"Interdisciplinary Symposium on Modern Density Functional Theory"
June 19th – 23rd, 2017, RIKEN, Japan

Nuclear Gamow-Teller excitation and β-decay study

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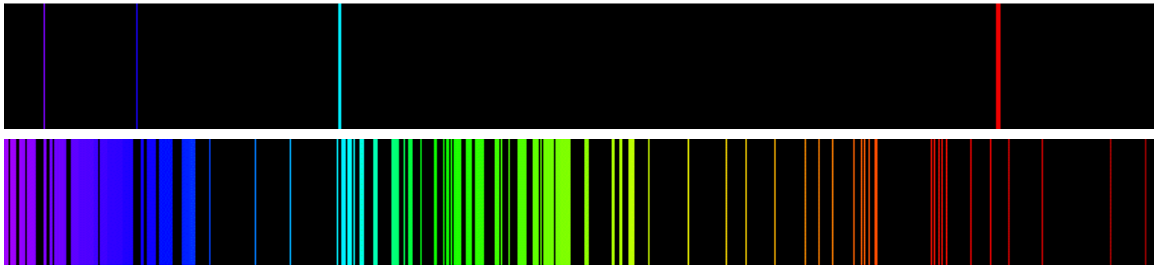
牛一斐

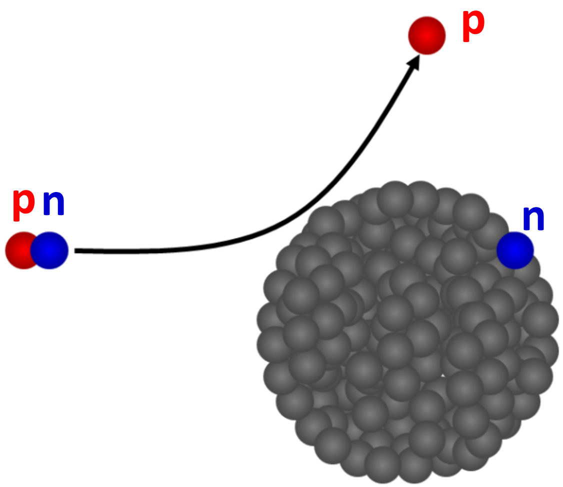
Nuclear Physics

Collaborators: G. Colo, E. Vigezzi, H. Sagawa, Z. M. Niu, C. L. Bai

Single Particle Excitation

- **Emission Spectra of atom H and Fe**





- **Single Neutron Spectrum in ^{209}Pb**

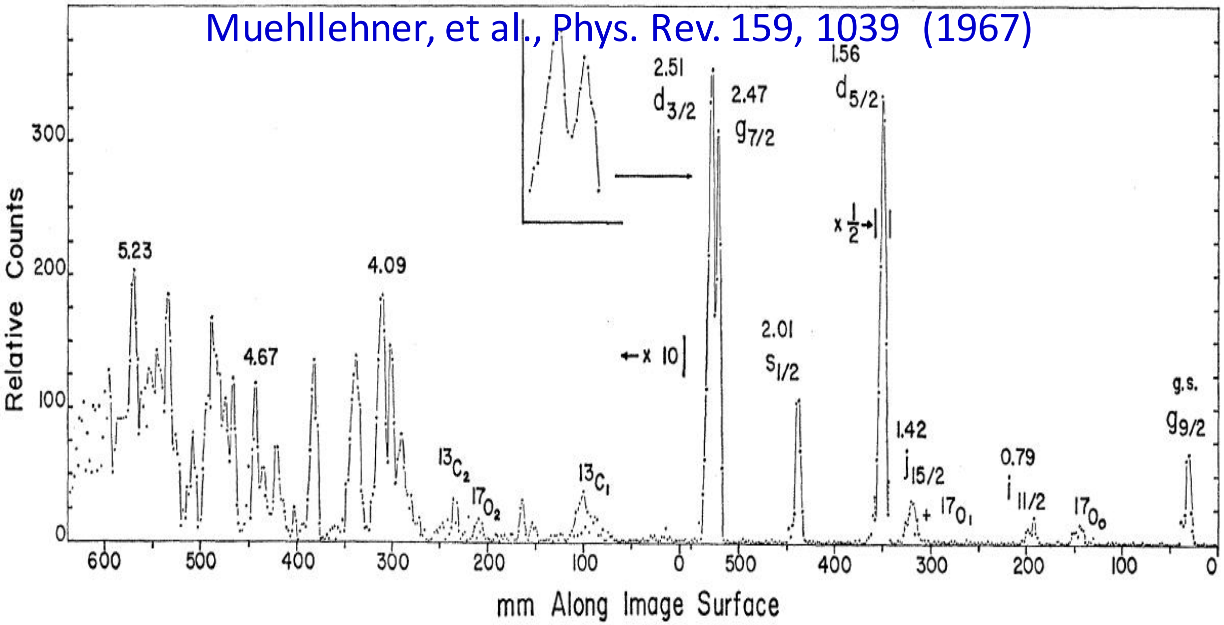
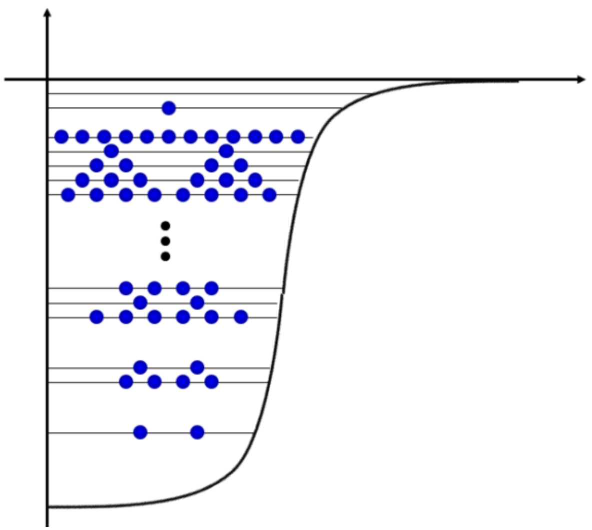
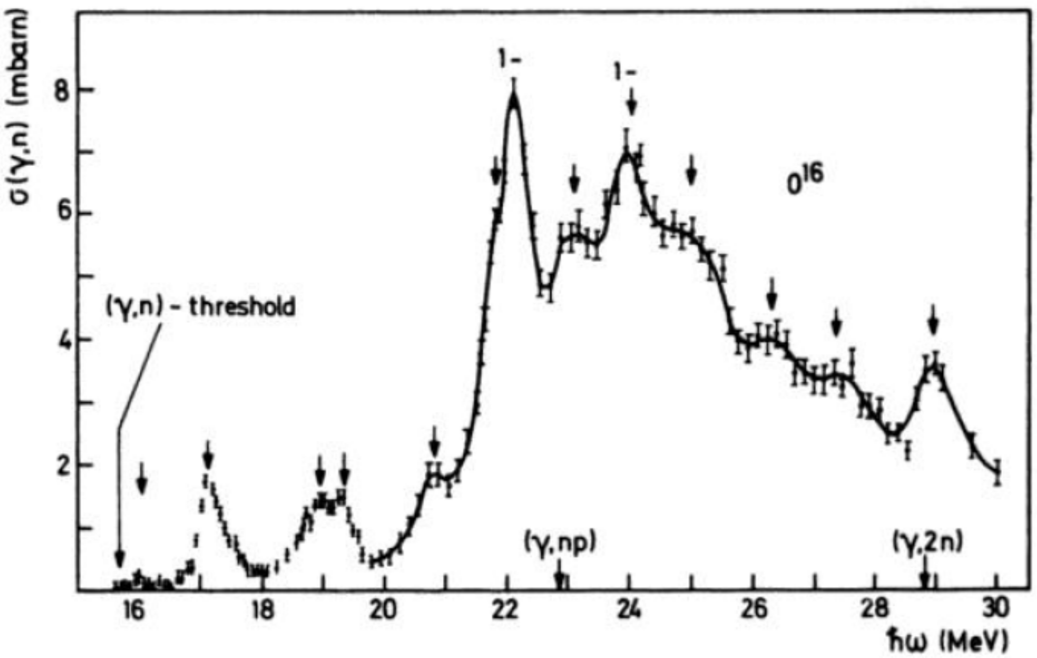


FIG. 3. Typical spectrum obtained in the reaction ^{208}Pb$(d,p)^{209}$Pb, at a deuteron energy of 20.1 MeV. $\theta=35°$.

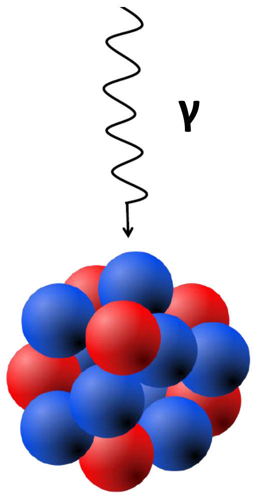


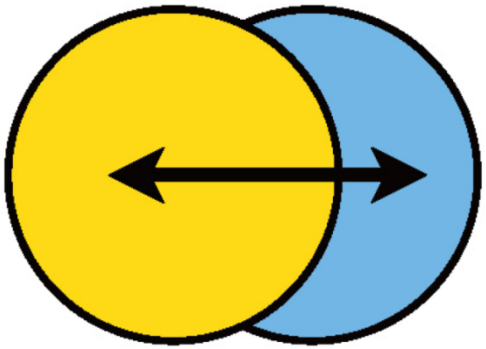
Collective Vibrations -- An Example

- **Giant Dipole Resonance in ^{16}O**



R. Bramblett, Phys. Rev. 133, B869 (1964)





- **Characteristics**
  - Broad resonance width ~ 5 MeV
  - Larger transition probabilities than s.p.
  - Excitation energy varies slowly and smoothly with mass number

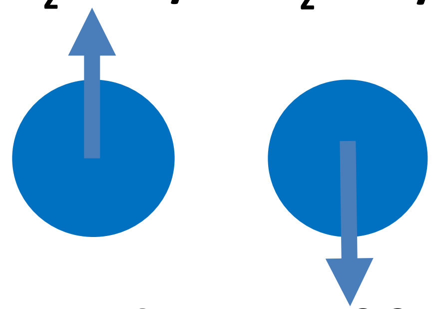
Richness in nuclear system

- **Degrees of freedom**

Fermion

✓ **Spin**

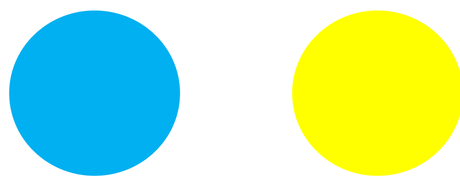
$s_z = +1/2$ $s_z = -1/2$ $s = 1/2$



nucleon

✓ **Isospin - Unique degree of freedom in nucleus!**

$t_z = +1/2$ $t_z = -1/2$ $t = 1/2$



nucleon

neutron   **proton**

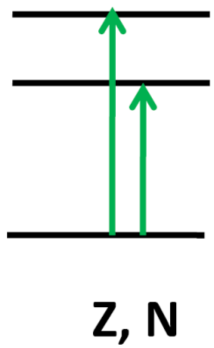
Similar mass   nearly the same interaction

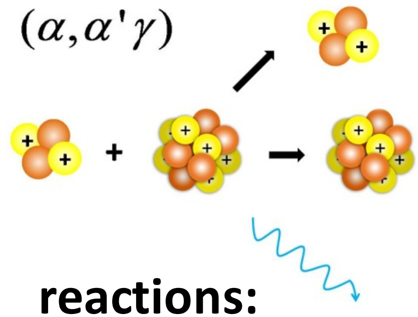
- **Interactions involved**

Strong   Electromagnetic   Weak

Various Modes of Collective Vibrations

- **Non-charge-exchange excitations**



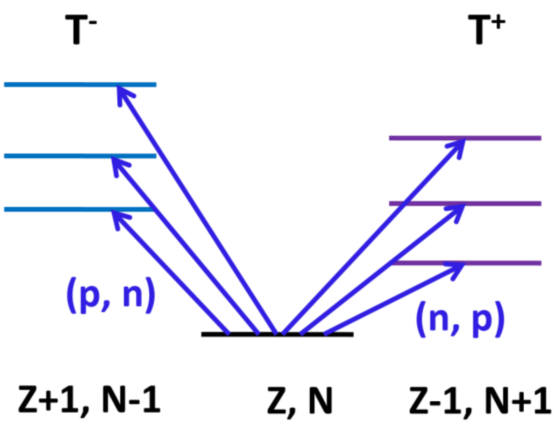


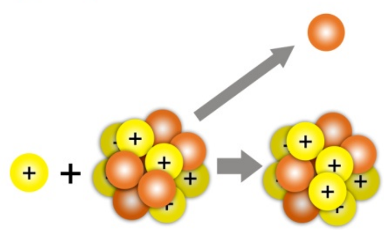
Strong
EM

reactions:
(α, α’ γ),
(γ, γ’), (γ, n) ,(e, e’)

- **Charge-exchange excitations**

(p,n) reaction





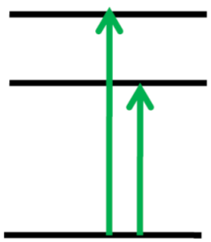
reactions:
(p,n) T^-, (n,p) T^+

Strong

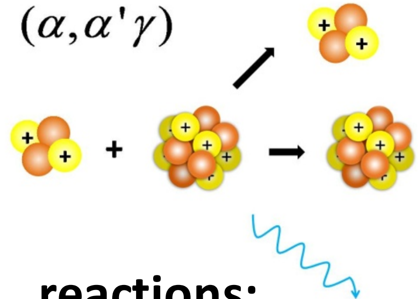
Bell	**Nucleus**
Hammers (iron,wooden)	**Operators/Probes (e, γ, α, p, n)**
Sounds of the Bell	**Modes of collective vibrations**

Various Modes of Collective Vibrations

- **Non-charge-exchange excitations**

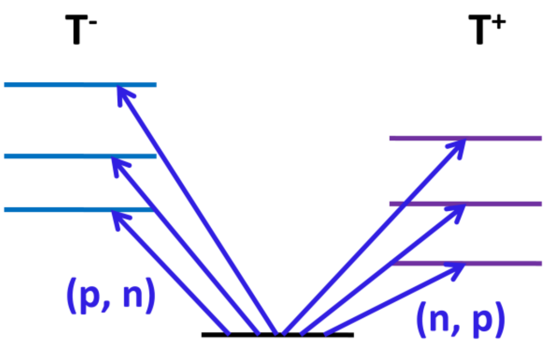


Z, N



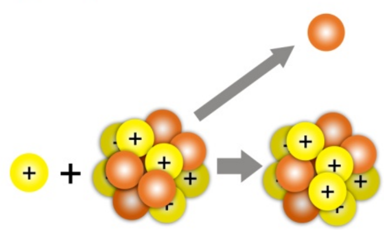
Strong **reactions:**
(α, α’ γ),
EM **(γ, γ’), (γ, n) ,(e, e’)**

- **Charge-exchange excitations**



Z+1, N-1 Z, N Z-1, N+1

(p,n) reaction



reactions:
(p,n) T$^-$, (n,p) T$^+$

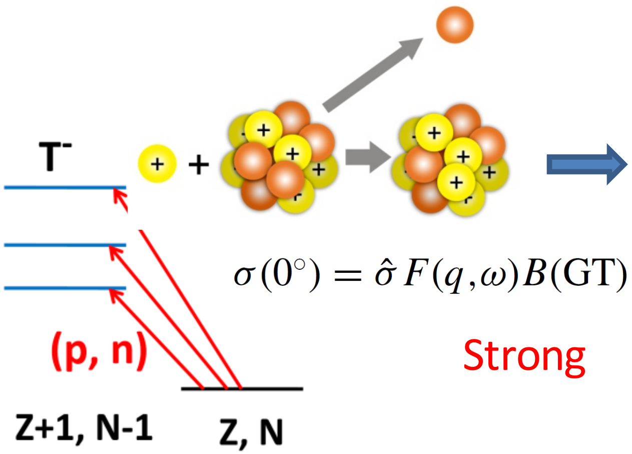
Strong

Bell	**Nucleus**
Hammers (iron,wooden)	**Operators/Probes (e, γ, α, p, n)**
Sounds of the Bell	**Modes of collective vibrations**

Gamow-Teller Resonance and β decay

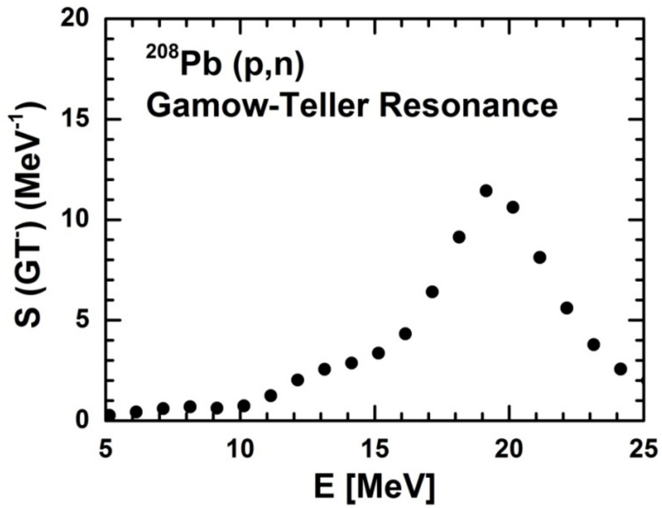
- **Gamow-Teller Resonance**

(p,n) reaction



$$\sigma(0^\circ) = \hat{\sigma}\, F(q,\omega) B(\mathrm{GT})$$

Wakasa, et al., PRC 85, 064606 (2012)



Gamow-Teller transition

ΔS=1 ΔL=0 ΔT=1

operator

$$\hat{O}_{\mathrm{GT}^-} = \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \tau_-(i)$$

Transition probability

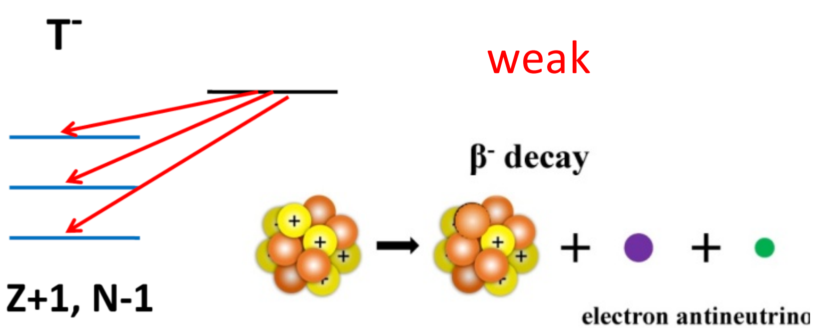
$$B(\mathrm{GT}^-) = \sum_{\nu} |\langle \nu | \hat{O} | 0 \rangle|^2$$

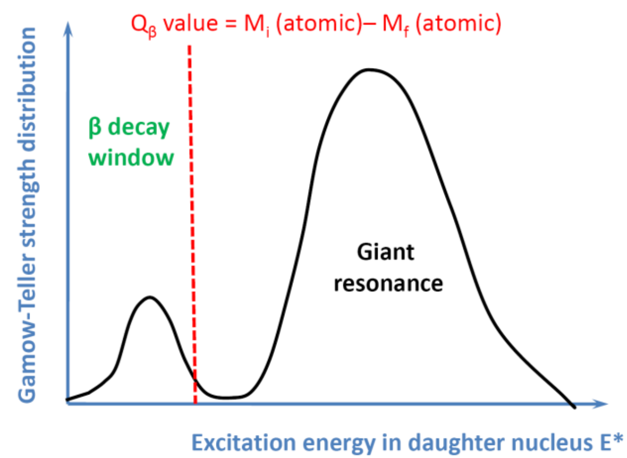
Transition strength S

= smoothed B

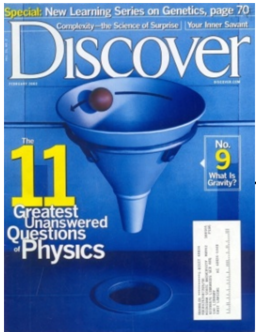
- **β-decay** **dominated by low-energy GT transition**

weak



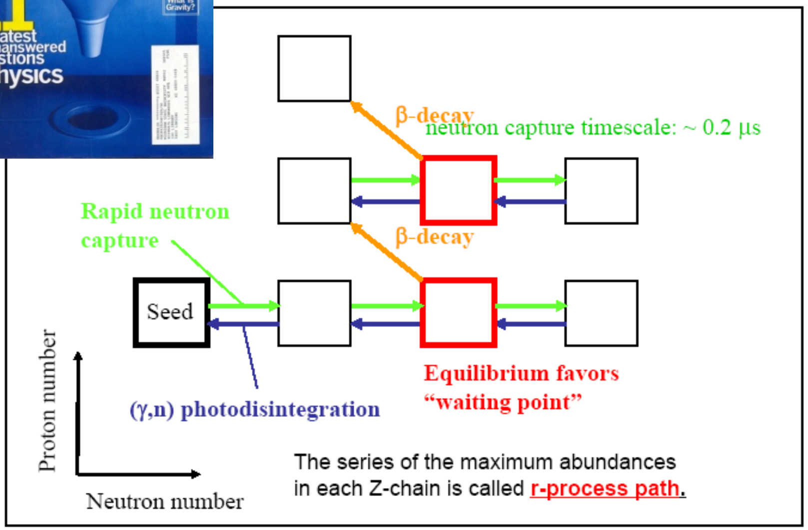


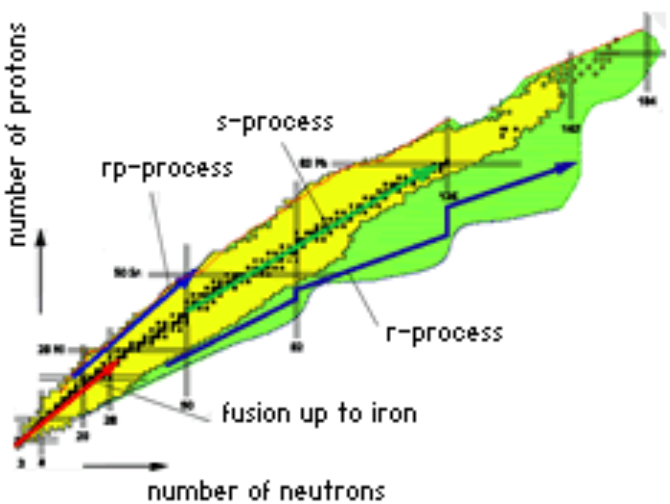
How Were the Heavy Elements Made?



The 11 greatest unanswered questions of physics

The r-process





Question 3

How were the heavy elements from iron to uranium made?

rapid neutron capture process (r-process)

↓

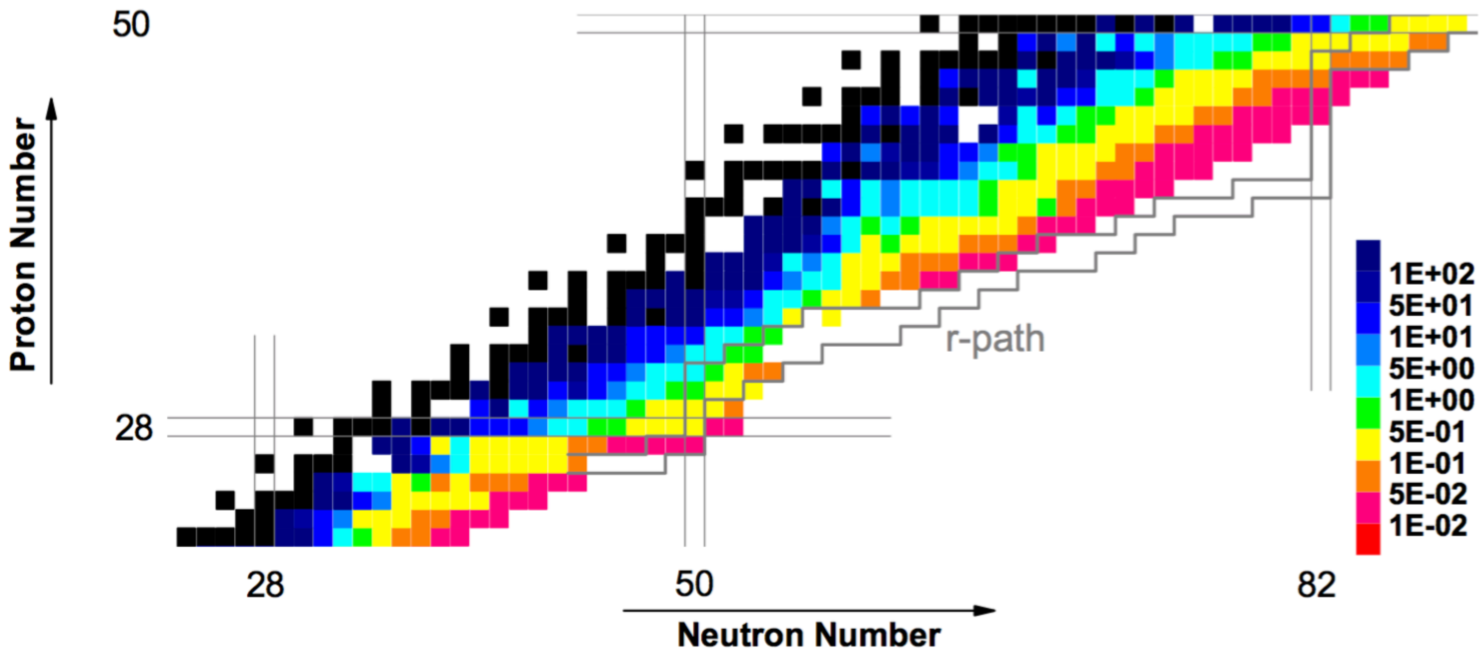
nuclear physics inputs:

mass, β-decay half-lives, …

↓

setting the time-scale of r-process

Experimental Investigations

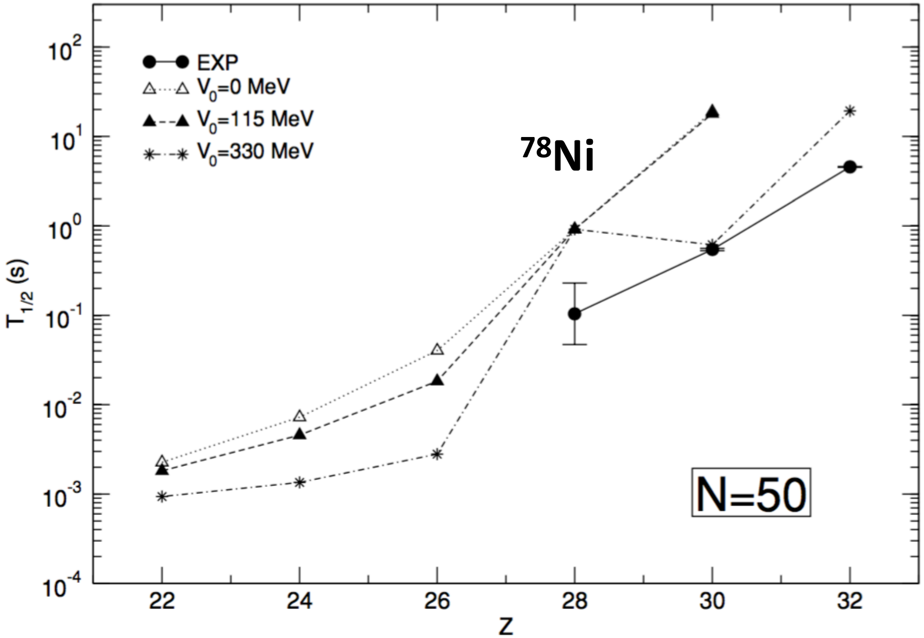


✓ development of radioactive ion-beam facilities ⇒ important advances
^{78}Ni and around *Hosmer, et al., PRL 94, 112501, 2005; Xu, et al., PRL 113, 032505, 2014*
very neutron rich Kr to Tc isotopes *Nishimura, et al., PRL 106, 052502, 2011*
Zn Ga isotopes *Madurga et al., PRL 109, 112501, 2012*
110 neutron-rich nuclei across N=82 shell gap *Lorusso, et al., PRL 114, 192501, 2015*

✓ provide a good test ground for theoretical models

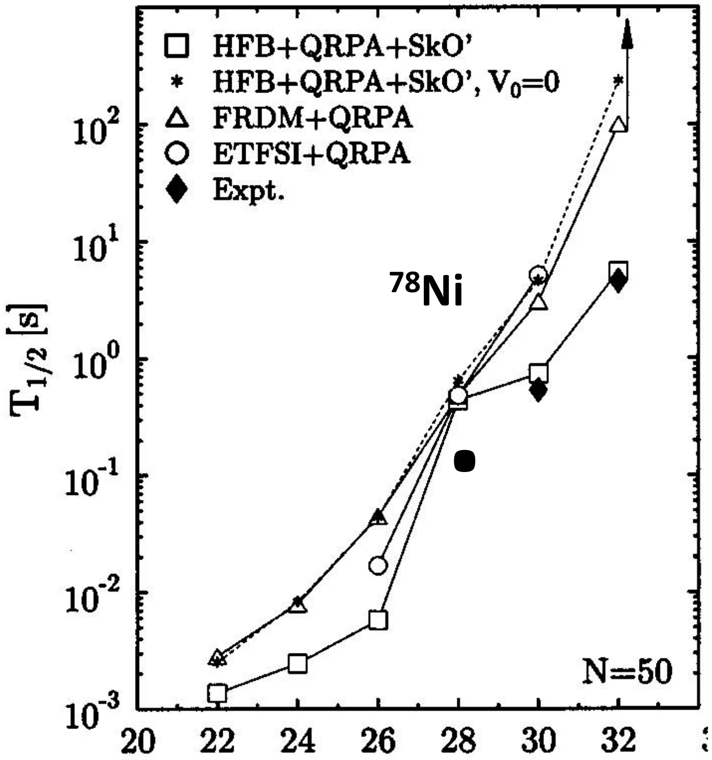
Lifetime Is a Hard Problem …

- **RHB+QRPA**



Niksic, et al., PRC 71, 014308 (2005)

- **Skyrme HFB+QRPA**



Engel, et al., PRC 60, 014302 (1999)

- **Half-lives are overestimated**
- **Due to the nuclear structure part – Gamow-Teller transition**

Density Functional Theory

- **Nucleus: quantum many-body system**
- **Density Functional Theory (DFT)**

$$E = \langle \Psi | \hat{H} | \Psi \rangle = \langle \Phi | \hat{H}_{eff} | \Phi \rangle = E[\hat{\rho}\,]$$

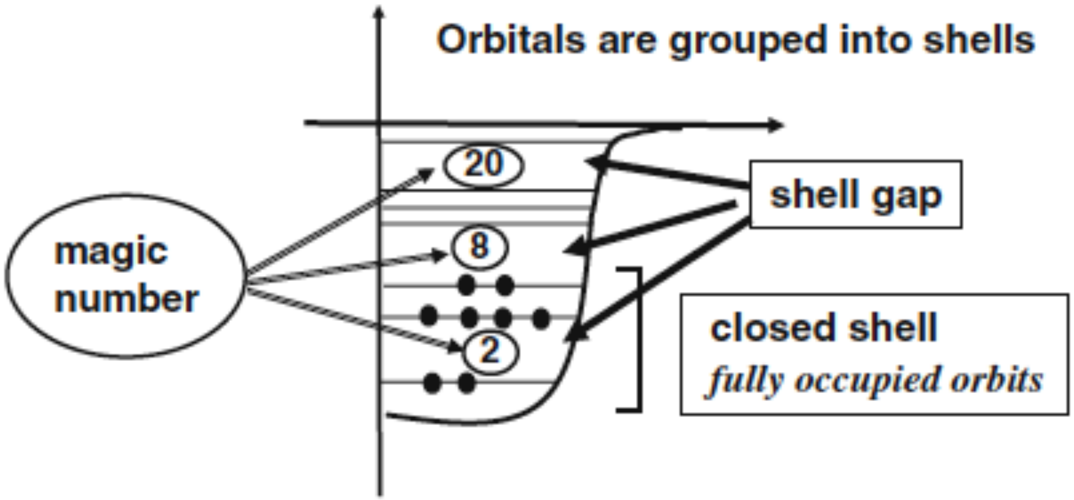
$|\Phi\rangle$ **Slater determinant** $\Leftrightarrow$ $\hat{\rho}$ **1-body density matrix**

$$|\Phi\rangle = \prod_{i=1}^{A} a_i^\dagger |-\rangle \qquad \hat{h} = \delta E / \delta \hat{\rho} \qquad \hat{h}|\varphi_i\rangle = \epsilon_i |\varphi_i\rangle$$

many-body problem

$\Downarrow$

one-body problem



Skyrme Density Functional

- **Effective Hamiltonian** $$\hat{H}_{eff} = \hat{T} + \hat{V}_{eff}$$
- **Skyrme Effective Interaction**

$$
\begin{aligned}
V(\mathbf{r}_1, \mathbf{r}_2) \;&=\; t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) \qquad \text{central term} \\
&+\frac{1}{2}t_1(1 + x_1 P_\sigma)[\mathbf{P}'^2\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{P}^2] \\
&+t_2(1 + x_2 P_\sigma)\mathbf{P}' \cdot \delta(\mathbf{r})\mathbf{P} \qquad \text{non-local term} \\
&+\frac{1}{6}t_3(1 + x_3 P_\sigma)[\rho(\mathbf{R})]^\alpha \delta(\mathbf{r}) \qquad \text{density-dependent term} \\
&+iW_0(\sigma_1 + \sigma_2) \cdot [\mathbf{P}' \times \delta(\mathbf{r})\mathbf{P}] \qquad \text{spin-orbit term}
\end{aligned}
$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \qquad \mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2),$$

$$\mathbf{P} = \frac{1}{2i}(\nabla_1 - \nabla_2), \qquad \mathbf{P}' \text{ cc of } \mathbf{P} \text{ acting on the left}$$

$$P_\sigma = (1 + \sigma_1 \cdot \sigma_2)/2.$$

- Around 11 parameters to fit the nuclear observables: t_i, x_i, W_0, α (SkM*, SLy5, …)
- Successful to describe almost the whole nuclear chart: g.s. and excited states

Random Phase Approximation (RPA)

- **RPA: widely used model for the description of collective vibration**
  - ✓ **Small oscillation: linear limit of time dependent DFT theory**

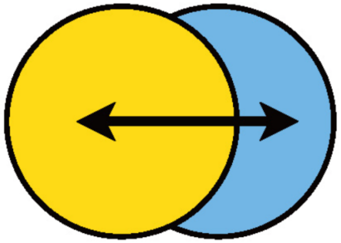
$$i\hbar\dot{\rho} = [h[\rho] + f(t), \rho] \qquad \rho(t) = \rho^{(0)} + \delta\rho(t)$$

keep the linear term

$$\begin{pmatrix} A_{mi,nj} & B_{mi,nj} \\ -B_{mi,nj} & -A_{mi,nj} \end{pmatrix} = \Omega_\nu \begin{pmatrix} X_{nj} \\ Y_{nj} \end{pmatrix}$$

$$\begin{aligned} \delta\rho &= \rho^{(1)} e^{-i\omega t} + \rho^{(1)\dagger} e^{i\omega t} \\ &= X e^{-i\omega t} - Y e^{i\omega t} \end{aligned}$$

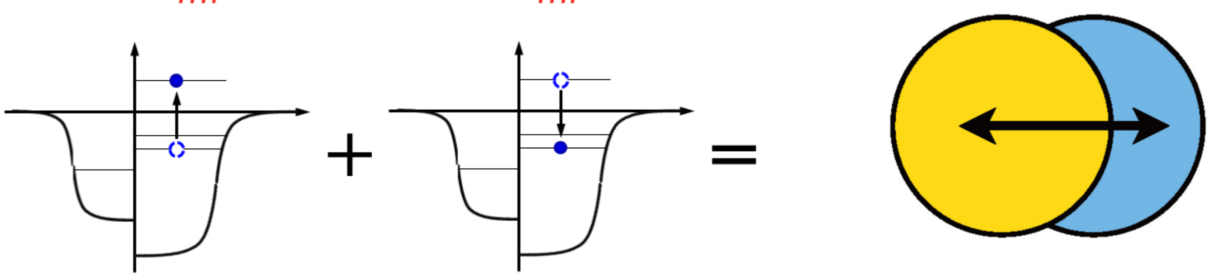
Harmonic oscillation



  - ✓ **Solution on basis**

The RPA excited state (collective vibration state) is generated by

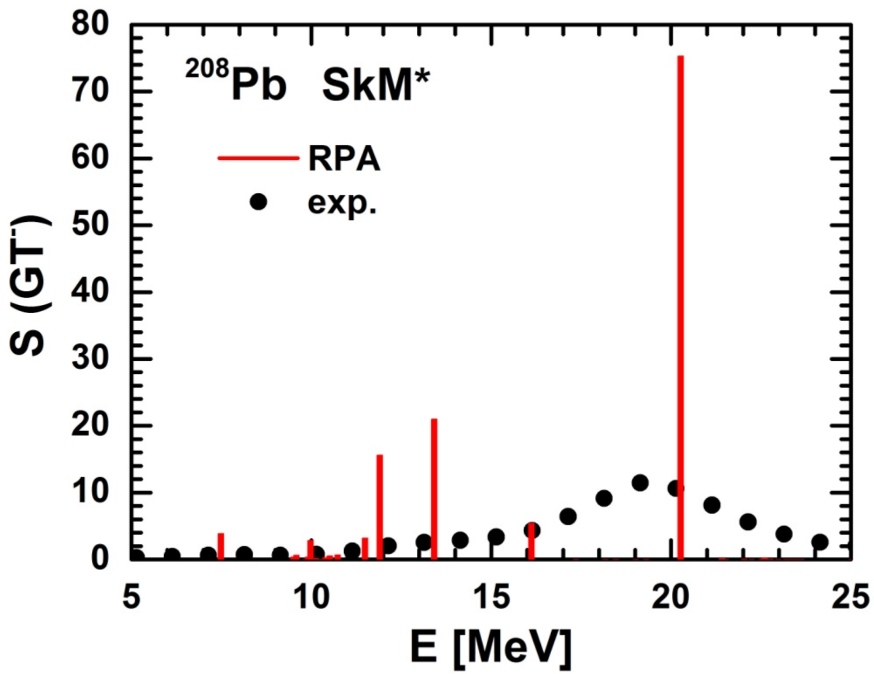
$$Q_\nu^\dagger = \sum_{mi} X_{mi}^\nu a_m^\dagger a_i - \sum_{mi} Y_{mi} a_i^\dagger a_m$$



- **RPA: magic nuclei**
- **Quasiparticle RPA (QRPA): superfluid nuclei**

Limits of (Q)RPA Description

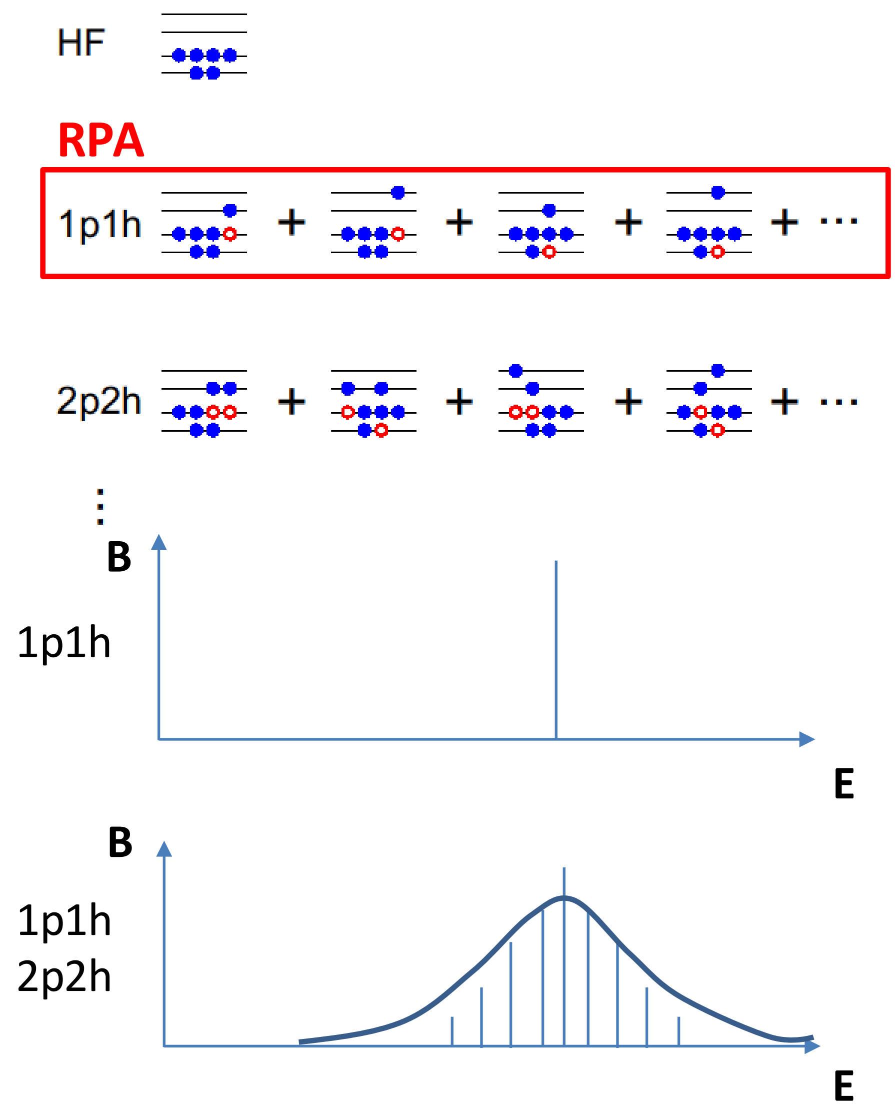
- **(Q)RPA cannot describe the spreading width**



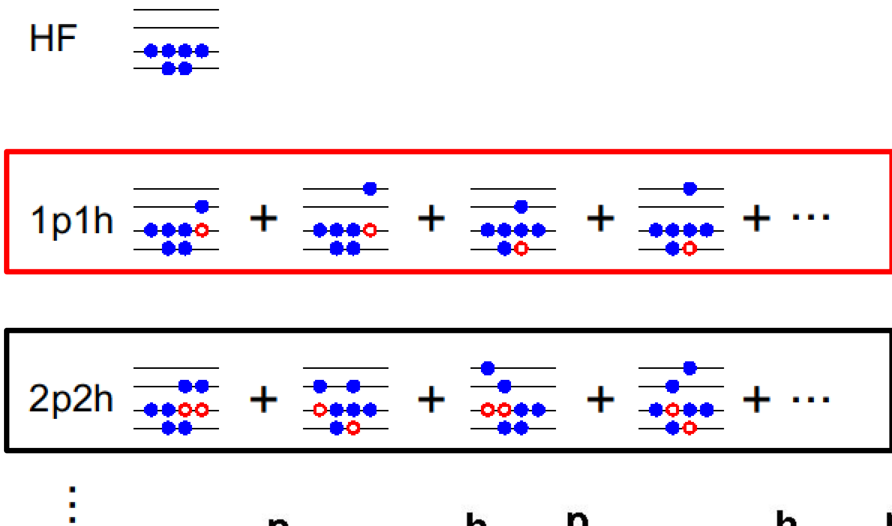
- **Spreading Width (Damping Width)**

  energy and angular momentum of coherent vibrations
  → more complicated states of 2p-2h, 3p-3h, … character

- **Correlations beyond RPA**

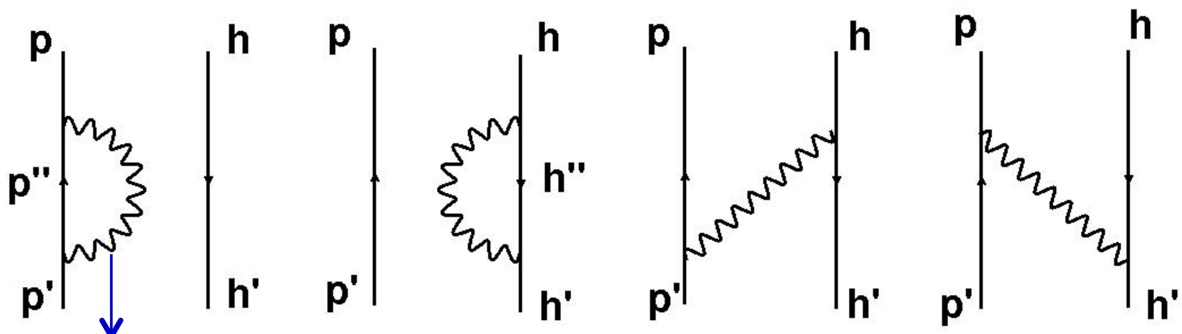


Solution: RPA + PVC



RPA

- **Second RPA** drozdz et al., PR 197, 1 (1990)
  Gambacurta et al., PRC 81, 054312 (2010)
- **RPA + PVC (particle vibration coupling)**



Low-lying vibration phonons |N>

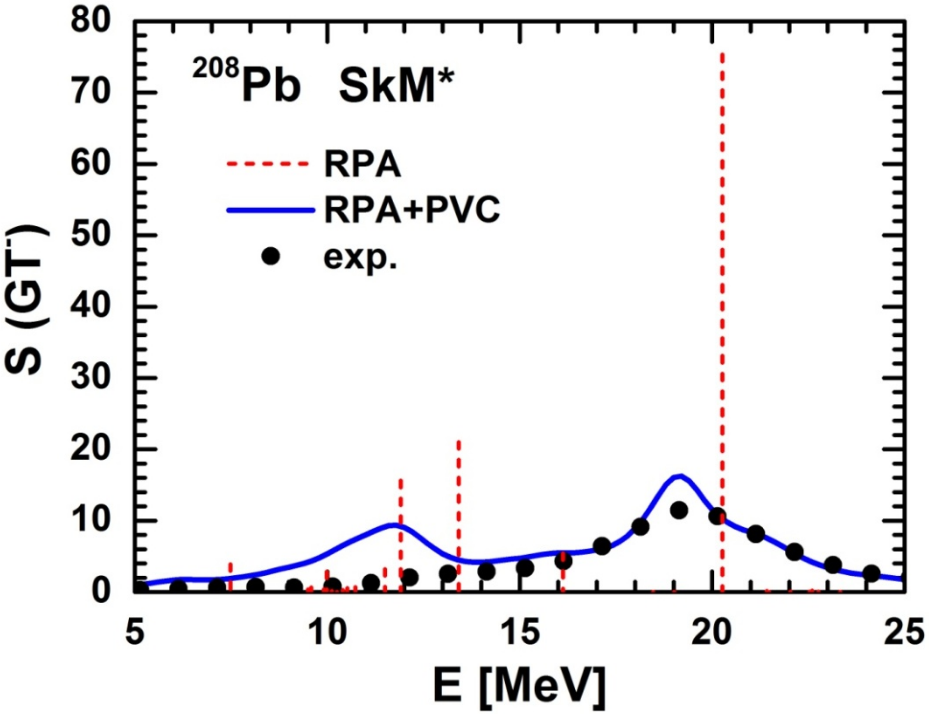
$$W^{\downarrow}_{ph,p'h'}(\omega) = \sum_N \frac{\langle ph|V|N\rangle\langle N|V|p'h'\rangle}{\omega - \omega_N}$$

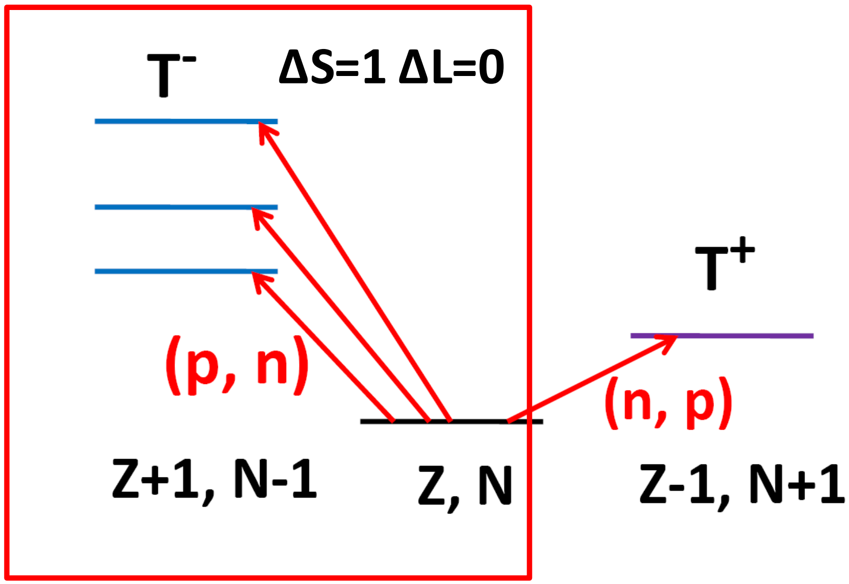
- RPA+PVC model based on Skyrme DFT
  Colo et al., PRC 50, 1496 (1994); Niu et al., PRC 85, 034314 (2012)
- RPA+PVC model based on relativistic DFT Litvinova et al., PRC 75,064308 (2007)

RPA+PVC: Gamow-Teller Resonance

- **Improved description of GT resonance in ^{208}Pb**

Y. F. Niu, G. Colo, and E. Vigezzi, **Phys. Rev. C** 90, 054328 (2014)



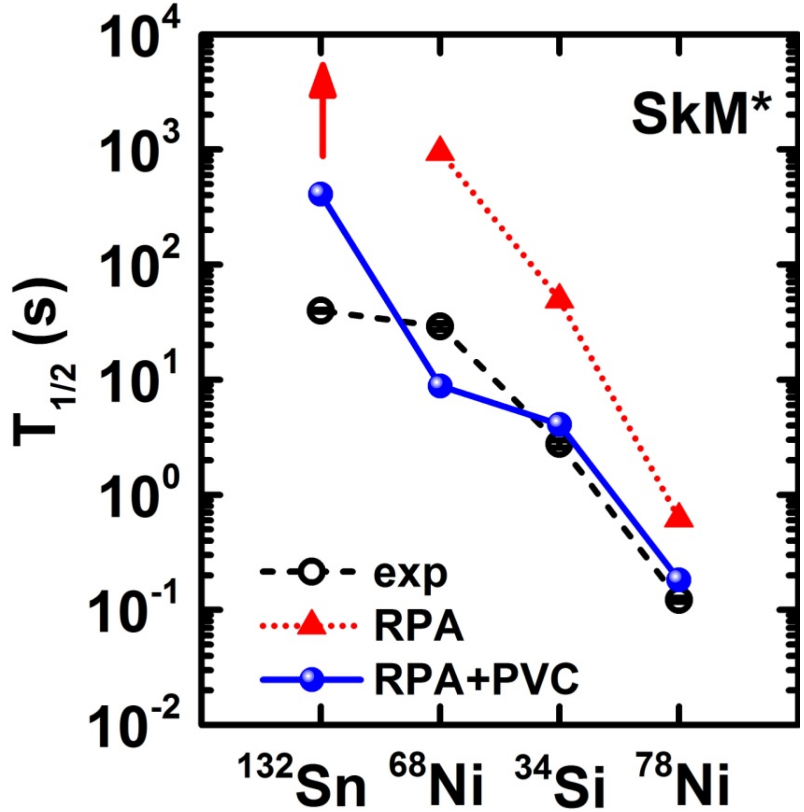


✓ **Develop a spreading width**

✓ **Reproduce resonance lineshape**

RPA+PVC: β-Decay Half-Lives

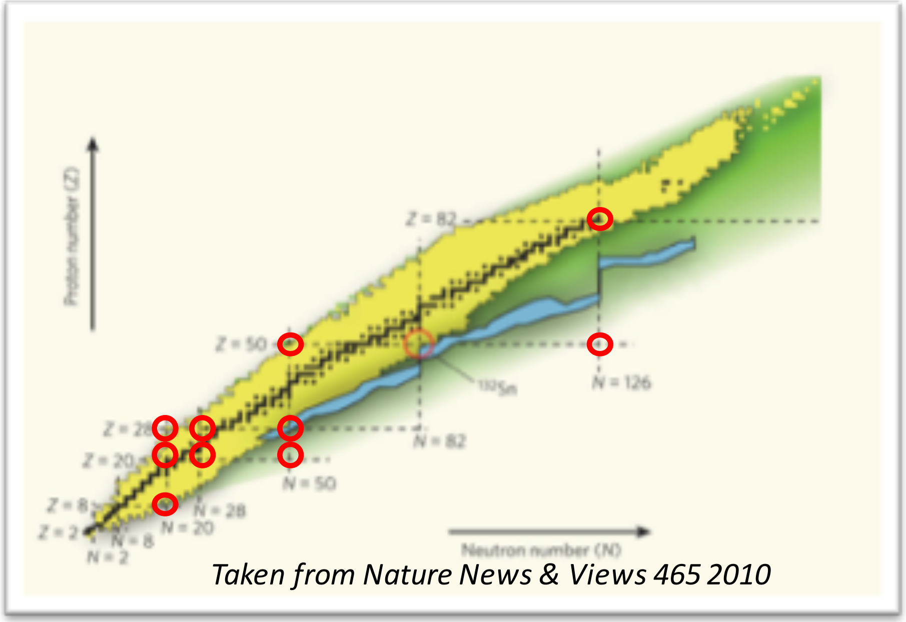
- **Improved description of β-decay half-lives**



✓ **Reduce half-lives systematically** ✓ **Reproduce β-decay half-lives**

Y.F. Niu, Z. M. Niu, G. Colo, and E. Vigezzi, **Phys. Rev. Lett.** 114, 142501 (2015).

RPA+PVC: only for magic nuclei…



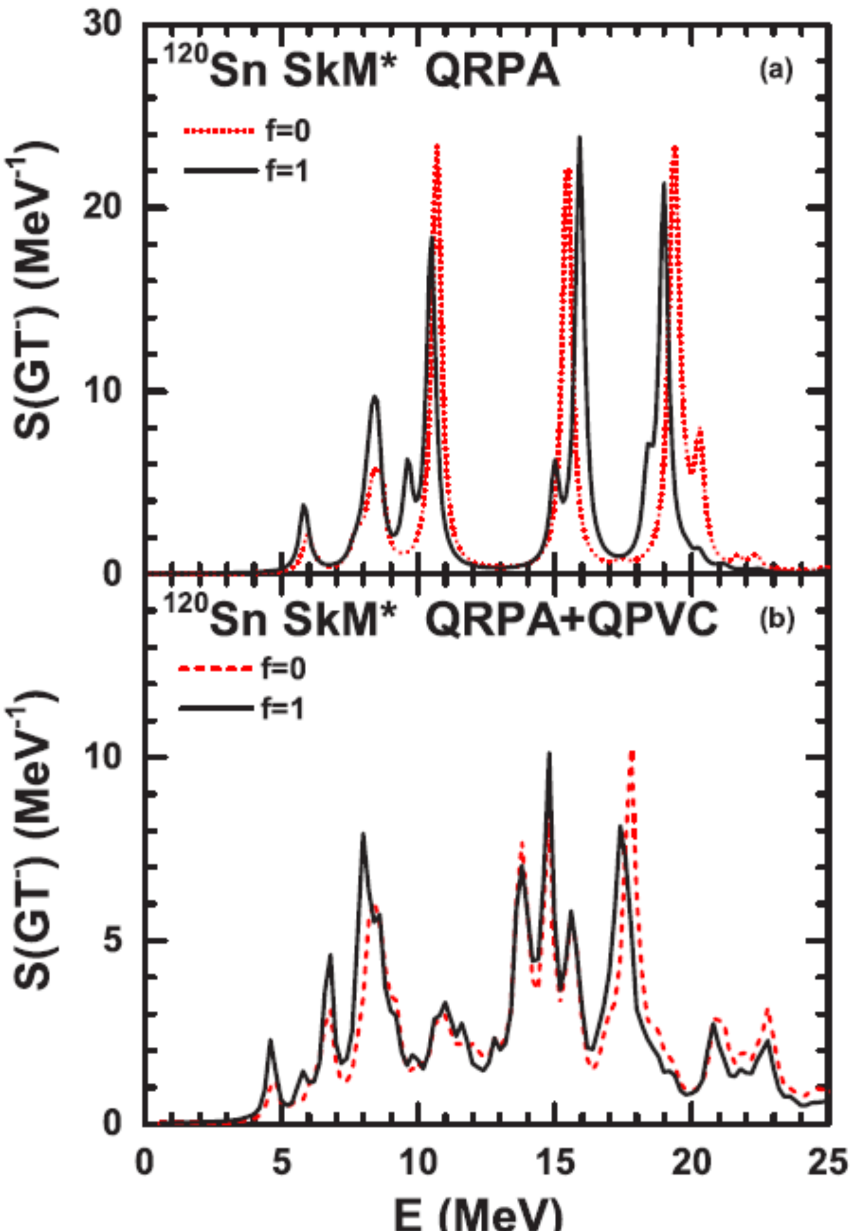
Taken from Nature News & Views 465 2010

- To include pairing correlations for superfluid nuclei

Quasiparticle RPA + quasiparticle vibration coupling
(**QRPA**) + (**QPVC**)

- ✓ for the study of Gamow-Teller resonance in superfluid nuclei
- ✓ for the study of β-decay half-lives of the whole isotopic chain

Isoscalar Pairing



Particle-particle interaction

$$V_{T=1}(\mathbf{r}_1,\mathbf{r}_2) = V_0 \frac{1-P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

$$V_{T=0}(\mathbf{r}_1,\mathbf{r}_2) = f\,V_0 \frac{1+P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

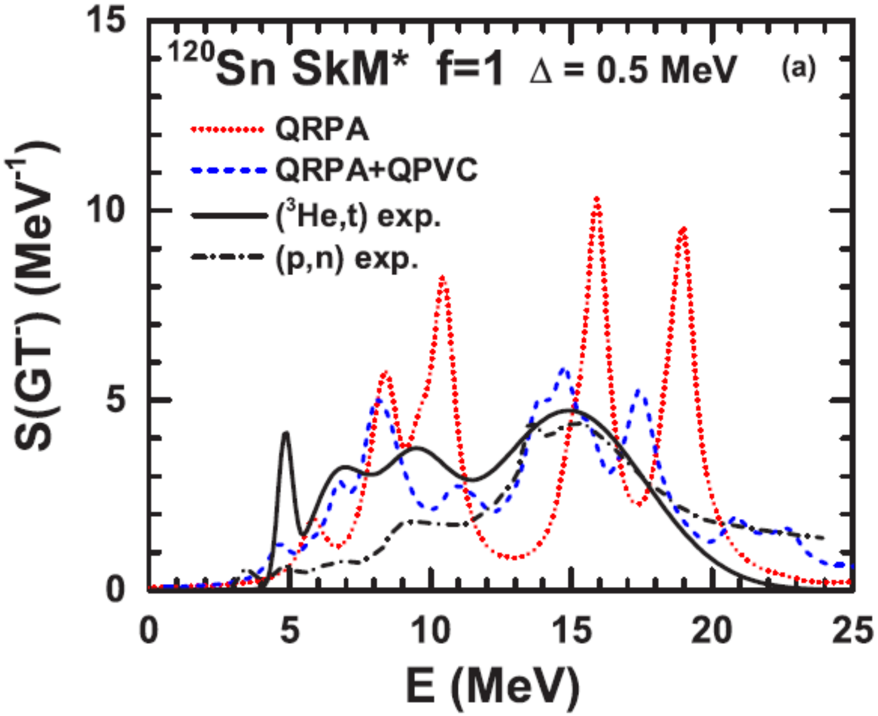
f = 0 : without isoscalar pairing
f = 1 : with isoscalar pairing

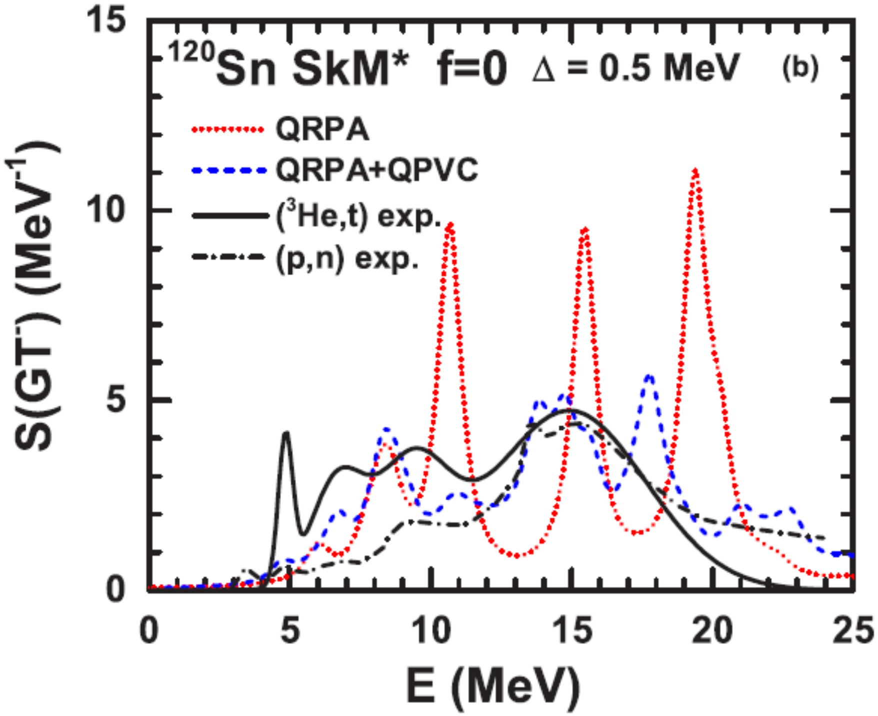
Effect of isoscalar pairing

- QRPA level
  - Increase the low-lying strength
  - Decrease the splitting between two high-lying states
- QRPA+QPVC level
  - Increase the low-lying strength
  - Similar profile

Niu, Colo, Vigezzi, Bai, Sagawa, PRC 94, 064328 (2016)

GT Strength Distribution

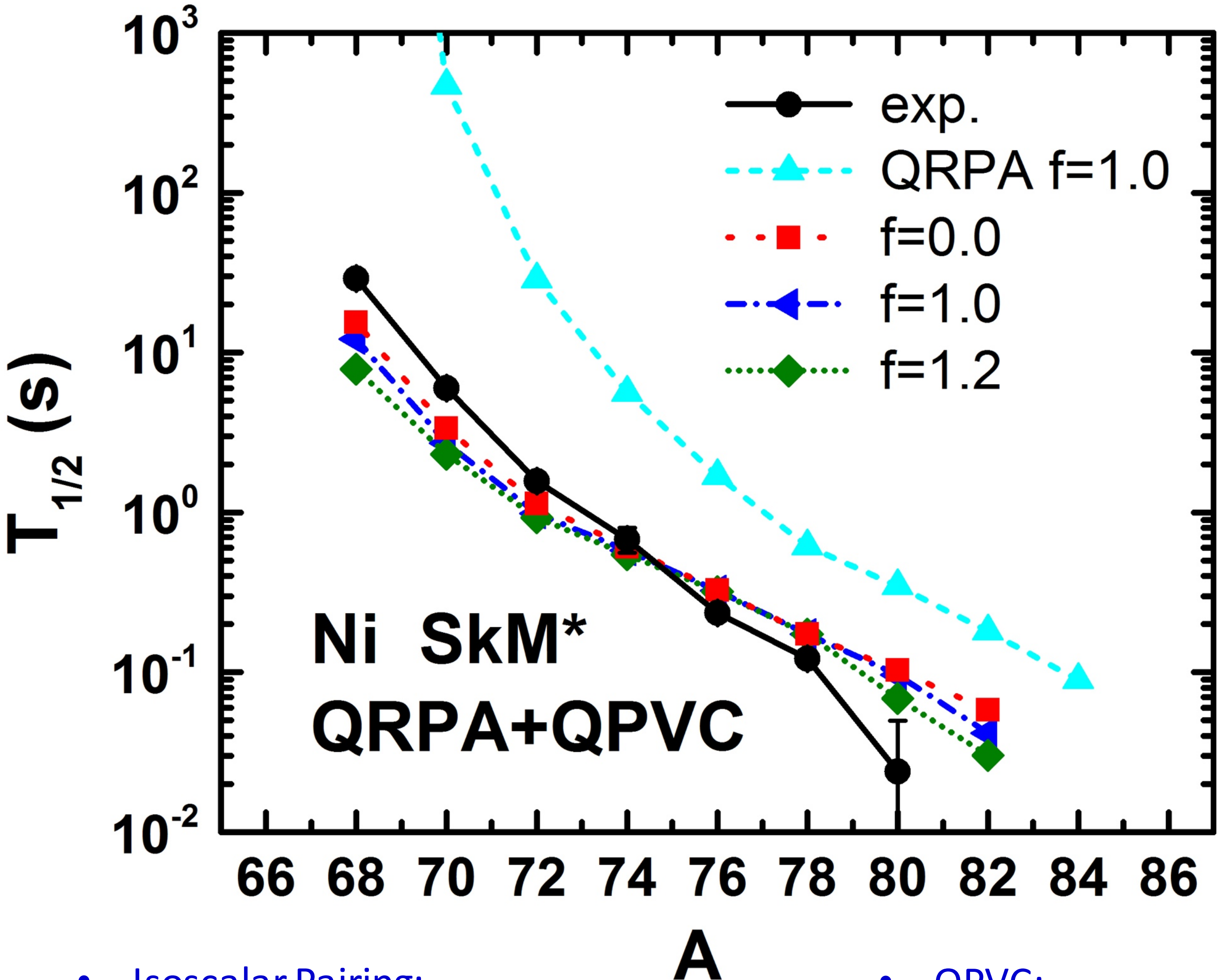




- (^{3}He,t) data: cross section × 1.6 so that the main GTR strength exhausts 65% sum rule
  Pham, et al., PRC 51, 526 (1995)
  overestimate the low-lying strength
- (p,n) data: normalized by unit cross section $\sigma(0^\circ) = \hat{\sigma}\, F(q,\omega) B(\mathrm{GT})$
  Sasano, et al., PRC 79, 024602 (2009)

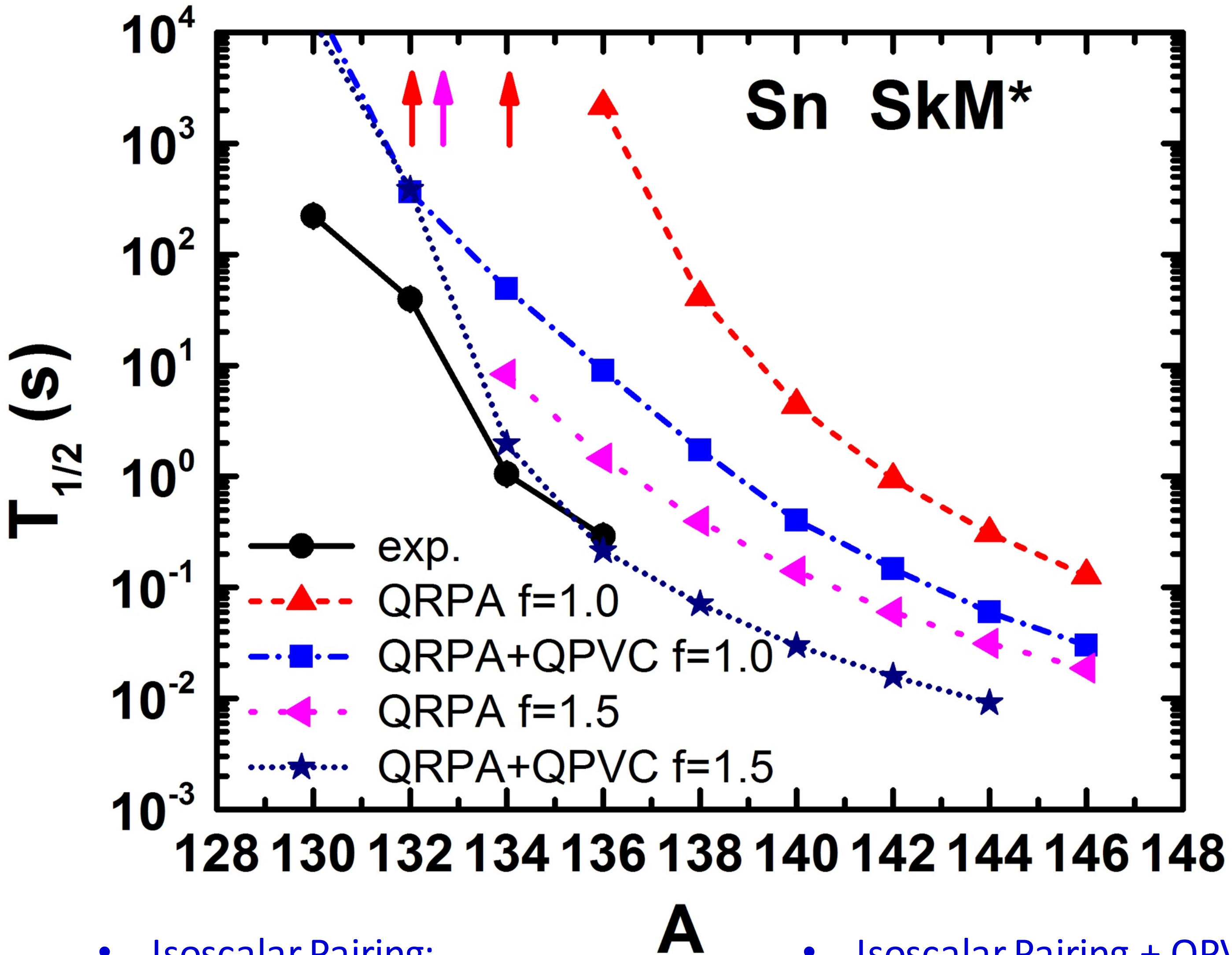
✓ QRPA + QPVC
  - Develop a width of 5.3 MeV (6.4 MeV from exp.), reproduce exp. profile in GTR
  - Overestimate the low-lying strength

β-Decay Half-Lives in Ni isotopes



- Isoscalar Pairing:
  not so effective for Ni isotopes
  (nuclei before N=50 closed shell)
- QPVC:
  reduce the half-lives

β-Decay Half-Lives in Sn isotopes



- Isoscalar Pairing: effective for Sn isotopes (nuclei above N=82 closed shell)
- Isoscalar Pairing + QPVC: reduce the half-lives

Summary and Perspectives

Summary

- Gamow-Teller transition is an important spin-isospin mode of nucleus; β-decay is mainly determined by low-energy GT transition
- Going beyond RPA: RPA+PVC
  - ✓ Spreading width
  - ✓ Reduce the half-lives
- Going beyond QRPA + pairing correlations: QRPA+QPVC
  - ✓ Effect of QPVC
  - ✓ Effect of isoscalar pairing

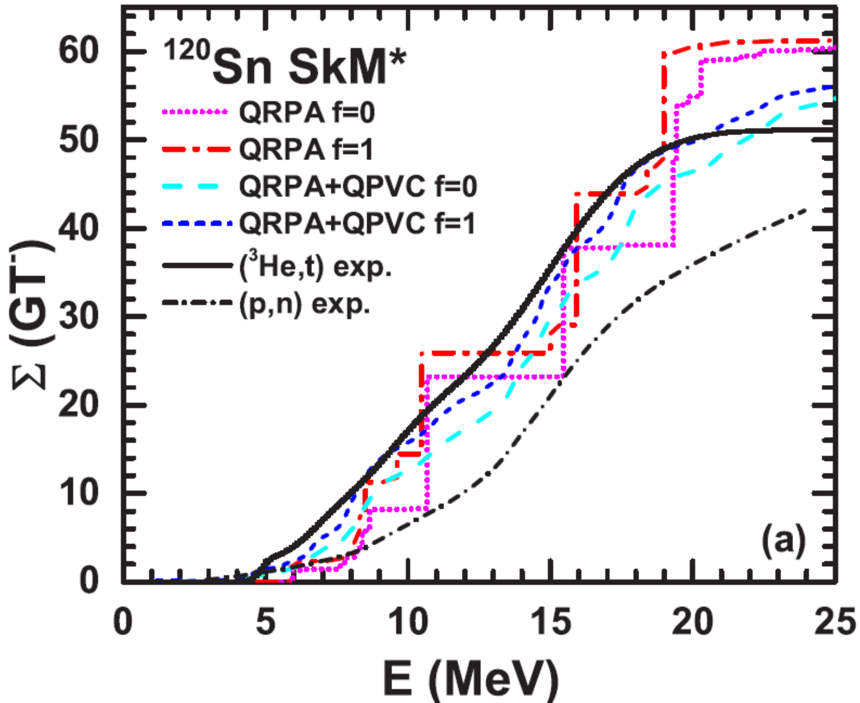
Perspectives

- QRPA+QPVC: systematic study of isotopic chain
- QRPA+QPVC: other weak interaction processes

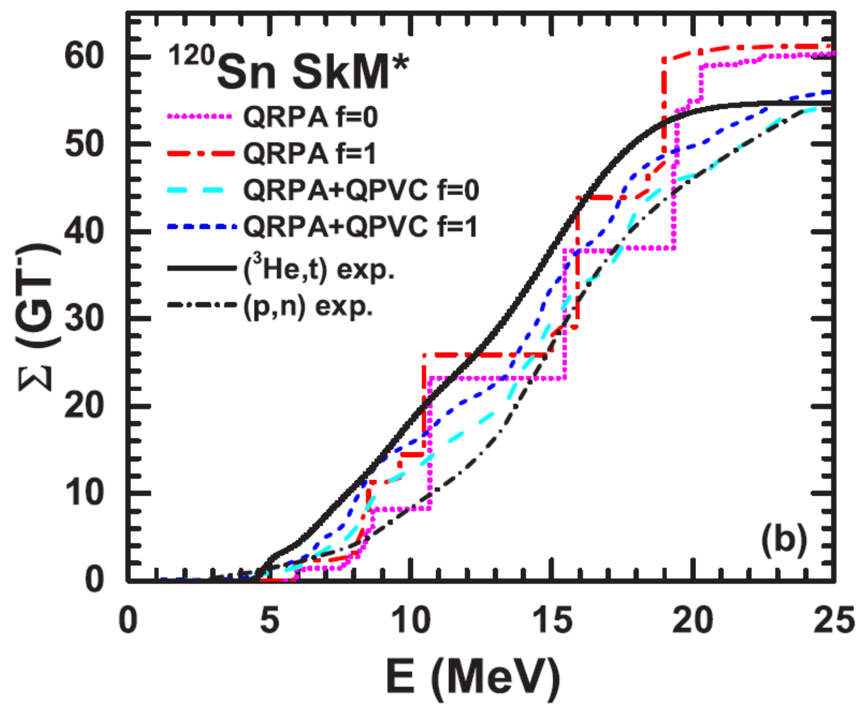
Thank you!

Cumulative Sum

Cumulative sum



Cumulative sum with normalized data



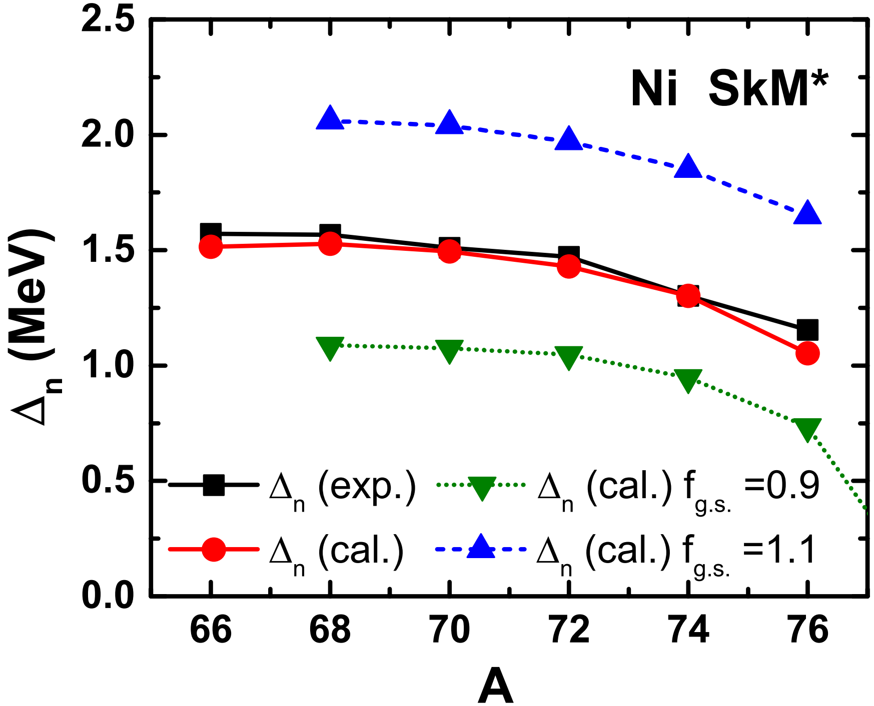
- f=0 -> f=1
  - ✓ low-lying strength is increased for both QRPA and QRPA + QPVC

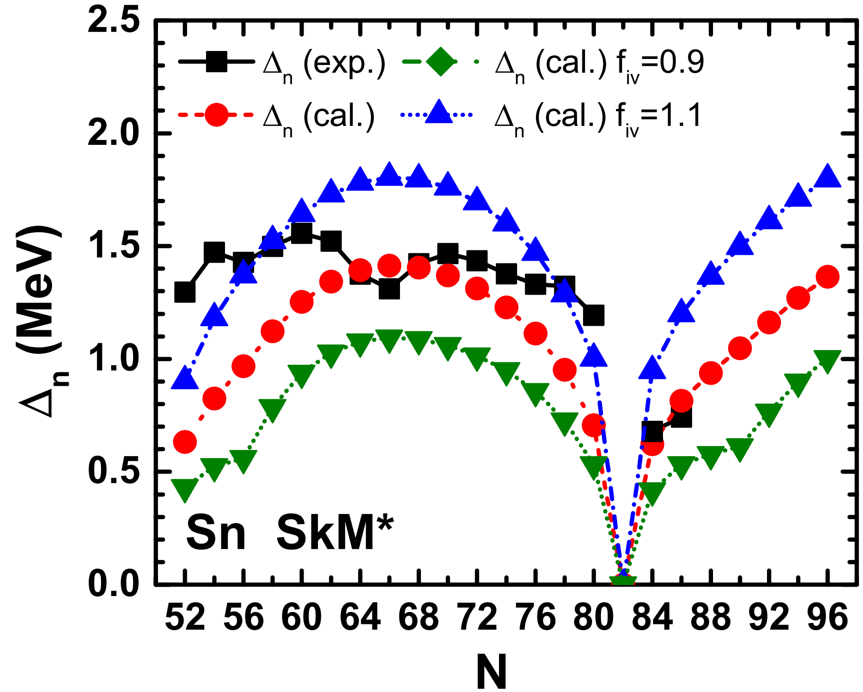
- QRPA -> QRPA+QPVC:
  - ✓ better reproduces the exp. profile
  - ✓ cumulative strength is quenched by 10% at E=25 MeV
  - ✓ QRPA+QPVC strength × 0.75 = exp. strength ((p,n) data) at E=25 MeV

Niu, Colo, Vigezzi, Bai, Sagawa, PRC 94, 064328 (2016)

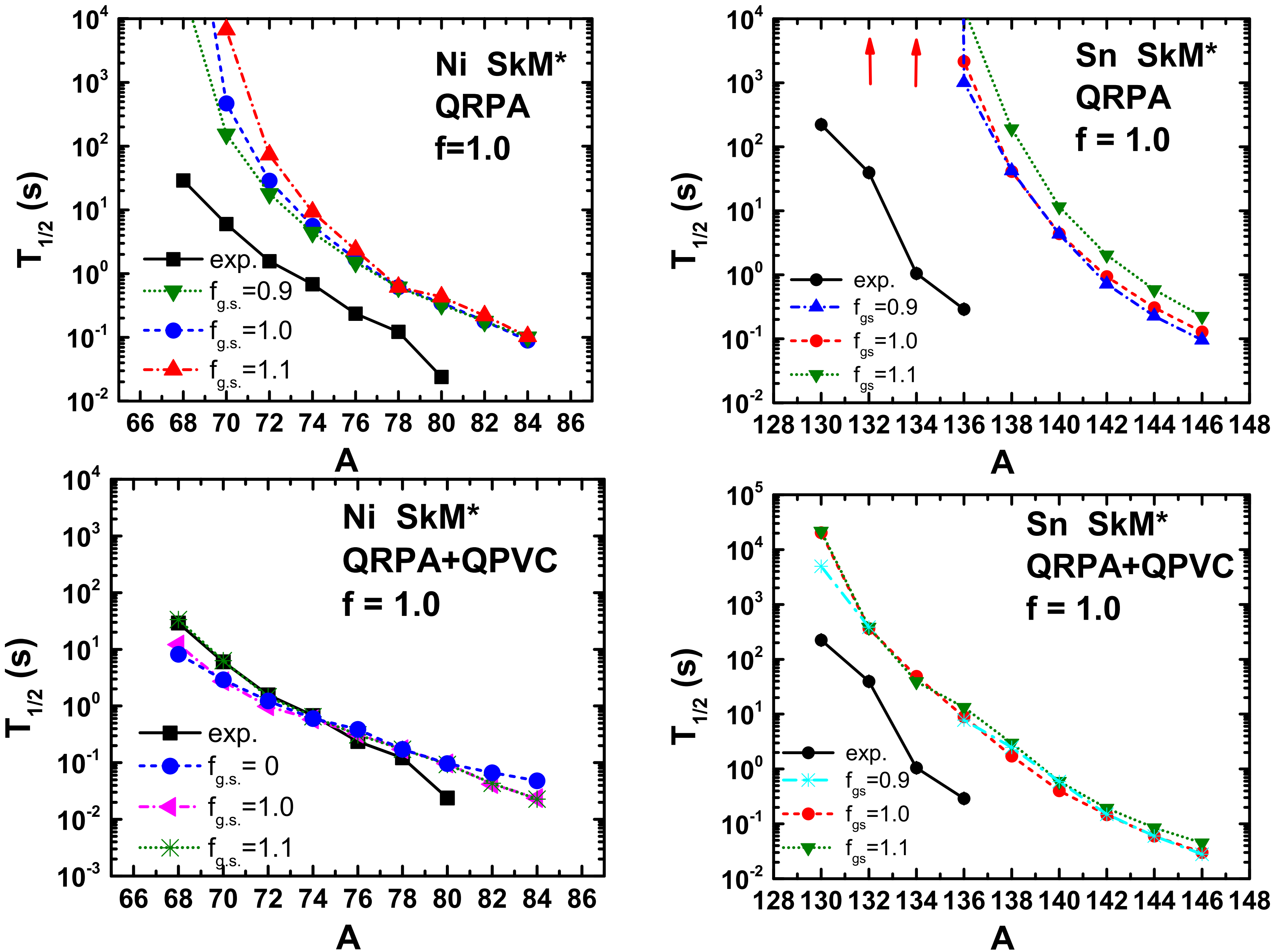
Pairing gap in Ni and Sn isotopes

- **Surface pairing Ecut = 100 MeV**
  - **Ni isotopes: V_0= -457 MeV fm^3**
  - **Sn isotopes: V_0= -520MeV fm^3**

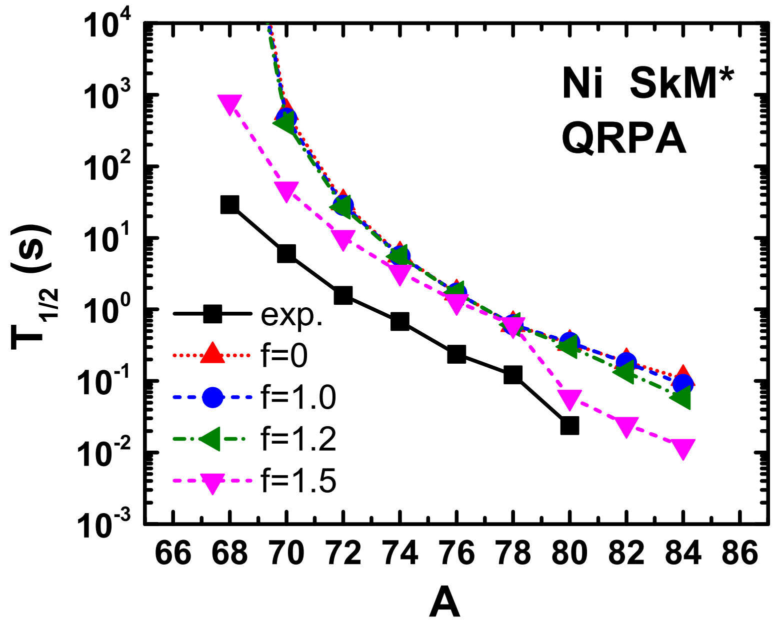


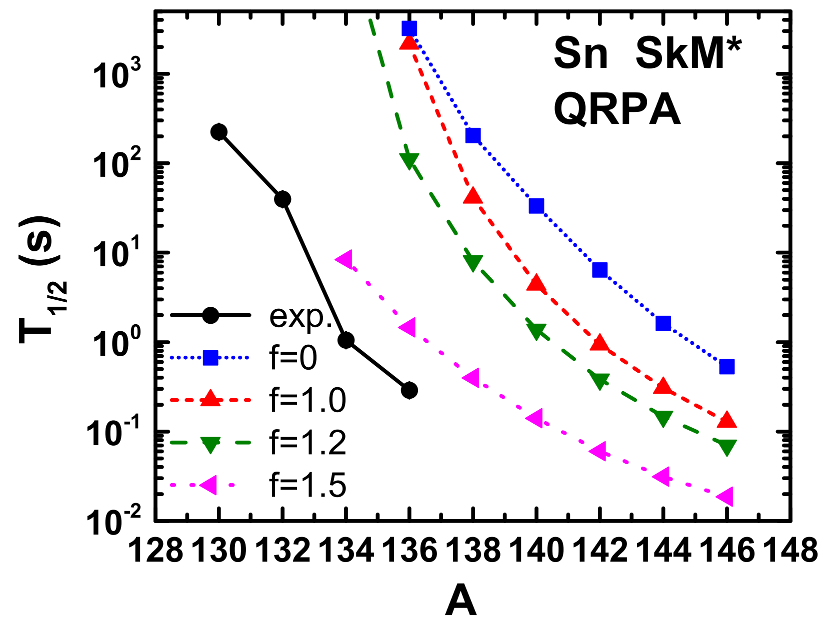


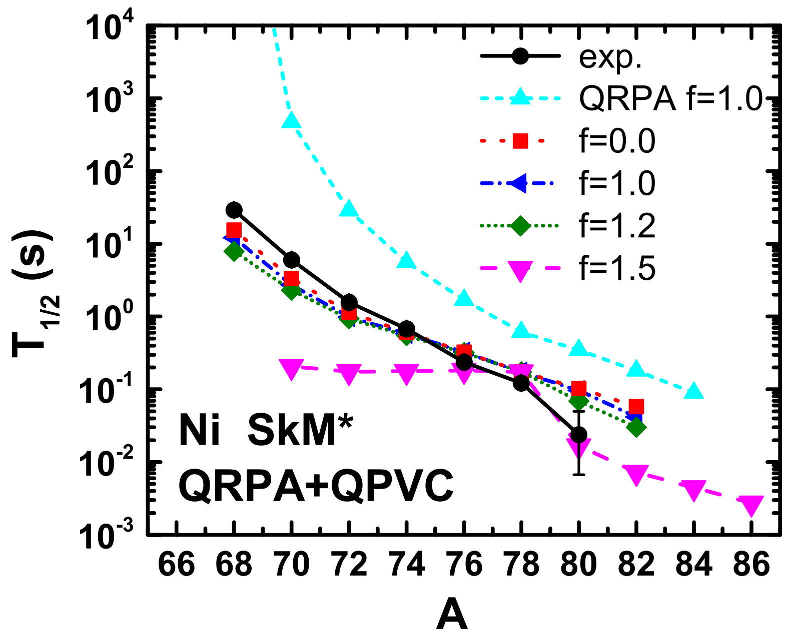
Effect of Isovector pairing

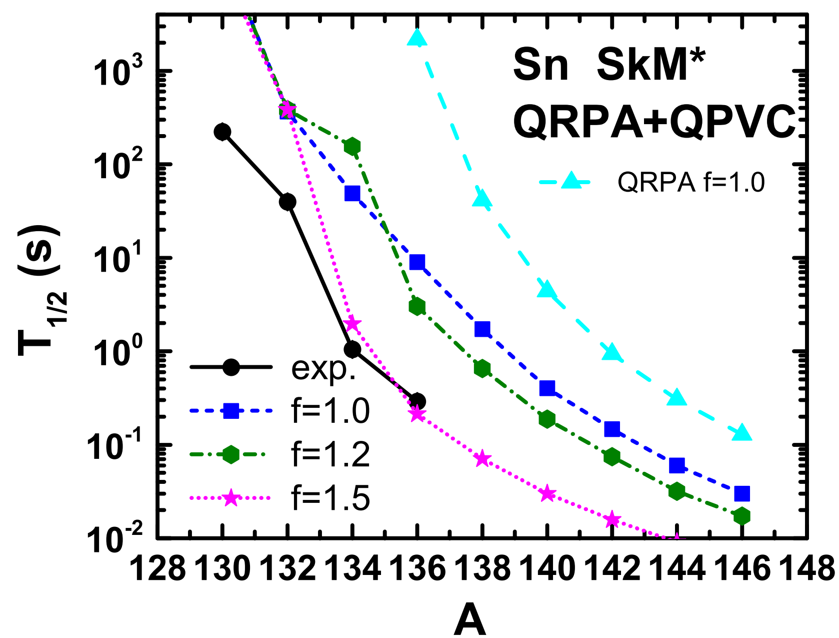


Effect of isoscalar pairing









QRPA+QPVC model

Step 1: HFB+QRPA calculation => Gamow-Teller response in QRPA level

Particle-particle interaction

$$V_{T=1}(\mathbf{r}_1,\mathbf{r}_2) = V_0\frac{1-P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

$$V_{T=0}(\mathbf{r}_1,\mathbf{r}_2) = f\,V_0\frac{1+P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_0}\right)\delta(\mathbf{r}_1-\mathbf{r}_2),$$

The QRPA equation will give the energy E_n , and wavefunction $X^{(n)}, Y^{(n)}$

Step 2: QPVC calculation => Gamow-Teller response in QRPA+QPVC level

The QRPA+QPVC equation reads

$$\begin{pmatrix} \mathcal{D}+\mathcal{A}_1(\omega) & \mathcal{A}_2(\omega) \\ -\mathcal{A}_3(\omega) & -\mathcal{D}-\mathcal{A}_4(\omega) \end{pmatrix}\begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix} = (\Omega_\nu - i\frac{\Gamma_\nu}{2})\begin{pmatrix} F^{(\nu)} \\ \bar{F}^{(\nu)} \end{pmatrix},$$

where $\mathcal{D}=E_n$, and the $\mathcal{A}_i$ matrices contain the spreading contributions, e.g.,

$$(\mathcal{A}_1)_{mn} = \sum_{ab,a'b'} W^{\downarrow}_{ab,a'b'}(E)X^{(m)}_{ab}X^{(n)}_{a'b'} + W^{\downarrow *}_{ab,a'b'}(-E)Y^{(m)}_{ab}Y^{(n)}_{a'b'},$$

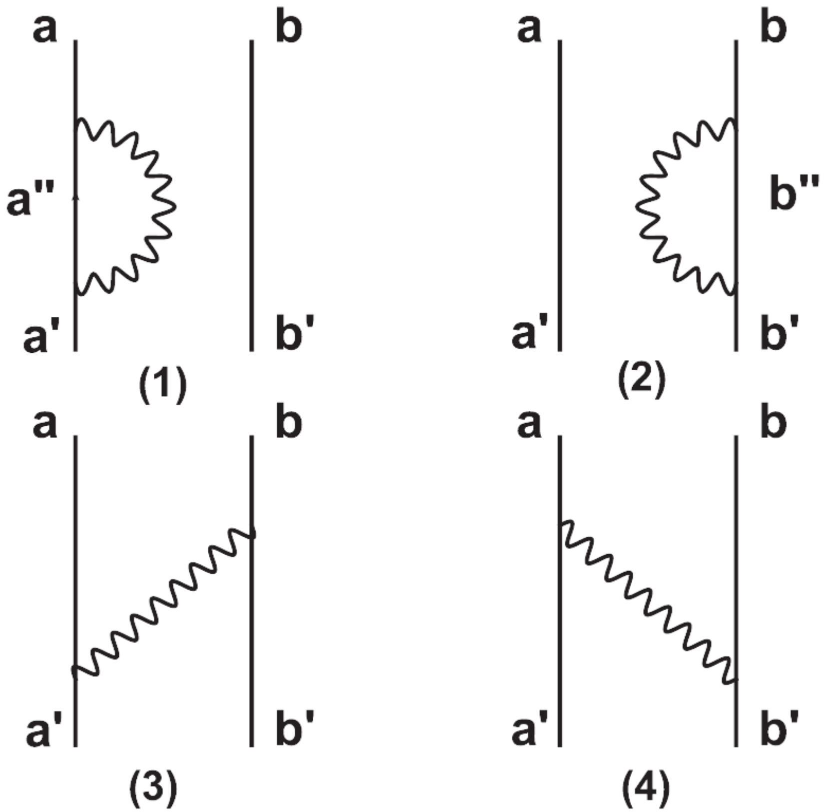
The GT strength function $S(E) = -\frac{1}{\pi}\mathrm{Im}\sum_\nu \langle 0|\hat{O}_{\mathrm{GT}\pm}|\nu\rangle^2 \frac{1}{E-\Omega_\nu+i\left(\frac{\Gamma_\nu}{2}+\Delta\right)}$

Spreading Terms

The matrix elements of the spreading term in the quasiparticle basis

$$W^{\downarrow}_{ab,a'b'} = \langle ab|V\frac{1}{E-\hat{H}}V|a'b'\rangle \;\; = \sum_{NN'}\langle ab|V|N\rangle\langle N|\frac{1}{E-\hat{H}}|N'\rangle\langle N'|V|a'b'\rangle,$$

where $|N\rangle = |a''b''\rangle \otimes |nL\rangle$ represents a doorway state, and a, b are quasi-particle states.



The vertex reduced matrix element:

$$\langle a||V||a'',nL\rangle = \frac{\hat{L}}{\sqrt{1+\delta_{cd}}}\sum_{cd}\Big[\widetilde{V}(cdLa'';a)X^{nL}_{cd} + (-1)^{j_a-j_{a''}+L}\widetilde{V}(cdLa;a'')Y^{nL}_{cd}\Big],$$

$$\widetilde{V}(cdLa'';a) = V^{Lph}_{ada''c}(u_a u_{a''} u_c v_d - v_a v_{a''} v_c u_d) + V^{Lph}_{aca''d}(u_a u_{a''} v_c u_d - v_a v_{a''} u_c v_d)(-)^{j_c-j_d+L} - V^{Lpp}_{aa''cd}(u_a v_{a''} u_c u_d - v_a u_{a''} v_c v_d)$$

Theoretical Investigations

- ab-initio approach: for very light nuclei *Barrett, et al. PPNP 69, 131, 2013*
- shell model: up to $A = 40 - 50$ or around magic regions assuming a frozen core

  Langanke, et al., ADNDT 79, 1, 2001; Suzuki, et al., PRC 85, 015802, 2012; Li, et al., JPG 41, 105102, 2014
- Random Phase Approximation (RPA)

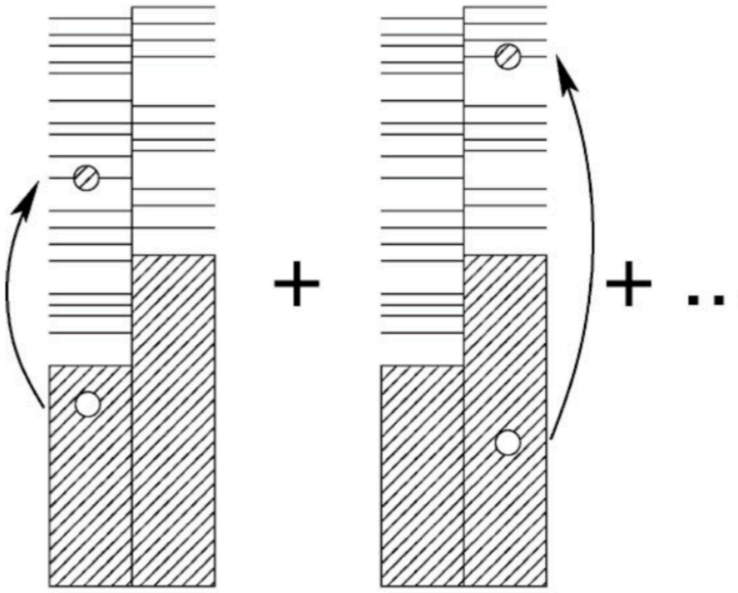
  ✓ non-self-consistent
  - Quasi-particle RPA (QRPA) based on FRDM *Moller, et al., ADNDT 66, 131, 1997*
  - DF3 + CQRPA *Borzov, PRC 67, 025802, 2003*

  ✓ self-consistent
  - QRPA based on Skyrme density functional

    Engel, et al., PRC 60, 014302, 1999
  - QRPA based on covariant density functional

    RHB + QRPA *Nikšić, et al., PRC 71, 014308, 2005; Marketin, et al., PRC 75, 024304, 2007*

    RHFB + QRPA *Niu, et al., PLB 723, 172, 2013*



RPA