*Interdisciplinary symposium on modern density functional theory*

*19-23 June 2017 Okochi Hall, RIKEN (Wako campus)*

# Covariant Density Functional Theory with Fock Terms

## — Role of the non-local Fock terms —

LONG, Wen Hui

School of Nuclear Science and Technology, Lanzhou University

# OUTLINE


1. **Introduction**
   - New Challenge and Opportunity in Nuclear Physcis
   - Covariant Density Functional theory
2. **Covariant density functional with Fock terms**
   - General Formalism for RHF Scheme
   - Relativistic Hartree-Fock-Bogoliubov method
   - Covariant density functionals with Fock terms
3. **Role of non-local Fock terms in describing nuclear structure**
   - New Balance induced by Fock terms in nuclear force
   - Neutron halo phenomena and Proton pseudo-spin symmetry
   - Nuclear tensor force components in Fock diagrams
4. **Conclusions and Perspectives**

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# Nucleus and Nuclear Structure

- Atomic Nucleus: one fundamental hierarchy of matter, Small (in unit of fm)
  - ☞ composed of neutrons and protons.
  - ☞ three types of force (strong, weak, electro-magnetic)
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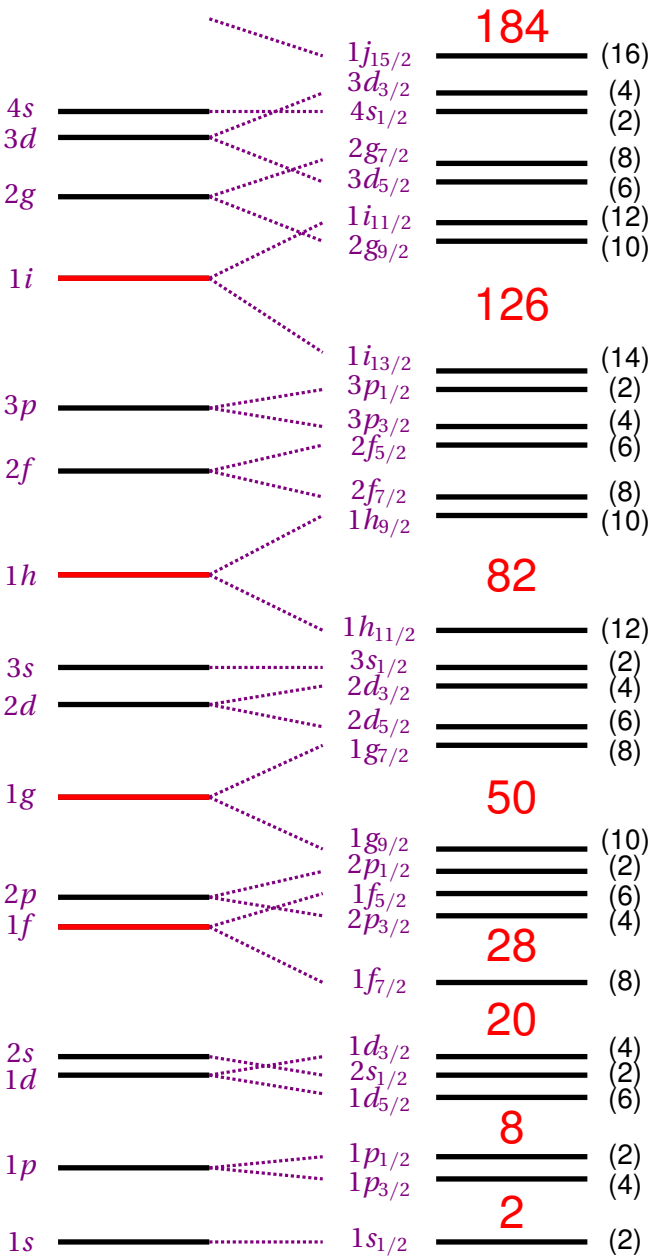
  - ☞ $nl_j$ denotes the orbits, $j = l \pm \frac{1}{2}$, e.g., $2s_{1/2}$.



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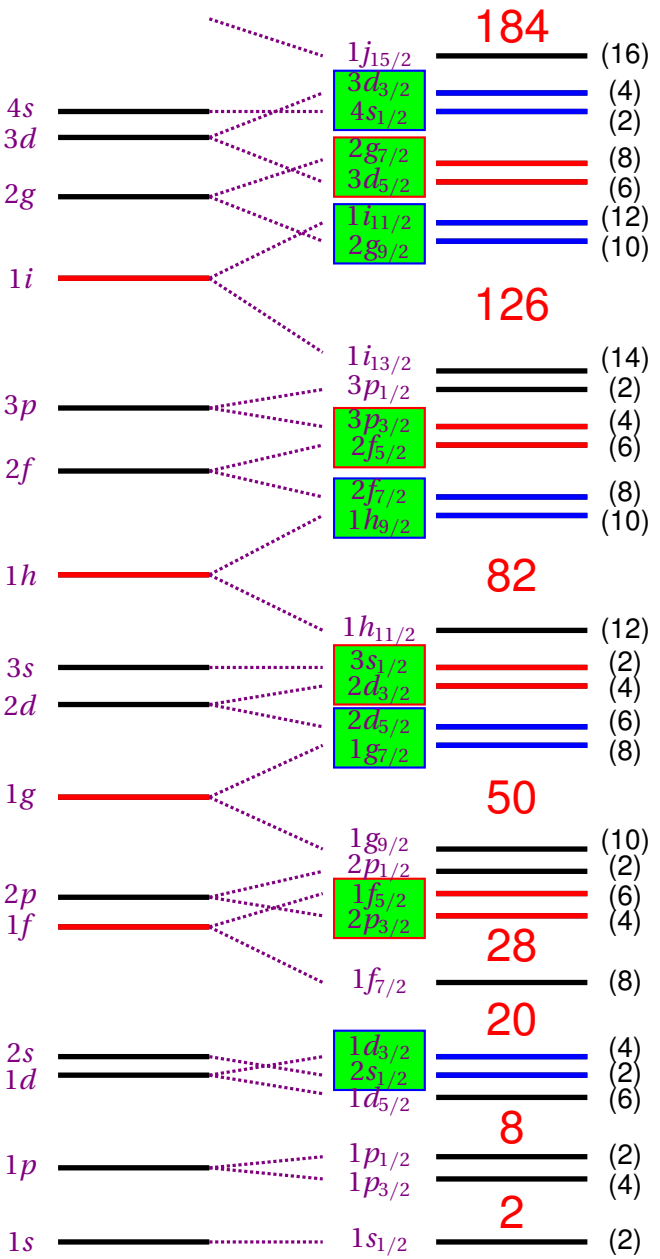
- Pseudo-spin symmetry (PSS): nearly degenerated orbits with quantum numbers $(n, l, j = l+1/2)$ and $(n' = n-1, l' = l+2, j' = l'-1/2)$.



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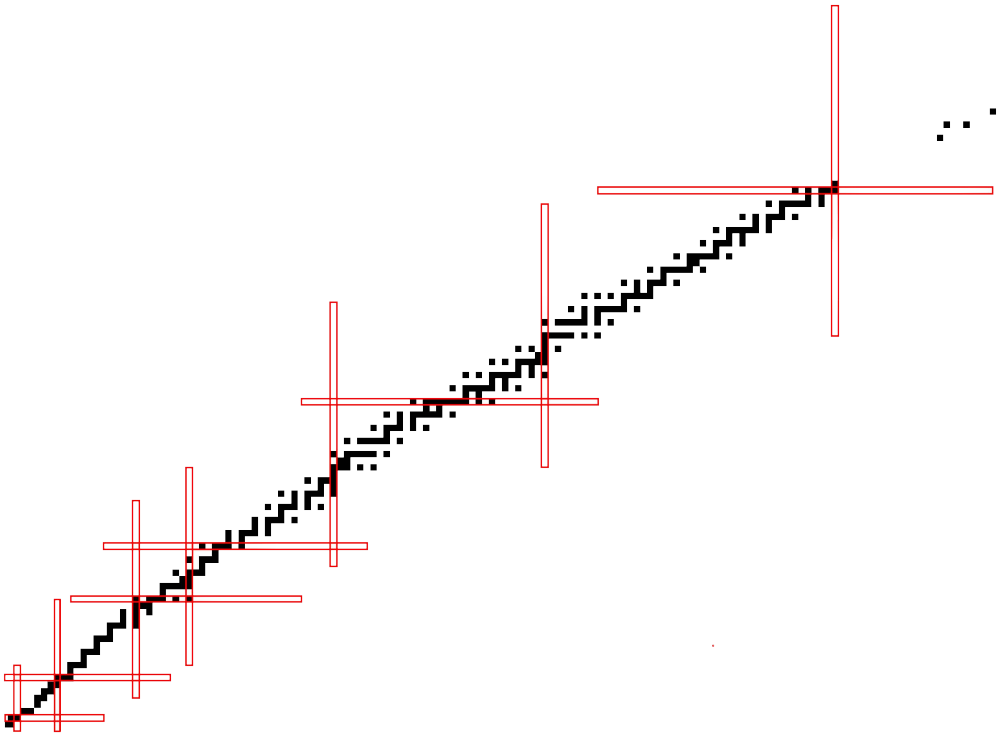

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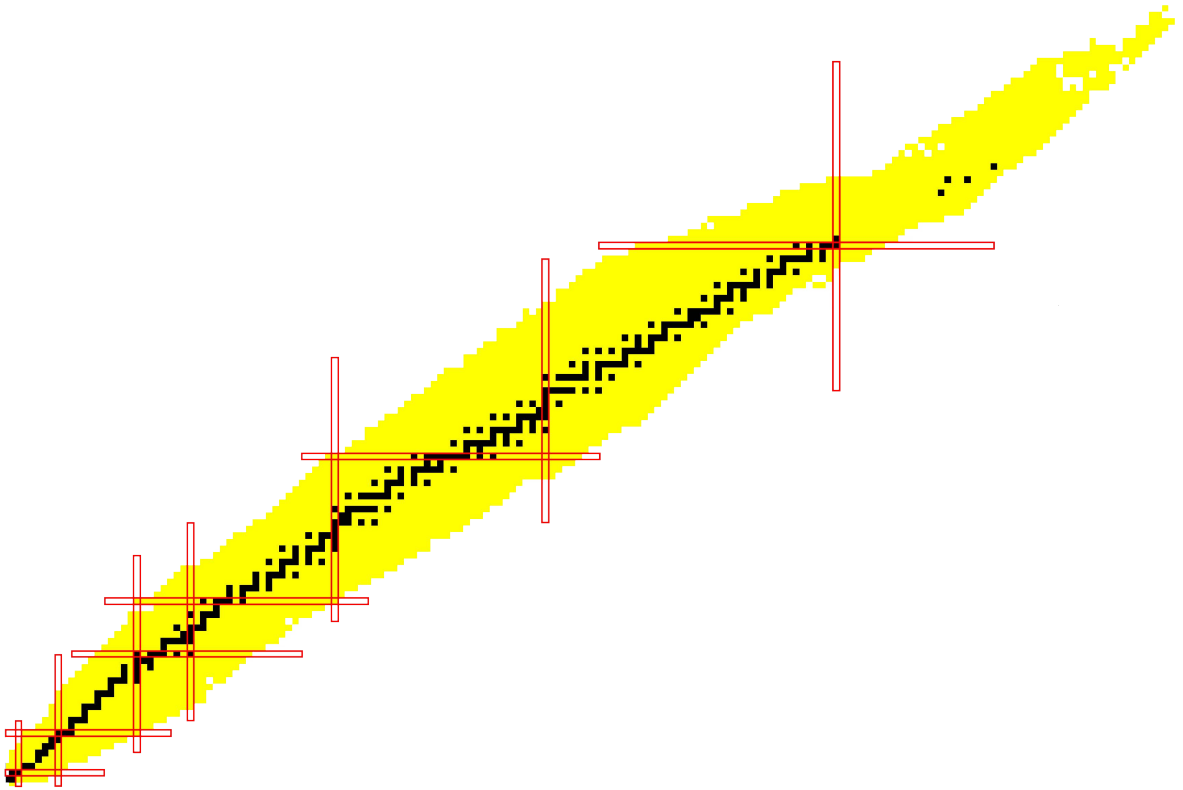

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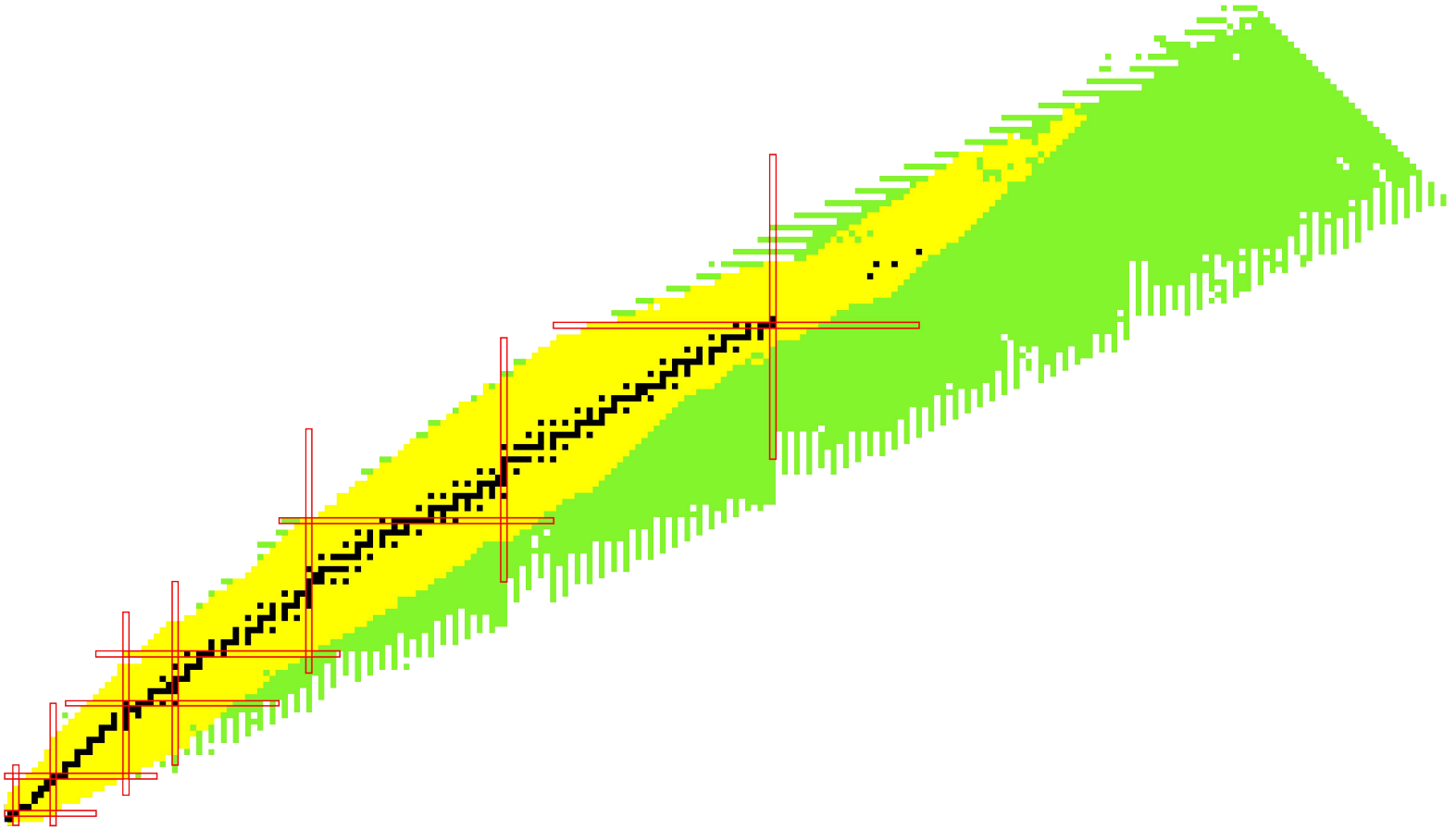

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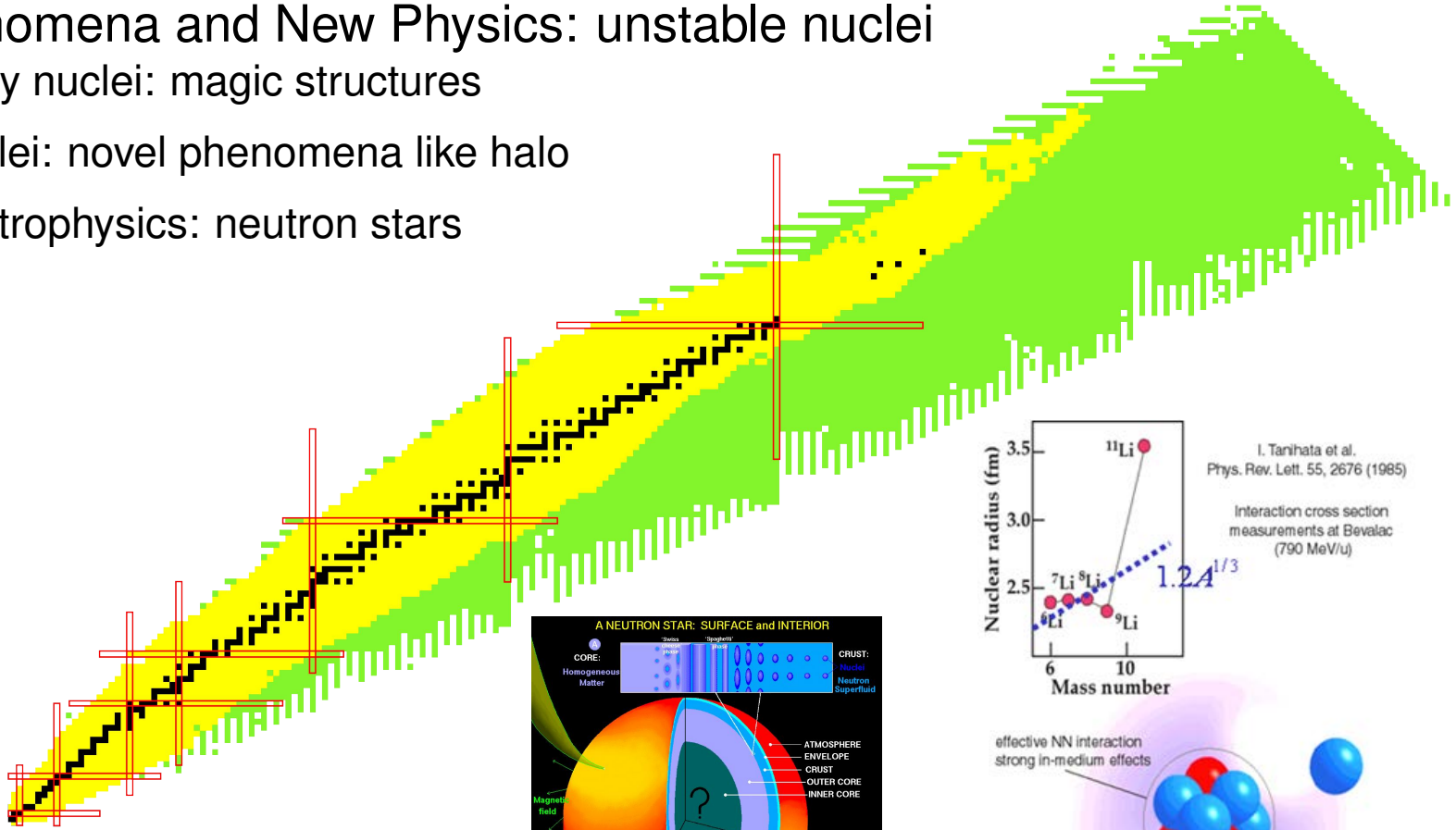
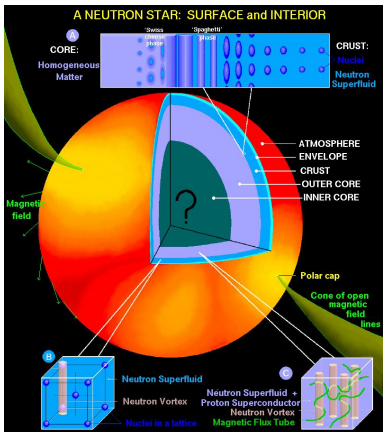
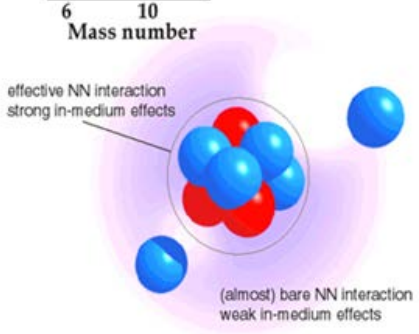
  - Q2: Exotic Nuclei: novel phenomena like halo
  - Q3: Nuclear astrophysics: neutron stars











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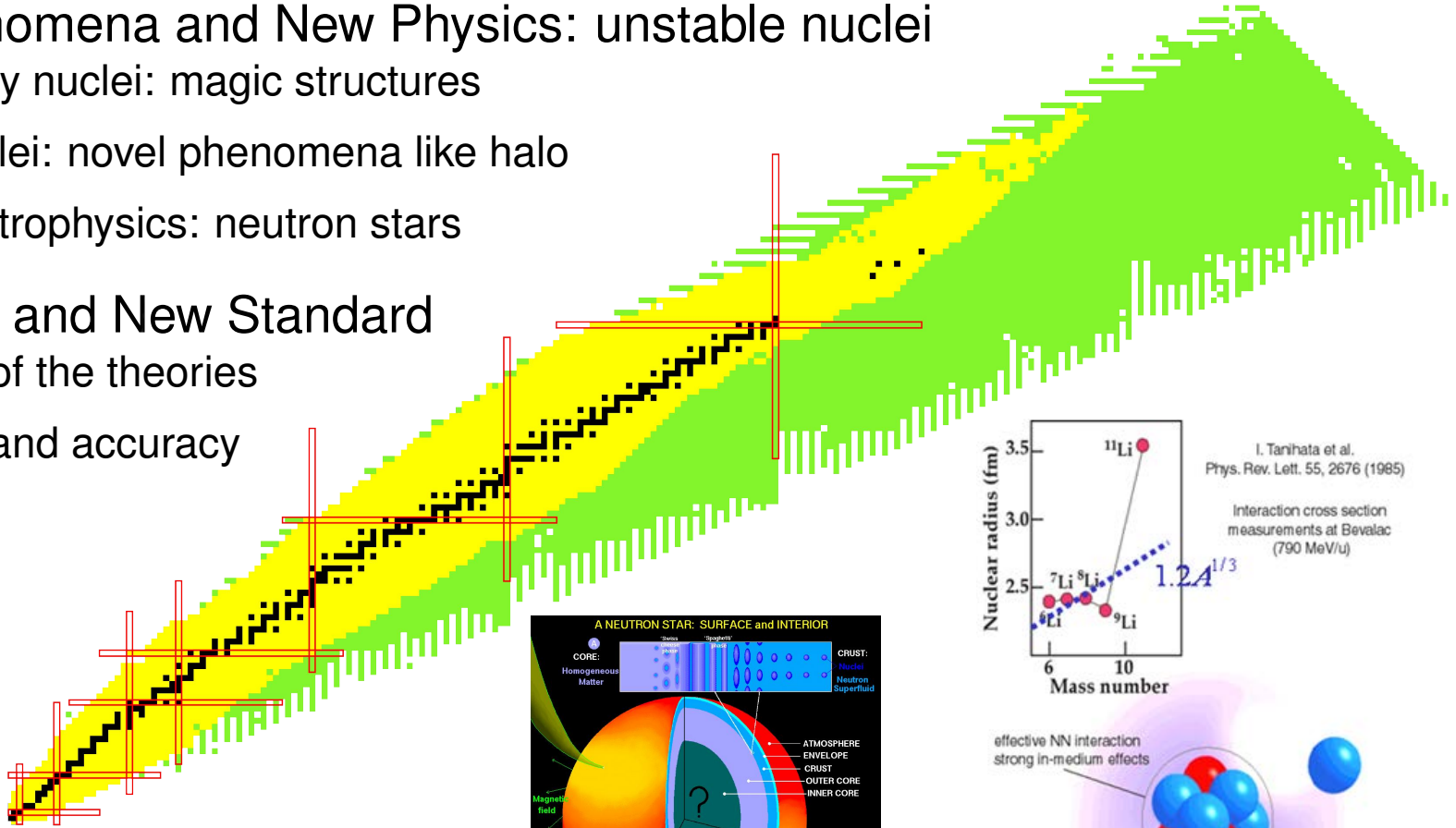

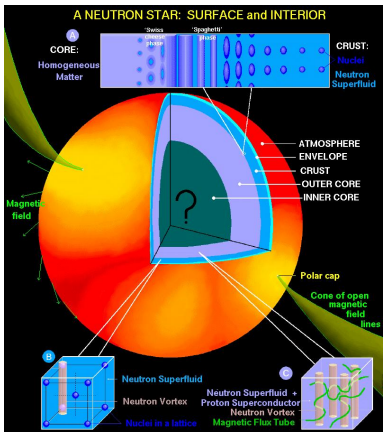





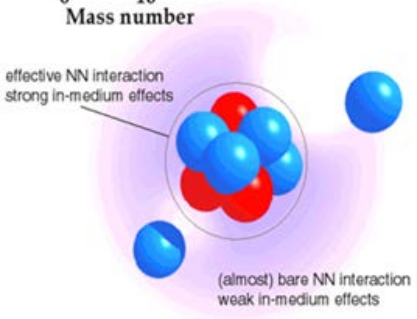

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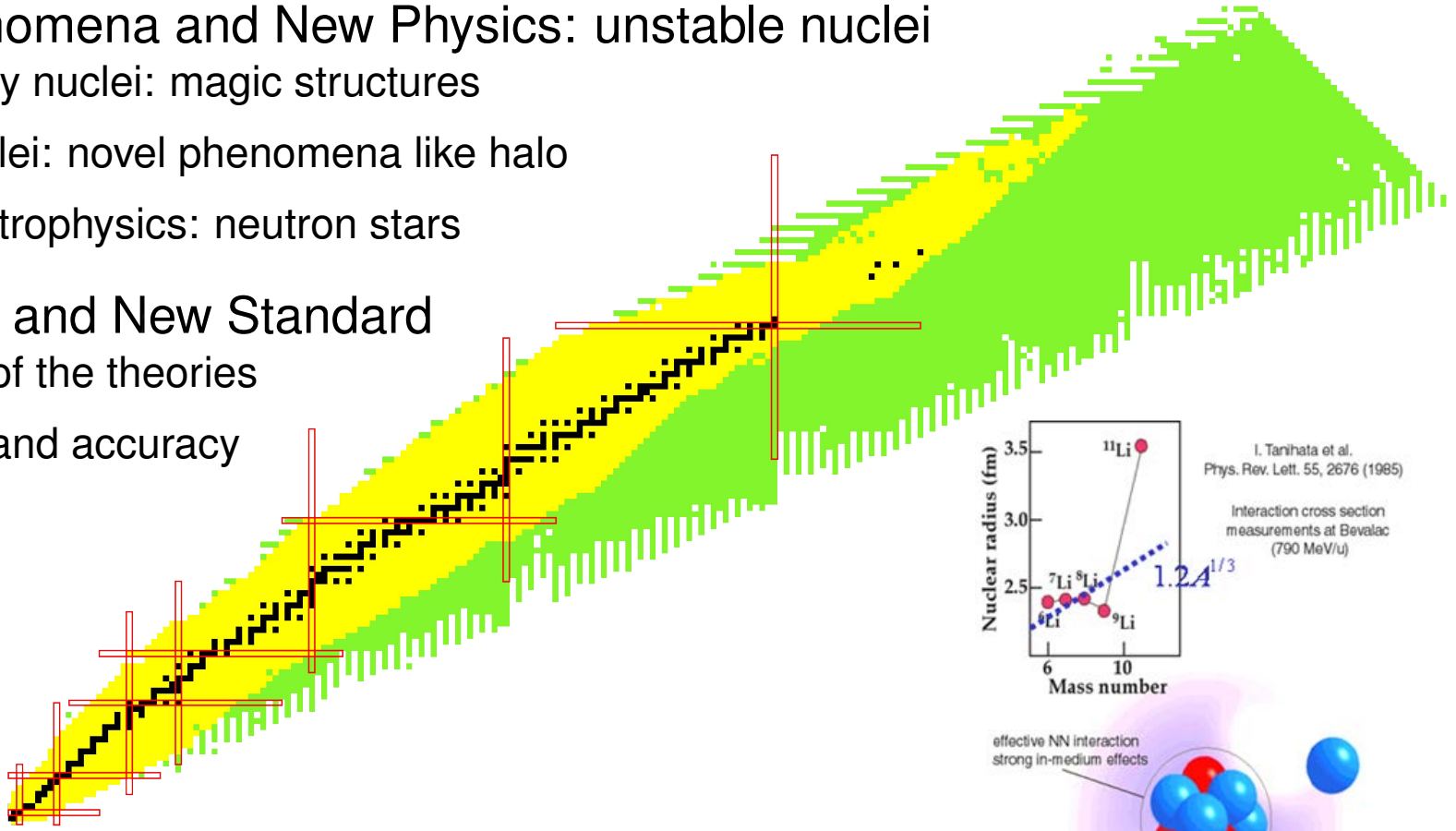
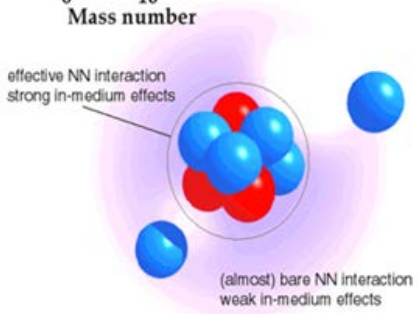




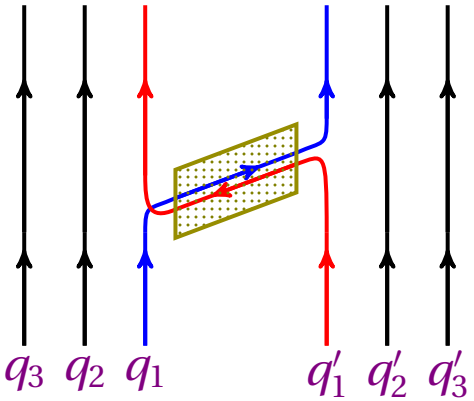




- Covariant Density Functional theory
  - Q1: Reliability: relativity and spirit of density functional theory
  - Q2: Exchange correlations introduced by Fock terms

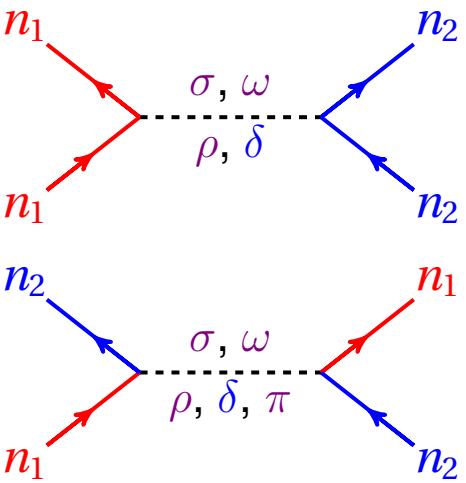
# Nuclear Force: Meson Exchange Diagram

- Nuclear Force: meson exchange diagram

Yukawa (1935)




Sketch of meson exchange diagram



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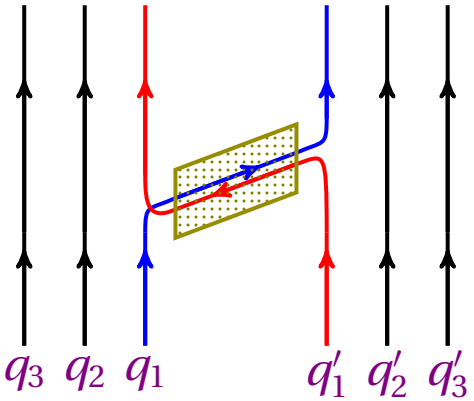


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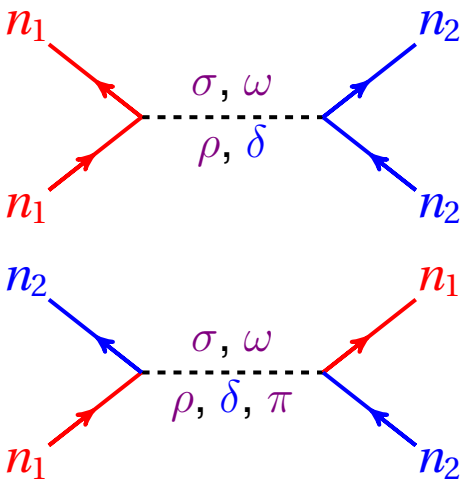
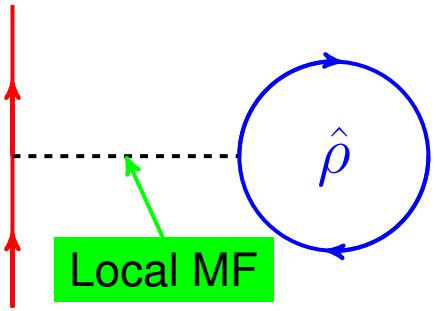
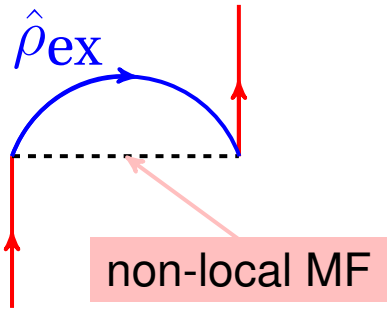



- Mean field (MF) approach: nucleon moving in the MF generated by others
  — Being consistent with principle of density functional theory




$\hat{\rho}$: local density

$\hat{\rho}_{ex}$: non-local density

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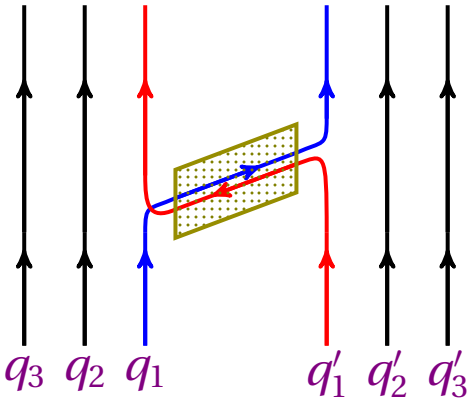


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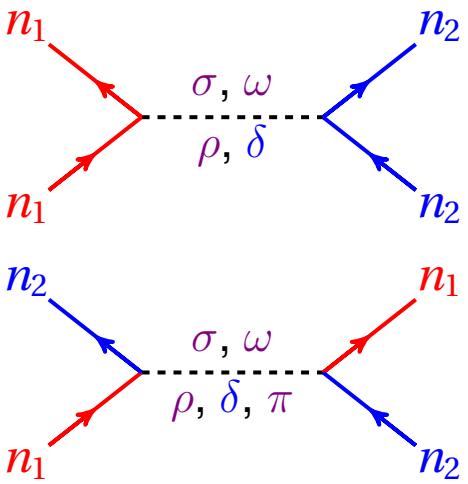


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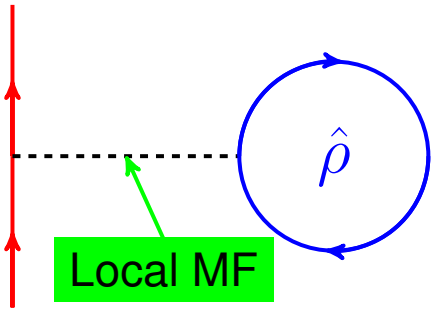


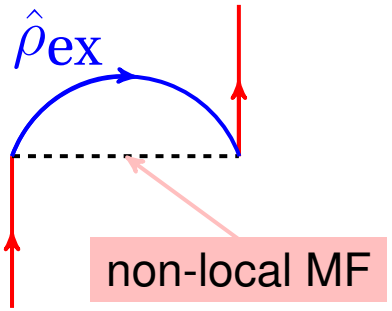


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- Complicated nuclear in-medium effects: non-perturbative nuclear force?

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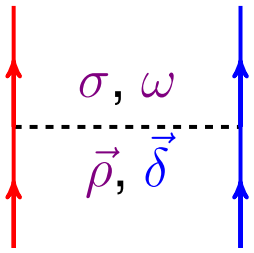
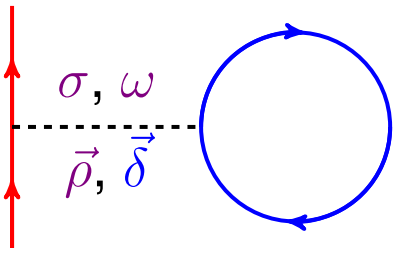
- CDF theory w/o Fock terms: relativistic mean field (RMF) theory
  - Walecka(1974), Serot(1986), Reihard(1989), Ring(1996), Bender(2003), Meng(2006)
  - ☞ Natural treatment of spin-orbit coupling: Covariant framework
  - ☞ Missing important ingredient like tensor force due to the limit of Hartree approach




Hartree terms




Local MF

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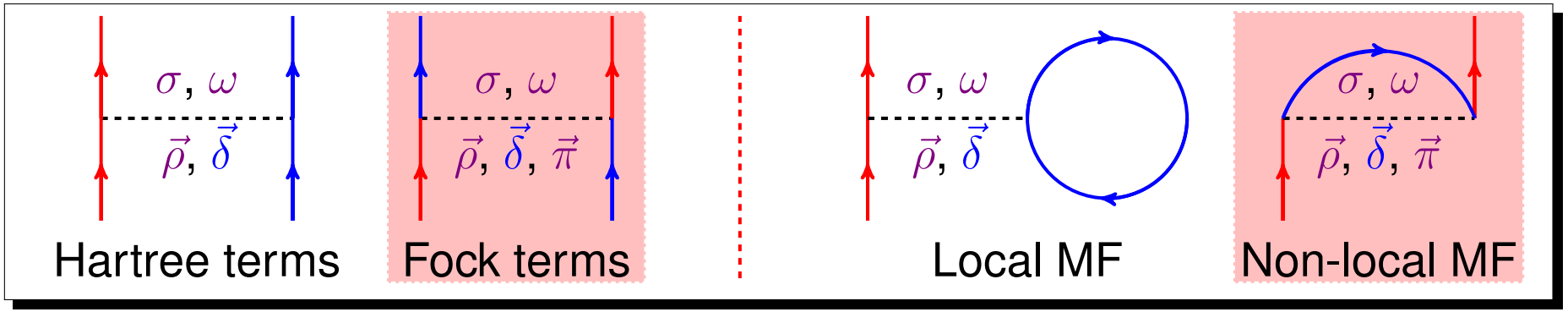
- CDF theory with Fock terms: relativistic Hartree-Fock (RHF) theory
  - Bouyssy (1987), Bernardos (1993), Shi (1995), Marcos (2004), Long (2006-2010).
  - ☞ Maintain the advantages of RMF theory, and include the tensor force naturally.
  - ☞ Non-local Fock terms are not easy to handle.



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  —Walecka, Ann. Phys. 83, 491 (1974); Serot & Walecka, Adv. Nucl. Phys. 16, 1 (1986).
- Medium effects: non-linear self-couplings of $\sigma$- and $\omega$-mesons

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- Various effective interactions: define a covariant density functional

  —PK1 & PKDD: W. H. Long, J. Meng, N. Van Giai, and S.-G. Zhou, PRC 69, 034319 (2004).

  —PC-PK1: P. W. Zhao, Z. P. Li, J. M. Yao, and J. Meng, PRC 82, 054319 (2010).

# CDF theory with Fock terms: RHF theory

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- Density-dependent relativistic Hartree-Fock (DDRHF) theory
  1. CDFs with Fock terms provide satisfied quantitative precision as RMF: PKO1-3、PKA1

  —PLB 640, 150 (2006); EPL 82 (2008) 12001; PRC 76, 034314 (2007).

  2. Relativistic Hartree-Fock-Bogoliubov (RHFB) theory: exotic and superheavy nuclei

  —W.H. Long, P. Ring, N. Van Giai, and J. Meng, PRC 81, 024308 (2010).

  —J.-P. Ebran, E. Khan, D. Pena Arteaga, and D. Wretenar, PRC 83, 064323 (2011).

  3. fully self-consistent RHF RPA and RHFB QRPA: nuclear isospin excitations

  —RHF+RPA: H.Z. Liang, N. Van Giai, J. Meng, PRL **101**, 122502 (2008).

  —RHFB+QRPA: Z.M. Niu, Y.F. Niu, H.Z. Liang, W.H. Long et al, PLB 723 (2013) 172.

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# Lagrangian Density

- Lagrangian density: Nucleon ($\psi$), Mesons ($\sigma$, $\omega$, $\rho$, $\pi$), Photon ($A$)

$$\mathscr{L} = \mathscr{L}_M + \mathscr{L}_\sigma + \mathscr{L}_\omega + \mathscr{L}_\rho + \mathscr{L}_\pi + \mathscr{L}_A + \mathscr{L}_I, \tag{1}$$

where the Lagrangians of the free fields $\mathscr{L}_\phi$ ($\phi = \psi, \sigma, \omega^\mu, \vec{\rho}^\mu, \vec{\pi}$ and $A^\mu$):

$$\mathscr{L}_M = \bar{\psi}\left(i\gamma^\mu\partial_\mu - M\right)\psi, \qquad \mathscr{L}_\rho = -\frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu, \tag{2}$$

$$\mathscr{L}_\sigma = +\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2, \qquad \mathscr{L}_\pi = +\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi}, \tag{3}$$

$$\mathscr{L}_\omega = -\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu, \qquad \mathscr{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{4}$$

and the interaction one between nucleon and mesons (photon) $\mathscr{L}_I$:

$$\mathscr{L}_I = \bar{\psi}\Big(-g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma^\mu\vec{\tau}\cdot\vec{\rho}_\mu - e\gamma^\mu\frac{1-\tau_3}{2}A_\mu + \frac{f_\rho}{2M}\sigma_{\mu\nu}\partial^\nu\vec{\rho}^\mu\cdot\vec{\tau} - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi}\cdot\vec{\tau}\Big)\psi, \tag{5}$$

with $\Omega^{\mu\nu} \equiv \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$, $\vec{R}^{\mu\nu} \equiv \partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu$, and $F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$.

# Hamiltonian

A. Bouyssy (1987)

- System Hamiltonian ($\phi = \sigma$-S, $\omega$-V, $\rho$-V, $\rho$-VT, $\rho$-T, $\pi$-PV and $A$-V):

$$H = \int d\boldsymbol{x}\bar{\psi}\left(-i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + M\right)\psi + \frac{1}{2}\int d\boldsymbol{x}d\boldsymbol{x}'\bar{\psi}(\boldsymbol{x})\bar{\psi}(\boldsymbol{x}')\Gamma_\phi D^\phi\psi(\boldsymbol{x}')\psi(\boldsymbol{x})$$

- Interaction matrices $\Gamma_\phi(x, x')$:

$$\Gamma_{\sigma\text{-S}} \equiv -g_\sigma(x)g_\sigma(x'), \qquad \Gamma_{A\text{-V}} \equiv \frac{e^2}{4}\left[\gamma_\mu\left(1-\tau_3\right)\right]_x\left[\gamma^\mu\left(1-\tau_3\right)\right]_{x'}, \tag{6}$$

$$\Gamma_{\omega\text{-V}} \equiv \left(g_\omega\gamma_\mu\right)_x\left(g_\omega\gamma^\mu\right)_{x'}, \qquad \Gamma_{\pi\text{-PV}} \equiv \frac{-1}{m_\pi^2}\left(f_\pi\vec{\tau}\gamma_5\gamma_\mu\partial^\mu\right)_x\cdot\left(f_\pi\vec{\tau}\gamma_5\gamma_\nu\partial^\nu\right)_{x'}, \tag{7}$$

$$\Gamma_{\rho\text{-V}} \equiv \left(g_\rho\gamma_\mu\vec{\tau}\right)_x\cdot\left(g_\rho\gamma^\mu\vec{\tau}\right)_{x'}, \qquad \Gamma_{\rho\text{-T}} \equiv \frac{1}{4M^2}\left(f_\rho\sigma_{\nu k}\vec{\tau}\partial^k\right)_x\cdot\left(f_\rho\sigma^{\nu l}\vec{\tau}\partial_l\right)_{x'}, \tag{8}$$

$$\Gamma_{\rho\text{-VT}} \equiv \frac{1}{2M}\left(f_\rho\sigma^{k\nu}\vec{\tau}\partial_k\right)_x\cdot\left(g_\rho\gamma_\nu\vec{\tau}\right)_{x'} + \left(g_\rho\gamma_\nu\vec{\tau}\right)_x\cdot\frac{1}{2M}\left(f_\rho\sigma^{k\nu}\vec{\tau}\partial_k\right)_{x'} \tag{9}$$

- Propagators $D_\phi\left(\boldsymbol{x}, \boldsymbol{x}'\right)$: neglecting the retardation effects

$$D_\phi\left(\boldsymbol{x}, \boldsymbol{x}'\right) = \frac{1}{4\pi}\frac{e^{-m_\phi\left|\boldsymbol{x}-\boldsymbol{x}'\right|}}{\left|\boldsymbol{x}-\boldsymbol{x}'\right|}, \qquad D_A\left(\boldsymbol{x}, \boldsymbol{x}'\right) = \frac{1}{4\pi}\frac{1}{\left|\boldsymbol{x}-\boldsymbol{x}'\right|} \tag{10}$$

# RHF energy density functional (EDF) A. Bouyssy(1987)

- Solutions of Dirac Eq.: $\left\{\varepsilon_k > 0, c_k, c_k^\dagger \text{ (Fermi sea)};\ \varepsilon_l < 0, c_l, c_l^\dagger \text{ (Dirac sea)}\right\}$
- Quantizing nucleon spinor:

$$\psi = \sum_k \psi_k(\boldsymbol{x}) e^{-i\varepsilon_k t} c_k + \sum_l \psi_l(\boldsymbol{x}) e^{-i\varepsilon_l t} d_l^\dagger,$$

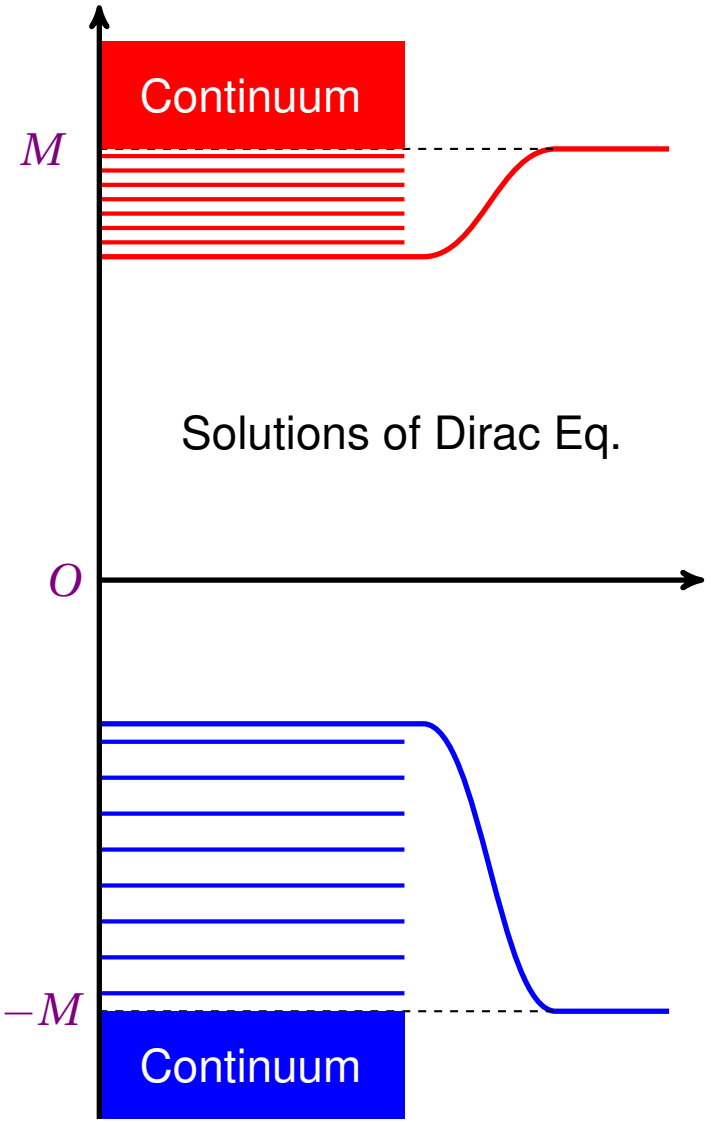
Ground state with no-sea approximation

$$|\Phi_0\rangle = \prod_{i=1}^{A} c_i^\dagger |0\rangle ,$$

RHF EDF: expectation of $H$ referring to $|\Phi_0\rangle$

$$E = \langle\Phi_0| H |\Phi_0\rangle = \langle\Phi_0| T |\Phi_0\rangle + \sum_\phi \langle\Phi_0| V_\phi |\Phi_0\rangle$$

$T$ and $V_\phi$ are the kinetic energy and potential energy terms, respectively.



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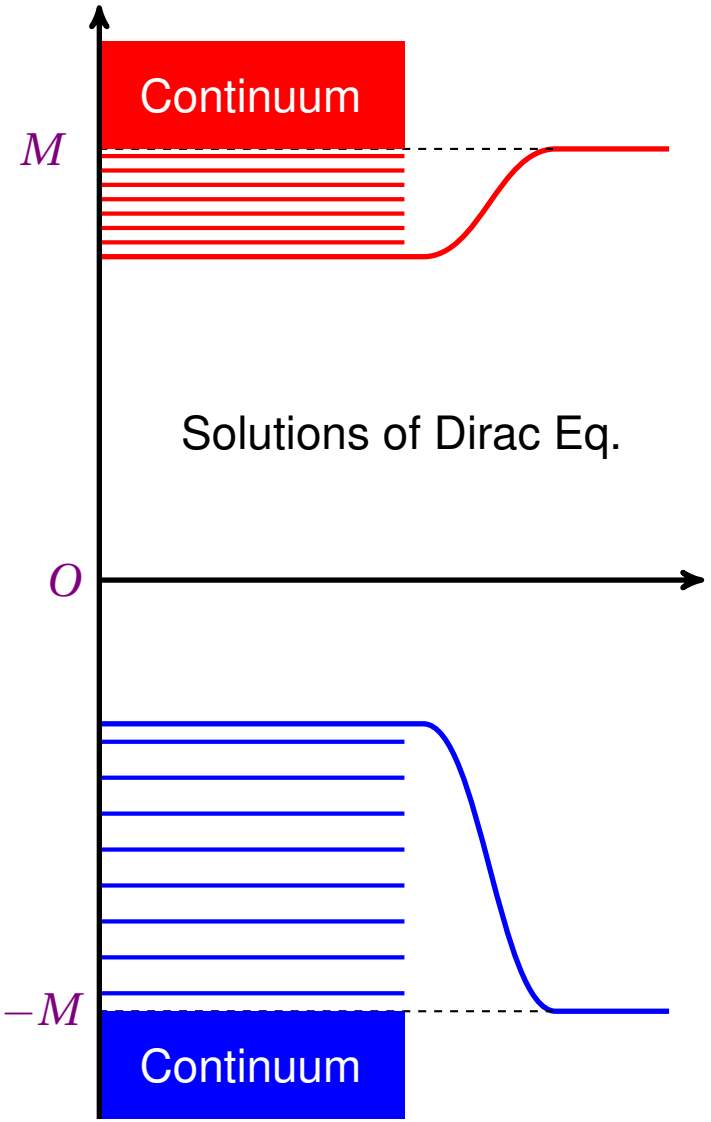
$T$ and $V_\phi$ are the kinetic energy and potential energy terms, respectively.

$$V_\phi = \frac{1}{2}\int d\boldsymbol{x}d\boldsymbol{x}' \sum_{\alpha\beta;\alpha'\beta'} c_\alpha^\dagger c_\beta^\dagger c_{\beta'} c_{\alpha'} \times \bar{\psi}_\alpha(\boldsymbol{x})\bar{\psi}_\beta(\boldsymbol{x}')\Gamma_\phi D_\phi \psi_{\beta'}(\boldsymbol{x}')\psi_{\alpha'}(\boldsymbol{x}),$$

Fock

Hartree



# Spherical RHF equation

W.H. Long (2010)

- Integro-differential Dirac equation

$$\int d\boldsymbol{r}' h(\boldsymbol{r},\boldsymbol{r}')\psi_\alpha(\boldsymbol{r}') = \varepsilon_a \psi_\alpha(\boldsymbol{r}), \qquad \psi_\alpha(\boldsymbol{r}) = \frac{1}{r}\begin{pmatrix} iG_a \mathcal{Y}^{l_a}_{j_a m_a}(\hat{\boldsymbol{r}}) \\ -F_a \mathcal{Y}^{l'_a}_{j_a m_a}(\hat{\boldsymbol{r}}) \end{pmatrix} \tag{11}$$

- Single-particle Dirac Hamiltonian $h = h^{\mathrm{kin}} + h^{\mathrm{D}} + h^{\mathrm{E}}$:

$$h^{\mathrm{kin}}(\boldsymbol{r},\boldsymbol{r}') = [\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta M]\,\delta(\boldsymbol{r},\boldsymbol{r}'), \tag{12a}$$

$$h^{\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}') = [\Sigma_T(\boldsymbol{r})\gamma_5 + \Sigma_0(\boldsymbol{r}) + \beta\Sigma_S(\boldsymbol{r})]\,\delta(\boldsymbol{r},\boldsymbol{r}'), \tag{12b}$$

$$h^{\mathrm{E}}(\boldsymbol{r},\boldsymbol{r}') = \begin{pmatrix} Y_G(\boldsymbol{r},\boldsymbol{r}') & Y_F(\boldsymbol{r},\boldsymbol{r}') \\ X_G(\boldsymbol{r},\boldsymbol{r}') & X_F(\boldsymbol{r},\boldsymbol{r}') \end{pmatrix} \tag{12c}$$

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- Local potentials: $\Sigma_{\mathrm{S}}$, $\Sigma_0$, and $\Sigma_{\mathrm{T}}$

$$\Sigma_{\mathrm{S}} = g_\sigma \sigma, \qquad \Sigma_{\mathrm{T}} = \frac{f_\rho}{2M}\left(\rho^{\mathrm{VT}} + \rho^{\mathrm{T}}\right)\tau_z \tag{13}$$

$$\Sigma_0 = g_\omega \omega + g_\rho\left(\rho^{\mathrm{V}} + \rho^{\mathrm{TV}}\right)\tau_z + e\frac{1-\tau_z}{2}A + \Sigma_R \tag{14}$$

Local MFs: $\sigma$, $\omega$, $\rho^{\mathrm{V}}$, and $A$, $\rho^{\mathrm{TV}}$ and $\rho^{\mathrm{VT}}$, $\rho^{\mathrm{T}}$, functionals of local densities.

# Exchange (Fock) Potentials

W.H. Long (2010)

- Non-local MFs: functionals of the non-local densities

$$X^{(\phi)}_{Ga}(r,r') = \sum_b \mathscr{T}^{\phi}_{ab}\,\hat{j}^2_b\; g_\phi(r) g_\phi(r') \underline{F_b(r) G_b(r')} \mathscr{R}^{X_G}_{ab}(m_\phi; r, r'),$$

$$X^{(\phi)}_{Fa}(r,r') = \sum_b \mathscr{T}^{\phi}_{ab}\,\hat{j}^2_b\; g_\phi(r) g_\phi(r') \underline{F_b(r) F_b(r')} \mathscr{R}^{X_F}_{ab}(m_\phi; r, r'),$$

$$Y^{(\phi)}_{Ga}(r,r') = \sum_b \mathscr{T}^{\phi}_{ab}\,\hat{j}^2_b\; g_\phi(r) g_\phi(r') \underline{G_b(r) G_b(r')} \mathscr{R}^{Y_G}_{ab}(m_\phi; r, r'),$$

$$Y^{(\phi)}_{Fa}(r,r') = \sum_b \mathscr{T}^{\phi}_{ab}\,\hat{j}^2_b\; g_\phi(r) g_\phi(r') \underline{G_b(r) F_b(r')} \mathscr{R}^{Y_F}_{ab}(m_\phi; r, r'),$$

$\mathscr{T}^{\phi}_{ab}$: $\delta_{\tau_a\tau_b}$ (isoscalar) and $2-\delta_{\tau_a\tau_b}$ (isovector).

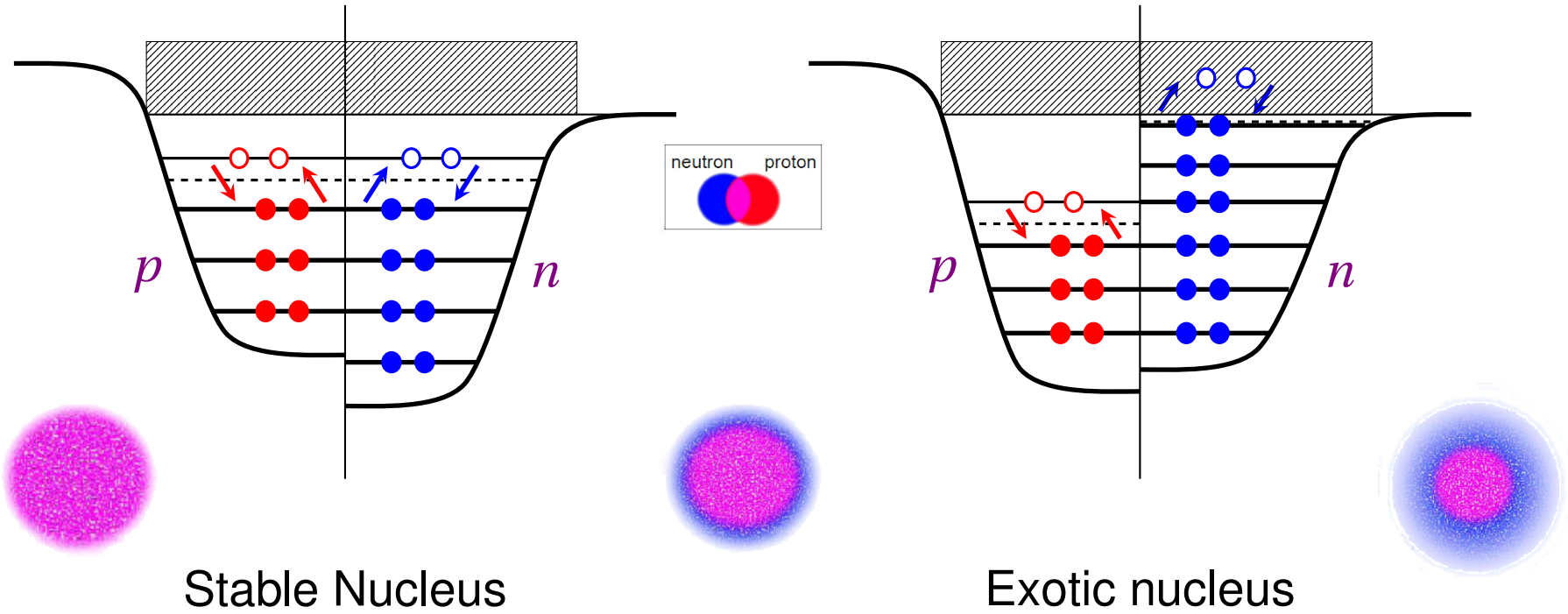
- The underlined terms can be taken as the non-local density component.
- $\sigma$-scalar coupling: $\mathscr{R}^{Y_G} = \mathscr{R}^{X_F} = -\mathscr{R}^{Y_F} = -\mathscr{R}^{X_G} = \mathscr{R}^{(\sigma)}$,

$$\mathscr{R}^{(\sigma)}_{ab}(m_\sigma; r, r') = \frac{1}{4\pi} {\sum_L}' C^{L0}_{j_a\frac{1}{2}j_b-\frac{1}{2}} C^{L0}_{j_a\frac{1}{2}j_b-\frac{1}{2}} R_{LL}(m_\sigma; r, r'). \tag{15}$$

The prime in Eq. (15) requires $L + l_a + l_b$ be even.

# Unstable nuclei: continuum effects

- Unstable exotic nuclei reveal lots of new physics: weakly bound mechanism, continuum, halo, etc.




Stable Nucleus

Exotic nucleus

- Bogoliubov scheme: unified treatment of pairing and mean field effects

— J. Meng, NPA 635, 3 (1998); S.-G. Zhou, J. Meng, and P. Ring, PRC 68, 034323 (2003).

- Bogoliubov scheme also has the advantages in exploring superheavy nuclei.

— J.J. Li, W.H. LONG, J. Margueron, N. Van Giai, PLB 732, 169 (2014).

# RHFB equation

- Bogoliubov transformation: particle $\{c_\alpha, c_\alpha^\dagger\}$ to quasi-particle $\{\beta_\alpha, \beta_\alpha^\dagger\}$

$$\begin{pmatrix} c_\alpha \\ c_\alpha^\dagger \end{pmatrix} = \mathcal{W} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix} = \begin{pmatrix} \psi_U & \psi_V^* \\ \psi_V & \psi_U^* \end{pmatrix} \begin{pmatrix} \beta_\alpha \\ \beta_\alpha^\dagger \end{pmatrix} \tag{16}$$

  where $\psi_U$ and $\psi_V$ are the quasi-particle spinors, and $\mathcal{W}^\dagger\mathcal{W} = 1$.

- RHFB equation: Chemical potential $\lambda$ H. Kucharek(1991)

$$\int d\boldsymbol{r}' \begin{pmatrix} h(\boldsymbol{r},\boldsymbol{r}') & \Delta(\boldsymbol{r},\boldsymbol{r}') \\ -\Delta(\boldsymbol{r},\boldsymbol{r}') & h(\boldsymbol{r},\boldsymbol{r}') \end{pmatrix} \begin{pmatrix} \psi_U(\boldsymbol{r}') \\ \psi_V(\boldsymbol{r}') \end{pmatrix} = \begin{pmatrix} \lambda + E & 0 \\ 0 & \lambda - E \end{pmatrix} \begin{pmatrix} \psi_U(\boldsymbol{r}) \\ \psi_V(\boldsymbol{r}) \end{pmatrix} \tag{17}$$

- Pairing potential, pairing tensor $\kappa$ and Pairing forces:

$$\Delta_\alpha(\boldsymbol{r},\boldsymbol{r}') = -\frac{1}{2}\sum_\beta V_{\alpha\beta}^{pp}(\boldsymbol{r},\boldsymbol{r}')\,\kappa_\beta(\boldsymbol{r},\boldsymbol{r}'), \quad \kappa_\alpha(\boldsymbol{r},\boldsymbol{r}') = \psi_{V_\alpha}^*(\boldsymbol{r})\psi_{U_\alpha}(\boldsymbol{r}') \tag{18}$$

$$V(\boldsymbol{r},\boldsymbol{r}') = V_0\delta(\boldsymbol{r}-\boldsymbol{r}')\frac{1}{4}(1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')\left(1-\frac{\rho(r)}{\rho_0}\right) \tag{19}$$

$$V(\boldsymbol{r},\boldsymbol{r}') = \sum_{i=1,2} e^{\left((r-r')/\mu_i\right)^2}\left(W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau\right) \tag{20}$$

# Numerical Algorithm

- Radial RHFB equations ($E_a$: Quasi-particle energy):

$$\left[\frac{d}{dr}+\frac{\kappa_a}{r}+\Sigma_{\rm T}\right]G_{U_a}-\left(E_a+\lambda-\Sigma_-\right)F_{U_a}+X_{U_a}+r\int r'dr'\Delta(r,r')F_{V_a}(r')=0,$$

$$\left[\frac{d}{dr}-\frac{\kappa_a}{r}-\Sigma_{\rm T}\right]F_{U_a}+\left(E_a+\lambda-\Sigma_+\right)G_{U_a}-Y_{U_a}+r\int r'dr'\Delta(r,r')G_{V_a}(r')=0,$$

$$\left[\frac{d}{dr}+\frac{\kappa_a}{r}+\Sigma_{\rm T}\right]G_{V_a}+\left(E_a-\lambda+\Sigma_-\right)F_{V_a}+X_{V_a}+r\int r'dr'\Delta(r,r')F_{U_a}(r')=0,$$

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$\Sigma_+\equiv\Sigma_0+\Sigma_{\rm S}$, $\Sigma_-\equiv\Sigma_0-\Sigma_{\rm S}-2M$, Integro terms $X_U$, $Y_U$, $X_V$, $Y_V$.

- Dirac Woods-Saxon (DWS) basis: $\{[\varepsilon_b, g_\beta(\boldsymbol{r},\tau)]\,;\varepsilon_b\gtrless 0\}$

$$\psi_U^\kappa=\sum_{p=1}^{N_F}U_p g_p^\kappa+\sum_{d=1}^{N_D}U_d g_d^\kappa,\qquad \psi_V^\kappa=\sum_{p=1}^{N_F}V_p g_p^\kappa+\sum_{d=1}^{N_D}V_d g_d^\kappa, \tag{21}$$

DWS basis is ONLY applied in solving the radial RHFB equations.

S.-G. Zhou, J. Meng, P. Ring, PRC 68, 034323 (2003); W.H. Long, P. Ring, N. Van Giai, J. Meng, PRC 81, 024308 (2010).

# Numerical Algorithm

- Radial RHFB equations ($E_a$: Quasi-particle energy):

$$\left[\frac{d}{dr}+\frac{\kappa_a}{r}+\Sigma_{\rm T}\right]G_{U_a}-(E_a+\lambda-\Sigma_-)\,F_{U_a}+X_{U_a}+r\int r'dr'\Delta(r,r')F_{V_a}(r')=0,$$

$$\left[\frac{d}{dr}-\frac{\kappa_a}{r}-\Sigma_{\rm T}\right]F_{U_a}+(E_a+\lambda-\Sigma_+)\,G_{U_a}-Y_{U_a}+r\int r'dr'\Delta(r,r')G_{V_a}(r')=0,$$

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# CDFs with Fock terms

- In-medium effects: coupling constants are taken as functionals of density
  1. Density dependent behavior — R. Brockmann(1992), H. Lenske(1995), C. Fuchs(1995)

$$g_i(\rho_v) = g_i(0)e^{-a_i\xi}, \qquad g_i = g_\rho, f_\rho, f_\pi, \qquad a_i = a_\rho, a_T, a_\pi; \tag{22}$$

$$g_i(\rho_v) = g_i(\rho_0)f_i(\xi), \qquad i = \sigma, \omega \tag{23}$$

where $\rho_v = \sqrt{j^\mu j_\mu}$, $\xi = \rho_v/\rho_0$, and

$$f_i(\xi) = a_i\frac{1+b_i(\xi+d_i)^2}{1+c_i(\xi+d_i)^2}; \quad \text{with } f_i(1) = 1, f_i''(0) = 0, f_\sigma''(1) = f_\omega''(1) \tag{24}$$

  2. Rearrangement terms $\Sigma_R^\mu$ induced by density dependence

$$\Sigma \to \Sigma + \gamma_\mu\Sigma_R^\mu, \tag{25}$$

$$\Sigma_R^\mu = \Sigma_{R,(\sigma)}^\mu + \Sigma_{R,(\omega)}^\mu + \Sigma_{R,(\rho)}^\mu + \Sigma_{R,(\pi)}^\mu \tag{26}$$

- CDFs with Fock terms (defined by 8$\sim$12 free parameters)
  1. PKO2: $\sigma$-S, $\omega$-V, and $\rho$-V couplings — W.H Long (2006)
  2. PKO1, PKO3: $\sigma$-S, $\omega$-V, $\rho$-V, and $\pi$-PV — W.H Long (2006)
  3. PKA1: $\sigma$-S, $\omega$-V, $\rho$-V, $\pi$-PV, and $\rho$-T — W.H Long (2007)

# Quantitative quality of RHF functional

Table 1: Deviations (rms) from the data given by the RHF functional PKO1, and the RMF ones PK1, PKDD, NL3 and DD-ME1. $\Delta_{S.O.}$ is the deviations of the spin-orbit splittings.

| | | PKO1 | PK1 | PKDD | NL3 | DD-ME1 |
|---|---|---|---|---|---|---|
| $\Delta_{E_b}$ | S.N. | 1.6177 | 1.8825 | 2.3620 | 2.2506 | 2.7561 |
| | Pb | 1.8995 | 2.0336 | 2.7007 | 2.0021 | 2.1491 |
| | Sn | 1.2665 | 1.9552 | 2.4567 | 1.6551 | 0.9168 |
| $\Delta_{S_{2n}}$ | Pb | 0.6831 | 0.9192 | 1.3139 | 0.9359 | 1.2191 |
| | Sn | 0.6813 | 0.7762 | 1.0629 | 0.8463 | 0.7646 |
| $\Delta_{r_c}$ | S.N. | 0.0269 | 0.0204 | 0.0188 | 0.0177 | 0.0163 |
| | Pb | 0.0056 | 0.0061 | 0.0060 | 0.0143 | 0.0150 |
| $\Delta_{S.O.}$ | O | 0.1761 | 0.2879 | 0.6817 | 0.2195 | 0.1107 |
| | Ca | 0.5078 | 0.6638 | 0.8159 | 0.7184 | 0.6041 |
| | Ni | 0.3959 | 0.9923 | 1.3287 | 1.3315 | 0.9029 |
| | Sn | 0.1650 | 0.3300 | 0.6913 | 0.4757 | 0.5408 |
| | Pb | 0.2014 | 0.3902 | 0.6370 | 0.4604 | 0.4588 |

S.N.: $^{16}$O, $^{40,48}$Ca, $^{56,58,68}$Ni, $^{90}$Zr, $^{112,116,124,132}$Sn, $^{182,194,204,208,214}$Pb, $^{210}$Po.

# OUTLINE

1 Introduction
- New Challenge and Opportunity in Nuclear Physcis
- Covariant Density Functional theory

2 Covariant density functional with Fock terms
- General Formalism for RHF Scheme
- Relativistic Hartree-Fock-Bogoliubov method
- Covariant density functionals with Fock terms

3 Role of non-local Fock terms in describing nuclear structure
- New Balance induced by Fock terms in nuclear force
- Neutron halo phenomena and Proton pseudo-spin symmetry
- Nuclear tensor force components in Fock diagrams

4 Conclusions and Perspectives

# Balance in Nuclear Force: PSS Restoration

- Balance between strong repulsion and attraction: be responsible for nuclear binding, spin-orbit effects, pseudo-spin symmetry (PSS) conservation
  - Lect. Notes Phys. 641 (2004) 1; PRL 78 (1997) 436; Phys. Rep. 570 (2015) 1.

| $^{208}$Pb | | Neutron | Proton | Total |
|---|---|---|---|---|
| Hartree | $\sigma$-S | -14320.86 | -10022.49 | -24343.35 |
| | $\omega$-V | 11520.81 | 7962.50 | 19483.32 |
| | $A$-V | 0.00 | 827.64 | 827.64 |
| | $\rho$-V | 98.42 | -65.12 | 33.30 |
| | $\rho$-VT | -1.65 | 1.08 | -0.57 |
| | $\rho$-T | -0.31 | 0.21 | -0.10 |

# Balance in Nuclear Force: PSS Restoration

- Balance between strong repulsion and attraction: be responsible for nuclear binding, spin-orbit effects, pseudo-spin symmetry (PSS) conservation
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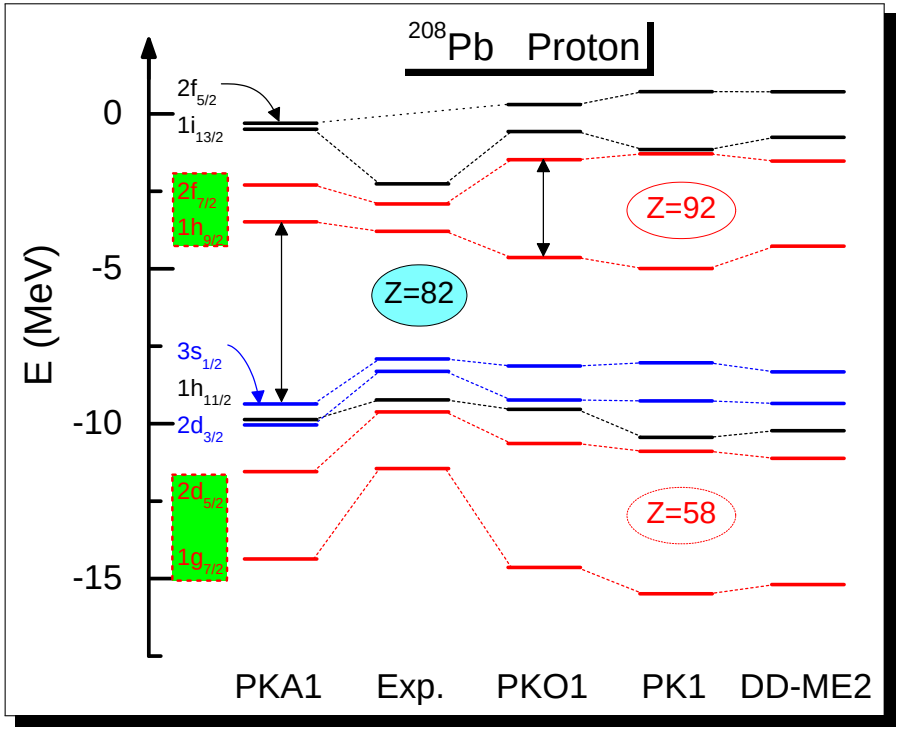
| Fock | $\sigma$-S | 3503.24 | 1774.28 | 5277.53 |
| | $\omega$-V | -2451.22 | -1265.20 | -3716.43 |
| | $A$-V | 0.00 | -29.02 | -29.02 |
| | | | | |
| | $\pi$ | -103.26 | -79.75 | -183.01 |

# Balance in Nuclear Force: PSS Restoration

- Balance between strong repulsion and attraction: be responsible for nuclear binding, spin-orbit effects, pseudo-spin symmetry (PSS) conservation
  - Lect. Notes Phys. 641 (2004) 1; PRL 78 (1997) 436; Phys. Rep. 570 (2015) 1.
- Balance changed with Fock terms, further changed by $\rho$-T in PKA1




—Phys. Rev. C 76, 034314 (2007).

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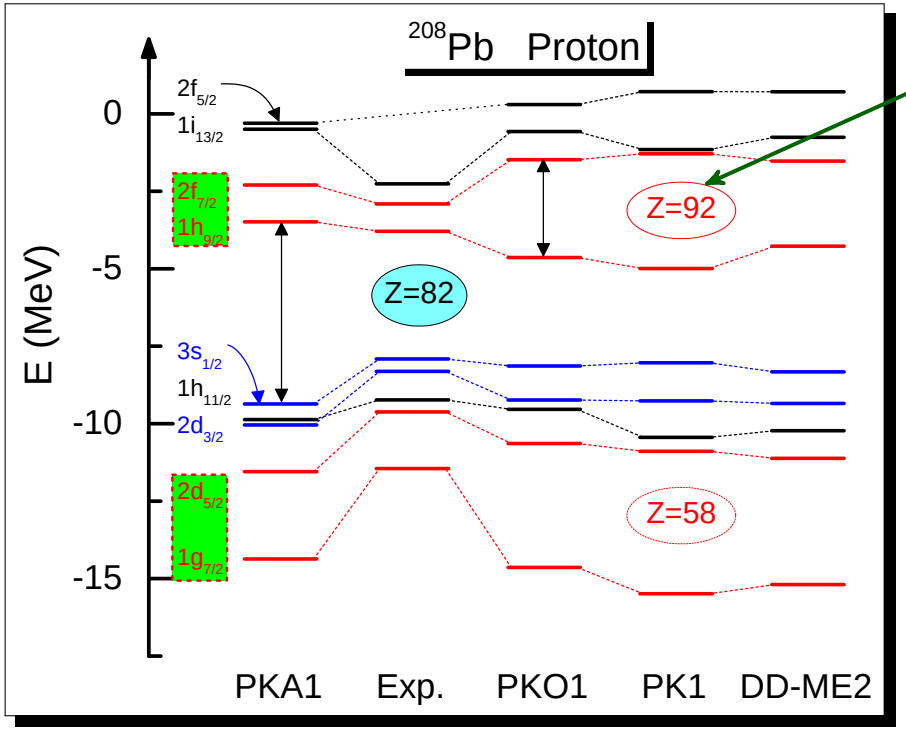
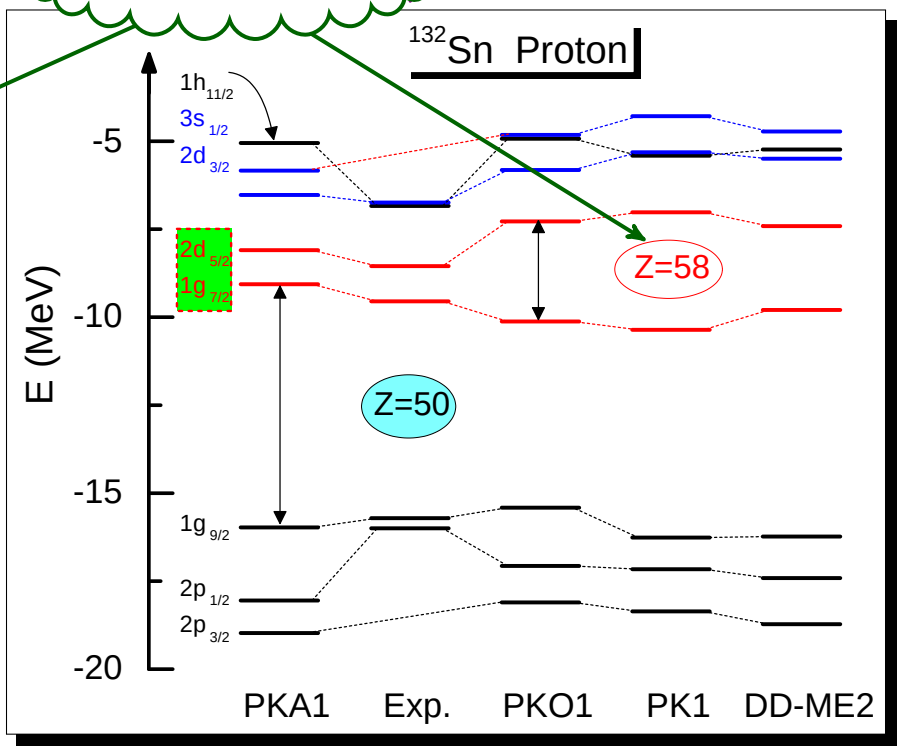
| Fock | $\sigma$-S | 3503.24 | 1774.28 | 5277.53 |
| | $\omega$-V | -2451.22 | -1265.20 | -3716.43 |
| | $A$-V | 0.00 | -29.02 | -29.02 |
| | $\rho$-V | -266.20 | -210.10 | -476.30 |
| | $\rho$-VT | 122.51 | 89.16 | 211.66 |
| | $\rho$-T | -687.90 | -531.94 | -1219.85 |
| | $\pi$ | -103.26 | -79.75 | -183.01 |

# Balance in Nuclear Force: PSS Restoration

- Balance between strong repulsion and attraction: be responsible for nuclear binding, spin-orbit effects, pseudo-spin symmetry (PSS) conservation
  - Lect. Notes Phys. 641 (2004) 1; PRL 78 (1997) 436; Phys. Rep. 570 (2015) 1.
- Balance changed with Fock terms, furt… -T in PKA1

Spurious Shells




—Phys. Rev. C 76, 034314 (2007).

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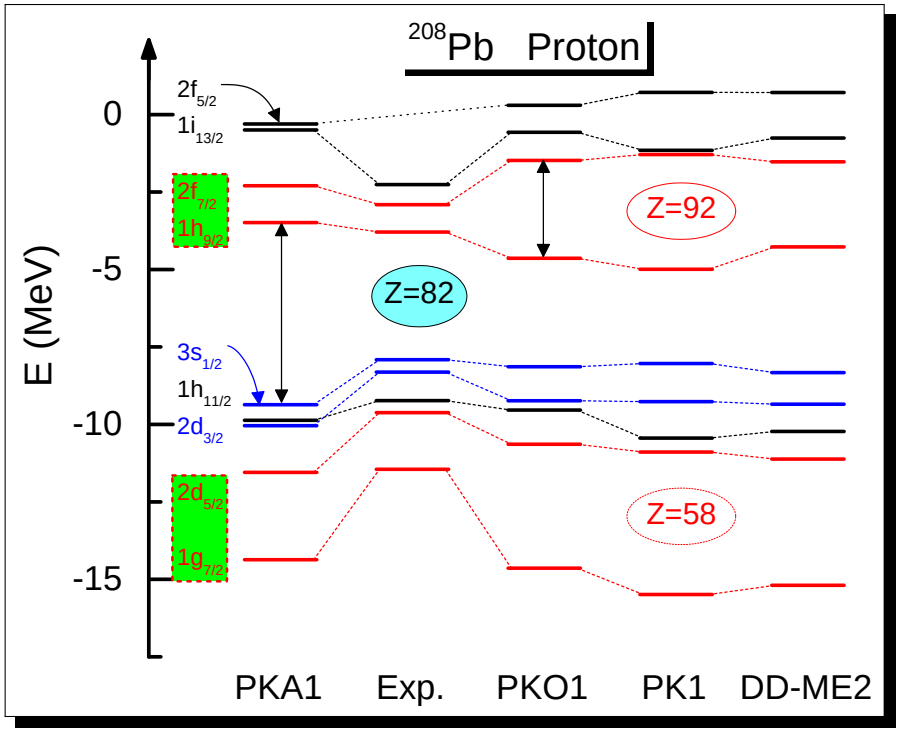


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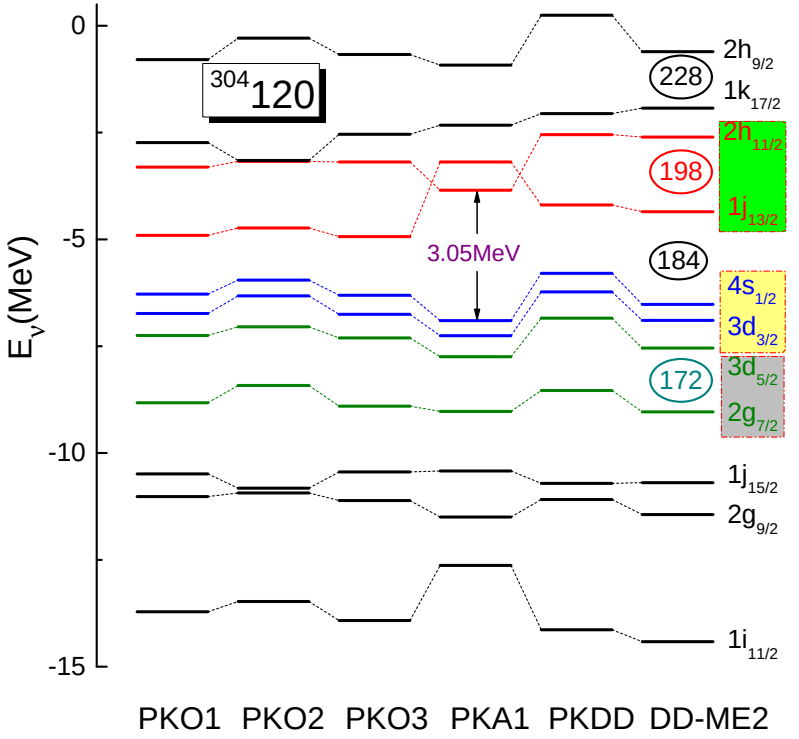



—Phys. Lett. B 732 (2014) 169.

- PKA1 predicts different superheavy magicities, due to the new balance.

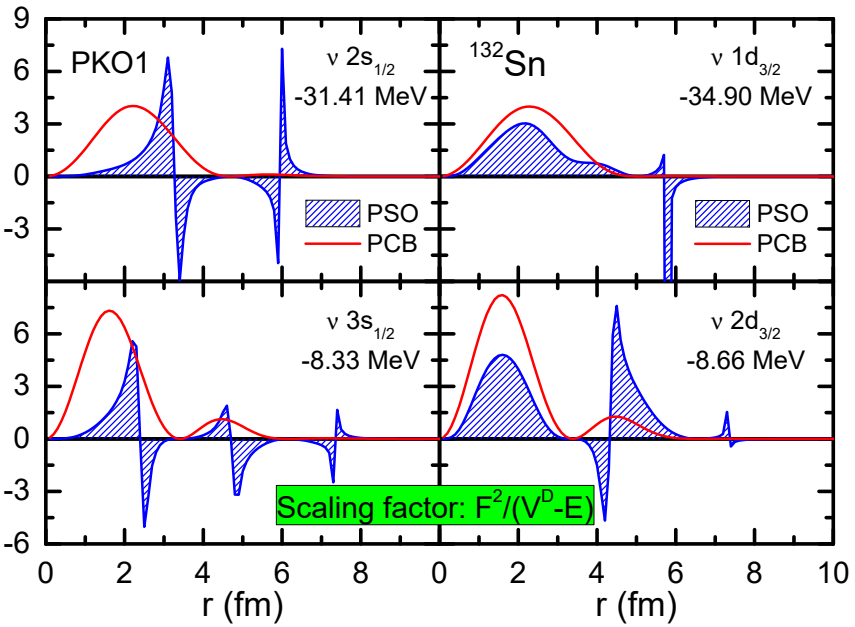
# Pseudo-spin symmetry (PSS)

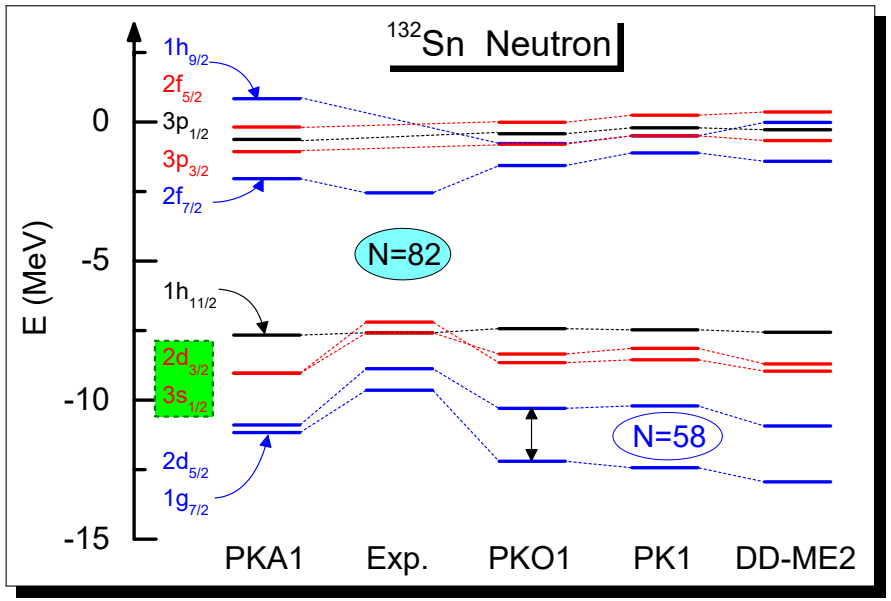
- Conservation condition in RMF:

$$V_{\mathrm{PCB}} >> V_{\mathrm{PSO}} \qquad (27)$$

J. Meng, et al., PRC 58, R628 (1998).

- For $d$ state, $V_{\mathrm{PSO}}$ is comparable to $V_{\mathrm{PCB}}$ because of Fock terms $V^{E}_{\mathrm{PSO}}$.




Long, Sagawa, Meng, Giai, PLB 639, 242(2006).

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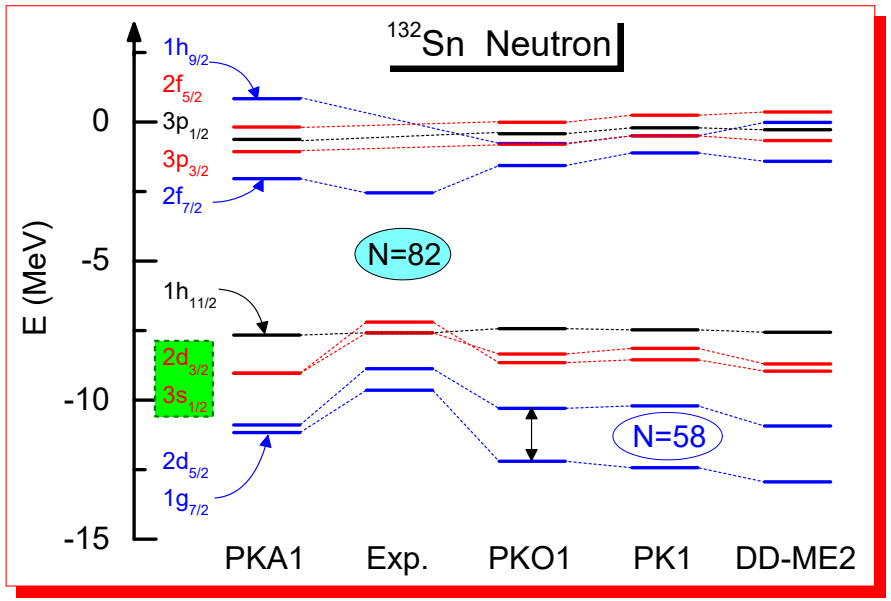
- $V^E_{\mathrm{PSO}}$ is counteracted by the other Fock contributions: like $V^E_1$.

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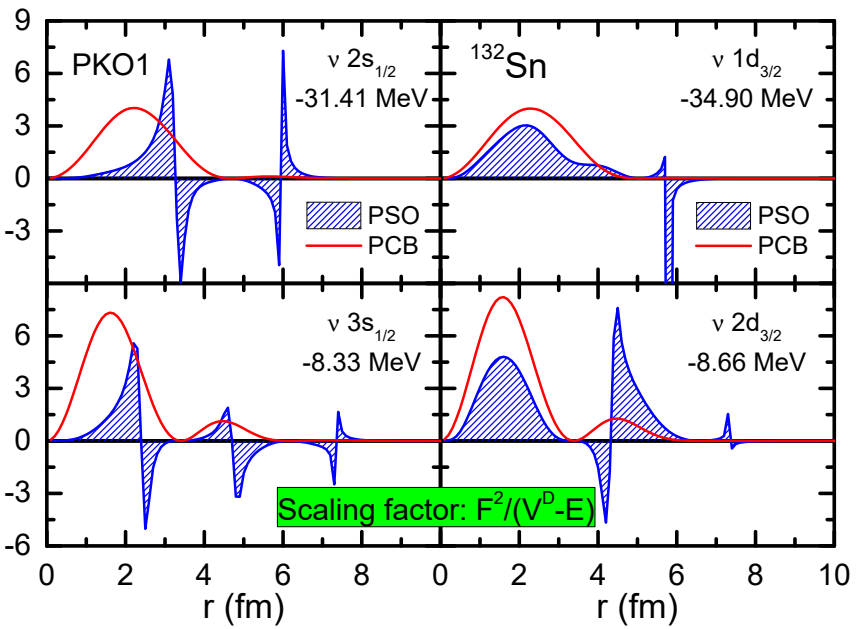
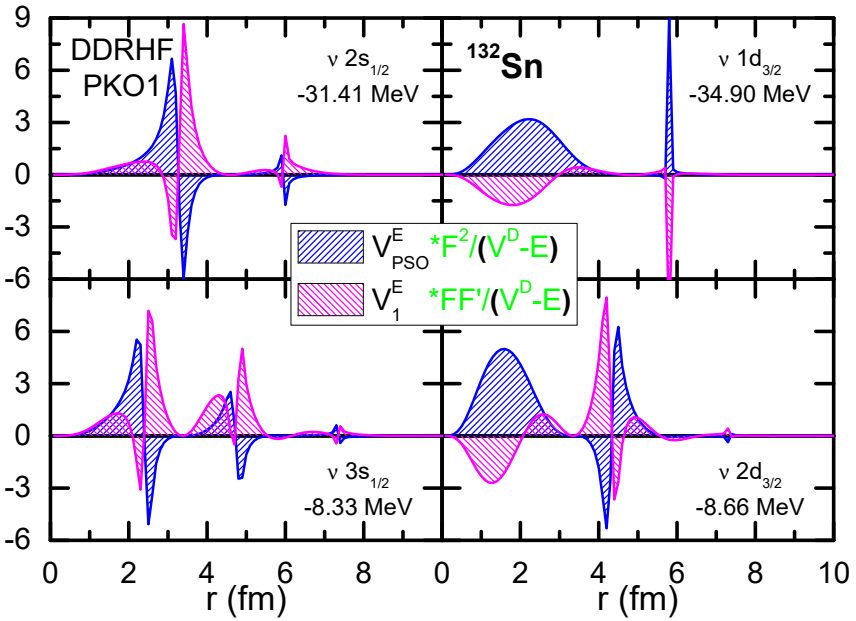
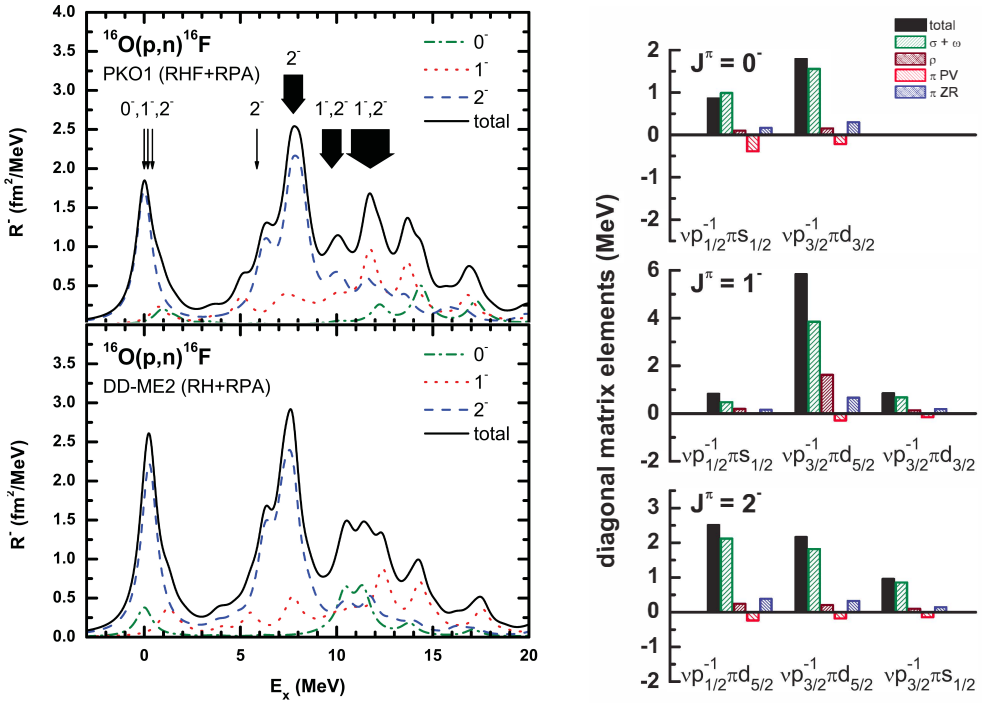


# Nuclear Isospin Excitation

- RHF+RPA: Full self-consistent description of Nuclear Isospin Excitation

—Liang, Giai, Meng, PRL 101, 122502 (2008).

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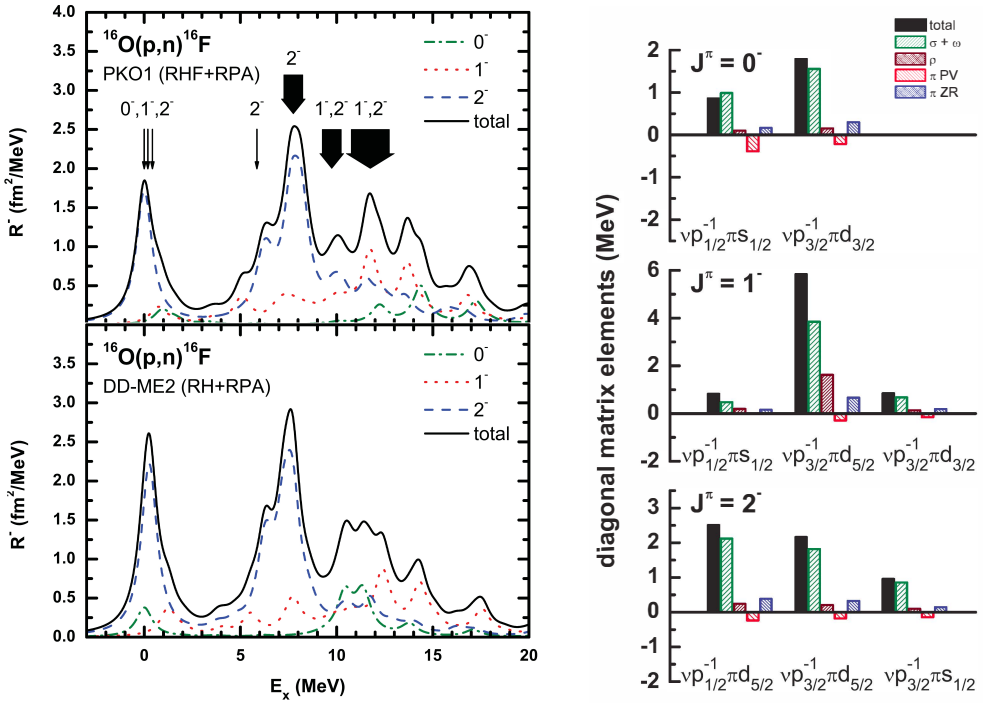
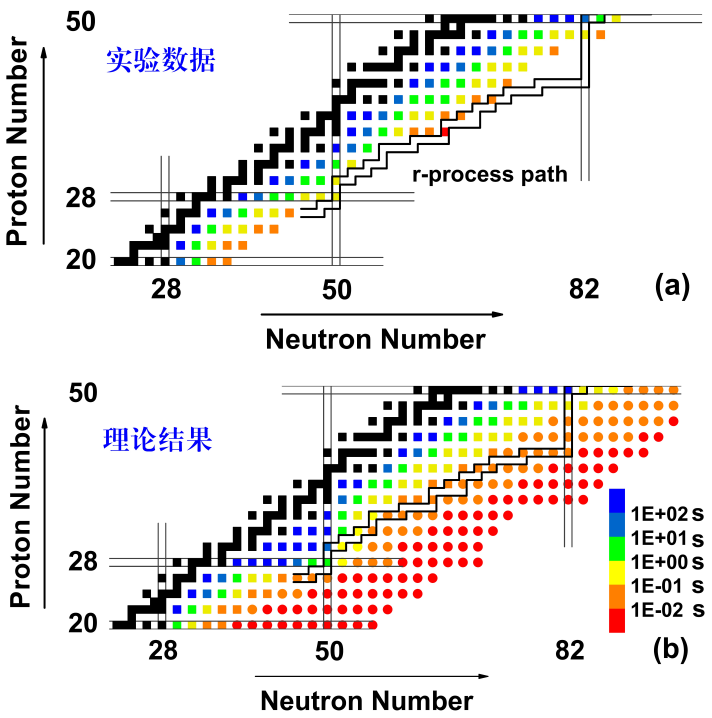



—Liang, Zhao, Meng, PRC 85, 064302 (2012).

- RHFB+QRPA: well reproduce the half-lives of $\beta$-decay

—Niu, et al, PLB 723, 172 (2013).

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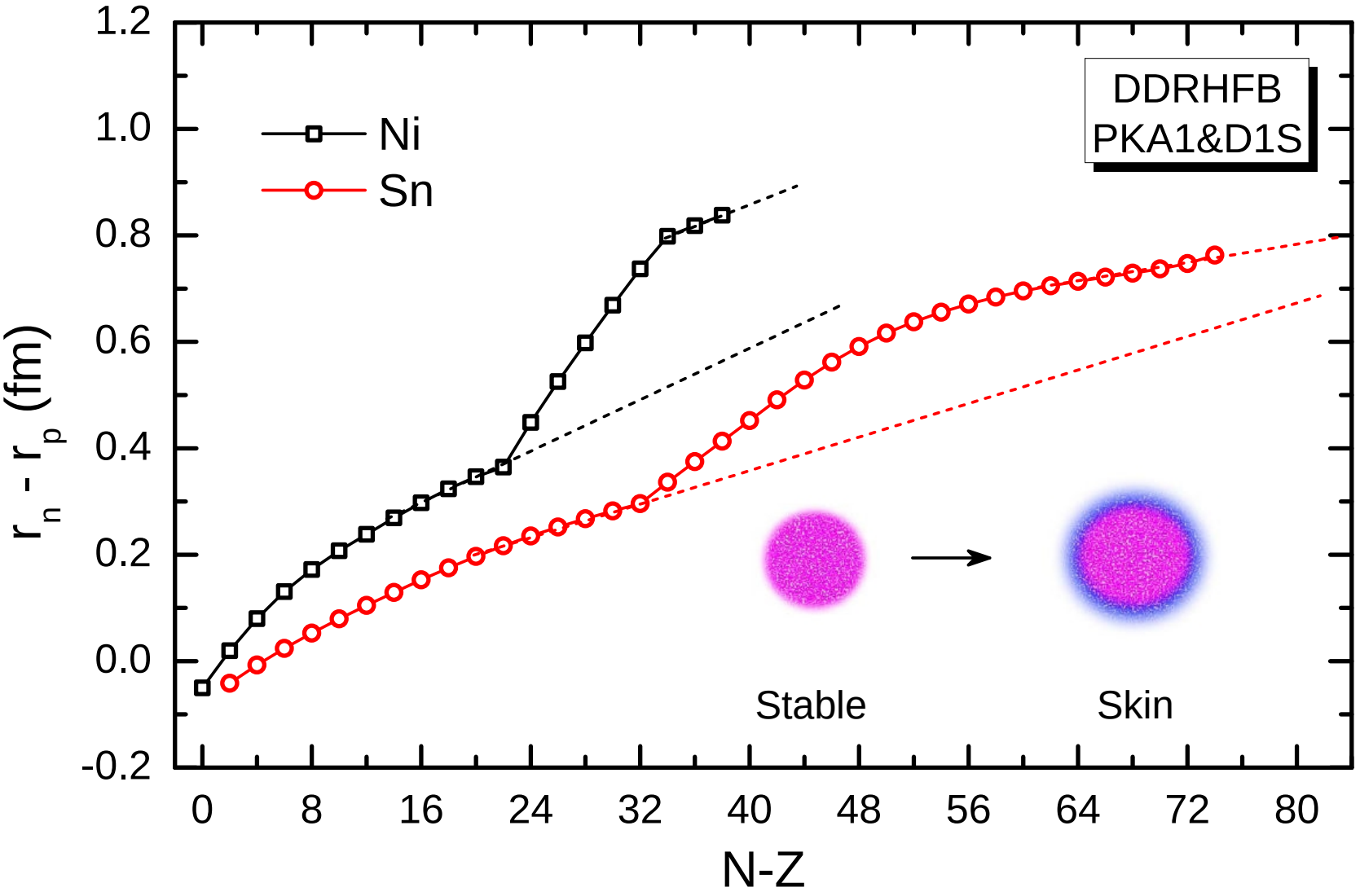
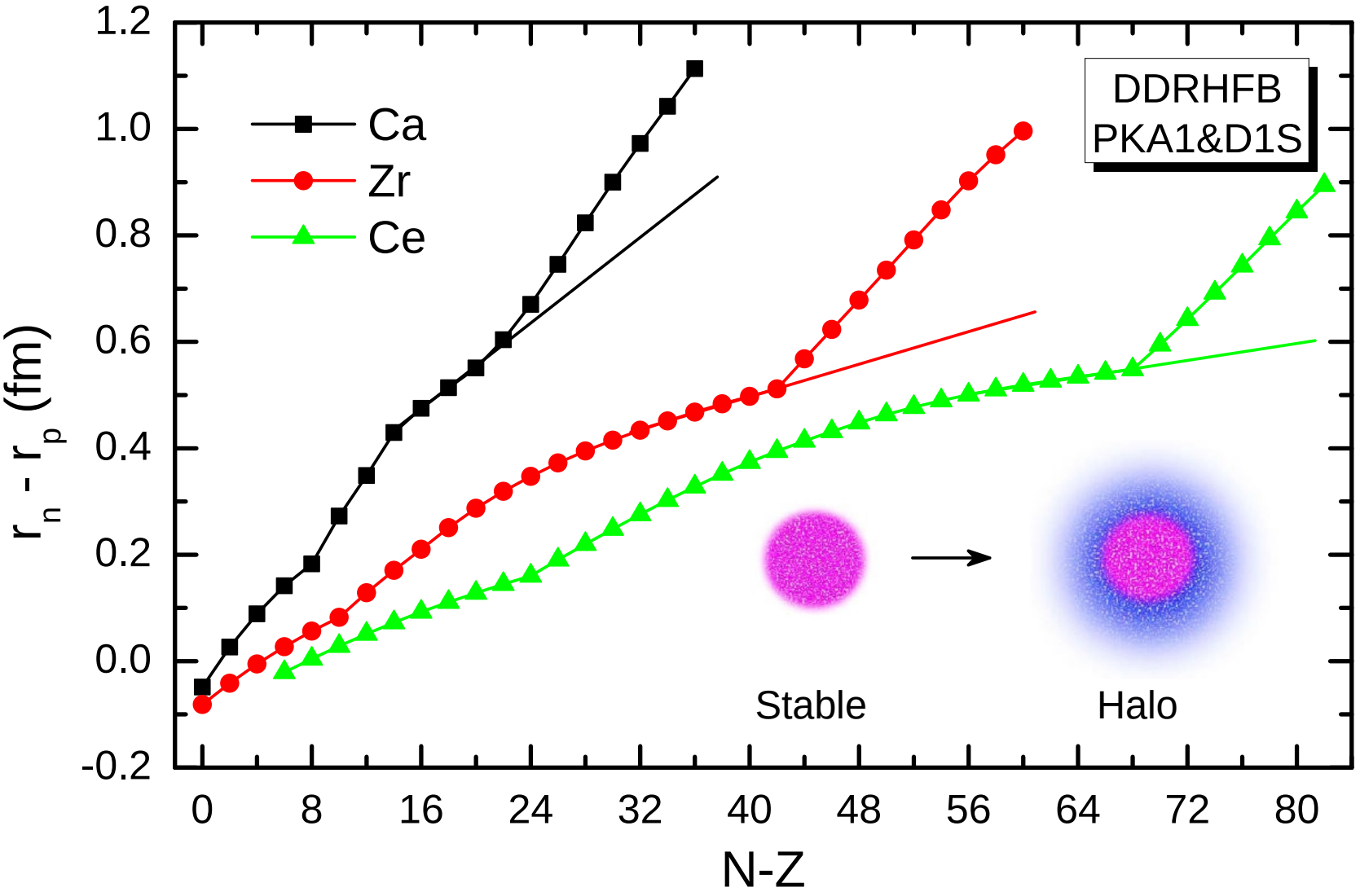
# Nuclear Halo Phenomena



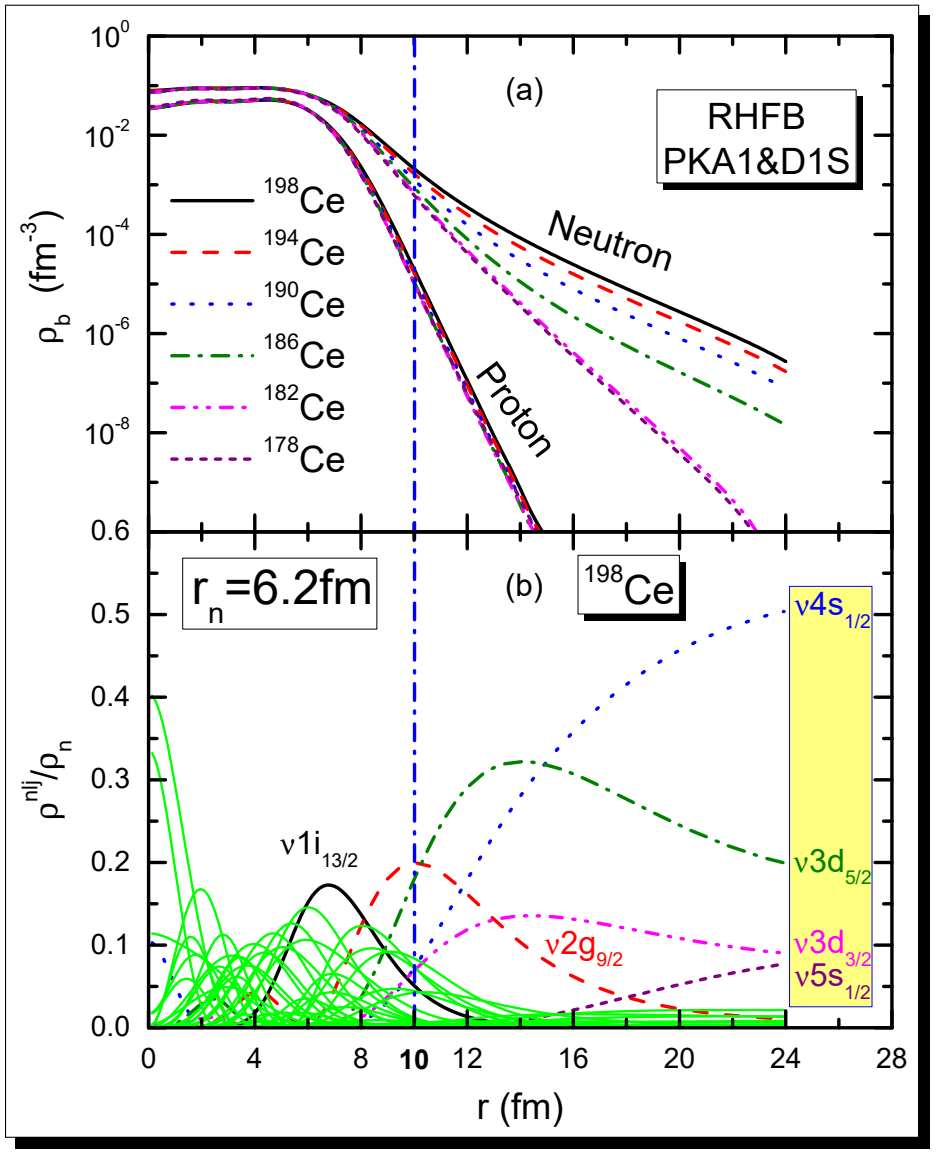

Neutron Skin: Ni and Sn;

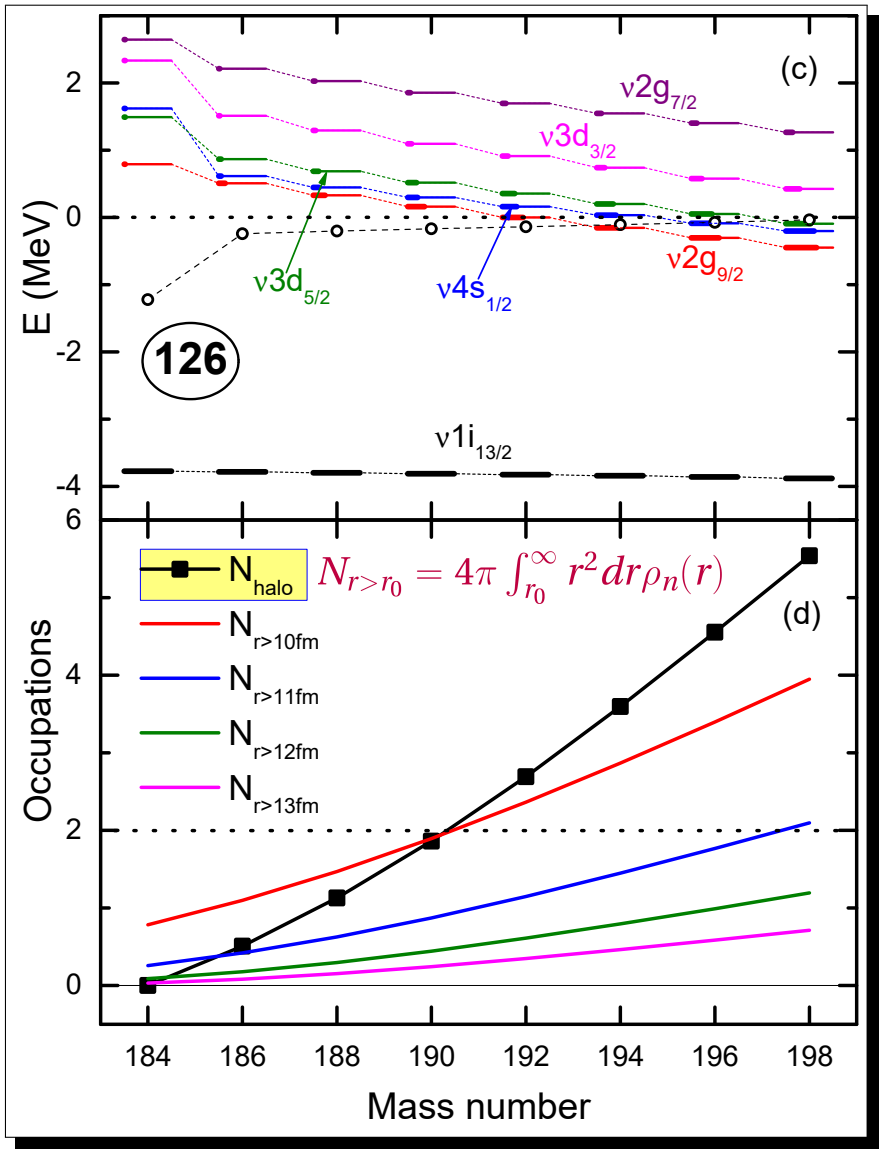
# Nuclear Halo Phenomena




Neutron Skin: Ni and Sn; Neutron halo: Ca, Zr and Ce

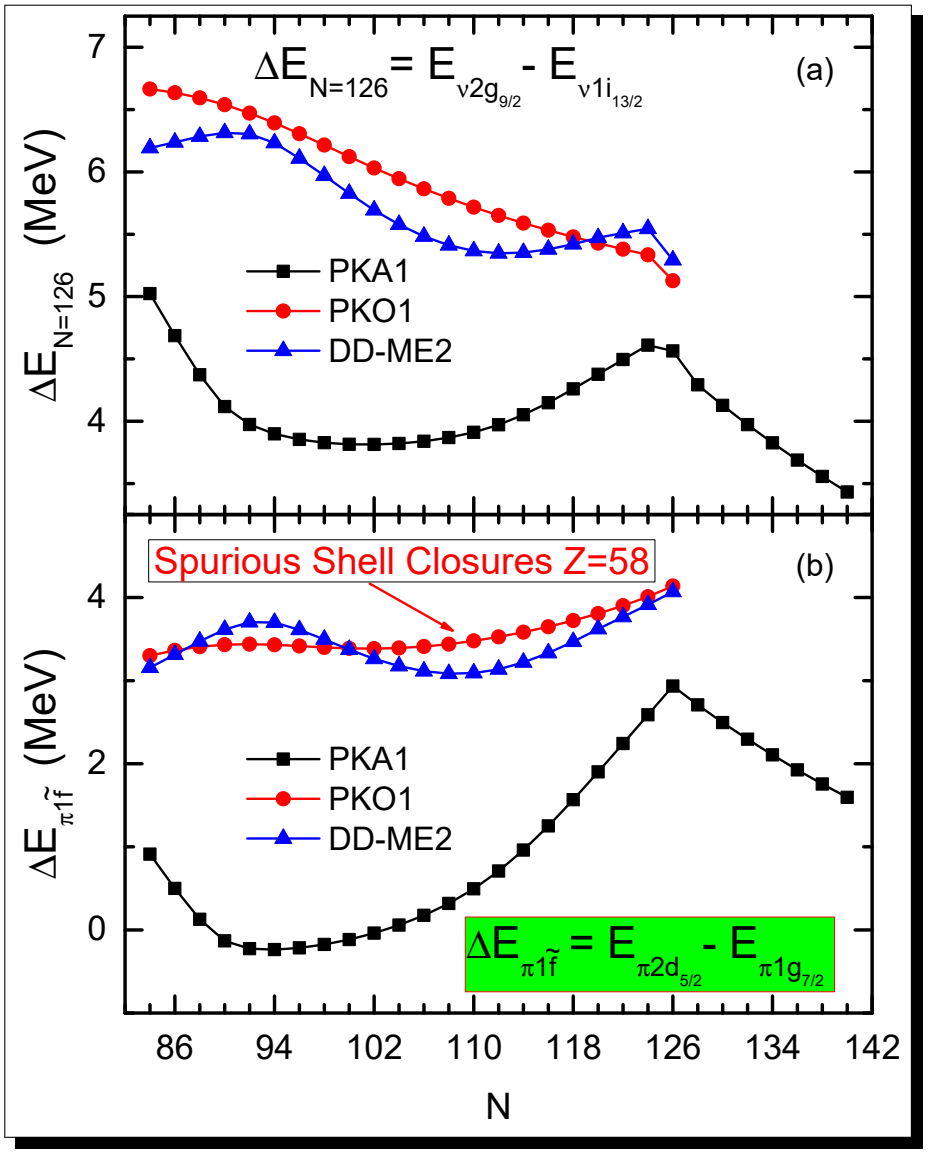
# Giant Halo in Cerium Isotopes

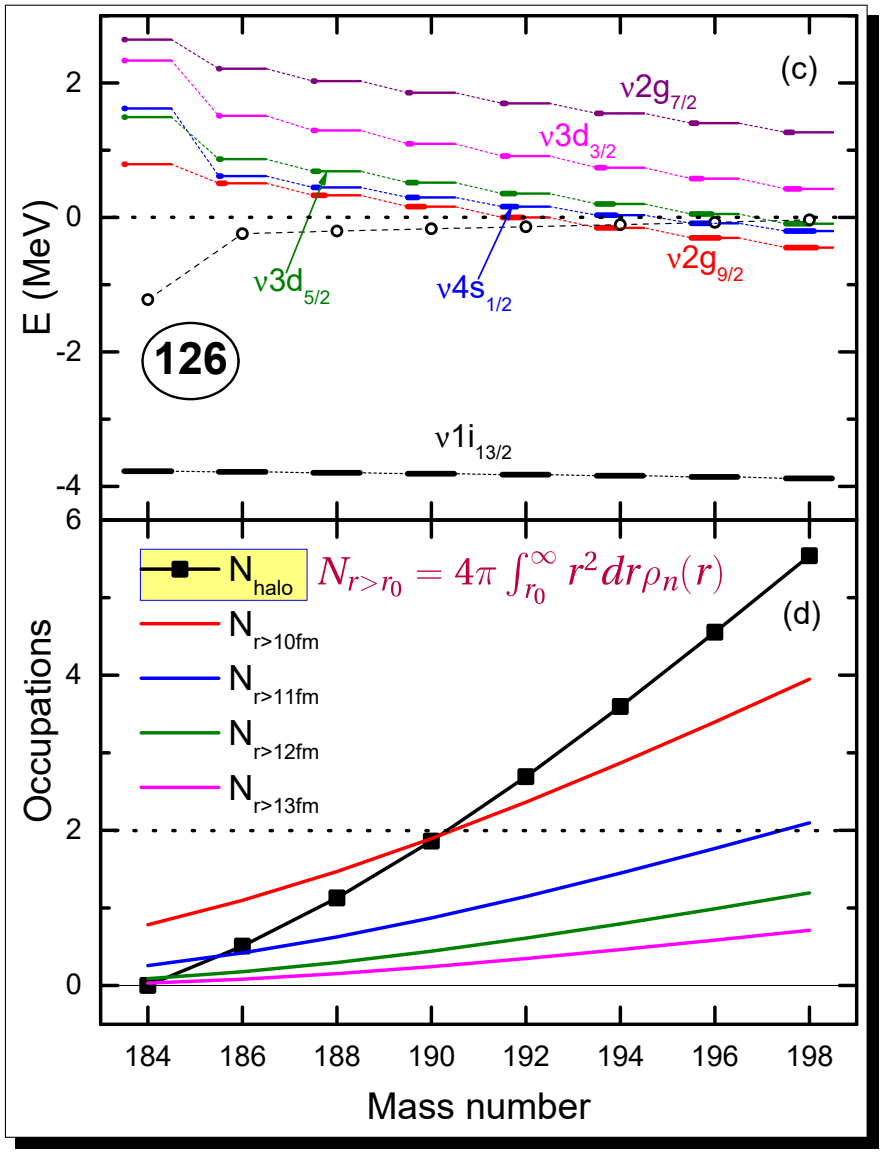



Neutron Halos: Halo nuclei $^{186-190}$Ce & Giant halo nuclei $^{192-198}$Ce

# Giant Halo in Cerium Isotopes

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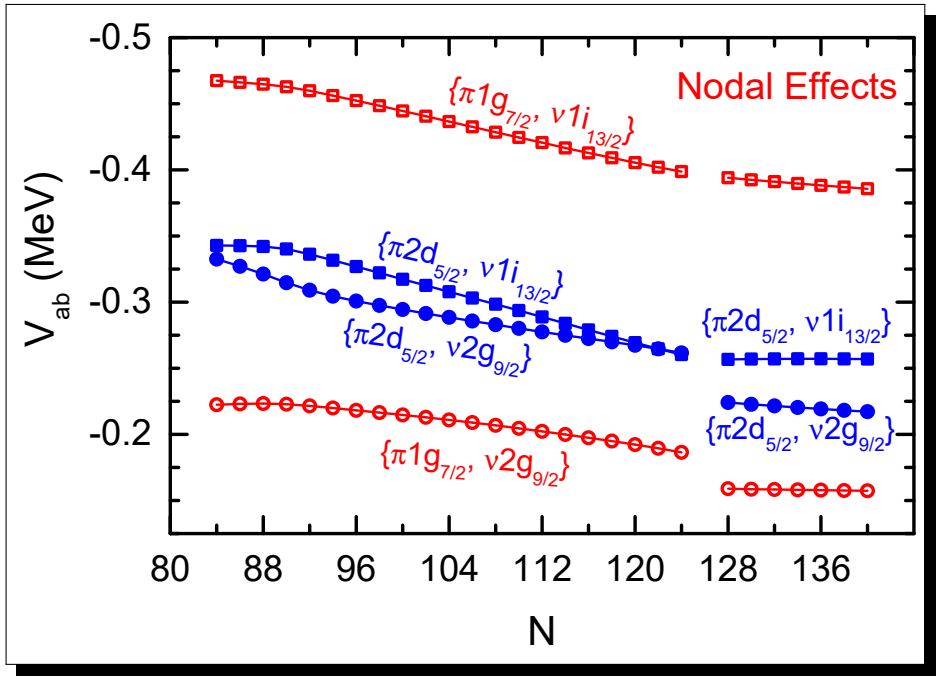
# Drip-Line Position, Halo vs Pseudo-Spin Symmetry

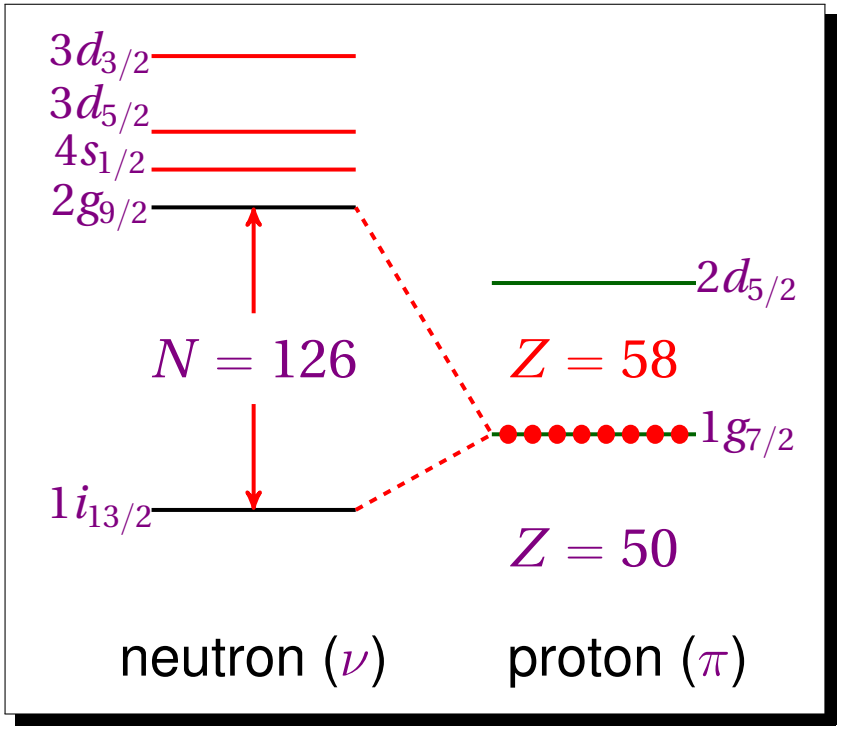
- RHFB-PKA1 predicted neutron drip-line at $N = 140$ for Ce isotopes.

  —PRC 81, 031302(R) (2010).

- Self-consistent relation between halo structure and Pseudo-spin symmetry

- Nodal Effect is particulary important.




Configuration sketch ($\nu$ & $\pi$) at drip line

- Better conserved PSS favors the halo formation

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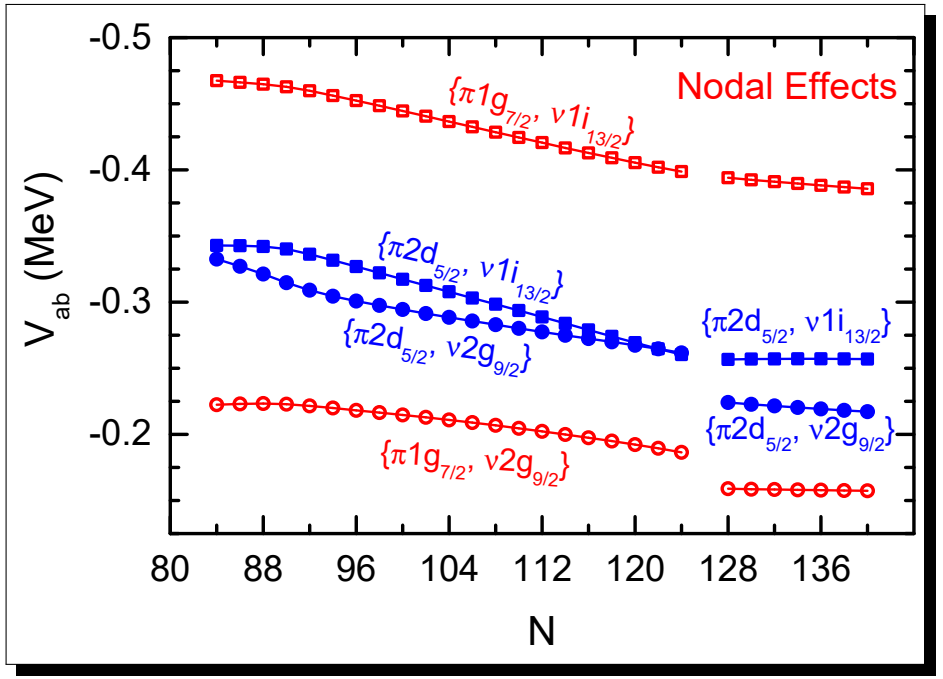


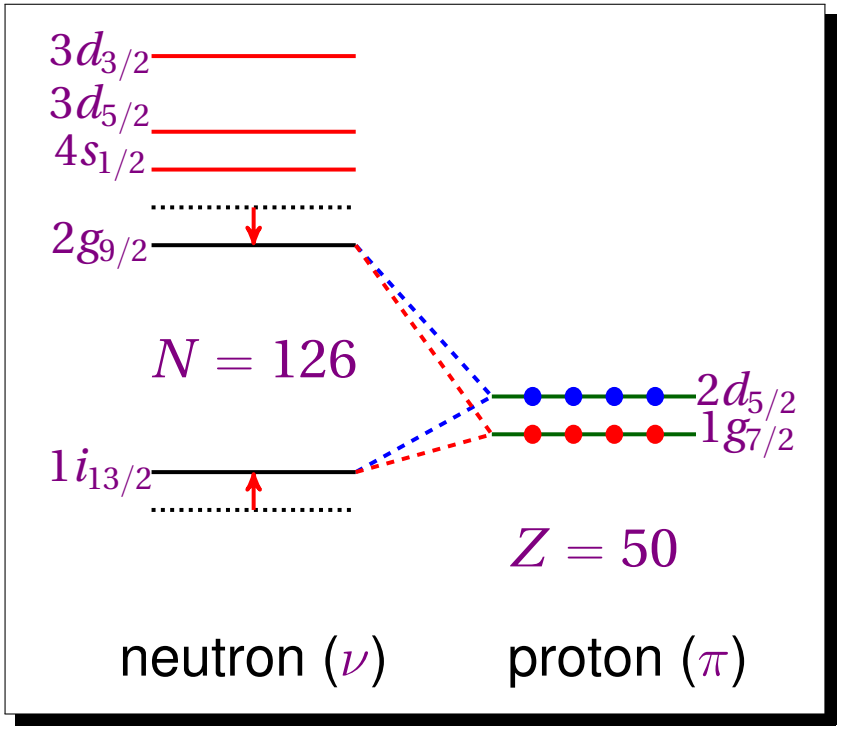


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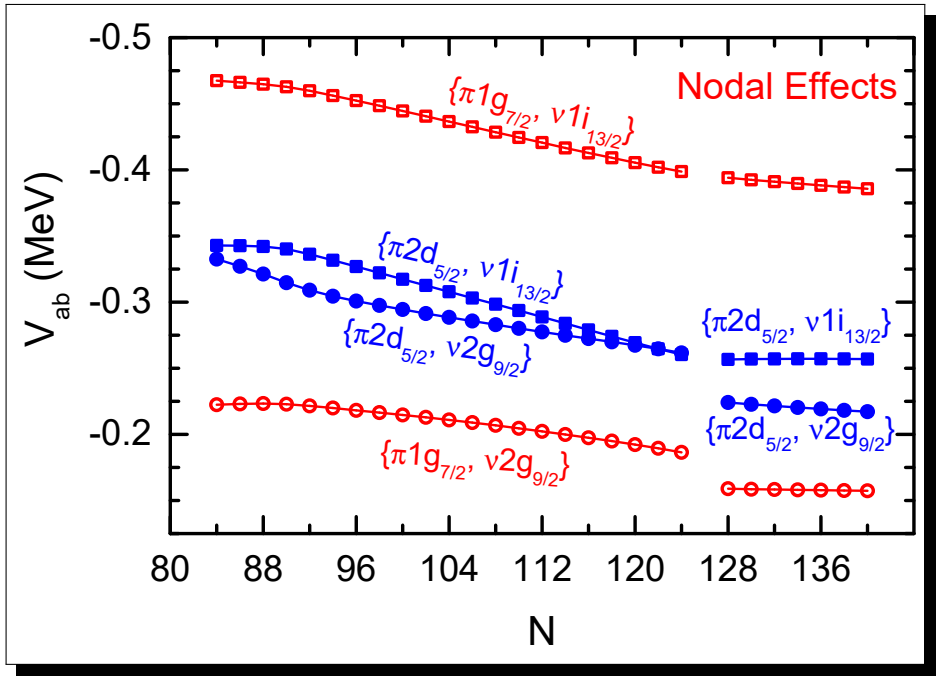
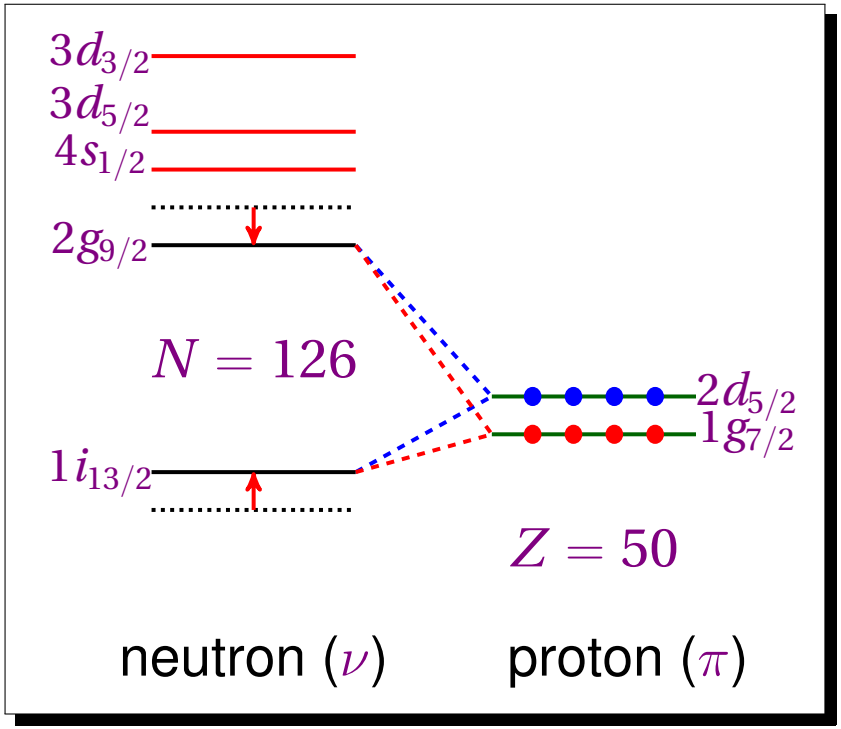



Configuration sketch ($\nu$ & $\pi$) at drip line

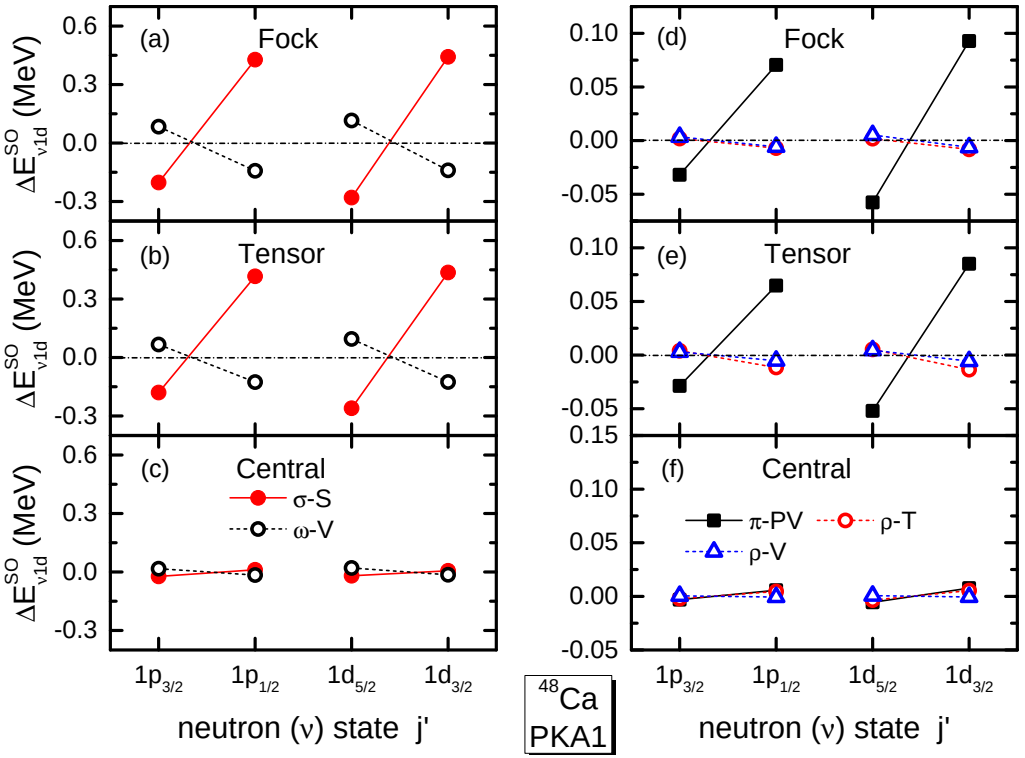
- Better conserved PSS favors the halo formation
- $\rho$-T: enhance the neutron-proton correlations

# Tensor Force Component in Fock terms

- Non-local Tensor force involved naturally with Fock terms plays essential role in nuclear structure.

$$V_\phi^T = \frac{1}{3}\frac{3(\gamma_0\Sigma_1\cdot\boldsymbol{q})(\gamma_0\Sigma_2\cdot\boldsymbol{q}) - (\gamma_0\Sigma_1)\cdot(\gamma_0\Sigma_2)\boldsymbol{q}^2}{m_\phi^2+\boldsymbol{q}^2}, \qquad \phi = \pi\text{-PV}, \sigma\text{-S} \qquad (28)$$

—L.J. Jiang, S. Yang, B.Y. Sun, W.H. Long, and H.Q. Gu, PRC 91, 034326 (2015).



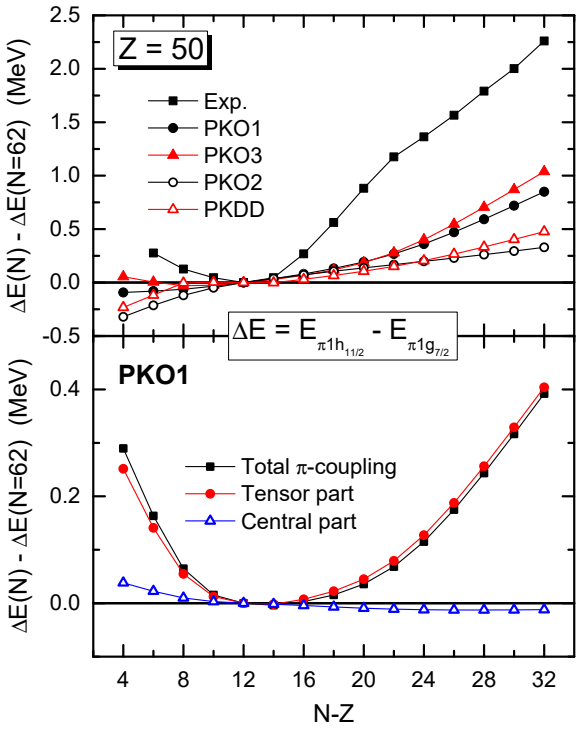

| $^{48}$Ca | $\hat{j}^2_{\gtrless}$ | $V^{T,\sigma}_{j\gtrless j'}$ ($10^{-1}$MeV) | | | |
|---|---|---|---|---|---|
| | | $\nu 1p_{1/2}$ | $\nu 1d_{5/2}$ | $\nu 1d_{3/2}$ | $\nu 1f_{7/2}$ |
| $\nu 1p_{3/2}$ | 4 | $-1.72$ | $+0.80$ | $-1.24$ | $+0.56$ |
| $\nu 1p_{1/2}$ | 2 | $+3.43$ | $-1.60$ | $+2.48$ | $-1.11$ |
| $\nu 1d_{5/2}$ | 6 | $-1.62$ | $+1.13$ | $-1.66$ | $+1.02$ |
| $\nu 1d_{3/2}$ | 4 | $+2.44$ | $-1.69$ | $+2.50$ | $-1.53$ |

Sum rule of nuclear tensor force:

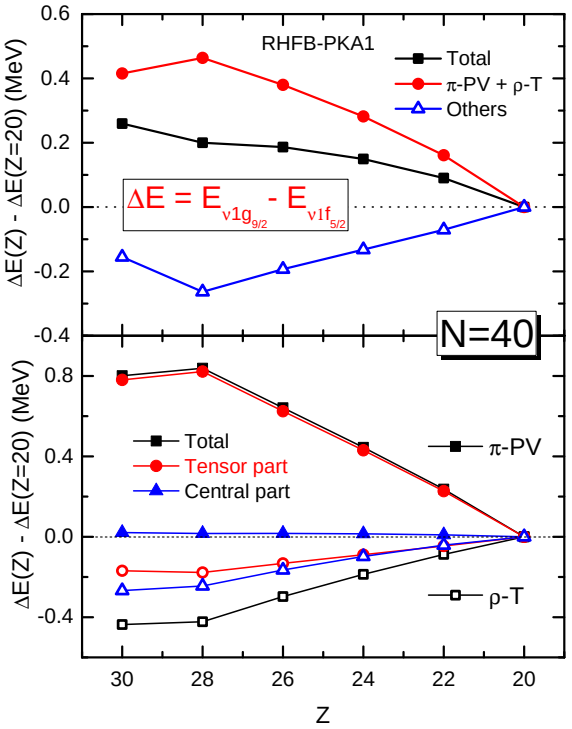
$$\hat{j}^2_> V^T_{j>j'} + \hat{j}^2_< V^T_{j<j'} = 0, \qquad (29)$$

# Isospin Nature: Tensor Effects vs Shell Evolution

- Isospin Evolution of the shells: $\rho$-T & $\pi$-PV couplings




— EPL 82, 12001 (2008).

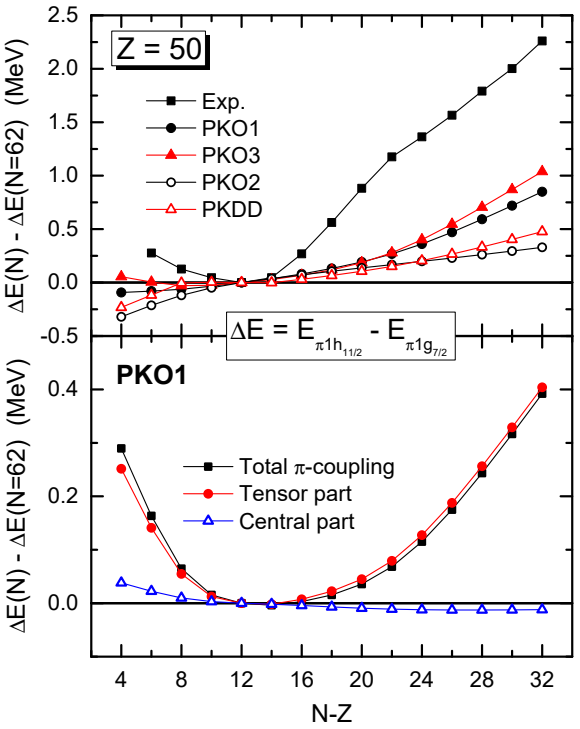



— PRC 87, 047301 (2013).

- Nuclear Tensor forces play the crucial role in the neutron (proton) shell evolution with respect to the change of proton (neutron) number.

# Isospin Nature: Tensor Effects vs Shell Evolution

- Isospin Evolution of the shells: $\rho$-T & $\pi$-PV couplings, non-local MF effects

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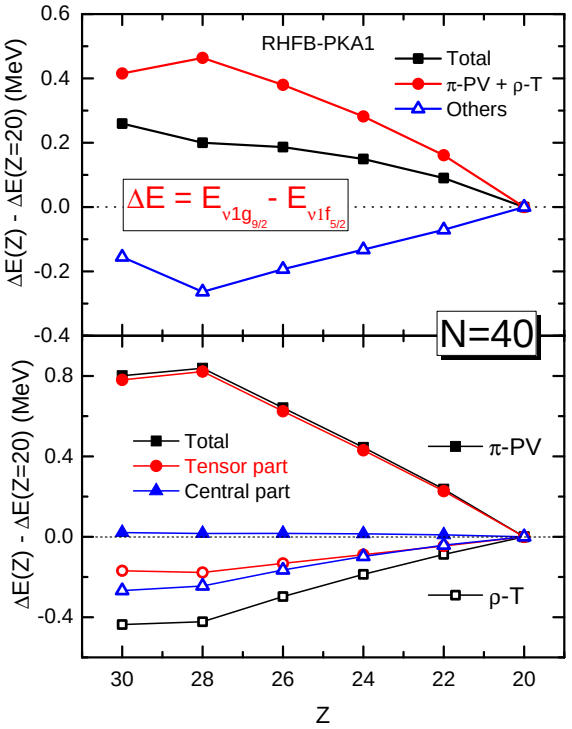
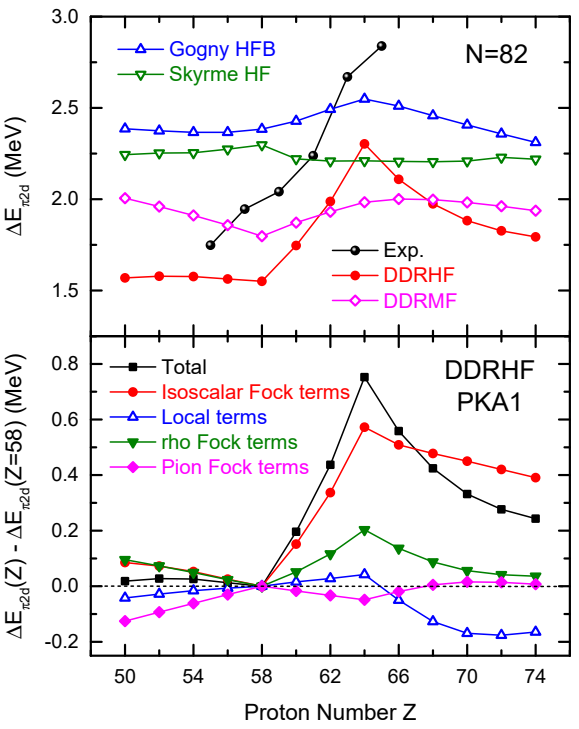



— PRC 87, 047301 (2013).




— PLB 680, 428 (2009).

- Nuclear Tensor forces play the crucial role in the neutron (proton) shell evolution with respect to the change of proton (neutron) number.

- Non-local mean fields are significant in the neutron (proton) shell evolution with respect to the change of neutron (proton) number.

# Saturation mechanism of nuclear matter

- Calculations with full energy density functional (EDF) and the one with EDF excluding tensor force components (values in parentheses)

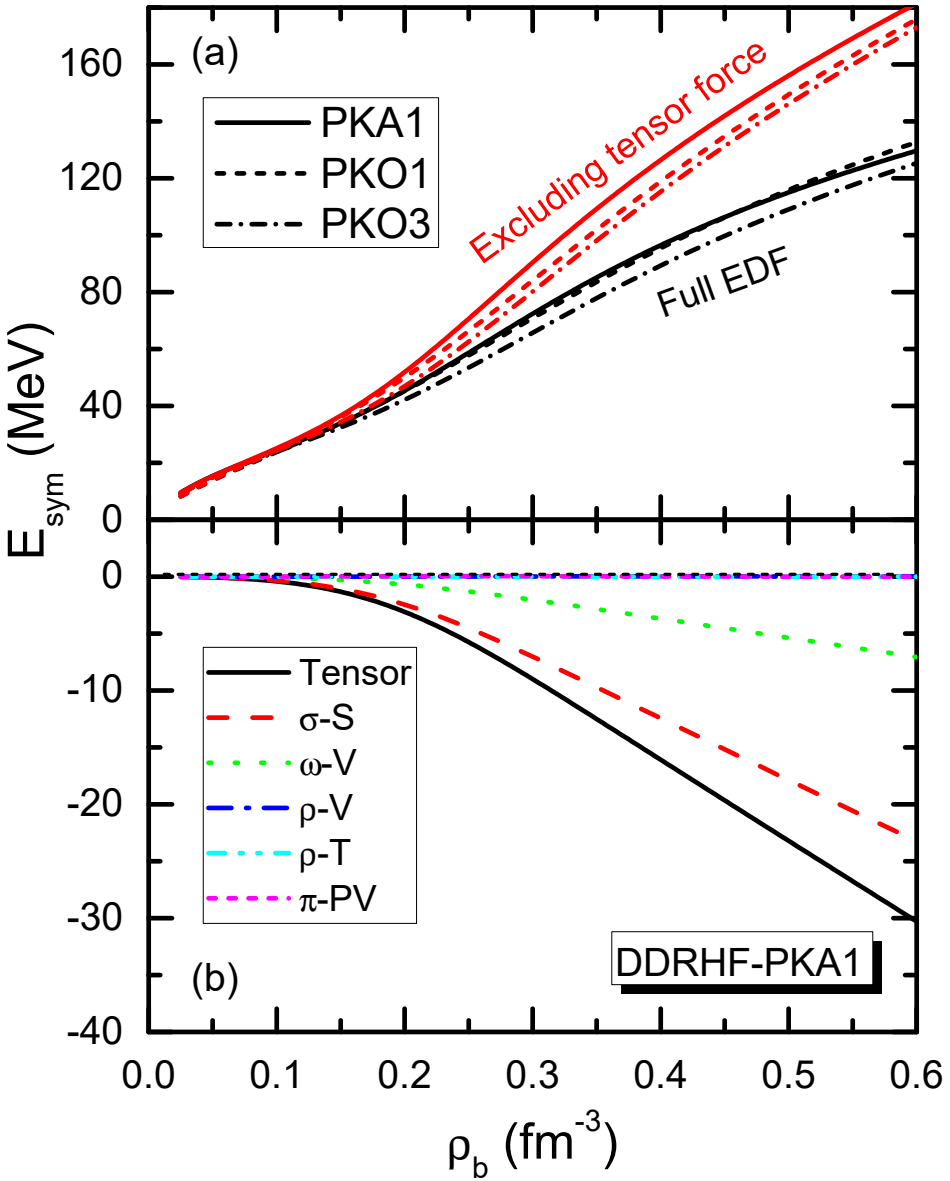
Table 2: Saturation density $\rho_0$, binding energy per nucleon $E/A$, incompressibility $K$, and the symmetry energy coefficient $J$ with its slope $L$ and curvature $K_{\rm sym}$.

| | $\rho_0$ (fm$^{-3}$) | $E/A$ (MeV) | $K$ (MeV) |
|---|---|---|---|
| PKA1 | 0.160 (0.148) | −15.83 (−14.18) | 229.96 (203.56) |
| PKO1 | 0.152 (0.140) | −16.00 (−14.21) | 250.24 (221.96) |
| PKO2 | 0.151 (0.139) | −16.03 (−14.31) | 249.60 (222.65) |
| PKO3 | 0.153 (0.140) | −16.04 (−14.22) | 262.47 (229.82) |

| | $J$ (MeV) | $L$ (MeV) | $K_{\rm sym}$ (MeV) |
|---|---|---|---|
| PKA1 | 36.02 (35.95) | 103.50 (115.49) | 212.90 (317.31) |
| PKO1 | 34.37 (33.50) | 97.70 (101.66) | 105.85 (158.87) |
| PKO2 | 32.49 (31.73) | 75.93 (81.12) | 77.51 (128.77) |
| PKO3 | 32.98 (32.26) | 83.00 (88.91) | 116.43 (176.39) |

— L.J. Jiang, S. Yang, J.M. Dong, and W.H. Long, PRC 91, 025802 (2015).

# Symmetry energy and neutron stars

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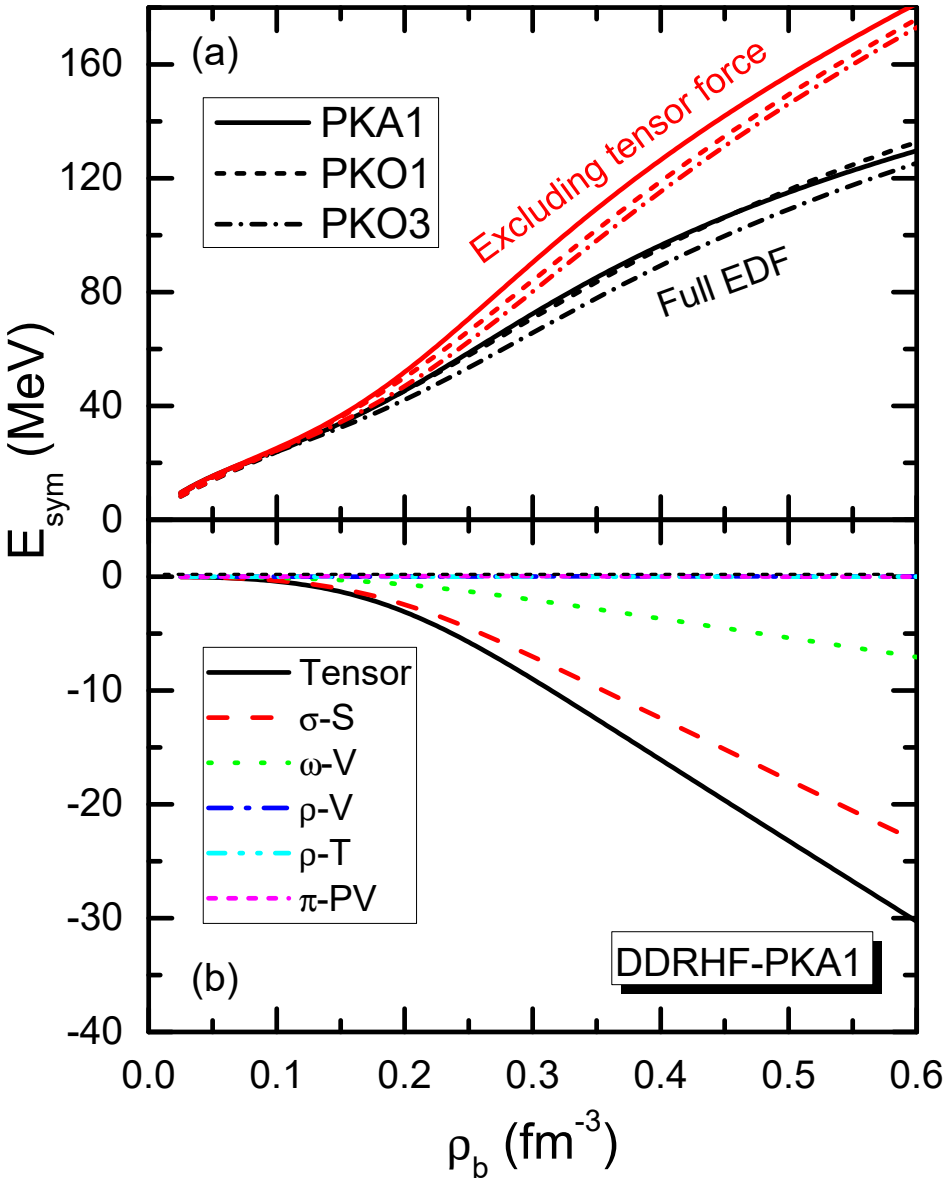


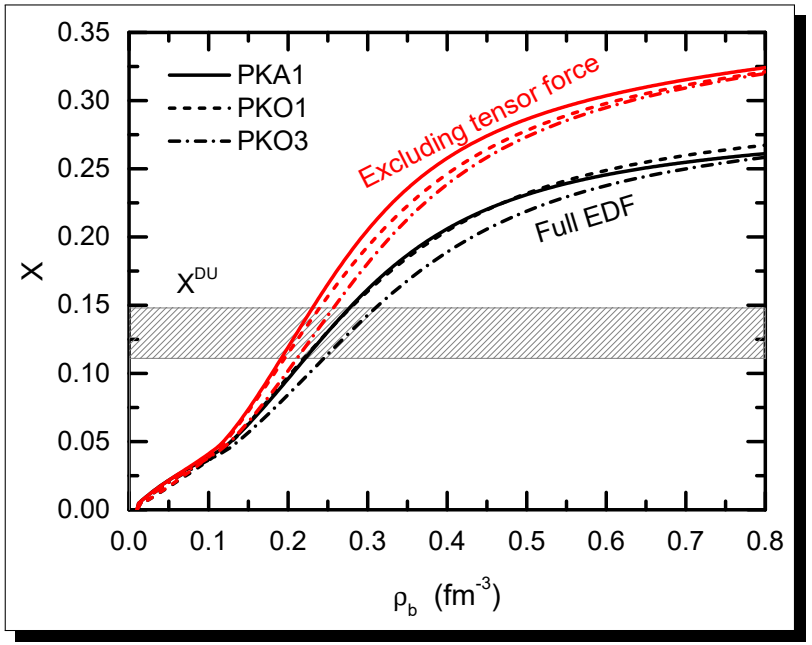


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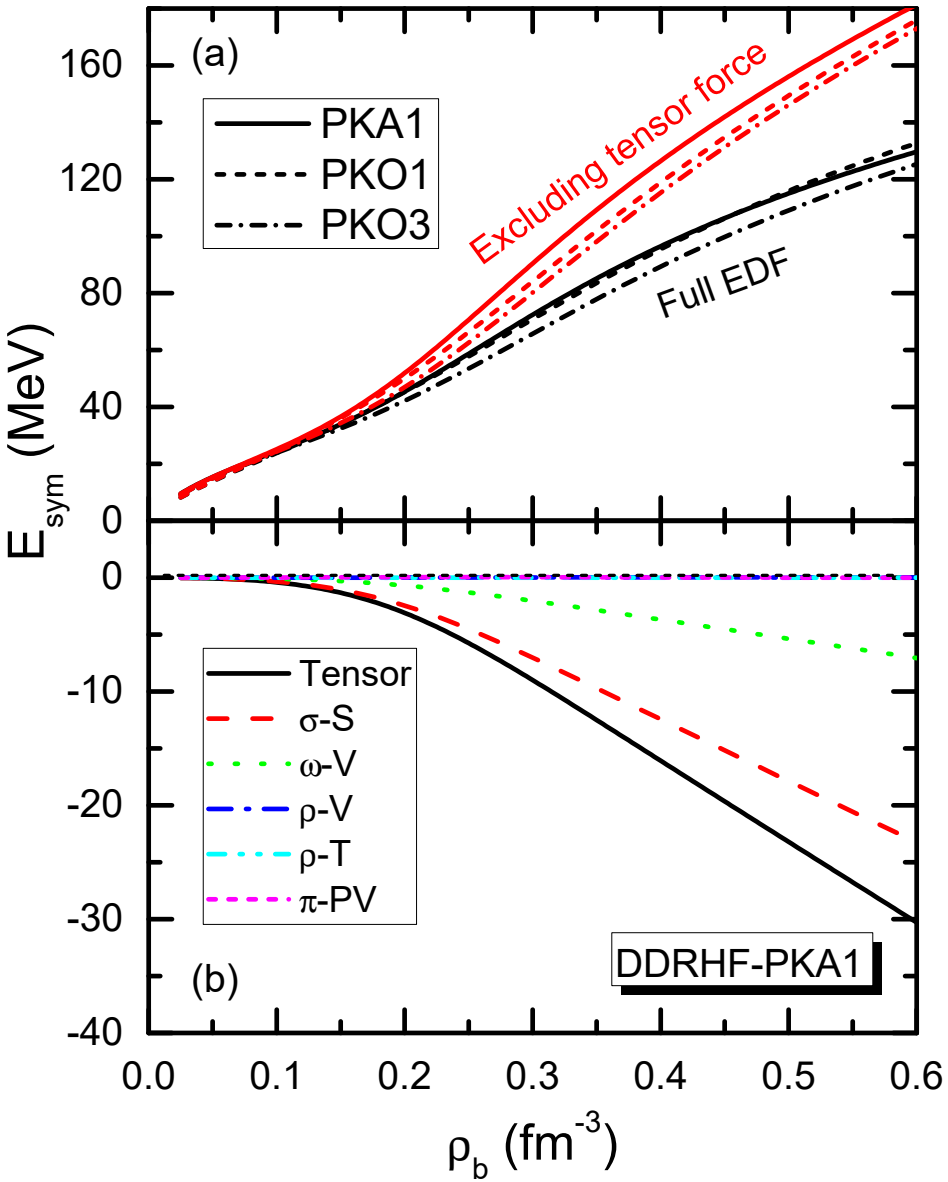


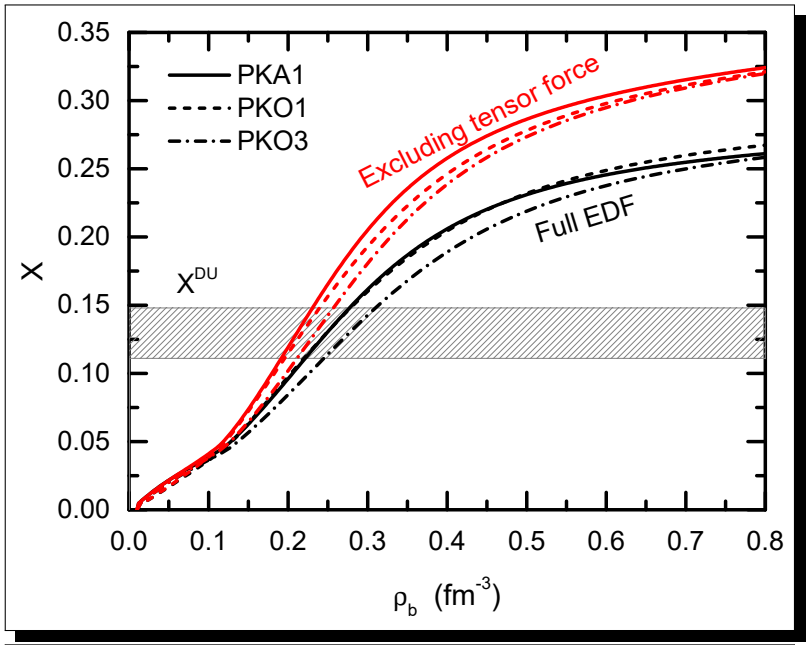


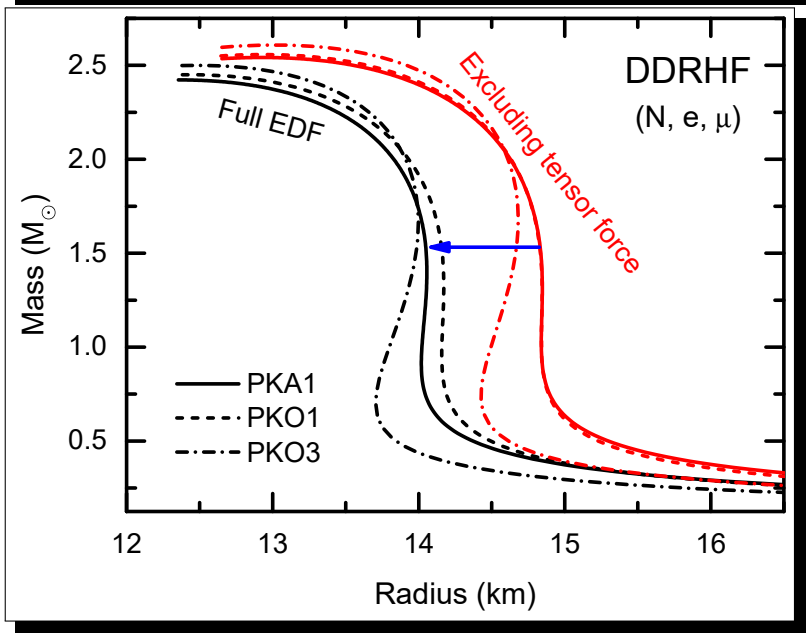


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# OUTLINE


1. Introduction
   - New Challenge and Opportunity in Nuclear Physcis
   - Covariant Density Functional theory
2. Covariant density functional with Fock terms
   - General Formalism for RHF Scheme
   - Relativistic Hartree-Fock-Bogoliubov method
   - Covariant density functionals with Fock terms
3. Role of non-local Fock terms in describing nuclear structure
   - New Balance induced by Fock terms in nuclear force
   - Neutron halo phenomena and Proton pseudo-spin symmetry
   - Nuclear tensor force components in Fock diagrams
4. **Conclusions and Perspectives**

# Conclusions

- The covariant density functional theory with Fock terms has been established for nuclear matter (neutron star physics as well), finite nuclei (spherical and axial deformed).

- Advantages brought about by the non-local Fock terms:
  - ☞ More reasonable balance in nuclear force: $\rho$-T Fock terms become comparable with rather strong $\omega$-repulsion and $\sigma$-attraction.
    Pseudo-spin symmetry restoration, halo formations, and nuclear isospin excitations
  - ☞ Important ingredient of nuclear force —— tensor force can be naturally introduced with the Fock terms: unified treatment of spin-orbit coupling and nuclear tensor force.
    Shell evolution, neutron star physics
  - ☞ Optimized balance in isovector channels: Fock terms present substantial contributions to the symmetry energy

- Perspectives: nature of nuclear force, microscopic mass table, $\cdots\cdots$

**Thank you for your attention!**