

# Spin-isospin responses in low energy: roles of many-body correlations

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# Unique features of atomic nucleus

Spin-isospin d.o.f: made of “four” types of fermions

Strongly-correlated nucleons: superfluidity/superconductivity, spin-singlet/triplet

Surface and shape: deformation

Distribution of densities in space

for the ground-state (static) w/ time-reversal inv.

particle (normal) density

$$\rho_\nu(\mathbf{r}) = \sum_{\sigma} \langle \psi_{\nu}^{\dagger}(\mathbf{r}\sigma) \psi_{\nu}(\mathbf{r}\sigma) \rangle$$

$$\rho_{\pi}(\mathbf{r}) = \sum_{\sigma} \langle \psi_{\pi}^{\dagger}(\mathbf{r}\sigma) \psi_{\pi}(\mathbf{r}\sigma) \rangle$$

pair (abnormal) density

$$\tilde{\rho}_{\nu}(\mathbf{r}) = \langle \psi_{\nu}(\mathbf{r} \downarrow) \psi_{\nu}(\mathbf{r} \uparrow) \rangle$$

$$\tilde{\rho}_{\pi}(\mathbf{r}) = \langle \psi_{\pi}(\mathbf{r} \downarrow) \psi_{\pi}(\mathbf{r} \uparrow) \rangle$$

assuming no S=1 superfluidity

onset of multipole deformation:

$$Q_{\lambda} = \int d\mathbf{r} r^{\lambda} Y_{\lambda}(\hat{r}) \psi^{\dagger}(\mathbf{r}) \psi(\mathbf{r}), \quad \langle Q_{\lambda} \rangle \neq 0$$

# Unique features of atomic nucleus

Spin-isospin d.o.f: "four" types of fermion

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## Distribution of densities in space/time for dynamics

particle (normal) density

$$\rho_0(\mathbf{r}t) = \sum_{\sigma} \sum_{\tau} \langle \psi^{\dagger}(\mathbf{r}\sigma\tau t) \psi(\mathbf{r}\sigma\tau t) \rangle$$

$$\rho_{1,\mu_{\tau}}(\mathbf{r}t) = \sum_{\sigma} \sum_{\tau,\tau'} \langle \psi^{\dagger}(\mathbf{r}\sigma\tau t) \psi(\mathbf{r}\sigma\tau' t) \rangle \langle \tau | \boldsymbol{\tau}_{\mu_{\tau}} | \tau' \rangle$$

$$s_{0,\mu_{\sigma}}(\mathbf{r}t) = \sum_{\sigma,\sigma'} \sum_{\tau} \langle \psi^{\dagger}(\mathbf{r}\sigma\tau t) \psi(\mathbf{r}\sigma'\tau t) \rangle \langle \sigma | \boldsymbol{\sigma}_{\mu_{\sigma}} | \sigma' \rangle$$

$$s_{1,\mu_{\sigma},\mu_{\tau}}(\mathbf{r}t) = \sum_{\sigma,\sigma'} \sum_{\tau,\tau'} \langle \psi^{\dagger}(\mathbf{r}\sigma\tau t) \psi(\mathbf{r}\sigma'\tau' t) \rangle \langle \sigma | \boldsymbol{\sigma}_{\mu_{\sigma}} | \sigma' \rangle \langle \tau | \boldsymbol{\tau}_{\mu_{\tau}} | \tau' \rangle$$

pair (abnormal) density

$$\tilde{\rho}_{0,\mu_{\sigma}}(\mathbf{r}t) = \frac{1}{2} \sum_{\sigma,\sigma'} \sum_{\tau} \langle \psi(\mathbf{r}\sigma\tau t) \psi(\mathbf{r}\bar{\sigma}'\bar{\tau} t) \rangle \langle \sigma | \boldsymbol{\sigma}_{\mu_{\sigma}} | \sigma' \rangle$$

$$\tilde{\rho}_{1,\mu_{\tau}}(\mathbf{r}t) = \frac{1}{2} \sum_{\sigma} \sum_{\tau,\tau'} \langle \psi(\mathbf{r}\sigma\tau t) \psi(\mathbf{r}\bar{\sigma}\bar{\tau}' t) \rangle \langle \tau | \boldsymbol{\tau}_{\mu_{\tau}} | \tau' \rangle$$

$$\psi(\mathbf{r}\bar{\sigma}\bar{\tau}) = (-2\sigma)(-2\tau)\psi(\mathbf{r} -\sigma -\tau)$$

# Spin-isospin response present only in nuclear system

$$s_{1,\mu_\sigma,\mu_\tau}(\mathbf{r}t) = \sum_{\sigma,\sigma'} \sum_{\tau,\tau'} \langle \psi^\dagger(\mathbf{r}\sigma\tau t) \psi(\mathbf{r}\sigma'\tau' t) \rangle \langle \sigma | \boldsymbol{\sigma}_{\mu_\sigma} | \sigma' \rangle \langle \tau | \boldsymbol{\tau}_{\mu_\tau} | \tau' \rangle$$

$$\rho(\mathbf{r}) + [\delta s_{1,\mu_\sigma,\mu_\tau}^\lambda(\mathbf{r}) e^{-i\omega_\lambda t} + \text{c.c}] \quad \text{in linear-response of TDDFT}$$

physical observables in spin-isospin response

$$\left| \int d\mathbf{r} f(\mathbf{r}) \delta s_{1,\mu_\sigma,\mu_\tau}^\lambda(\mathbf{r}) \right|^2$$

Gamow-Teller strength distribution: w/ isospin-change  $\mu_\tau = \pm 1$

$$S_{\mu_\sigma}^{\mu_\tau}(\omega) = \sum_\lambda \left| \int d\mathbf{r} \delta s_{1,\mu_\sigma,\mu_\tau}^\lambda(\mathbf{r}) \right|^2 \delta(\omega - \omega_\lambda)$$

GT matrix element to low-frequency states:  
key quantity to beta-decay rate

# Skyrme energy-density functional approach

Energy functional:  $\mathcal{E} = \int dr \mathcal{H}(r)$

Energy density:  $\mathcal{H} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{Skyrme}} + \mathcal{H}_{\text{em}}$

Skyrme energy density:  $\mathcal{H}_{\text{Skyrme}} = \sum_{t=0,1} \sum_{t_3=-t}^t \left( \mathcal{H}_{tt_3}^{\text{even}} + \mathcal{H}_{tt_3}^{\text{odd}} \right)$

$$\mathcal{H}_{tt_3}^{\text{even}} = C_t^\rho \rho_{tt_3}^2 + C_t^{\Delta\rho} \rho_{tt_3} \Delta\rho_{tt_3} + C_t^\tau \rho_{tt_3} \tau_{tt_3} + C_t^{\nabla J} \rho_{tt_3} \nabla \cdot \mathbf{J}_{tt_3} + C_t^{J^{\leftarrow\rightarrow}} \mathbf{J}_{tt_3}^2$$

$$\mathcal{H}_{tt_3}^{\text{odd}} = C_t^s \mathbf{s}_{tt_3}^2 + C_t^{\Delta s} \mathbf{s}_{tt_3} \cdot \Delta \mathbf{s}_{tt_3} + C_t^T \mathbf{s}_{tt_3} \cdot \mathbf{T}_{tt_3} + C_t^{\nabla s} (\nabla \cdot \mathbf{s}_{tt_3})^2 + C_t^j \mathbf{j}_{tt_3}^2 + C_t^{\nabla j} \mathbf{s}_{tt_3} \cdot \nabla \times \mathbf{j}_{tt_3}$$

Poorly known (poorly constrained):

T-odd Skyrme energy density

vanishes for ground-state of even-even nuclei

Isovector (t=1) coupling constants

less information on nuclei with neutron (proton) excess

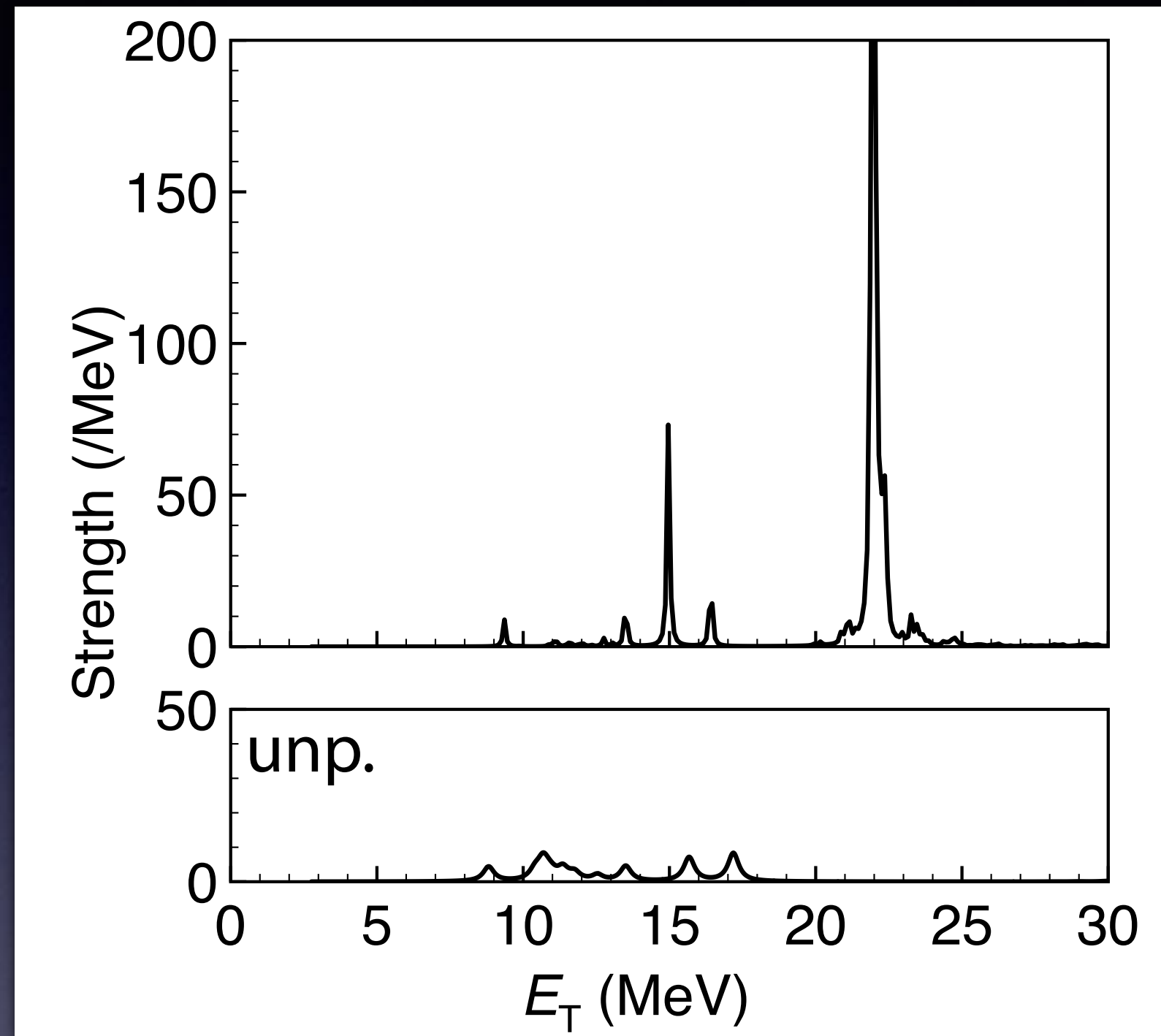
vector-isovector density



Gamow-Teller

# Many-body correlations essential in low energy

$^{208}\text{Pb}$ , SGII

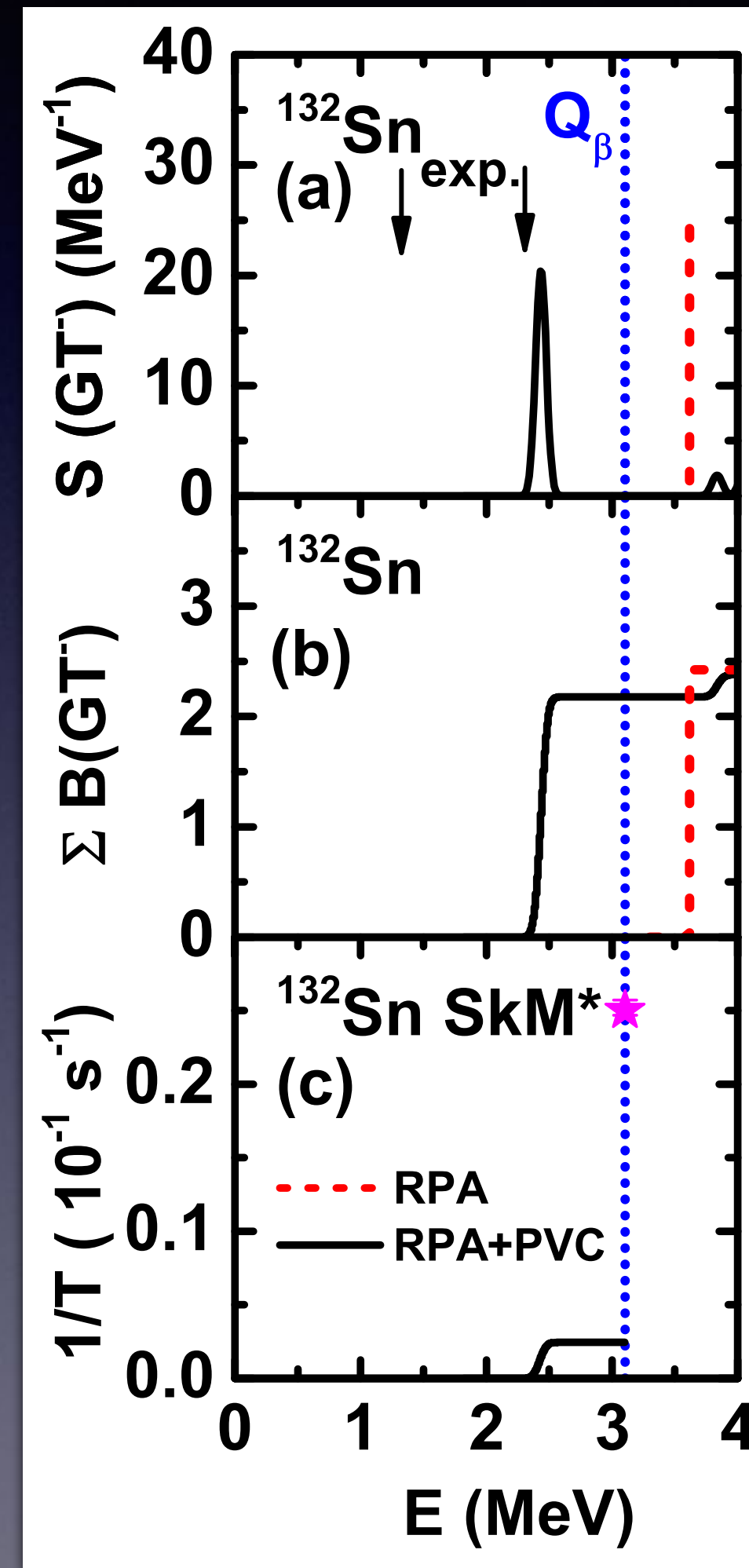


Most of the strengths are gathered in the high-energy giant resonance

Tiny low-lying strengths

$^{132}\text{Sn}$

spherical and normal-fluid nucleus



RPA(LR-TDDFT)

single peak



Phonon coupling

spreading  
lowering

$T_{1/2}: \infty \rightarrow \text{finite}$

# Many-body correlations essential in low energy

Kohn-Sham + RPA (LR-TDDFT)



deformation  
superfluidity

MB correlations showing up in open-shell nuclei

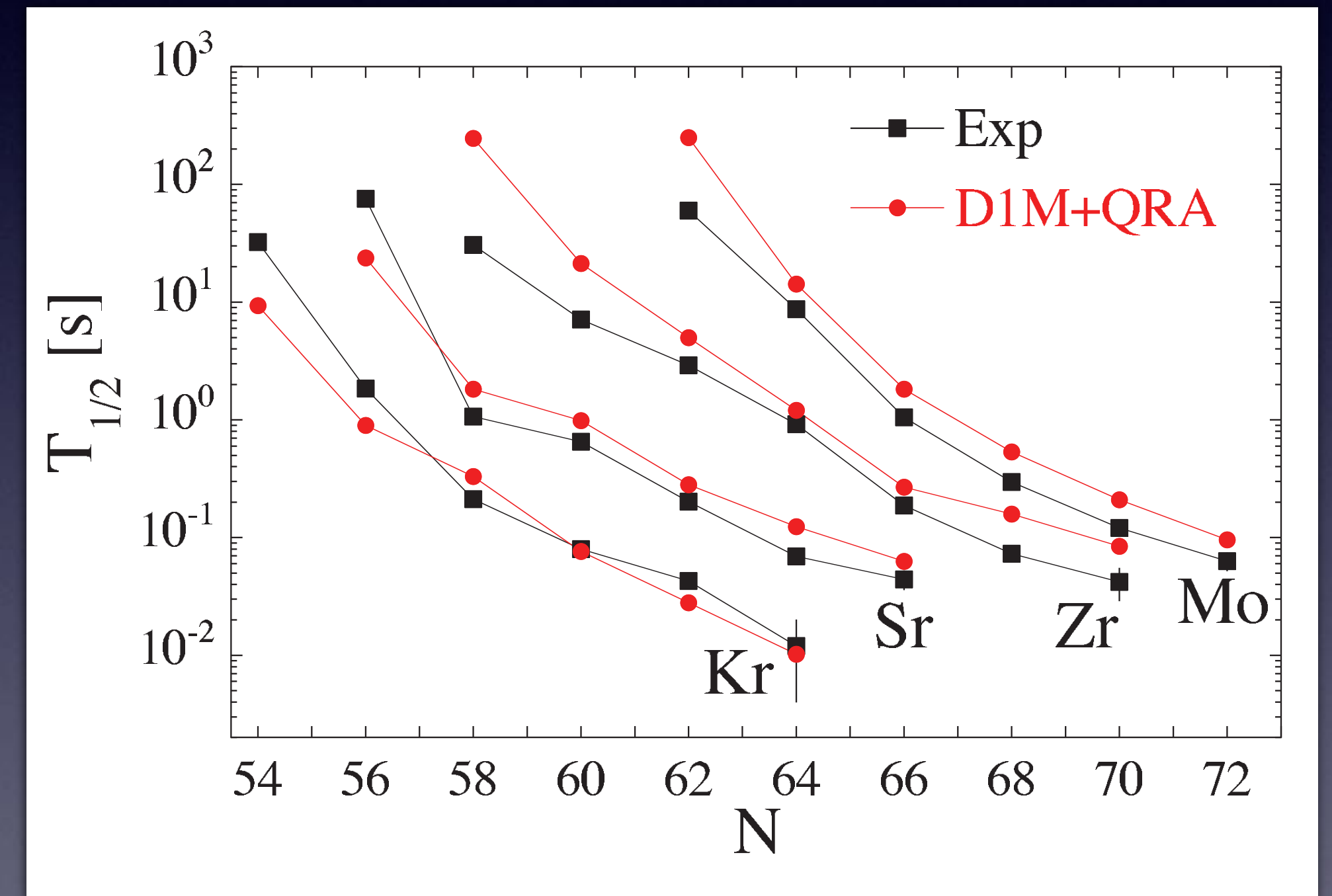
Deformed Kohn-Sham-Bogoliubov  
+ Quasiparticle RPA (LR-TDDFT)

M. T. Mustonen and J. Engel, PRC87(2013)064302

KY, PTEP2013(2013)113D02

M. Martini et al., PRC89(2014)044306

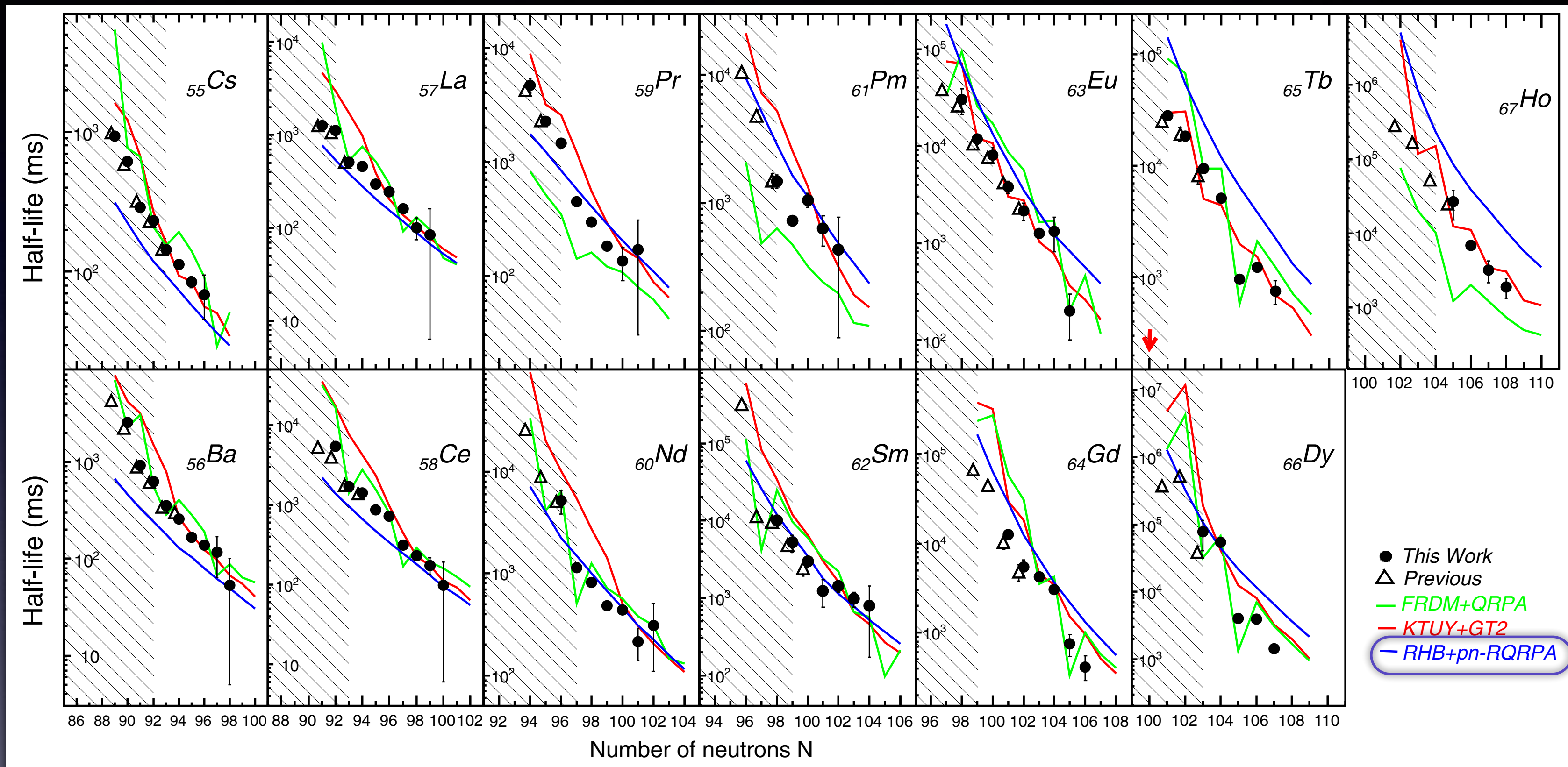
$\beta$ -decay half-lives of deformed nuclei



M. Martini et al., PRC89(2014)044306

# $\beta$ -decay study at RIBF for the rare-earth elements production of the r-process

J. Wu, S. Nishimura et al., PRL118(2017)072701



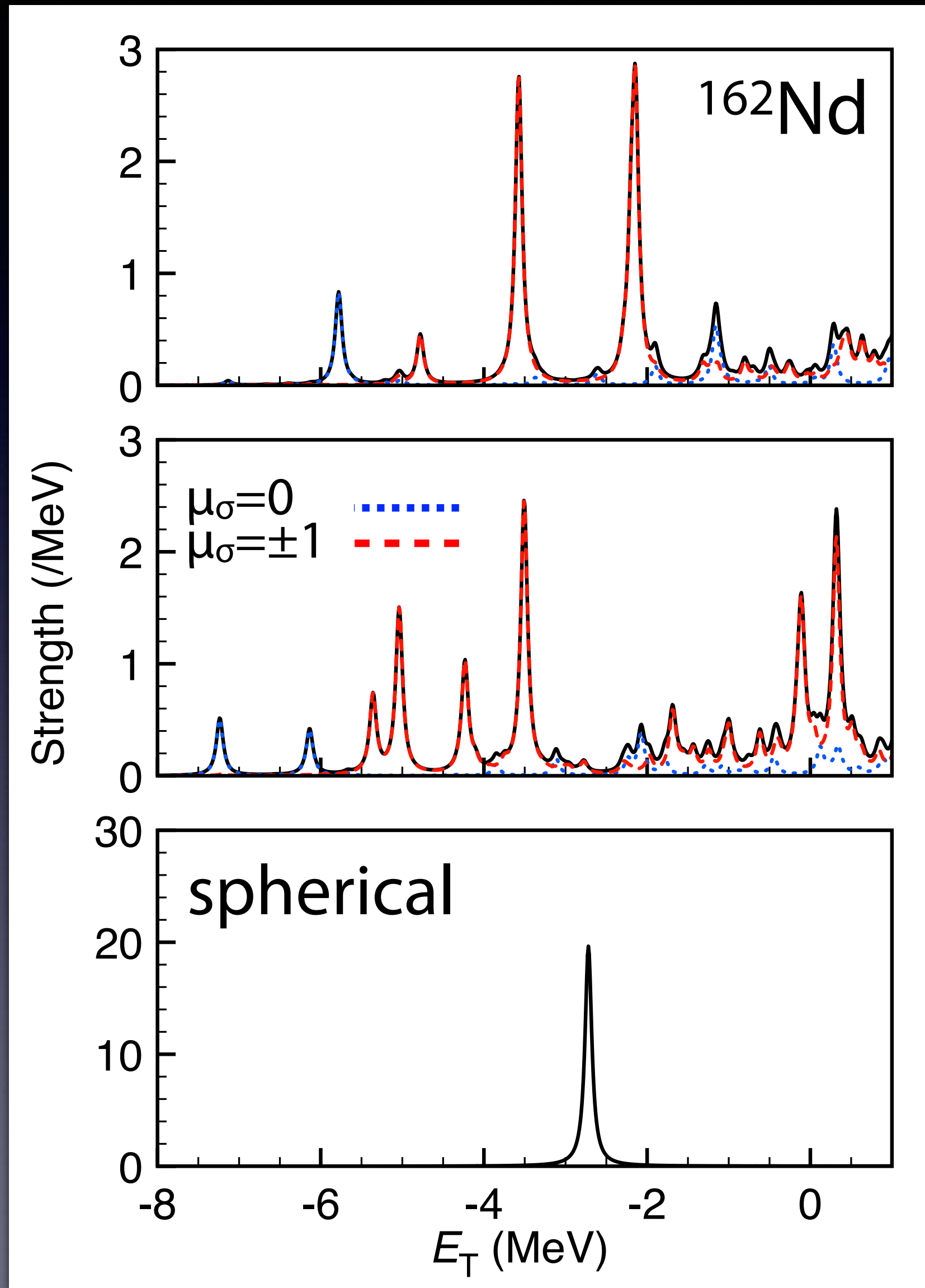
Rare-earth nuclei  
far from the magic numbers  
strongly deformed in space/  
gauge space

role of MB correlations?

Nuclear DFT cal.  
assuming the spherical shape  
reasonably produces the obs.



# Pairing and deformation for low-lying GT states



$$T_{1/2} = 0.27 \text{ s}$$

↑  
superfluidity

Energies are shifted higher  
pairing gaps

Low-lying strengths are reduced  
distortion of Fermi surfaces

Exp.

$$T_{1/2} = 0.31(20) \text{ s}$$

$$T_{1/2} = 0.14 \text{ s}$$

↑  
deformation

Quadrupole deformation  $\approx 2^+$  phonon condensation  
non-perturbed phonon coupling  
fragmentation and appearance of low-lying states

$$T_{1/2} = 0.53 \text{ s}$$

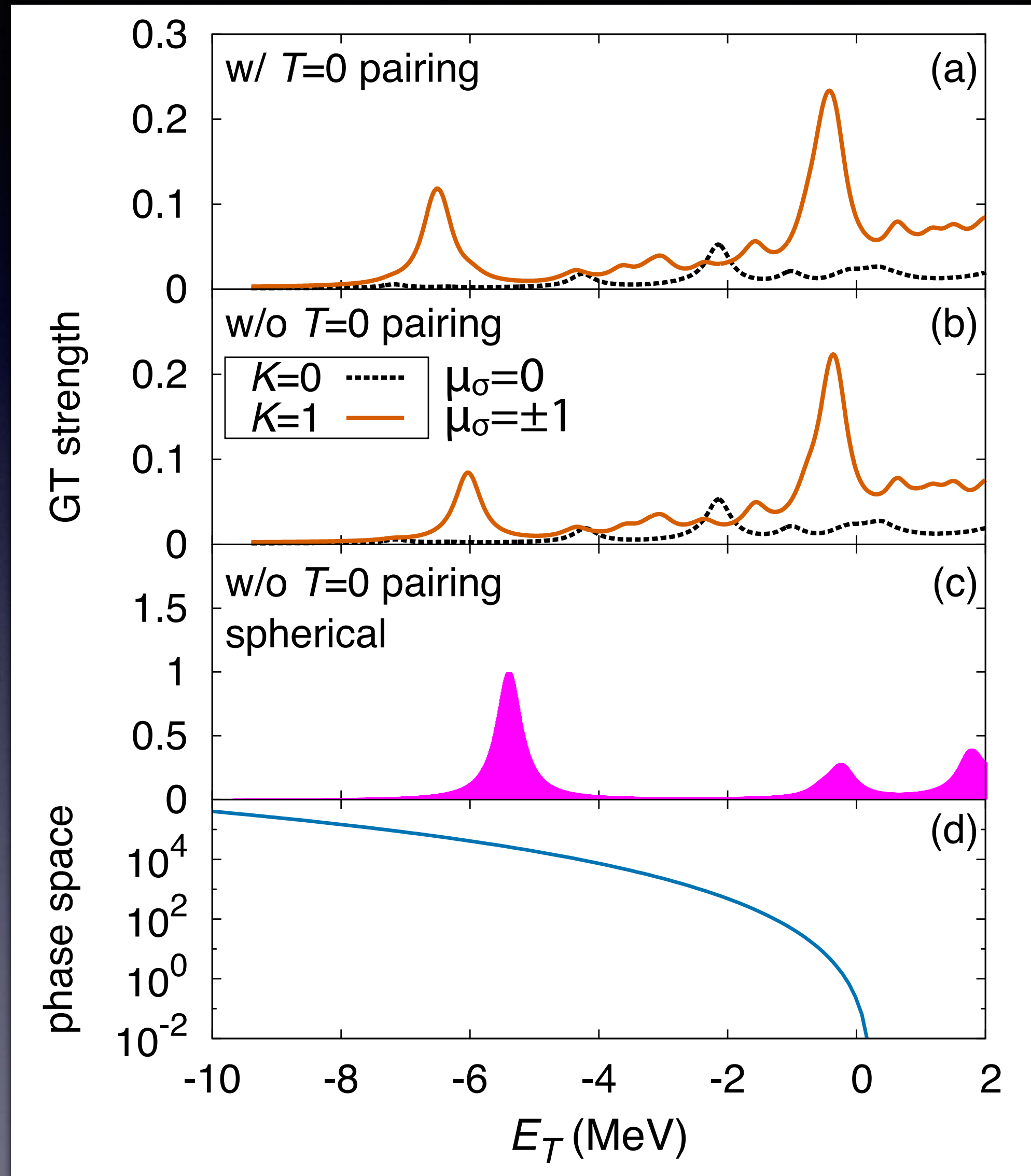
Strengths are concentrated on a single state w/ high energy

$$\nu 1h_{9/2} \rightarrow \pi 1h_{11/2}$$

# Pairing and deformation for low-lying GT states

S. Nishimura et al., PRL106(2011)052502

$^{106}\text{Zr}$



SLy4

$$T_{1/2} = 0.21 \text{ s}$$

$$T_{1/2} = 0.41 \text{ s}$$



deformation  
superfluidity

$$T_{1/2} = 0.07 \text{ s}$$

Exp.

$$T_{1/2} = 0.186(11) \text{ s}$$

Effect of MB correlations on  $\beta$ -decay rate depends very much on the nuclide (shell structure)

## Summary

TDDFT gives an intuitive picture of nuclear dynamics  
by looking at the density distributions

Linear response is a powerful method to investigate vibration of densities

allowing the breaking of symmetries: rotational symmetry in space/gauge space,  
we can include the many-body correlations in a simple way

(Q)RPA on top of the ordered (deformed) state takes the non-perturbed phonon  
coupling effect into account