

数値解析による

非一様カイラル相転移

Nobutoshi Yasutake (Chiba Inst. Tech.)

**Lee Tong Gyu, Toshiki Maruyama, Toshitaka Tatsumi
Takeshi Mizushima**

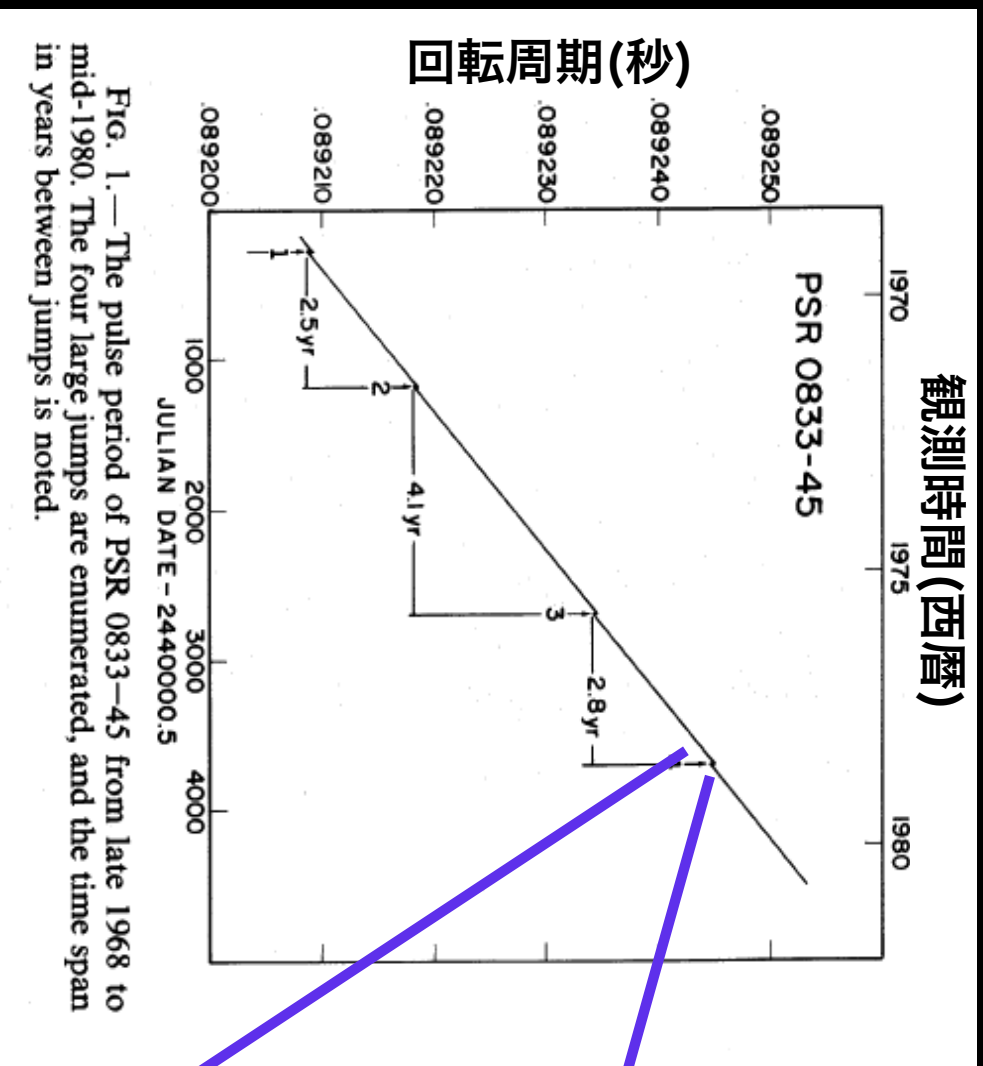


Pulsar Astrophysics: The Next Fifty Years

IAU Symposium 337 - 4th-8th September 2017 - Jodrell Bank Observatory, University of Manchester

パルサーグリッチ

一定の割合でスピンダウンしているパルサーが、あるとき不連続的にスピンアップする現象
(蟹パルサー、帆座パルサー etc.)



spin up 時間(~2分)

緩和時間(10h ~ 1ヶ月)

緩和時間が長い (常流動の陽子、中性子散乱の特徴的なタイムスケールは 10^{-11} s)
ことが内部物質の超流動の存在を示唆

常流動

超流動/超伝導



外殻 ($4 \times 10^{11} \text{g/cc}$ 以下)

原子核同士が格子を組んでいる。

内殻 ($4 \times 10^{11} \text{g/cc} \sim 0.5 \rho_0$)

neutron drip \sim subnuclear density

中性子の海の中に、中性子過剰核の非一様構造。

中心核 ($0.5 \rho_0 \sim$)

ほぼ中性子に占められる。陽子は約5%。さらに高密度の場合は、ハイペロンやクォークなどが存在する可能性も有り。

中性子星

これらが別々の角速度で回転している。渦糸の芯には中性子過剰原子核がピン止めされている。このピン止めが外れ内殻超流動物質の角運動量が外殻へ伝搬する。

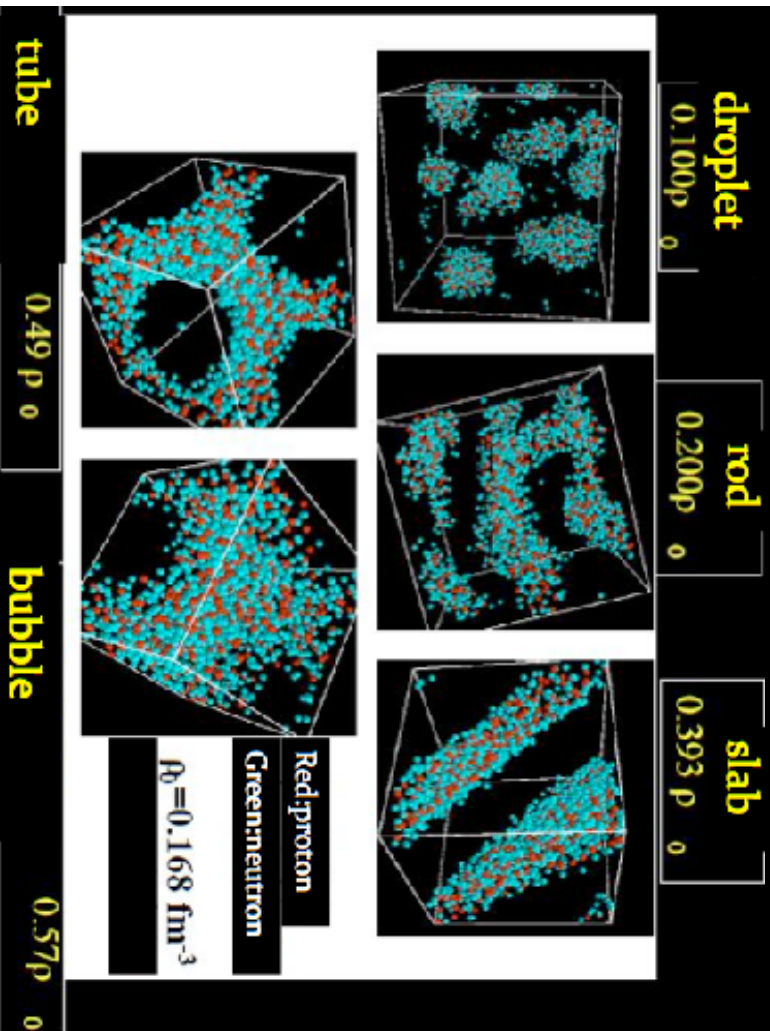
Anderson & Itoh 1975

中性子の超流動起源のグリッチ否定論文が出るなどして問題とされていた。

Chamel PRC(2012)

内部磁場や物質、温度の進化を考えるとときに避けては通れない。

Channel に対しての答え → “パイスタ構造”



中性子dripの第一原理計算

Sonoda et al. 2008

BdG equation

$$\begin{pmatrix} \tilde{H}'_{\mathbf{q}}(\mathbf{r}) & \tilde{\Delta}(\mathbf{r}) \\ \tilde{\Delta}(\mathbf{r})^* & -\tilde{H}'_{-\mathbf{q}}(\mathbf{r}) \end{pmatrix} \begin{pmatrix} \tilde{u}_i(\mathbf{r}) \\ \tilde{v}_i(\mathbf{r}) \end{pmatrix} = \epsilon_i \begin{pmatrix} \tilde{u}_i(\mathbf{r}) \\ \tilde{v}_i(\mathbf{r}) \end{pmatrix}. \quad (5)$$

$$\tilde{H}'_{\mathbf{q}}(\mathbf{r}) = \frac{1}{2m} (-i\nabla + \mathbf{Q} + \mathbf{k})^2 + V_{\text{ext}}(\mathbf{r}) - \mu,$$

$$\tilde{\Delta}(\mathbf{r}) = -g \sum_i \tilde{u}_i(\mathbf{r}) \tilde{v}_i^*(\mathbf{r}),$$

$$V_{\text{ext}}(\mathbf{r}) = V_K (e^{iKz} + e^{-iKz}),$$

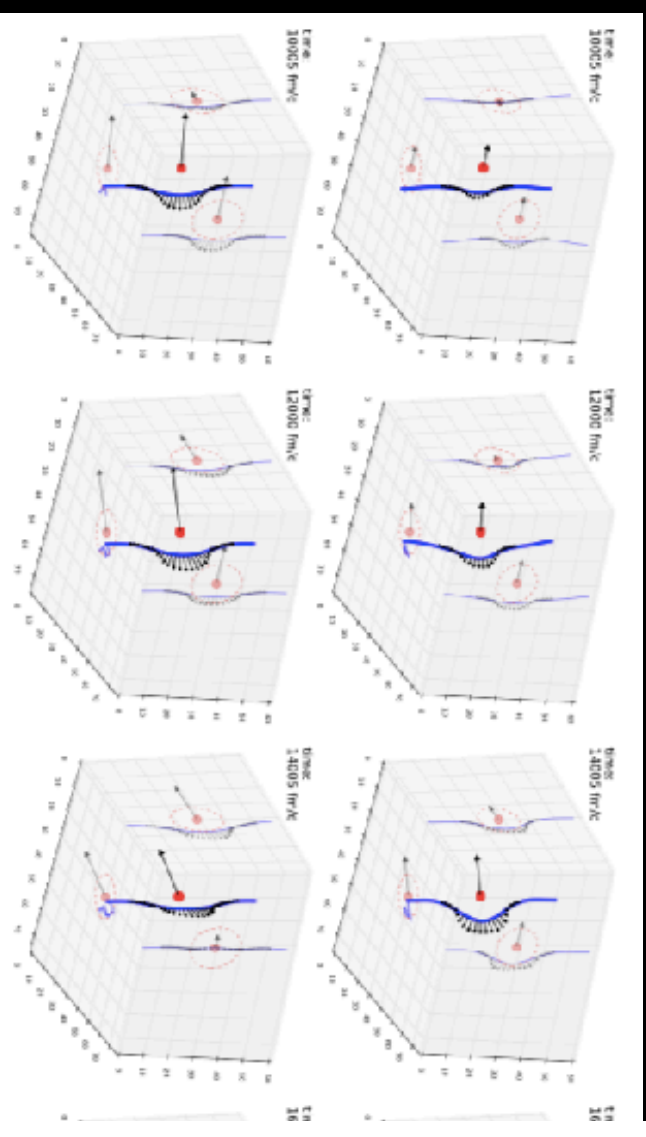
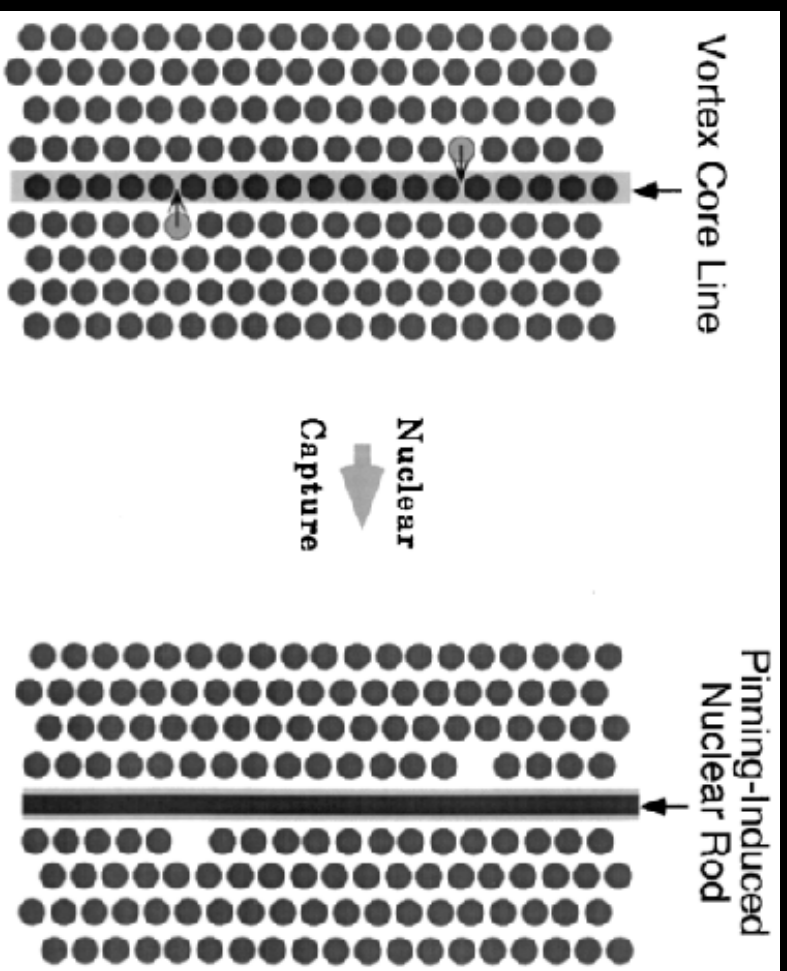
一次元的な外場
(板状)を仮定

パイスタ構造中を考えると超流動計算

→ クラスト超流動起源のグリッチ否定不可

Watanabe & Pethick (2017) PRL

常識 → 非常識？



ピン止めが非一様構造 (rod) を誘発する

Mochizuki, Oyamatsu, Izuyama, 1995, ApJ

ちゃんと超流動計算 (TDDFT)

→渦は原子核に捉われるというわけではない

Wlazlowski et al.(2016) PRL

常識ついでに

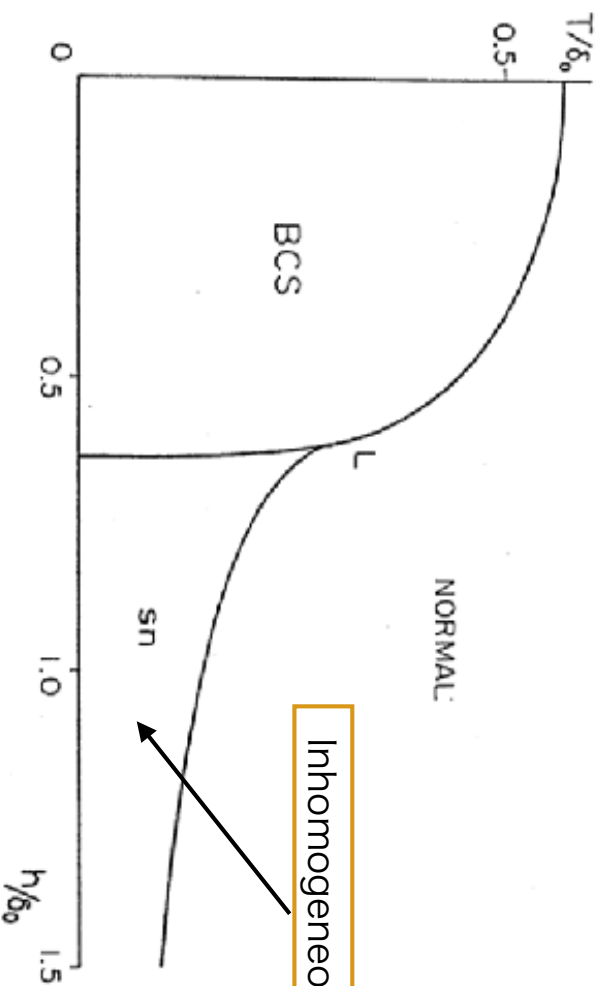
低温で弱磁場になると超流動になる。

→不十分。物性では既知。

Inhomogeneous phase in condensed matter physics

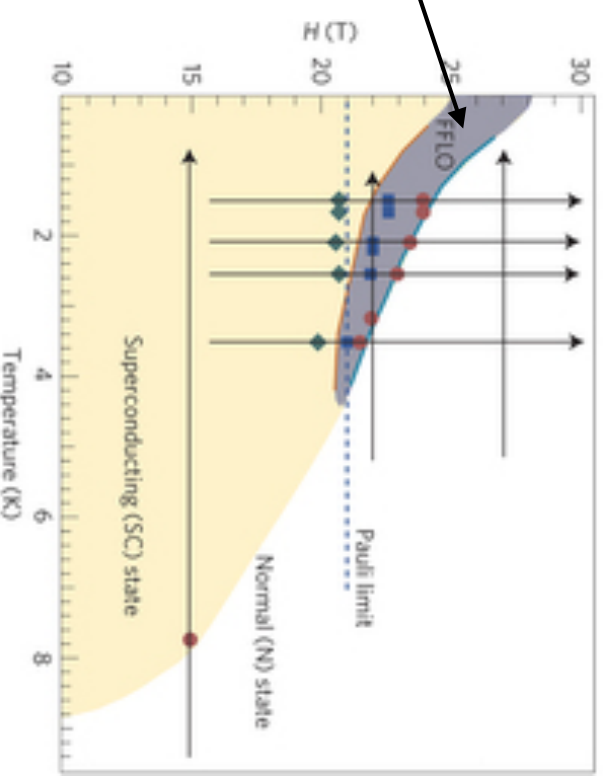
BdG equationを空間的な自由度を与えて解くと...

“Superconductivity under a ferromagnetic molecular field”



Machida-Nakanishi (1984) PRB

“Evidence of Andreev bound states as a hallmark of the FLO phase in k -(BEDT-TTF)₂Cu(NCS)₂”



Mayaffre et al.(2014)Nature

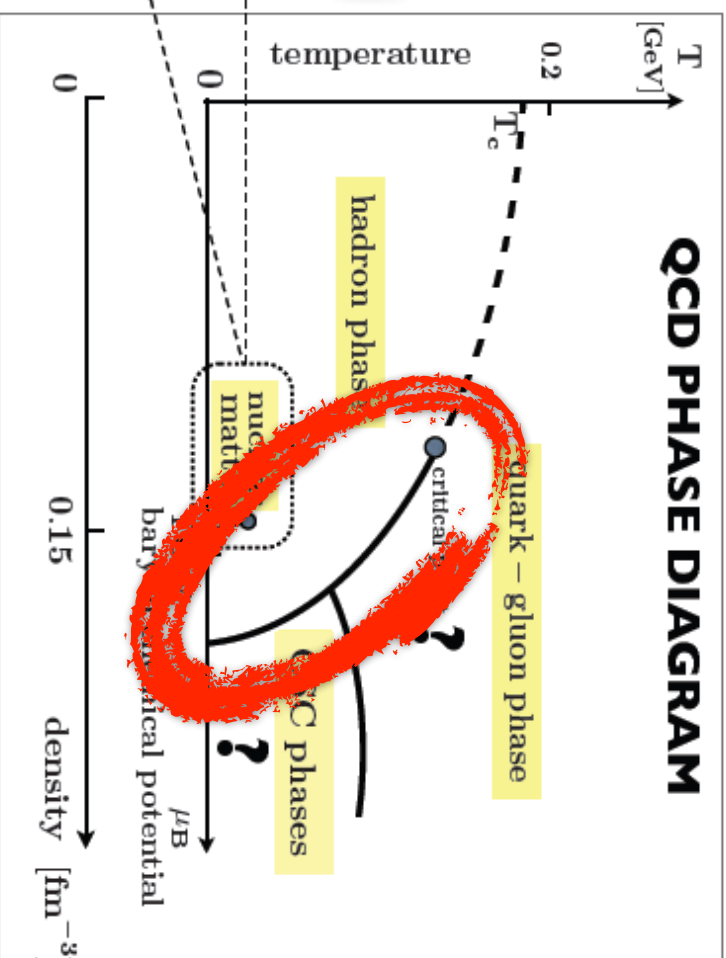
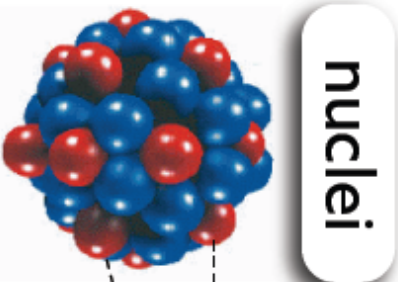
前置き終わり

BdG equation

$$\begin{pmatrix} \tilde{H}'_{\mathbf{Q}}(\mathbf{r}) & \tilde{\Delta}(\mathbf{r}) \\ \tilde{\Delta}(\mathbf{r})^* & -\tilde{H}'_{-\mathbf{Q}}(\mathbf{r}) \end{pmatrix} \begin{pmatrix} \tilde{u}_i(\mathbf{r}) \\ \tilde{v}_i(\mathbf{r}) \end{pmatrix} = \epsilon_i \begin{pmatrix} \tilde{u}_i(\mathbf{r}) \\ \tilde{v}_i(\mathbf{r}) \end{pmatrix}. \quad (5)$$

1 Prelude: PHASES and STRUCTURES of QCD

... the goal:



Scales
in
nuclear matter

- momentum scale:
- **Fermi momentum**
- NN distance:
- energy per nucleon:
- compression modulus:

$$k_F \approx 1.4 \text{ fm}^{-1} \sim 2m_\pi$$
$$d_{NN} \approx 1.8 \text{ fm} \approx 1.3 m_\pi^{-1}$$
$$E/A \approx -16 \text{ MeV}$$
$$K = (260 \pm 30) \text{ MeV}$$



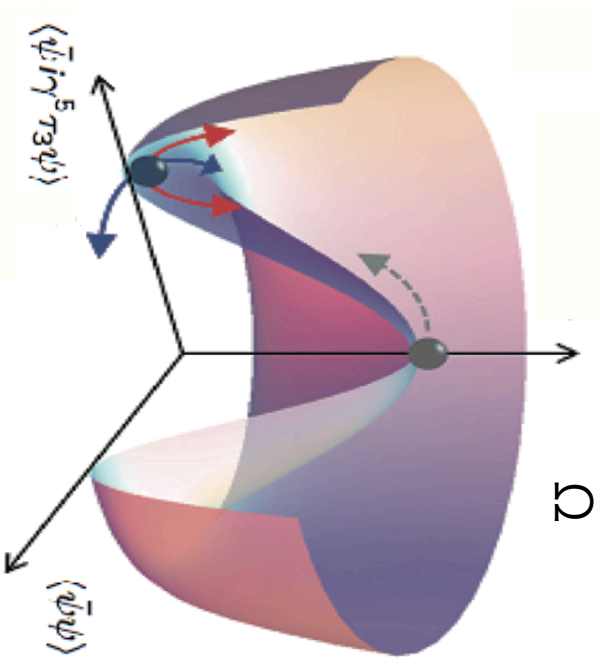
Chiral phase transition with Spatial dependency

NJL model

$$\mathcal{L}_{NJL} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi + G \left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2 \right)$$

Condensations can have spatial dependency, generally.

$$\langle \bar{\psi}\psi \rangle = S(\mathbf{x}), \quad \langle \bar{\psi}i\gamma^5\tau_3\psi \rangle = P(\mathbf{x})$$



Impact on astrophysics

- Cooling ratio of inhomogeneous phase (DCCDW)

$$\epsilon_{\text{DCCDW}} \simeq 6.1 \times 10^{26} (\rho_B / \rho_0)^{2/3} Y_e^{2/3} T_9^6 \quad (\text{erg} \cdot \text{cm}^{-3} \cdot \text{s}^{-1}), \quad \text{Muto \& Tatsumi (2014)}$$

→ It can be seen by Xray observations.



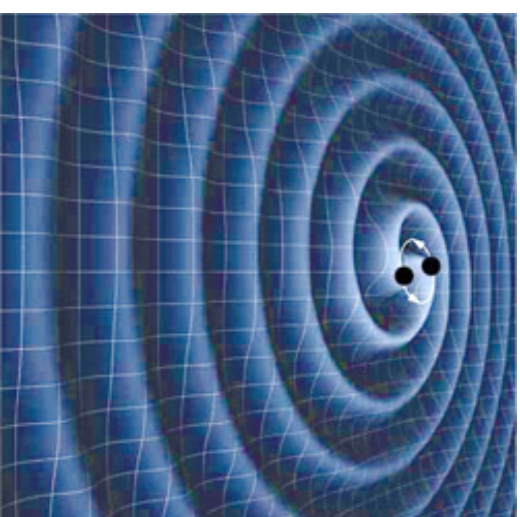
- Ferromagnetism (DCCDW)

Yoshiike & Tatsumi (2015)

- EOS of inhomogeneous phase

→ Inhomogeneous phase will change EOS.

→ It will be observed by **gravitational wave**.



1 + 1 Dimension

NJL₂ model (Gross-Neveu model)

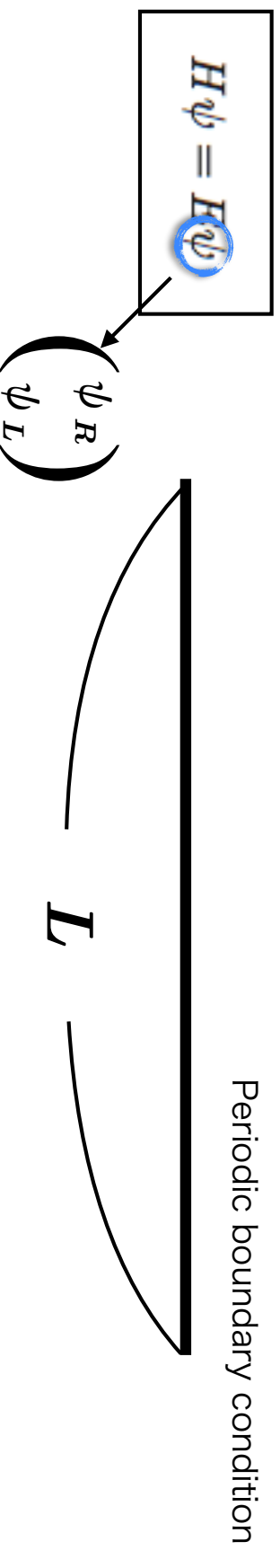
$$\mathcal{L}_{\text{NJL}} = \bar{\psi} i \not{\partial} \psi + \frac{g^2}{2} [(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma^5 \psi)^2] \quad \mathcal{L}_{\text{GN}} = \bar{\psi} i \not{\partial} \psi + \frac{g^2}{2} [(\bar{\psi} \psi)^2]$$

$$\Leftrightarrow \mathcal{L} = \bar{\psi} \left[i \not{\partial} - \frac{1}{2}(1 - \gamma^5) \Delta - \frac{1}{2}(1 + \gamma^5) \Delta^* \right] \psi - \frac{1}{2g^2} |\Delta|^2,$$

Corresponding Hamiltonian

$$H = -i \gamma^5 \frac{d}{dx} + \gamma^0 \left(\frac{1}{2}(1 - \gamma^5) \Delta - \frac{1}{2}(1 + \gamma^5) \Delta^* \right) \quad \text{Or} \quad H = \begin{pmatrix} -i \frac{d}{dx} & \Delta(x) \\ \Delta^*(x) & i \frac{d}{dx} \end{pmatrix}$$

\Leftrightarrow Bogoliubov-de Gennes (BdG) equation **物性などで超流動を記述する式**



BdG eq. で予言された非一様超流動状態
→ 実験で発見

Dirac eq. で予言された非一様カイラル相転移
→ 観測(GW?)で発見されるはず

Numerical calculation of inhomogeneous chiral phase

$$\langle \bar{\psi} \psi \rangle = S(\mathbf{x}), \quad \langle \bar{\psi} i\gamma^5 \tau_3 \psi \rangle = P(\mathbf{x})$$

Chiral density wave: $S(\mathbf{x}) \sim A \cos(\mathbf{q} \cdot \mathbf{x})$, $P(\mathbf{x}) \sim A \sin(\mathbf{q} \cdot \mathbf{x})$
 Dirac solutions: $\delta(\mathbf{x}) \sim \sum_{\mathbf{q}} \cos(\mathbf{q} \cdot \mathbf{x})$, $\delta(\mathbf{x}) \sim \sum_{\mathbf{q}} \sin(\mathbf{q} \cdot \mathbf{x})$

NJL₂ model

$$\mathcal{L}_{\text{NJL}} = \bar{\psi} i \not{\partial} \psi + \frac{g^2}{2} [(\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma^5 \psi)^2]$$

Periodic boundary condition



Dirac (BdG) equation by pseudo spectral method

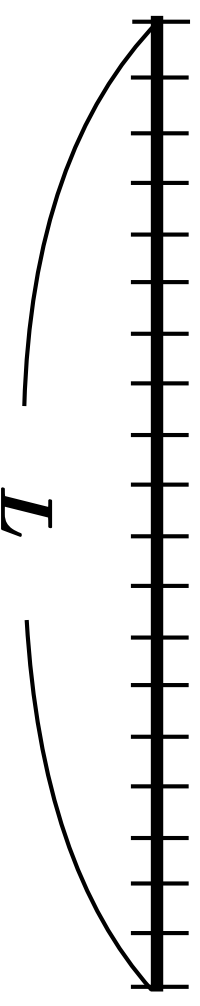
$$\hat{\sigma}(\mathbf{x}) = \sum_{i=1}^N \langle \bar{\psi}_i \psi_i \rangle = \sum_{n=-10}^{10} c_n e^{i n x}$$

$$\hat{\eta}(\mathbf{x}) = \sum_{i=1}^N \langle \bar{\psi}_i i \gamma^5 \psi_i \rangle = \sum_{n=-10}^{10} d_n e^{i n x}$$

Cf) Heintz et al. (2016)

$$H \psi = E \psi$$

Periodic ($N \rightarrow 0$)
 0 1 2 3 $N (\sim 10^2)$



Energy spectrum

$$\Delta(x)$$

Dual Chiral Density Wave (DCDW)

$$S(\mathbf{x}) \sim \Delta \cos(\mathbf{q} \cdot \mathbf{x}), \quad P(\mathbf{x}) \sim \Delta \sin(\mathbf{q} \cdot \mathbf{x})$$



⇔ FF type (Fulde-Ferrell 1964)

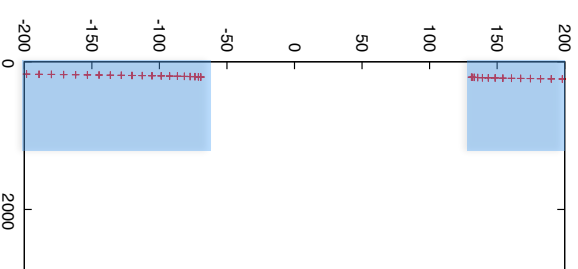
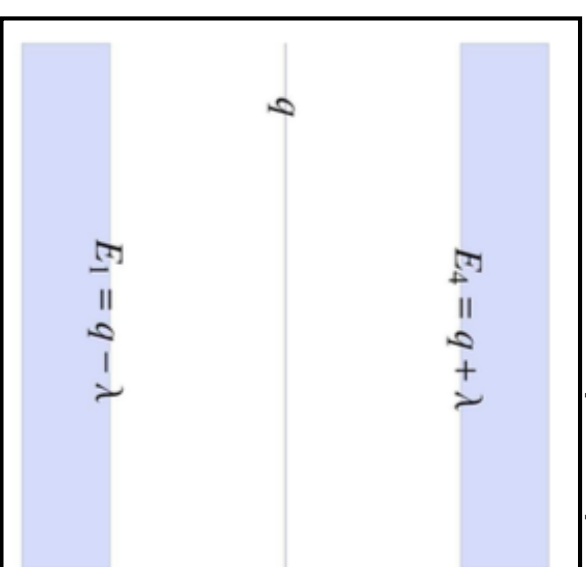
Real Kink Cristal (RKC)

$$S(\mathbf{x}) \sim \Delta \nu \operatorname{sn}(\Delta z, \nu), \quad P(\mathbf{x}) = 0$$

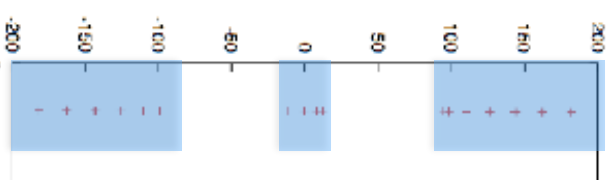
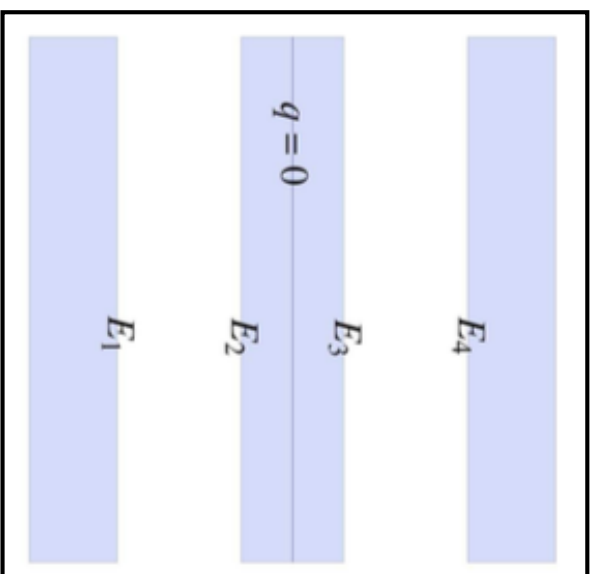


⇔ LO type (Larkin-Ovchinnikov 1964)

Schematic pictures of analytic solutions
Basar et al. (2009)



Our numerical calculation

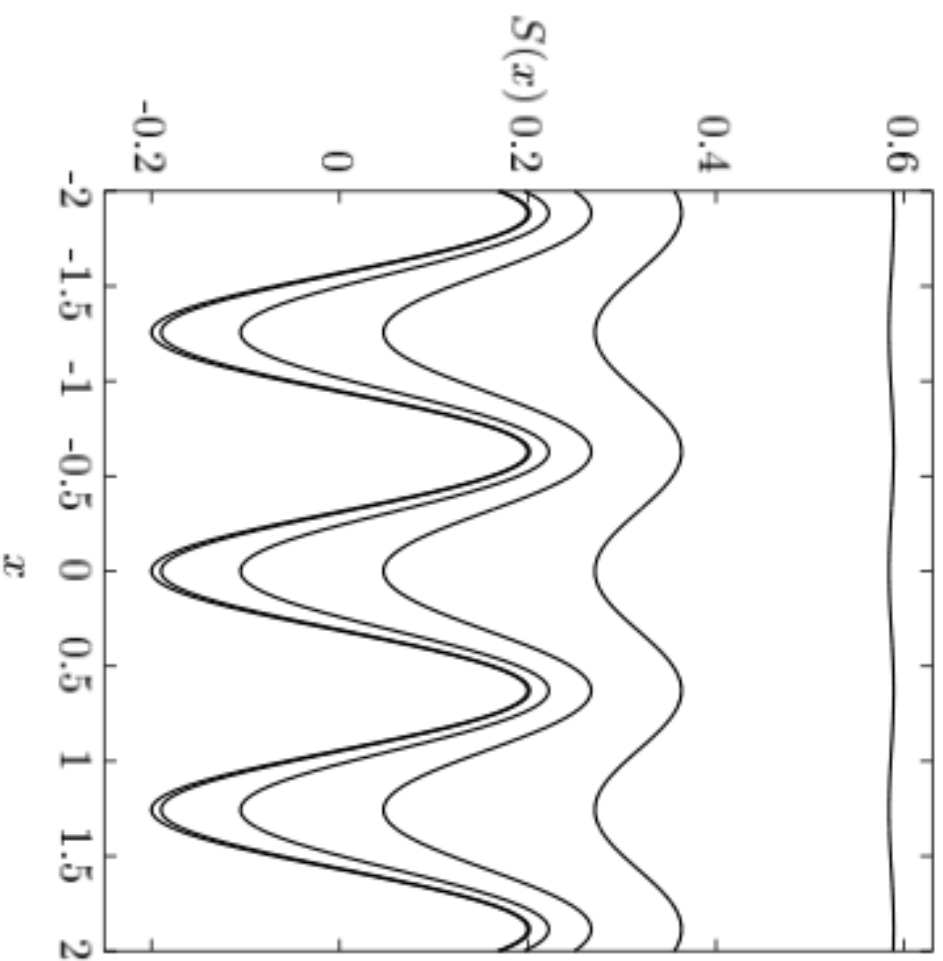


Mass dependency on Δ

Real Kink Cristal (RKC)

$$S(\mathbf{x}) \sim \Delta \nu \operatorname{sn}(\Delta z, \nu), \quad P(\mathbf{x}) = 0$$

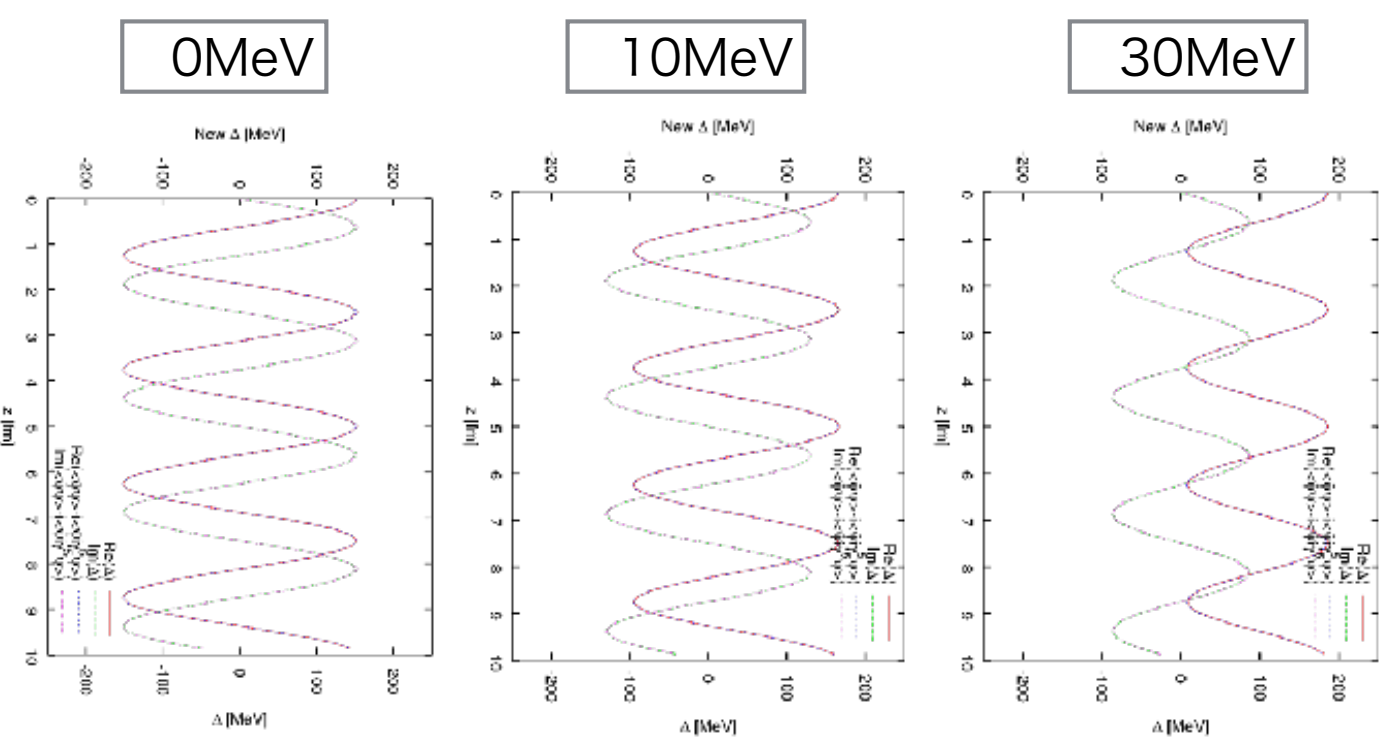
(Thies & Ulrich 2006)



カレント質量

Numerical approach $S(\mathbf{x}), P(\mathbf{x}) \neq 0$
Our calculation

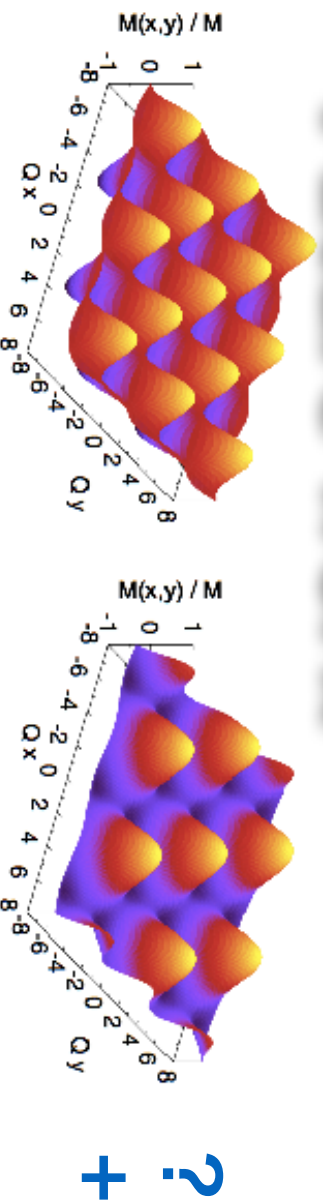
カレント質量



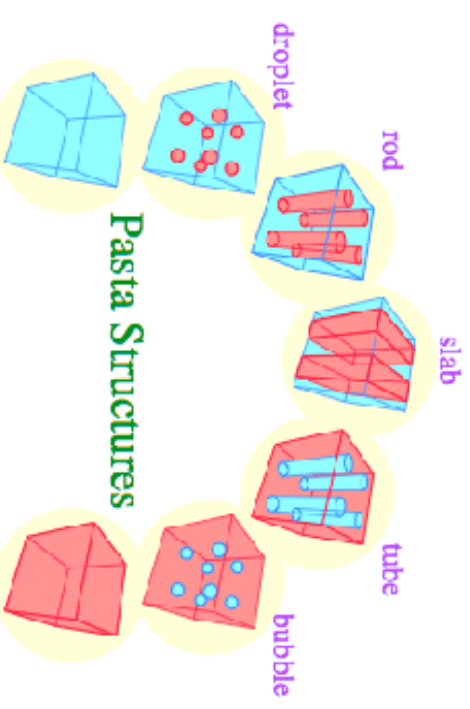
現状の結果

- 数値的な手法(スペクトル法)を用いてDirac eq.を解き、カイラル相転移(EOS)を解いた。
 - 超流動(BdG eq)で理論的に予言された非一様相転移が実験に見つかったようにカイラル相転移においても観測されることを期待。
- Heintz et al. (2016) + カリント質量の効果 自己無撞着解を数値的に得るまでにはできなかった。
- 超流動(グリッチ), EOS(MR関係やQCDC相転移), 冷却, 磁場の起源と進化, 全てを変える。

Future work



$$M(x, y) = M \cos(Q_x) \cos(Q_y), \quad M(x, y) = \frac{M}{3} \left[2 \cos(Q_x) \cos\left(\frac{1}{\sqrt{3}} Q_y\right) + \cos\left(\frac{2}{\sqrt{3}} Q_y\right) \right],$$



NY, Maruyama, Tatsumi (2009) PRD

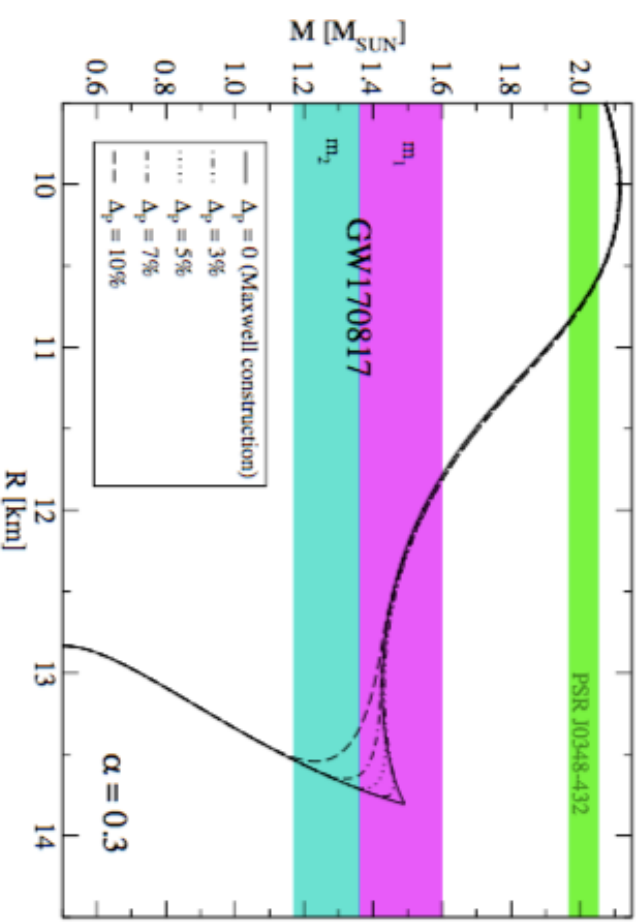
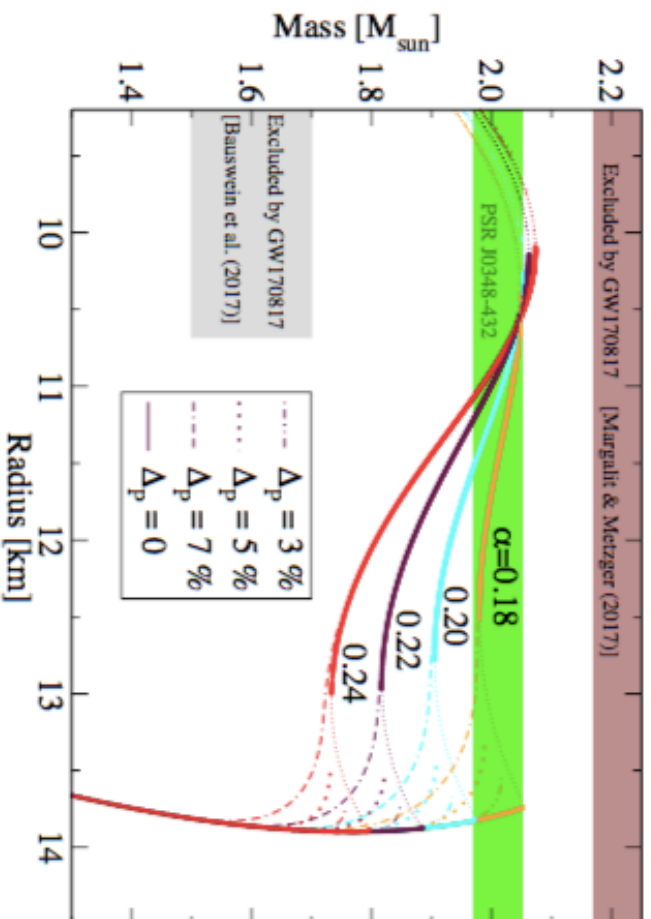
NY, Lastwieski, Sanjin, Blaschke,
Maruyama, Tatsumi (2014) PRC

Carignano & Buballa (2012) PRD

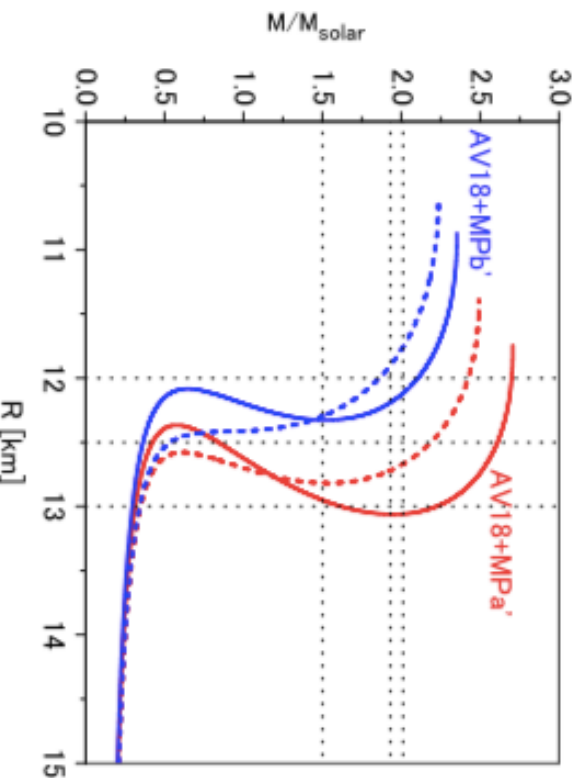
Watanabe & Pethick(2017) PRL と同じ式を厳密に解くことに等しい

• Quark hadron pasta EOS

Ayriyan et al.(2017) arXiv 1711.03926



- BHFや変分原理 (Lattice QCDやJPARCの相互作用を直接使えるEOS)



Yamamoto et al.(2017) arXiv 1708.06163

Maxwell eq. should be modified

Incera (CSQCD2016)

Axion Electrodynamics in DCDW in B

Ferrer & VI, 1512.03972 [nucl-th]

Anomalous E.C. due to the spectral asymmetry

$$\nabla \cdot \mathbf{E} = J_0 + \kappa \nabla \theta \cdot \mathbf{B},$$

$$\nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J}_V + \kappa \left(\frac{\partial \theta}{\partial t} \mathbf{B} + \nabla \theta \times \mathbf{E} \right),$$

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0,$$

$$0 = qz/2.$$

$$\kappa = \frac{e^2}{2\pi^2}$$

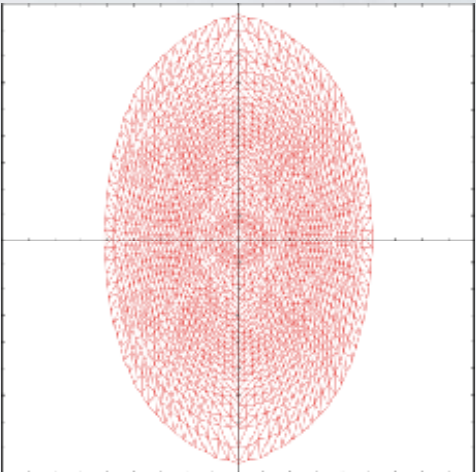
Anomalous Hall current: dissipationless, \perp to both B and E

We can see the anomaly from NSS !

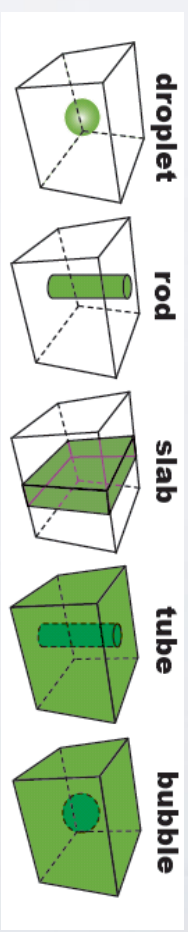
“DEFORMED STAR/PASTA DUALITY”

NY, FUJISAWA, YAMADA (2014) MNRAS LETTER, (2016) MNRAS

Deformed stars
(non-spherical stars)



Pasta structures which are non-uniform structures in phase transitions; neutron drip, quark-hadron phase transition, etc.)



Hydro-static equilibrium

$$\frac{\delta E[\xi]}{\delta \xi} = \nabla P + \rho \nabla \phi - \frac{\rho j^2}{\omega^3} \mathbf{e}_\omega = 0.$$

Chemical equilibrium

$$\begin{aligned} \mu_n &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_C^Q, & \mu_u &= \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_C^Q, \\ \mu_n &= \mu_n = \mu_B, & \mu_p &= \mu_B + \mu_{G,S}, & \mu_{\Sigma^-} + \mu_p &= 2\mu_B, \\ \mu_L^{H(Q)} &= \mu_{\Sigma_c}^{H(Q)}, & \mu_C^{H(Q)} &= \mu_L^{H(Q)} - \mu_e^{H(Q)}, \end{aligned}$$

ex) quark-hadron phase transition

repulsion = pressure and rotation
attraction = gravitation



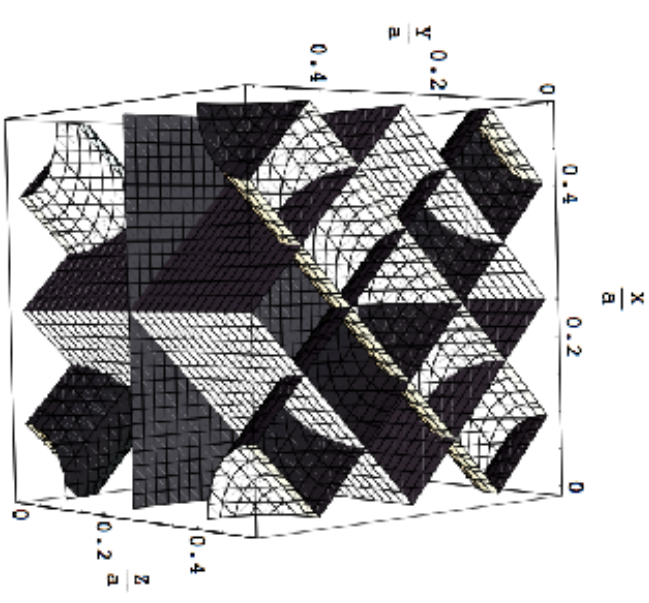
repulsion = Coulomb interaction
attraction = surface tension

Other topics of inhomogeneous phase

- Color SC crystals by GL approach

Rajagopal & Sharma (2006)

$$\begin{aligned} & \epsilon_{2\alpha\beta}\epsilon_{2ij} 2\Delta \left[\cos \frac{2\pi}{a} (x+y+z) + \cos \frac{2\pi}{a} (-x-y+z) \right] \\ & + \epsilon_{3\alpha\beta}\epsilon_{3ij} 2\Delta \left[\cos \frac{2\pi}{a} (-x+y+z) + \cos \frac{2\pi}{a} (x-y+z) \right] \end{aligned}$$



- FLO of SC in multi dimension (2D)

Matsuda & Shimahara (2007)

hexagonal: $\Delta(\mathbf{r}) = \Delta_1 (\cos(\mathbf{q}_1 \cdot \mathbf{r}) + \cos(\mathbf{q}_2 \cdot \mathbf{r}) + \cos(\mathbf{q}_3 \cdot \mathbf{r}))$

square: $\Delta(\mathbf{r}) = \Delta_1 (\cos(qx) + \cos(qy))$

triangular: $\Delta(\mathbf{r}) = \Delta_1 (e^{i\mathbf{q}_1 \cdot \mathbf{r}} + e^{i\mathbf{q}_2 \cdot \mathbf{r}} + e^{i\mathbf{q}_3 \cdot \mathbf{r}})$

