Structure and reactions of N=7 isotones: the role of core degrees of freedom

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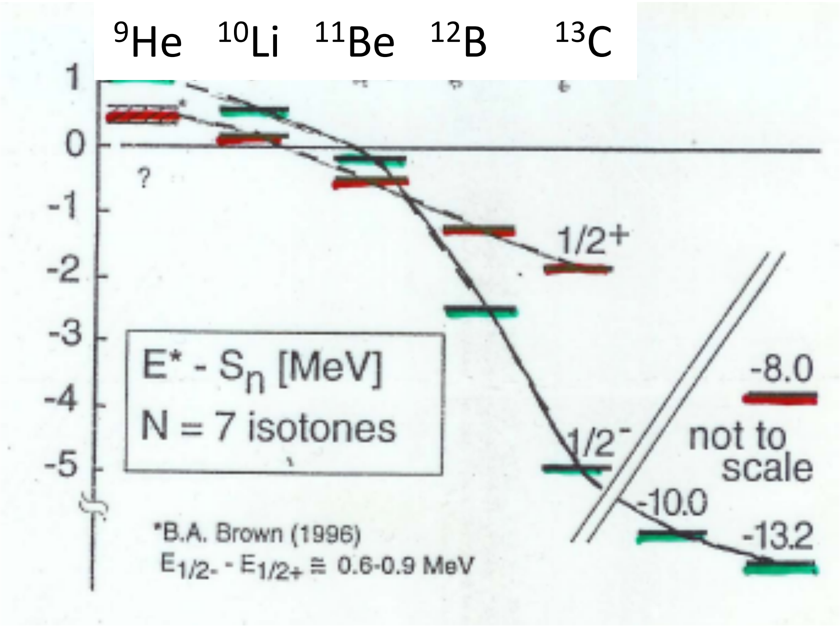
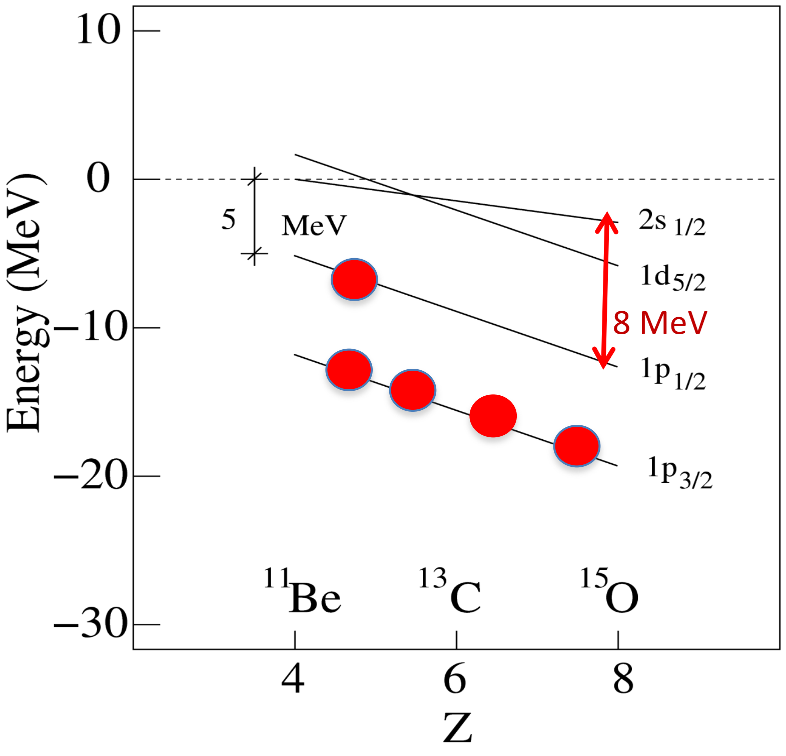
Parity inversion in N=7 isotones

Experimental systematics



Mean-field results favour ½⁻

(Sagawa,Brown,Esbensen PLB 309(93)1)



A possible explanation of parity inversion: dynamical coupling between the core and the loosely bound neutron

VOLUME 70, NUMBER 10 PHYSICAL REVIEW LETTERS 8 MARCH 1993

Structure of Exotic Neutron-Rich Nuclei

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(Received 13 November 1992)

A new framework, the variational shell model, is proposed to describe the structure of neutron-rich unstable nuclei. An application to ^{11}Be is presented. Contrary to the failure of the spherical Hartree-Fock model, the anomalous $\frac{1}{2}^+$ ground state and its neutron halo are reproduced with the Skyrme (SIII) interaction. This state is bound due to dynamical coupling between the core and the loosely bound neutron, which oscillates between the $2s_{1/2}$ and the $1d_{5/2}$ orbits.

N. Vinh Mau, Nucl. Phys. A 592 (1995) 43
G.F. Esbensen and H. Sagawa, Phys. Rev C 51 (1995)1274
F.M. Nunes and I. Thompson, Nucl. Phys. A 703 (2002) 593
G. Blanchon et al., Phys Rev. C 82 (2010) 034313
Myo et al, PRC 86 (2012) 024318

Admixture of $d_{5/2}$ x 2^+ configuration in the $1/2^+$ g.s. of ^{11}Be is about 20%

Calculated ground state

$$|1/2+\rangle = \sqrt{0.87}|s_{1/2}\rangle + \sqrt{0.13}|d_{5/2} \otimes 2+\rangle$$

Exp.:

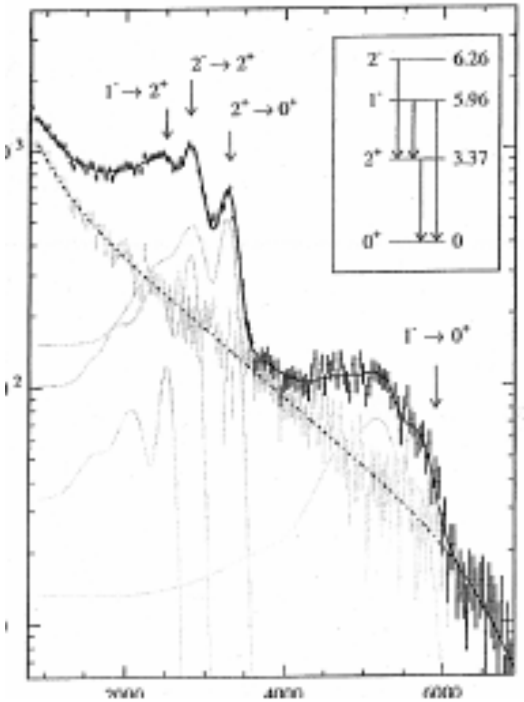
J.S. Winfield et al., Nucl.Phys. **A683** (2001) 48

$$|1/2+\rangle = \sqrt{0.84}|s_{1/2}\rangle + \sqrt{0.16}|d_{5/2} \otimes 2+\rangle$$

^{11}Be(p,d)^{10}Be in inverse kinematic detecting both the ground state and the 2+ excited state of ^{10}Be.

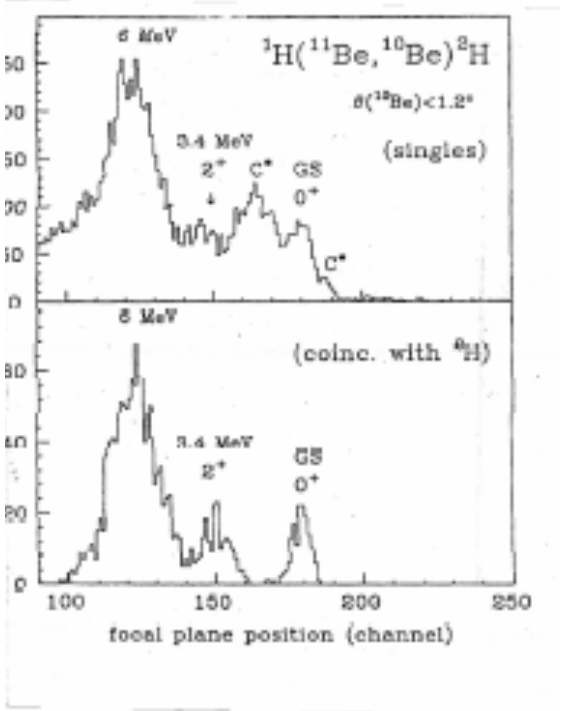
9Be(11Be,10Be+ γ) X

T. Aumann et al.
PRL 84(2000)35



p(11Be,10Be)d

S. Fortier et al.
Phys. Lett.B461(1999)22



PVC: Vibrational Core (even-even) + One particle (neutron)

$$H = H_{coll} + H_{sp} + H_{PVC}$$

$$H_{coll} = \sum_{\lambda\mu\nu} \hbar\,\omega_{\lambda,\nu}\left[\Gamma^{+}_{\lambda\mu,\nu}\Gamma_{\lambda\mu,\nu} + 1/2\right]$$

$$H_{sp} = -\hbar^2/2\mu\, d^2/d\vec{r}^{\,2} + V(r) + V_{ls}(r)$$

$$H_{PVC} = \sum_{\lambda\mu\nu} \delta V_{\lambda\nu}(r) Y_{\lambda\mu}(\hat{r})\left[\Gamma^{+}_{\lambda\mu,\nu} + (-1)^{\mu}\Gamma_{\lambda\mu,\nu}\right]$$

$$\delta V_{\lambda\nu}(r) \approx -r dV/dr\, \beta_{\lambda,\nu}$$

$$H\Psi_a = \tilde{E}\Psi_a$$

$$\Psi_a = \left[\psi_a^x + \left[\psi_b^C \cdot \Gamma^{+}_{\lambda}\right]_{j_a} + \ldots\right]\Phi^A$$

$$\psi_a^x = (R_a^x(r)/r)\Theta_{j_a m_a}$$

$$\left[\psi_b^C \cdot \Gamma^{+}_{\lambda}\right]_{j_a} = \left(R_b^C(r)/r\right)\left[\Theta_{j_b} \cdot \Gamma^{+}_{\lambda}\right]_{j_a}$$

Standard Coupled Channel Equation

$$\begin{bmatrix} \left[-\hbar^2/2\mu\, d^2/dr^2 + V_a(r)\right] & \Xi_{a,b\lambda}(-\beta_\lambda\, r dV/dr) \\ \Xi_{a,b\lambda}(-\beta_\lambda\, r dV/dr) & \left[-\hbar^2/2\mu\, d^2/dr^2 + V_b(r) + \hbar\omega\right] \end{bmatrix} \begin{bmatrix} R_a^x(r) \\ R_b^C(r) \end{bmatrix} = \tilde{E} \begin{bmatrix} R_a^x(r) \\ R_b^C(r) \end{bmatrix}$$

$$with\ \ \Xi_{a,b\lambda} = \langle \Theta_{j_a m_a} \sum_{\mu} Y_{\lambda\mu}\left[\Gamma^{+}_{\lambda\mu} + (-1)^{\mu}\Gamma_{\lambda\mu}\right]\left[\Theta_{j_b} \cdot \Phi_\lambda\right]_{j_a m_a} \rangle \sim \langle j_b\,, 1/2\,; \lambda 0 \mid j_a, 1/2 \rangle$$

Energy dependent Self-energy

$$R_a^x(r)=\sum_i x_{ai}\,R_{ai}^{WS}(r) \qquad i\in non-occ.$$
$$R_b^C(r)=\sum_i C_{bi}\,R_{bi}^{WS}(r) \qquad i\in non-occ.$$

$$\begin{pmatrix} e_{a1} & 0\ldots & h_{a1,b1\lambda} & h_{a1,b2\lambda}\ldots \\ 0 & e_{a2}\ldots & h_{a2,b1\lambda} & h_{a2,b2\lambda}\ldots \\ h_{a1,b1\lambda} & h_{a2,b1\lambda}\ldots & e_{b1}+\hbar\omega & 0\ldots \\ h_{a1,b2\lambda} & h_{a2,b2\lambda}\ldots & 0 & e_{b2}+\hbar\omega\ldots \end{pmatrix}\begin{pmatrix} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2}\ldots \end{pmatrix}=\tilde{E}\begin{pmatrix} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2}\ldots \end{pmatrix}$$

Projecting onto the single-particle basis

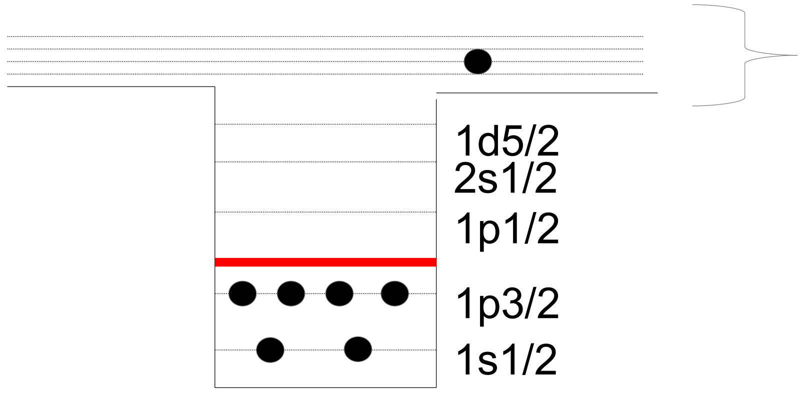
$$\begin{pmatrix} e_{a1}+\Sigma_{11}(\tilde{E}) & \Sigma_{12}(\tilde{E})\ldots \\ \Sigma_{21}(\tilde{E}) & e_{a2}+\Sigma_{22}(\tilde{E})\ldots \end{pmatrix}\begin{pmatrix} x_{a1} \\ x_{a2}\ldots \end{pmatrix}=\tilde{E}\begin{pmatrix} x_{a1} \\ x_{a2}\ldots \end{pmatrix}$$

SINGLE-PARTICLE HAMILTONIAN MATRIX (energy **dependent**), where

$$\Sigma_{ij}(\tilde{E})=\sum_{bk}\frac{h_{ai,bk,\lambda}\,h_{aj,bk,\lambda}}{\tilde{E}-(E_{bk}+\hbar\omega_\lambda)}$$

$$\boxed{h_{ai,bk\lambda}=\Xi_{a,b\lambda}\,\beta_\lambda\int R_{ai}^{WS}(r)(-r\,dV/dr)\,R_{bk}^{WS}(r)\,dr}$$

The simplest way to take care of the Pauli principle:
expand on a basis excluding ***occupied*** single-particle states



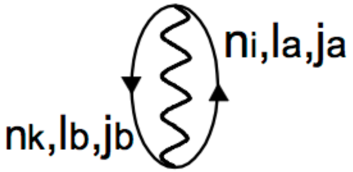
$$\begin{bmatrix} [-\hbar^2/2\mu\, d^2/dr^2 + V_a(r)] & \Xi_{a,b\lambda}(-\beta_\lambda\, r dV/dr) \\ \Xi_{a,b\lambda}(-\beta_\lambda\, r dV/dr) & [-\hbar^2/2\mu\, d^2/dr^2 + V_b(r) + \hbar\omega] \end{bmatrix} \begin{bmatrix} R_a^x(r) \\ R_b^C(r) \end{bmatrix} = \tilde{E} \begin{bmatrix} R_a^x(r) \\ R_b^C(r) \end{bmatrix}$$

$$with\ \ \Xi_{a,b\lambda} = \langle \Theta_{j_a m_a} \sum_\mu Y_{\lambda\mu} [\Gamma^{+}_{\lambda\mu} + (-1)^\mu \Gamma_{\lambda\mu}][\Theta_{j_b} \cdot \Phi_\lambda]_{j_a m_a} \rangle \sim \langle j_b, 1/2; \lambda 0 \,|\, j_a, 1/2 \rangle$$

$$R_a^x(r) = \sum_{i=1}^{N} x_{ai} R_{ai}^{WS}(r) \qquad i \in non-occ \qquad\qquad R_b^C(r) = \sum_{i=1}^{N} C_{bi} R_{bi}^{WS}(r) \qquad i \in non-occ$$

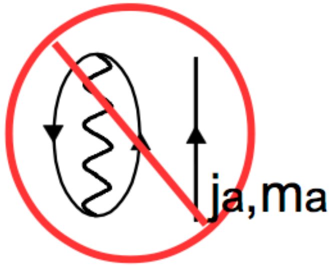
Matrix elements due to GSC Pauli rearrangement

The contribution of a given p-h configuration to the GS Correlation Energy is (*B&MII*)



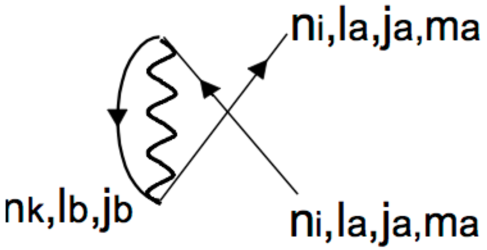
$$\delta E = \frac{\left(-h_{ai,bk,\lambda}\sqrt{(2j_a+1)}\right)^2}{0-(E_{ai}+E_{bk}+\hbar\omega_\lambda)} < 0$$

The presence of a new neutron (scattering- or bound-like) inhibits some of these correlations, producing an energy modification of the core state...



$$-\frac{\delta E}{2j_a+1} > 0$$

This is the meaning/value of the NFT self energy diagram (*B&MII, eq.6.225*)



$$(-1)\frac{\left(-h_{ai,bk,\lambda}\sqrt{2j_a+1}\right)^2 \langle ((j_{a1},j_{a2})J=0, j_{a3}; j_a | (j_{a1},j_{a3})J=0, j_{a2}; j_a)\rangle}{E_{ai}-(2E_{ai}+E_{bk}+\hbar\omega_\lambda)}$$

$$.=\frac{(h_{ai,bk,\lambda})^2}{(E_{ai}+E_{bk}+\hbar\omega_\lambda)}$$

Include the effect of ground state correlations

$$\Psi_a = \left[\psi_a^x + \left[\psi_b^C \cdot \Gamma_\lambda^{+\cdot}\right]_a\right]\Phi_{GS}$$

We add new terms, to take into account ground state correlations

$$\Psi_a = \left[\psi_a^x + \left[\psi_b^C \cdot \Gamma_\lambda^{+\cdot}\right]_{j_a} - \psi_a^y - \left[\psi_c^D \cdot \Gamma_\lambda\ \right]_{j_a} + \ldots\right]\Phi_{GS}$$

$$u_a^x(r) = \sum_i x_{ai}\, R_{ai}^{WS}(r);\, e_{ai} > e_F$$

$$u_b^C(r) = \sum_i C_{bi}\, R_{bi}^{WS}(r);\, e_{bi} > e_F$$

$$v_a^y(r) = \sum_i y_{ai}\, R_{ai}^{WS}(r);\, e_{ai} < e_F$$

$$v_c^D(r) = \sum_i D_{ci}\, R_{ci}^{WS}(r);\, e_{ci} < e_F$$

$$\begin{pmatrix} H_p - e_F & \Xi_{a,b\lambda} f(r) & 0 & \Xi_{a,c\lambda} f(r) \\ \Xi_{a,b\lambda} f(r) & H_p - e_F + \hbar\omega & \Xi_{a,b\lambda} f(r) & 0 \\ 0 & \Xi_{a,b\lambda} f(r) & (H_p - e_F) & -\Xi_{a,c\lambda} f(r) \\ \Xi_{a,c\lambda} f(r) & 0 & -\Xi_{a,c\lambda} f(r) & (H_p - e_F) - \hbar\omega \end{pmatrix} \begin{pmatrix} u_a^x \\ u_b^C \\ -v_a^y \\ -v_c^D \end{pmatrix} = \tilde{E} \begin{pmatrix} u_a^x \\ u_b^C \\ -v_a^y \\ -v_c^D \end{pmatrix}$$

Ingredients of our calculation for ^{11}Be

F. Barranco et al, PRL 119 (2017) 082501

Fermionic degrees of freedom:

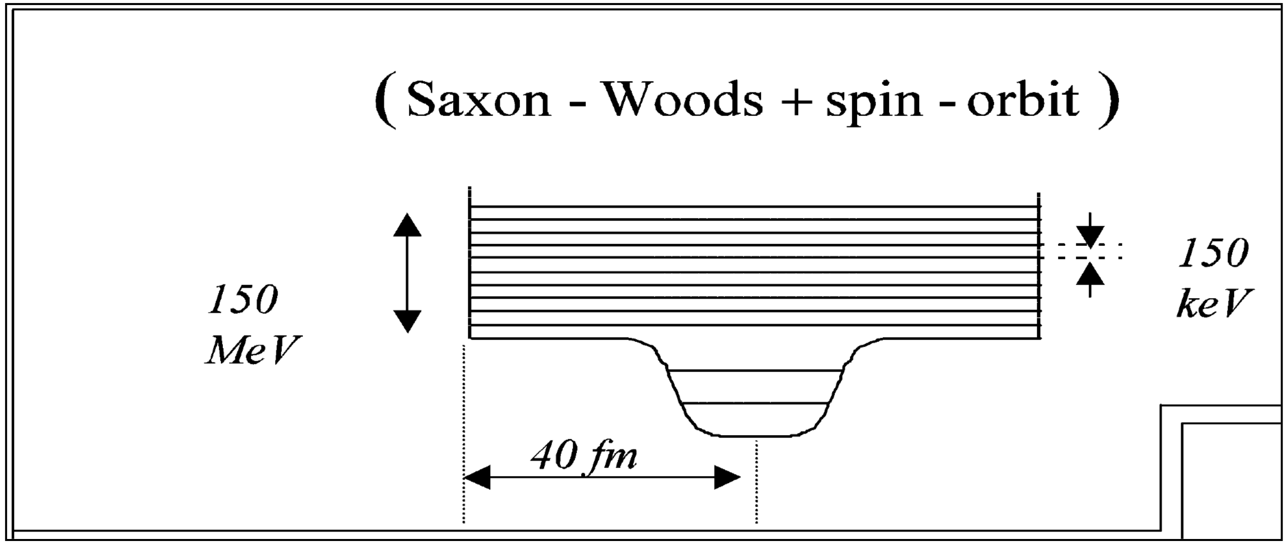
- $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{5/2}$ Wood-Saxon levels in a box

•**Bosonic degrees of freedom**:

- 2+ (3-, pair vib.) QRPA solutions tuned to reproduce available exp. data:

•B(E2) = 10.4 ±1.2 e^2 fm^4

• $\beta_{em} = 1.12$ $\beta_n \approx 0.8$



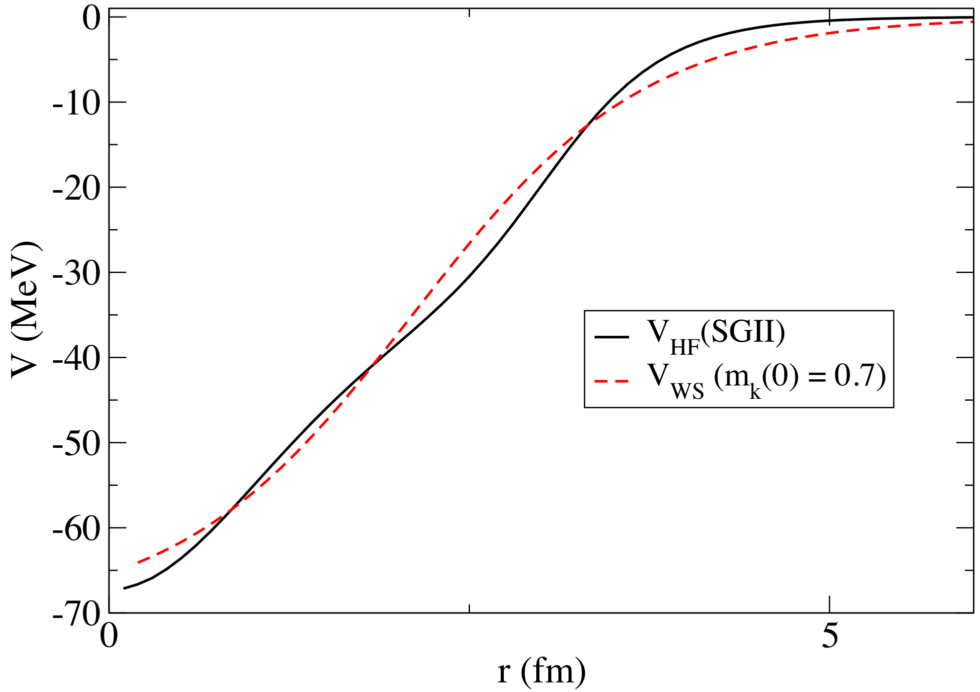
Parameter optimization

We perform the many-body calculation starting from a Woods-Saxon potential,
with a spatially dependent effective mass, with

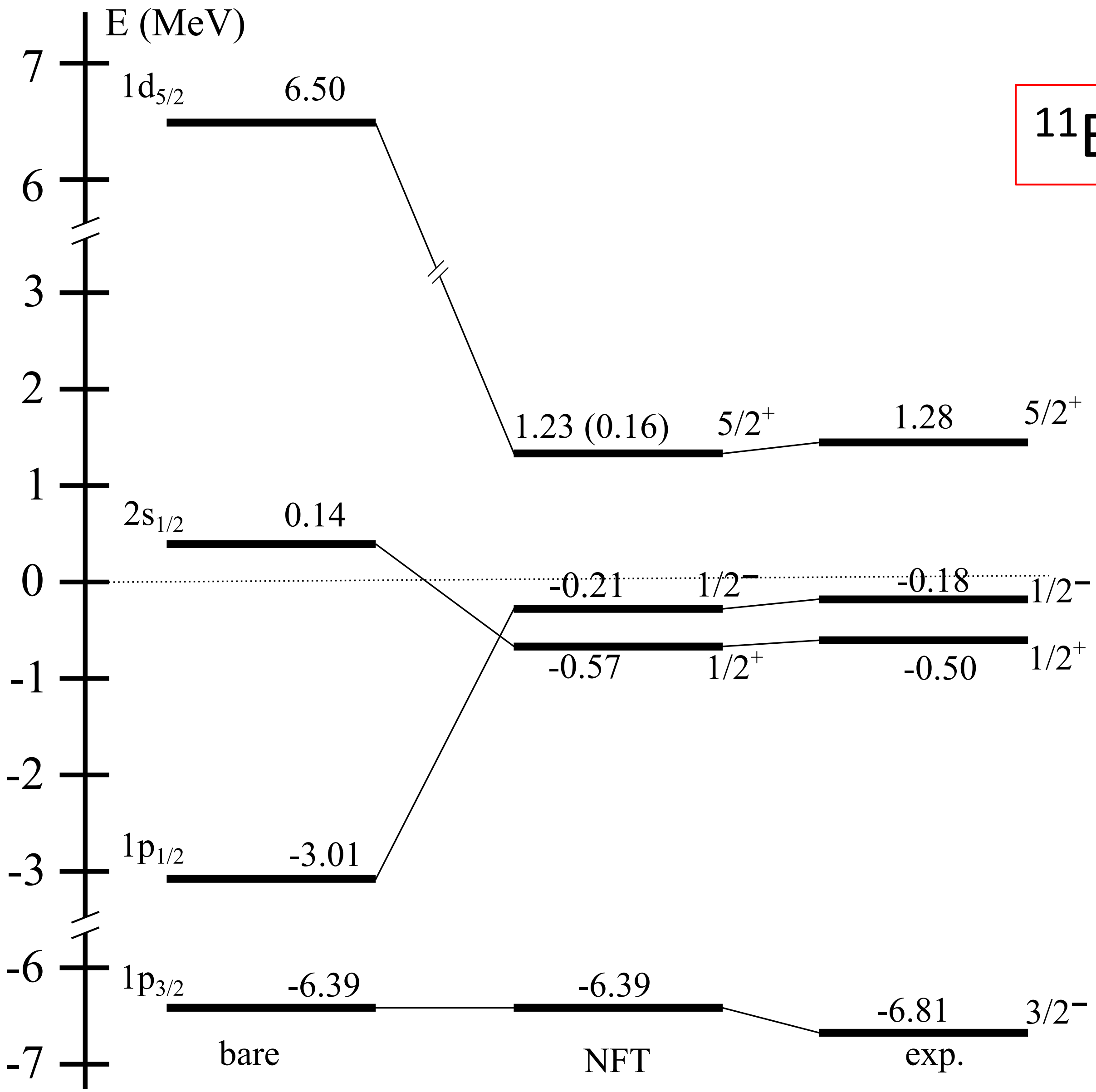
$m_k(r=0) = 0.7$ m , m_k(r >> R) = **μ** =0.91 m

The following parameters are fitted to obtain the best agreement of the renormalized energies with the experimental $1/2^+$,$1/2^-$ and $5/2^+$ states in ^{11}Be and $3/2^-$ in ^{9}Be:

- Depth, diffuseness,radius, strength of spin-orbit coupling



^{11}Be

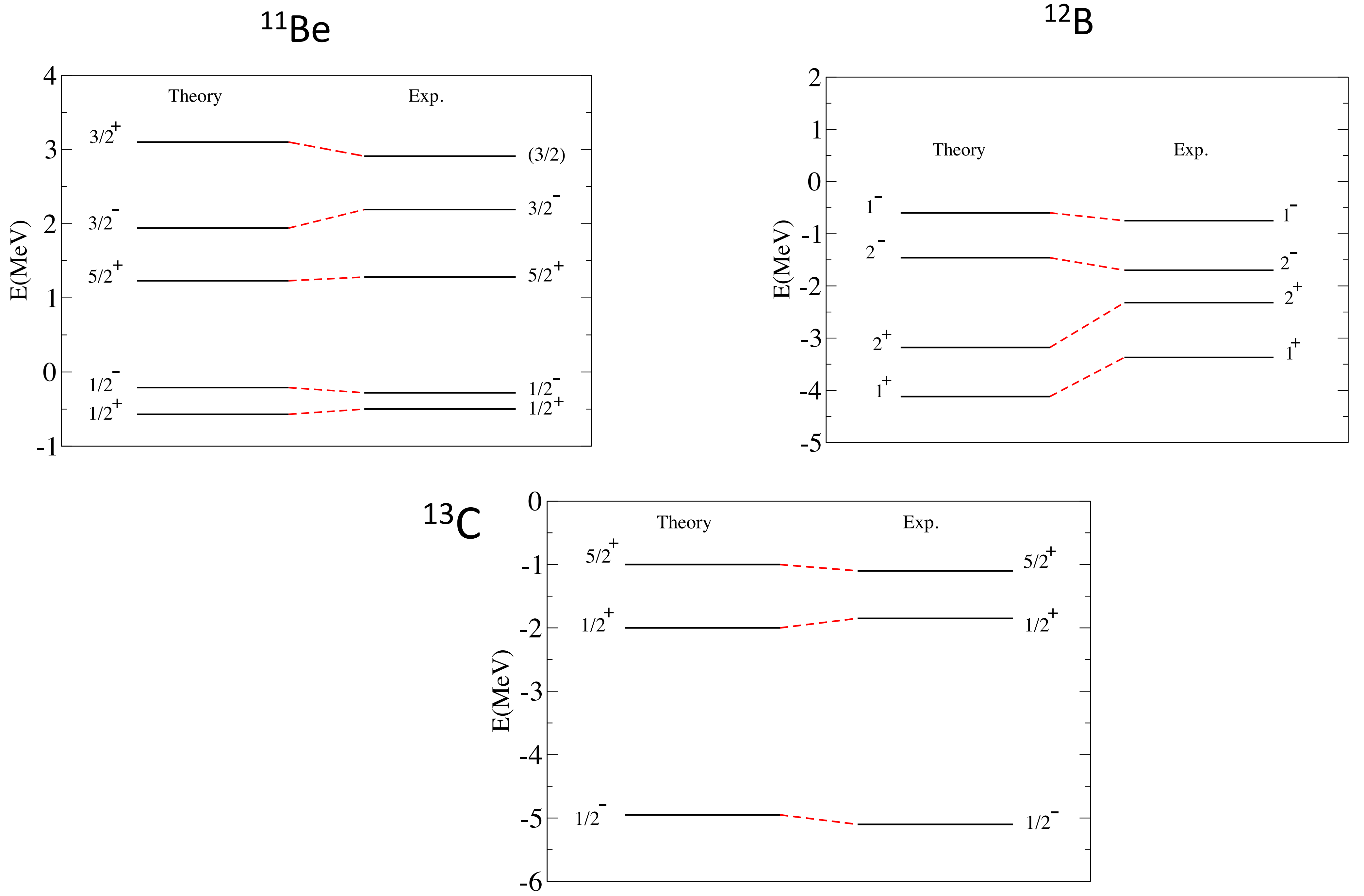


Bare

Renorm.

Exp.

F. Barranco et al., PRL 119 (2017) 082501

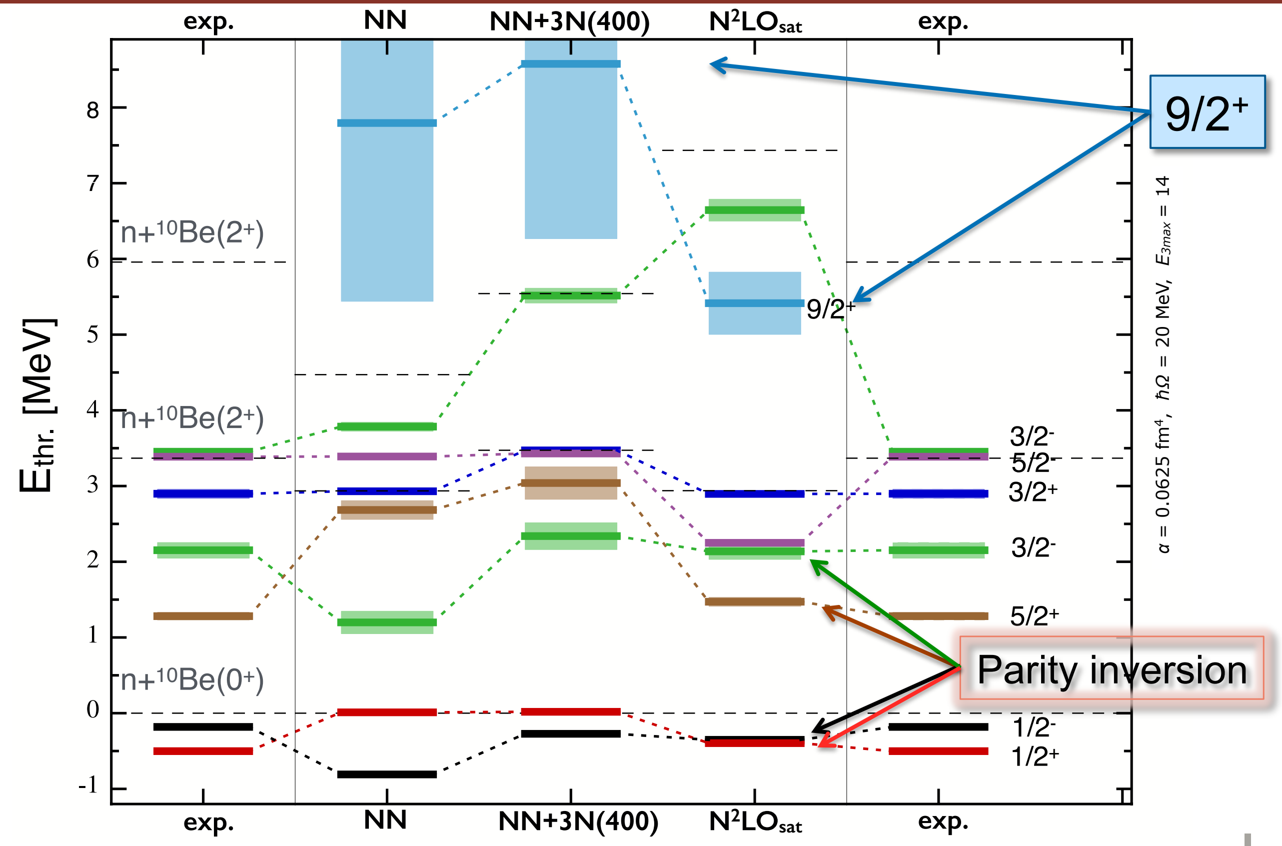


11Be
12B
13C
Theory
Exp.
E(MeV)
3/2+
3/2-
5/2+
1/2-
1/2+
(3/2)
1-
2-
2+
1+

The description of the experimental results from complementary approaches is of great interest

PRL **117,** 242501 (2016) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2016

Can *Ab Initio* Theory Explain the Phenomenon of Parity Inversion in ^{11}Be?



A. Calci, P. Navratil, R. Roth, J. Dohet-Eraly, S. Quaglioni, G. Hupin, PRL 117, 242501 (2016)

Wavefunctions of renormalized states

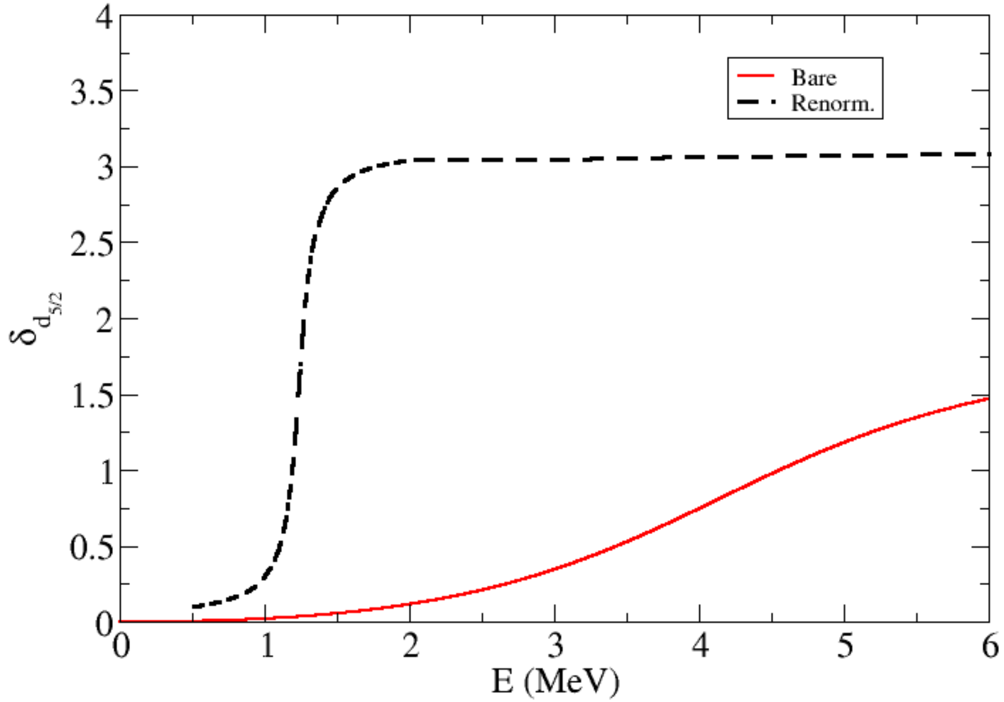
$$|\widetilde{1/2^+}\rangle = \sqrt{0.82}|s_{1/2}\rangle + \sqrt{0.17}|(d_{5/2}\otimes 2^+)_{1/2^+}\rangle$$

$$|\widetilde{1/2^-}\rangle = \sqrt{0.84}|p_{1/2}\rangle + \sqrt{0.14}|((p_{1/2},p_{3/2}^{-1})_{2^+}\otimes 2^+)_{0^+},p_{1/2}\rangle$$

$$|\widetilde{5/2^+}\rangle = \sqrt{0.56}|d_{5/2}\rangle + \sqrt{0.21}|(s_{1/2}\otimes 2^+)_{5/2^+}\rangle + \sqrt{0.22}|(d_{5/2}\otimes 2^+)_{5/2^+}\rangle$$

$5/2^+$: a ‘Fano resonance’ (Orrigo et al. PLB 633 (2006)469)

The renormalized 5/2+ phase shift is very different from the d5/2 phase shift In the bare potential.

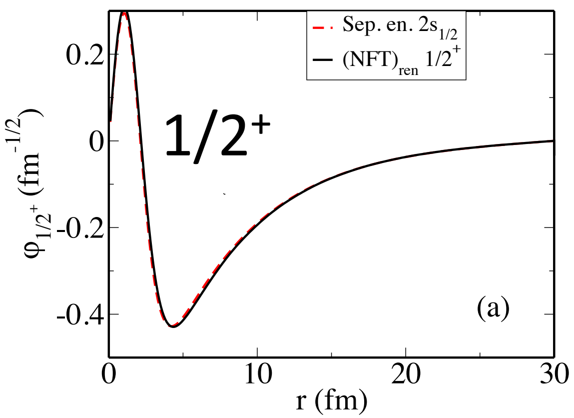


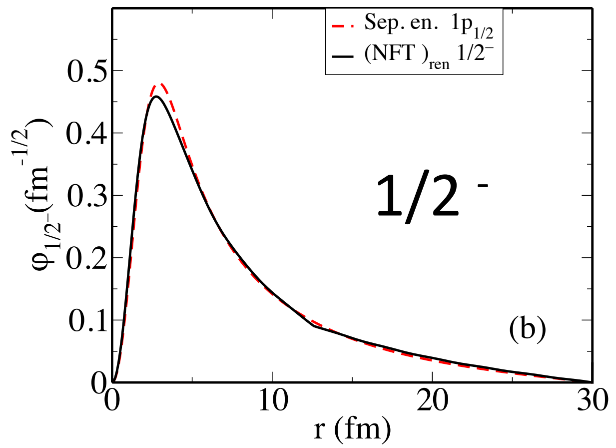
Test of the single-particle component of the many-body wavefunction

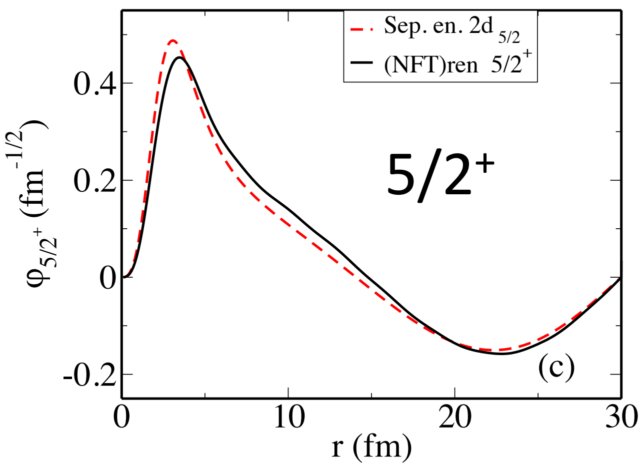
$^{10}Be(d,p)^{11}Be$ at E_d = 21.4 MeV

K.T. Schmitt et al., PRC88 (2012) 064612

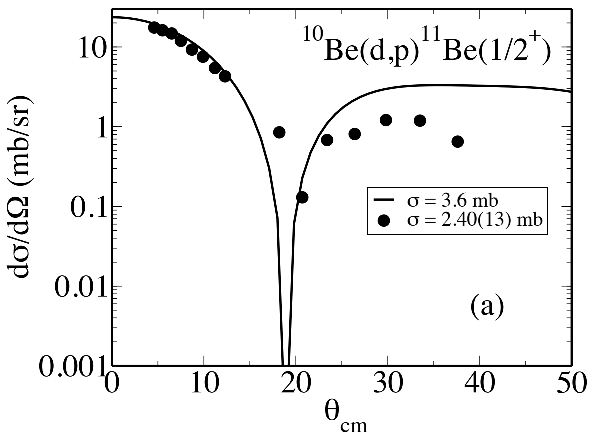
Form factors

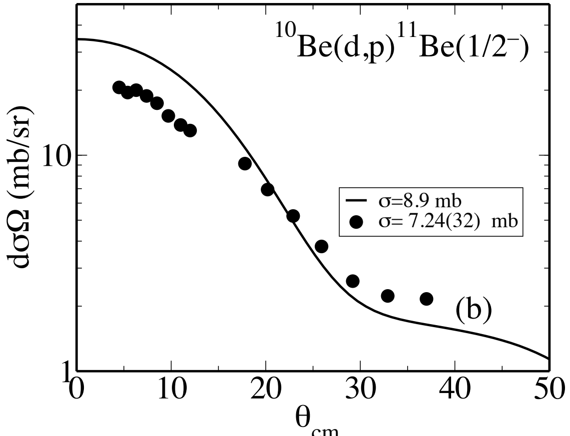


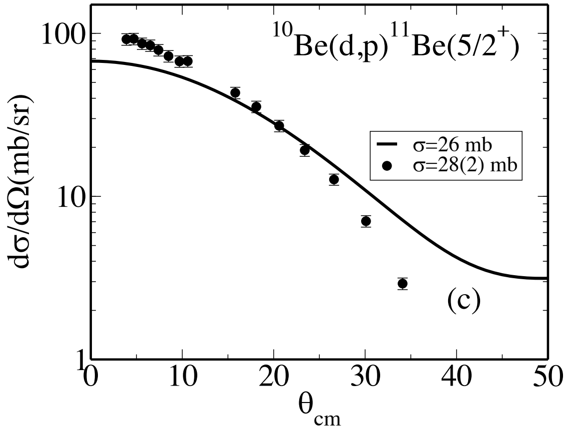




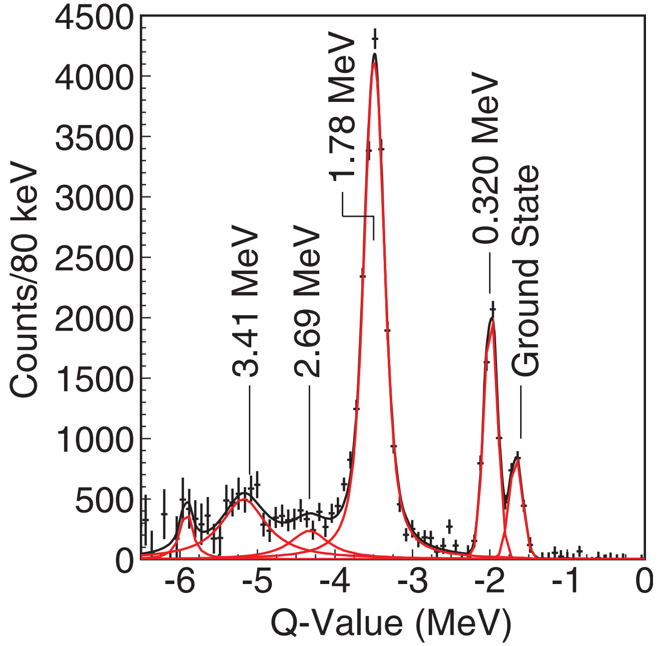
Cross sections

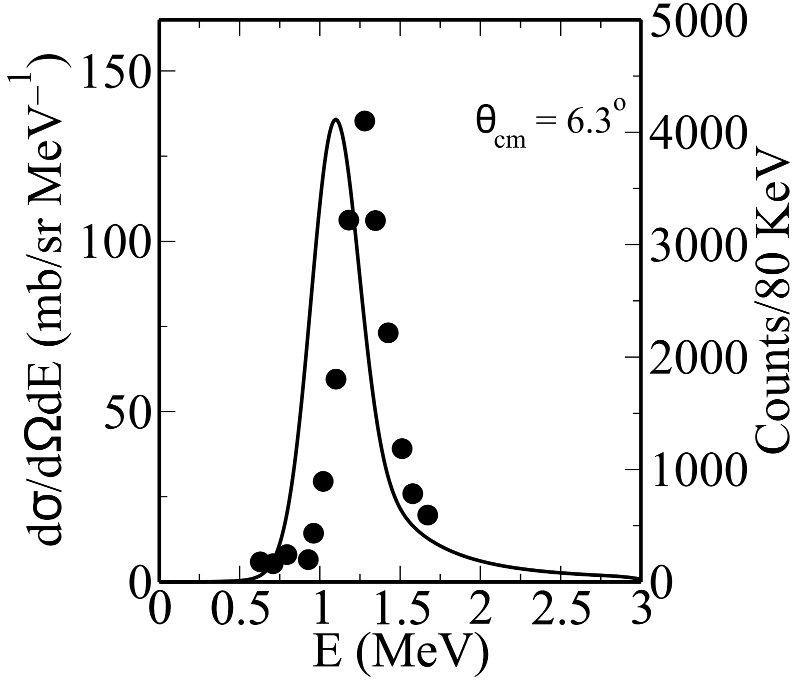


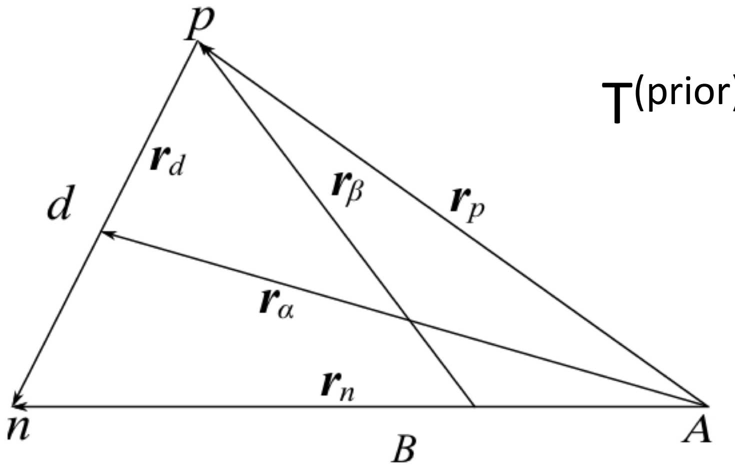




The lineshape of $^{10}Be(d,p)^{11}Be(5/2^{+})$ spectrum is well reproduced by theory





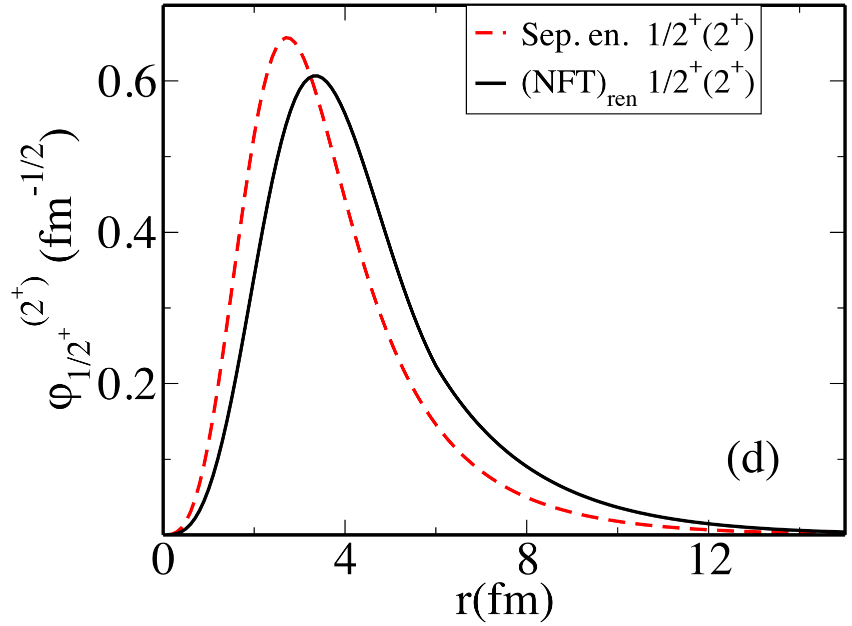


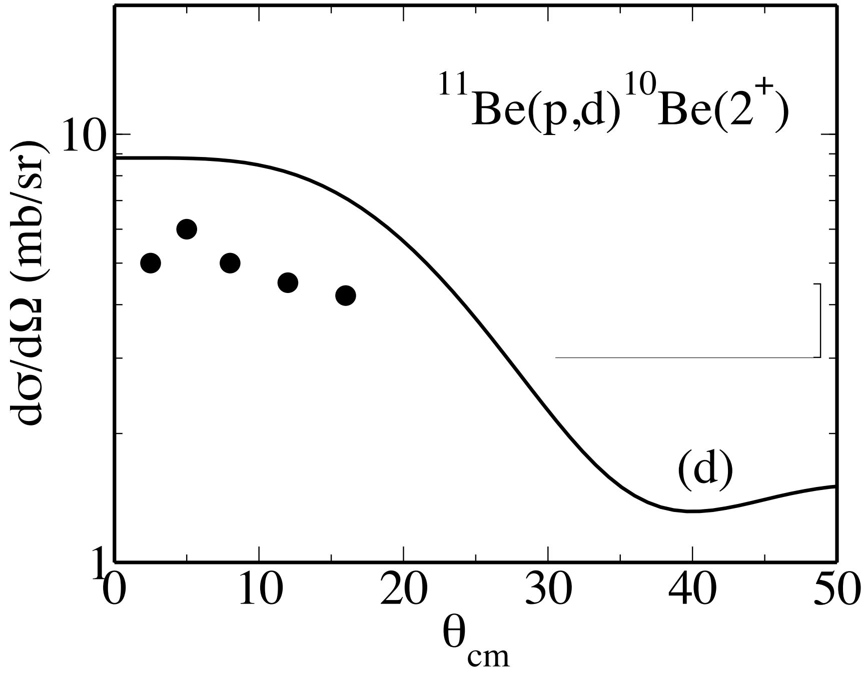
$$T^{(\mathrm{prior})} = \int d\boldsymbol{r}_d \int d\boldsymbol{r}_n \chi_\beta^{*(-)}(\boldsymbol{r}_d, \boldsymbol{r}_n)\psi_n^*(\boldsymbol{r}_n)V_{nA}(\boldsymbol{r}_n)\psi_d(\boldsymbol{r}_d)\chi_\alpha^{(+)}(\boldsymbol{r}_d, \boldsymbol{r}_n).$$

^{11}Be(1/2$^+$)(p,d)^{10}Be(2$^+$)

Test of the collective component of the many-body wavefunction

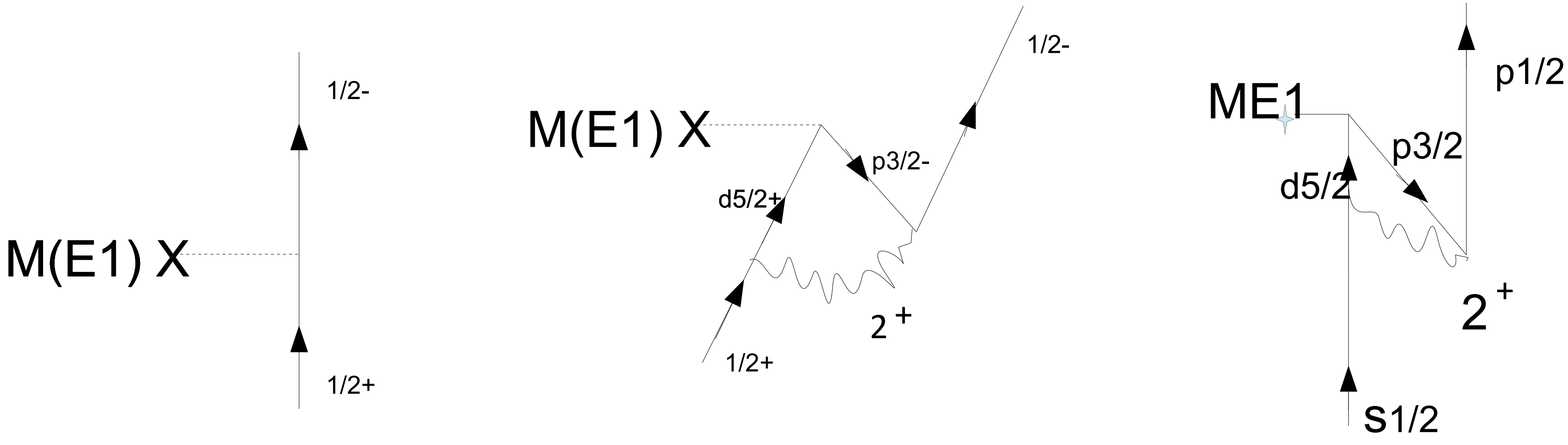
$$|\widetilde{1/2^+}\rangle = \sqrt{0.82}|s_{1/2}\rangle + \sqrt{0.17}|(d_{5/2} \otimes 2^+)_{1/2^+}\rangle$$





J.S. Winfield et al., Nucl.Phys. **A683** (2001) 48

Strength of the dipole transition between ½⁺ and ½⁻ states



1.95 e fm

-0.26 e fm

-0.19 e fm

B(E1) (th.) = 0.11 e^2 fm^2

B(E1) (exp.) = 0.102 ± 0.002 e^2 fm^2

This result is sensitive to the details of the mean field potential

Isotopic shift of the charge radius

$(\langle r^2 \rangle_{10Be})^{1/2} = 2.361 \pm 0.017$ fm

$(\langle r^2 \rangle_{11Be})^{1/2} = 2.466 \pm 0.015$ fm

Single-particle picture: S=1

Many-body picture: S=0.83

$(\langle r^2 \rangle)^{1/2}_{1s1/2} = 7.1$ fm

$(\langle r^2 \rangle)^{1/2}_{d5/2,coll} = 3.0$ fm

$$\langle r^2 \rangle_{11Be} = \left(\langle r^2 \rangle_{10Be} + \left(\frac{\langle r^2 \rangle^{1/2}_{1s1/2}}{11} \right)^2 \right) \times S^2 + (1-S^2) \times \left(\langle r^2 \rangle_{10Be} \left(1 + \frac{2}{4\pi} \beta_\pi^2 \right) + \left(\frac{\langle r^2 \rangle^{1/2}_{d5/2\,coll}}{11} \right)^2 \right) = .$$

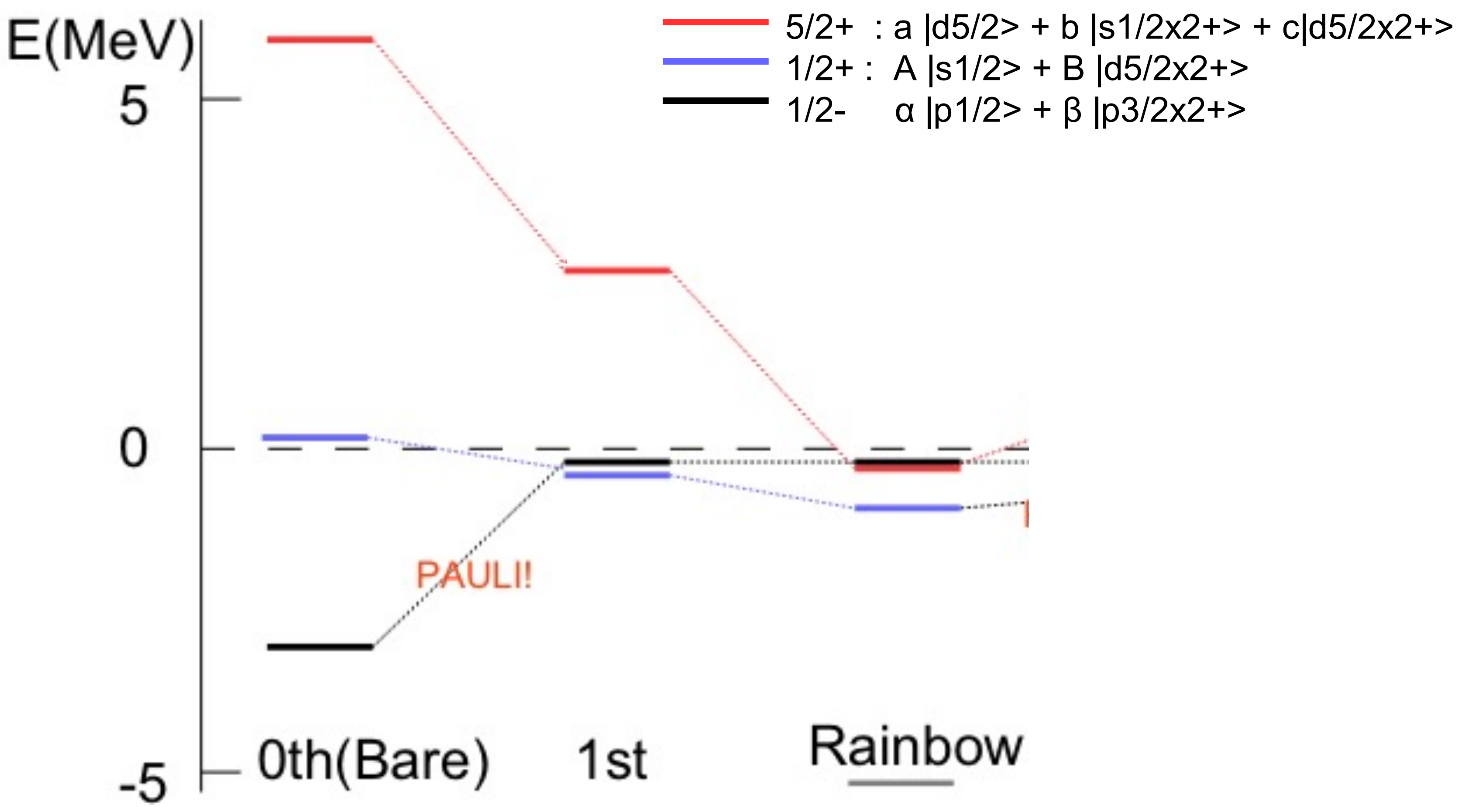
$$\langle r^2 \rangle_{10Be} + \left(\frac{\langle r^2 \rangle^{1/2}_{1s1/2}}{11} \right)^2 \times S^2 + (1-S^2) \times \left(\left(\frac{\langle r^2 \rangle^{1/2}_{d5/2\,coll}}{11} \right)^2 + \langle r^2 \rangle_{10Be} \frac{2}{4\pi} \beta_\pi^2 \right)$$

$\Delta \langle r^2 \rangle^{1/2}_{11Be}$ (th.) = 0.12 fm / 0.27 fm

$\Delta \langle r^2 \rangle^{1/2}_{11Be}$ (exp.)= 0.11 fm

Taking properly into account the empirical coupling between single-particle and collective degrees of freedom leads to a quantitative description of various structure properties and of reactions of ^{11}Be

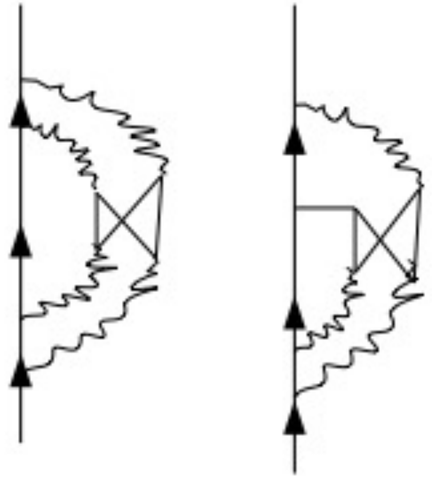
The initial $2s_{1/2}$ state is not bound . After the first diagonalization, the ½+ state becomes bound, $1/2^{+} = 0.95\, s_{1/2} + 0.3\, d_{5/2} \times 2^{+}$ while the 5/2+ state contains an admixture of the $2s_{1/2} \times 2+$ configuration.
By iterating, in the calculation of the dressed $5/2^{+}$ state we take into account the 2-phonon contribution and the fact that the $1/2^{+}$ state is localized.

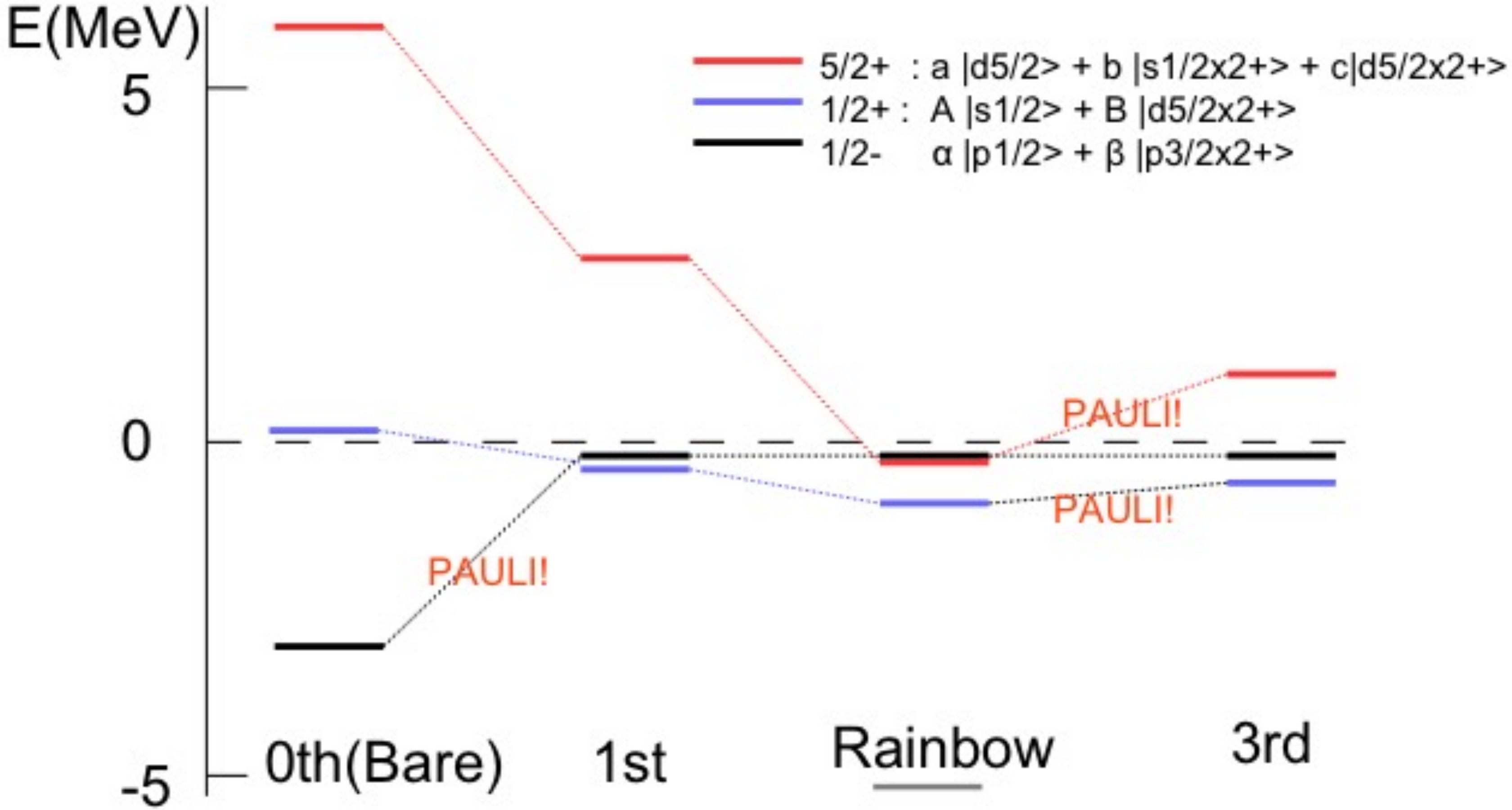


V_0=70MeV
a=0.81fm
R=2.1fm

Two Phonon Anharmonicities

Butterfly Diagrams





Aim of the talk:

To present the variety of **many body effects** at the basis of the structure
of 11Be **states of quasi-particle character**, by using the framework of **Nuclear Field Theory**.

It will be shown that in order to
Simultaneously explaining:
O spectra,
O one neutron transfer absolute dσ/dEdΩ cross sections in 10Be(d,p)11Be and in
….11Be(p,d)10Be reactions,
O dipole transitions and
O isotopic rms charge radius change

up to **3p-2h** configurations are unavoidable, and that antisymmetrization (**Pauli principle**) between neutron and core, and of the 2p-2h core configurations, play a crucial role, as well as the coupling to the **single-particle continuum.**

By incorporing coupling to the single-particle continuum, a **common framework for the study of structure and reactions**, based on the Nuclear Field Theory emerges. A formulation in terms of **Generalized Coupled Channels allows for the proper inclusion of Pauli principle.**

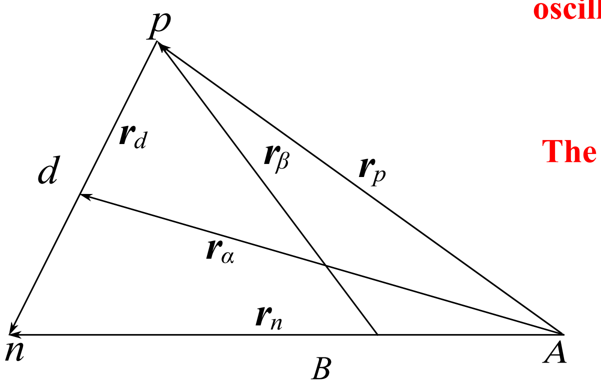
Stripping reaction to unbound state: *A*(*d*, *p*)*B*

- The transition matrix of the **post-form** representation for the (*d*, *p*) reaction within the distorted-wave Born approximation (DWBA):

$$
\begin{aligned}
T_{\mathrm{DWBA}}^{(\mathrm{post})} &= \left\langle \chi_\beta^{(-)}\psi_n \middle| V_{pn} + V_{pA} - U_\beta \middle| \psi_d \chi_\alpha^{(+)} \right\rangle \\
&\sim \left\langle \chi_\beta^{(-)}\psi_n \middle| V_{pn} \middle| \psi_d \chi_\alpha^{(+)} \right\rangle \\
&= \int d\boldsymbol{r}_d \int d\boldsymbol{r}_n \chi_\beta^{*(-)}(\boldsymbol{r}_d, \boldsymbol{r}_n) \underbrace{\psi_n^*(\boldsymbol{r}_n)}_{\textbf{oscillate}} \underbrace{V_{pn}(\boldsymbol{r}_d)\psi_d(\boldsymbol{r}_d)}_{\textbf{attenuate}} \chi_\alpha^{(+)}(\boldsymbol{r}_d, \boldsymbol{r}_n).
\end{aligned}
$$

The remnant term

$$\boxed{V_{pA} - U_\beta = V_{pA} - U_{pB} \sim 0}$$



V_{pn}: deuteron binding potential

The r_n integration does not converge!

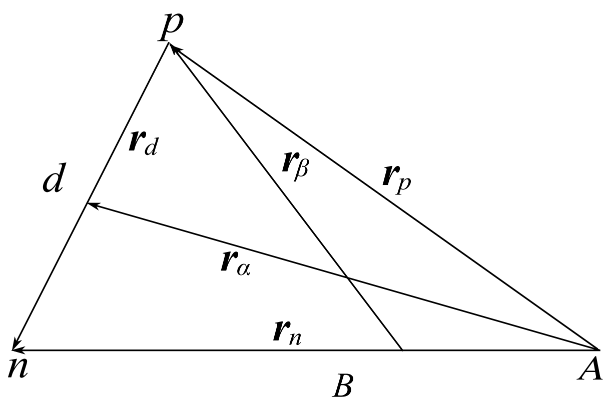
T. Fukui

■ The prior form

$$T^{(\mathrm{prior})}_{\mathrm{DWBA}} = \left\langle \chi^{(-)}_{\beta}\psi_n \middle| V_{nA} + V_{pA} - U_\alpha \middle| \psi_d \chi^{(+)}_\alpha \right\rangle$$

$$= \left\langle \chi^{(-)}_{\beta}\psi_n \middle| V_{nA} \middle| \psi_d \chi^{(+)}_\alpha \right\rangle \qquad \leftarrow \text{If the remnant term } \boxed{V_{pA} - U_\alpha = 0}$$

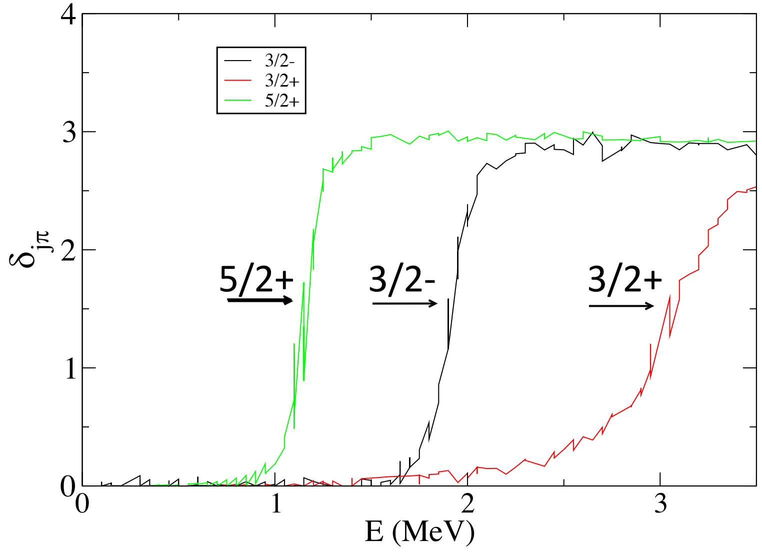
$$= \int d\boldsymbol{r}_d \int d\boldsymbol{r}_n \chi^{*(-)}_{\beta}(\boldsymbol{r}_d, \boldsymbol{r}_n) \underbrace{\psi^*_n(\boldsymbol{r}_n)}_{\text{oscillate}} \underbrace{V_{nA}(\boldsymbol{r}_n)}_{\text{attenuate}} \underbrace{\psi_d(\boldsymbol{r}_d)}_{\text{attenuate}} \chi^{(+)}_{\alpha}(\boldsymbol{r}_d, \boldsymbol{r}_n).$$



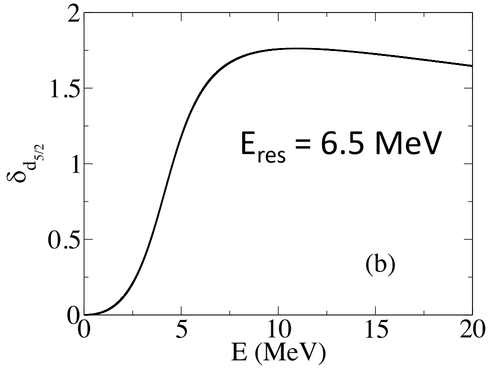
**These respectively attenuate for two independent coordinates.
→ The integration does converge.**

V_{nA}: interaction for the *n*-*A* unbound state

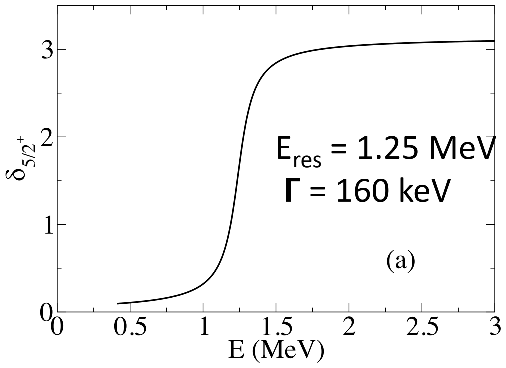
Calculated phase shifts, ^{11}Be



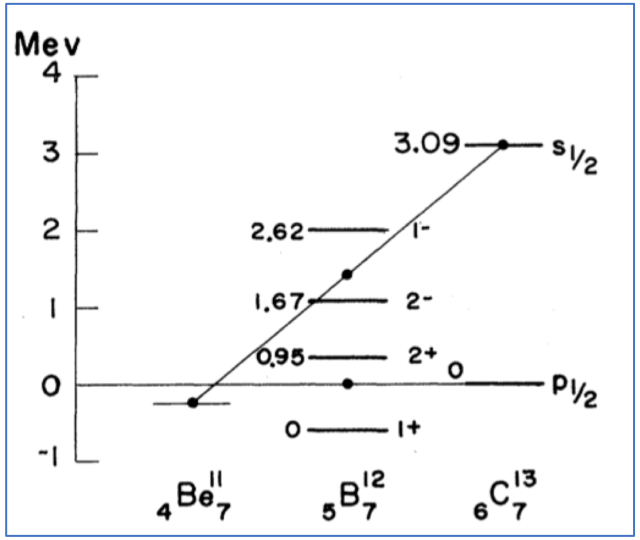
d5/2 phase shift in the bare potential

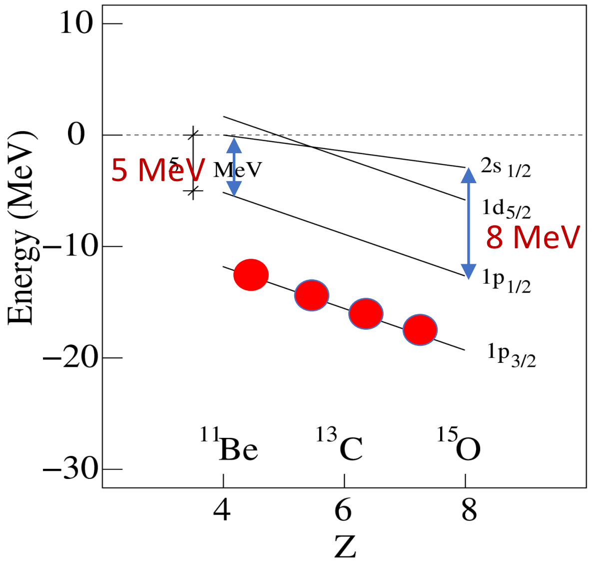


Renormalized 5/2+ phase shift



Parity inversion in N=7 isotones is not reproduced by spherical non relativistic mean field calculations, although the mean field includes most of the effect of the neutron-proton interaction





Typical spherical mean-field results with Skyrme forces (Sagawa,Brown,Esbensen PLB 309(93)1)

Structure and reactions of N=7 isotones: the role of core degrees of freedom

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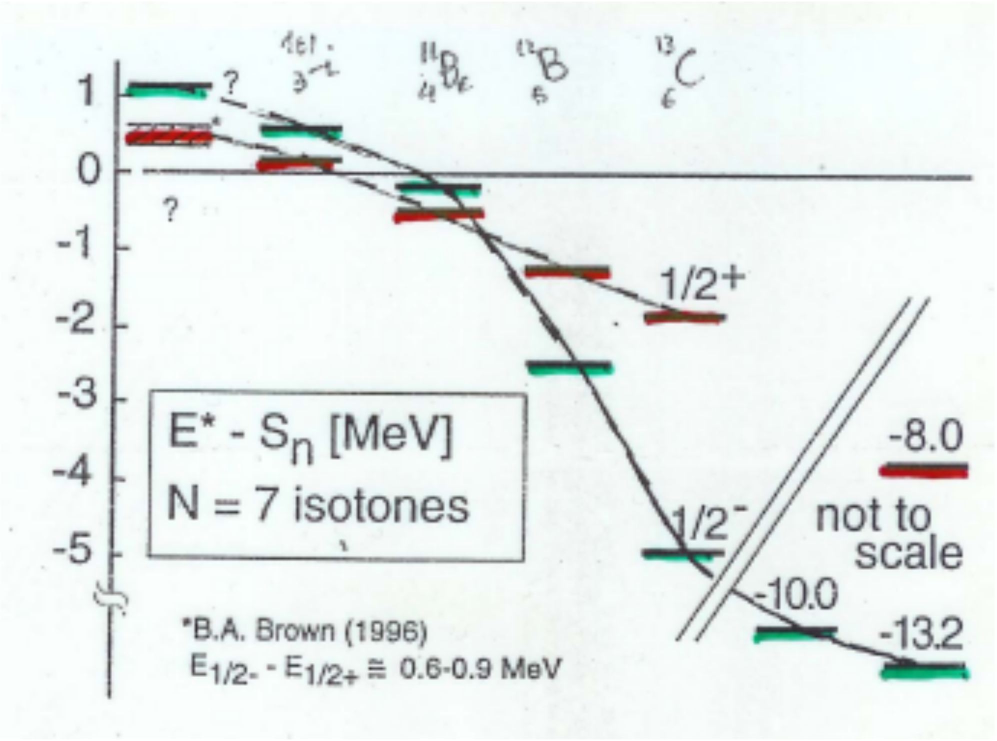
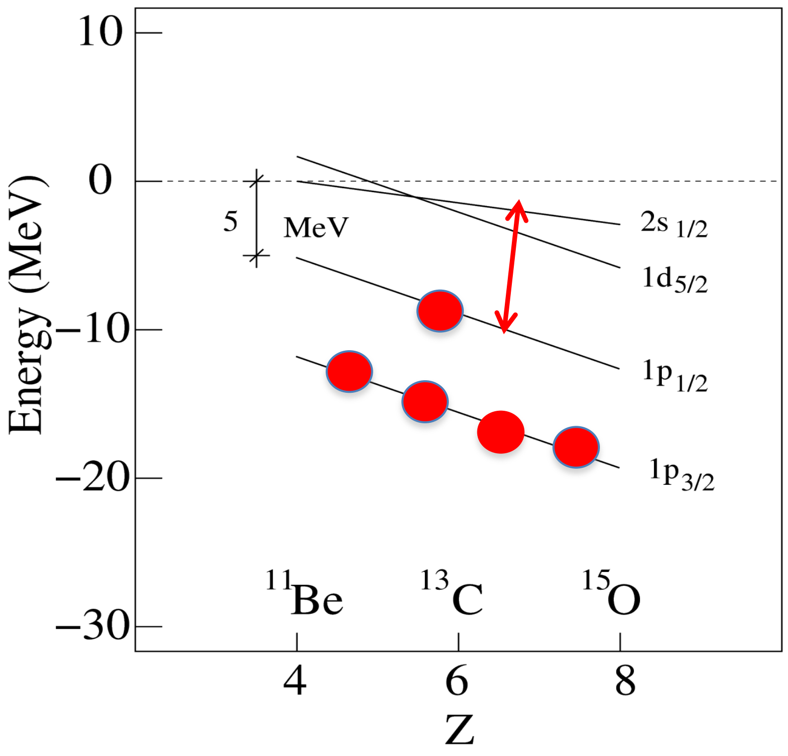
Parity inversion in N=7 isotones

Experimental systematics

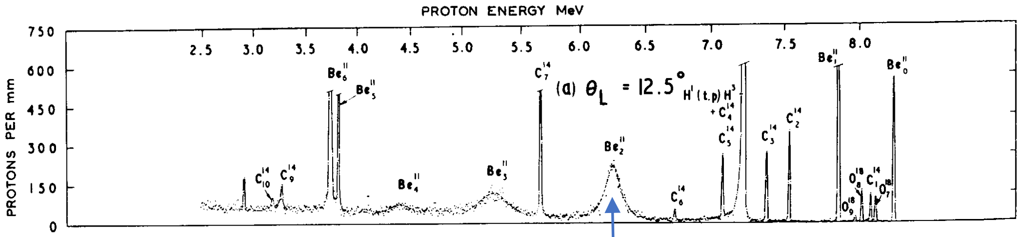


Mean-field results

(Sagawa,Brown,Esbensen PLB 309(93)1)



The usually quoted value of width of the 5/2+ resonance (100 keV) is derived from $^{9}Be(t,p)^{11}Be$ spectra



Pullen et al., Nucl. Phys. 36 (1962)1

The width from $^{10}Be(d,p)^{11}Be(5/2^{+})$ spectra is much larger and is well reproduced by theory