Assessing the foundation of the Trojan Horse Method

C.A. Bertulani

TEXAS A&M UNIVERSITY COMMERCE

CB, Hussein, Typel, Phys. Lett. B 776, 217 **(2018)**

Transfer reactions

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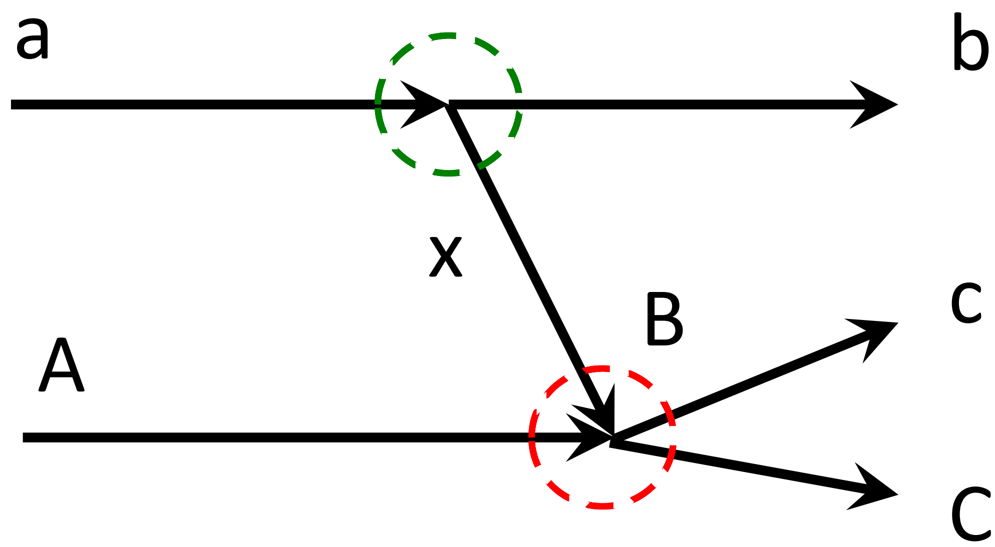
Consider the reaction

B

$$\overbrace{A + (x}^{B}\underbrace{{}+b)}_{\text{bound (a)}} \rightarrow b + c + C$$

bound (a)



Transfer reactions

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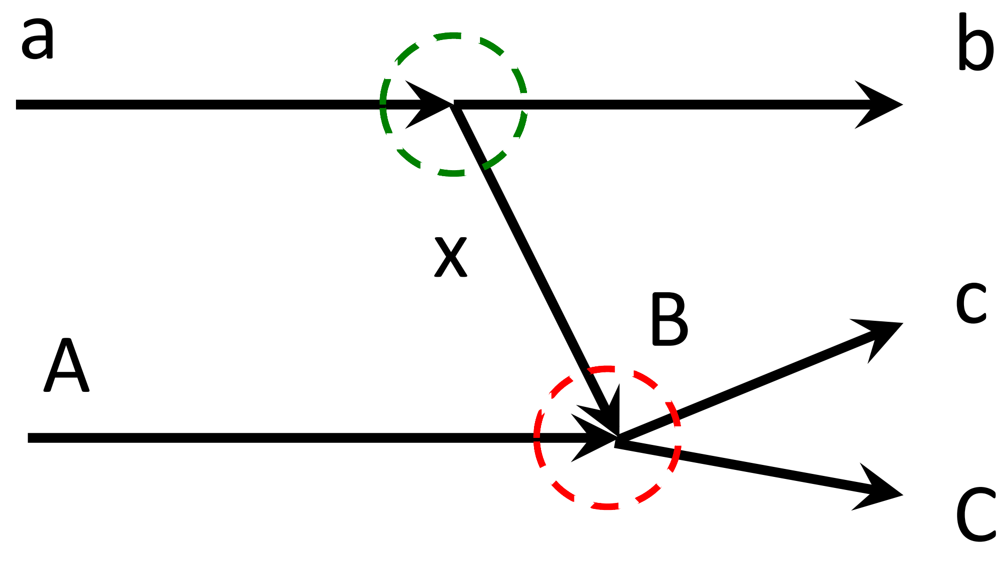
$$\overbrace{A + (x+b)}^{B} \rightarrow b + c + C$$

with $(x+b)$ bound (a)



Need to solve (neglect anti-symmetrization)

$$\left[E - \left(K_x + K_b + K_A + V_{xb} + U_{xA} + U_{bA}\right)\right]\left|\Psi^{(+)}_{3B}\Phi_A\phi_a\right\rangle = 0$$

$\left|\Psi^{(+)}_{3B}\right\rangle$ = full 3-body x+b+A wavefunction

A brief history - 1

Optical Background Representation

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DWBA – Prior form

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Trojan Horse Method:

Specific direct reaction induced by a tertiary beam

See, e.g. $^{12}C+^{12}C$ fusion with THM: Tumino et al, Nature 2018

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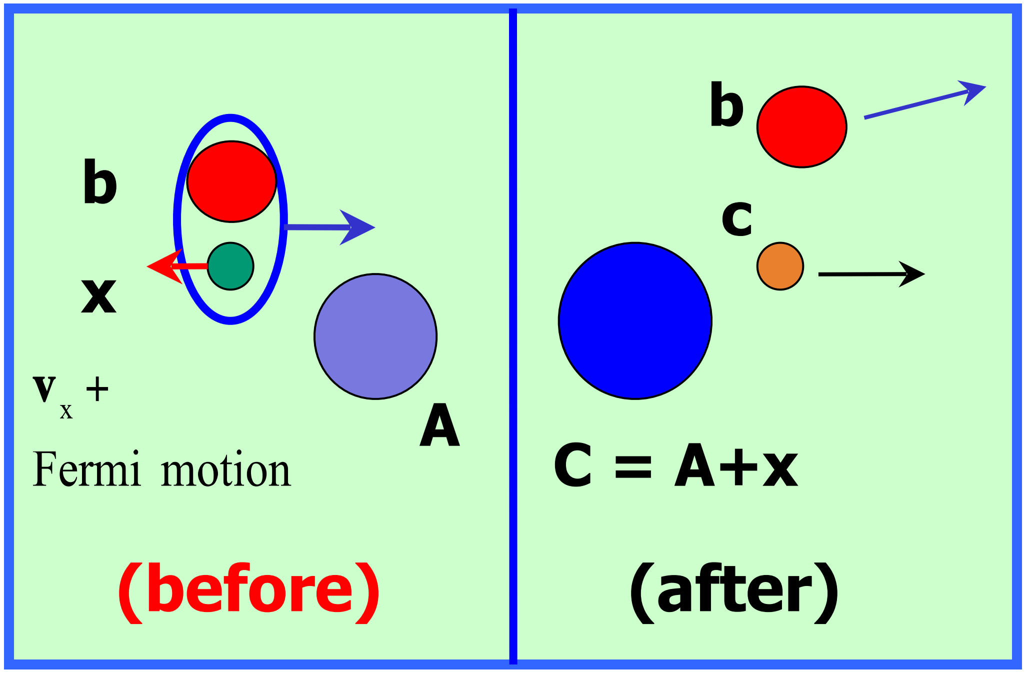
Measuring

$$A + (b+x) \rightarrow b + c + C$$

➡ $A + x \rightarrow C + c$

(astrophysics)



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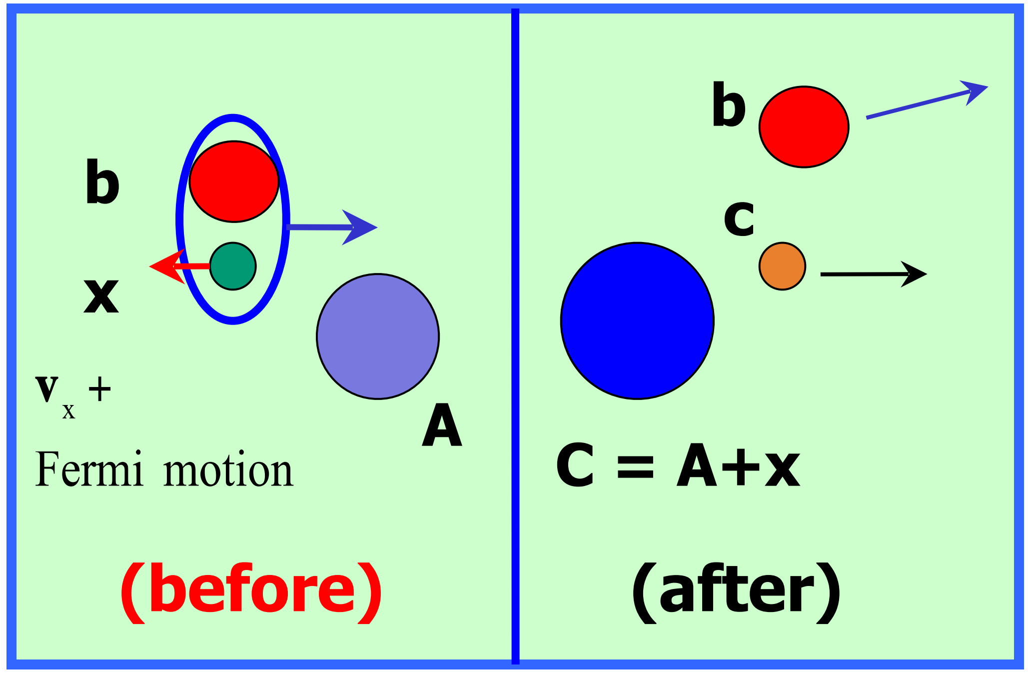

$$\sigma_{xA\rightarrow cC} = \frac{\pi}{k_x^2}\sum_l (2l+1)\left|S_{lc}\right|^2 \sim \frac{S_0}{E} e^{-2\pi\eta(E)}$$ (astrophysics)

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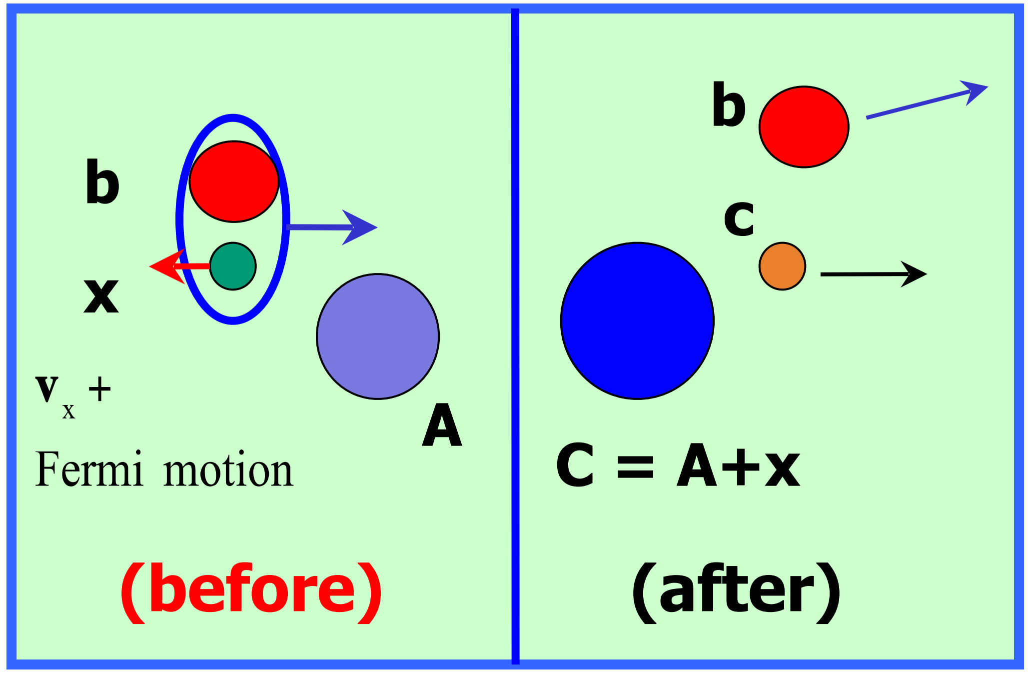

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(Trojan horse)

$$\frac{d^3\sigma}{d\Omega_b d\Omega_c dE_b} = \frac{m_a m_b m_c}{(2\pi)^5 \hbar^6} \frac{k_b k_c}{k_a} \left|\sum_l T_{lm}(k_a, k_b, k_x) S_{lc} Y_{lm}(k_c)\right|^2$$

Trojan horse method

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Post-form DWBA

$$T_{\ell m} = \left\langle \chi_b^{(-)} \Psi_{xA}^{(-)} \middle| V_{bx} \middle| \Psi_{bxA}^{(+)} \phi_a \right\rangle \approx \left\langle \chi_b^{(-)} \chi_x^{(-)} \middle| V_{bx} \middle| \chi_a^{(+)} \phi_a \right\rangle$$

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surface-dominated x+A w.f.

$$k_x \to 0 \; (\eta \to 0) \quad \begin{cases} \chi_x^{(-)} \sim \dfrac{1}{k_x R} Y_{lm} e^{\pi\eta} K_{2l+1}\left(\sqrt{\dfrac{8R}{a_B}}\right) \\ S_{l_c} \sim e^{-\pi\eta} \end{cases}$$

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surface-dominated $x+A$ w.f.

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$$\Rightarrow \quad \frac{d^3\sigma}{d\Omega_b \, d\Omega_c \, dE_b} \to \text{const.}$$

(Trojan horse)

G. Baur, *Phys. Lett. B 178, 135* ***(1986)***

Typel, Baur, Ann. Phys. (NY) 305, 228 **(2003)**

Trojan horse method: quasi-free kinematics

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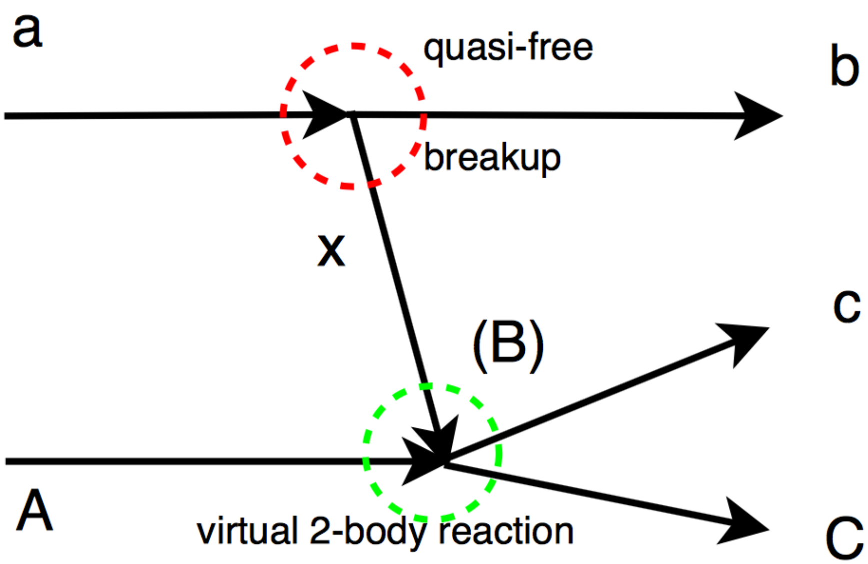
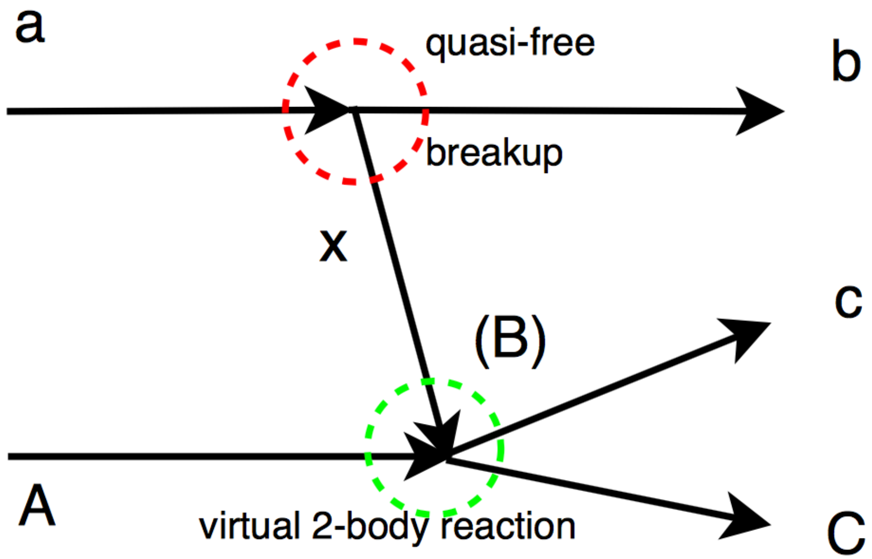


$$\frac{d\sigma}{dE_c d\Omega_c d\Omega_C} \propto KF \left|\phi\left(\mathbf{k}_{bx}\right)\right|^2 \frac{d\sigma^{HOES}}{d\Omega_{cm}}$$

Quasi-free mechanism

b acts as a spectator: small contribution to the reaction

Trojan horse method: quasi-free kinematics



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Quasi-free mechanism

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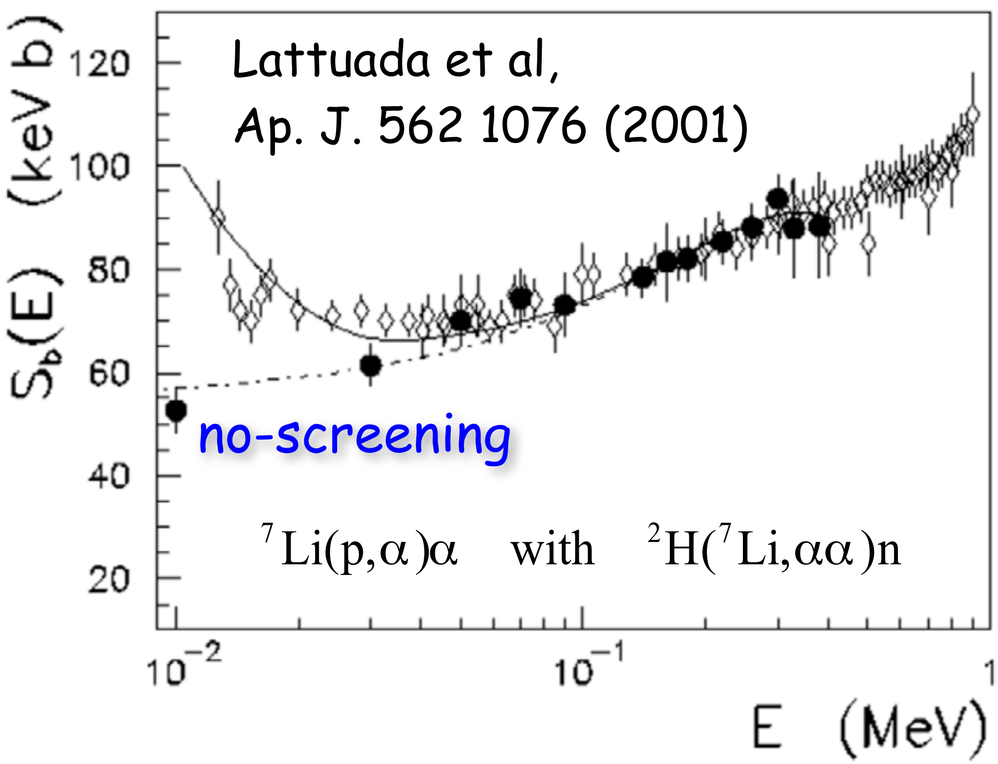
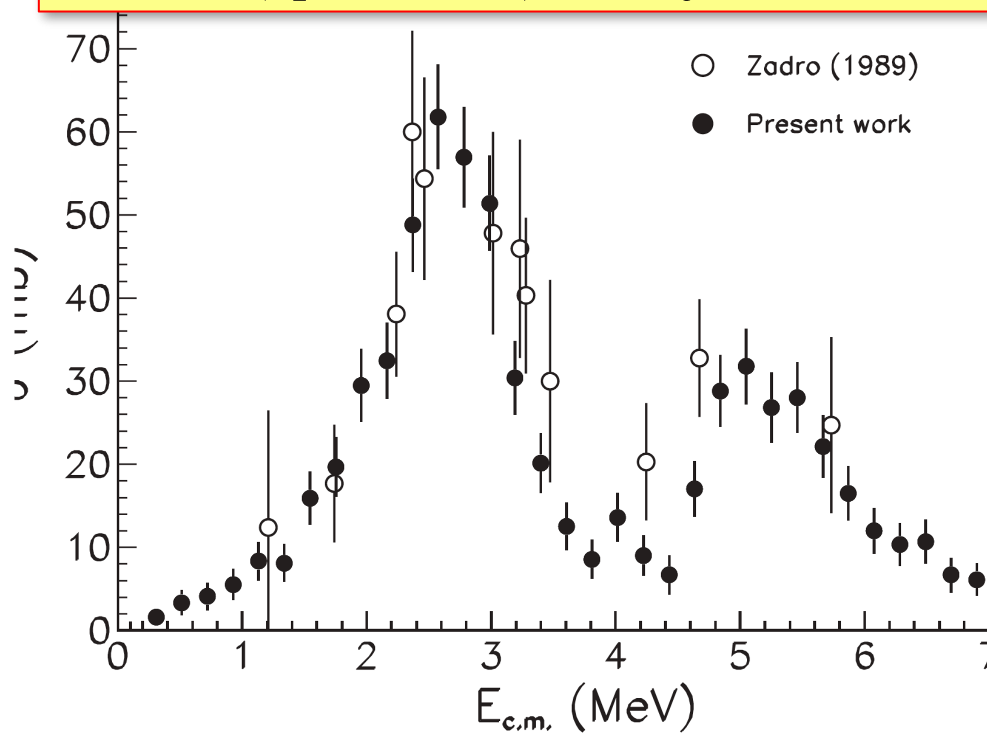
Same reaction, using ^{3}He (filled circles) and deuteron (open circles) as Trojan horse.





Pizzone et al, PRC 83, 045801 (2011)

Inclusive Non-Elastic Breakup Cross Section (INEB)

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Consider the reaction

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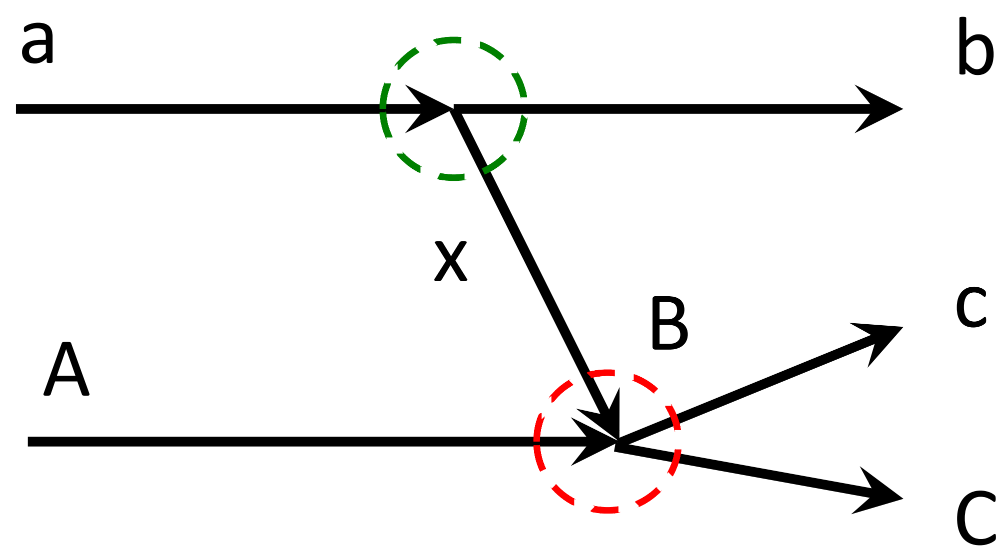

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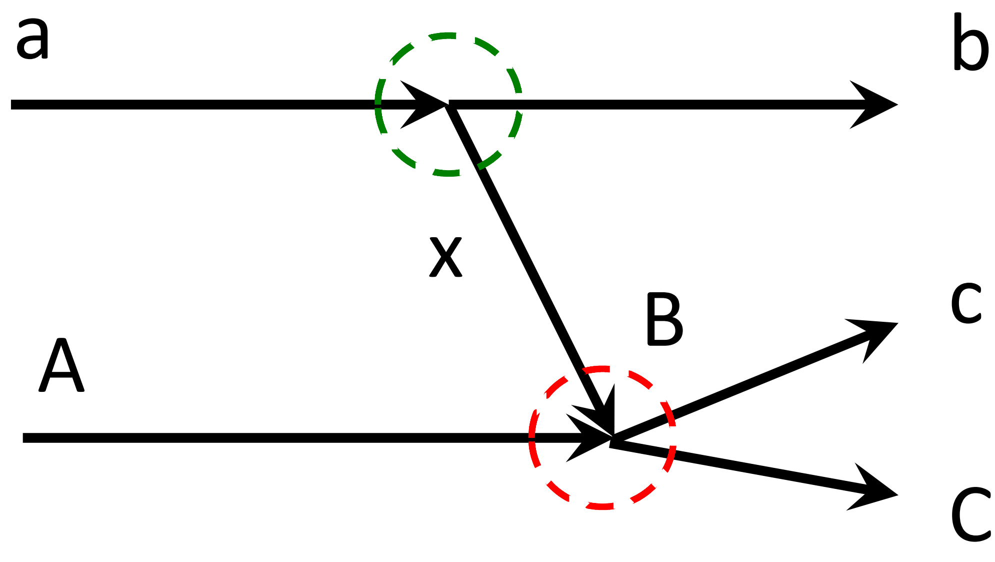

INEB

dens. states

$$\frac{d\sigma}{dE_b d\Omega_b} = \rho_b(E_b)\, \sigma_R^x$$

$$\sigma_R^x = -\frac{k_x}{E_x}\left\langle \hat{\rho}_x(\mathbf{r}_x) \middle| W_x(\mathbf{r}_x) \middle| \hat{\rho}_x(\mathbf{r}_x) \right\rangle \qquad \hat{\rho}_x(\mathbf{r}_x) = \left(\chi_b^{(-)} \middle| \Psi_{3B}^{(+)} \right\rangle$$

imaginary part opt. pot.

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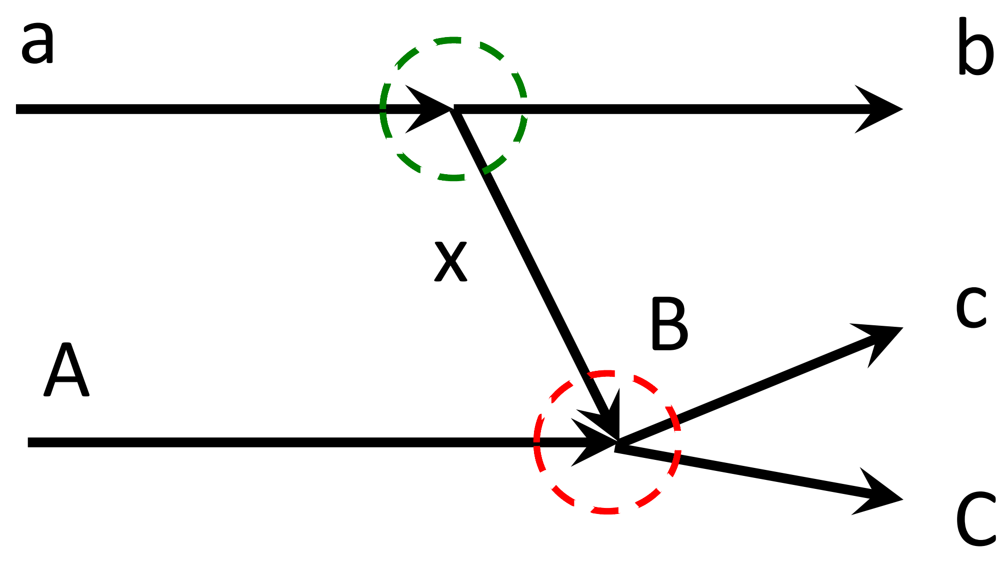

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imaginary part opt. pot.

Ichimura, Austern, Vincent, PRC 32, 431 **(1985)** – post form

$$\left|\Psi_{3B}^{(+)}\right\rangle = \left(E - K_b - U_b - K_x - U_x + i\varepsilon\right)^{-1} V_{xb}\left|\phi_{bx}\chi_{bx}^{(+)}\right\rangle$$

Inclusive Non-Elastic Breakup Cross Section (INEB)

$$\Rightarrow \quad \hat{\rho}_x^{IAV}\left(\mathbf{r}_x\right) = G_x^{(+)}\left(E_x\right)\left(\chi_b^{(-)}\left|V_{bx}\right|\phi_a\chi_a^{(+)}\right\rangle$$

$$G_x^{(+)}\left(E_x\right) = \left(E_x - K_x - U_x + i\varepsilon\right)^{-1} \qquad E_x = E - E_b$$

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Structure of W_x

Consider the scattering **x+A** with dynamics governed by $\mathbf{H_x}$

open x+A channels

closed compound x+A channels

$$P_x + Q_x = 1, \quad P_x Q_x = Q_x P_x = 0$$

$$P_x^2 = P_x, \quad Q_x^2 = Q_x$$

Structure of W_x

➡

Structure of W_x

$$\Rightarrow \left(E_x - P_x H_x P_x - P_x H_x Q_x \frac{1}{E_x - Q_x H_x Q_x} Q_x H_x P_x \right) P_x \left| \Psi_{xA}^{(+)} \right\rangle = 0$$

Exact but useless!

Structure of W_x

$$\left(E_x - P_x H_x P_x - P_x H_x Q_x \frac{1}{E_x - Q_x H_x Q_x} Q_x H_x P_x\right) P_x \left|\Psi^{(+)}_{xA}\right\rangle = 0$$

Exact but useless!

- **Average** over CN states
- Define optical **x+A open channels** $\left|\tilde{\Psi}^{(+)}_{xA}\right\rangle$
- Split P_x into $P_x^{(0)}$**(elastic breakup of b+x+A)** + $P_x^{(D)}$**(all open non-elastic direct channels)**

Structure of W_x

$$\left(E_x - P_x H_x P_x - P_x H_x Q_x \frac{1}{E_x - Q_x H_x Q_x} Q_x H_x P_x\right) P_x \left|\Psi^{(+)}_{xA}\right\rangle = 0$$

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- **Average** over CN states
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$$\left(E_x - H_x^{(\text{eff})} - H_x^{(\text{eff})} P_x^{(D)} G_x^{(+),D} P_x^{(D)} H_x^{(\text{eff})}\right) P_x \left|\bar{\Psi}^{(+)}_{xA}\right\rangle = 0$$

Structure of W_x

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$$W_x = \mathrm{Im}\, U_x = W_x^{CN} + W_x^{D}$$

$$\mathrm{Im}\, H_x^{(eff)} \qquad \mathrm{Im}\left[H_x^{(eff)} P_x^{(D)} G_x^{(+),D} P_x^{(D)} H_x^{(eff)} \right]$$

$$G_x^{(+),D} = \left[E_x - P_x^{(D)} H_x^{(eff)} P_x^{(D)} + i\varepsilon \right]^{-1}$$

Structure of W_x

$$W_x = \mathrm{Im}\,U_x = W_x^{CN} + W_x^{D}$$

$$\mathrm{Im}\,H_x^{(\mathrm{eff})} \qquad \mathrm{Im}\left[H_x^{(\mathrm{eff})} P_x^{(D)} G_x^{(+),D} P_x^{(D)} H_x^{(\mathrm{eff})}\right]$$

$$G_x^{(+),D} = \left[E_x - P_x^{(D)} H_x^{(\mathrm{eff})} P_x^{(D)} + i\varepsilon\right]^{-1}$$

Spectral decomposition

$$\mathrm{Im}\,G_x^{(+),D} = -\pi \sum_D \int \frac{d\mathbf{k}_D}{(2\pi)^3} \left|\chi_{\mathbf{k}_D}^{(-)}\right) \delta\left(E_x - E_{k_D}\right) \left(\chi_{\mathbf{k}_D}^{(-)}\right|$$

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All interactions coupling the ground to direct inelastic states

$$W_x^D = -\pi \sum_f \frac{d\mathbf{k}_f}{\left(2\pi\right)^3} V_{(0,f)} \left|\chi_f^{(-)}\left(\mathbf{k}_f\right)\right\rangle \left\langle \chi_f^{(-)}\left(\mathbf{k}_f\right)\right| V_{(0,f)} \delta\left(E_x - E_f\right)$$

$$V_{(0,f)} = P_x^{(0)} V P_x^{(D)}$$

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Consider a particular $x+A$ channel, i.e., $y+B$

$$\frac{d^2\sigma_b^{\mathrm{INEB},(D)}}{dE_b d\Omega_b dE_y d\Omega_y} = -\pi\rho_b(E_b)\rho_y(E_y)\frac{k_x}{E_x}$$

$$\times\left|\left\langle \chi_y^{(-)}(\mathbf{k}_y)\right|V_{(x,y)}G_x^{(+)}\left|\left(\chi_b^{(-)}\right|V_{xb}\left|\phi_a\chi_a^{(+)}\right\rangle\right|^2$$

transition to y channel

propagation of x

elastic breakup

The Trojan Horse formula

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Approximation: **maximum survival probability for b** $$\hat{S}_b(\mathbf{r}_x)\approx\phi_a(\mathbf{r}_b)$$

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$$\Rightarrow\quad \frac{d^2\sigma_b^{\mathrm{INEB},(D)}}{dE_b\,d\Omega_b}=\frac{d^2\sigma_b^{(\mathrm{THM})}}{dE_b\,d\Omega_b}=K_{(\mathrm{THM})}\left|\phi(\mathbf{k}_b)\right|^2\sigma_{(x+A\to y+B)}$$

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