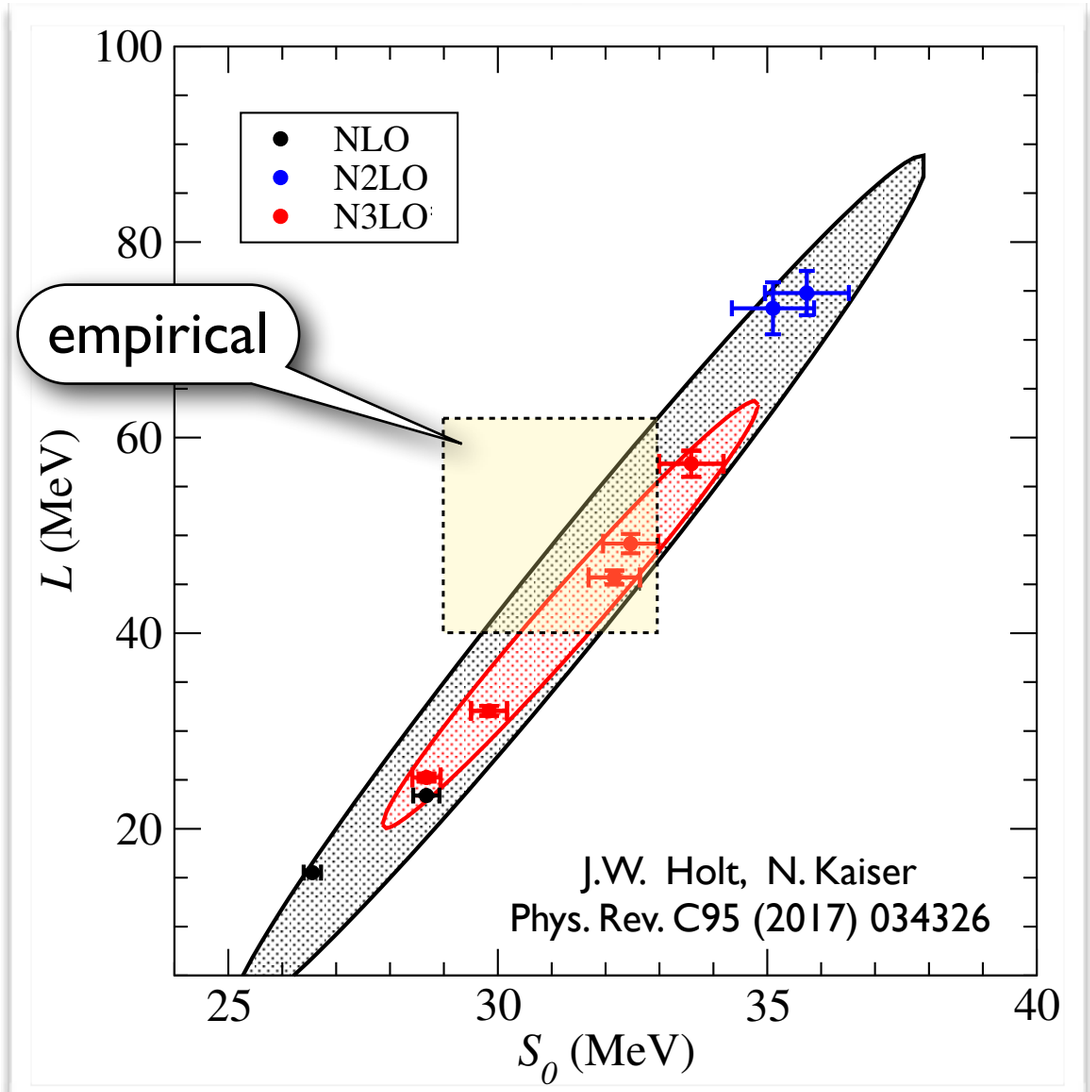
symmetric nuclear matter

$$S_0 \equiv S(\rho = \rho_0)$$

$$L = 3\rho \frac{\partial S(\rho)}{\partial \rho}\Big|_{\rho_0}$$

- **Key quantities for neutron-rich nuclei and astrophysics**

N3LO chiral NN interactions + N2LO 3-body forces



7. NUCLEAR SKYRME PHENOMENOLOGY

- **Effective interaction**
  (to be used e.g. in Hartree-Fock-Bogoliubov calculations)

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j \qquad \vec{p}_{ij} = -\frac{i}{2}(\vec{\nabla}_i - \vec{\nabla}_j) \qquad \vec{r} = \frac{1}{2}\,(\vec{r}_i + \vec{r}_j)$$

$$v_{ij} = t_0\,\delta(\vec{r}_{ij}) + \frac{1}{2}t_1\left[p_{ij}^2\,\delta(\vec{r}_{ij}) + \delta(\vec{r}_{ij})\,p_{ij}^2\right] + t_2\,\vec{p}_{ij}\cdot\delta(\vec{r}_{ij})\,\vec{p}_{ij} +$$
$$+\ iW_0\,(\vec{\sigma}_i + \vec{\sigma}_j)\cdot\vec{p}_{ij}\times\delta(\vec{r}_{ij})\,\vec{p}_{ij}$$
$$+\ \text{exchange terms}\ [\,t_n \to t_n(1 + x_n P_\sigma)]\ +\ \text{pairing}$$

- **Additional density-dependent terms :**

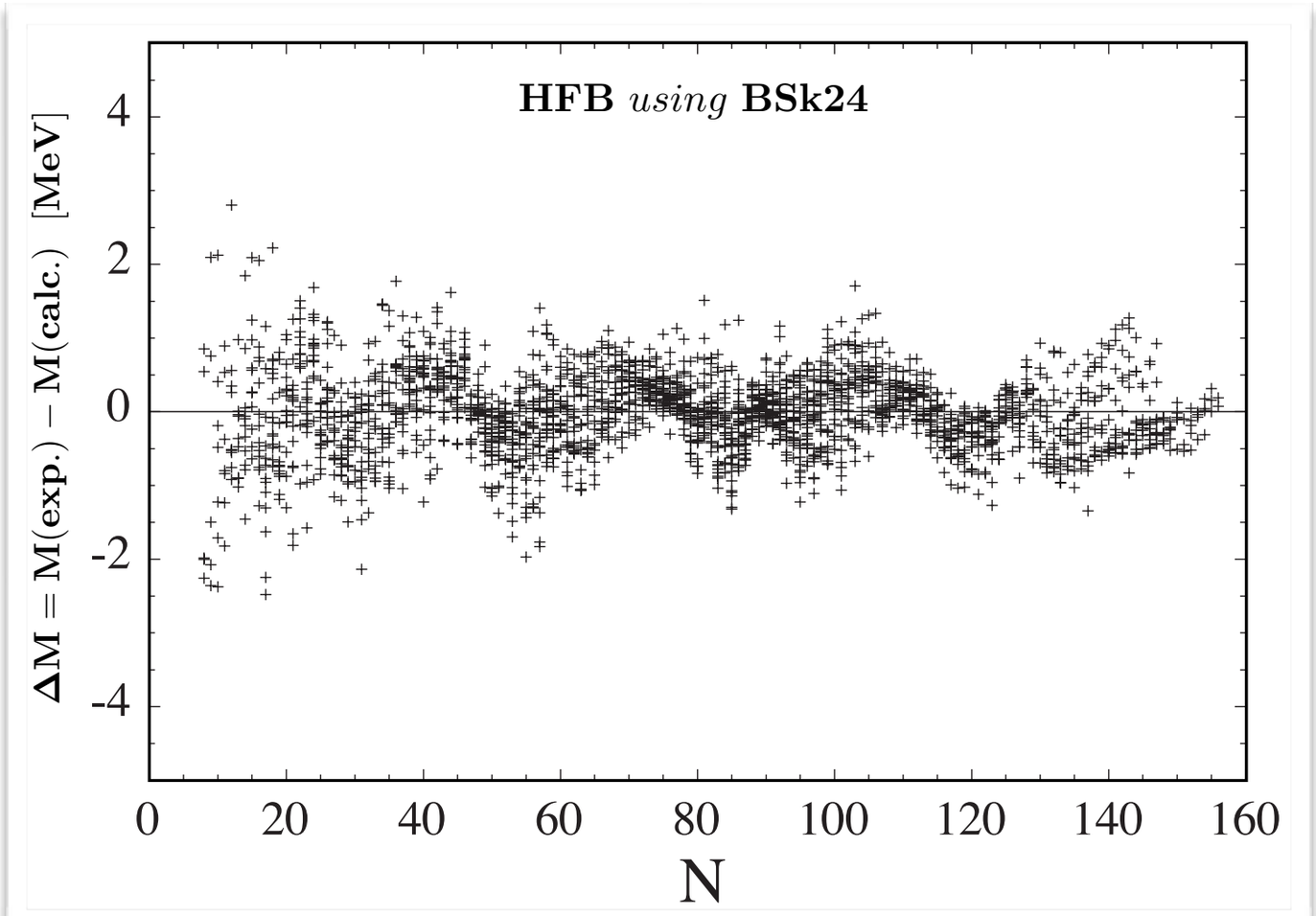
$$t_0 \to t_0 + \frac{1}{6}t_3\,\rho^a(\vec{r}\,) \qquad t_1\left\{p_{ij}^2\,,\delta(\vec{r}_{ij})\right\} \to \left\{p_{ij}^2\,,\left[t_1 + t_4\,\rho^b(\vec{r}\,)\right]\delta(\vec{r}_{ij})\right\}$$
$$t_2\,\vec{p}_{ij}\cdot\delta(\vec{r}_{ij})\,\vec{p}_{ij} \to \vec{p}_{ij}\cdot\left[t_2 + t_5\,\rho^c(\vec{r}\,)\right]\delta(\vec{r}_{ij})\,\vec{p}_{ij}$$

- **Precision fits to nuclear masses, radii, density profiles, equations of state** (dense nuclear matter and neutron stars)

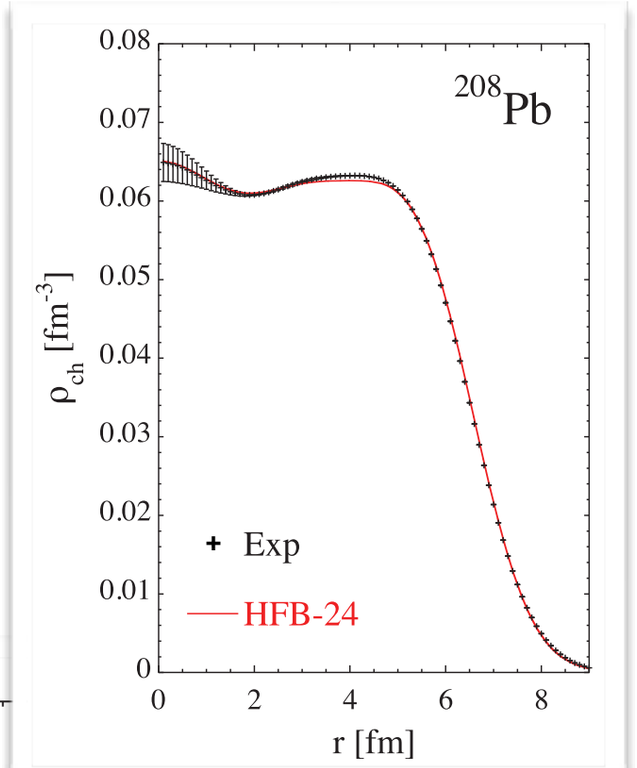
S. Goriely, N. Chamel, J.M. Pearson : Phys. Rev. C88 (2013) 024308

Selected RESULTS

- Total # of parameters: **20**



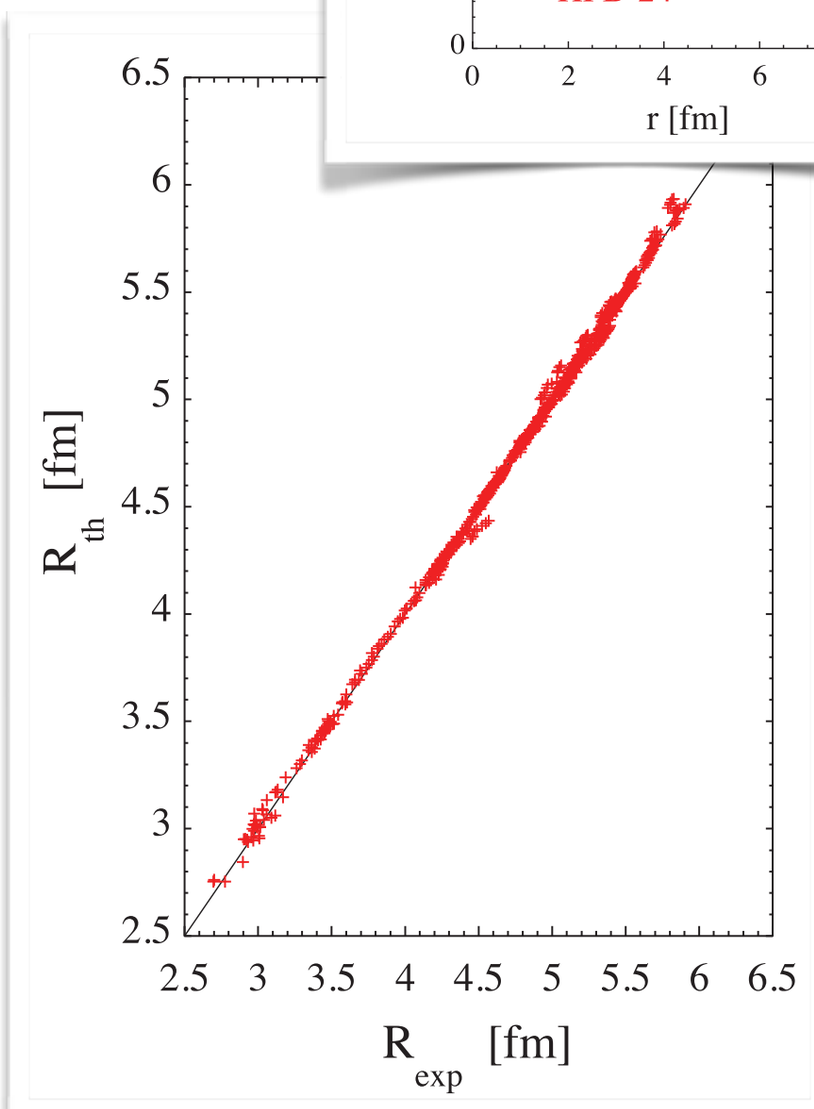
- **Nuclear charge density profiles**



- **Fits to 2353 nuclear masses r.m.s. deviation < 0.6 MeV**

S. Goriely,
N. Chamel,
J.M. Pearson
Phys. Rev. C88 (2013) 024308

- **Systematics of nuclear charge radii**

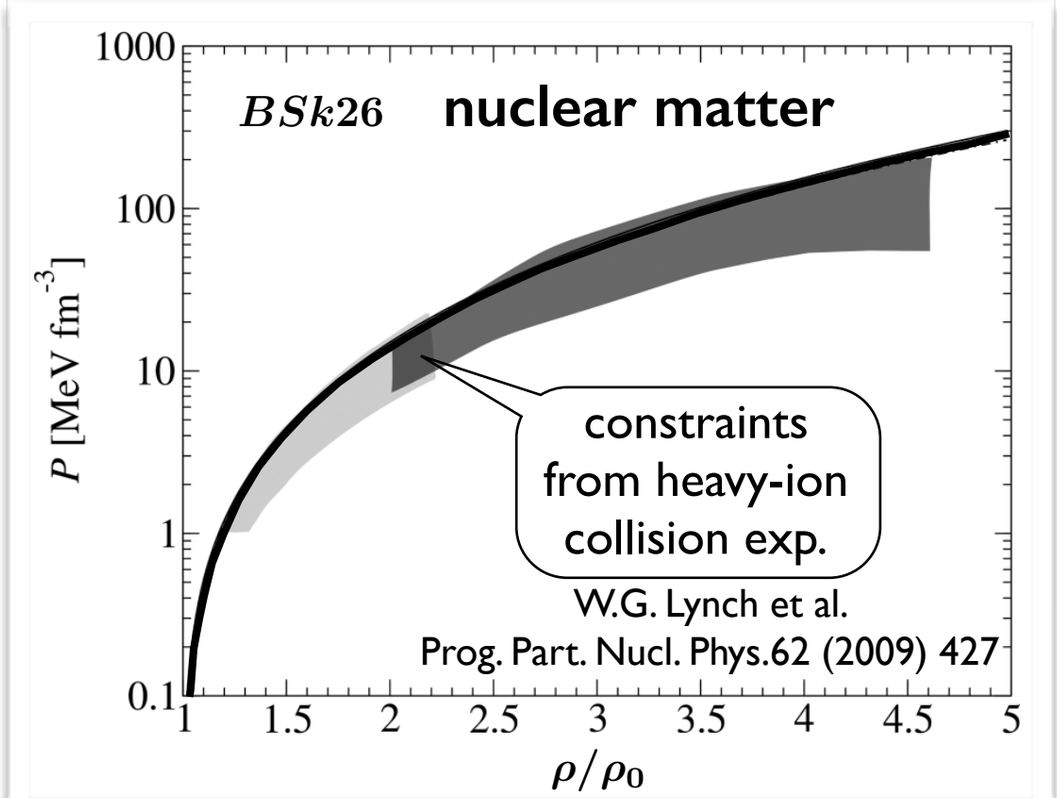


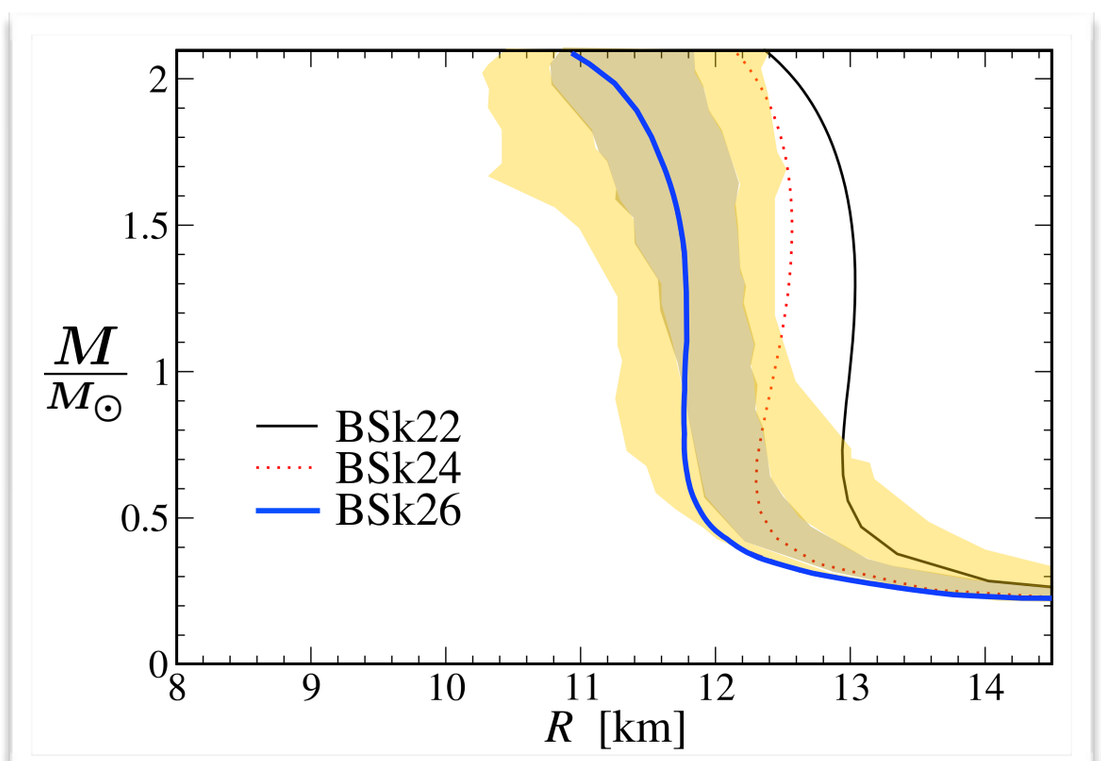
Selected RESULTS (contd.)

S. Goriely, N. Chamel, J.M. Pearson
Phys. Rev. C88 (2013) 024308
J.M. Pearson, N. Chamel, A.F. Fantina, S. Goriely
Eur. Phys. J. A50 (2014) 43

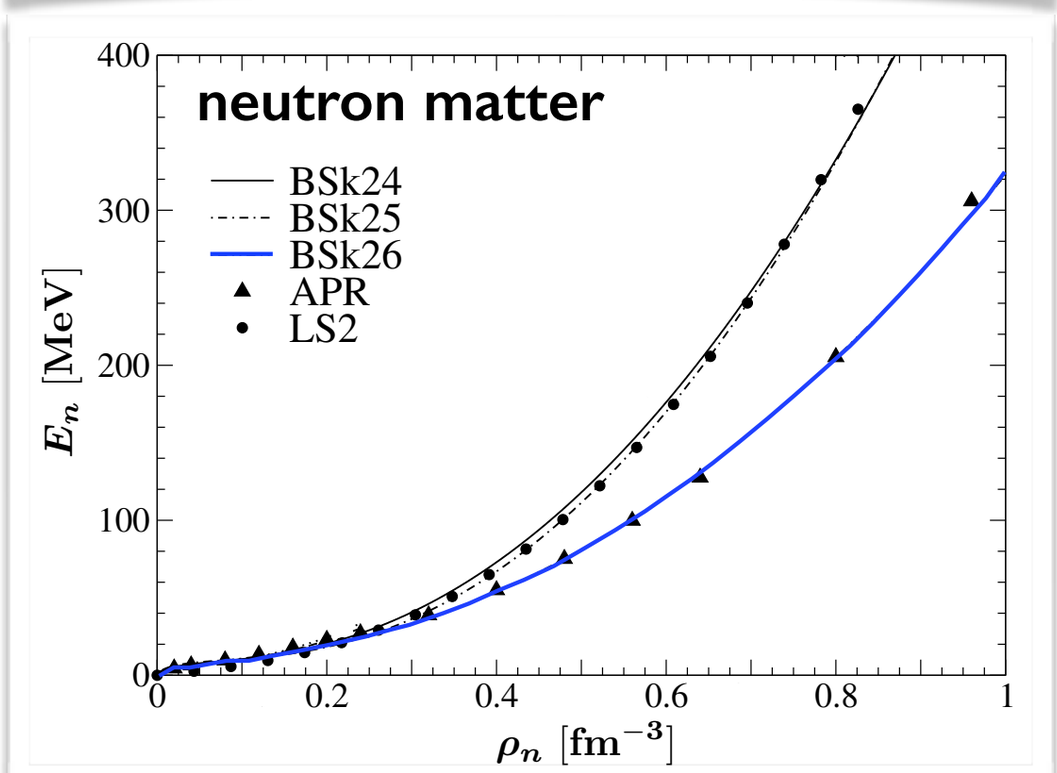
- **Equations of state:**
  Dense nuclear and neutron matter

- **Mass-radius relation**
  of neutron stars





Constraints on radii : A.W. Steiner, J.M. Lattimer, E.F. Brown
Astroph. J. 722 (2010) 33



APR : A. Akmal, V.R. Pandharipande, D.G. Ravenhall
Phys. Rev. C 58 (1998) 1804

8. From CHIRAL EFFECTIVE FIELD THEORY to ENERGY DENSITY FUNCTIONALS

- For **finite nuclear systems**, derive **energy density functional** from nuclear interactions based on **chiral pion-nucleon dynamics**
- Starting point :
  density matrix expansion for inhomogeneous symmetric nuclear matter

$$\sum_{\alpha} \Psi_{\alpha}\left(\vec{r}+\vec{a}/2\right)\ \Psi_{\alpha}^{\dagger}\left(\vec{r}-\vec{a}/2\right) =$$

$$\left[\frac{3}{ak_F}\rho - \frac{a}{2k_F}\left(\tau - \frac{3}{5}\rho\, k_F^2 - \frac{1}{4}\nabla^2\rho\right) + \frac{3i}{2ak_F}\vec{\sigma}\cdot\left(\vec{a}\times\vec{J}\right)\right] j_1(ak_F) + \ldots$$

- **Local nucleon density** $\rho(\vec{r}) = \dfrac{2\,k_F^3(\vec{r})}{3\pi^2} = \sum_{\alpha} \Psi_{\alpha}^{\dagger}(\vec{r})\,\Psi_{\alpha}(\vec{r})$
- **Kinetic energy density** $\tau(\vec{r}) = \sum_{\alpha} \vec{\nabla}\Psi_{\alpha}^{\dagger}(\vec{r})\cdot\vec{\nabla}\Psi_{\alpha}(\vec{r})$
- **Spin-orbit density** $\vec{J}(\vec{r}) = i\sum_{\alpha} \Psi_{\alpha}^{\dagger}(\vec{r})\,\vec{\sigma}\times\vec{\nabla}\Psi_{\alpha}(\vec{r})$

B. Gebremariam, S.K. Bogner, T. Duguet : Phys. Rev. C82 (2010) 014305

NUCLEAR ENERGY DENSITY FUNCTIONAL

- N = Z systems -

- **up to 2nd order in the gradient expansion :**

$$\mathcal{E}[\rho,\tau,\vec{J}] = \frac{E(\rho)}{A}\rho + \left(\frac{\tau}{2M_N} - \frac{3\rho\,k_F^2}{10M_N}\right)\left[1 - \frac{k_F^2}{2M_N^2} + 2M_N\,F_\tau(\rho)\right]$$
$$+ (\vec{\nabla}\rho)^2\,F_\nabla(\rho) + \vec{\nabla}\rho\cdot\vec{J}\,F_{so}(\rho) + \vec{J}^2\,F_J(\rho)$$

homogeneous nuclear matter

- **Effective mass :** $M_N^*(\rho) = M_N\left[1 - \frac{k_F^2}{2M_N^2} + 2M_N\,F_\tau(\rho)\right]^{-1}$

$$F_\tau(\rho) = \frac{1}{2k_F}\frac{\partial U(p,k_F)}{\partial p}\bigg|_{p=k_F} = -\frac{k_F}{3\pi^2}\,f_1(k_F)$$

p-wave Fermi liquid parameter

- **Relation to original Skyrme parameters :**

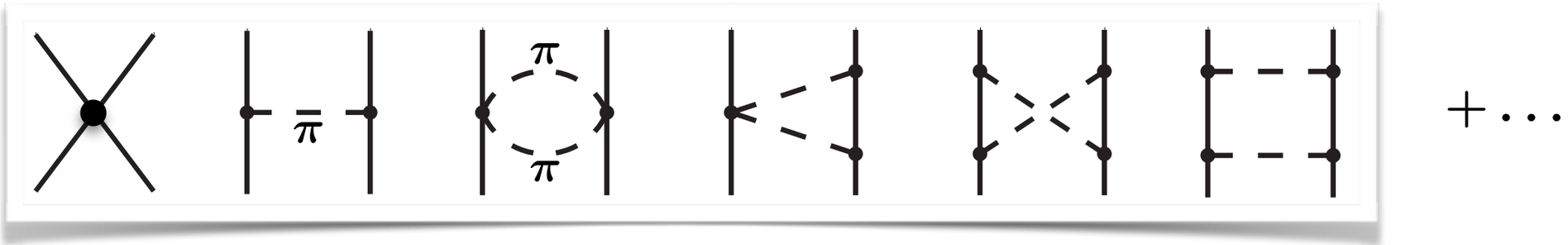
$$F_\tau = \frac{\rho}{16}(3t_1 + 5t_2) \qquad F_\nabla = \frac{1}{64}(9t_1 - 5t_2)$$

$$F_{so} = \frac{3}{4}W_0 \qquad F_J = \frac{1}{32}(t_1 - t_2)$$

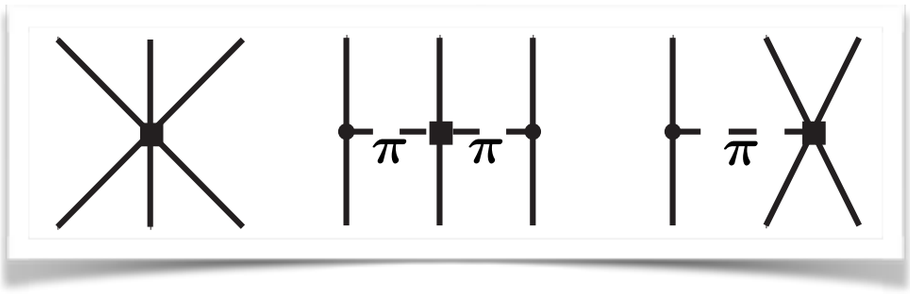
NUCLEAR ENERGY DENSITY FUNCTIONAL from CHIRAL 2- and 3-NUCLEON INTERACTIONS

J.W. Holt, N. Kaiser, W.W. : Eur. Phys. J. A47 (2011) 128

- **N3LO chiral NN interactions**



- **N2LO three-body forces**



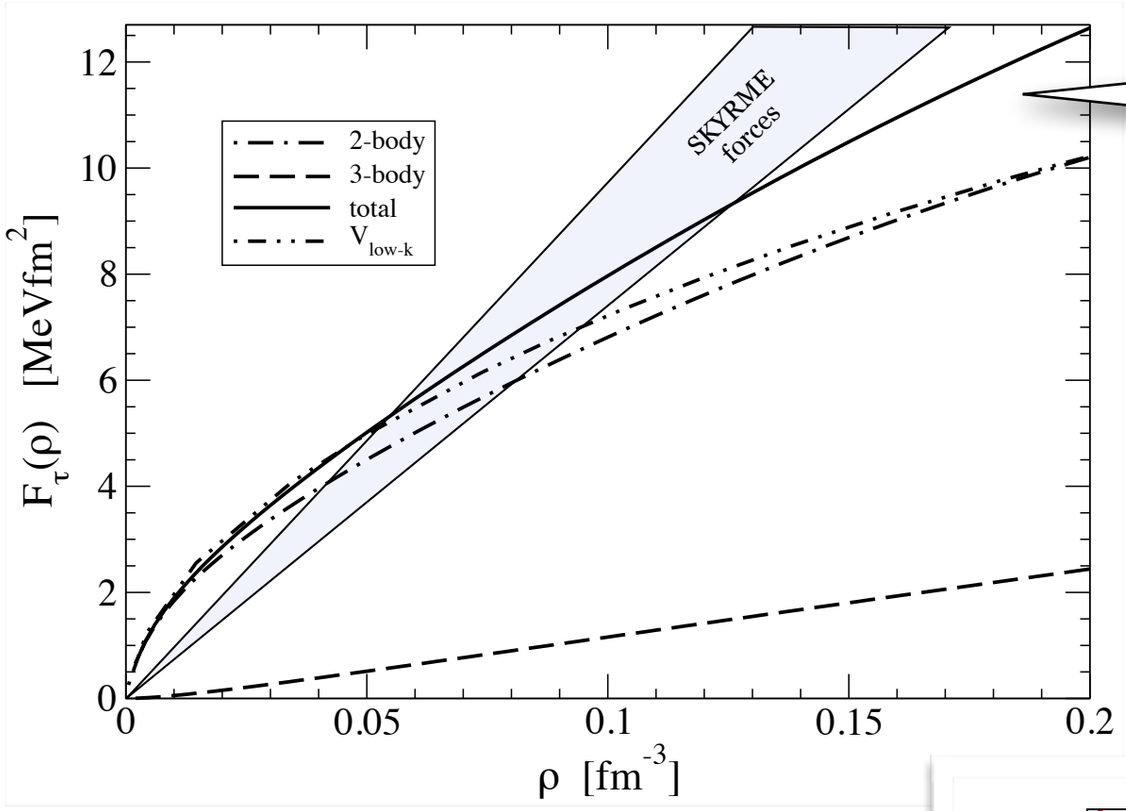
- **Contact terms + multipion exchange finite range interactions**

- **Finite range forces** $=$

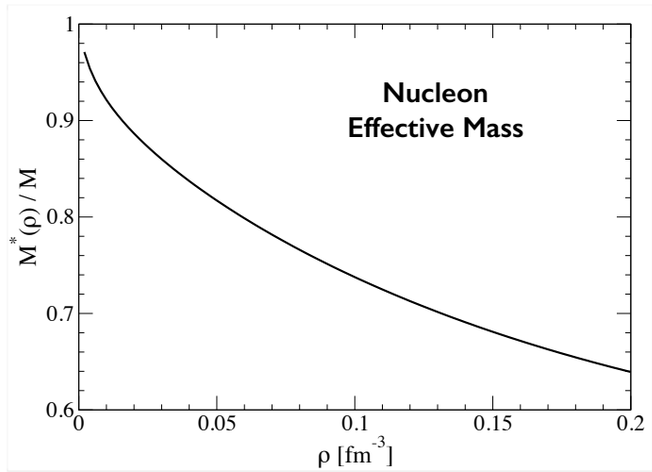
$$V_C(q) + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_C(q) + [V_S(q) + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_S(q)] \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$+[V_T(q) + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_T(q)] \, \vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_2 \cdot \vec{q}$$
$$+[V_{SO}(q) + \vec{\tau}_1 \cdot \vec{\tau}_2 \, W_{SO}(q)] \, i(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{q} \times \vec{p}\,) \, ,$$

generate strongly density dependent functions $F_\tau(\rho)$, $F_\nabla(\rho)$, $F_{so}(\rho)$, $F_J(\rho)$

at **Hartree-Fock** level



$$F_\tau(\rho) = \frac{1}{2k_F} \left. \frac{\partial U(p, k_F)}{\partial p} \right|_{p=k_F}$$



(density gradient)^2 term

- **note importance of three-body correlations**

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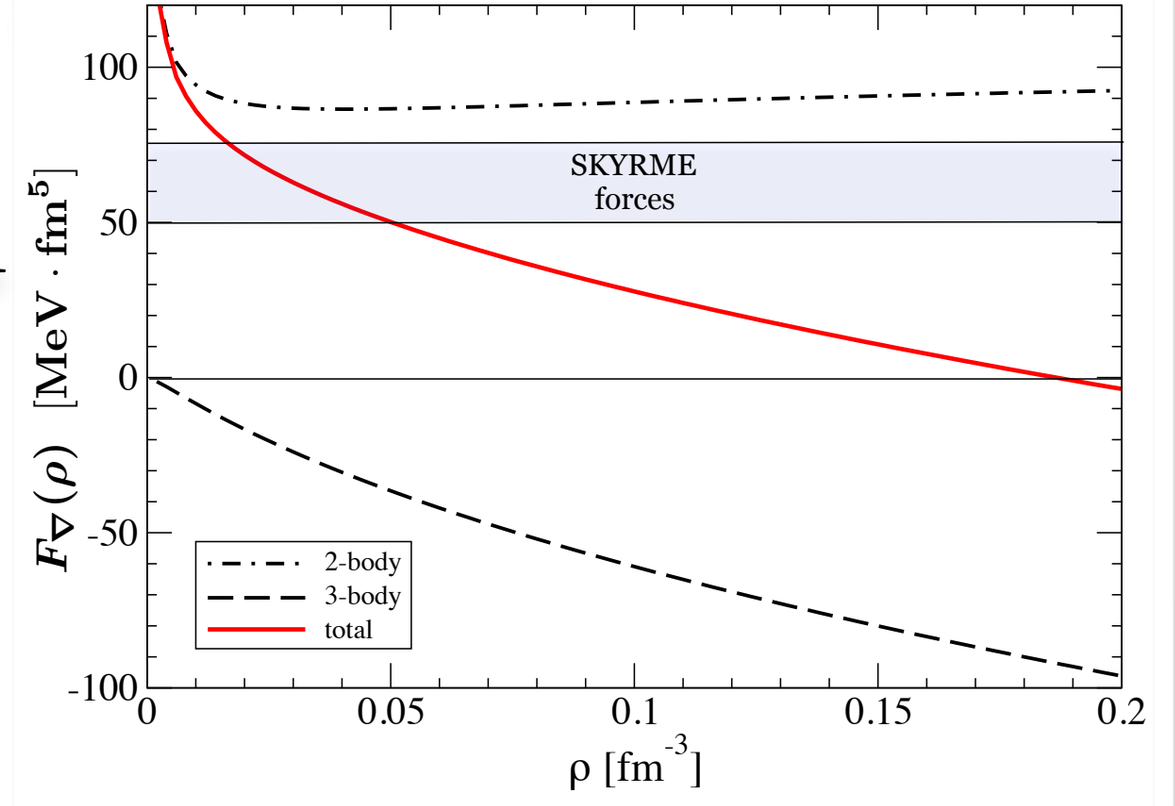

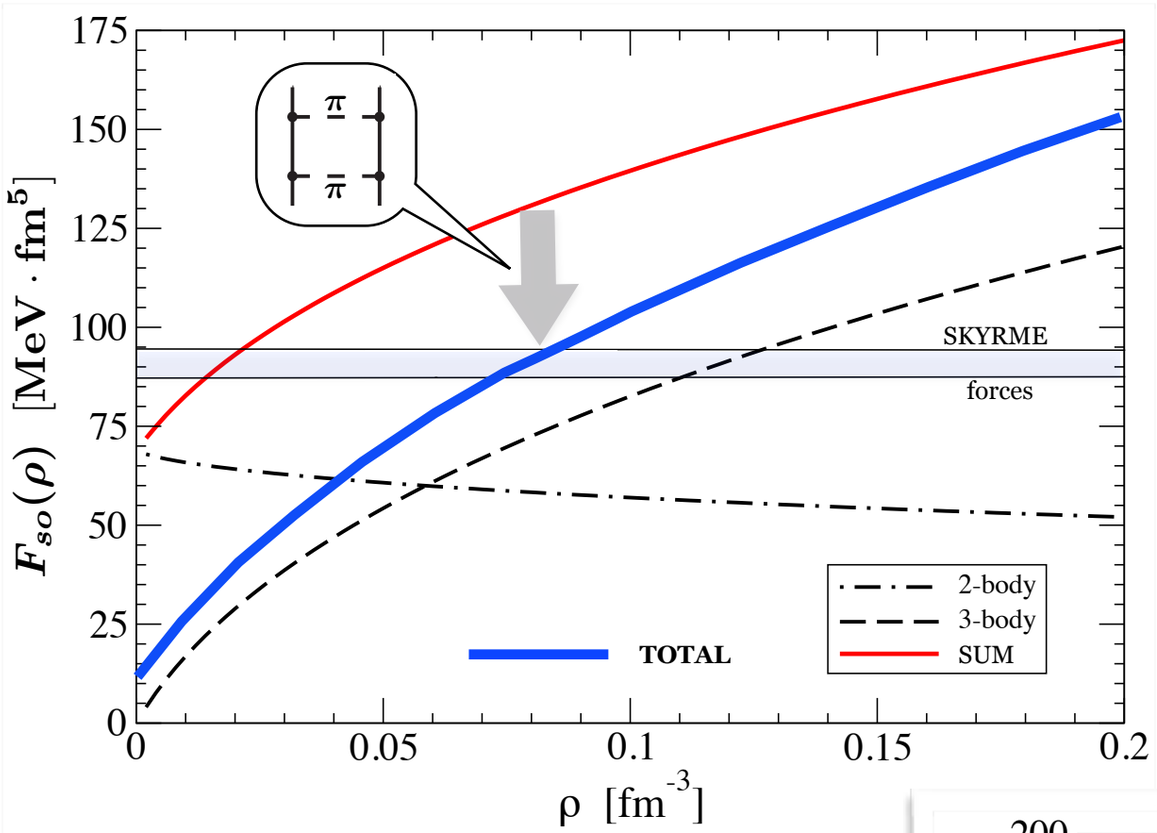

Spin – orbit coupling

- Contributions from :
  - short-range dynamics
  - three-nucleon forces
  - iterated pion-exchange (2nd order tensor force)

Quadratic spin – orbit interaction

- governed by one-pion exchange

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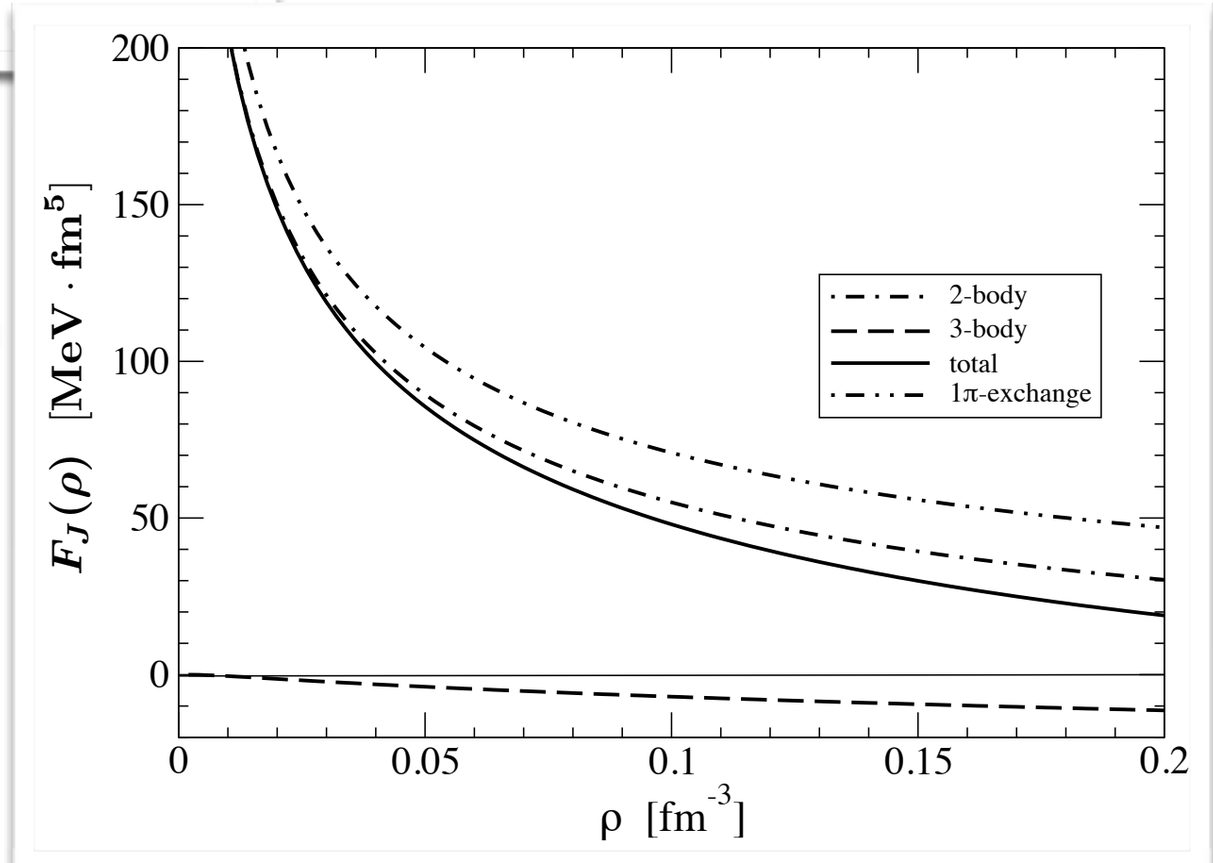

SUMMARY & CONCLUSIONS

- Systematic framework at the interface of **QCD** (with light quarks) and the physics of **hadrons, nuclei** and **nuclear forces** :

CHIRAL EFFECTIVE FIELD THEORY
based on spontaneously broken
Chiral Symmetry of **Low-Energy QCD**

- **Chiral EFT ⟷ DFT connection**
  - New **density dependent strength functions** of energy functional from **finite-range forces** governed by **chiral (pion) dynamics**
- New constraints from **massive neutron stars** for the **equation-of-state** of **dense & cold baryonic matter** :
  - Mass - radius relation: **stiff equation of state** required ! **No** ultrahigh densities ($\varrho_{\mathbf{core}} \lesssim 5\,\varrho_{\mathbf{0}}$)
  - "**Conventional**" (non-exotic) **EoS works** remarkably well (nuclear effective field theory + advanced many-body methods)