# Origin of the magnetospheric X-ray radiation from the rotation powered pulsars

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# パルサーからのX線の起源について

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Because it is rotation powered,
$\Omega$, $\mu$ ($B_d$), a must predict uniquely
Lx, spectrum and everything.

This spirit is ill.

Things are much more complicated:
e.g.
Toroidal magnetic field (multipole field)
Coupling with the NS evolution
Metastable states of the magnetosphere

+ magnetar, CCO,XINS,…
➔Comprehensive study

# Introduction

Challenges

- ◆GeV gamma-ray pulses

outer gap vs current sheet ?

- ◆ variation
- Radio ON/OFF, null, mode change, RRAT
- High-B PSR; radio OFF on magnetar bursts
- Magnetar; radio ON on busts

neither $\Omega$, $\mu$, $\alpha$, torque/current change

➔not local but global

mechanism?

meta stable states,

toroidal fields(multipole fields)

- ◆ Lx-Lrot plot shows a large scatter

origin?

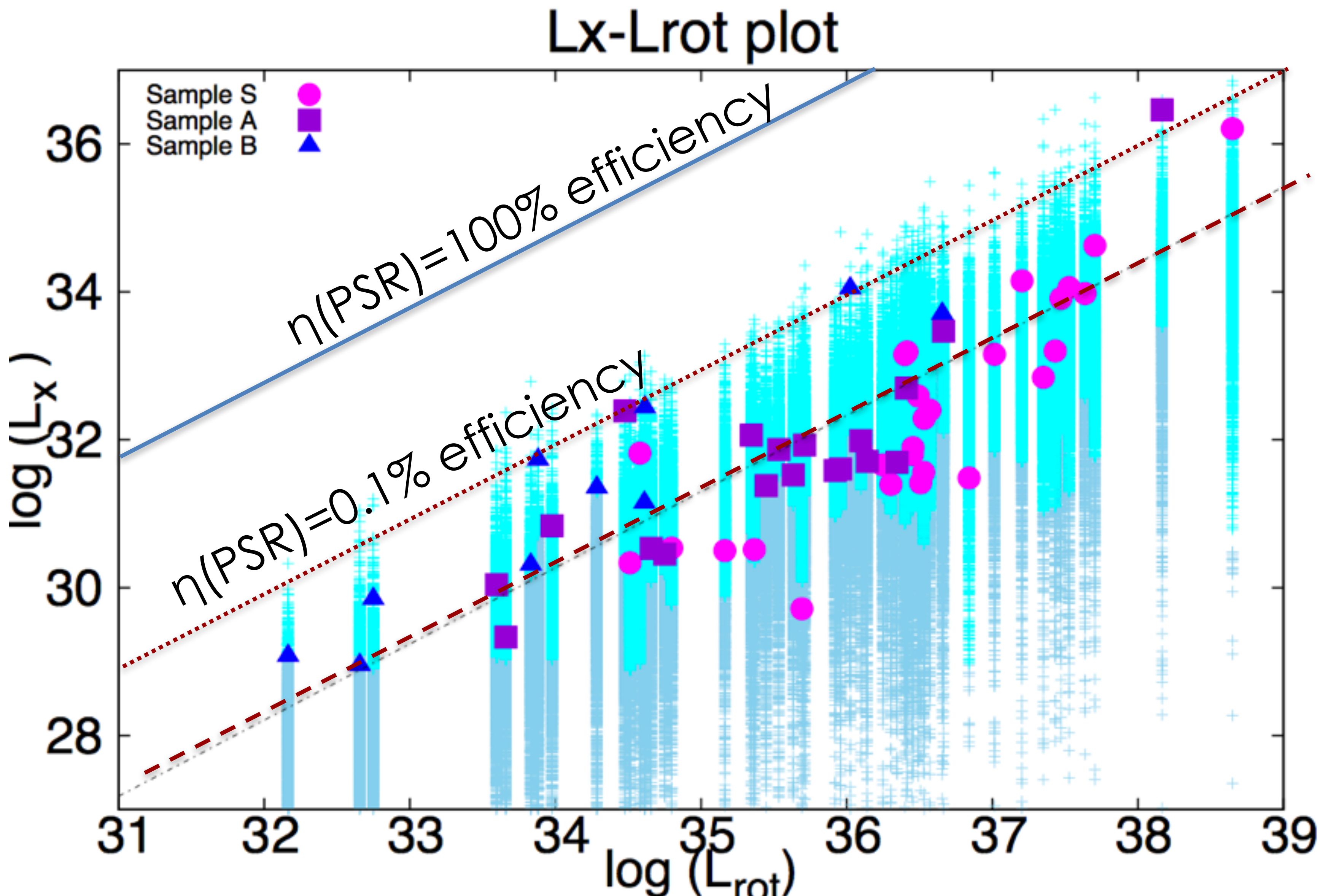
High efficiency Lx/Lrot: soft $\gamma$-ray pulsar hot pulsar

Low efficiency; ??????? Linked with PWN

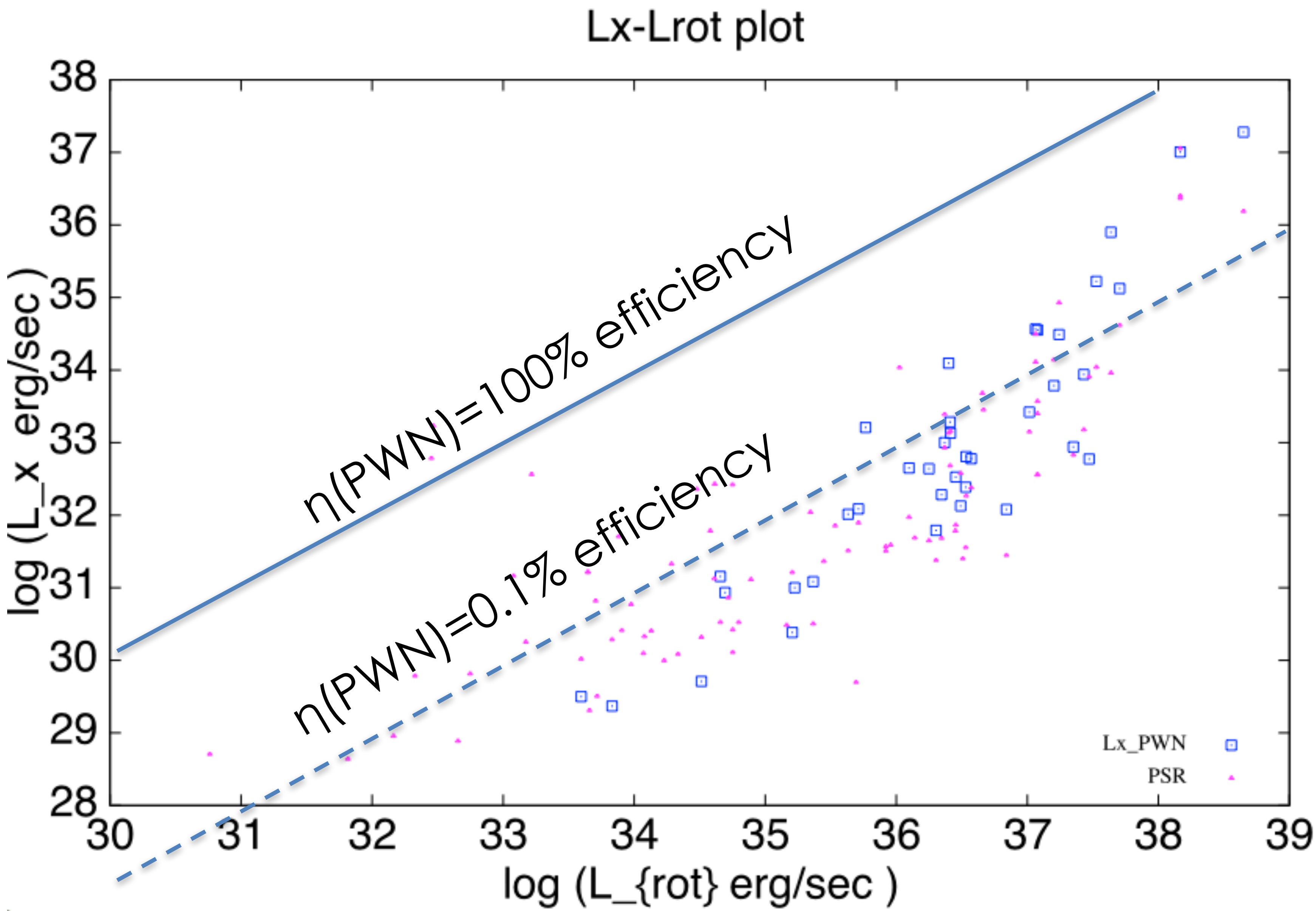
Lx/Lrot ∼ $10^{-3}$ = const., but large scatter

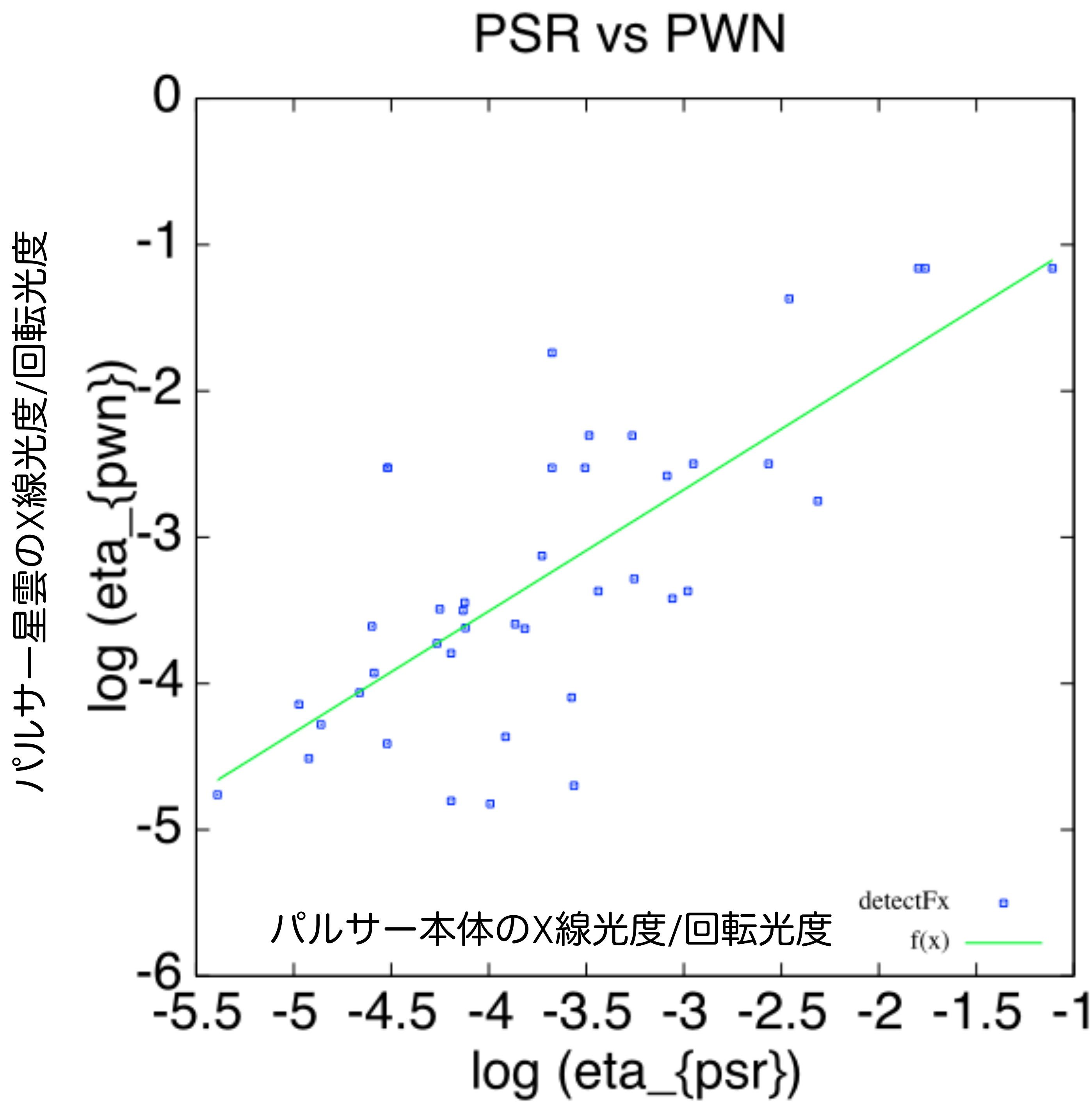
Lx bright ➔ soft gamma-ray pulsars, hot pulsars

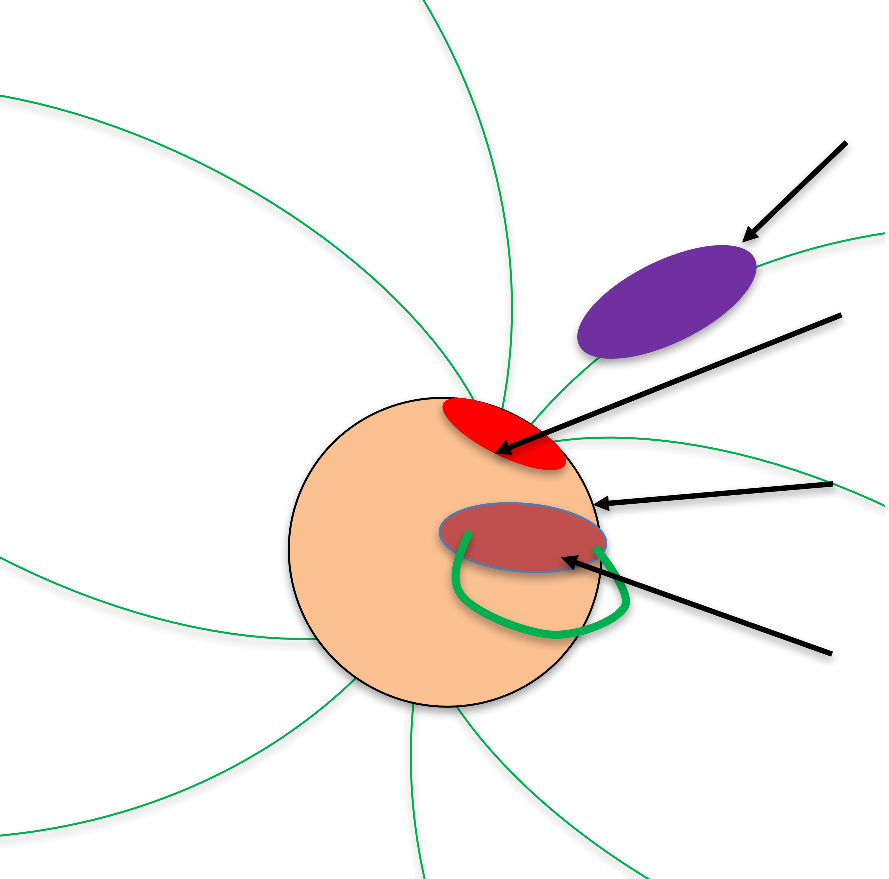
Lx dim ➔ ????



# A hint; Lx(PWN) has also large scatter




PSR vs PWN
0
-1
-2
-3
-4
-5
-6
-5.5
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
log (eta_{pwn})
log (eta_{psr})
パルサー星雲のX線光度/回転光度
パルサー本体のX線光度/回転光度
detectFx
f(x)



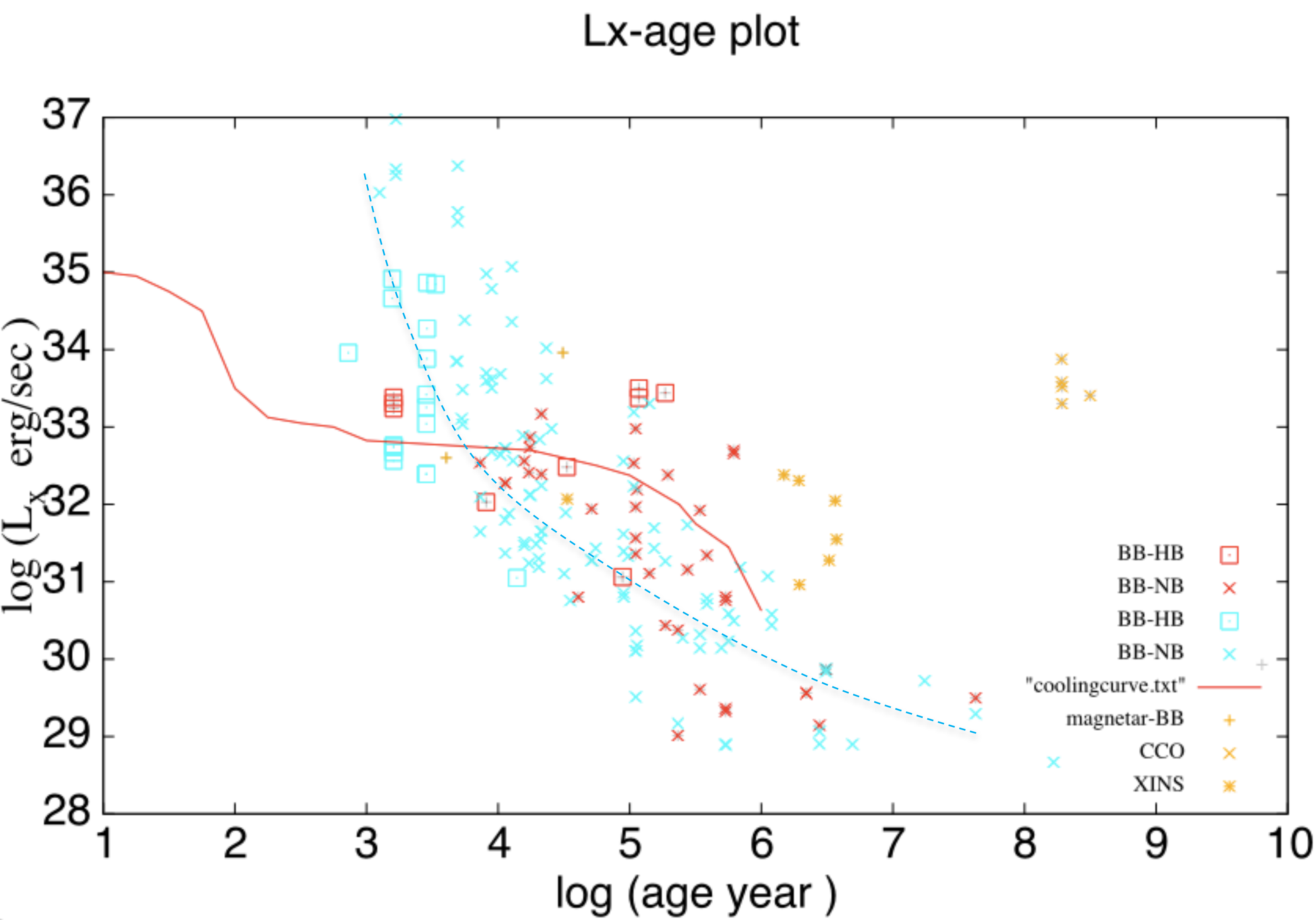
- Magnetospheric emission; non thermal
- Polar cap heating: thermal small area
- Cooling radiation; thermal large area
- Magnetic heating; thermal

Presumption

X-ray spectrum shall be decomposed eg. Thermal / non-thermal and hopefully phase resolved.

➔XMM, Chandra, NuSTAR, NICER etc.

# Decomposition of spectrum is a very strong tool!



Aim

# Origin of the magnetospheric X-ray radiation

What determines Lx(magnetosphere)?

The mechanism must be linked with Lx(PWN)

Aim

# Origin of the magnetospheric X-ray radiation

Outer Gap X

Polar Caps ○

We revisit the full polar cap pair cascade model

" Full Polar Cap Cascade Scenario: Gamma-Ray and X-Ray Luminosities from Spin-powered Pulsars"
Zhang, B., & Harding, A.K. 2000, ApJ, 532, 1150

Syn. R

"Quantized synchrotron radiation in strong magnetic fields"
Harding, A.~K., & Preece, R. 1987, ApJ, 319, 939

Res. IC

"Magentic compton-induced pair cascade model for gamma-ray pulsars"
Sturner, Steven J., Dermer, Charles D., Michel, F. Curtis 1995, ApJ, 445, 736

"On the polar cap cascade pair multiplicity of young pulsars"
Timokhin, A.N., & Harding, A.K. 2015, ApJ, 810, 144

Aim

# Origin of the magnetospheric X-ray radiation

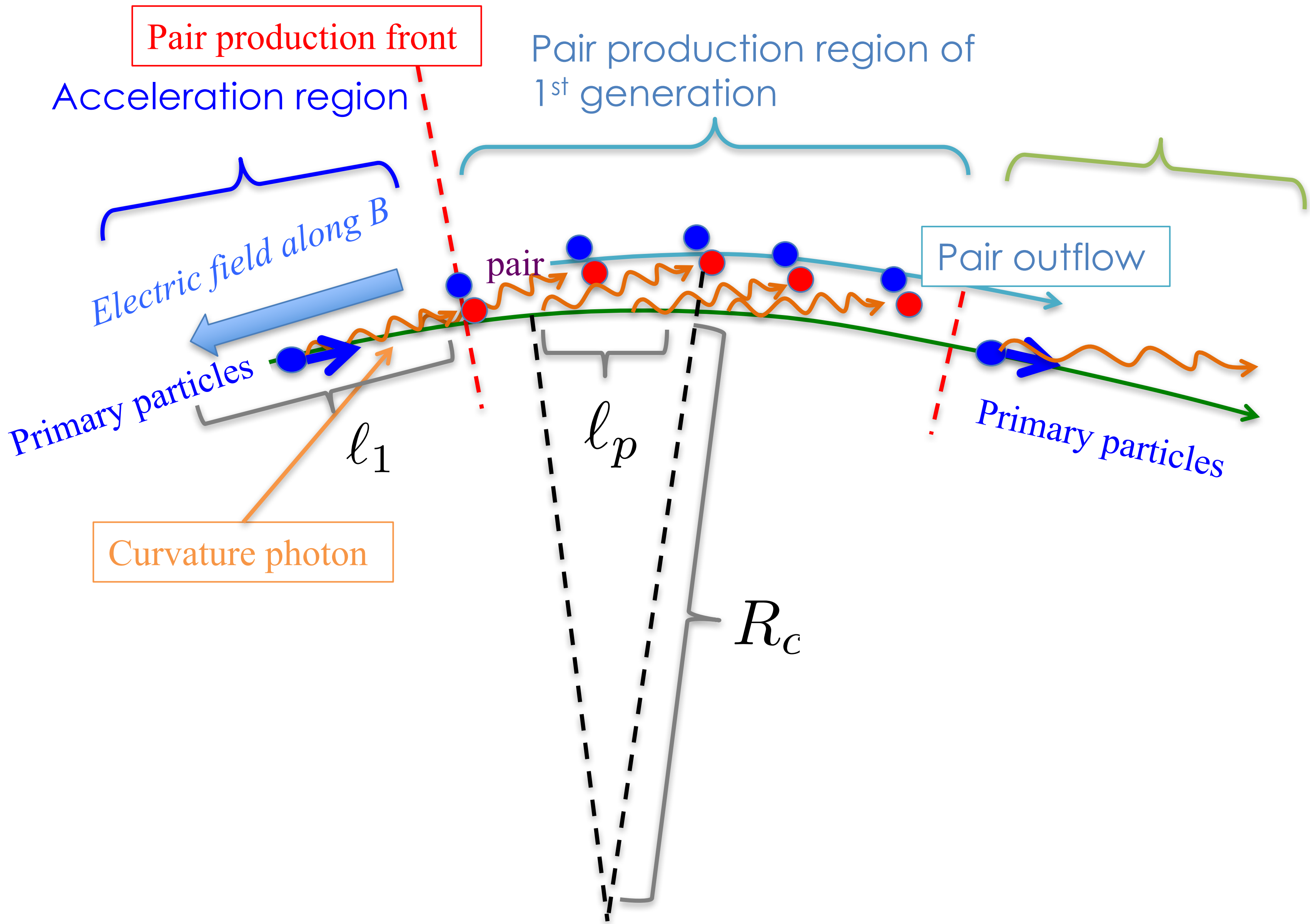
Outer Gap X

Polar Caps ◯

We revisit the full polar cap pair cascade model

Obtain Lx, its spectrum
as function of μ(or $B_d$), Ω, a, B
(multipole/toroidal field), $R_c$ etc.

# Polar Cap Model



# Energy Flow in the cascade

Power of the particle accelerator

$$L_1 = V_1 I = \begin{cases} \text{escaping particles} \\ L_{curv} = \begin{cases} \text{escaping gamma-ray} \\ L_p = \begin{cases} \text{escaping pairs} \\ L_{\rm x} \\ L_h \text{hard X or soft-}\gamma \end{cases} \end{cases} \end{cases}$$

observation

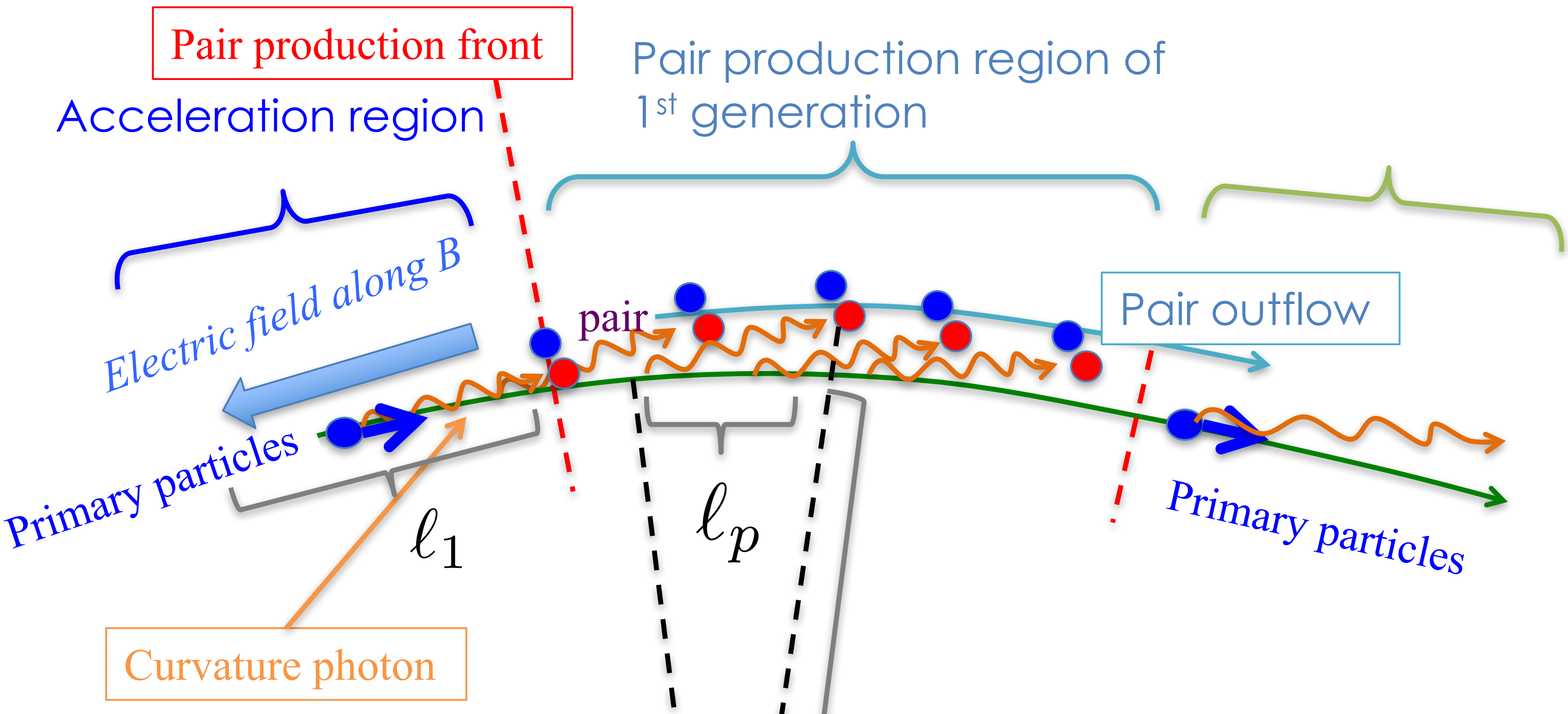
probably this part dominates particle parts

Pair Luminosity at 1st generation

$$L_p = \dot{N}_p mc^2 (\gamma_p - 1)$$

*Let us calculate $L_p$ first, then obtain $L_x$*

# Polar Cap Model




$$\dot{N}_p = 2\mathcal{M}\frac{iI_0}{e}$$

$$I_0 = \mu\Omega^2/c$$

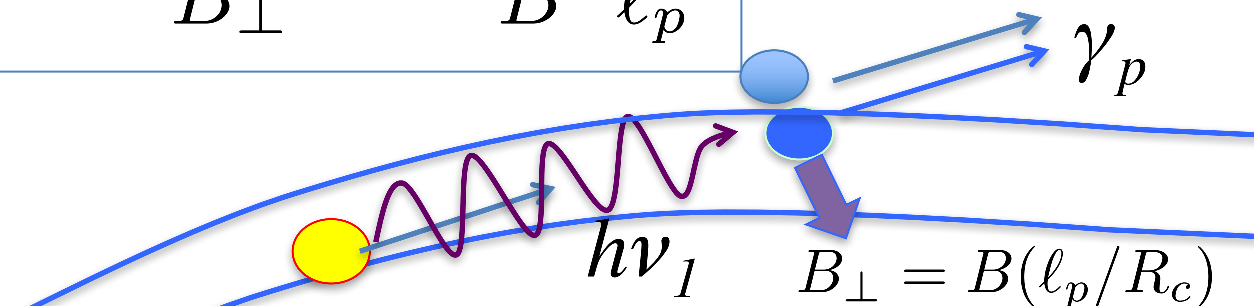
$$\ell_{pc} = \min\left(\tau c,\ \ R\right)$$

$$\mathcal{M} = \frac{P_{curv}}{h\nu_1}\frac{\ell_{pc}}{c} = \frac{4\alpha}{9}\frac{\ell_{pc}}{R_c}\gamma_1$$

$$\tau c = \frac{3}{2\gamma_1^3}\frac{R_c^2}{r_e}$$

# Magnetic Pair Creation

$$\gamma_p = \frac{h\nu_1}{2mc^2} = \chi \frac{B_q}{B_\perp} = \chi \frac{B_q}{B} \frac{R_c}{\ell_p}$$




## Primary photon = curvature rad.

$$\frac{h\nu_1}{2mc^2} = \frac{3}{4} \frac{\hbar/mc}{R_c} \gamma_1^3$$

condition I

$$\gamma_1^3 = \frac{4\chi}{3} \frac{B_q}{B} \frac{R_c^2}{\ell_p(\hbar/mc)}$$

large B ➔ short pair mean free path if $\gamma_1$ const.

# Accelerator model : Space Charge Limited Flow

$$\nabla \cdot \boldsymbol{E}_{\parallel} = 4\pi(\rho_e - \rho_{gj})$$

$$\rho_{gj} = \Omega B / 2\pi c$$

$$E_{\parallel} = \frac{2\Delta j \Omega B}{c} \ell_1$$

condition II

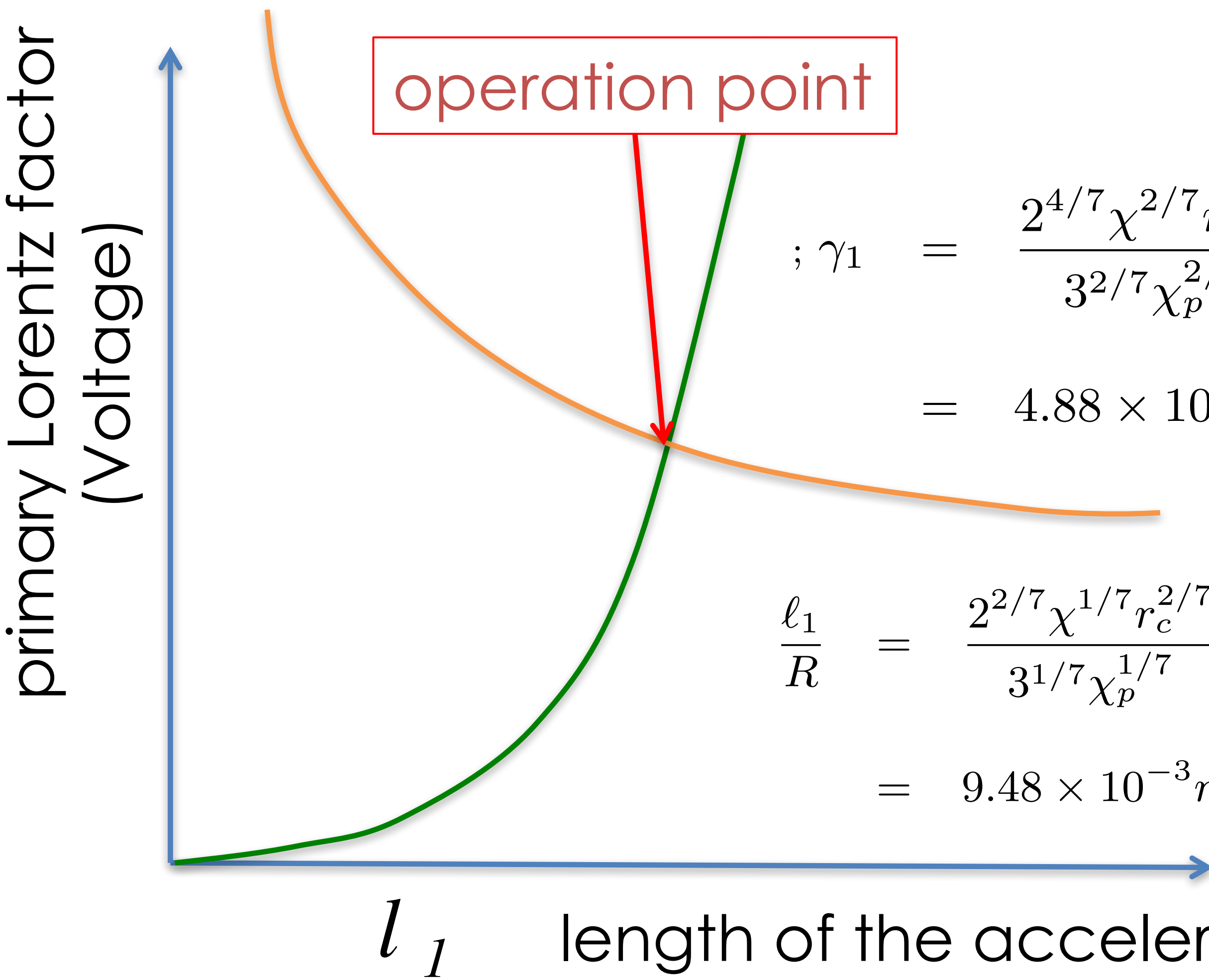
$$V_1 \approx \frac{1}{2} E_{\parallel} \ell_1 = \Delta j \frac{\Omega B}{c} \ell_1^2$$

*with* $$\ell_p \approx 0.14 \ell_1$$ pair production front condition I

$$\gamma_1^3 \propto \frac{R}{\ell_1}$$

condition II

$$\gamma_1 \propto \left(\frac{\ell_1}{R}\right)^2$$

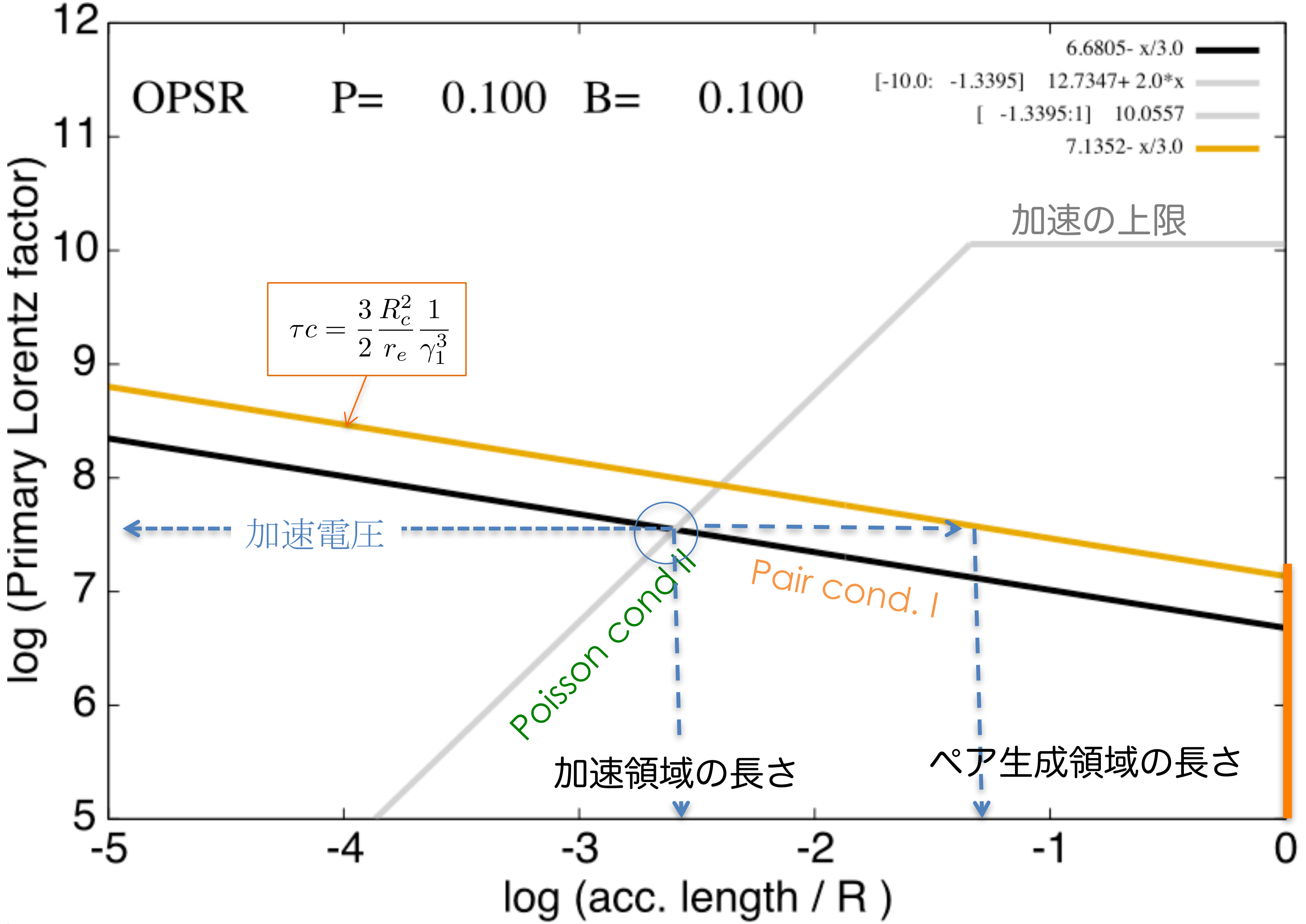



$$; \gamma_1 = \frac{2^{4/7}\chi^{2/7} r_c^{4/7}}{3^{2/7}\chi_p^{2/7}} \left(\frac{B_q}{B}\right)^{1/7} \left[\frac{R_L R^2}{(\hbar/mc)^3}\right]^{1/7}$$

$$= 4.88 \times 10^7 r_c^{4/7} \left(\frac{0.1 B_q}{B}\right)^{1/7} P^{1/7}$$

$$\frac{\ell_1}{R} = \frac{2^{2/7}\chi^{1/7} r_c^{2/7}}{3^{1/7}\chi_p^{1/7}} \left(\frac{B_q}{B}\right)^{4/7} R_L^{4/7} R^{-6/7} (\hbar/mc)^{2/7}$$

$$= 9.48 \times 10^{-3} r_c^{2/7} \left(\frac{0.1 B_q}{B}\right)^{4/7} P^{4/7}$$



OPSR P= 0.100 B= 0.100
6.6805- x/3.0
[-10.0: -1.3395] 12.7347+ 2.0*x
[ -1.3395:1] 10.0557
7.1352- x/3.0
加速の上限
$\tau c = \frac{3}{2}\frac{R_c^2}{r_e}\frac{1}{\gamma_1^3}$
加速電圧
Poisson cond II
Pair cond. I
加速領域の長さ
ペア生成領域の長さ
log (Primary Lorentz factor)
log (acc. length / R )

# Standard Polar Cap or Not?

Primary particle energy $\gamma_1$ saturated by curvature radiation drag force

$$\tau c = \frac{3}{2}\frac{R_c^2}{r_e}\frac{1}{\gamma_1^3} \quad \begin{matrix} > \\ < \end{matrix} \quad \ell_1 = \frac{\ell_p}{\chi_p} = \frac{4\chi}{3\chi_p}\frac{B_q}{B}\frac{R_c^2}{\hbar/mc}\frac{1}{\gamma_1^3}$$

normal polar cap

$$\frac{9}{8}\frac{\chi_p}{\alpha\chi}\frac{B}{B_q} = 573\frac{B}{B_q} > 1$$

otherwise saturated primary energy ➔partial screened gap?

# Standard Polar Cap or Not?

All the accelerator power goes to the pair luminosity or not?

$$\tau c = \frac{3}{2}\frac{R_c^2}{r_e}\frac{1}{\gamma_1^3}$$

<R Yes

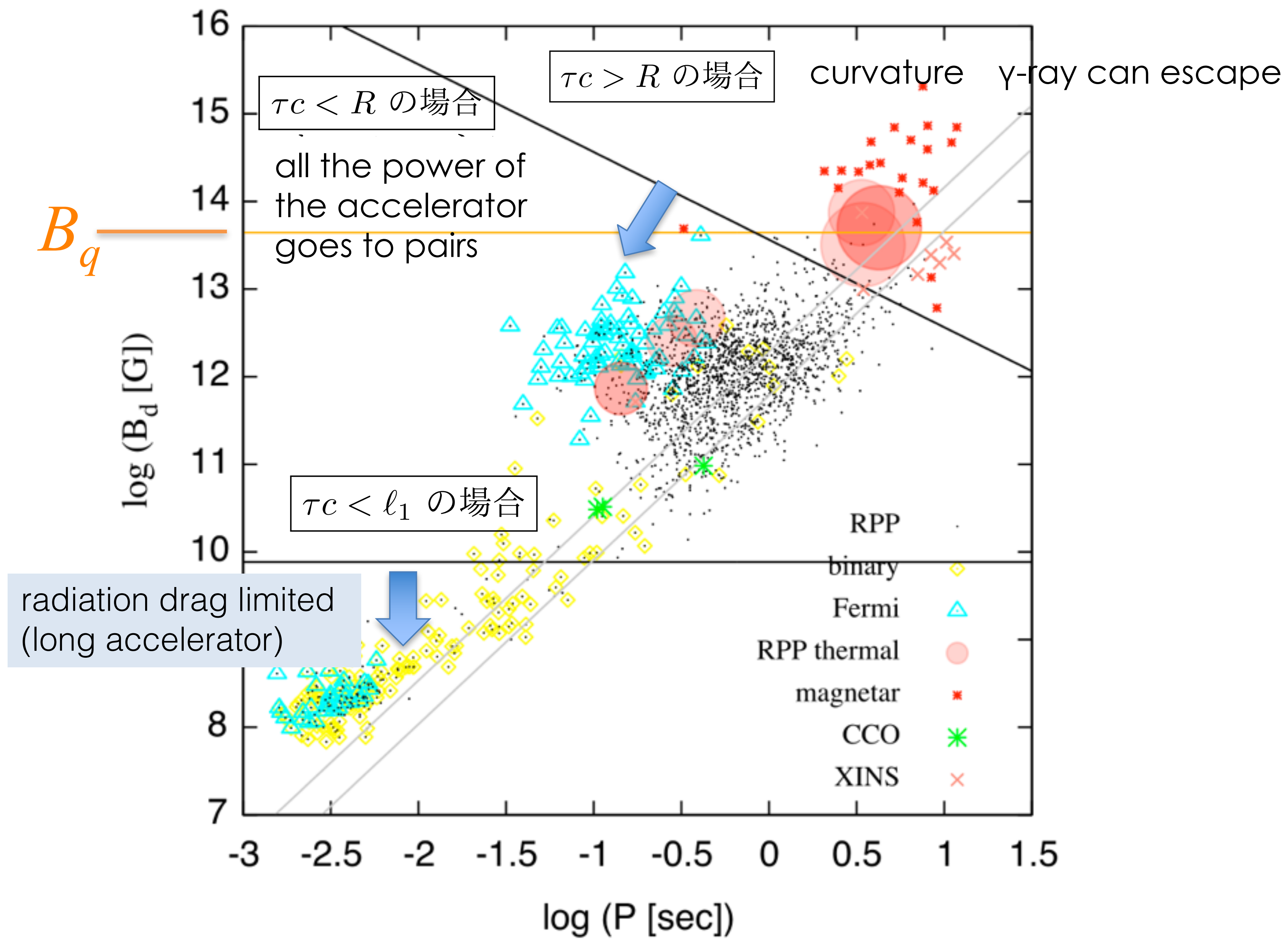
>R No

$$\frac{\tau c}{R} = \frac{3^{13/7}\chi_p^{6/7} r_c^{2/7}}{2^{19/7}\alpha\chi^{6/7}}\left(\frac{B}{B_q}\right)^{3/7}\left(\frac{R_L}{\hbar/mc}\right)^{4/7}\left(\frac{R}{\hbar/mc}\right)^{-6/7}$$

$$= 2.98\times 10^{-1}\left(\frac{B}{0.1B_q}\right)^{3/7} P^{4/7} > 1$$

V1 < emf : limit of the unipolar induction

$$\frac{\ell_1}{R_{pc}} = 0.167\left(\frac{B}{B_q}\right)^{-4/7} P^{15/14} \sim 1$$



$\tau c < R$ の場合
$\tau c > R$ の場合
curvature
γ-ray can escape
all the power of
the accelerator
goes to pairs
$B_q$
$\tau c < \ell_1$ の場合
radiation drag limited
(long accelerator)
log (B_d [G])
log (P [sec])
16
15
14
13
12
11
10
8
7
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
RPP
binary
Fermi
RPP thermal
magnetar
CCO
XINS

# Pair Luminosity at the 1$^{st}$ generation

# Energy Flow in the cascade

How much fraction is radiated?

accelerator

$$L_1 = V_1 I = \begin{cases} \text{escaping particles} \\ L_{curv} = \begin{cases} \text{escaping gamma-ray} \\ L_p = \begin{cases} \text{escaping pairs} \\ L_x \\ L_h \text{hard X or soft-}\gamma \end{cases} \end{cases} \end{cases}$$

← observation

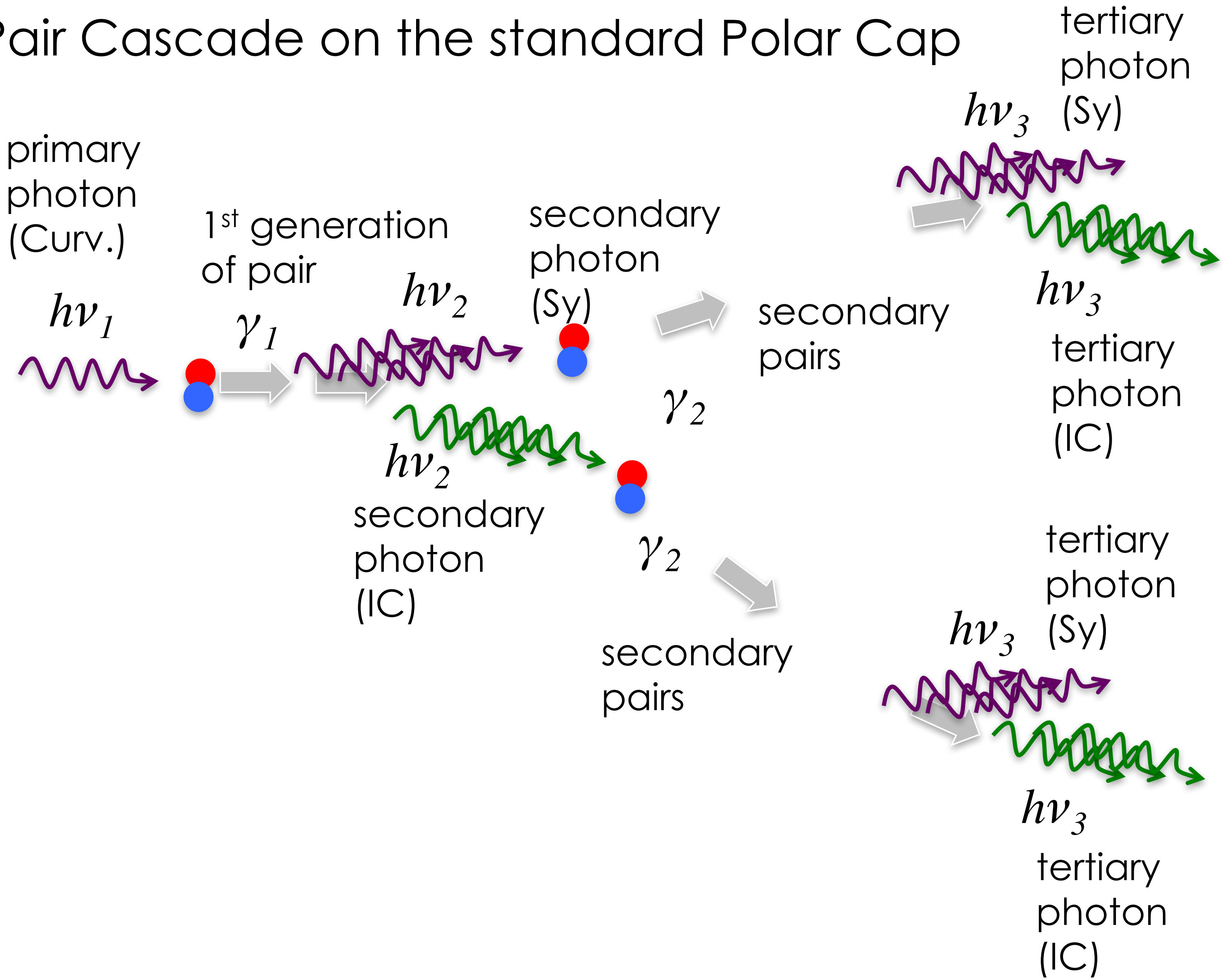
probably this part dominates particle parts

Pair Luminosity at 1st generation

$$L_p = \dot{N}_p mc^2 (\gamma_p - 1)$$

*Let us calculate $L_p$ first, then obtain $L_x$*

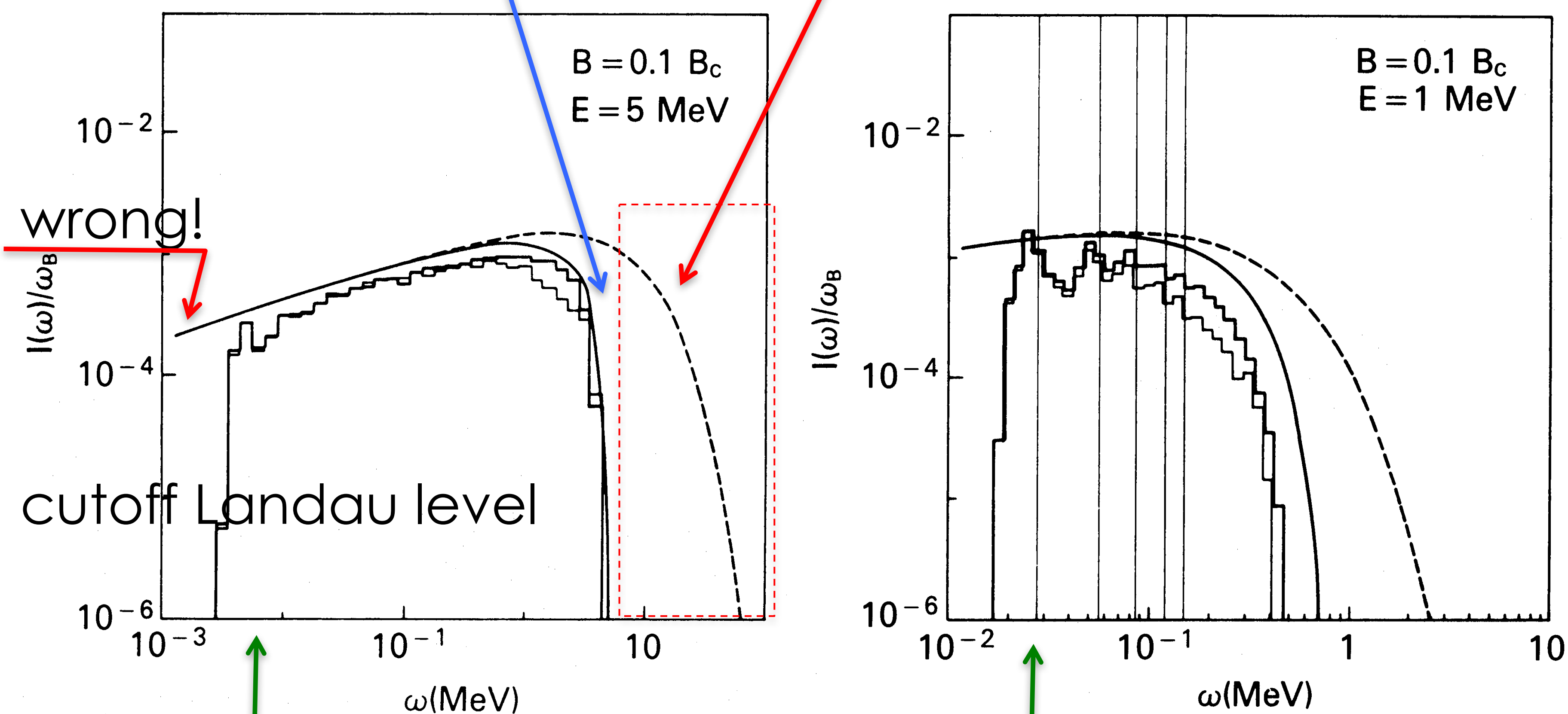
# Pair Cascade on the standard Polar Cap



# Synchrotron Radiation in high-B pulsars

cut off by $(\gamma\text{-}1)mc^2$

cutoff energy of classical formula critical frequency (wrong!)

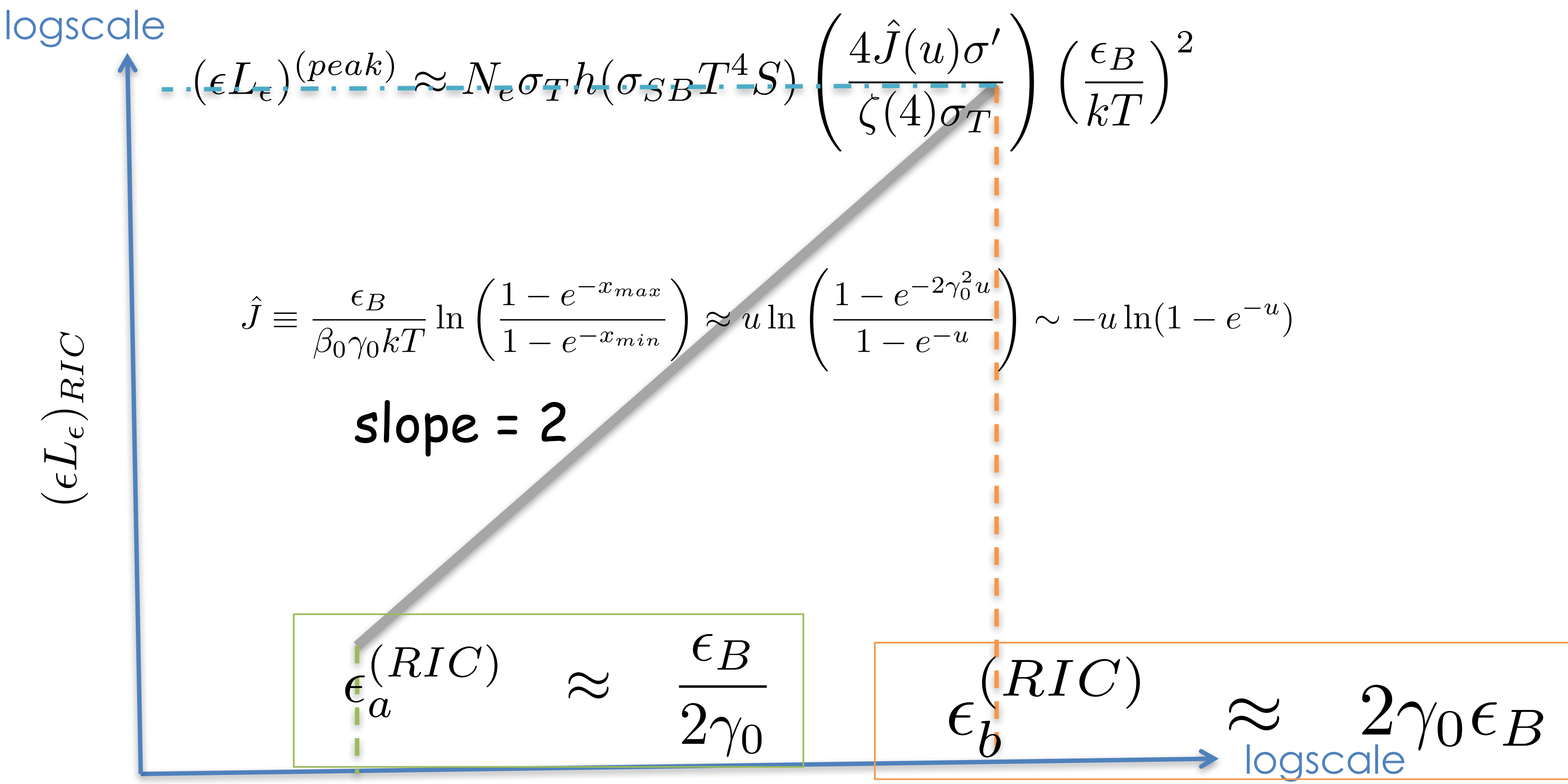



$$\epsilon_B = \hbar eB/mc = \hbar\omega_B = mc^2(B/B_q)$$

# SED of Resonant Inverse Compton Scattering

$$\epsilon_B = \hbar e B/mc = \hbar\omega_B = mc^2(B/B_q)$$

case BB(kT)+mono energetic beam $\gamma_0$



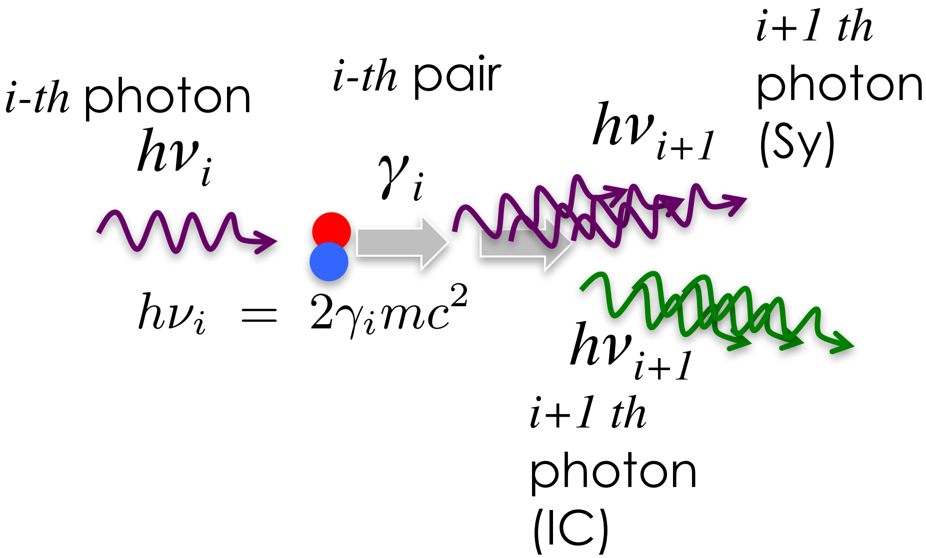

$$(\epsilon L_\epsilon)^{(peak)} \approx N_e \sigma_T h(\sigma_{SB} T^4 S)\left(\frac{4\hat{J}(u)\sigma'}{\zeta(4)\sigma_T}\right)\left(\frac{\epsilon_B}{kT}\right)^2$$

$$\hat{J} \equiv \frac{\epsilon_B}{\beta_0\gamma_0 kT}\ln\left(\frac{1-e^{-x_{max}}}{1-e^{-x_{min}}}\right) \approx u\ln\left(\frac{1-e^{-2\gamma_0^2 u}}{1-e^{-u}}\right) \sim -u\ln(1-e^{-u})$$

$$\epsilon_a^{(RIC)} \approx \frac{\epsilon_B}{2\gamma_0}$$

$$\epsilon_b^{(RIC)} \approx 2\gamma_0\epsilon_B$$

## Generation relation




### Cascade rule

**Synchrotron blanch**

$$h\nu_{i+1} = \frac{3\chi}{4} h\nu_i$$

**RIC blanch**

$$h\nu_{i+1} = \eta_{\parallel} \frac{B}{B_q} h\nu_i$$

$$\eta_{\parallel} = \frac{1}{\left(1 + [\chi B_q/B]^2\ \right)^{1/2}}$$

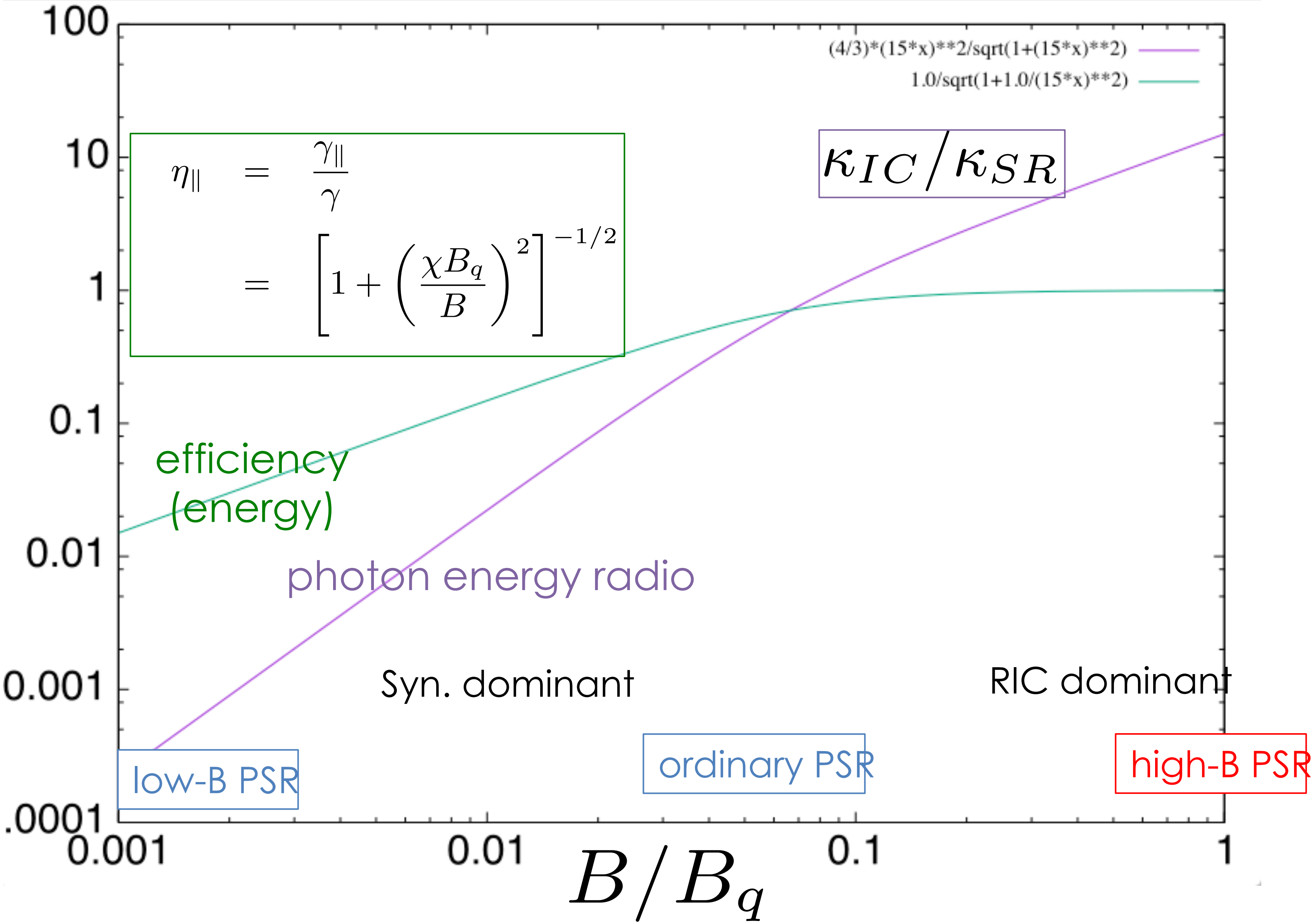
$$h\nu_{i+1} = \kappa h\nu_i$$

とおいて、$\kappa_{SR}$ と$\kappa_{IC}$を定義

### Energy blanching

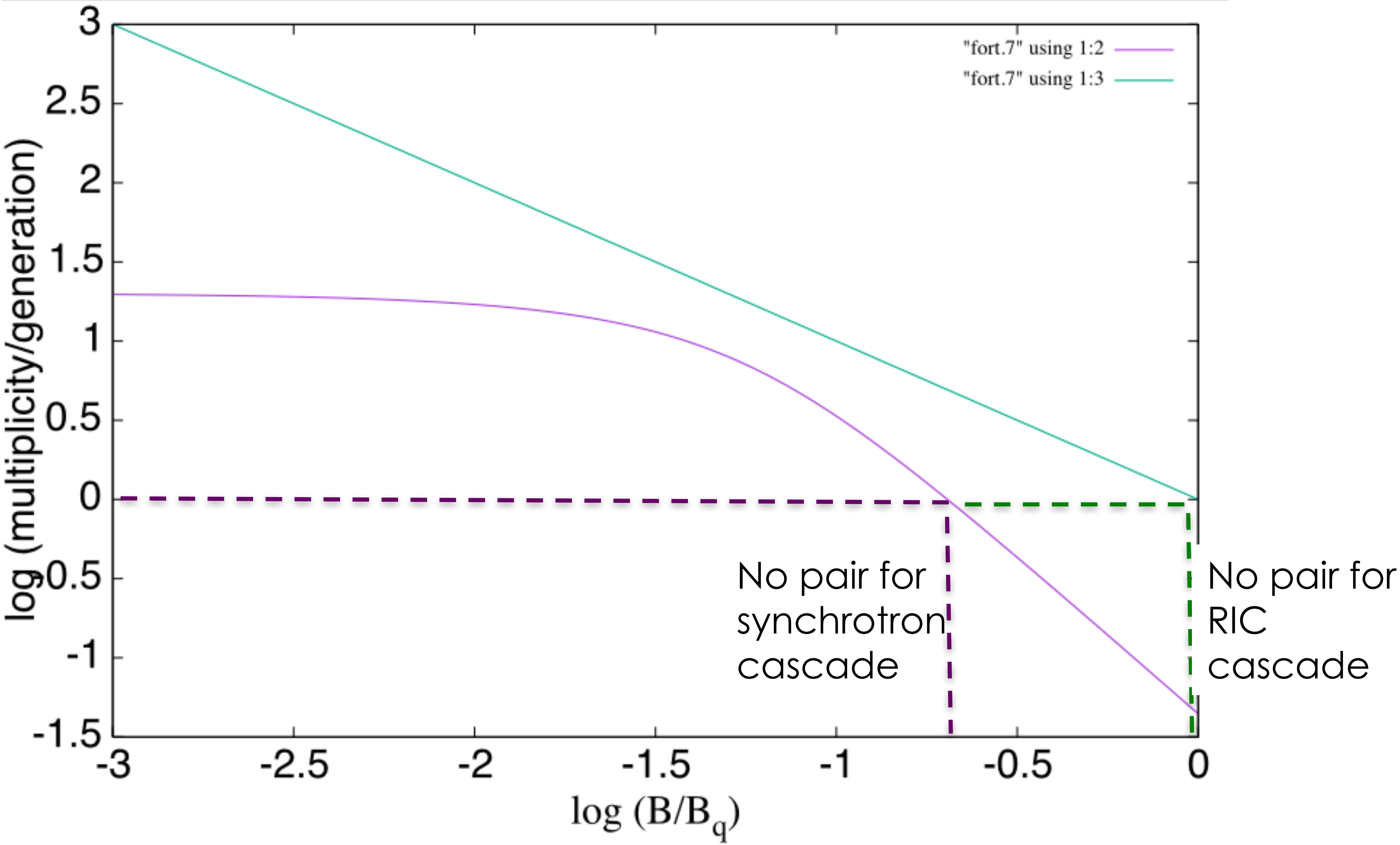
$$h\nu_i = \begin{cases} \eta_{\perp} h\nu_i = (1 - \eta_{\parallel}) h\nu_i & \text{Synchrotron photon energy} \\ \eta_{\parallel} h\nu_i & \text{RIC photon energy} \end{cases}$$

# Resonant IC vs Synchrotron Radiation



## Multiplicity per generation

$$\mu = \begin{cases} \frac{4}{3\chi}(1-\eta_{\parallel}) = \frac{4}{3\chi}\left(1 - \frac{1}{[1+(\chi B_q/B)^2]^{1/2}}\right) & \text{Synchrotron induced pairs} \\ B_q/B & \text{RIC induced pairs} \end{cases}$$



# Cascade and its termination

$h\nu_{i+1} = \kappa h\nu_i$ とすると

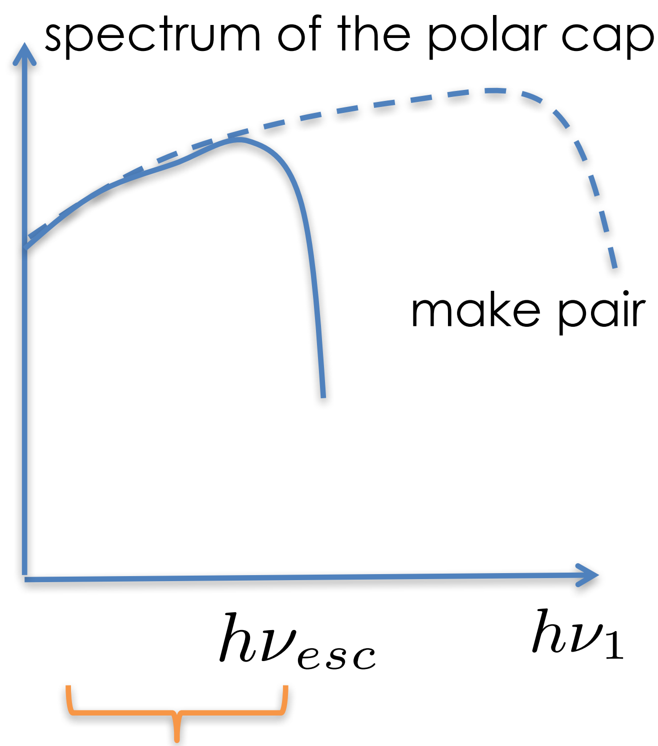
$$h\nu_k = \kappa^{k-1} h\nu_1$$

escape photon without pair creatoin

$$h\nu_{esc} = 2\chi \frac{B_q}{B} \frac{R_c}{\ell_p} mc^2 = 2\chi \frac{B_q}{B} \frac{R_c}{R/3} mc^2$$

$h\nu_{esc}$

$$\frac{h\nu_{esc}}{2mc^2} = 138 \left( \frac{0.1 B_q}{B} \right) P^{1/2} r_c$$

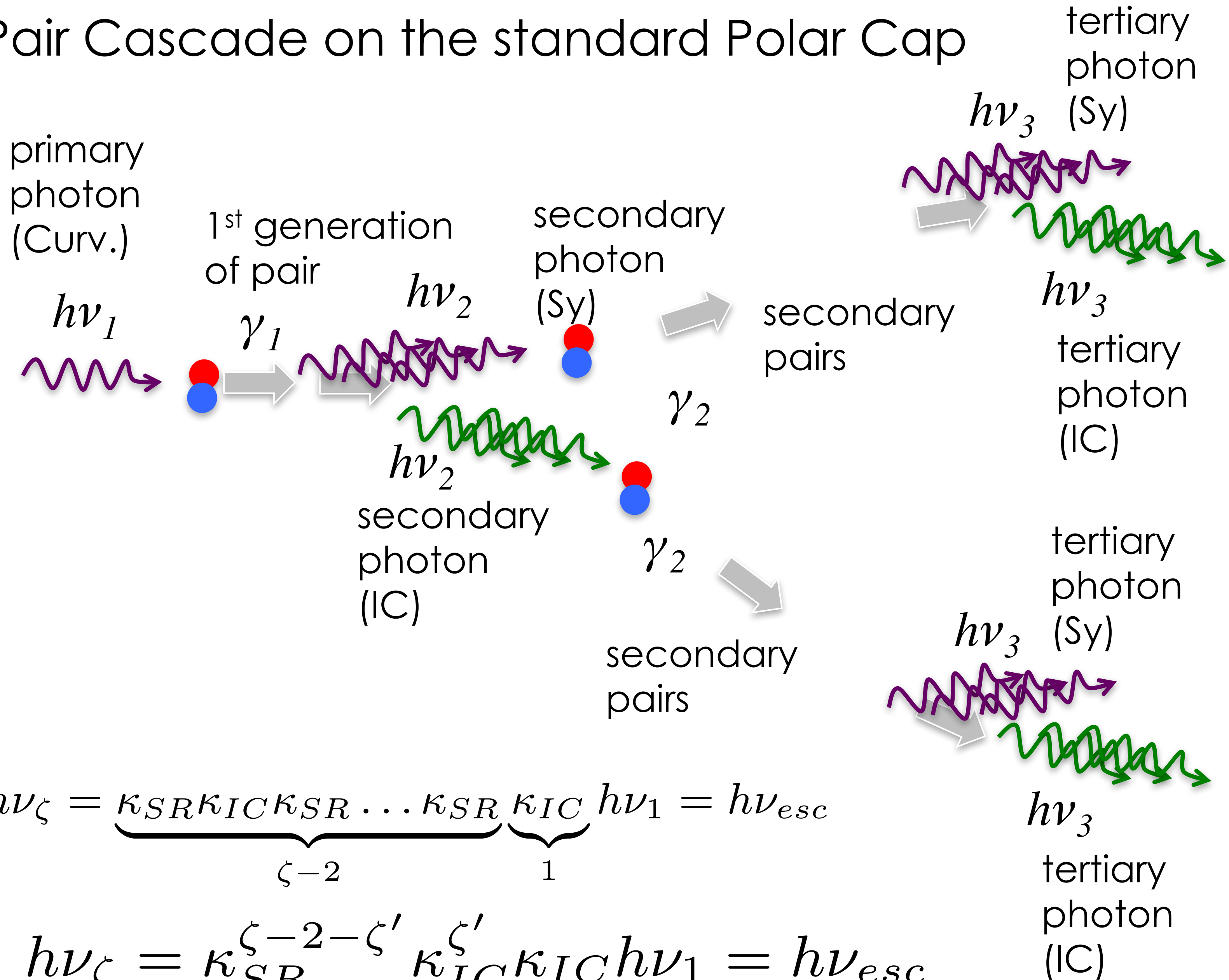



$$h\nu_\zeta = \kappa^{\zeta-1} h\nu_1 = h\nu_{esc}$$

escape
i.e. observed

$\zeta$ : generation parameter

# Pair Cascade on the standard Polar Cap




$$h\nu_\zeta = \underbrace{\kappa_{SR}\kappa_{IC}\kappa_{SR}\ldots\kappa_{SR}}_{\zeta-2}\underbrace{\kappa_{IC}}_{1} h\nu_1 = h\nu_{esc}$$

$$h\nu_\zeta = \kappa_{SR}^{\zeta-2-\zeta'}\kappa_{IC}^{\zeta'}\kappa_{IC} h\nu_1 = h\nu_{esc}$$

# RICを放射するすべてのブランチを数え上げる

( SR + RIC )$^{\zeta\text{-}1}$

$$\eta_{IC}(\zeta') = \eta_\perp^{\zeta-2-\zeta'}\eta_\parallel^{\zeta'+1}$$

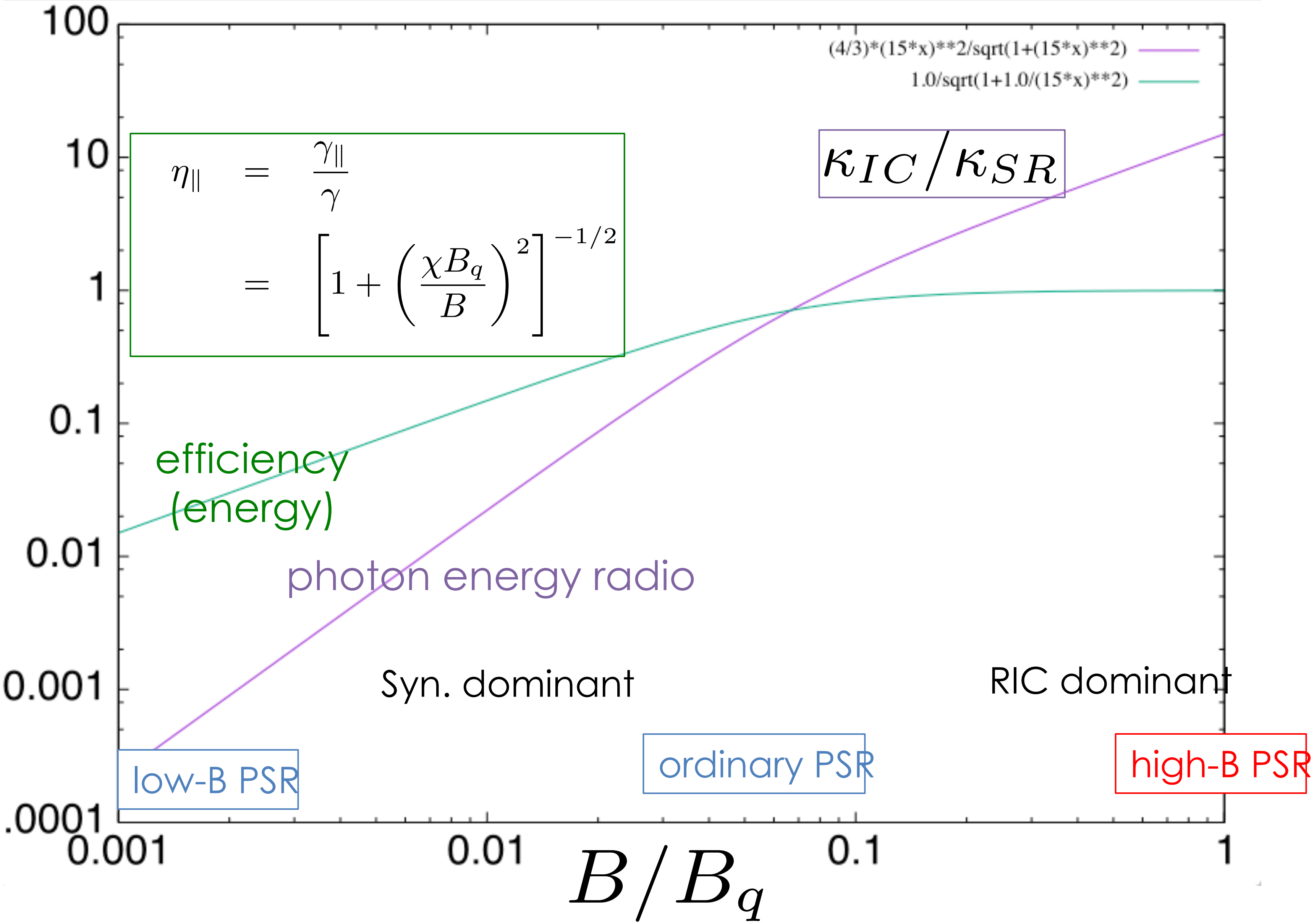
$$\kappa_{SR}^{\zeta}\left(\frac{\kappa_{IC}}{\kappa_{SR}}\right)^{\zeta'} = \frac{h\nu_{esc}}{h\nu_1}\frac{\kappa_{SR}^2}{\kappa_{IC}}$$

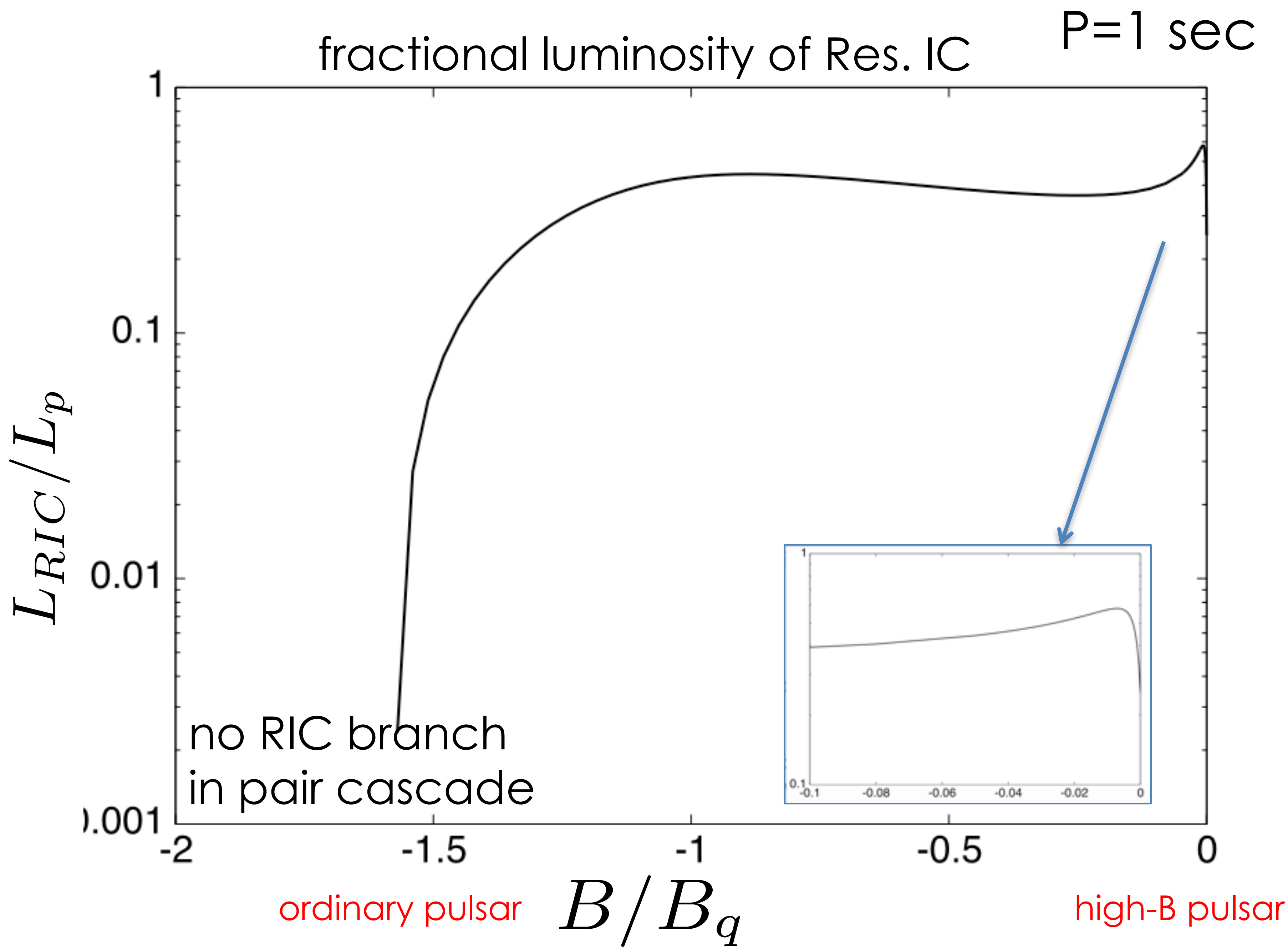
$$\begin{aligned} a\,\zeta + b\,\zeta' &= c \\ a &= \ln\kappa_{SR} \\ b &= \ln(\kappa_{IC}/\kappa_{SR}) \\ c &= \ln\left(\frac{h\nu_{esc}}{h\nu_1}\frac{\kappa_{SR}^2}{\kappa_{IC}}\right) \end{aligned}$$

Fractional Luminosity of RIC

$$\begin{aligned} \eta_{IC} &= \sum_{\zeta'=0}^{(c-2a)/(a+b)} \frac{a+b}{a}\frac{\Gamma(\zeta-1)}{\Gamma(\zeta'+1)\Gamma(\zeta-1-\zeta')}\eta_\perp^{\zeta-2-\zeta'}\eta_\parallel^{\zeta'+1} \\ &\approx \int_0^{(c-2a)/(a+b)} \frac{a+b}{a}\frac{\Gamma(\zeta-1)}{\Gamma(\zeta'+1)\Gamma(\zeta-1-\zeta')}\eta_\perp^{\zeta-2-\zeta'}\eta_\parallel^{\zeta'+1}d\zeta' \\ &= (1+\frac{b}{a})\eta_\perp^{c/a}\eta_\parallel\int_0^{(c-2a)/(a+b)} \frac{\Gamma(-\frac{b}{a}\zeta'+\frac{c}{a}-1)}{\Gamma(\zeta'+1)\Gamma(-(\frac{b}{a}+1)\zeta'+\frac{c}{a})}\eta_\perp^{-(b/a+1)\zeta'}\eta_\parallel^{\zeta'}d\zeta' \end{aligned}$$

# Resonant IC vs Synchrotron Radiation





P=1 sec
fractional luminosity of Res. IC
1
0.1
0.01
).001
$L_{RIC}/L_p$
-2
-1.5
-1
-0.5
0
$B/B_q$
no RIC branch
in pair cascade
ordinary pulsar
high-B pulsar

# In the Future

We obtain an analytic form of the RIC luminosity for the standard polar cap model.

In the next step,

1. get spectrum (so Lx) and multiplicity as function of P, B and Rc to understand large variation in Lx/Lrot and correlation between Lx(PSR) and Lx(PWN)
2. Confirm by numerical simulations.
3. How about non standard polar cap model, i.e., high-B pulsars and magnetars, MSPSRs
4. Advance precise observations so that clearer spectrum (thermal / non-thermal) as functions of phase