

Spin and Spin-Isospin Responses in N=Z nuclei and Isoscalar Spin-triplet Pairing

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Hiroyuki Sagawa RIKEN/University of Aizu



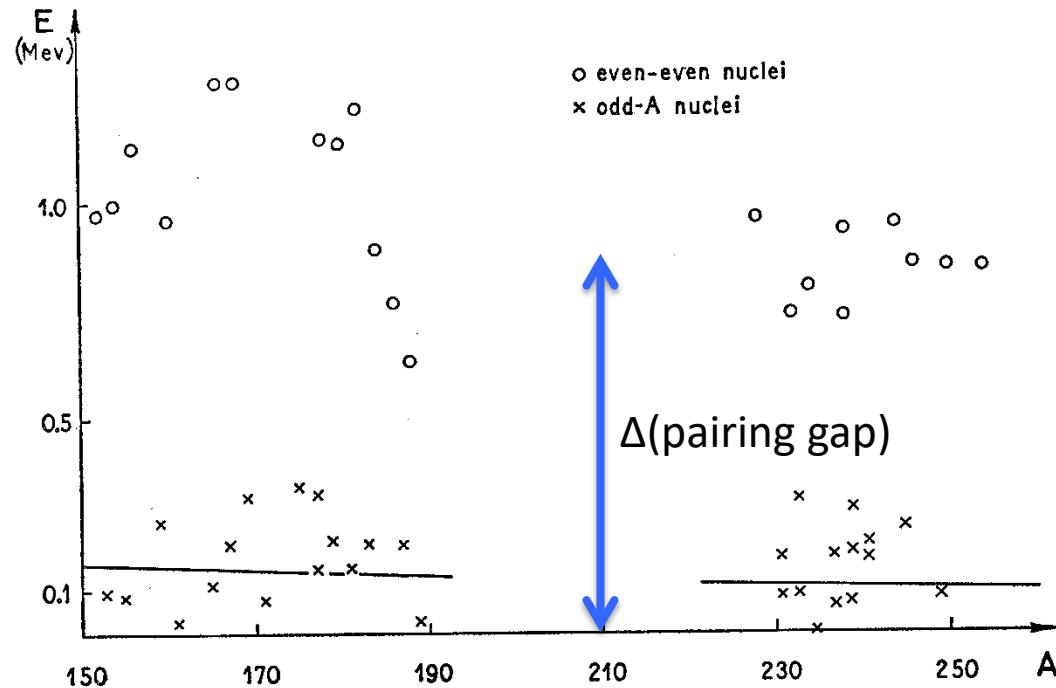
Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. BOHR, B. R. MOTTELSON, AND D. PINES*

Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

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The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.



BCS (Bardeen-Cooper-Schrieffer:1957)
Theory of Pairing correlations in metallic superconductor

T=1 S=0 pairing and T=0 S=1 pairing interactions

T=1 pairing (n-n, p-p pairing correlations) → spin singlet superfluid

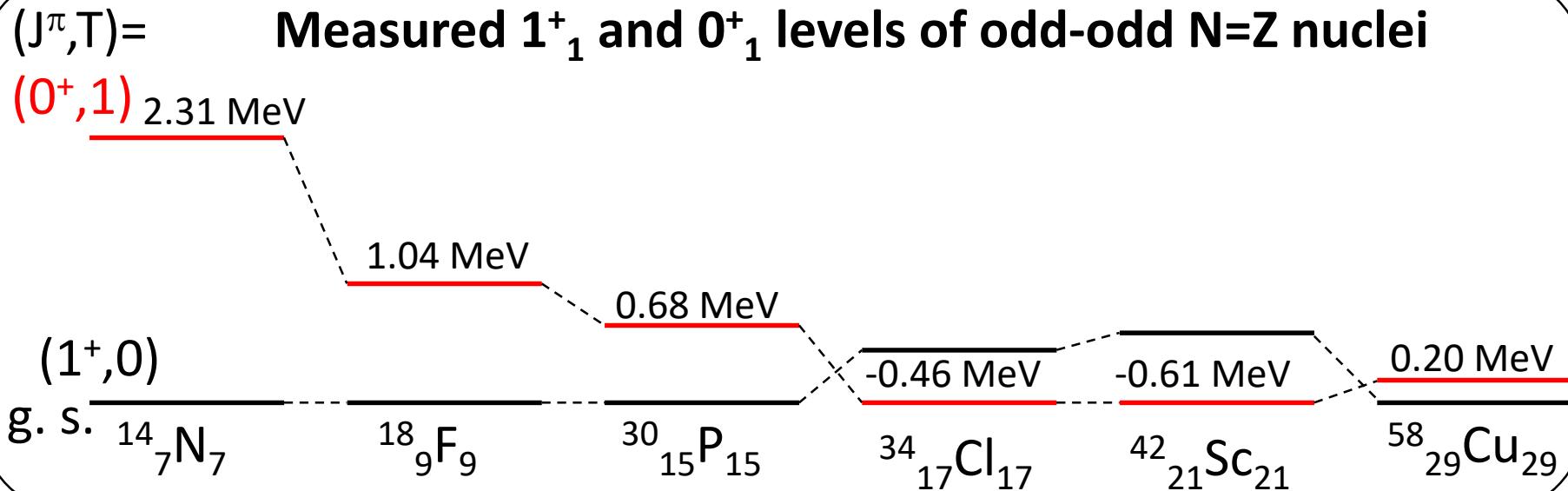
- mass (odd-even staggering)
- energy spectra (gap between the first excited state and the ground state in even-even nuclei)
- moment of inertia
- n-n or p-p Pair transfer reactions
- fission barrier (large amplitude collective motion)

Strong T=0 pairing (p-n pairing with S=1) → spin triplet superfluid ?

- deuteron ($T=0, S=1$) is bound, but not di-neutron ($T=1, S=0$)
- $N=Z$ Wigner energy (still controversial)
- Energy spectra in nuclei with $N=Z$ ($T=0$ and $J=1$)
- n-p pair transfer reaction
- low-energy super-allowed Gamow-Teller transition in $N=Z$ and $N=Z+2$ between SU(4) supermultiples

- n-p Pair correlations studied by 3-body model
 - ✓ T=0, 1 two channels
 - ✓ T=0, S=1 is attractive stronger than T=1, S=0 pair
cf. deuteron, matrix elements in shell models
 - ✓ In finite nuclei N>Z , the strong spin-orbit coupling may quench or even kill T=0 pairing

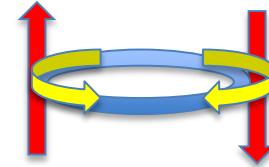
when l is larger , the spin-orbit is larger and T=0 pair correlations decrease



Two particle systems

T=1, S=0 pair

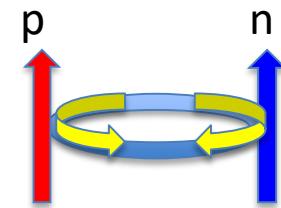
$$|(L=S=0)J=0, T=1\rangle \Rightarrow |(j=j')J=0, T=1\rangle \quad p(n) \quad p(n)$$



T=0, S=1 pair

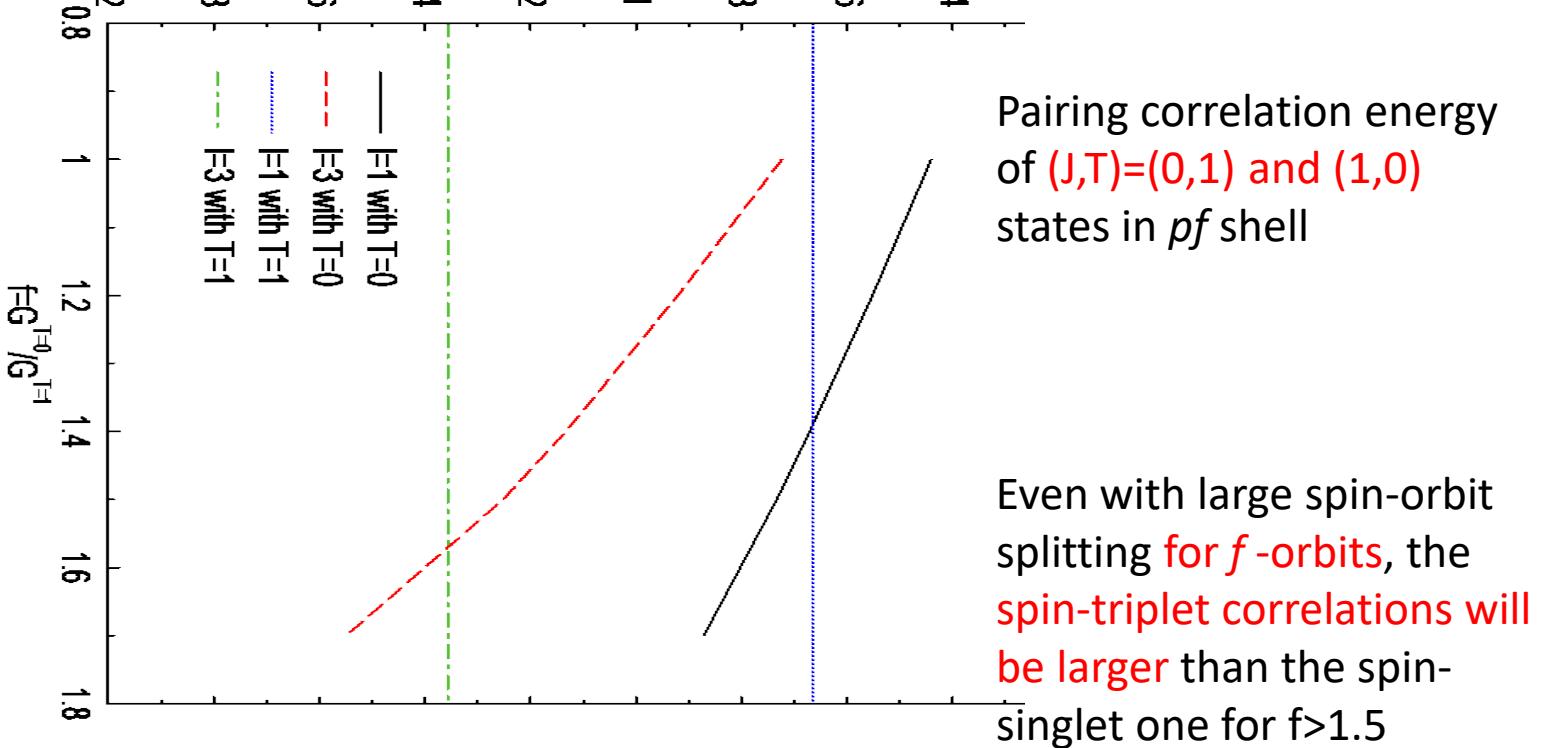
$$|(L=0, S=1)J=1, T=0\rangle \Rightarrow$$

$$a|(l=l' j=j')J=1, T=0\rangle + b|(l=l', j=j \pm 1)J=1, T=0\rangle$$



The total wave function should be anti-symmetric in spin-isospin-relative angular momentum quantum space.

If there is strong spin-orbit splitting, it is difficult to make (T=0, S=1)pair.



HS, Y. Tanimura and K. Hagino, PRC87, 034310 (2013)
TABLE I. Strengths of triplet and singlet interactions from shell-model fits and their ratios. See text for details.

| Source | v_s (MeV fm 3) | v_t (MeV fm 3) | Ratio |
|---------------------|----------------------|----------------------|-------|
| <i>sd</i> shell [8] | 280 | 465 | 1.65 |
| <i>fp</i> shell [9] | 291 | 475 | 1.63 |

G.F. Bertsch and Y. Luo, PRC81, 064320 (2010)

IS and IV M1 response and T=0 spin-triplet pairing correlations

HS, T. Suzuki and M. Sasano (Phys. Rev. C94, 041303(R) (2016)

HS and T. Suzuki, to be published (2017)

Exp. Data, Matsubara, et al., PRL115, 102501(2015)
High energy resolution proton inelastic scattering with $E_p = 295\text{MeV}$

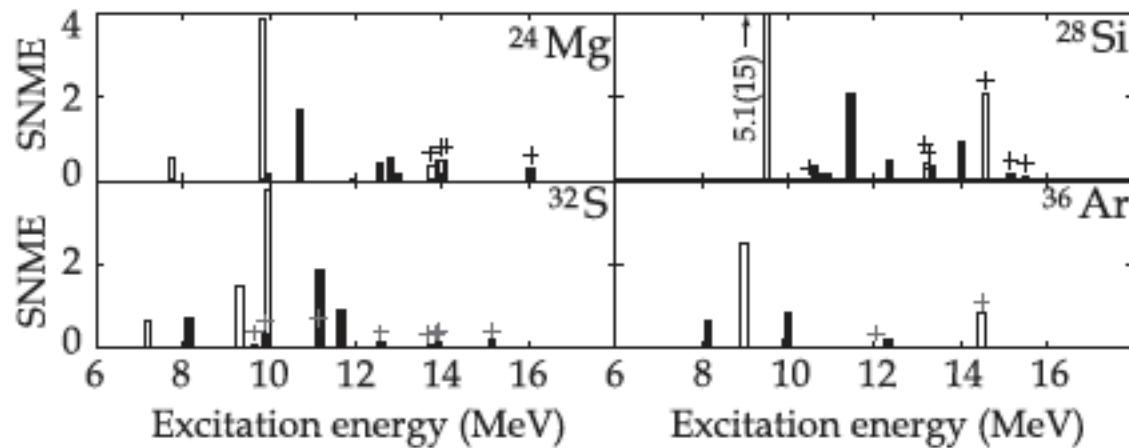


FIG. 2. Observed distributions of IS and IV-spin-*M1* SNME [open (filled) bars represent IS (IV) transitions]. The bars labeled + indicate states with a less confident spin assignment.

IS Channel

IS pairing=1.1xUSDB

- USDB: the original interaction with the bare spin operator

→ USDB1: the IS spin-triplet pairing matrix is enhanced multiplying a factor 1.1 on the relevant matrix elements of USDB interaction. The bare spin operator is adopted.

IS pairing=1.2xUSDB

IS effective g

$$g_{\text{eff}}/g_{\text{bare}} = 0.9$$

USDB2:the IS spin-triplet pairing matrix is enhanced multiplying a factor 1.1 on the relevant matrix elements of USDB interaction. The IS spin operator is 10% quenched.

USDB3:the IS spin-triplet pairing matrix is enhanced multiplying a factor 1.2 on the relevant matrix elements of USDB interaction. The IS spin operator is 10% quenched.

IV Channel

IS pairing=1.1xUSDB

IS pairing=1.2xUSDB

- USDB: the original interaction with the bare spin operator
- USDBq1:the IS spin-triplet pairing matrix is enhanced multiplying a factor 1.1 on the relevant matrix elements of USDB interaction. The effective IV spin operator is adopted.
- USDBq2:the IS spin-triplet pairing matrix is enhanced multiplying a factor 1.2 on the relevant matrix elements of USDB interaction. The effective IV spin operator is adopted.

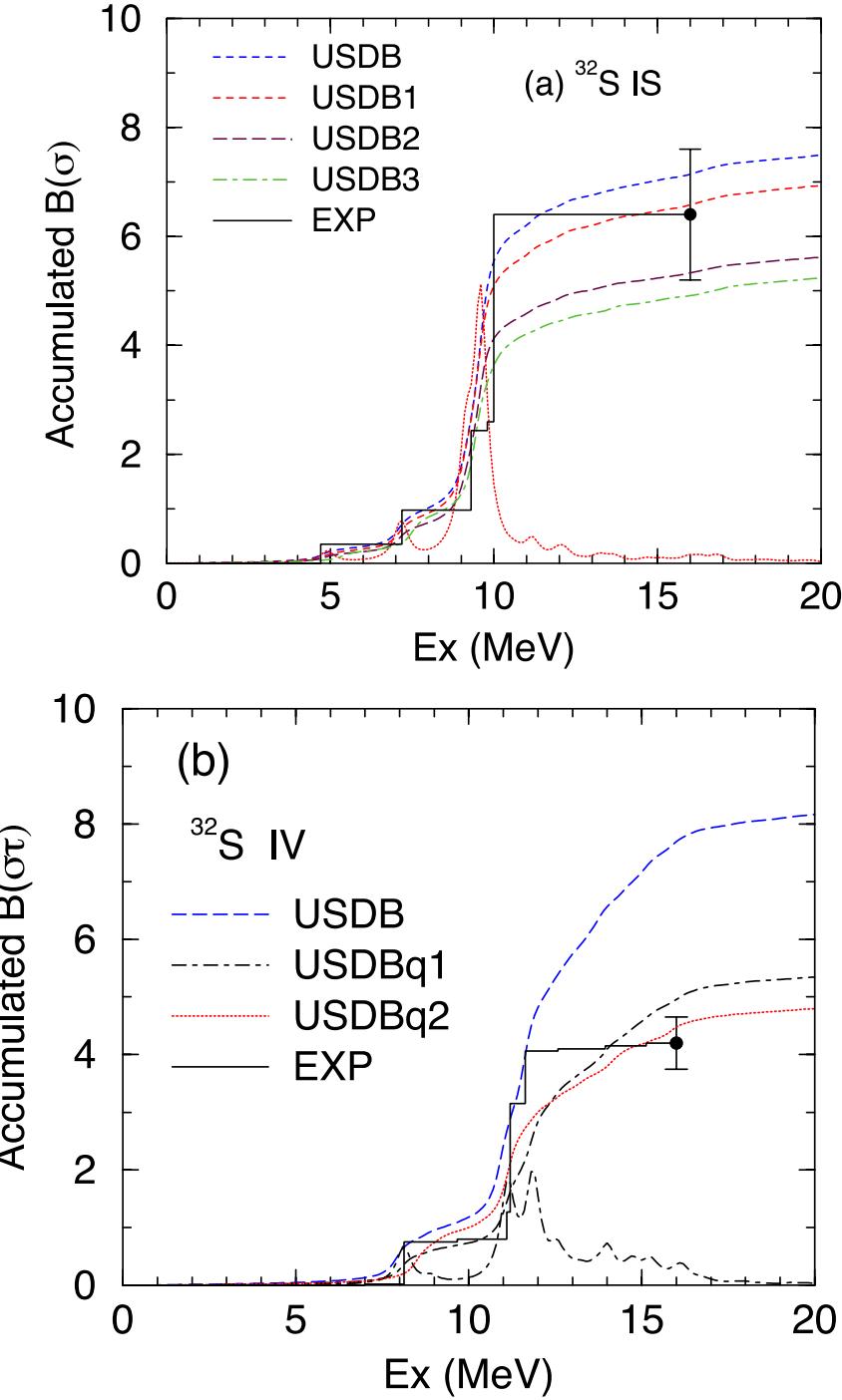
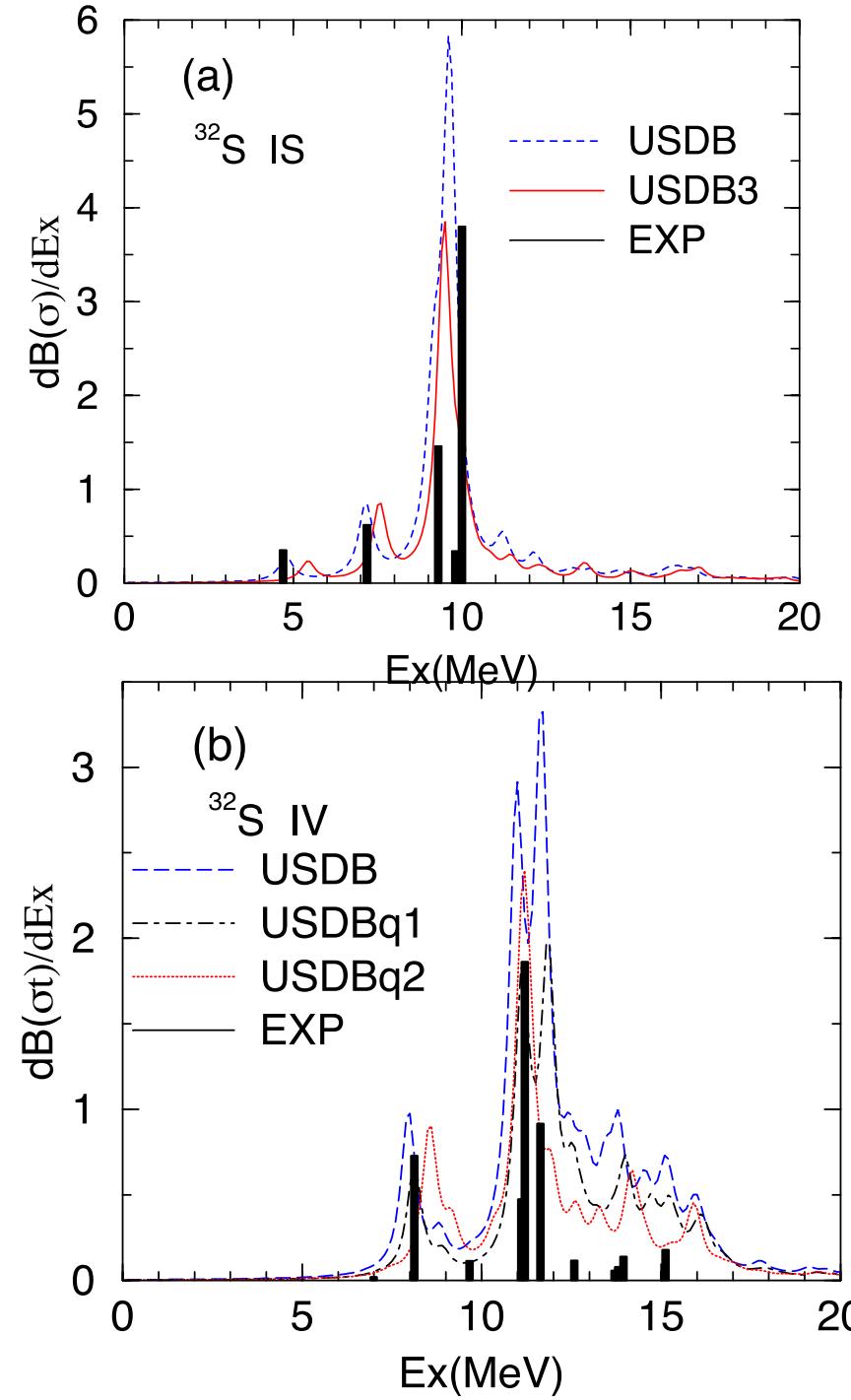
$$\hat{O}_{IV}^{eff} = f_s^{IV} \vec{\sigma} \tau_z + f_l^{IV} \vec{l} \tau_z + f_p^{IV} \sqrt{8\pi} [Y_2 \times \vec{\sigma}]^{(\lambda=1)} \tau_z \quad (10)$$

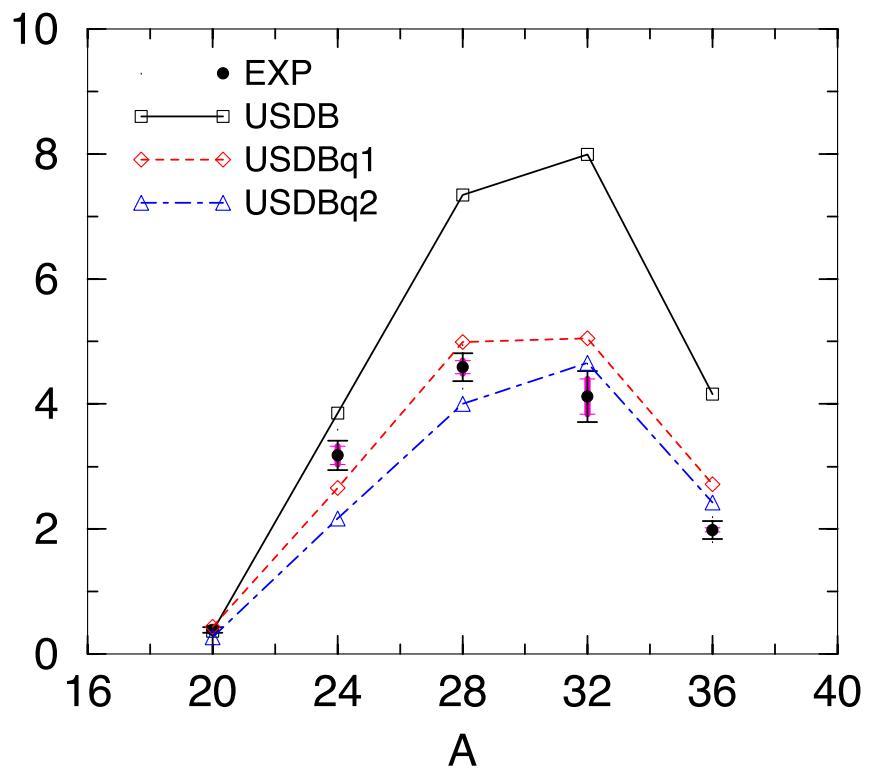
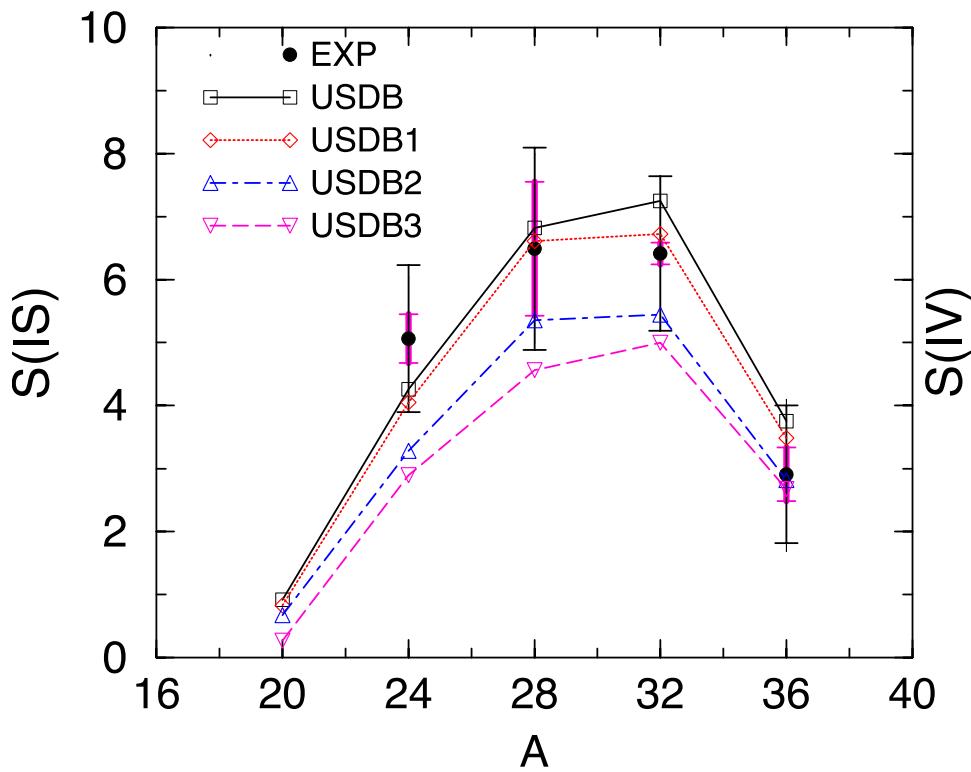
where $f_i^{IS(IV)}$ ($i = s, l, p$) are the effective coefficients of $IS(IV)$ spin, orbital an spin-tensor operators. The summation of index i in Eq. (1) is discarded in the effective operators. For the IV part, Towner obtained the corrections for the spin, orbital and the spin-tensor operators of GT transitions of $1d$ -orbit as

$$\hat{O}_{GT}^{eff} = (1 + \delta g_s) \vec{\sigma} t_{\pm} + \delta g_l \vec{l} t_{\pm} + \delta g_p \sqrt{8\pi} [Y_2 \times \vec{\sigma}]^{(\lambda=1)} t_{\pm} \quad (11)$$

with

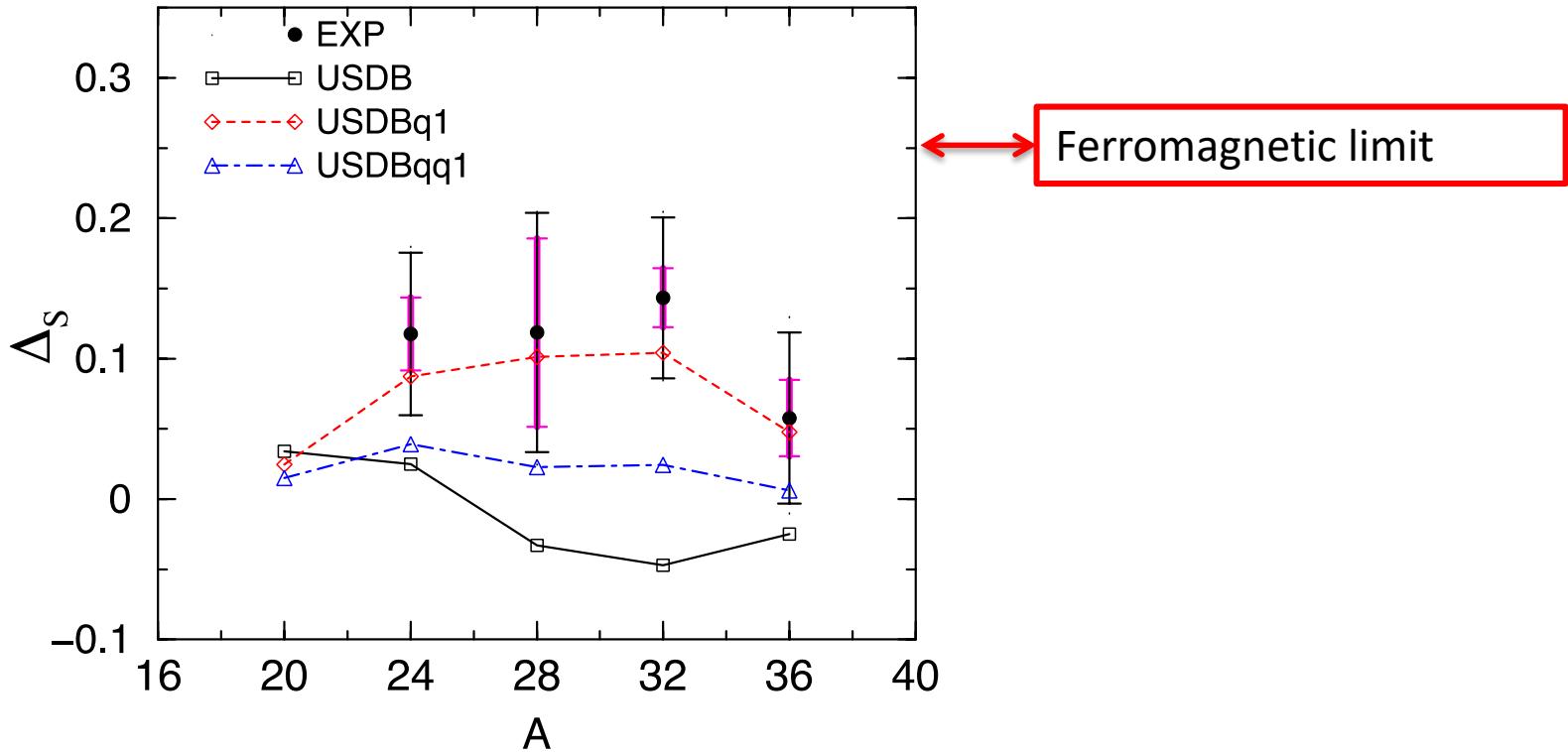
$$\delta g_s = -0.139, \quad \delta g_l = 0.0103, \quad \delta g_p = 0.0283 \quad (12)$$





$$S(\vec{\sigma}) = \sum_f \frac{1}{2J_i + 1} |\langle J_f || \hat{O}_{IS} || J_i \rangle|^2,$$

$$S(\vec{\sigma}\tau_z) = \sum_f \frac{1}{2J_i + 1} |\langle J_f || \hat{O}_{IV} || J_i \rangle|^2.$$



$$\begin{aligned}
 \Delta_{spin} &= \frac{1}{16} (S(\vec{\sigma}) - S(\vec{\sigma}\tau_z)) \\
 &= \sum_f \langle J_i | \sum_i \frac{\vec{\sigma}_n(i) + \vec{\sigma}_p(i)}{4} | J_f \rangle \langle J_f | \sum_i \frac{\vec{\sigma}_n(i) + \vec{\sigma}_p(i)}{4} | J_i \rangle \\
 &\quad - \sum_f \langle J_i | \sum_i \frac{\vec{\sigma}_n(i) - \vec{\sigma}_p(i)}{4} | J_f \rangle \langle J_f | \sum_i \frac{\vec{\sigma}_n(i) - \vec{\sigma}_p(i)}{4} | J_i \rangle \\
 &= \langle J_i | \vec{S}_p \cdot \vec{S}_n | J_i \rangle,
 \end{aligned} \tag{8}$$

Summary

1. Strong quenching in the IV spin response which is consistent with magnetic moments and Gamow-Teller beta-decay matrix.
2. IS spin sum rule strength shows a smaller quenching than IV spin ones.
3. Strong spin-triplet pairing gives a better agreement with empirical spin and spin-isospin correlations in $N=Z$ nuclei.
4. How Spin-triplet superfluidity can be realized in nuclear many-body system?

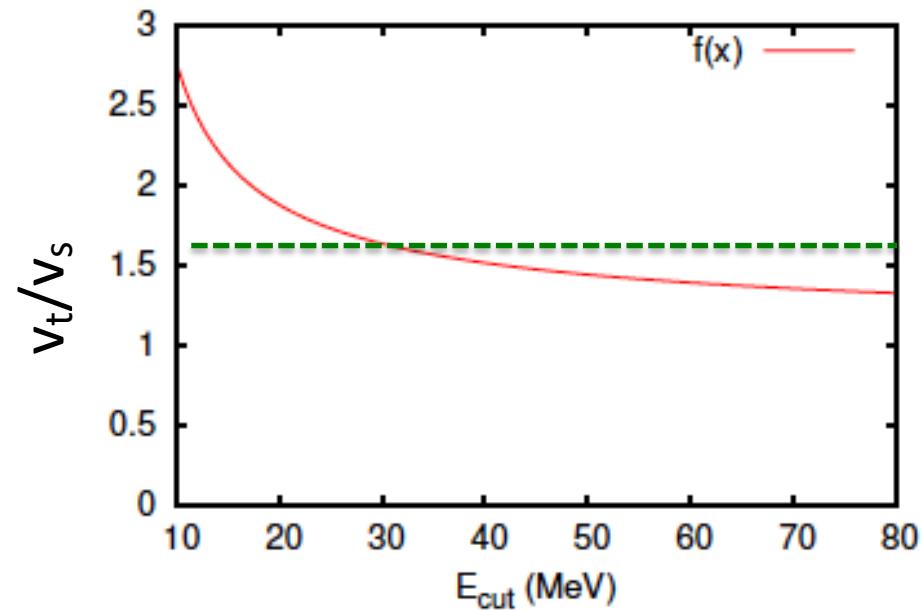
pn pairing interaction

$$V_{np}(\mathbf{r}_1, \mathbf{r}_2) = \hat{P}_s v_s \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[1 + x_s \left(\frac{\rho(r)}{\rho_0} \right)^\alpha \right] \\ + \hat{P}_t v_t \delta(\mathbf{r}_1 - \mathbf{r}_2) \left[1 + x_t \left(\frac{\rho(r)}{\rho_0} \right)^\alpha \right]$$

$$\hat{P}_s = \frac{1}{4} - \frac{1}{4} \boldsymbol{\sigma}_p \cdot \boldsymbol{\sigma}_n, \quad \hat{P}_t = \frac{3}{4} + \frac{1}{4} \boldsymbol{\sigma}_p \cdot \boldsymbol{\sigma}_n.$$

$$v_s = \frac{2\pi^2 \hbar^2}{m} \frac{2a_{pn}^{(s)}}{\pi - 2a_{pn}^{(s)} k_{\text{cut}}},$$

$$v_t = \frac{2\pi^2 \hbar^2}{m} \frac{2a_{pn}^{(t)}}{\pi - 2a_{pn}^{(t)} k_{\text{cut}}},$$



$$a_{pn}^{(s)} = -23.749 \text{ fm} \text{ and } a_{pn}^{(t)} = 5.424 \text{ fm}$$

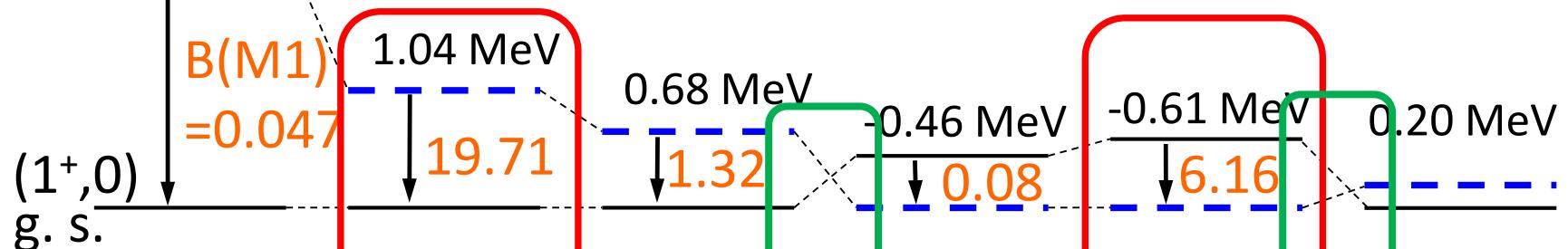
$$E_{\text{cut}} = k_{\text{cut}}^2 / 2m$$

1. E_{0+} - E_{1+} and $B(M1)$

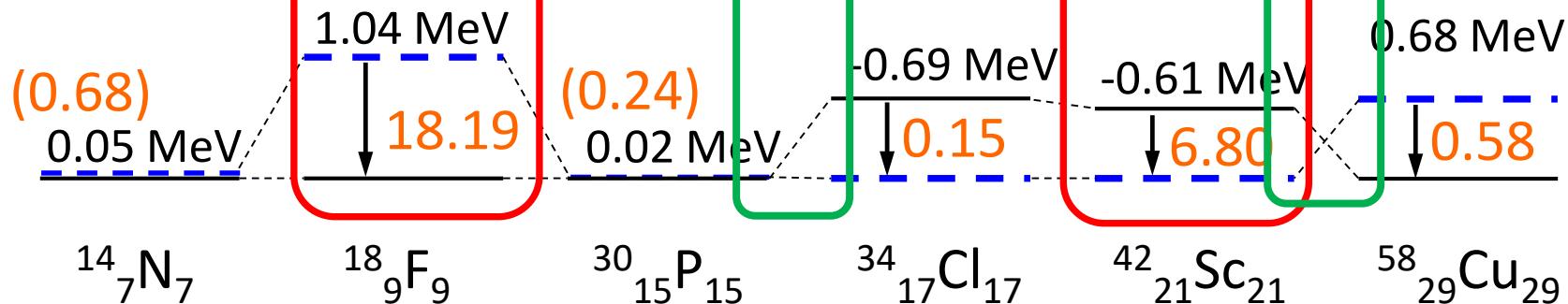
(a) Experiment <http://www.nndc.bnl.gov/>

$(J^\pi, T) =$

$(0^+, 1^-)$ 2.31 MeV



(b) Three-body model



The inversion of 1^+ and 0^+ shows a clear manifestation of the competition between spin-orbit and the spin-triplet pairing.

✓ Inversion of 1^+ and 0^+

✓ $^{18}\text{F}, ^{42}\text{Sc}$

■ Large $B(M1)$

■ Accurate E_{0+} - E_{1+} (^{42}Sc)