

# Manifestation of $\alpha$ -clustering in Be isotopes via $\alpha$ -knockout reaction

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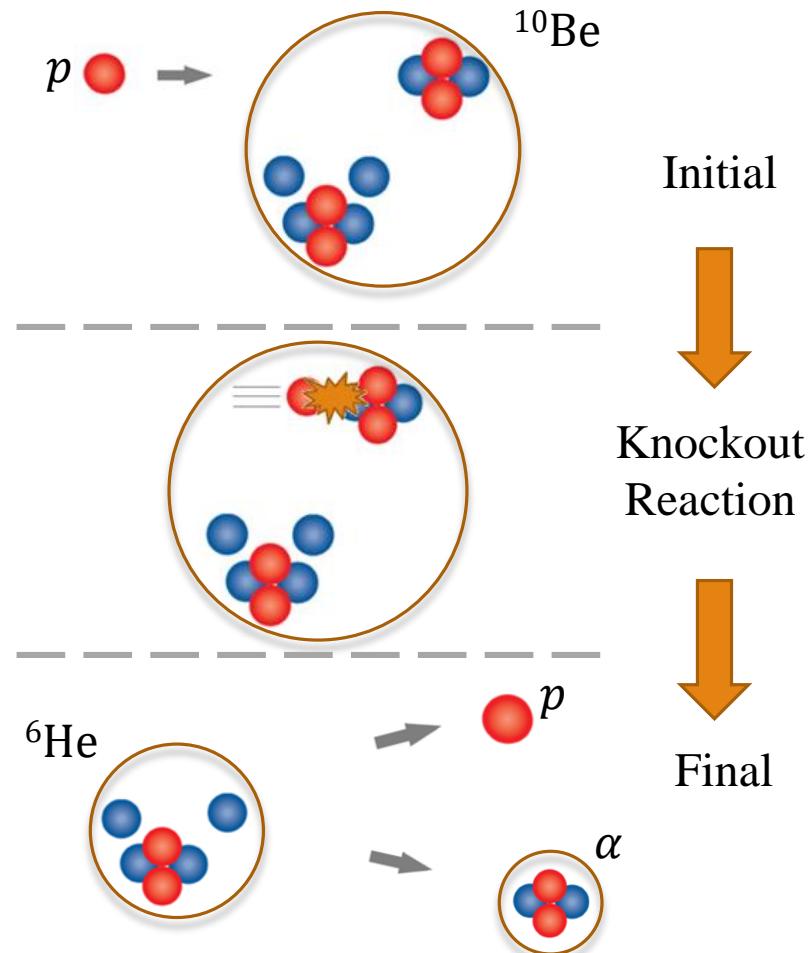
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# How to probe the $\alpha$ -cluster ?

The  $\alpha$  knockout reaction

- *Well established theory* employing the DWIA framework
- *Peripherality* only the surface region contributes
- *Clean*



# DWIA framework for $\alpha$ -knockout reaction

- The *transition amplitude* of  $\alpha$ -knockout reaction

$$T_{\mathbf{K}_0 \mathbf{K}_1 \mathbf{K}_2} = \langle \chi_{1, \mathbf{K}_1}^{(-)}(\mathbf{R}_1) \chi_{2, \mathbf{K}_2}^{(-)}(\mathbf{R}_2) | t_{p\alpha}(s) | \chi_{0, \mathbf{K}_0}^{(+)}(\mathbf{R}_0) \phi_\alpha(\mathbf{R}_2) \rangle$$

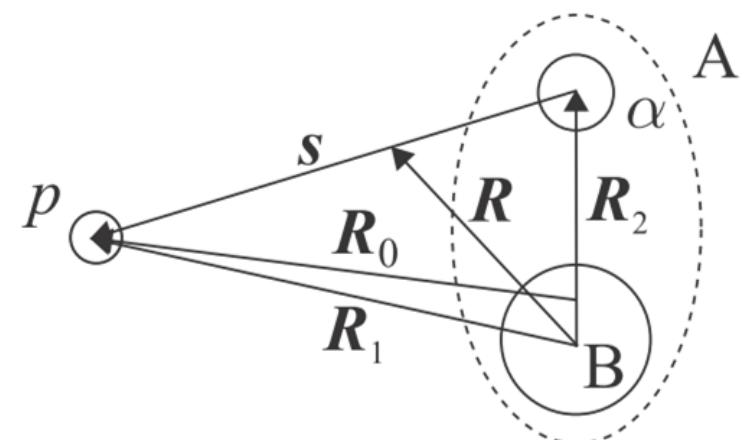
- With the *asymptotic momentum approximation* [1]

$$\bar{T}_{\mathbf{K}_0 \mathbf{K}_1 \mathbf{K}_2} = \int d\mathbf{R} F_{\mathbf{K}_0 \mathbf{K}_1 \mathbf{K}_2}(\mathbf{R}) \phi_\alpha(\mathbf{R})$$

- where

$$F_{\mathbf{K}_0 \mathbf{K}_1 \mathbf{K}_2}(\mathbf{R}) = \chi_{1, \mathbf{K}_1}^{*(-)}(\mathbf{R}_1) \chi_{2, \mathbf{K}_2}^{*(-)}(\mathbf{R}_2) \chi_{0, \mathbf{K}_0}^{(+)}(\mathbf{R}_0) e^{-i\mathbf{K}_0 \cdot \mathbf{R} A_\alpha / A}$$

- $\chi_{1, \mathbf{K}_1}^{(-)}, \chi_{2, \mathbf{K}_2}^{(-)}$  distorted w. f. in initial channel
- $\chi_{0, \mathbf{K}_0}^{(+)}$  distorted w. f. in final channel



# Triple differential cross section for $\alpha$ -knockout reaction

- The triple *differential cross section (TDX)*

$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} = F_{kin} C_0 \frac{d\sigma_{p\alpha}}{d\Omega_{p\alpha}}(\theta_{p\alpha}, E_{p\alpha}) |\bar{T}_{K_0 K_1 K_2}|^2$$

- $F_{kin}$ : kinematical factor

$$F_{kin} = J_L \frac{K_1 K_2 E_1 E_2}{\hbar^4 c^4} \left[ 1 + \frac{E_2}{E_B} + \frac{E_2}{E_B} \frac{(K_1 \cdot K_2)}{K_2^2} \right]$$

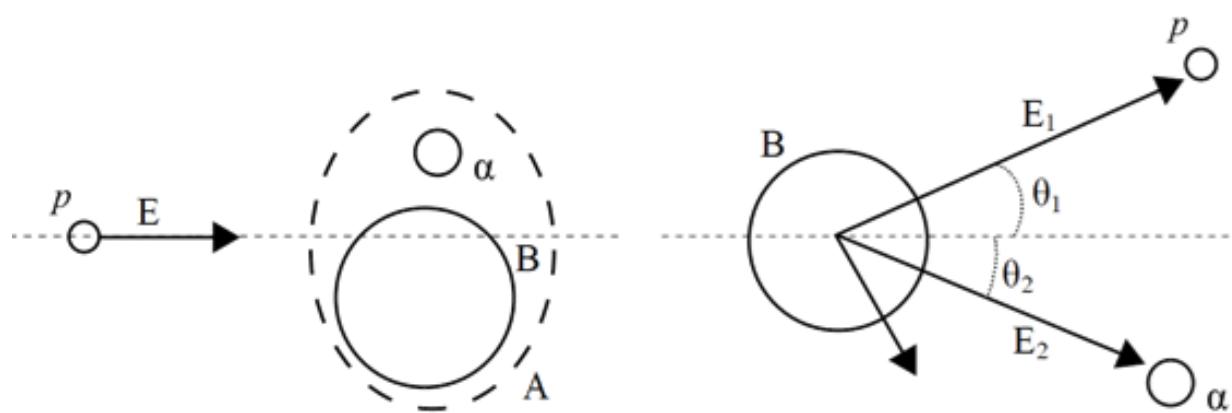
- $C_0 = \frac{E_0}{(\hbar c)^2 K_0} \frac{\hbar^4}{(2\pi)^3 \mu_{p\alpha}^2}$

- $J_L$ : Jacobian

- Scattering energy

$$E_{p\alpha} = \frac{\hbar^2 \kappa'^2}{2\mu_{p\alpha}}$$

- $\kappa'$ : asymptotic  $p\alpha$  relative momentum



# THSR description of Be and He

- *THSR wave function* for nuclei composed of  $\alpha$ -clusters and valence neutrons [1,2]

$$|\Psi\rangle = (c_\alpha^\dagger)^m (c_n^\dagger)^n |\text{vac}\rangle$$

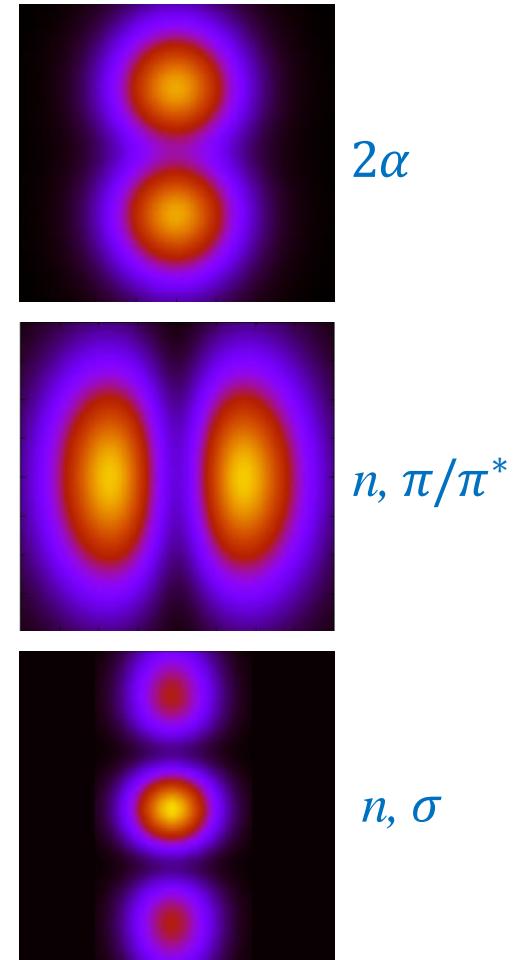
- *$\alpha$ -clusters*

$$\begin{aligned} c_\alpha^\dagger = & \int d^3\mathbf{R} \exp\left(-\frac{R_x^2 + R_y^2}{\beta_{\alpha,xy}^2} - \frac{R_z^2}{\beta_{\alpha,z}^2}\right) \int d^3\mathbf{r}_1 \cdots d^3\mathbf{r}_4 \\ & \times \psi(\mathbf{r}_1 - \mathbf{R}) a_{\sigma_1,\tau_1}^\dagger(\mathbf{r}_1) \cdots \psi(\mathbf{r}_4 - \mathbf{R}) a_{\sigma_4,\tau_4}^\dagger(\mathbf{r}_4) \end{aligned}$$

- *valance neutrons*

$$\begin{aligned} c_n^\dagger = & \int d^3\mathbf{R}_n \exp\left(-\frac{R_{n,x}^2 + R_{n,y}^2}{\beta_{n,xy}^2} - \frac{R_{n,z}^2}{\beta_{n,z}^2}\right) f(\mathbf{R}_n) \int d^3\mathbf{r}_n \\ & \times \psi(\mathbf{r}_n - \mathbf{R}) a_{\sigma n,\tau=\downarrow}^\dagger(\mathbf{r}_n) \end{aligned}$$

- $f(\mathbf{R}_n)$ : factors to adjust neutron w. f.  $\phi_n = c_n^\dagger |0\rangle$  into *p-orbits* or molecular  *$\pi$ -orbit* and  *$\sigma$ -orbits* [1,2]



[1] M. Lyu *et al.*, *Phys. Rev. C* **91**, 14313 (2015) & *Phys. Rev. C* **93**, 54308 (2016)..

[2] M. Lyu *et al.*, arXiv:1706.06538 [nucl-th] (2017).

# Approximation for reduced width amplitude

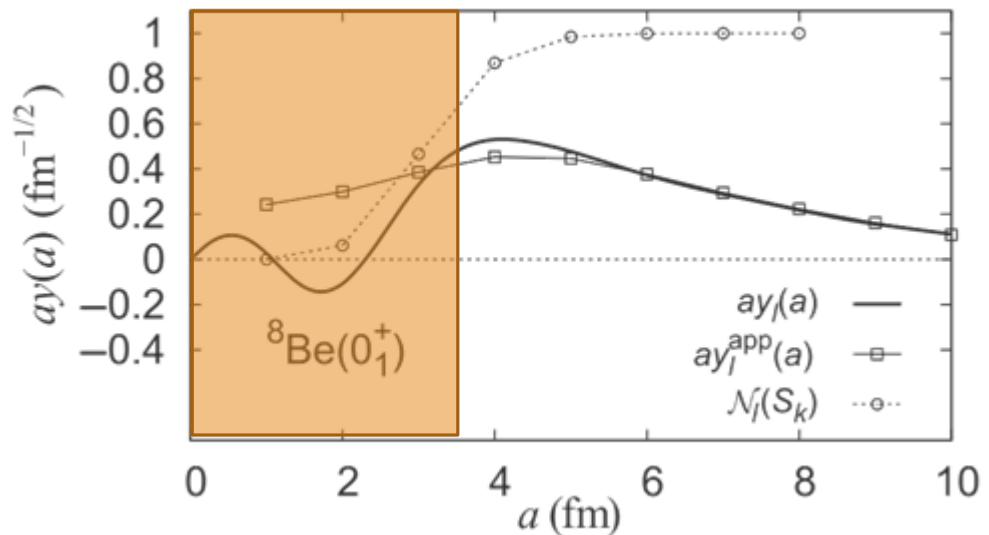
- The *reduced width amplitude (RWA)* of  $\alpha$ -cluster

$$y_l(a) \equiv \frac{1}{M} \left\langle \frac{\delta(r - a)}{r^2} Y_{00}(\hat{r}) \phi_\alpha \phi_A \phi_{c.m.} | \Phi \right\rangle$$

- is approximated by [1]

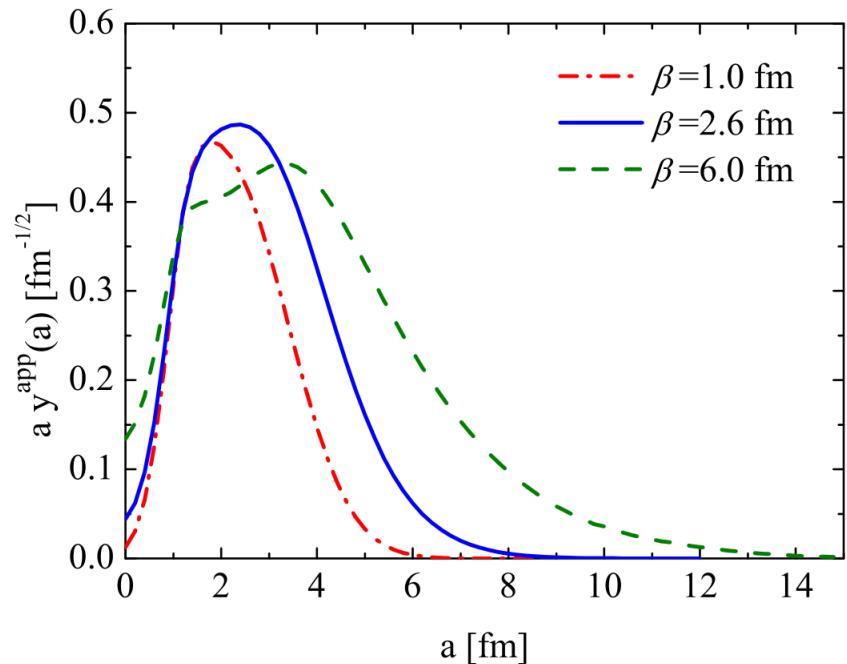
$$|ay(a)| \approx ay^{\text{app}}(a) = \frac{1}{\sqrt{2}} \left( \frac{n_B n_\alpha}{n_A \pi b^2} \right)^{\frac{1}{4}} \left\langle \Phi_A | \Phi_{\text{BB}}^{(0+)} \left( \Phi_B^{(0+)}, \alpha; S = a \right) \right\rangle$$

- $\Phi_A$ : THSR w. f. of target A
- $\Phi_{\text{BB}}^{(0+)}(\Phi_B, \alpha; S = a)$ : Brink-Bloch-type w. f. of residual B and  $\alpha$ -cluster
- $\Phi_B^{(0+)}$ : THSR w. f. of residual B



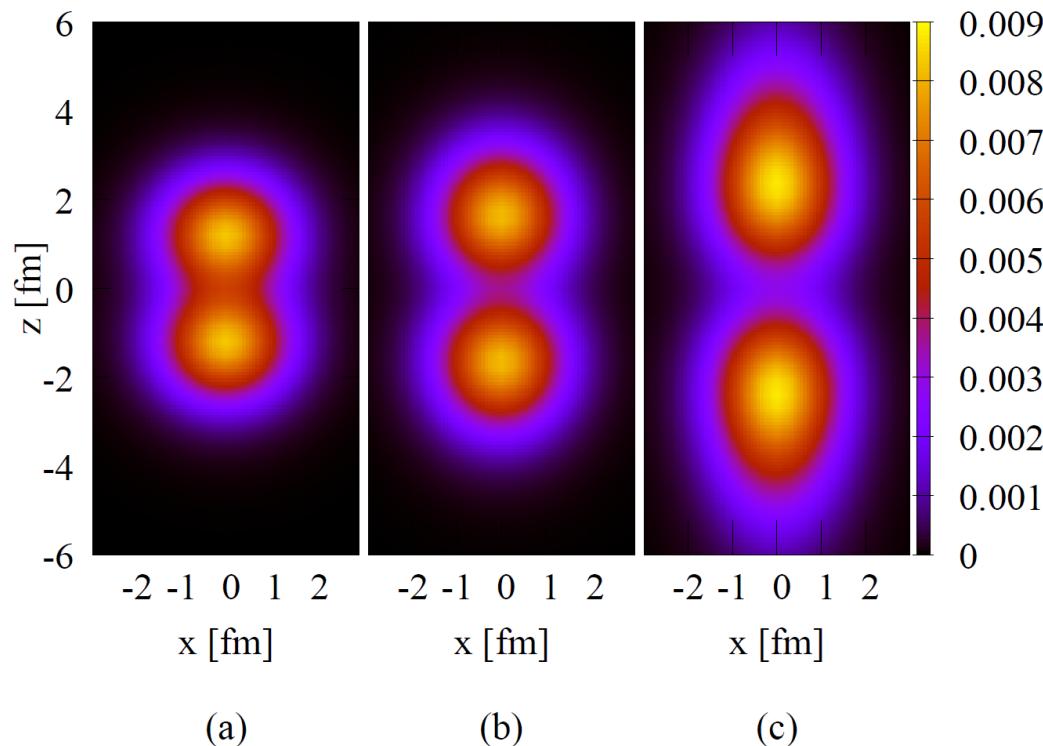
# $\alpha$ -cluster RWA of $^{10}\text{Be}$

- *physical  $^{10}\text{Be}$  nucleus*
  - $\beta_\alpha=2.6 \text{ fm}$  (optimized)  
(molecular like)
  - $E=-61.4 \text{ MeV}$
  - $R_c=2.31 \text{ fm}$  (Exp: 2.35 fm)
- *artificial  $^{10}\text{Be}$  nucleus*
  - $\beta_\alpha=1.0 \text{ fm}$   
(shell-model limit)
  - $\beta_\alpha=6.0 \text{ fm}$   
(gas like)



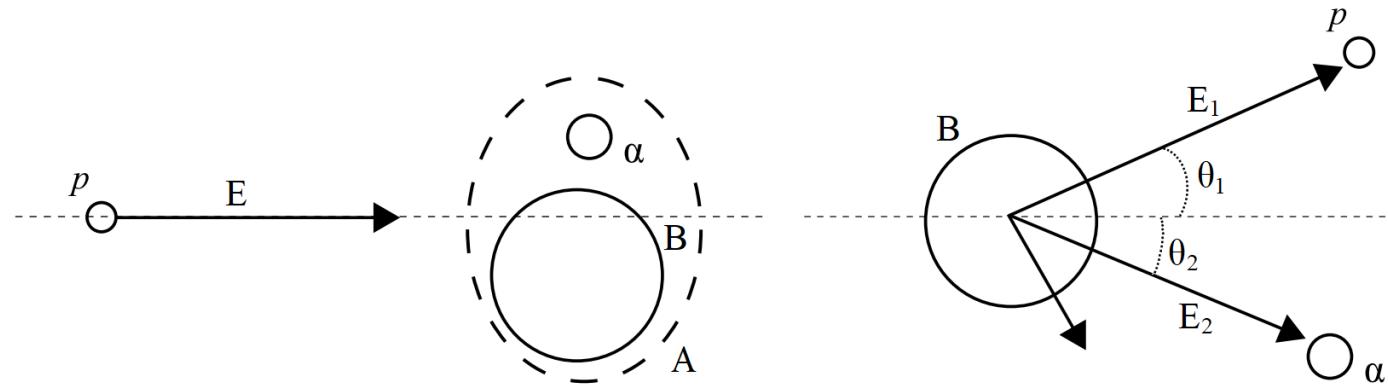
**Figure:** Reduced width amplitude of  $^{10}\text{Be}$

# Density distribution of $^{10}\text{Be}$



- (a):  $\beta_\alpha = 1.0$  fm (*shell-model limit*)
- (b):  $\beta_\alpha = 2.6$  fm (*physical, molecular-like*)
- (c):  $\beta_\alpha = 6.0$  fm (*gas like*)

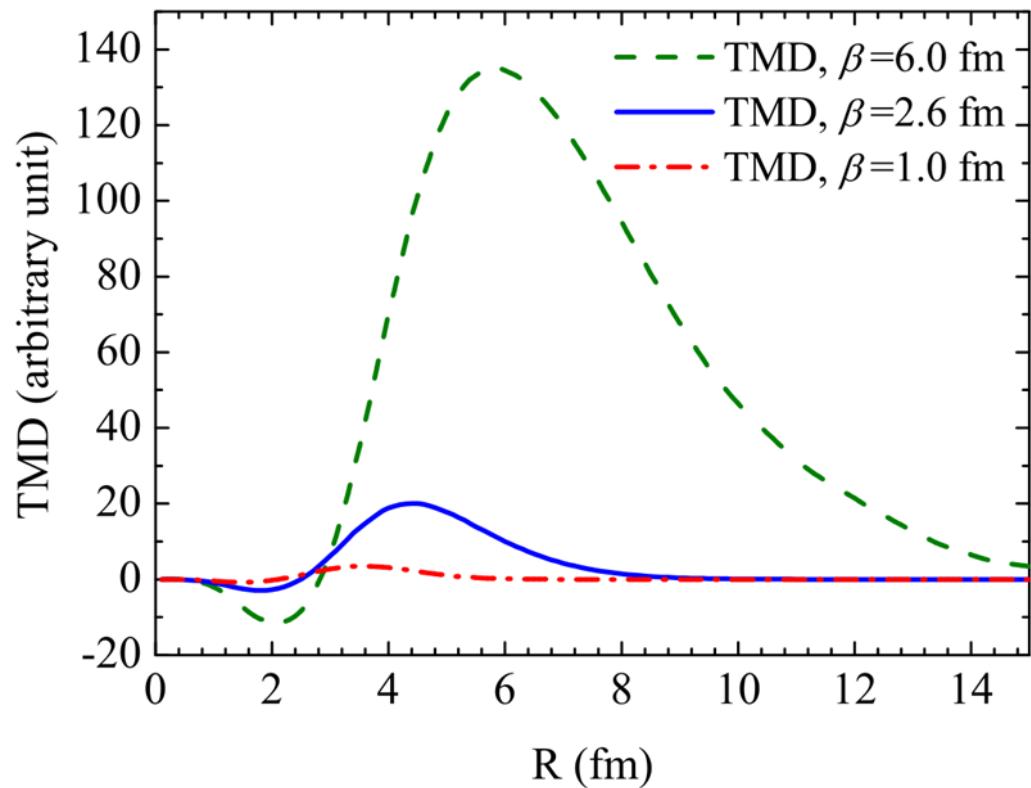
# Kinematics for $\alpha$ -knockout reaction of $^{10,12}\text{Be}$



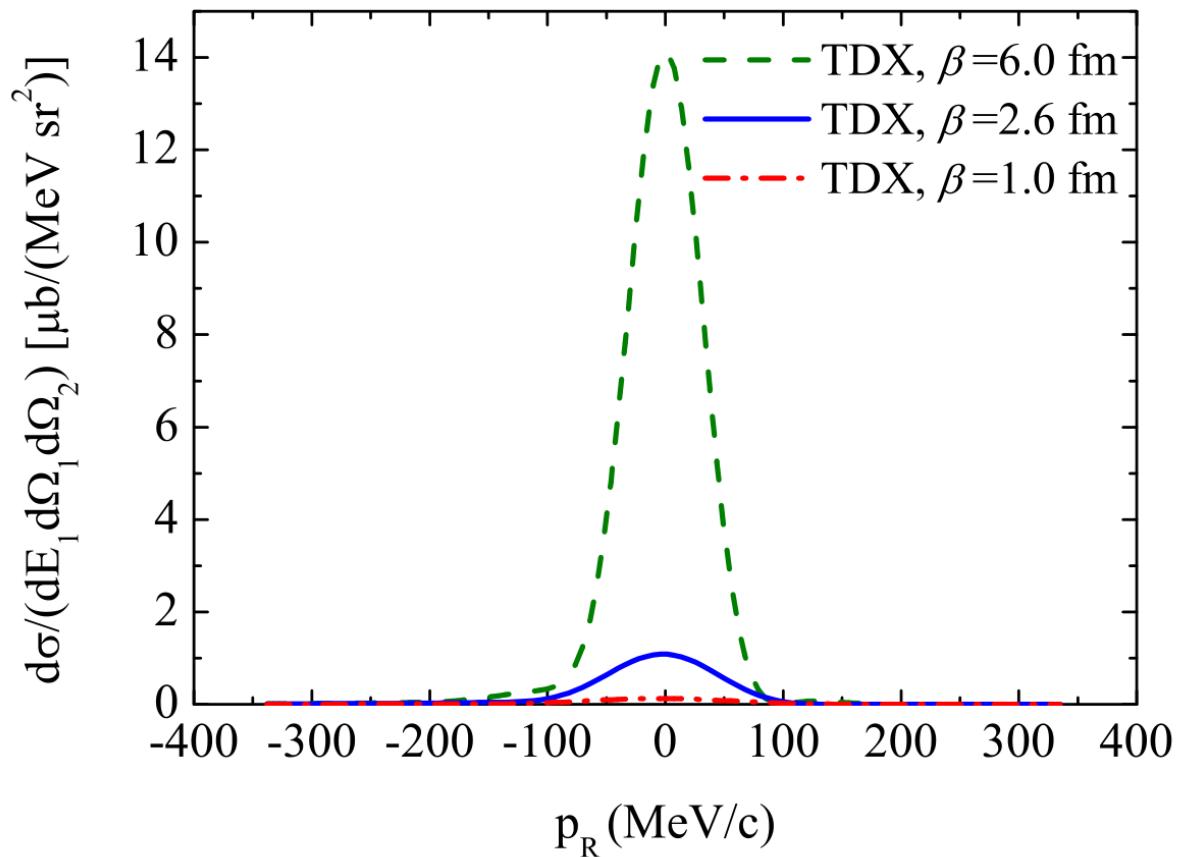
- *Incident proton*
  - $E = 250 \text{ MeV}$
- *Outgoing proton*
  - $E_1 = 180 \text{ MeV}$
  - $(\theta_1, \phi_1) = (60.9^\circ, 0^\circ)$
- *Outgoing  $\alpha$* 
  - $\theta_2$ : varied around  $51^\circ$
  - $\phi_2$ :  $180^\circ$

# Transition matrix density (TMD)

- **TMD**: Transition strength as a function of R
- TMD is defined as  $\delta(R)$  where
$$\int dR \delta(R) \propto \frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2}$$
- mostly contributed by the surface region



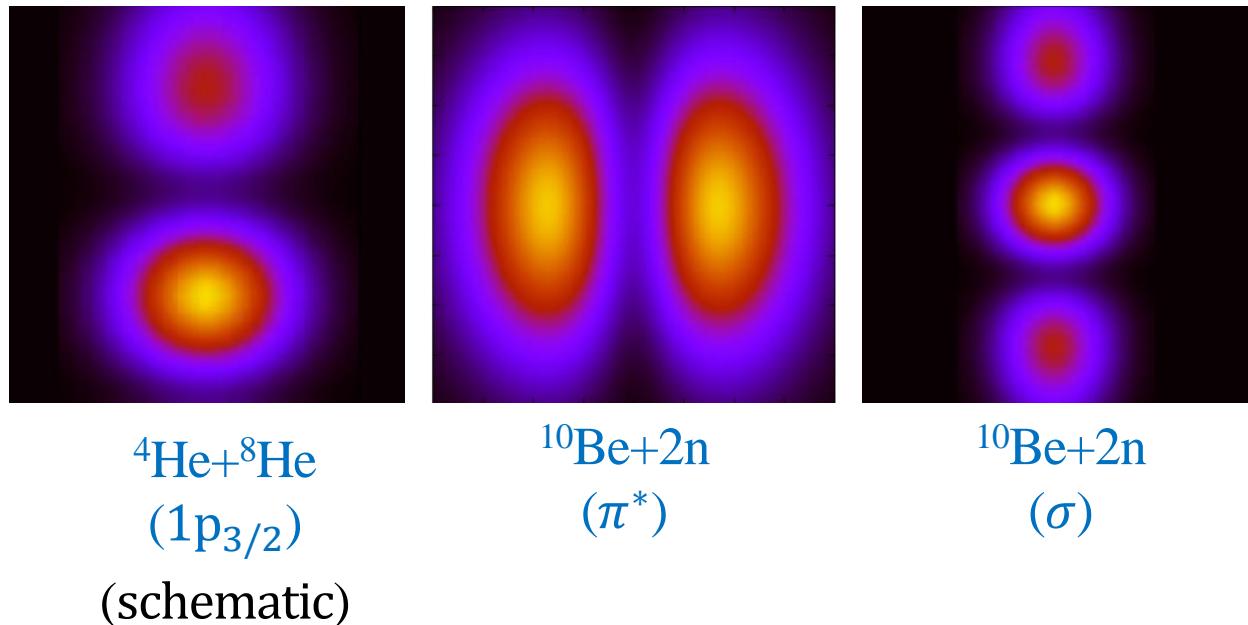
# TDX for the $^{10}\text{Be}(\text{p},\text{pa})^6\text{He}$ reaction



$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} = F_{kin} C_0 \frac{d\sigma_{p\alpha}}{d\Omega_{p\alpha}} (\theta_{p\alpha}, E_{p\alpha}) \left| \bar{T}_{K_0 K_1 K_2} \right|^2$$

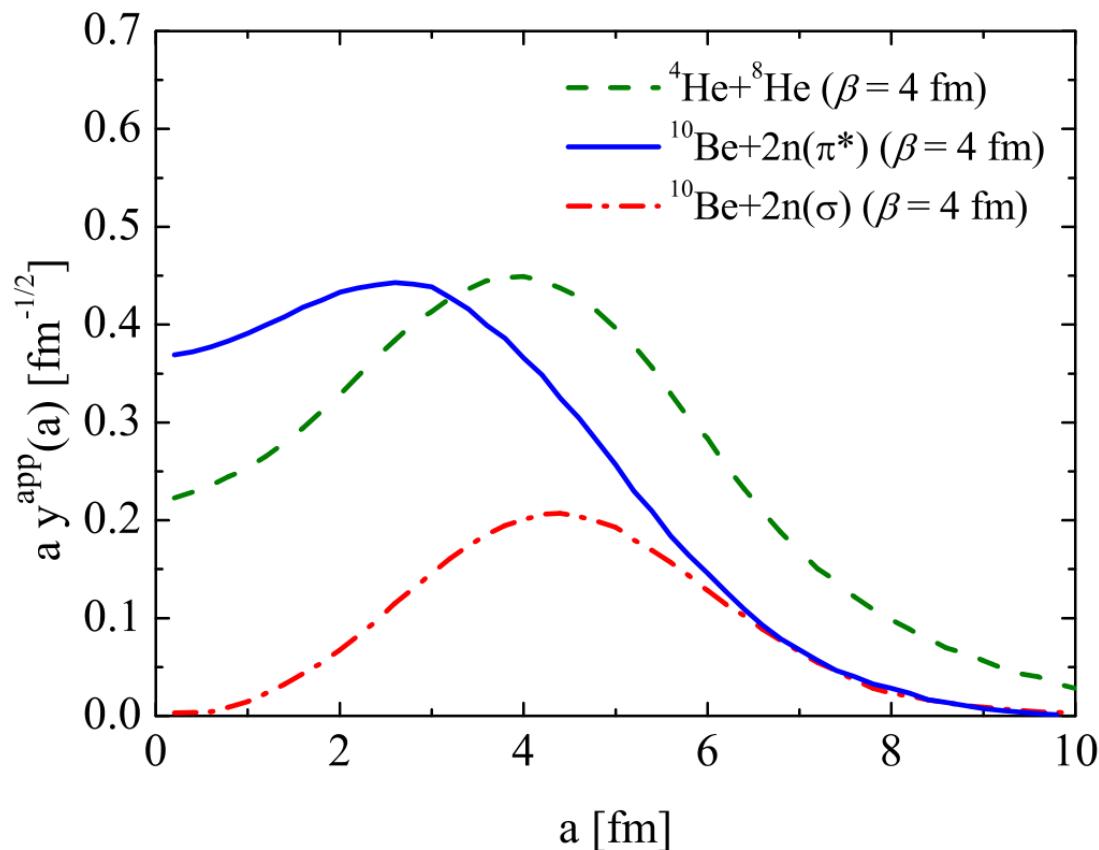
# Coupling of clustering configurations in $^{12}\text{Be}$

- Coupling of  $^4\text{He}+^8\text{He}$ ,  $^{10}\text{Be}+2n(\pi^*)$  and  $^{10}\text{Be}+2n(\sigma)$  configurations
- Breaking of neutron magic number  $N=8$
- *Probing* of  $\alpha$ -cluster and clustering configurations in  $^{12}\text{Be}$

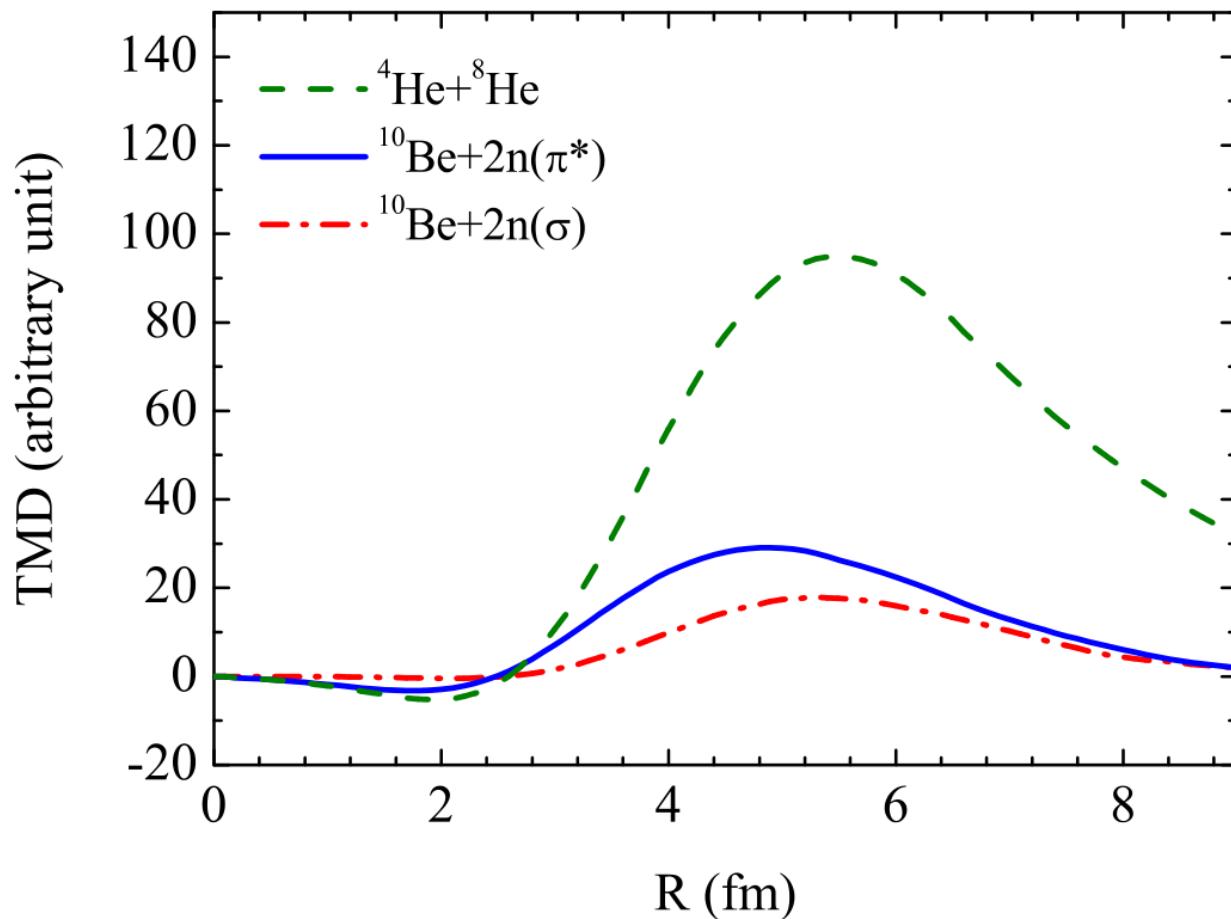


# $\alpha$ -cluster RWA of $^{12}\text{Be}$ in three configurations

- $^4\text{He}+^8\text{He}$  configuration
  - $\beta_\alpha=4.0$  fm (optimized)
- $^{10}\text{Be}+2n(\pi^*)$  configuration
  - $\beta_\alpha=4.0$  fm (optimized)
- $^{10}\text{Be}+2n(\sigma)$  configuration
  - $\beta_\alpha=4.0$  fm  
(adjusted, preliminary)

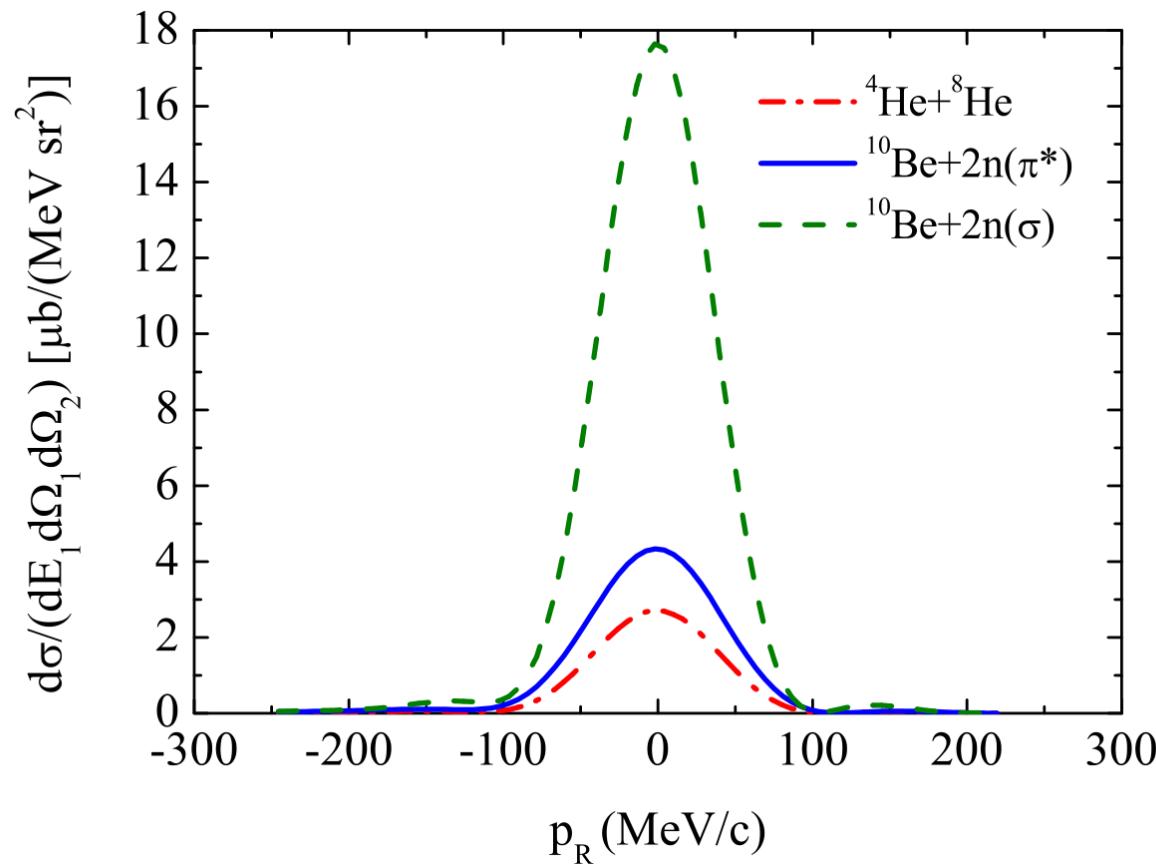


# Transition matrix density (TMD)



$$\int dR \delta(R) \propto \frac{d^3 \sigma}{dE_1 d\Omega_1 d\Omega_2}$$

# TDX for the $^{12}\text{Be}(\text{p},\text{pa})^{8}\text{He}$ reaction



$$\frac{d^3\sigma}{dE_1 d\Omega_1 d\Omega_2} = F_{kin} C_0 \frac{d\sigma_{p\alpha}}{d\Omega_{p\alpha}} (\theta_{p\alpha}, E_{p\alpha}) \left| \bar{T}_{K_0 K_1 K_2} \right|^2$$

# Summary

- Proposed **fully microscopic** framework for  $\alpha$ -knockout reaction by integrating microscopic clustering model into DWIA framework
- Approximation of **RWA** extracted from the THSR description for  $^{10}\text{Be}$  and three configurations of  $^{12}\text{Be}$
- Observables (**TDX**) predicted for future experiment
- For  $^{10}\text{Be}$ , results compared for the shell-model limit, molecular-like and gas-like states.
- For  $^{12}\text{Be}$ , results compared for  $^4\text{He}+^8\text{He}$ ,  $^{10}\text{Be}+2\text{n}(\pi^*)$  and  $^{10}\text{Be}+2\text{n}(\sigma)$  configurations.
- TDX found to be **highly sensitive** to the **extent of clustering**, and also to the **cluster/molecular configurations**.

*Many thanks for all collaborators  
and  
thank you very much for your attention!*