Generalized distribution amplitudes in two-photon process

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12th meeting on High-energy QCD and nucleon structure
KEK Tokai campus, Tokai, Japan, July 7, 2017
https://sites.google.com/a/quark.kj.yamagata-u.ac.jp/nucllecture/

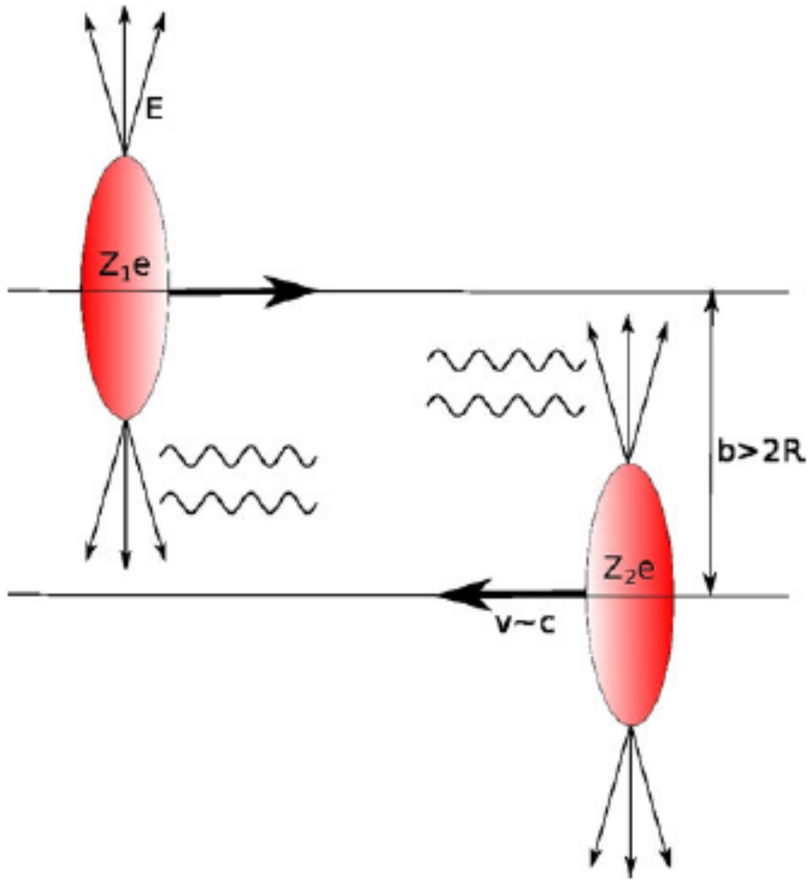
July 7, 2017

Ultra-Peripheral Collision (UPC)

INT Workshop INT-17-65W

Probing QCD in Photon-Nucleus Interactions at RHIC and LHC: the Path to EIC

February 13 - 17, 2017



Motivations

- **3D structure of hadrons**
- **Nucleon spin structure**
- **Exotic hadrons**

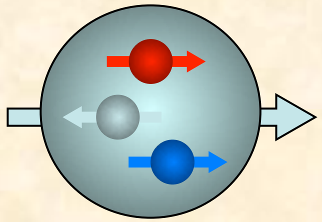
…

Hadron tomography: 3D structure functions are (can be) investigated at at high-energy lepton and hadron facilities (BNL, JLab, Fermilab, CERN, J-PARC, KEKB, GSI, IHEP@China & Russia, EIC, LHeC, ILC, ...).

Here, I discuss hadron tomography by $\gamma\gamma \to h\bar{h}$, experimentally possible at KEKB and ILC.

Recent progress on origin of nucleon spin

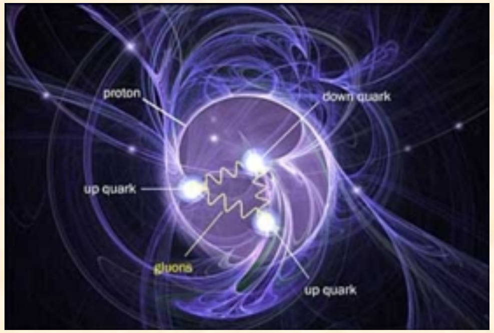
"old" standard model



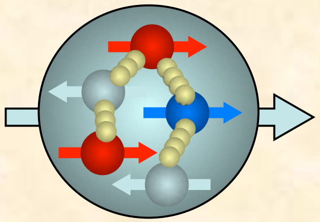
$$p_{\uparrow} = \frac{1}{3\sqrt{2}}\left(uud\left[2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right] + \text{permutations} \right)$$

$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

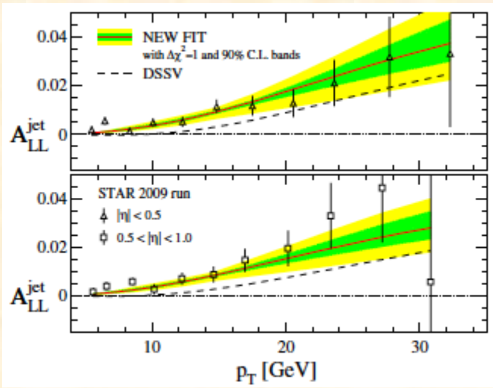
$$\Delta\Sigma = \sum_i \int dx \left[\Delta q_i(x) + \Delta\bar{q}_i(x) \right] \rightarrow 1\ (100\%)$$

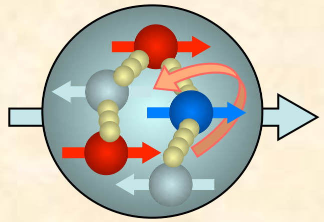


Scientific American (2014)



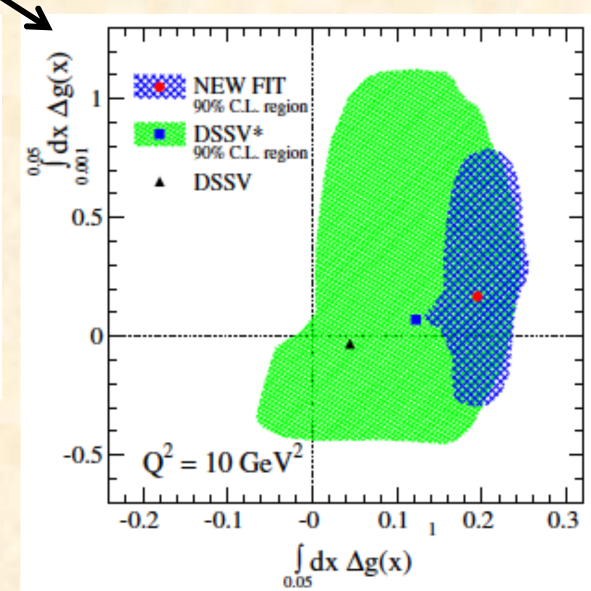
gluon spin





angular momentum

need to understand 3D structure



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_{q,g}$$

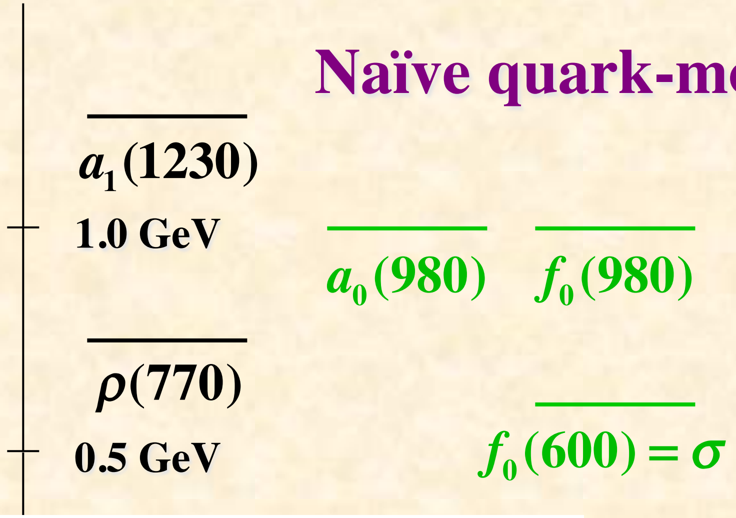
Progress in exotic hadrons

$q\bar{q}$ Meson
q^3 Baryon

$q^2\bar{q}^2$ Tetraquark
$q^4\bar{q}$ Pentaquark
q^6 Dibaryon
…
$q^{10}\bar{q}$ e.g. Strange tribaryon
…
gg Glueball
…

- $\Theta^+(1540)$???: LEPS
  Pentaquark?
  $uudd\bar{s}$?
- Kaonic nuclei?: KEK-PS, …
  Strange tribaryons, …
  $K^- pnn$, $K^- ppn$?
  $K^- pp$?
- X (3872), Y(3940): Belle
  Tetraquark, $D\bar{D}$ molecule
  $c\bar{c}$
  $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$
  $D^+(c\bar{d})D^-(\bar{c}d)$?
- $D_{sJ}(2317)$, $D_{sJ}(2460)$: BaBar, CLEO, Belle
  Tetraquark, DK molecule
  $c\bar{s}$
  $D^0(c\bar{u})K^+(u\bar{s})$
  $D^+(c\bar{d})K^0(d\bar{s})$?
- Z (4430): Belle
  Tetraquark, …
  $c\bar{c}u\bar{d}$, D molecule?
- $P_c(4380)$, $P_c(4450)$: LHCb
- …
  $u\bar{c}udc$, $\bar{D}(u\bar{c})\Sigma_c^*(udc)$, $\bar{D}^*(u\bar{c})\Sigma_c(udc)$ molecule?

Exotic hadrons: Scalar mesons $J^P = 0^+$ at $M \sim 1$ GeV



$$\sigma = f_0(600) = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$f_0(980) = s\bar{s} \quad \rightarrow \text{denote } f_0 \text{ in this talk}$$

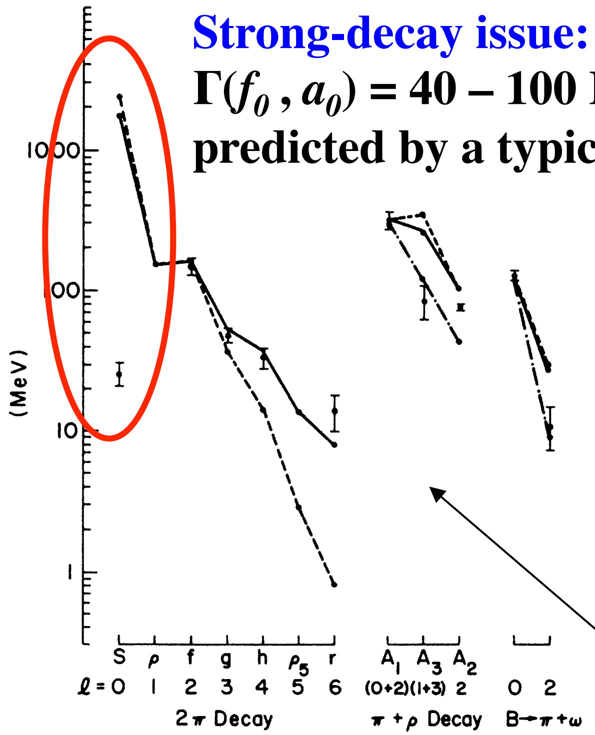
$$a_0(980) = u\bar{d}, \quad \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad d\bar{u}$$

Naive model: $m(\sigma) \sim m(a_0) < m(f_0)$

$\Updownarrow$ **contradiction**

Experiment: $m(\sigma) < m(a_0) \sim m(f_0)$

Strong-decay issue: The experimental widths $\Gamma(f_0, a_0) = 40 - 100$ MeV are too small to be predicted by a typical quark model.



These issues could be resolved if f_0 is a tetraquark ($q q \bar{q} \bar{q}$) or a $K\bar{K}$ molecule, namely an "exotic" hadron.

Radiative decay: F. E. Close, N. Isgur, and SK, Nucl. Phys. B389 (1993) 513.

SK and V. R. Pandharipande, Phys. Rev. D38 (1988) 146.

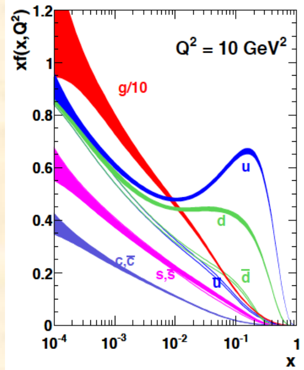
Wigner distribution and various structure functions

Form factor

$\int dx\, d^2k_T$

PDF (Parton Distribution Function)

$\int d^2k_T,\ \Delta \to 0$



GTMD $W(x, \vec{k}_T, \Delta) \xrightarrow[\Delta^+ \to 0]{}$ **Wigner** $W(x, \vec{k}_T, \vec{r}_T)$

3D world

$\int d^2k_T$

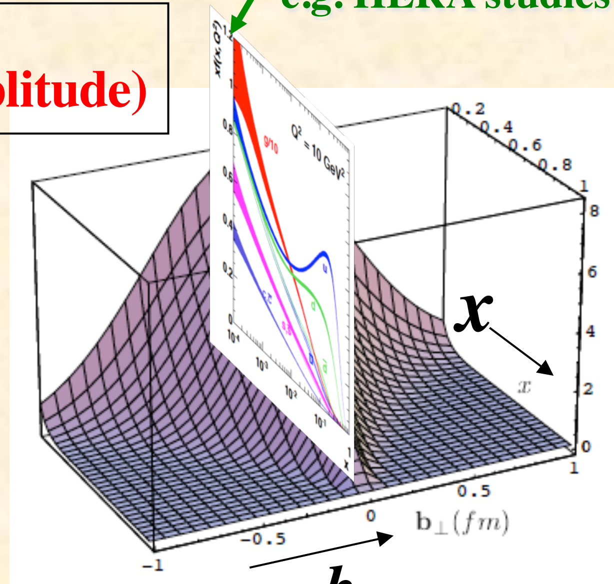
e.g. HERA studies

GPD (Generalized Parton Distribution)

$\xrightarrow[s \leftrightarrow t]{}$ **GDA (Generalized Distribution Amplitude)**

$\Delta \to 0$

TMD (Transverse Momentum Dependent) parton distribution



Our work by the two-photon process $\gamma\gamma \to h\bar{h}$.

References on following tomography topics

GPDs at J-PARC

SK, M. Strikman, K. Sudoh, PRD 80 (2009) 074003.

T. Sawada, Wen-Chen Chang, S. Kumano, Jen-Chieh Peng, S. Sawada, K. Tanaka, PRD 93 (2016) 114034. → Tanaka's talk

GPDs and GDAs (including exotic hadrons)

H. Kawamura, SK, PRD 89 (2014) 054007.

SK, Q.-T. Song, O. Teryaev, research in progress.

My talk

Related topics: Constituent counting rule:

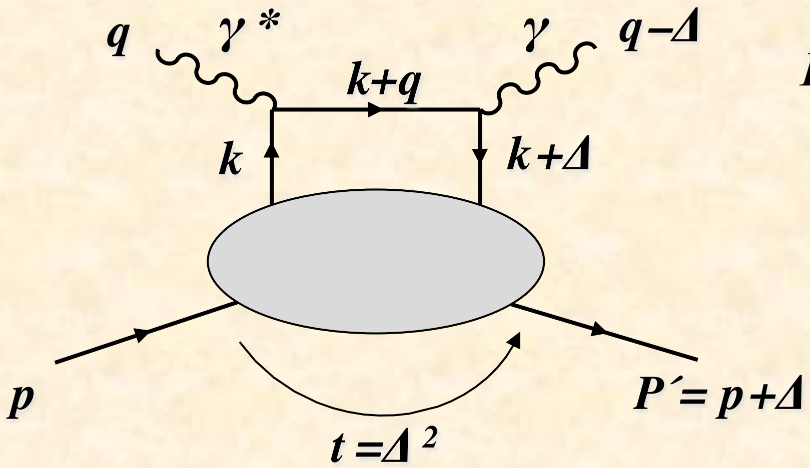
H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010.

W.-C. Chang, SK, T. Sekihara, PRD 93 (2016) 034006.

GPDs for exotic hadrons at hadron facilities

H. Kawamura, SK, PRD 89 (2014) 054007.

Generalized Parton Distributions (GPDs)



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2p\cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewdness parameter $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

Forward limit: PDFs $H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \frac{1}{2}\Delta q + L_q$

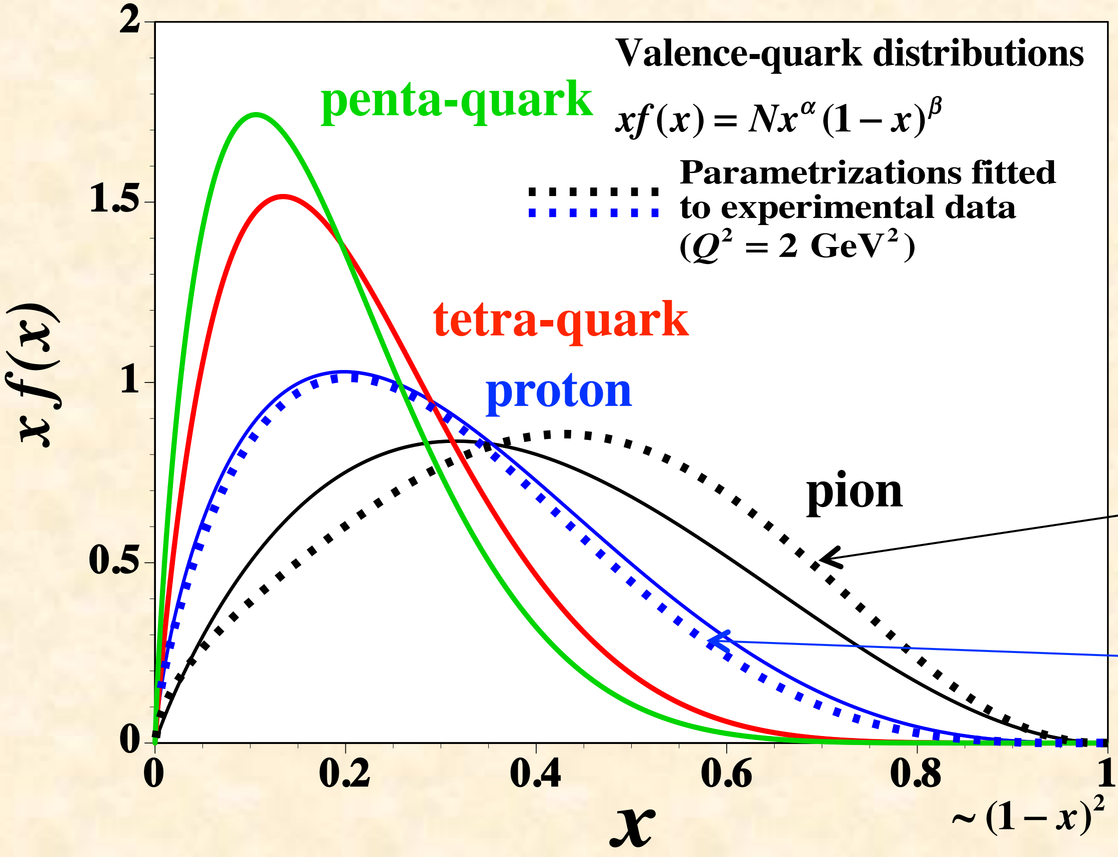
Simple function of GPDs

$$H_q^h(x,t) = f(x)F(t,x)$$

M. Guidal, M.V. Polyakov, A.V. Radyushkin, M. Vanderhaeghen, PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n}(1-x)^{\beta_n}$

- **Valence-quark number sum rule (charge and baryon numbers):** $\int_0^1 dx\, f(x) = n$
- **Constituent conting rule at** $x \to 1$: $\beta_n = 2n - 3 + 2\Delta S$ **(**n **= number of constituents)**
- **Momentum carried by quarks** $\langle x \rangle_q \simeq \int_0^1 dx\, x f(x)$



$\sim (1-x)^2$ for π

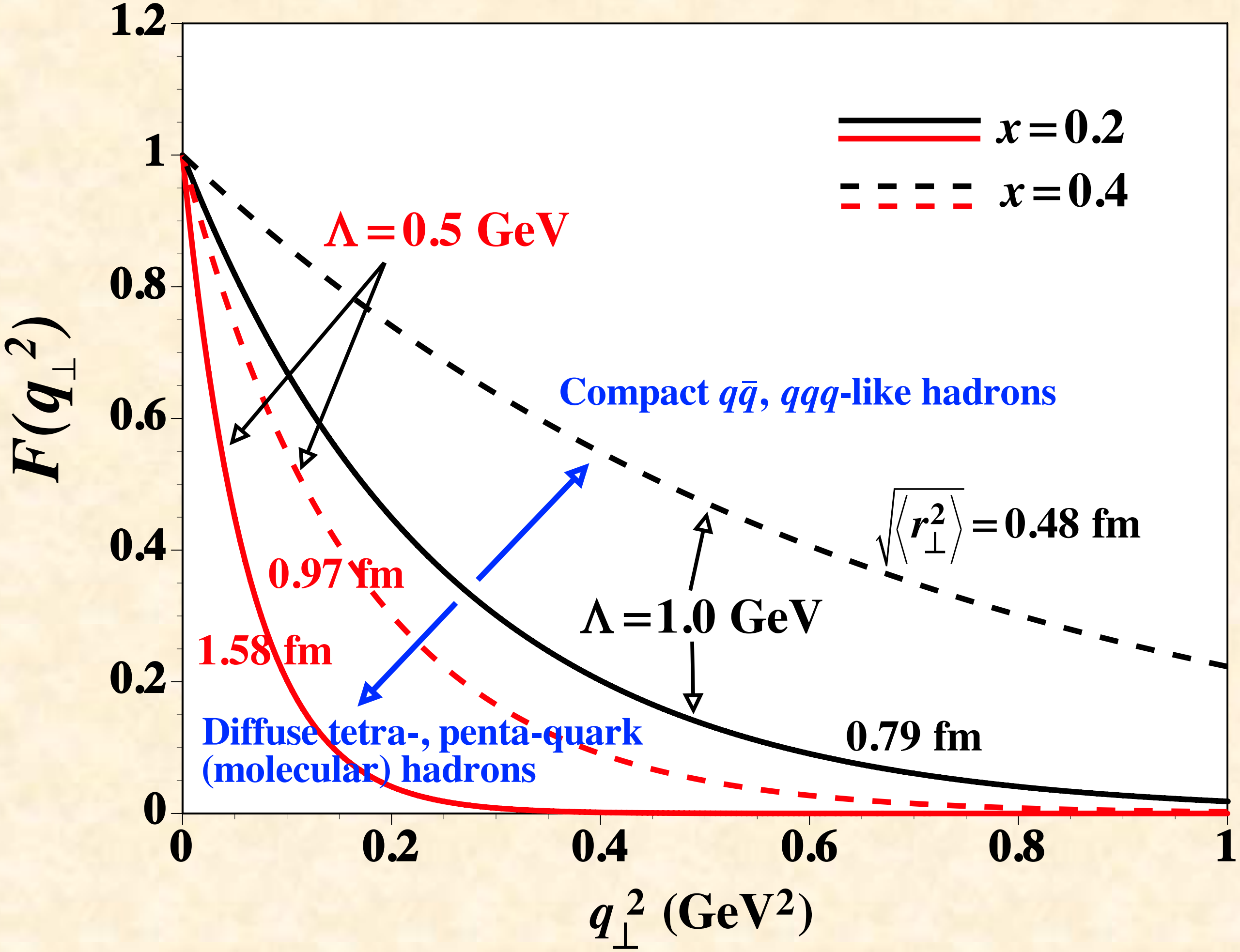
Gedankenexperiment, but … read our paper for studying exotics in high-energy processes at KEK-B, Linear Collider, ….

π: M. Aicher, A. Schafer, W. Vogelsang, PRL 105 (2010) 252003.

p: A. D. Martin, R. G. Roberts, W. J. Stirling, PLB 636, 259 (2006)

Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$



Generalized Distribution Amplitudes (GDAs)

and KEKB/ILC project

H. Kawamura, SK, PRD 89 (2014) 054007.

SK, Q.-T. Song, O. Teryaev, research in progress.

GPDs for exotic hadrons !?

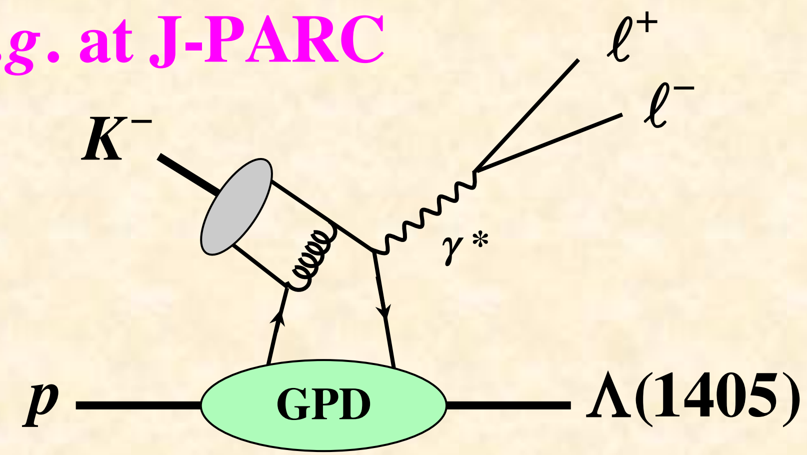
Because stable targets do not exit for exotic hadrons, it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or

→ $s \leftrightarrow t$ crossed qunatity = GDAs at KEKB, Linear Collider

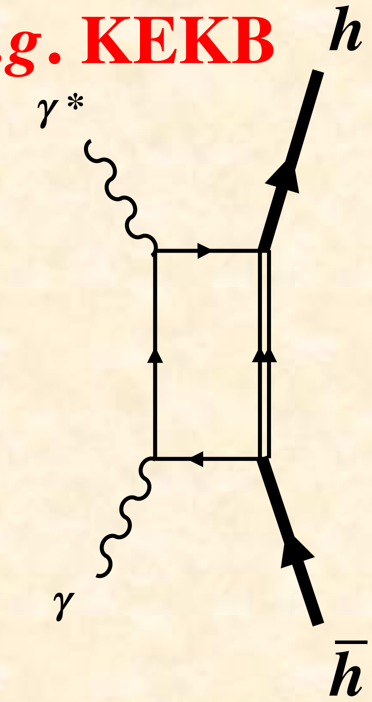
***e.g.* at J-PARC**



$K^-(\bar{u}s) + p(uud) \to \Lambda_{1405}(uud\bar{u}s) + \gamma^*$

Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate

***e.g.* KEKB**



GPD $H_q^h(x,\xi,t)$ and GDA $\Phi_q^{hh}(z,\zeta,W^2)$

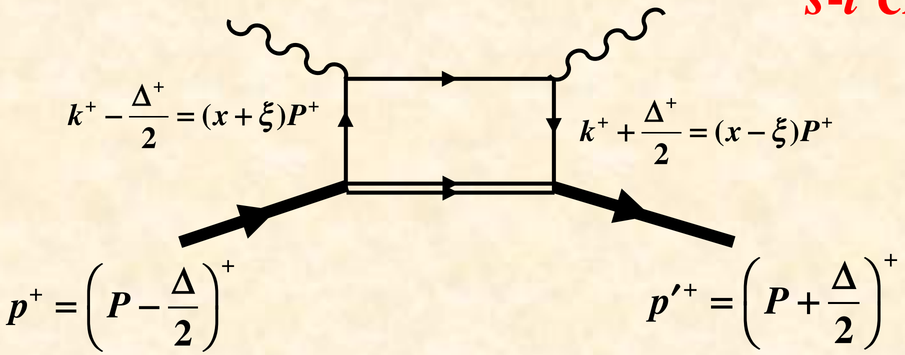
GPD: $H_q(x,\xi,t)=\int\frac{dy^-}{4\pi}e^{ixP^+y^-}\langle\pi^+(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|\pi^+(p)\rangle\big|_{y^+=0,\vec y_\perp=0}$, $P^+=\frac{(p+p')^+}{2}$

GDA: $\Phi_q(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle\pi^+(p)\pi^-(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|0\rangle\big|_{y^+=0,\vec y_\perp=0}$

DA: $\Phi_q^\pi(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle\pi(p)|\bar\psi(-y/2)\gamma^+\gamma_5\psi(y/2)|0\rangle\big|_{y^+=0,\vec y_\perp=0}$

$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

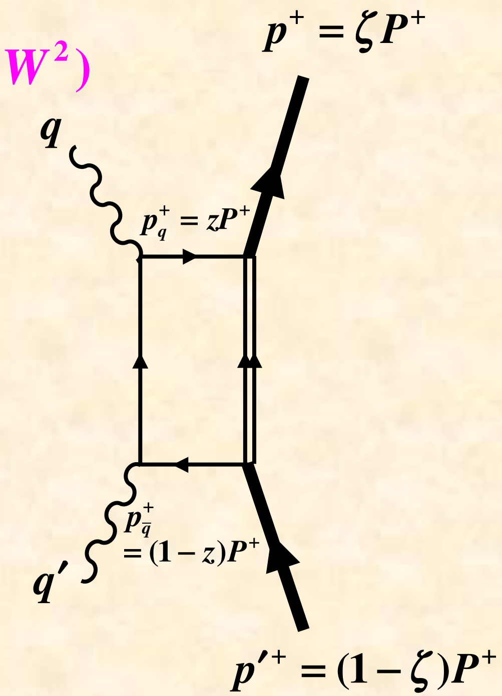
***s*-*t* crossing**



$z\Leftrightarrow\frac{1-x/\xi}{2}$

$\zeta\Leftrightarrow\frac{1-1/\xi}{2}$

$W^2\Leftrightarrow t$



$P=\frac{p+p'}{2}$, $\Delta=p'-p$

Bjorken variable: $x=\frac{Q^2}{2p\cdot q}$

Momentum transfer squared: $t=\Delta^2$

Skewdness parameter: $\xi=\frac{p^+-p'^+}{p^++p'^+}=-\frac{\Delta^+}{2P^+}$

Bjorken variable for $\gamma\gamma^*$: $z=\frac{Q^2}{2q\cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar h$: $\zeta=\frac{p^+}{P^+}=\frac{1+\beta\cos\theta}{2}$

Invariant mass of $h\bar h$: $W^2=(p+p')^2$

Cross section: form factor dependence

$$\Phi_q^{h\bar{h}(I=0)}(z,\zeta,W^2) \propto F_h(W^2)$$

Ordinal $q\bar{q}$

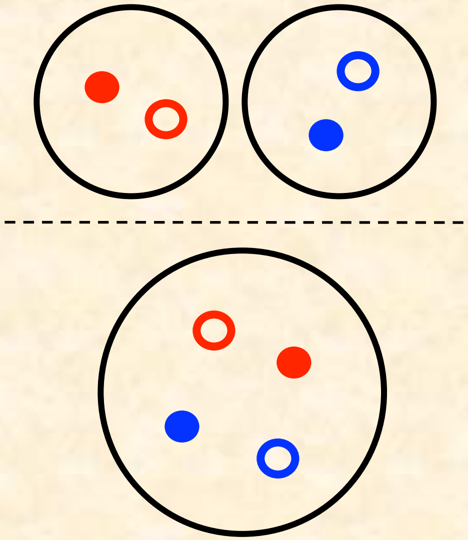
$$F_h(W^2) = \frac{1}{\left[1+(W^2-4m_h^2)/\Lambda^2\right]^{n-1}}$$

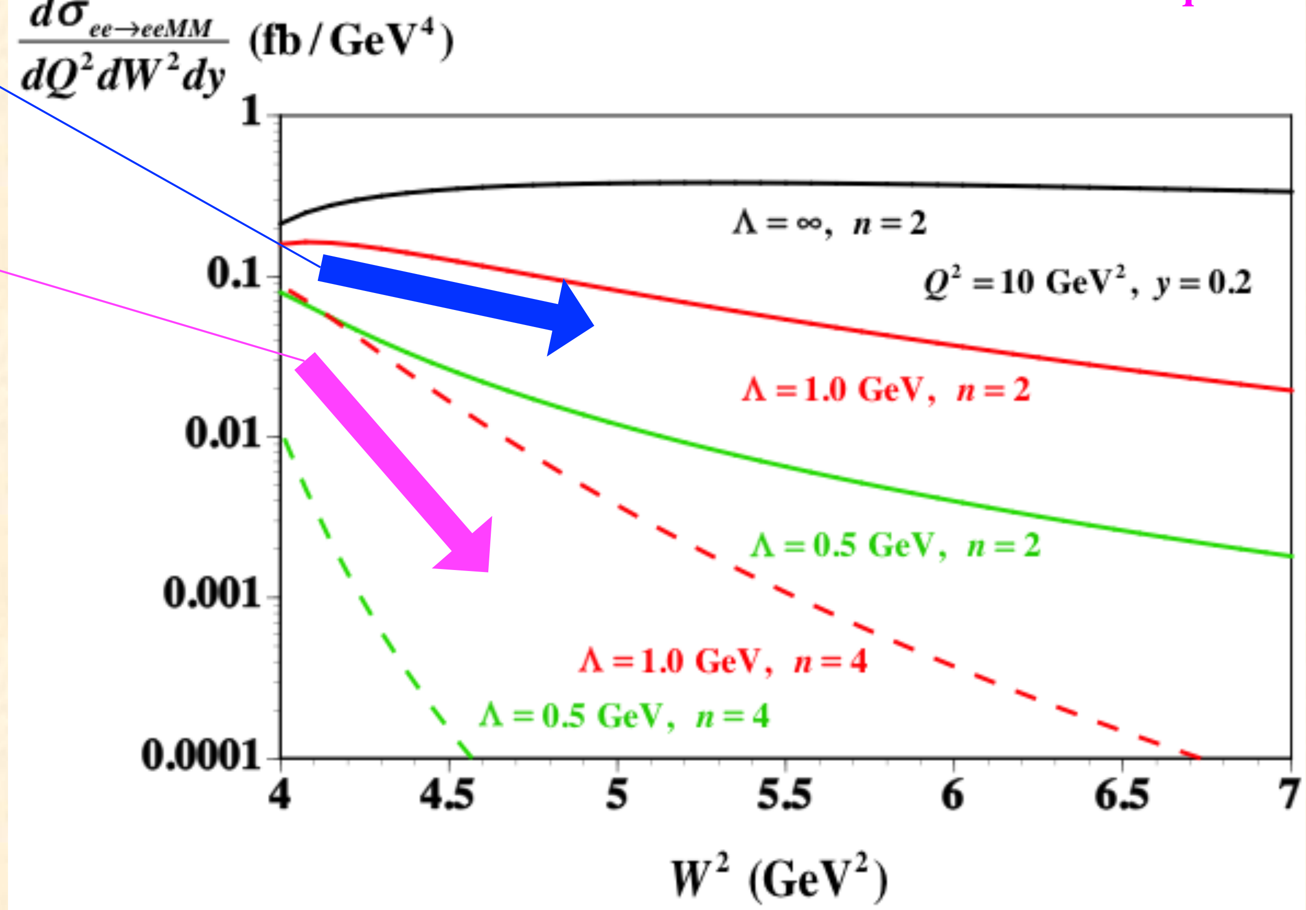
Constituent-counting rule

$n=2$: ordinary meson

$n=4$: molecule or tetra-quark

Molecule $K\bar{K}$ or tetra-quark $qq\bar{q}\bar{q}$





Generalized Distribution Amplitudes (GDAs) for pion

from KEKB measurements

SK, Q.-T. Song, O. Teryaev, research in progress.

KEKB-Belle measurement (2016)

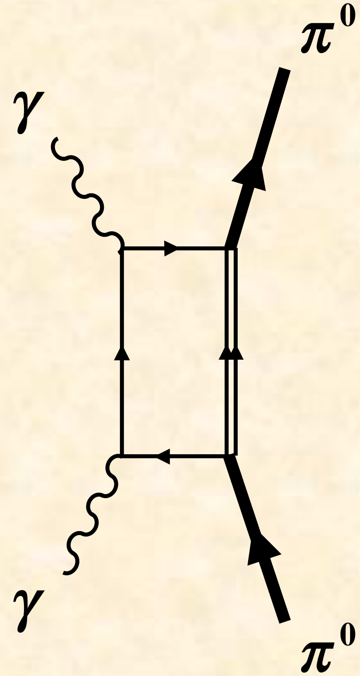
M. Masuda *et al*., Phys. Rev. D 93 (2016) 032003 (arXiv:1508.06757).

PHYSICAL REVIEW D **93,** 032003 (2016)

Study of π^0 pair production in single-tag two-photon collisions

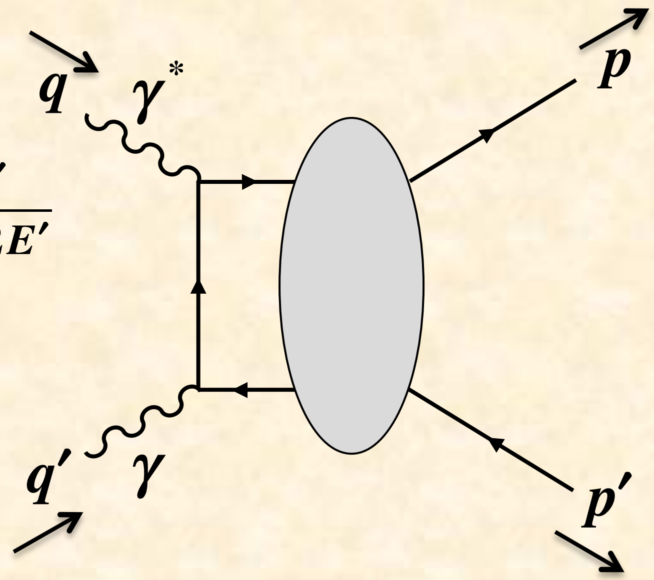
M. Masuda,[68] S. Uehara,[15,11] Y. Watanabe,[26] H. Nakazawa,[46] A. Abdesselam,[62] I. Adachi,[15,11] H. Aihara,[69] S. Al Said,[62,30] D. M. Asner,[54] H. Atmacan,[40] V. Aulchenko,[4,52] T. Aushev,[42] V. Babu,[63] I. Badhrees,[62,29] A. M. Bakich,[61] E. Barberio,[39] P. Behera,[19] B. Bhuyan,[18] J. Biswal,[25] A. Bobrov,[4,52] G. Bonvicini,[75] A. Bozek,[49] M. Bračko,[37,25] T. E. Browder,[14] D. Červenkov,[5] V. Chekelian,[38] A. Chen,[46] B. G. Cheon,[13] K. Chilikin,[41] R. Chistov,[41] K. Cho,[31] V. Chobanova,[38] S.-K. Choi,[12] Y. Choi,[60] D. Cinabro,[75] J. Dalseno,[38,64] M. Danilov,[41] N. Dash,[17] J. Dingfelder,[3] Z. Doležal,[5] Z. Drásal,[5] D. Dutta,[63] S. Eidelman,[4,52] D. Epifanov,[69] H. Farhat,[75] J. E. Fast,[54] T. Ferber,[8] B. G. Fulsom,[54] V. Gaur,[63] N. Gabyshev,[4,52] A. Garmash,[4,52] R. Gillard,[75] F. Giordano,[78] R. Glattauer,[22] Y. M. Goh,[13] P. Goldenzweig,[27] B. Golob,[35,25] J. Haba,[15,11] K. Hayasaka,[44] H. Hayashii,[45] X. H. He,[55] W.-S. Hou,[48] T. Iijima,[44,43] K. Inami,[43] A. Ishikawa,[67] R. Itoh,[15,11] Y. Iwasaki,[15] I. Jaegle,[14] D. Joffe,[28] K. K. Joo,[6] T. Julius,[39] K. H. Kang,[33] E. Kato,[67] T. Kawasaki,[51] D. Y. Kim,[59] J. B. Kim,[32] J. H. Kim,[31] K. T. Kim,[32] M. J. Kim,[33] S. H. Kim,[13] Y. J. Kim,[31] B. R. Ko,[32] S. Korpar,[37,25] P. Križan,[35,25] P. Krokovny,[4,52] T. Kumita,[71] A. Kuzmin,[4,52] Y.-J. Kwon,[77] J. S. Lange,[9] D. H. Lee,[32] I. S. Lee,[13] C. Li,[39] L. Li,[57] Y. Li,[74] J. Libby,[19] D. Liventsev,[74,15] P. Lukin,[4,52] D. Matvienko,[4,52] K. Miyabayashi,[45] H. Miyata,[51] R. Mizuk,[41,42] G. B. Mohanty,[63] S. Mohanty,[63,73] A. Moll,[38,64] H. K. Moon,[32] T. Mori,[43] R. Mussa,[24] E. Nakano,[53] M. Nakao,[15,11] T. Nanut,[25] Z. Natkaniec,[49] M. Nayak,[19] N. K. Nisar,[63] S. Nishida,[15,11] S. Ogawa,[66] P. Pakhlov,[41] G. Pakhlova,[42] B. Pal,[7] C. W. Park,[60] H. Park,[33] T. K. Pedlar,[36] R. Pestotnik,[25] M. Petrič,[25] L. E. Piilonen,[74] J. Rauch,[65] E. Ribežl,[25] M. Ritter,[38] A. Rostomyan,[8] S. Sandilya,[63] L. Santelj,[15] T. Sanuki,[67] Y. Sato,[43] V. Savinov,[56] O. Schneider,[34] G. Schnell,[1,16] C. Schwanda,[22] Y. Seino,[51] K. Senyo,[76] O. Seon,[43] M. E. Sevior,[39] V. Shebalin,[4,52] C. P. Shen,[2] T.-A. Shibata,[70] J.-G. Shiu,[48] B. Shwartz,[4,52] F. Simon,[38,64] Y.-S. Sohn,[77] A. Sokolov,[23] E. Solovieva,[42] M. Starič,[25] M. Sumihama,[10] T. Sumiyoshi,[71] U. Tamponi,[24,72] K. Tanida,[58] Y. Teramoto,[53] T. Uglov,[42] Y. Unno,[13] S. Uno,[15,11] C. Van Hulse,[1] P. Vanhoefer,[38] G. Varner,[14] A. Vinokurova,[4,52] V. Vorobyev,[4,52] A. Vossen,[20] M. N. Wagner,[9] C. H. Wang,[47] M.-Z. Wang,[48] P. Wang,[21] K. M. Williams,[74] E. Won,[32] J. Yamaoka,[54] Y. Yamashita,[50] S. Yashchenko,[8] H. Ye,[8] Y. Yusa,[51] C. C. Zhang,[21] Z. P. Zhang,[57] V. Zhilich,[4,52] V. Zhulanov,[4,52] and A. Zupanc[25]

(The Belle Collaboration)



Research in progress to extract $\Phi_q^{\pi\pi}(z,\zeta,W^2)$, $\cdots$.

Cross section for $\gamma\gamma^* \to \pi^0\pi^0$



$$d\sigma = \frac{1}{4\sqrt{(q\cdot q')^2 - q^2 q'^2}}(2\pi)^4\delta^4(q+q'-p-p')\overline{\sum_{\lambda,\lambda'}}|\mathcal{M}|^2\frac{d^3p}{(2\pi)^3 2E}\frac{d^3p'}{(2\pi)^3 2E'}$$

$$q = (q^0,\, 0,\, 0,\, |\vec{q}|),\;\; q' = (|\vec{q}|,\, 0,\, 0,\, -|\vec{q}|),\;\; q'^2 = 0 \text{ (real photon)}$$

$$p = (p^0,\, |\vec{p}|\sin\theta,\, 0,\, |\vec{p}|\cos\theta),\;\; p = (p^0,\, -|\vec{p}|\sin\theta,\, 0,\, -|\vec{p}|\cos\theta)$$

$$\beta = \frac{|\vec{p}|}{p^0} = \sqrt{1-\frac{4m_\pi^2}{W^2}}$$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}\overline{\sum_{\lambda,\lambda'}}|\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu},\quad T^{\mu\nu} = i\int d^4\xi e^{-i\xi\cdot q}\left\langle \pi(p)\pi(p')\middle| TJ_{em}^\mu(\xi)J_{em}^\nu(0)\middle|0\right\rangle$$

$$\mathcal{M} = e^2 A_{\lambda\lambda'} = 4\pi\alpha A_{\lambda\lambda'}$$

$$A_{\lambda\lambda'} = \frac{1}{e^2}\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = -\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')g_T^{\mu\nu}\sum_q\frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

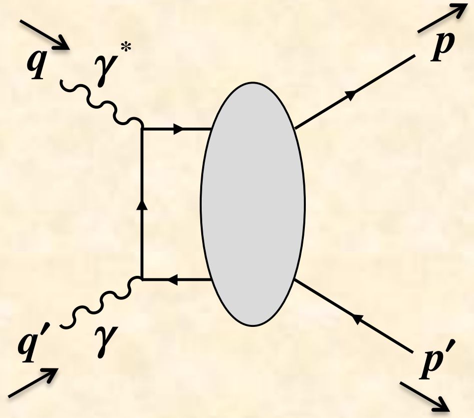
$$A_{++} = \sum_q\frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2),\quad \varepsilon_\mu^+(q)\varepsilon_\nu^+(q')g_T^{\mu\nu} = -1$$

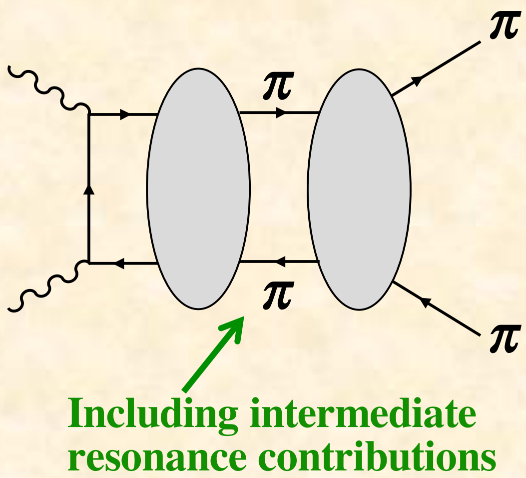
$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}|A_{++}|^2$$

GDA parametrization for pion

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}\,|A_{++}|^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$





- GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha(1-z)^\beta(2z-1)\zeta(1-\zeta)F_q^\pi(s)$$

- In addition, there exist resonance contributions to the cross section.

$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18N_f z^\alpha(1-z)^\alpha(2z-1)\left[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)\right]$$

$$\tilde{B}_{nl}(W) = \bar{B}_{nl}(W)\exp(i\delta_l), \quad P_2(x) = \frac{1}{2}(3x^2-1)$$

$$\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta) = B_{10}(W)+B_{12}(W)P_2(2\zeta-1)$$

$$B_{10}(0) = -B_{12}(0) = -\frac{10R_\pi}{9N_f}$$

R_π = momentum fraction carried by quarks

$$\bar{B}_{10}(W) = \text{resonance}\ [f_0(500)\equiv\sigma,\ f_0(980)\equiv f_0] + \text{continuum}$$

$$= \frac{5g_{\sigma\pi\pi}f_\sigma M_\sigma\Gamma_\sigma/3}{(M_\sigma^2-W^2)^2+\Gamma_\sigma^2 M_\sigma^2} + \frac{5g_{f_0\pi\pi}f_{f_0}M_{f_0}\Gamma_{f_0}/3}{(M_{f_0}^2-W^2)^2+\Gamma_{f_0}^2 M_{f_0}^2} - \frac{3-\beta^2}{2}\frac{10R_\pi}{9N_f}F_q^\pi(W^2)$$

$$\bar{B}_{12}(W) = \text{resonance}[f_2(1270)] + \text{continuum} = \frac{10g_{f_2\pi\pi}f_{f_2}M_{f_2}^3\Gamma_{f_2}/9}{(M_{f_2}^2-W^2)^2+\Gamma_{f_2}^2M_{f_2}^2} + \beta^2\frac{10R_\pi}{9N_f}F_q^\pi(W^2)$$

$f_0(500)$ or σ [g] **was $f_0(600)$** $\qquad I^G(J^{PC}) = 0^+(0^{++})$

Mass m = (400–550) MeV
Full width Γ = (400–700) MeV

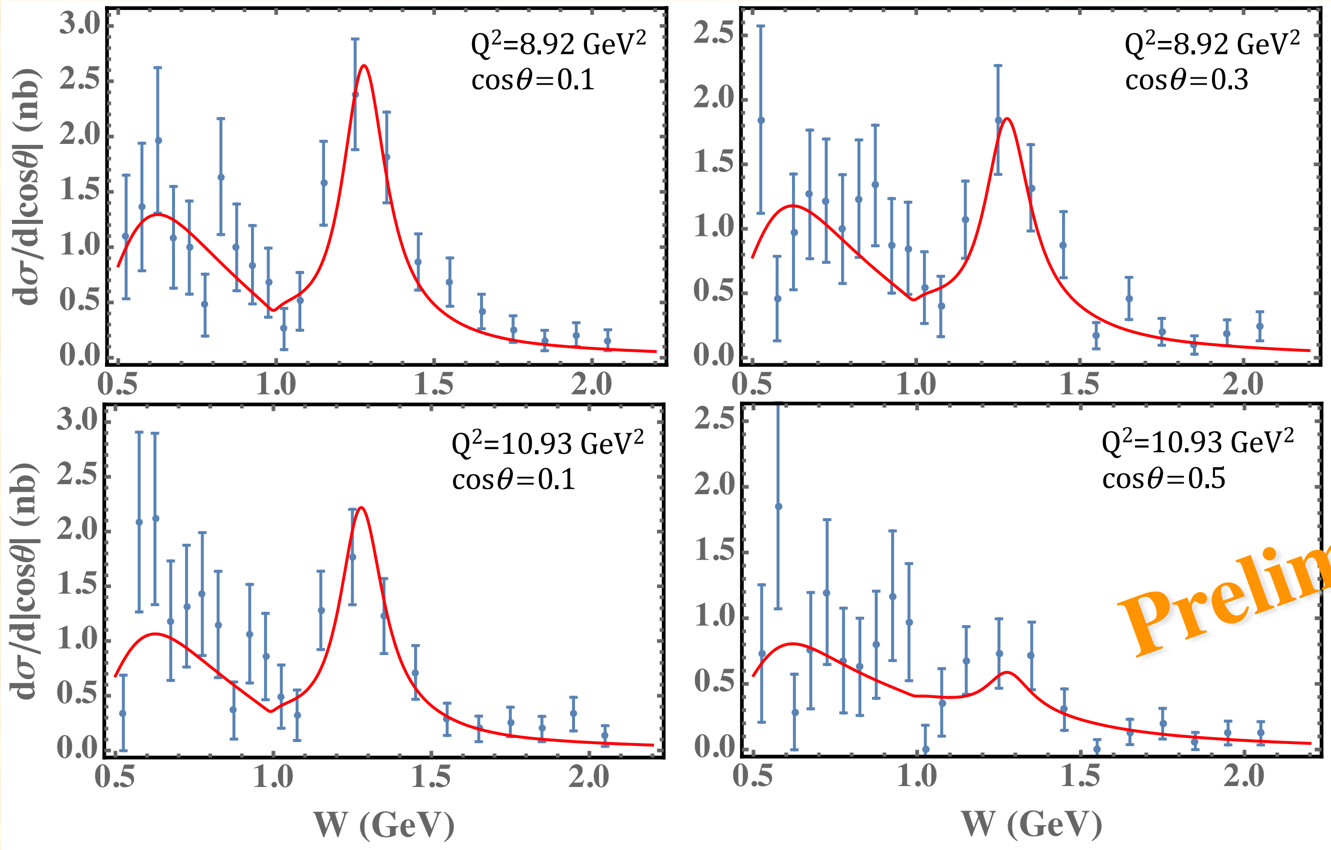
$f_0(980)$ [j] $\qquad I^G(J^{PC}) = 0^+(0^{++})$

Mass m = 990 ± 20 MeV
Full width Γ = 10 to 100 MeV

$f_2(1270)$ $\qquad I^G(J^{PC}) = 0^+(2^{++})$

Mass m = 1275.5 ± 0.8 MeV
Full width $\Gamma = 186.7^{+2.2}_{-2.5}$ MeV (S = 1.4)

Analysis of Belle data on $\gamma\gamma^* \to \pi^0\pi^0$



Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

$Q^2 = 8.92,\ 10.93\ \mathrm{GeV}^2$

Preliminary

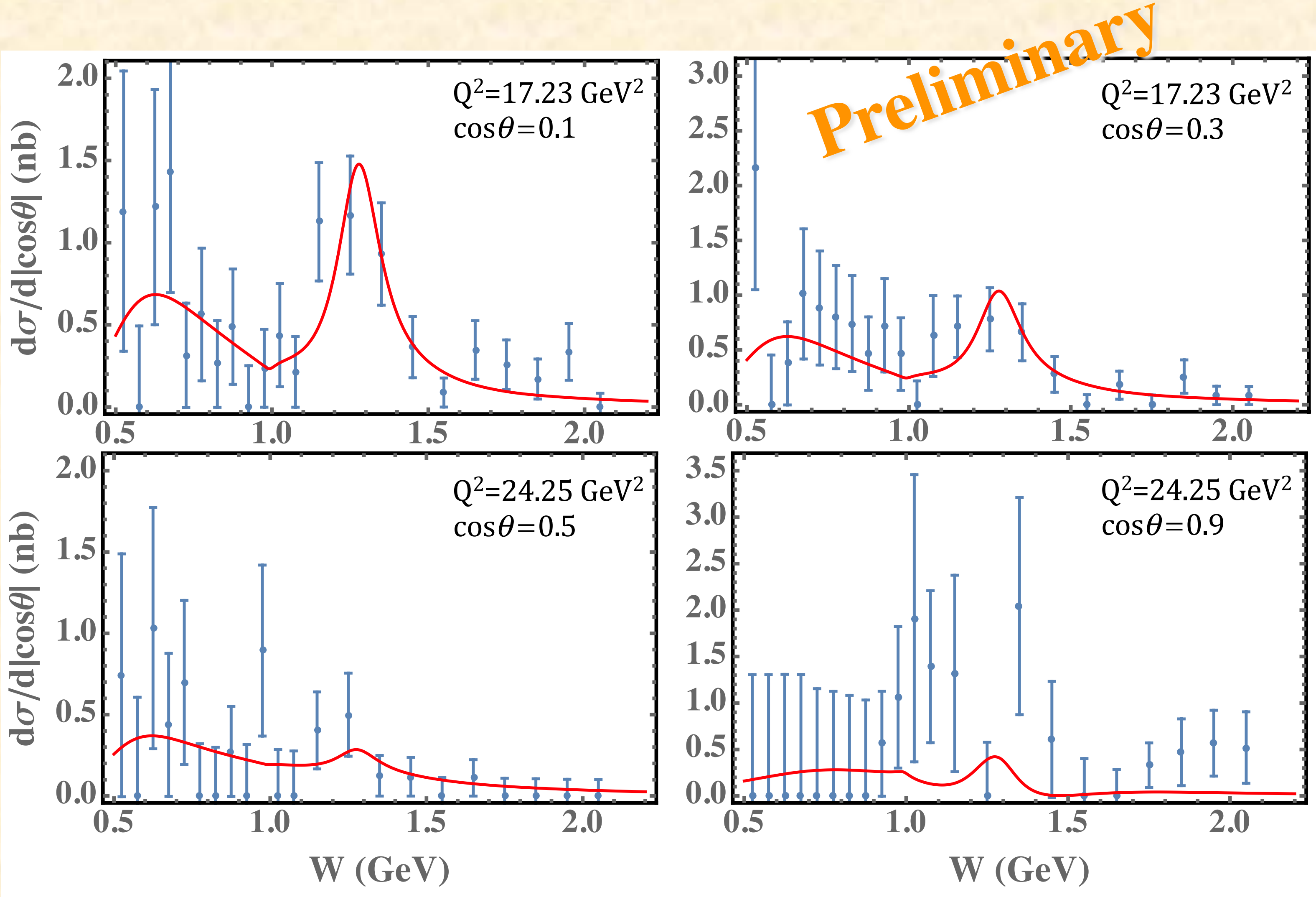
$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m^2}{s}}\,|A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\,\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N z^{\alpha}(1-z)^{\alpha}(2z-1)^{\alpha}\left[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos\theta)\right], \quad \tilde{B}_{nl}(W) = \bar{B}_{nl}(W)\exp(i\delta_l)$$

$$\bar{B}_{10}(W) = \frac{5g_{\sigma\pi\pi} f_\sigma M_\sigma \Gamma_\sigma / 3}{(M_\sigma^2 - W^2)^2 + \Gamma_\sigma^2 M_\sigma^2} + \frac{5g_{f_0\pi\pi} f_{f_0} M_{f_0}\Gamma_{f_0}/3}{(M_{f_0}^2 - W^2)^2 + \Gamma_{f_0}^2 M_{f_0}^2} - \frac{3-\beta^2}{2}\frac{10R_\pi}{9N_f}(1+aW^2)\left[F_\pi(W^2)\right]^m, \quad F_\pi(W^2) = \frac{1}{\left[1+(W^2-4m_\pi^2)/\Lambda^2\right]^{n_\pi - 1}}, \quad n_\pi = 2$$

$$\bar{B}_{12}(W) = \frac{10 g_{f_2\pi\pi} f_{f_2} M_{f_2}^3 \Gamma_{f_2}/9}{(M_{f_2}^2 - W^2)^2 + \Gamma_{f_2}^2 M_{f_2}^2} + \beta^2\frac{10R_\pi}{9N_f}(1+bW^2)\left[F_\pi(W^2)\right]^m$$

$Q^2 = 17.23,\ 24.25\ \mathrm{GeV}^2$

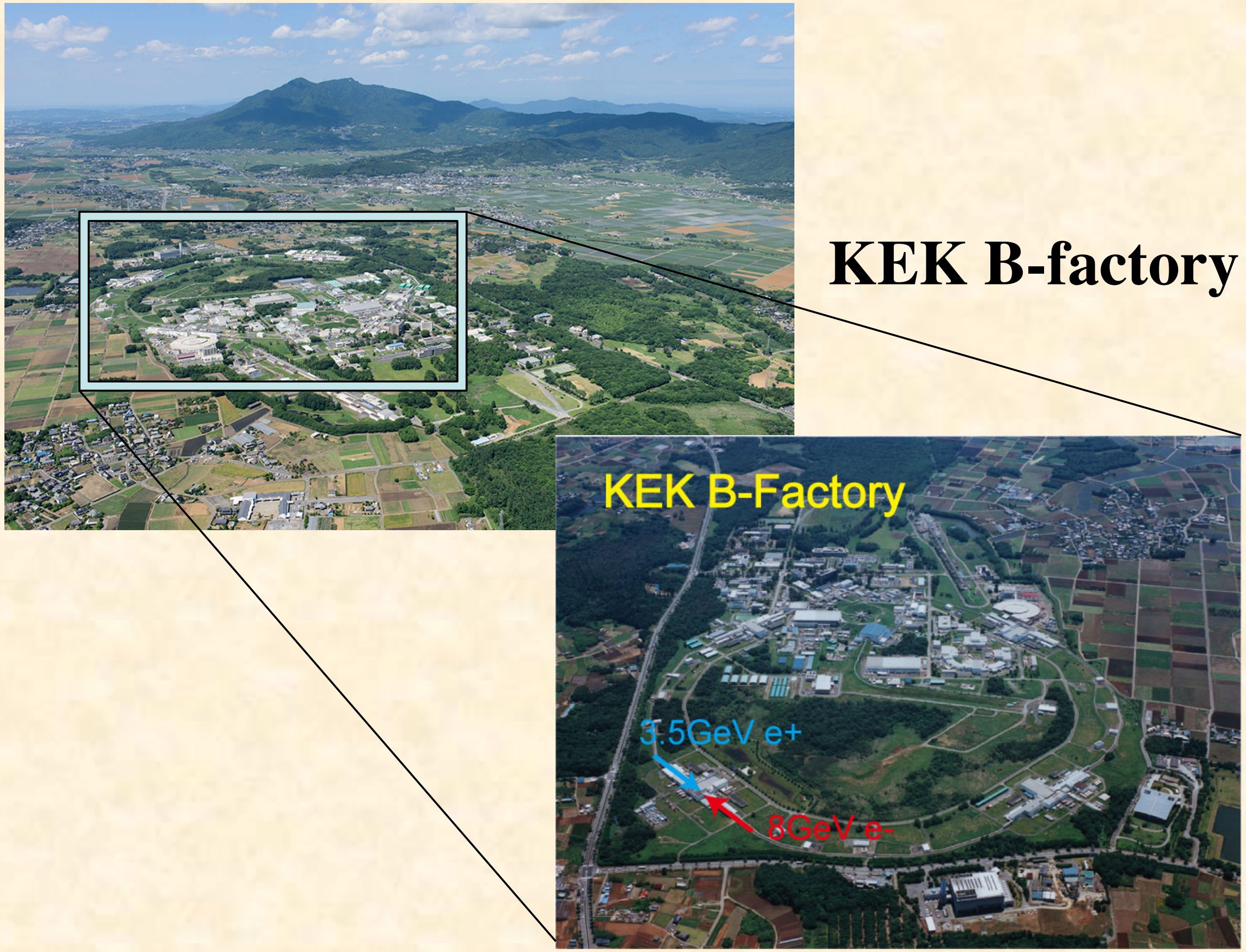


Detailed results will be reported soon for publication.

Prospects & Summary

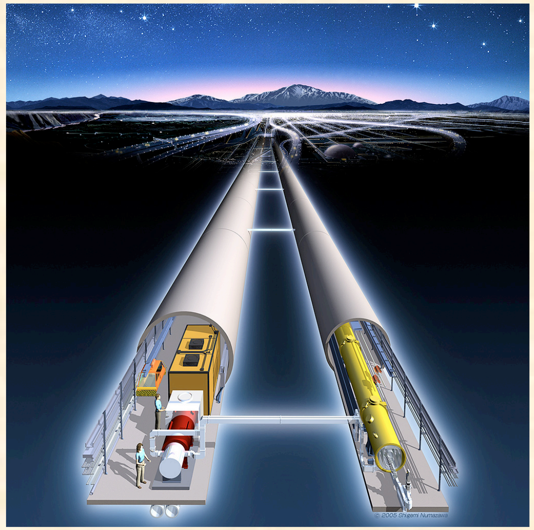
Experimental studies of GDAs in future

$\gamma\gamma \to h\bar{h}$ **for internal structure of exotic hadron candidate** h

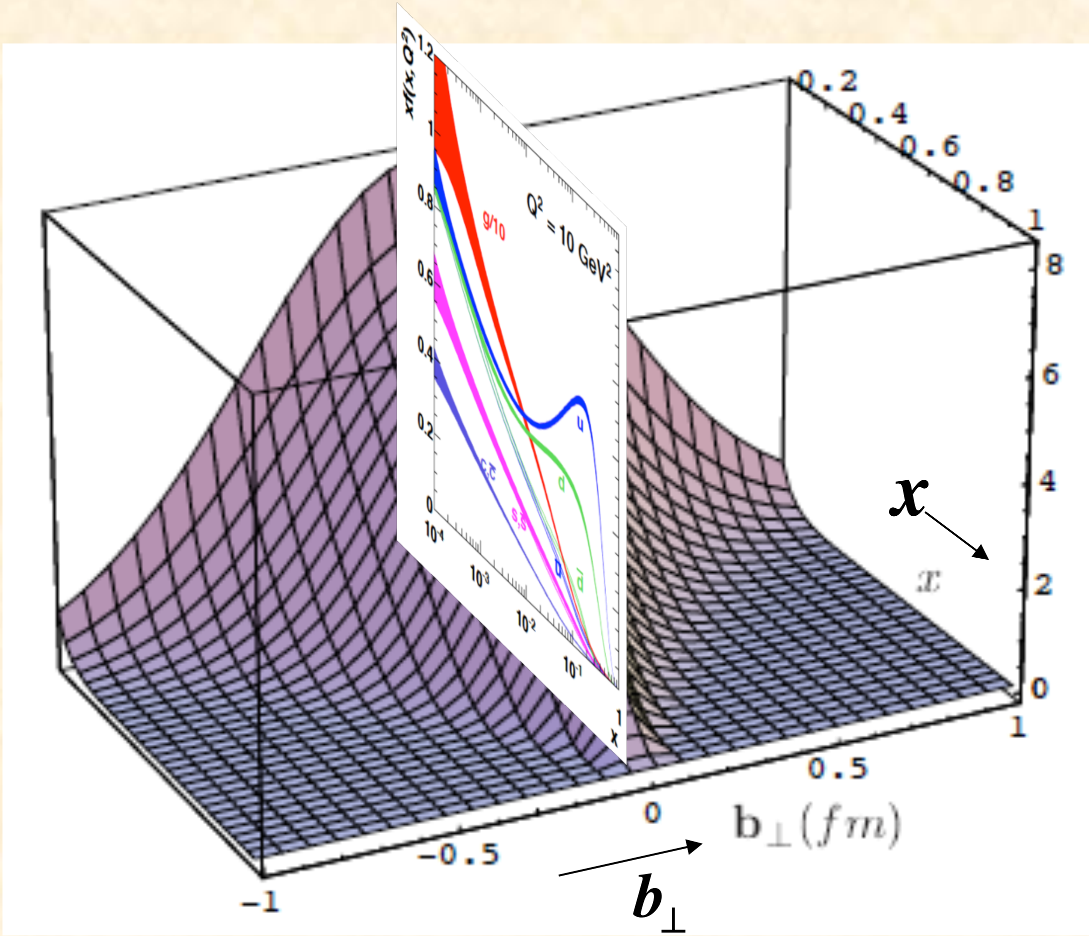


KEK B-factory

Linear Collider ?

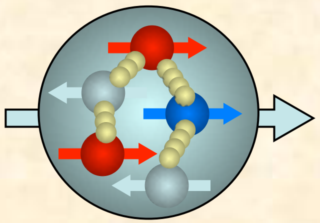


3D view of hadrons

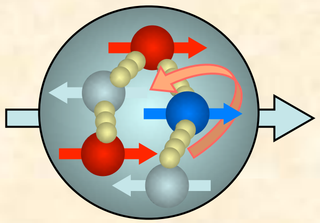


Origin of nucleon spin …

By the tomography, we determine



or



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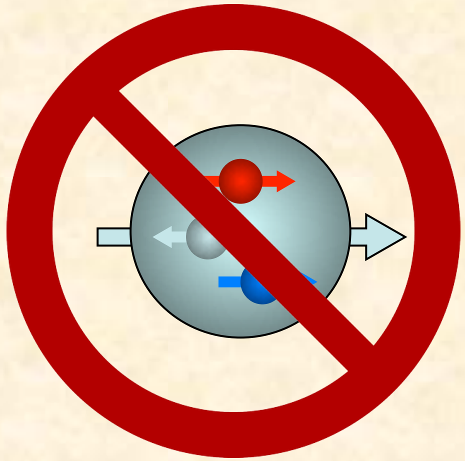
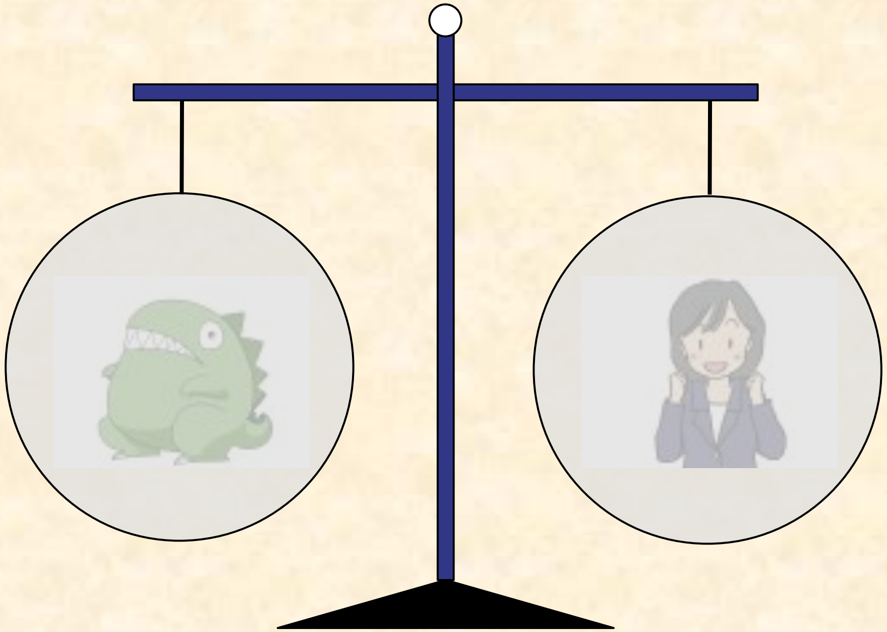


Search for exotic hadrons …



It is difficult to determine whether or not a hadron is exotic by low-energy observables, masses, decay widths, …

(Already, history of a half century)

By the tomography, we determine

 or  .

Summary

Hadron tomography studies are important
for solving the origin of the nucleon spin,
for probing internal structure of exotic hadrons.

GPDs / TMDs

Recently, GPDs and TMDs have been extensively investigated.

GDAs

3D structure of hadrons can be studied by GDAs (*s*⇔*t* of GPDs).
It is interesting to probe time-like form factors, and
the GDAs can be also investigated for unstable (exotic) hadrons.
Our analysis is the first trial to extract the GDAs from actual experimental measurements on $\gamma+\gamma^* \rightarrow \pi^0+\pi^0$. We will provide our "optimum" GDAs for public use.

Experimental projects on GDAs

KEKB, ILC, …

The End

The End