Corrections to the eikonal description of elastic scattering and breakup of halo nuclei

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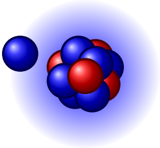
- **Halo nuclei** exhibit a very large matter radius
  Compact core + one or two loosely-bound neutrons
  Ex : $^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be}+\mathrm{n}$, $^{15}\mathrm{C} \equiv {}^{14}\mathrm{C}+\mathrm{n}$



  Short-lived : studied through **reactions processes** (**elastic scattering**, **breakup**,...)

  $\Rightarrow$ **Need an accurate reaction model to infer reliable information**

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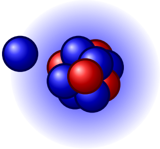
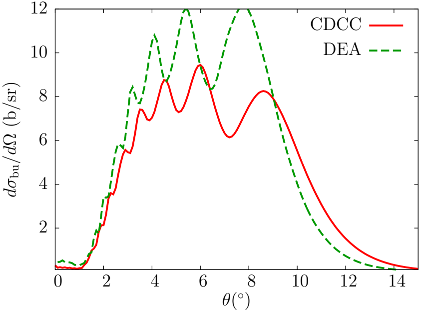


  Short-lived : studied through **reactions processes**
  (**elastic scattering**, **breakup**,...)
  ⇒ **Need an accurate reaction model to infer reliable information**
- **The eikonal approximation :**
  ⊕ reduced computational time
  ⊕ simple interpretation of the reaction
- Some experimental facilities will provide RIBs at ~ 10A MeV
  (e.g. HIE-ISOLDE @ CERN and ReA12 @ MSU)

⇒**Is it valid at these energies ?**

- **Coulomb dominated reactions :**

$^{15}\mathrm{C} + {}^{208}\mathrm{Pb} \rightarrow {}^{14}\mathrm{C} + \mathrm{n} + {}^{208}\mathrm{Pb}$ @ $20A$ MeV



[T. Fukui, K. Ogata and P. Capel. PRC **90**, 034617 (2014)]

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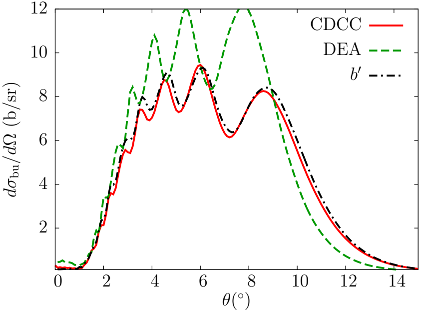

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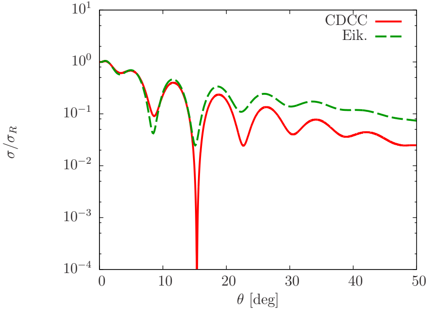

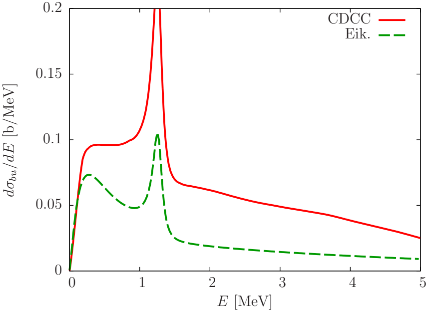

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→ Eikonal dampens the oscillations

Introduction

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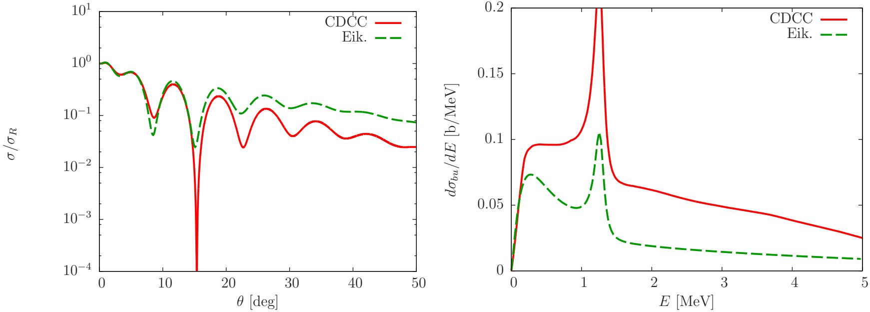

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$\Rightarrow$ **Can a nuclear correction fix these issues ?**

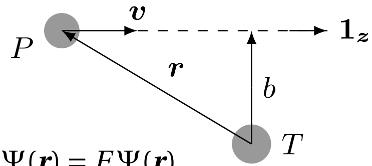
Outline

1. Eikonal model
2. Semi-classical correction
3. Exact continued S-matrix correction
4. Summary

Eikonal model

Assumptions :

- Spinless and structureless particles
- Central potentials



Schrödinger equation : $\left[-\frac{\hbar^2}{2\mu}\Delta_{\boldsymbol{r}} + V(r)\right]\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r})$

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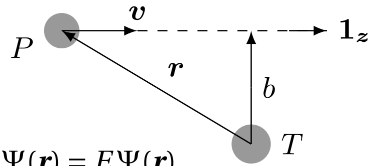

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Eikonal approximation : [R. J. Glauber, *High energy collision theory*, (1959)]

- At high energy, $\Psi \approx$ plane wave

  Factorization : $\Psi(\boldsymbol{r}) = e^{ikz}\widehat{\Psi}(\boldsymbol{r})$ with $|\Delta_{\boldsymbol{r}}\widehat{\Psi}| \ll k\left|\frac{\partial}{\partial z}\widehat{\Psi}\right|$

  $\Rightarrow$ **Solutions** : $\boxed{\Psi(b,z) = e^{ikz}e^{i\chi_0(b,z)}}$

  with $\chi_0(b,z) = -\frac{1}{\hbar v}\int_{-\infty}^{z} V(b,z')dz'$

- **Easy** interpretation : P follows a **straight-line**
- **Fast** computations
- **Limited** to high energies

$\Rightarrow$ **Need to better account for the deflection**

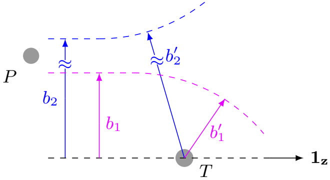
Idea : $\chi_0(b) \rightarrow \chi_0(b'')$, with b'' the **complex** distances of closest approach [Analysis of two-body collisions in A. Vitturi *et al.* , PRC **56**, 1511, (1997).]

① Real part of the distances b' :

trajectories at b_1 is nuclear dominated

at b_2 is Coulomb dominated

$\rightarrow$ b' computed exactly



[CH, P. Capel, *Proc. of the 55th International Winter Meeting on Nuclear Physics*, (2017).]

Semi-classical correction

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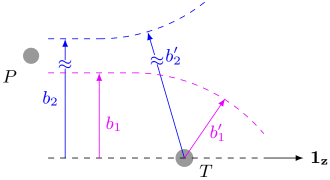

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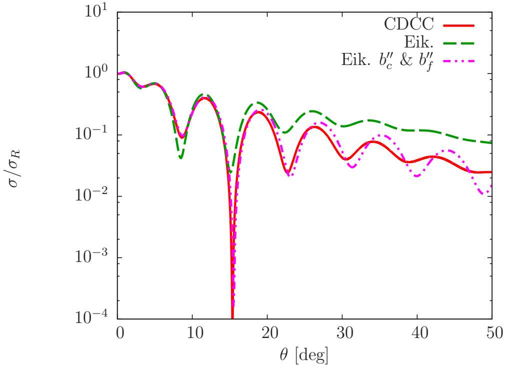
② Complex distances b'' :

$\rightarrow$ b'' approximated by a **perturbation formula**

$$b'' = b' - i \frac{\mathrm{Im}\{V(b')\}}{2\, E_0 \frac{b^2}{b'^3} - \left[\frac{\partial}{\partial r}\left(\mathrm{Re}\{V\}\right)\right]_{r=b'}}$$

[D. M. Brink, *Semi-classical methods in nucleus-nucleus scattering*, (1985).]

Elastic scattering $^{11}\mathrm{Be}+^{12}\mathrm{C}$ 10A MeV



[CH, P. Capel, PRC **96**, 054607, (2017).]

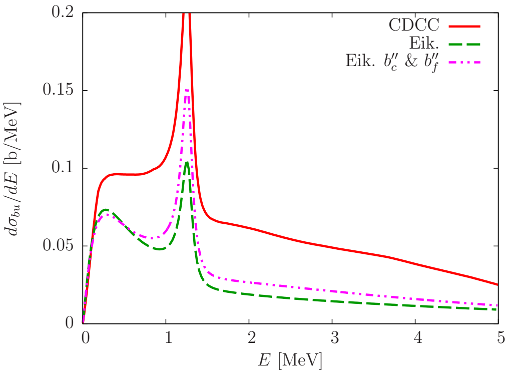
⊖ Overcorrection of the oscillations at large angles

⊕ More absorption at large angles

→ **Very accurate**

What about breakup observables ?

Breakup of ^{11}Be+^{12}C @ 10A MeV



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⊖ Underestimation of the magnitude

⇒ **No significant accuracy gain**

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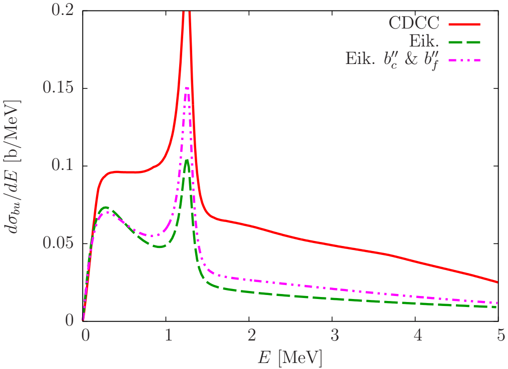

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Use of another correction ?

Exact continued S-matrix

Developed for elastic scattering of 1-body projectile

[S. J. Wallace, PRD **8**, 1846, (1973).]

Partial-wave expansion : $F(\theta) = \frac{1}{2iK}\sum_{l=0}^{+\infty}(2l+1)P_l(\cos\theta)[S_l - 1]$

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with $\mathbf{S}_0^{FB}(b) = \frac{1}{b}\sum_{k=0}^{\infty}\frac{1}{(2k)!}b_k\left(-\frac{b}{2}\frac{\mathrm{d}}{\mathrm{d}b}\right)\left(\frac{1}{K}\frac{\mathrm{d}}{\mathrm{d}b}\right)^{2k} be^{2i\delta(b)}$

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Extension to 3-body collisions [J. M. Brooke *et al.* , PRC **59**, 1560, (1999).]

$$\chi_{cT}(b_{cT}) \rightarrow 2\delta_{l_{cT}(b_{cT})} \text{ and } \chi_{fT}(b_{fT}) \rightarrow 2\delta_{l_{fT}(b_{fT})}$$

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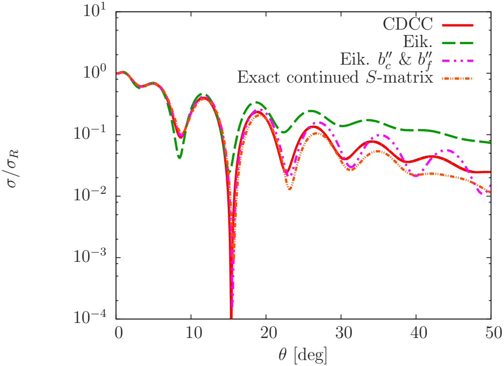

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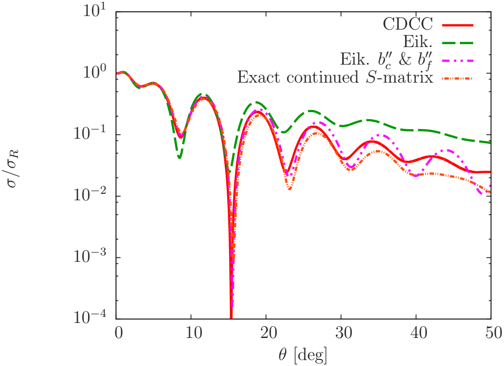

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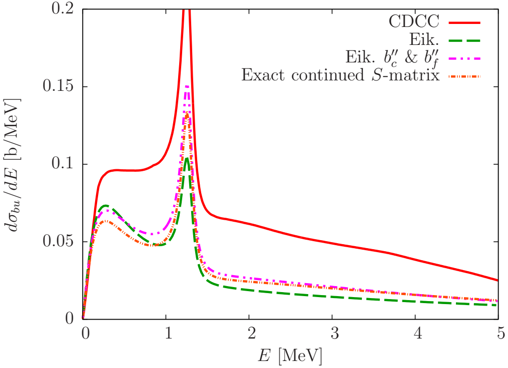

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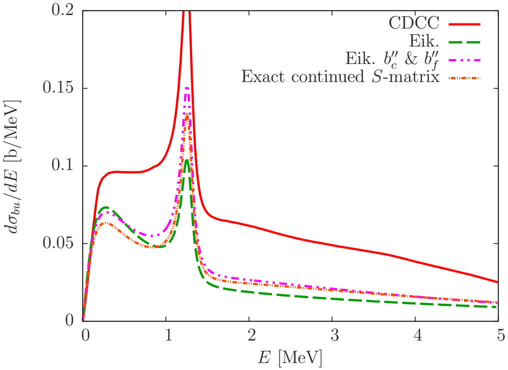

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⇒ **Improve the L-couplings within eikonal model ?**

Summary

Eikonal model : fast, easy but valid only at high energies

- Coulomb dominated reactions : use of b' shift
- Nuclear dominated reactions → **Can it be corrected ?**

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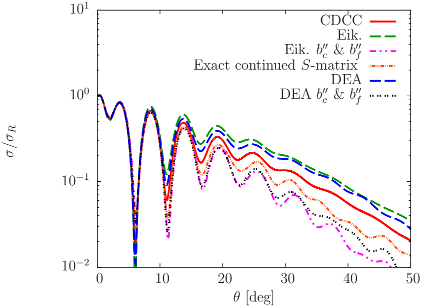
⇒ **Need to improve the couplings between the « trajectories » within the eikonal model**

Elastic scattering of ^{11}Be off ^{12}C

20A MeV



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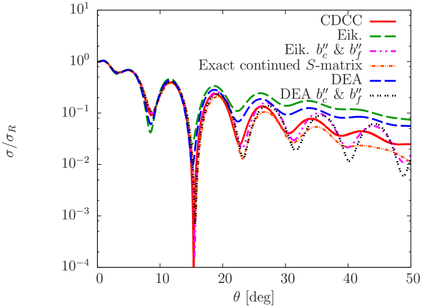
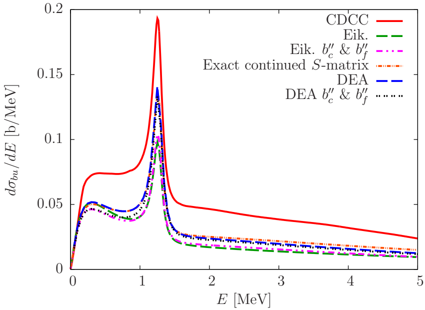


Breakup of ^{11}Be impinging on ^{12}C

20A MeV



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