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Canada's national laboratory
for particle and nuclear physics
and accelerator-based science

Microscopic optical potential for proton elastic scattering off light exotic nuclei

DREB2018 – 10th International conference on direct reactions with exotic beams

Matsue – June 05, 2018

Matteo Vorabbi | TRIUMF

Collaborators:
Michael Gennari, Angelo Calci, Petr Navratil (TRIUMF)

Outline

1. Consistent calculation of microscopic optical potentials for intermediate energies derived from nucleon-nucleon chiral interaction
   - Derivation of the model
   - Approximations
   - **Calculation of non-local one-body density matrices**
     (see Michael Gennari poster)

2. Scattering observables
   - Stable nuclei
   - ^{6}He and ^{8}He halo nuclei
   - Predictions for ^{10}Be, ^{10}C, ^{14}C, and ^{14}O

- Lippmann-Schwinger equation for nucleon-nucleus (NA) scattering

$$T = V + VG_0(E)T$$

- Separation of the LS equation

$$T = U + UG_0(E)PT$$

$$U = V + VG_0(E)QU$$

- Transition operator for the elastic scattering

$$T_{\mathrm{el}} \equiv PTP = PUP + PUPG_0(E)T_{\mathrm{el}}$$

- Spectator expansion [Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]

$$U = \sum_{i=1}^{A} \tau_i + \sum_{i,j\neq i}^{A} \tau_{ij} + \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{ijk} + \cdots$$

Free propagator

$$G_0(E) = (E - H_0 + i\epsilon)^{-1}$$

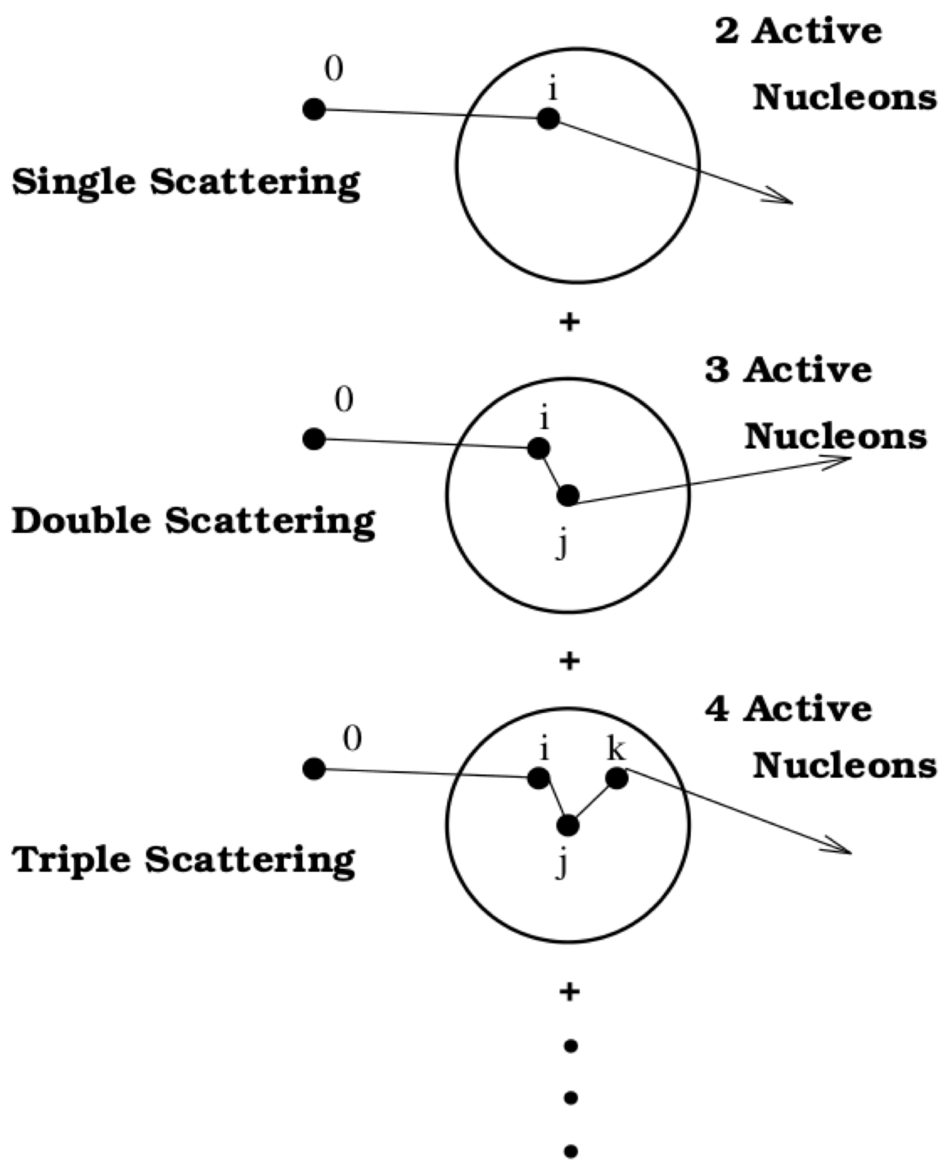
Free Hamiltonian

$$H_0 = h_0 + H_A$$

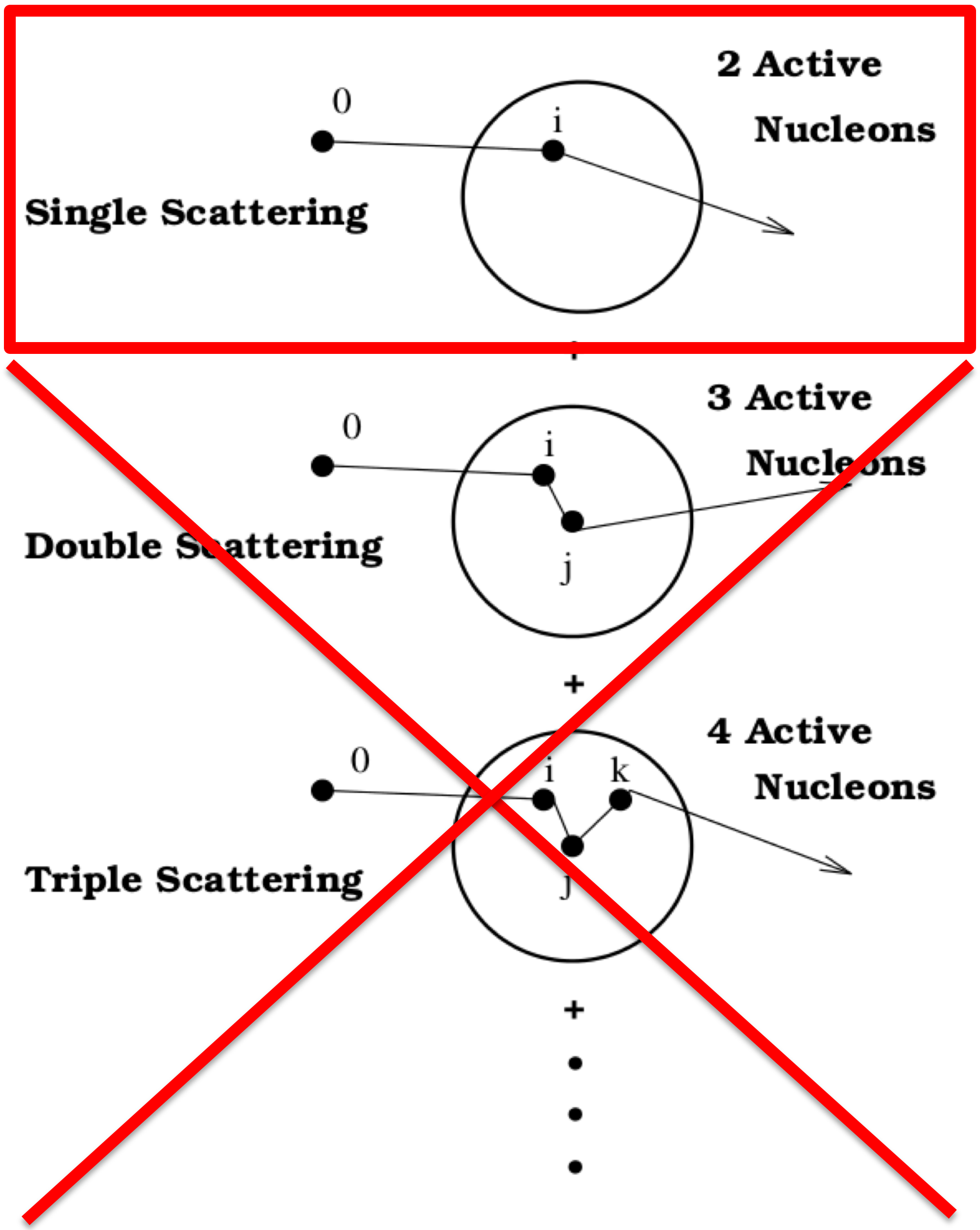
External interaction

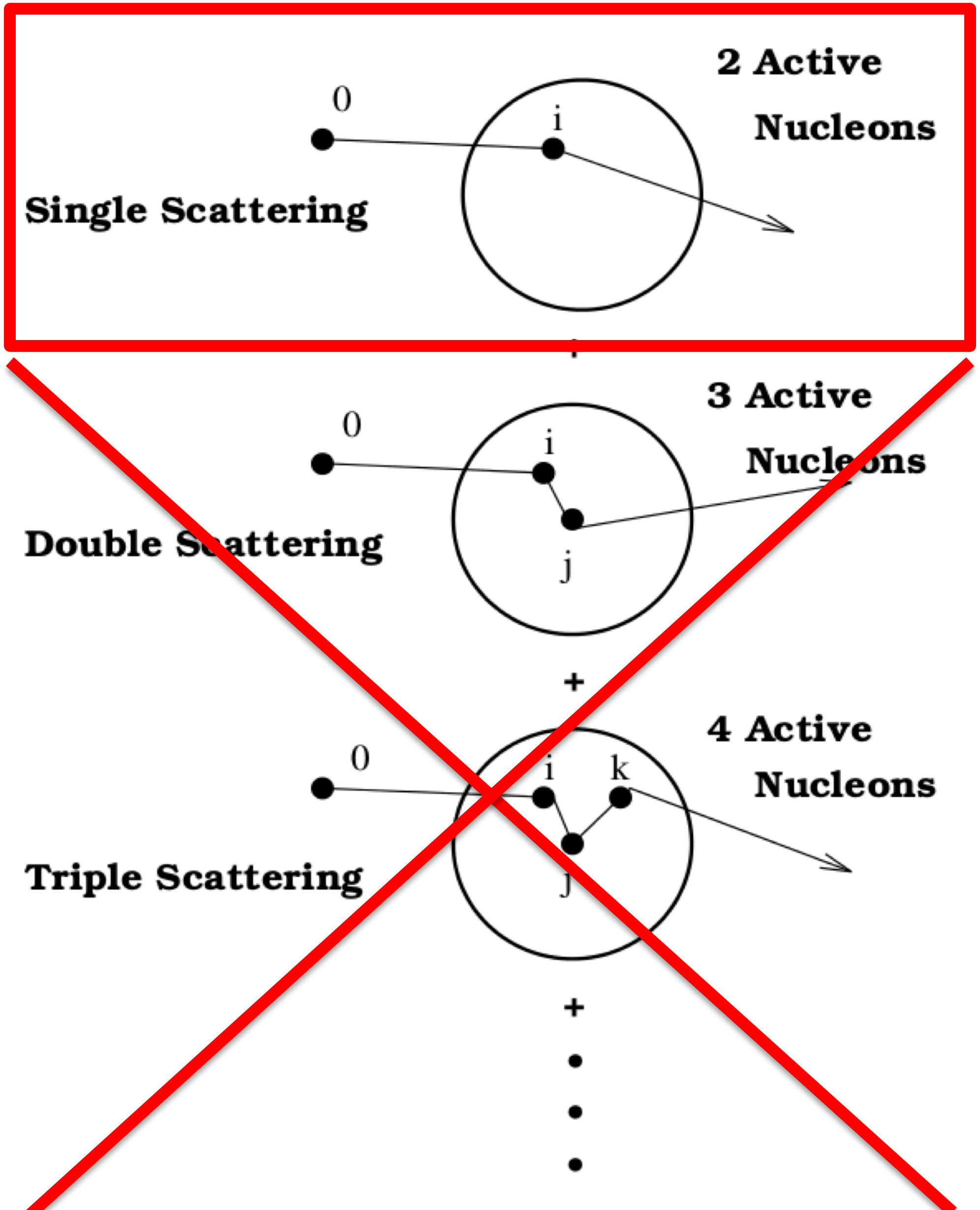
$$V = \sum_{i=1}^{A} v_{0i}$$

Approximations



Approximations





Impulse approximation

- Optical potential operator

$$U = \sum_{i=1}^{A} t_{0i}$$

- The free NN t matrix

$$t_{0i} = v_{0i} + v_{0i} g_i t_{0i}$$

- The free NN propagator

$$g_i = \frac{1}{E - h_0 - h_i + i\epsilon}$$

- The elastic scattering amplitude

$$T_{\rm el}(\boldsymbol{k}', \boldsymbol{k}; E) = U(\boldsymbol{k}', \boldsymbol{k}; E) + \int d^3p \frac{U(\boldsymbol{k}', \boldsymbol{p}; E)\, T_{\rm el}(\boldsymbol{p}, \boldsymbol{k}; E)}{E - E(p) + i\epsilon}$$

- The first-order optical potential

$$U(\boldsymbol{q}, \boldsymbol{K}; E) = \sum_{\alpha=n,p} \int d^3\boldsymbol{P}\, \eta(\boldsymbol{P}, \boldsymbol{q}, \boldsymbol{K}) t_{p\alpha} \left[\boldsymbol{q}, \frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{K} - \boldsymbol{P}\right); E\right]$$
$$\times \rho_\alpha \left(\boldsymbol{P} - \frac{A-1}{2A}\boldsymbol{q}, \boldsymbol{P} + \frac{A-1}{2A}\boldsymbol{q}\right)$$

Momentum transfer

$$\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$$

Total momentum

$$\boldsymbol{K} = \frac{1}{2}(\boldsymbol{k}' + \boldsymbol{k})$$

- The elastic scattering amplitude

$$T_{\mathrm{el}}(\boldsymbol{k}', \boldsymbol{k}; E) = U(\boldsymbol{k}', \boldsymbol{k}; E) + \int d^3p \frac{U(\boldsymbol{k}', \boldsymbol{p}; E)\, T_{\mathrm{el}}(\boldsymbol{p}, \boldsymbol{k}; E)}{E - E(p) + i\epsilon}$$

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POSTER: Michael Gennari

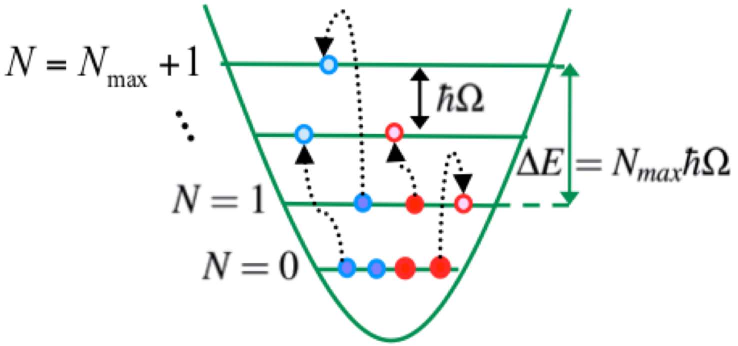
Nonlocal translationally invariant density

- Hamiltonian

$$\hat{H} = \frac{1}{A}\sum_{i<j=1}^{A}\frac{(\hat{\boldsymbol{p}}_i - \hat{\boldsymbol{p}}_j)^2}{2m} + \sum_{i<j=1}^{A}\hat{V}_{ij}^{NN} + \sum_{i<j<k=1}^{A}\hat{V}_{ijk}^{3N}$$

- Expansion over a complete set of antisymmetric A-nucleon harmonic oscillator basis states

$$|\Psi_A^{J^\pi T}\rangle = \sum_{N=0}^{N_{\max}}\sum_i c_{Ni}^{J^\pi T}|ANiJ^\pi T\rangle$$



- $N_{\max}$ excitations above the lowest configuration

- The basis is further characterized by the frequency Ω

- Extension of: Navratil, PRC **70**, 014317 (2004)

- Non-local nuclear density operator

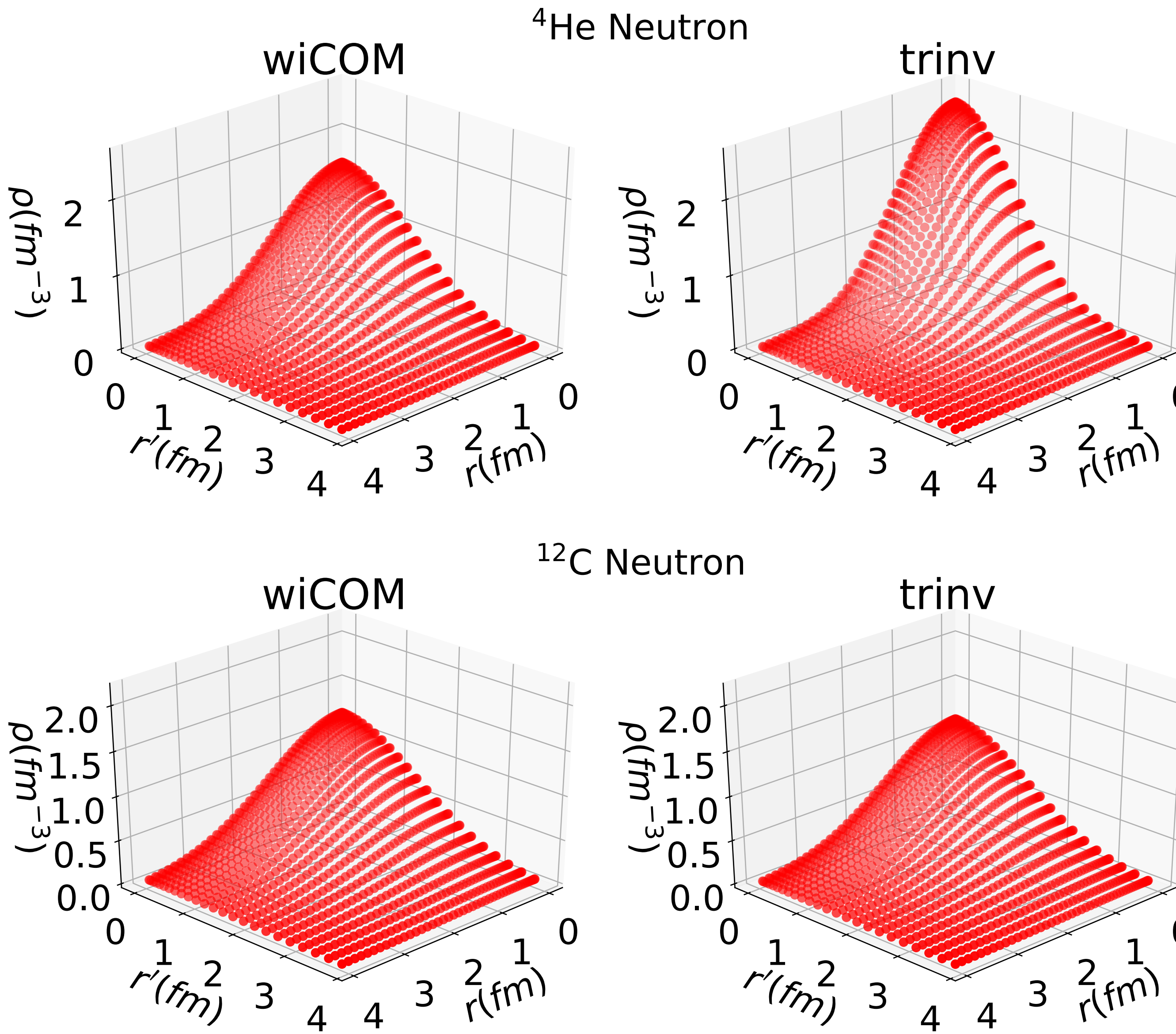
$$\rho_{\mathrm{op}} = \sum_{i=1}^{A} \delta(\boldsymbol{r} - \boldsymbol{r}_i)\delta(\boldsymbol{r}' - \boldsymbol{r}'_i)$$

- The matrix elements between a general initial and final state are obtained in the Cartesian coordinate single-particle Slater determinant basis

- Removal of the COM component is required
  - Enabled by the factorization of the Slater determinant and Jacobi eigenstates

Non-local densities

Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)



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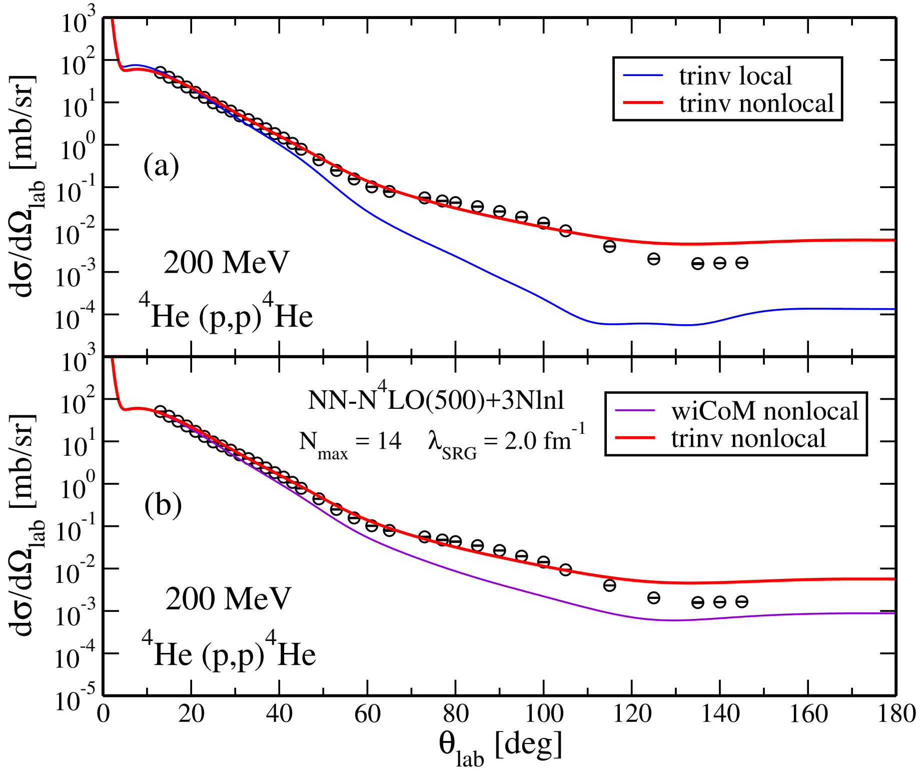
Scattering observables – Stable nuclei

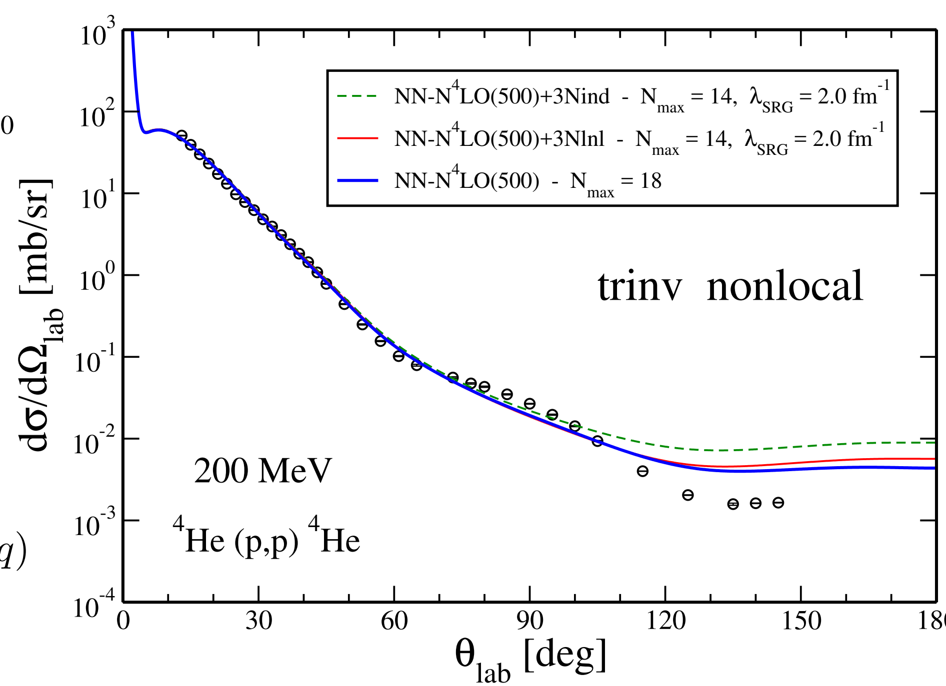


Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

FF from local density

$$\rho_\alpha(q) = 4\pi \int_0^\infty dr r^2 j_0(qr) \rho_\alpha(r)$$

Navratil, PRC **70**, 014317 (2004)

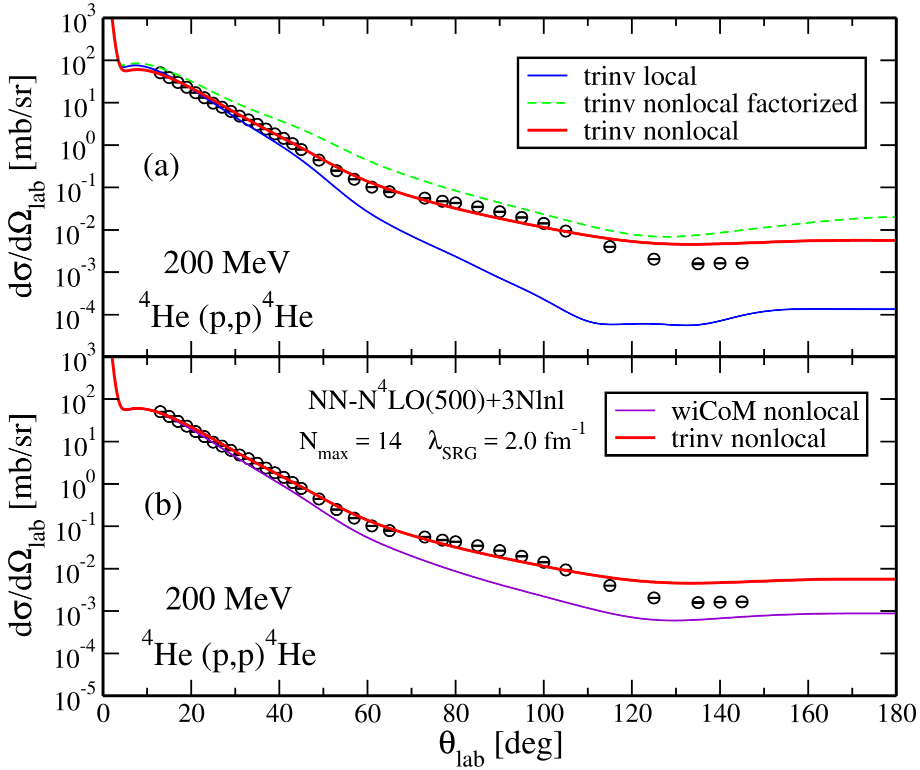


Factorized optical potential

$$U(\boldsymbol{q}, \boldsymbol{K}; E) = \eta(\boldsymbol{q}, \boldsymbol{K}) \sum_{\alpha=n,p} t_{p\alpha} \left[\boldsymbol{q}, \frac{A+1}{2A} \boldsymbol{K}; E \right] \rho_\alpha(q)$$

Scattering observables – Stable nuclei

Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

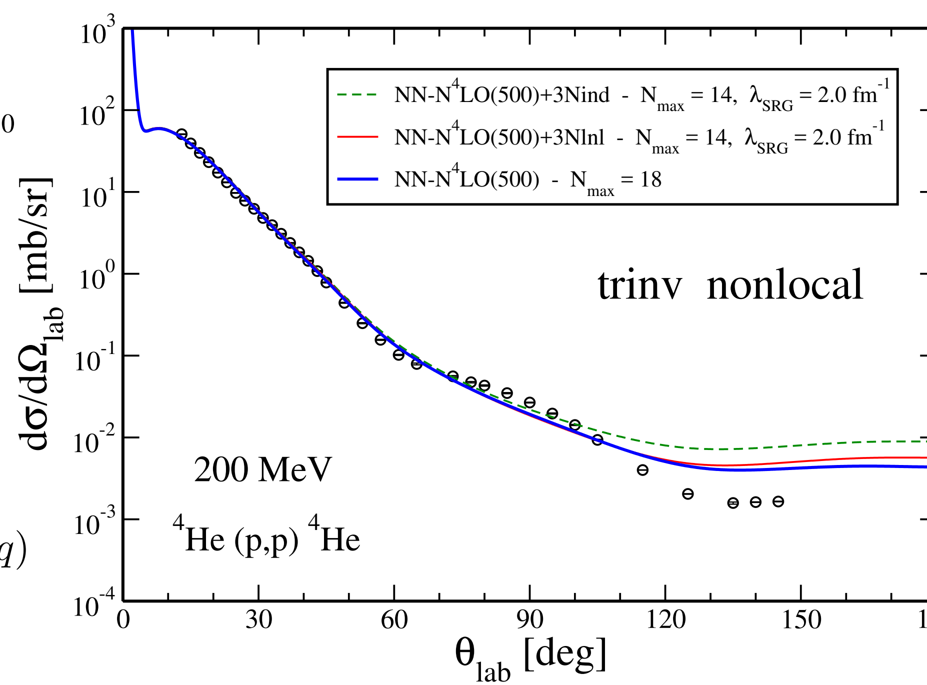


FF from nonlocal density

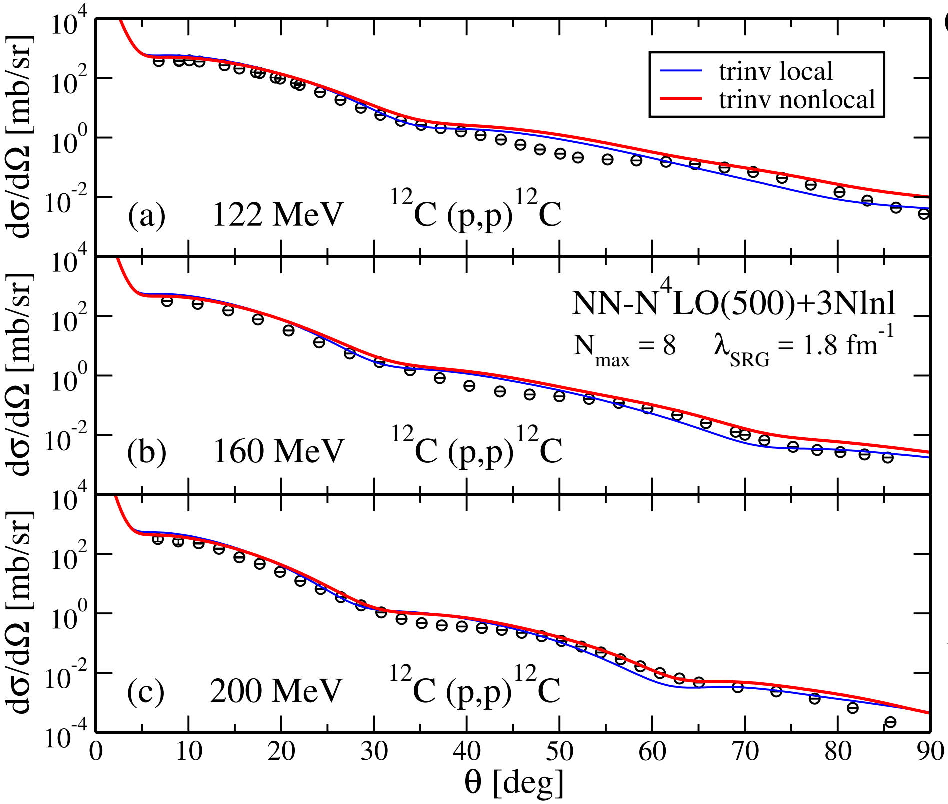
$$\rho_\alpha(q) = \int d\boldsymbol{P}\ \rho_\alpha\left(\boldsymbol{P} - \frac{A-1}{2A}\boldsymbol{q}, \boldsymbol{P} + \frac{A-1}{2A}\boldsymbol{q}\right)$$

Factorized optical potential

$$U(\boldsymbol{q}, \boldsymbol{K}; E) = \eta(\boldsymbol{q}, \boldsymbol{K}) \sum_{\alpha=n,p} t_{p\alpha}\left[\boldsymbol{q}, \frac{A+1}{2A}\boldsymbol{K}; E\right]\ \rho_\alpha(q)$$

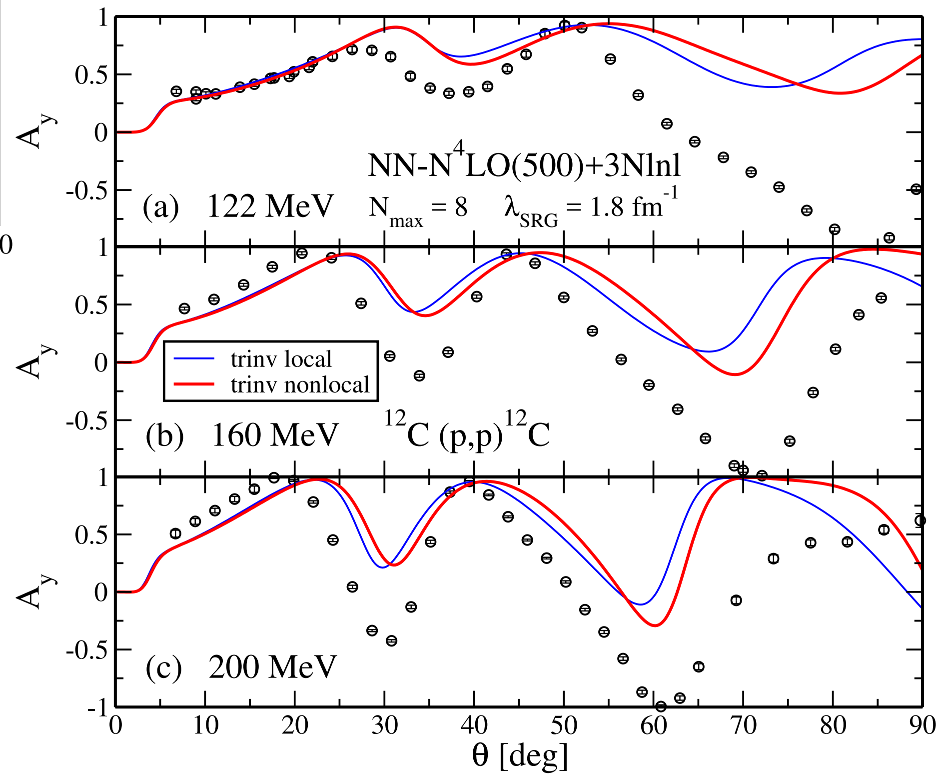


Scattering observables – Stable nuclei



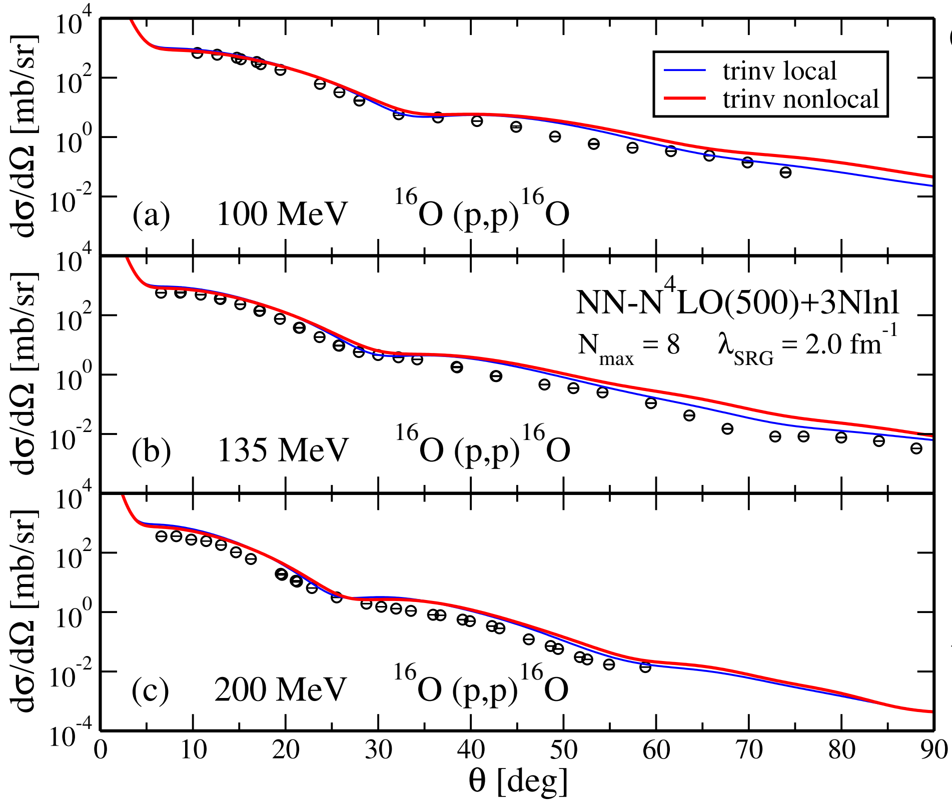
Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

Good description of differential cross sections

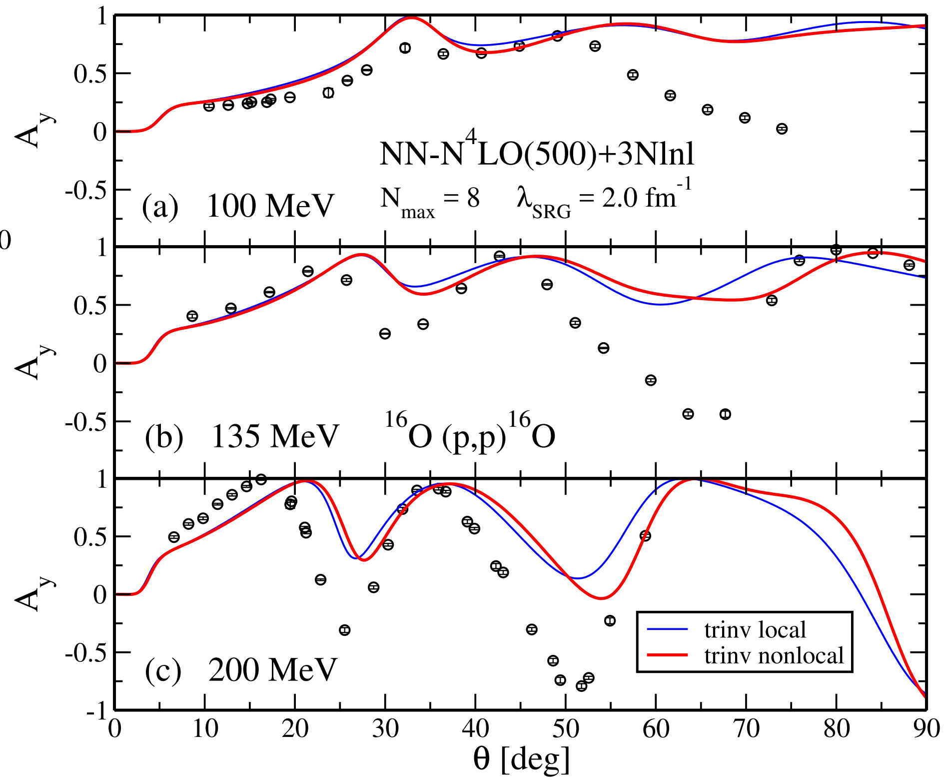


Reproduction of the general trend of the A_y

Scattering observables – Stable nuclei

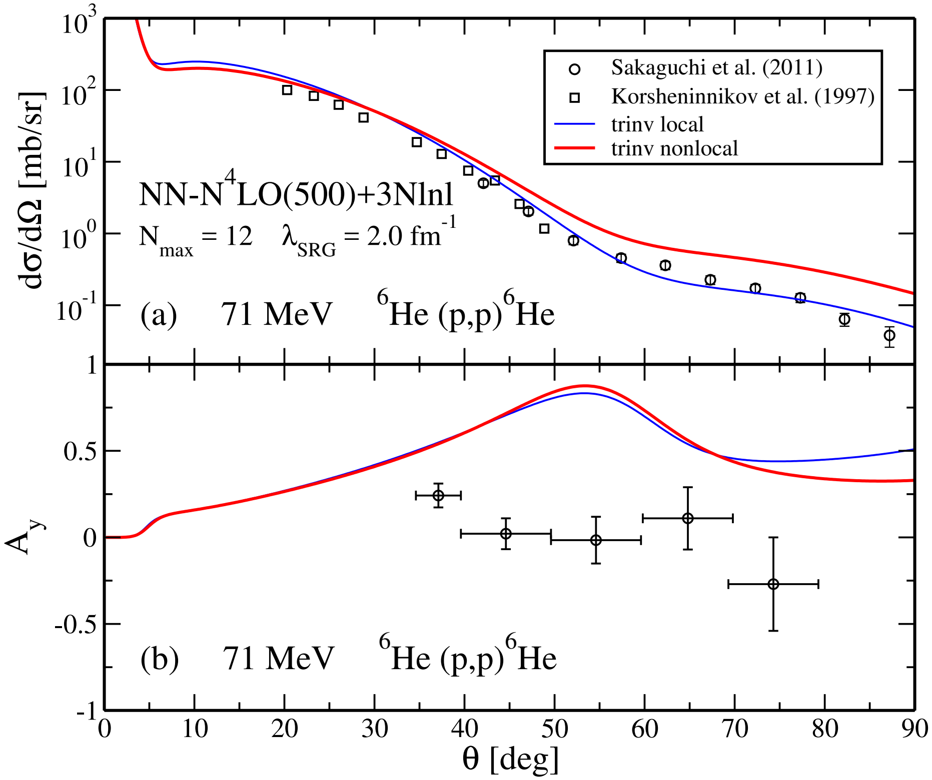


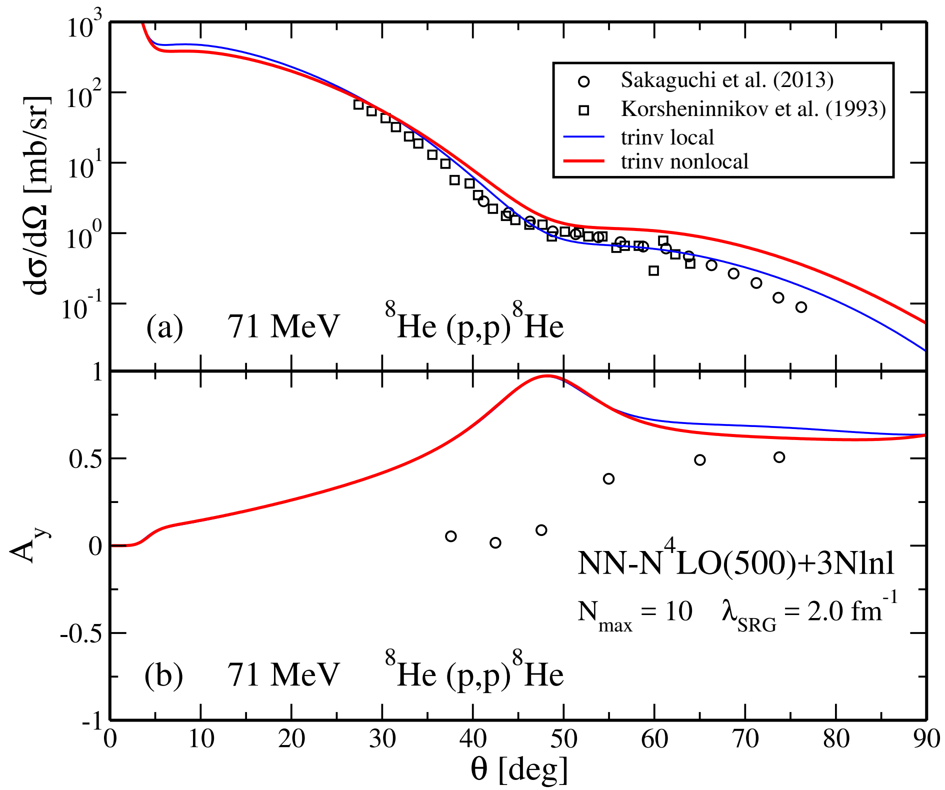
Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)



Scattering observables – ^{6}He and ^{8}He nuclei

Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

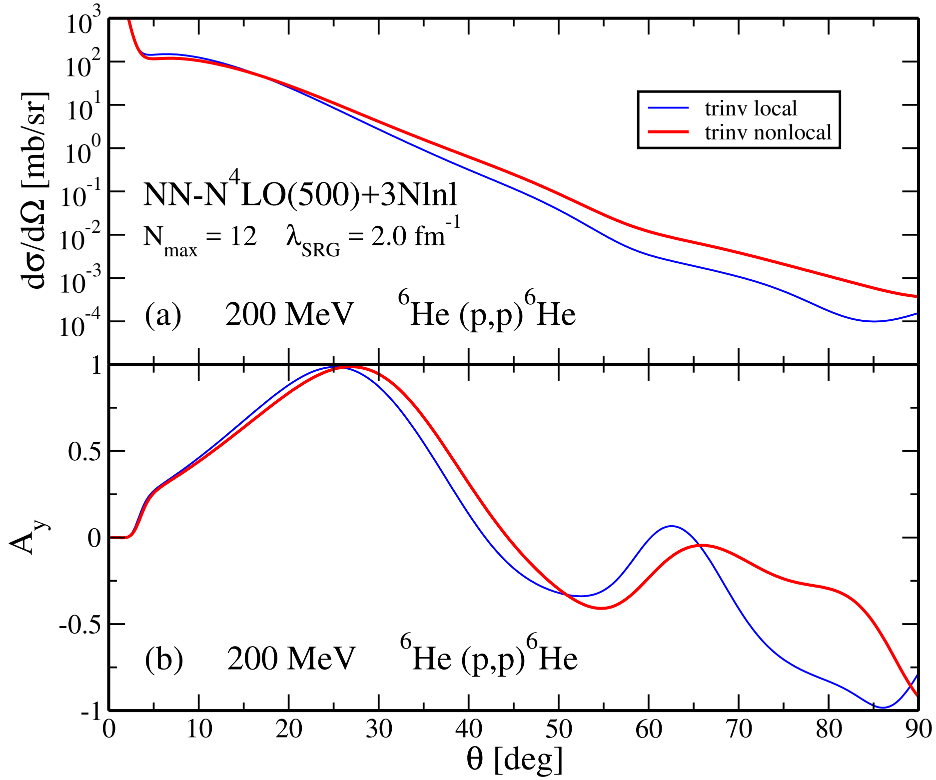


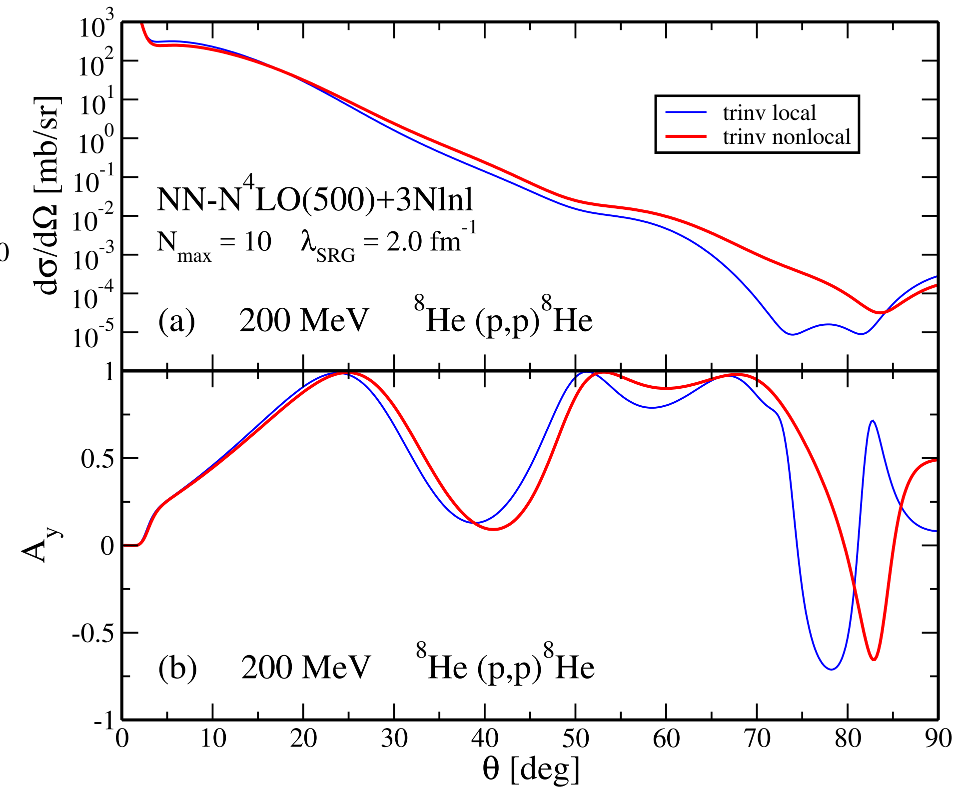


Reasonable description of the differential cross section

Scattering observables – ^{6}He and ^{8}He nuclei

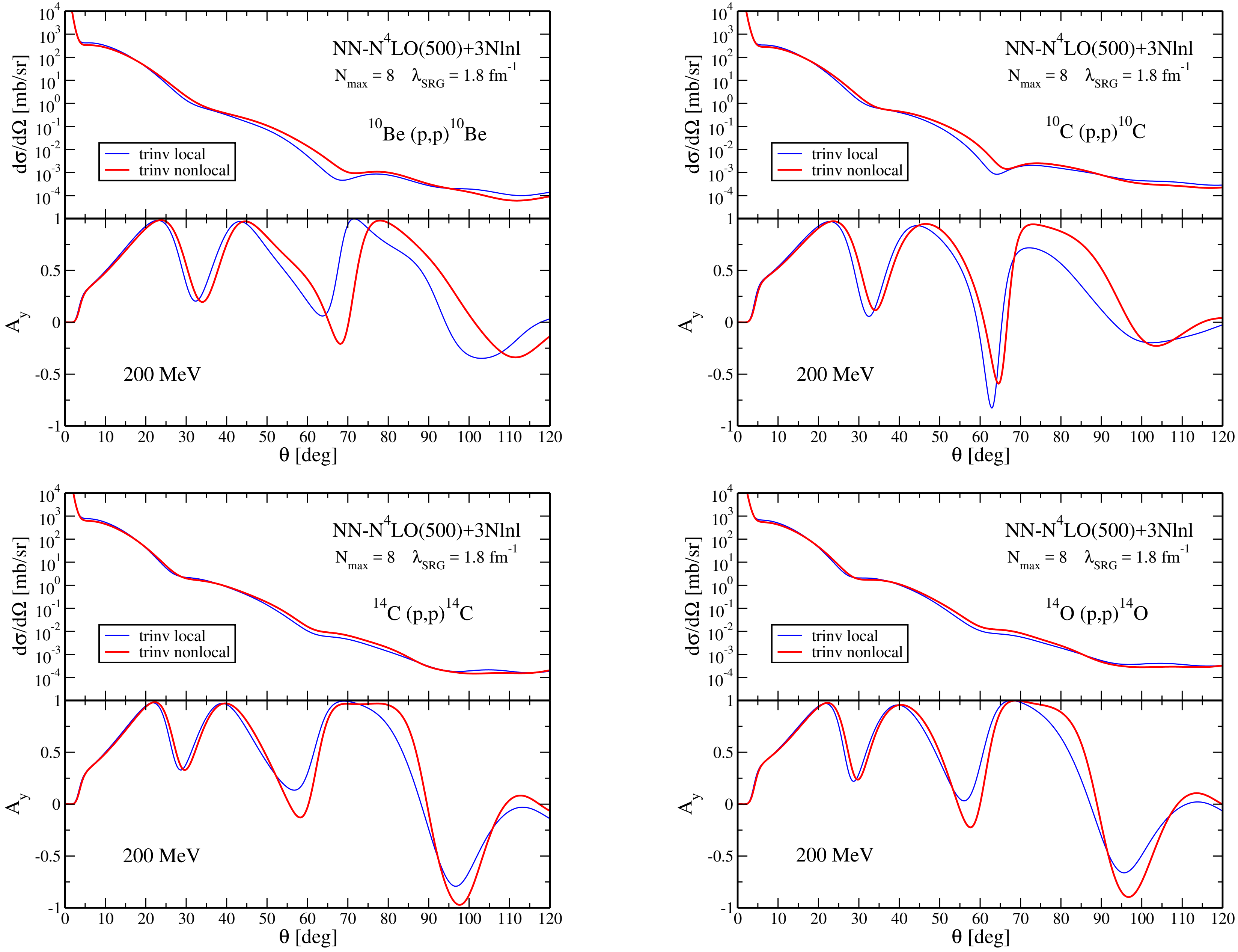
Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)





Different behavior of A_y after 40 deg

Predictions for other nuclei



Summary

- Good description of the data, especially for ^{12}C
- Importance of nonlocality for ^{4}He

Outlook

- Improvement of optical potential
  - Inclusion of the three-nucleon interaction
  - Inclusion of medium effects
  - Inclusion of cluster dynamics for halo nuclei
  - Extension to even-odd and odd-odd nuclei
- Calculation of the (e,e’p) quasi-elastic reactions with microscopic nonlocal optical potentials

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Backup Slides

- Translationally invariant non-local densities

$$\langle A\lambda_j J_j M_j \,|\rho_{op}^{trinv}(\vec{r}-\vec{R},\vec{r}\,'-\vec{R})|\, A\lambda_i J_i M_i\rangle$$

$$= \left(\frac{A}{A-1}\right)^{\frac{3}{2}} \sum \frac{1}{\hat{J}_f}(J_i M_i K k|J_f M_f)$$

$$\times \left(M^K\right)^{-1}_{nln'l',n_1l_1n_2l_2} \left(Y_l^*(\widehat{\vec{r}-\vec{R}})\, Y_{l'}^*(\widehat{\vec{r}\,'-\vec{R}})\right)^{(K)}_k$$

$$\times R_{n,l}\Big(\sqrt{\frac{A}{A-1}}|\vec{r}-\vec{R}|\Big) R_{n',l'}\Big(\sqrt{\frac{A}{A-1}}|\vec{r}\,'-\vec{R}|\Big)$$

$$\times (-1)^{l_1+l_2+K+j_2-\frac{1}{2}}\,\hat{j}_1\hat{j}_2 \left\{\begin{array}{ccc} j_1 & j_2 & K \\ l_2 & l_1 & \frac{1}{2}\end{array}\right\}$$

$$\times\, {}_{SD}\langle A\lambda_f J_f||\,(a^\dagger_{n_1,l_1,j_1}\,\tilde{a}_{n_2,l_2,j_2})^{(K)}\,||A\lambda_i J_i\rangle_{SD}$$

- Translationally invariant non-local densities

$$\langle A\lambda_j J_j M_j \,|\rho_{op}^{trinv}(\vec{r}-\vec{R},\vec{r}\,'-\vec{R})|\, A\lambda_i J_i M_i\rangle$$
$$= \left(\frac{A}{A-1}\right)^{\frac{3}{2}} \sum \frac{1}{\hat{J}_f}(J_i M_i K k|J_f M_f)$$
$$\times \left(M^K\right)^{-1}_{nln'l',n_1l_1n_2l_2} \left(Y_l^*(\widehat{\vec{r}-\vec{R}})\, Y_{l'}^*(\widehat{\vec{r}\,'-\vec{R}})\right)^{(K)}_k$$
$$\times R_{n,l}\left(\sqrt{\frac{A}{A-1}}|\vec{r}-\vec{R}|\right) R_{n',l'}\left(\sqrt{\frac{A}{A-1}}|\vec{r}\,'-\vec{R}|\right)$$
$$\times (-1)^{l_1+l_2+K+j_2-\frac{1}{2}}\, \hat{j}_1\hat{j}_2 \begin{Bmatrix} j_1 & j_2 & K \\ l_2 & l_1 & \frac{1}{2} \end{Bmatrix}$$
$$\times \;_{SD}\langle A\lambda_f J_f||\,(a^\dagger_{n_1,l_1,j_1}\,\tilde{a}_{n_2,l_2,j_2})^{(K)}\,||A\lambda_i J_i\rangle_{SD}$$

- Ground-state density for even-even nuclei

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \sum_l \rho_l(r,r')(-1)^l \frac{\sqrt{2l+1}}{4\pi} P_l(\cos\omega)$$

- Translationally invariant non-local densities

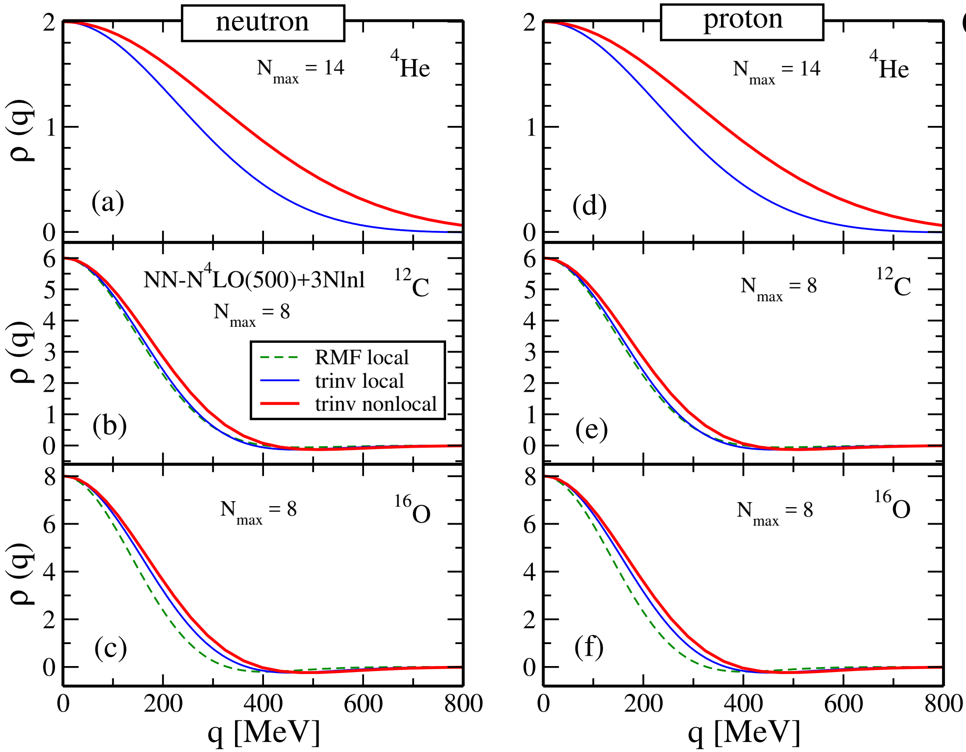
$$\langle A\lambda_j J_j M_j | \rho_{op}^{trinv}(\vec{r}-\vec{R},\vec{r}\,'-\vec{R}) | A\lambda_i J_i M_i\rangle$$
$$= \left(\frac{A}{A-1}\right)^{\frac{3}{2}} \sum \frac{1}{\hat{J}_f}(J_i M_i K k | J_f M_f)$$
$$\times \left(M^K\right)^{-1}_{nln'l',n_1 l_1 n_2 l_2} \left(Y_l^*(\widehat{\vec{r}-\vec{R}})\, Y_{l'}^*(\widehat{\vec{r}\,'-\vec{R}})\right)^{(K)}_k$$
$$\times R_{n,l}\left(\sqrt{\frac{A}{A-1}}|\vec{r}-\vec{R}|\right) R_{n',l'}\left(\sqrt{\frac{A}{A-1}}|\vec{r}\,'-\vec{R}|\right)$$
$$\times (-1)^{l_1+l_2+K+j_2-\frac{1}{2}}\, \hat{j}_1 \hat{j}_2 \begin{Bmatrix} j_1 & j_2 & K \\ l_2 & l_1 & \frac{1}{2} \end{Bmatrix}$$
$$\times\ {}_{SD}\langle A\lambda_f J_f || \left(a^\dagger_{n_1,l_1,j_1}\, \tilde{a}_{n_2,l_2,j_2}\right)^{(K)} || A\lambda_i J_i\rangle_{SD}$$

Angular part

- Ground-state density for even-even nuclei

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \sum_l \rho_l(r,r')(-1)^l \frac{\sqrt{2l+1}}{4\pi} P_l(\cos\omega)$$

Form factors (FF)



Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

FF from local density

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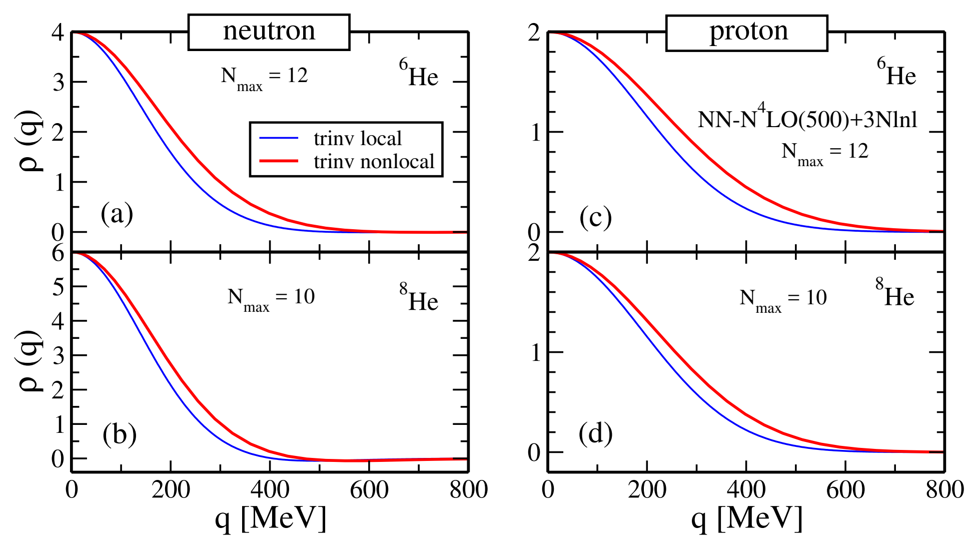
PRC **70**, 014317 (2004)

FF from non-local density

$$\rho_\alpha(q) = \int d^3\boldsymbol{P}\ \rho_\alpha\left(\boldsymbol{P} - \frac{A-1}{2A}\boldsymbol{q}, \boldsymbol{P} + \frac{A-1}{2A}\boldsymbol{q}\right)$$



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Scattering observables – Stable nuclei

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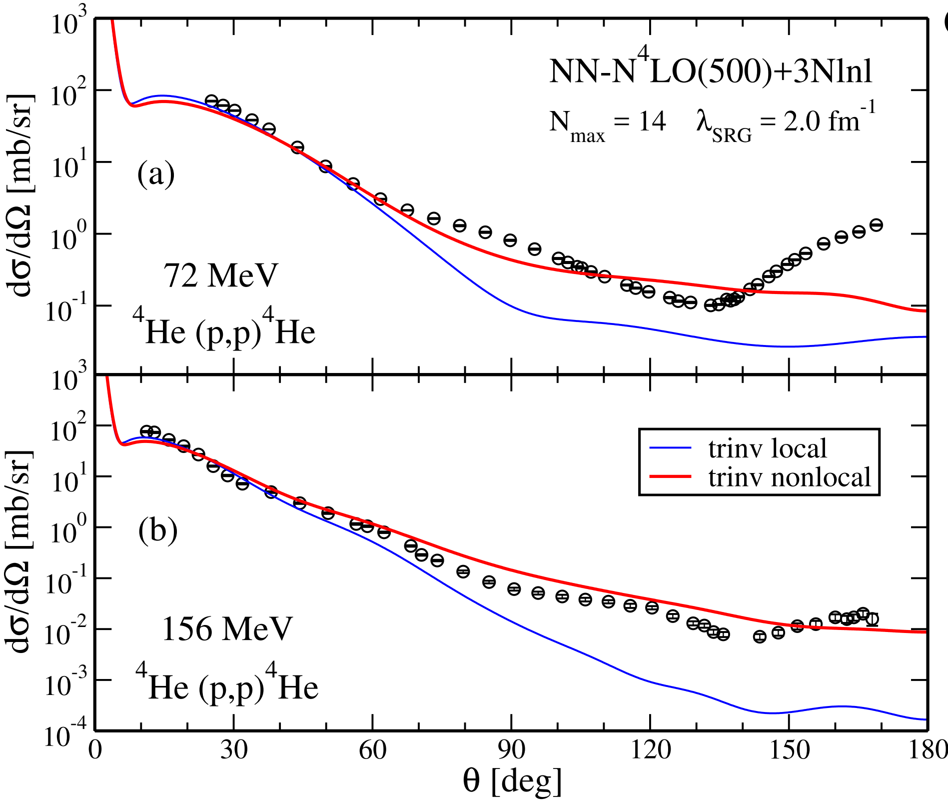

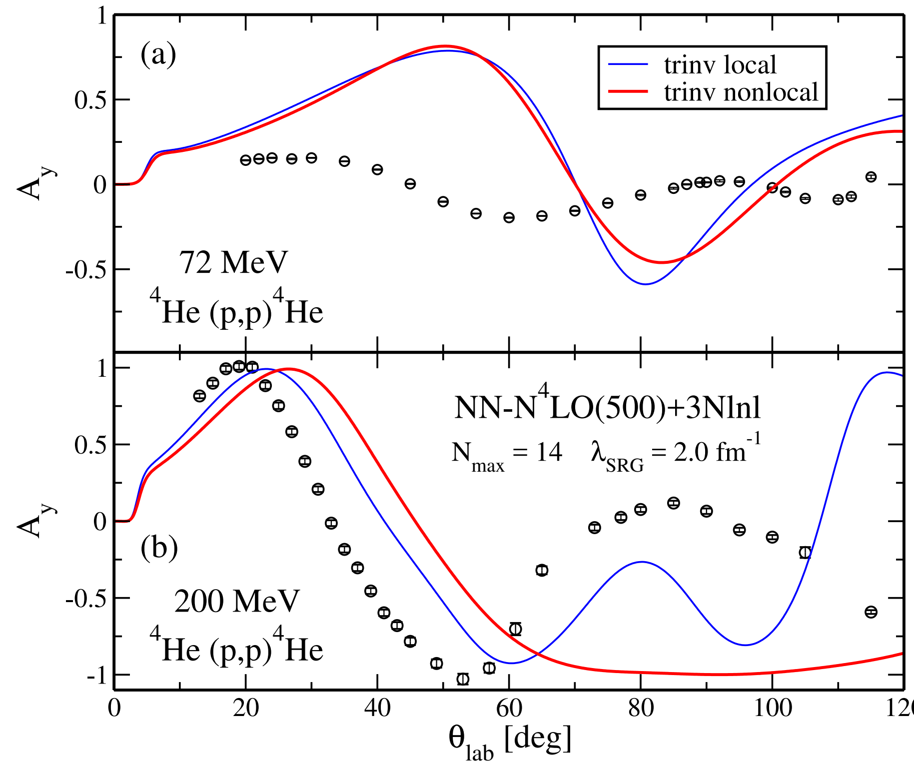

Comparison with RMF

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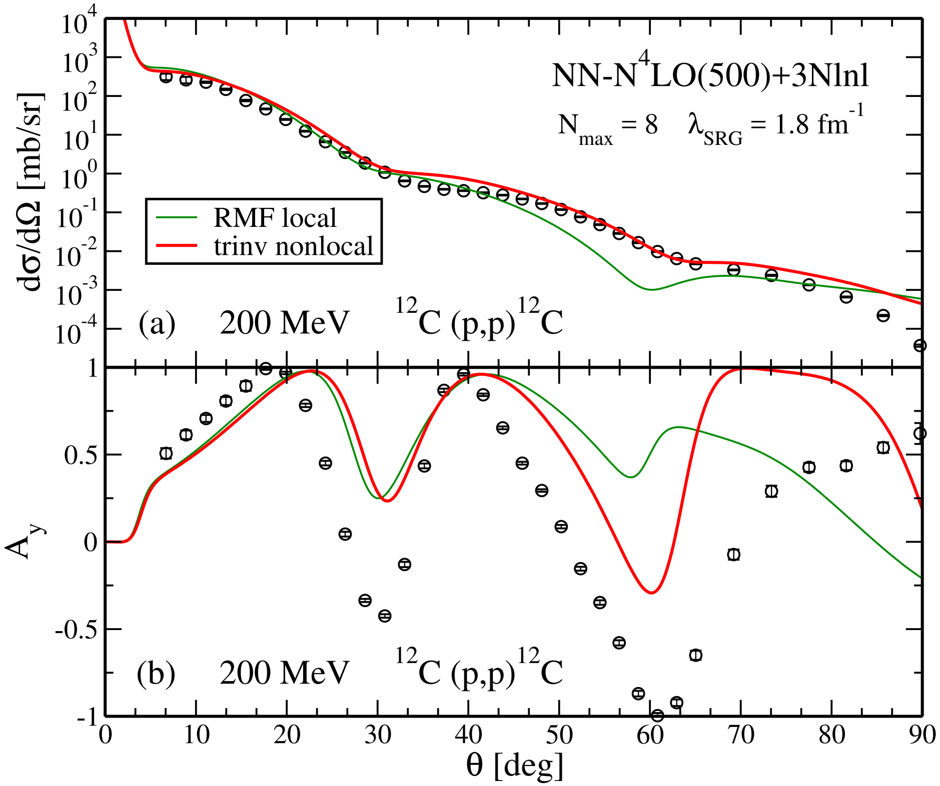

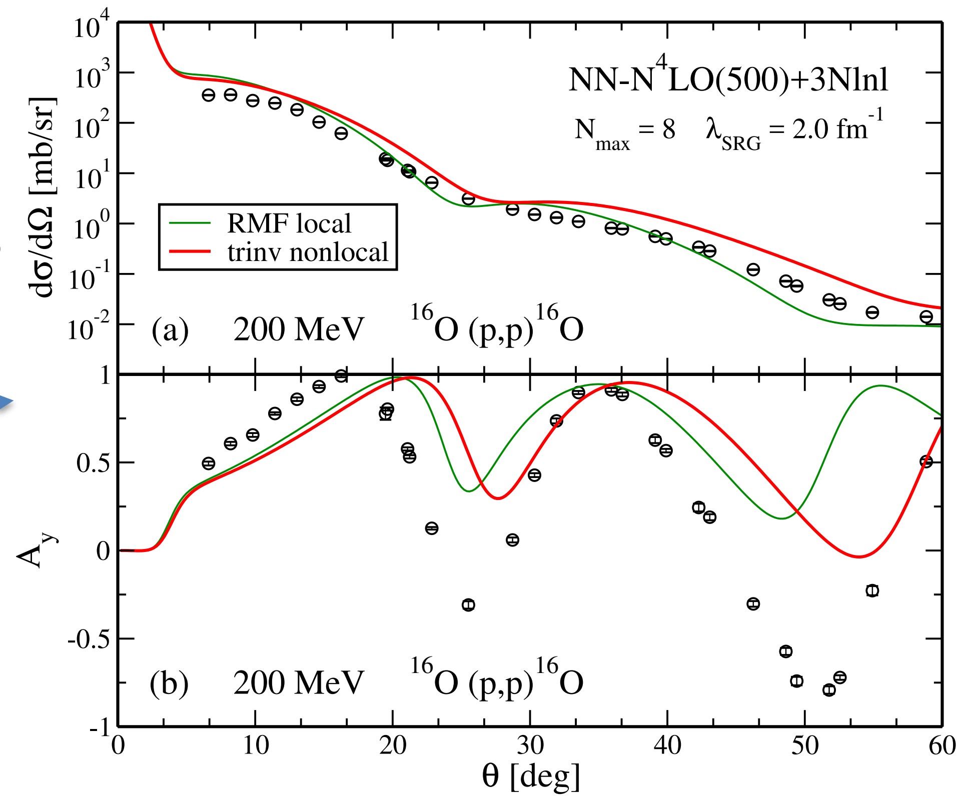

Shift with respect the RMF results