ρ^0 pair production in two-photon process

Kazuhiro Tanaka (Juntendo U/KEK)

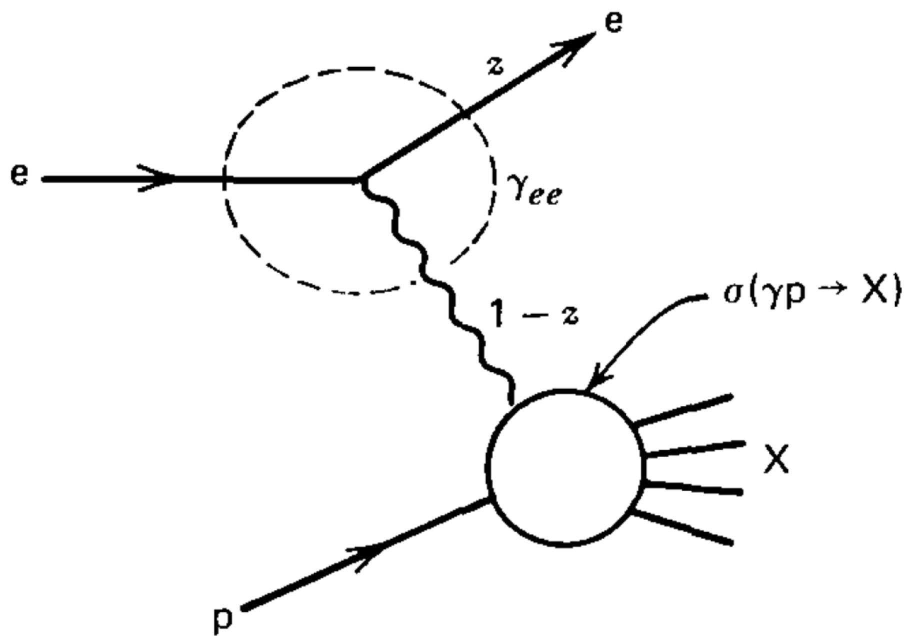
1. UPC and $\gamma \to \rho^0$ forward transition amplitude (impact factor)

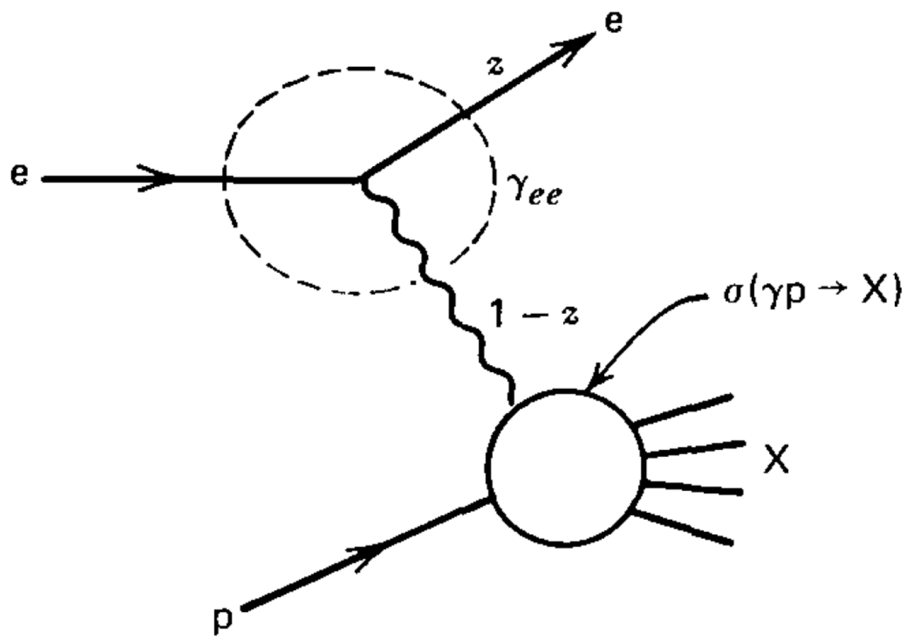
2. QCD calculation of the $\gamma \to \rho^0$ impact factor

QCD factorization

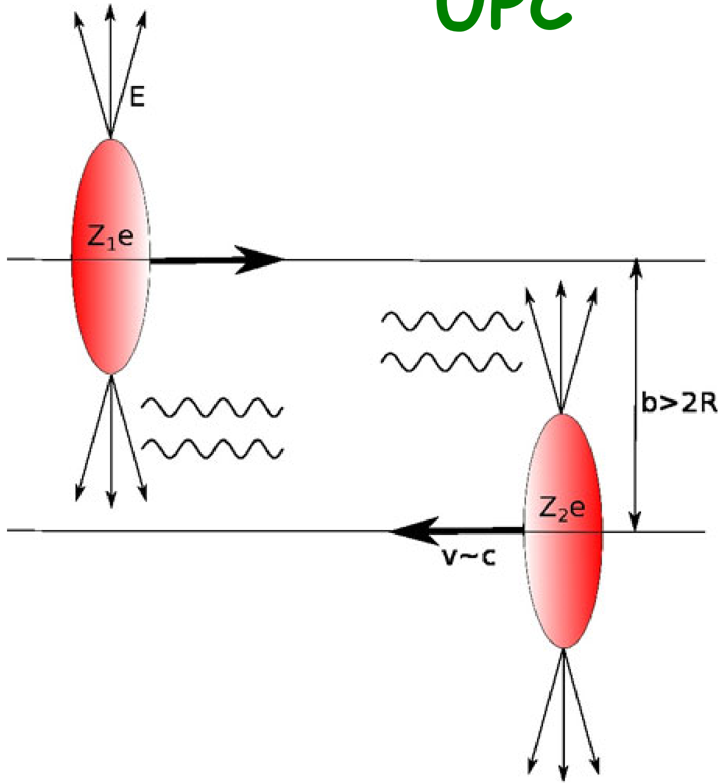
light-cone sum rule (LCSR)

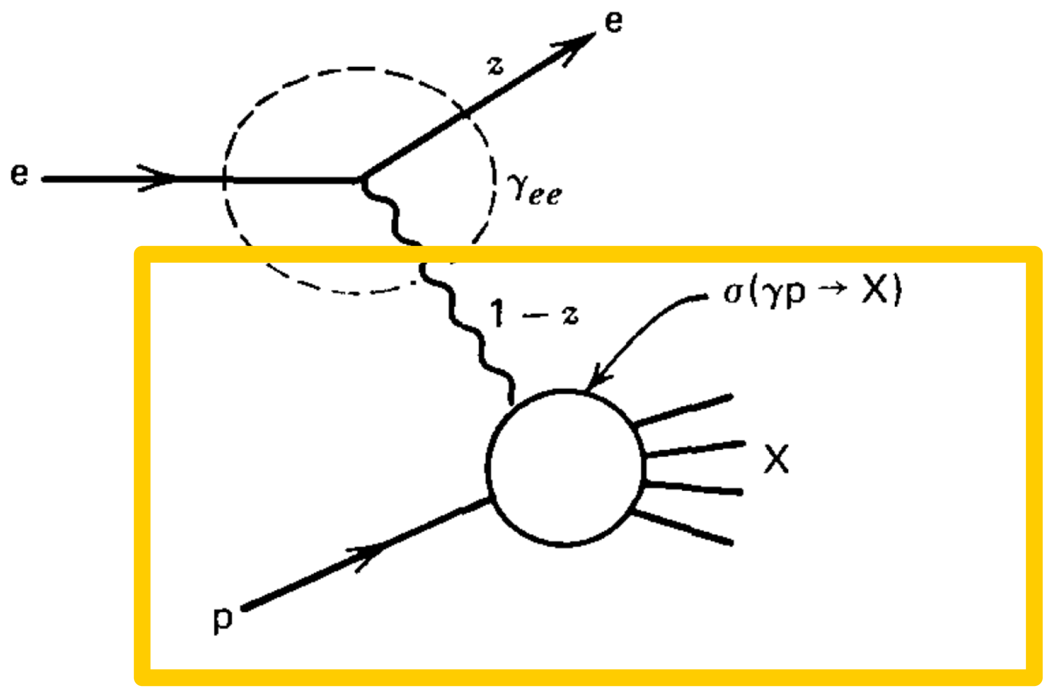
3. Application to two-photon process $\gamma\gamma \to \rho^0\rho^0$


e
e
z
γ_{ee}
$1-z$
$\sigma(\gamma p \to X)$
X
p

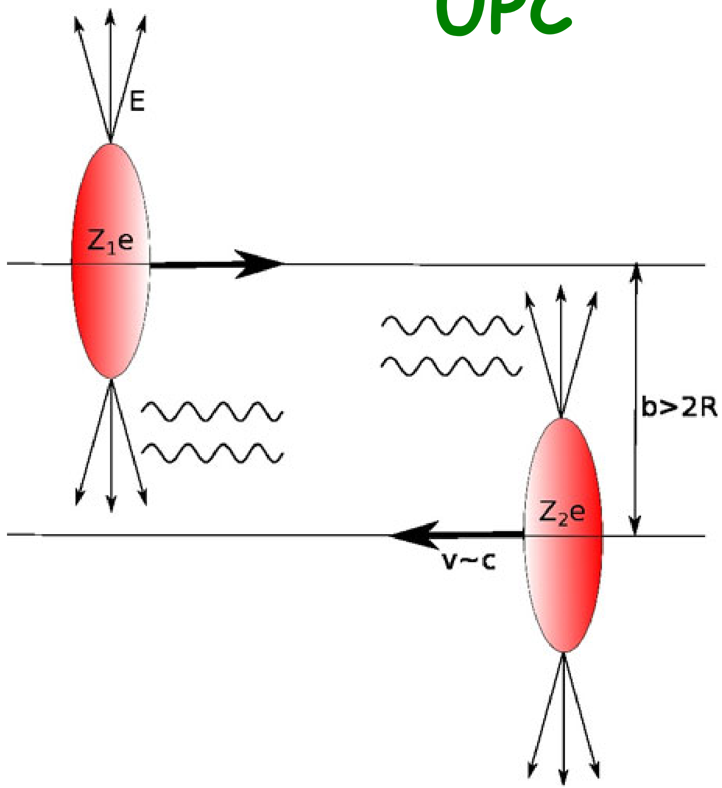


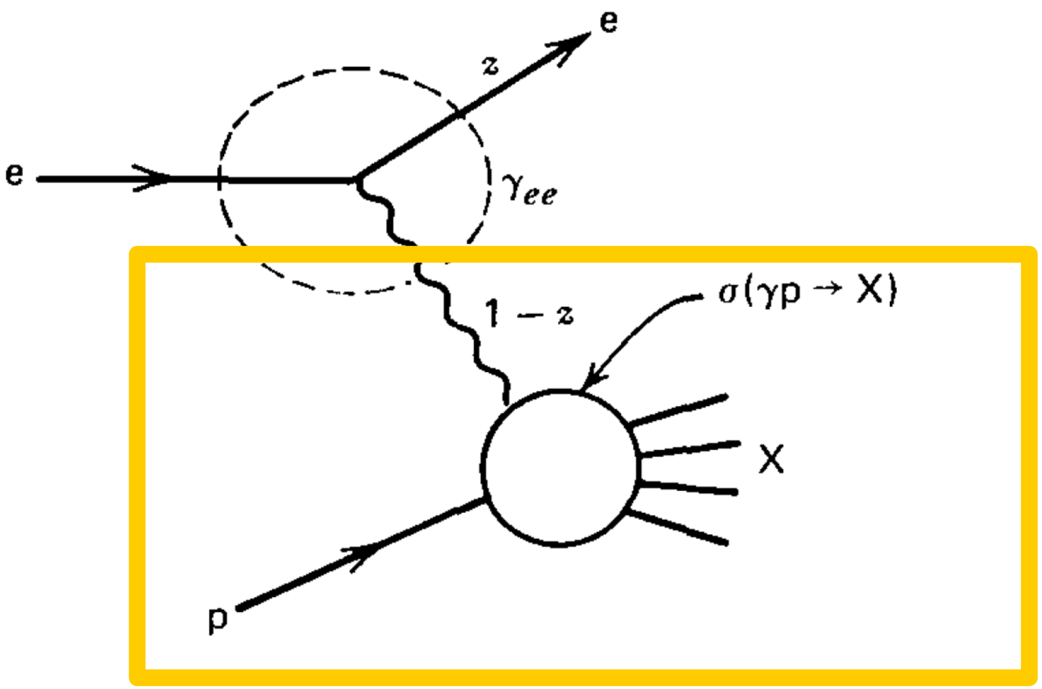
"UPC"



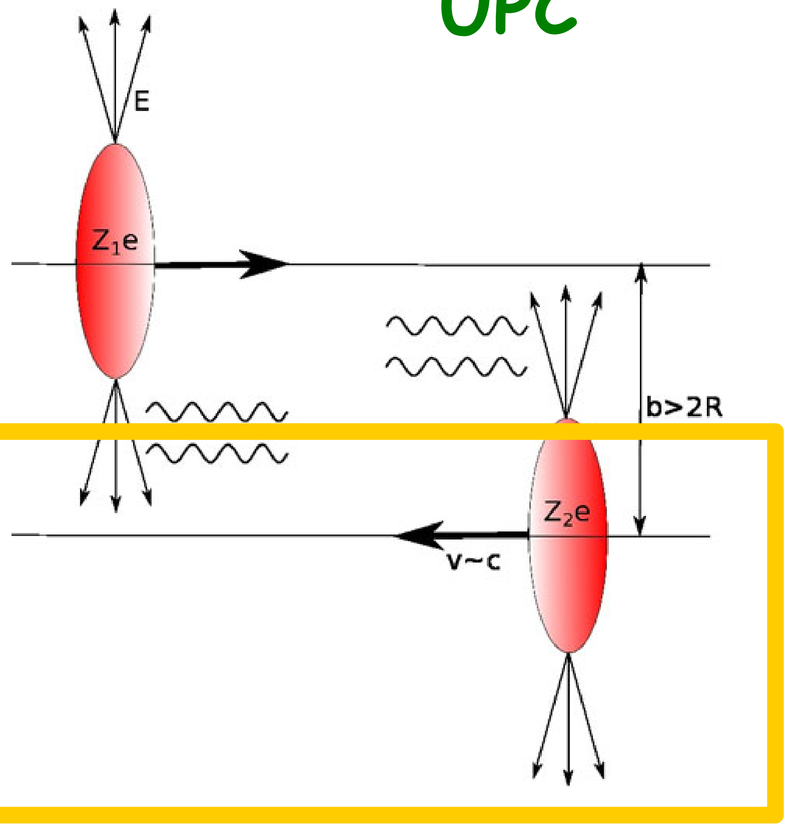


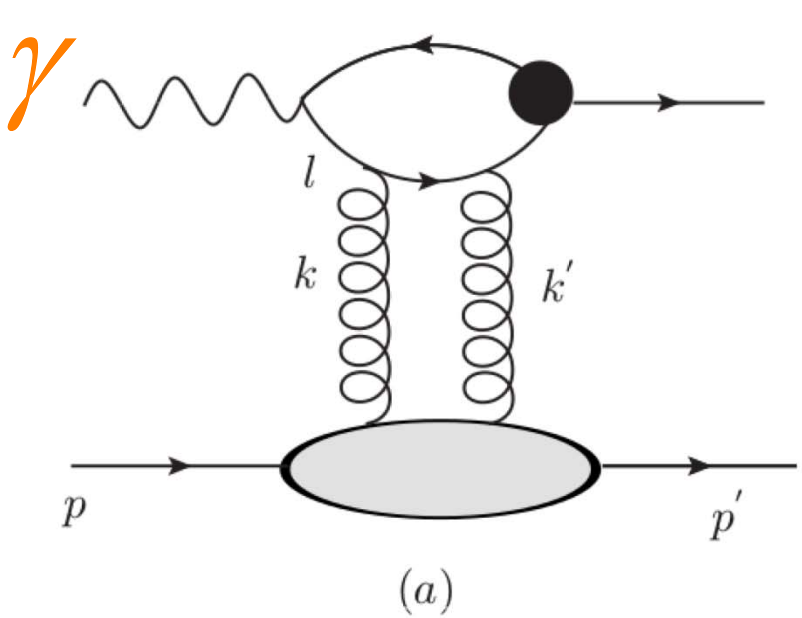
"UPC"



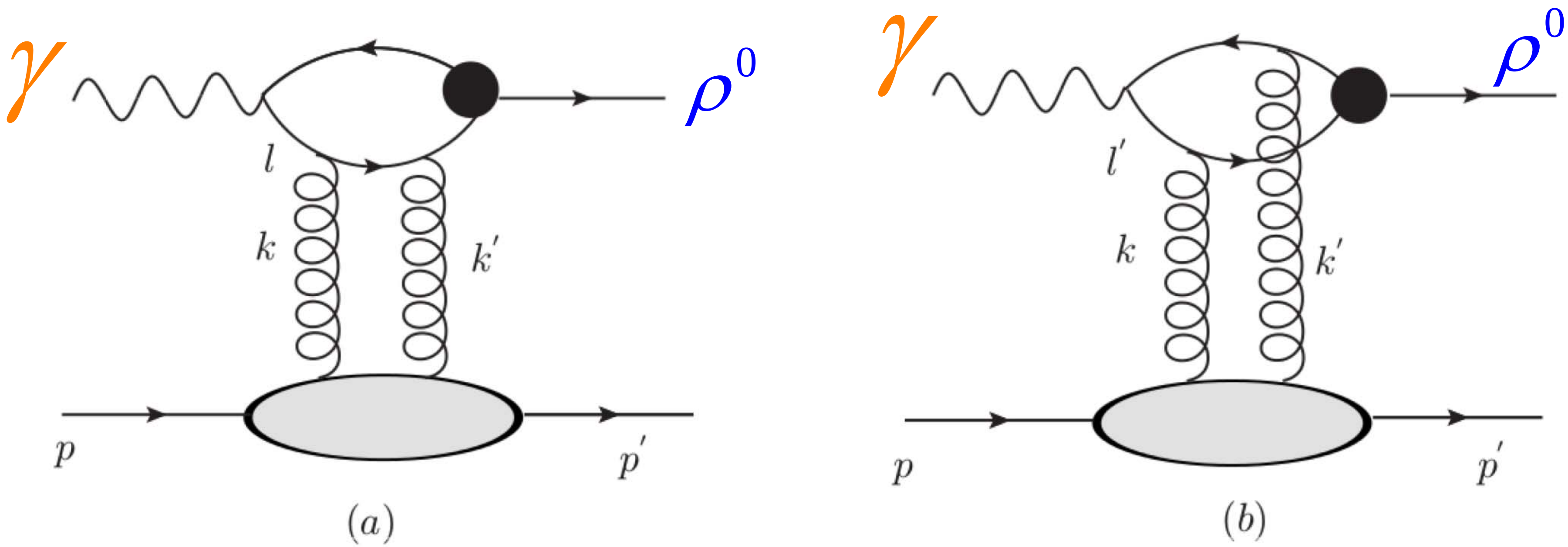


"UPC"





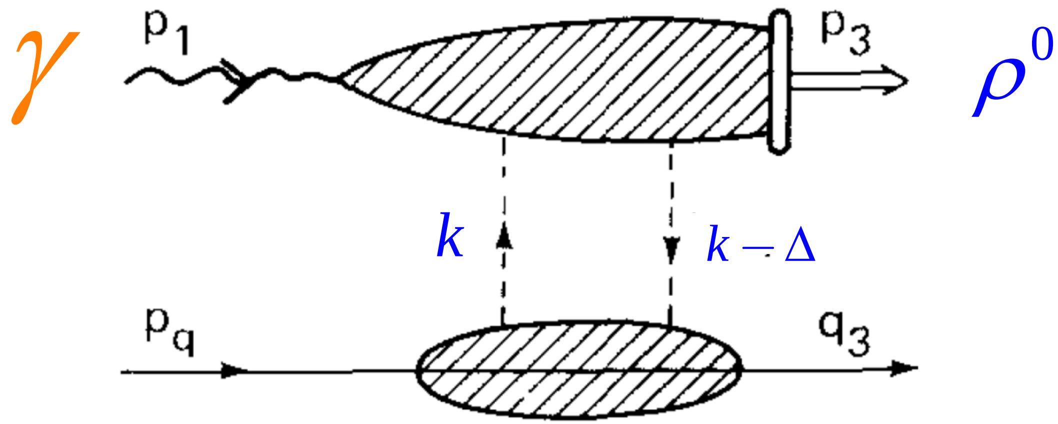
γ
l
k
k′
p
p′
(a)
γ
l′
k
k′
p
p′
(b)



γ
ρ0
l
k
k′
p
p′
(a)
γ
ρ0
l′
k
k′
p
p′
(b)

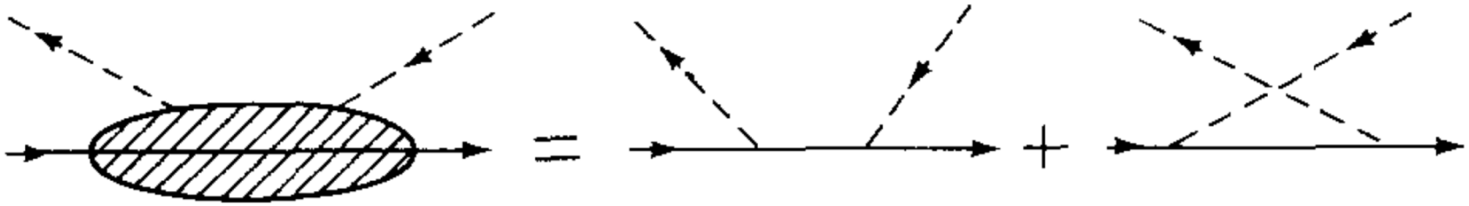
$$s \gg -t \gg \Lambda_{QCD}^2$$

$$t = \Delta^2 = -\frac{s}{2}(1-\cos\theta)$$



$$s_{\gamma q} = (p_1 + p_q)^2, \; t = (p_3 - p_1)^2$$

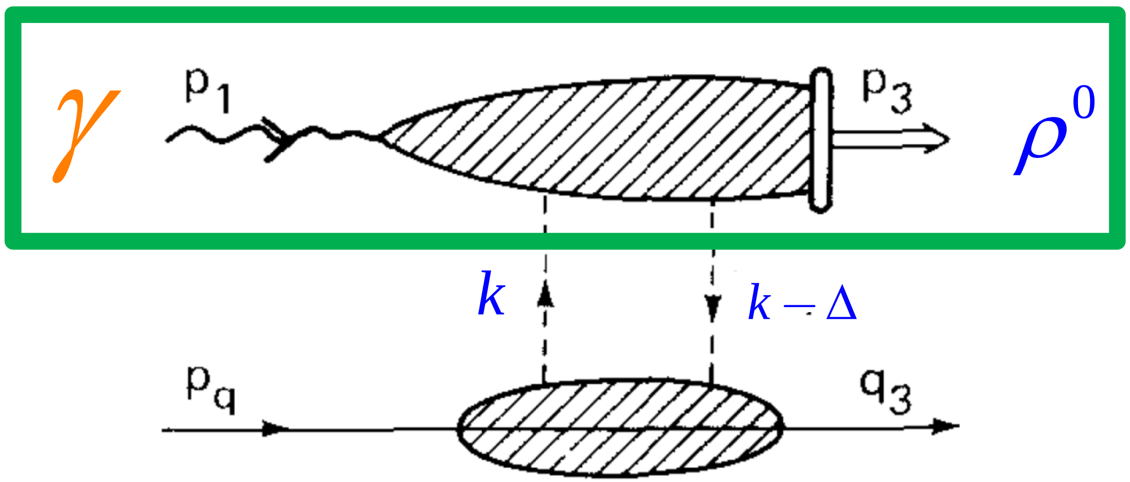
(a)



(b)

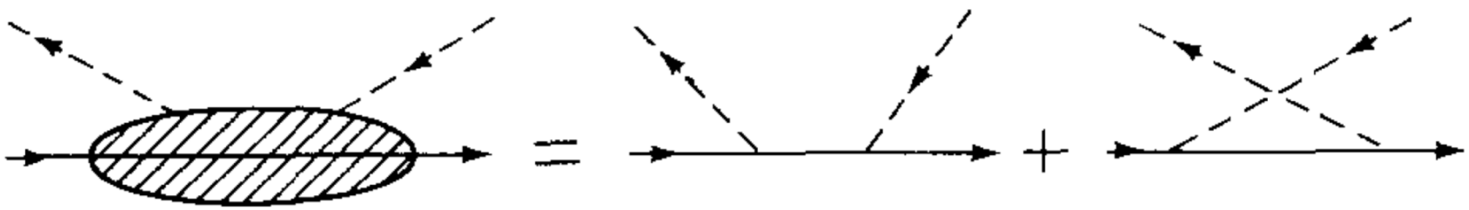
$$s \gg -t \gg \Lambda^2_{QCD}$$

$$t = \Delta^2 = -\frac{s}{2}(1 - \cos\theta)$$

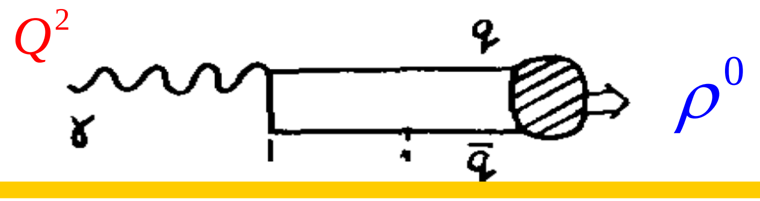


$s_{\gamma q} = (p_1 + p_q)^2,\ t = (p_3 - p_1)^2$

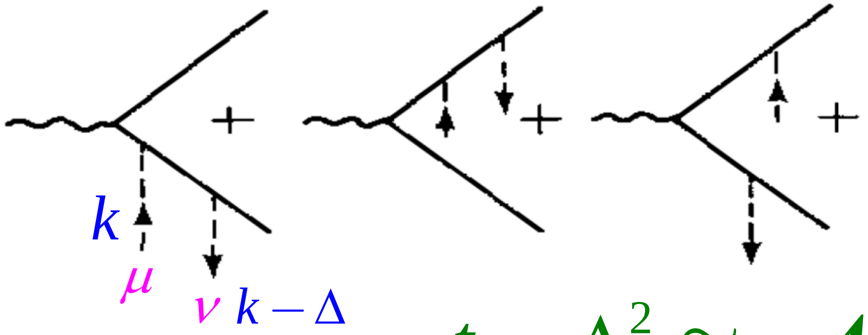
(a)



(b)

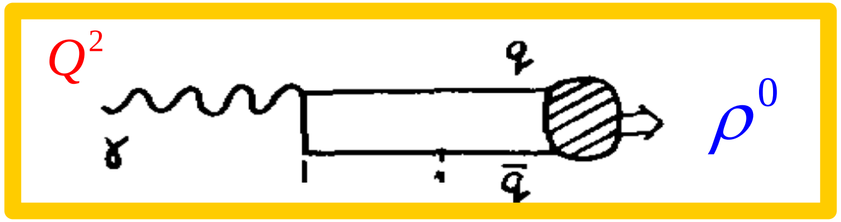


"Impact factor"



$$\left(k,\ \mu \rightarrow k-\Delta,\ \nu\right)$$

$$t = \Delta^2 \simeq -\boldsymbol{\Delta}_\perp^2$$



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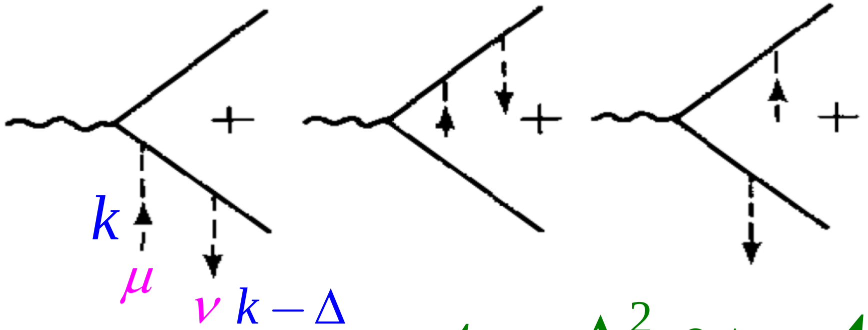
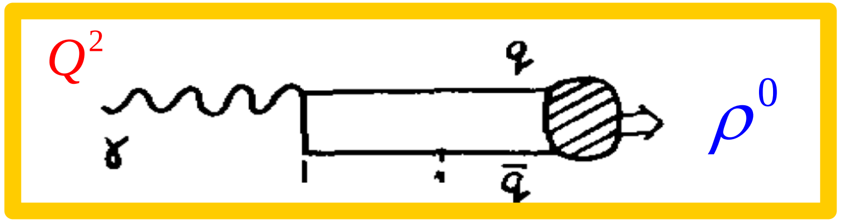
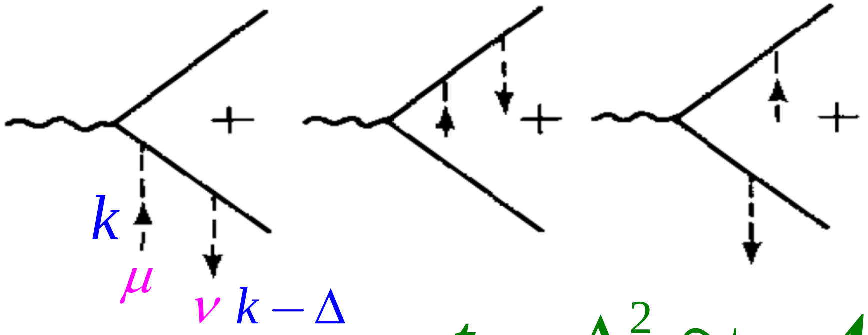


$$(k,\ \mu \rightarrow k-\Delta,\ \nu) \qquad \otimes \Phi$$

$$t = \Delta^2 \simeq -\boldsymbol{\Delta}_\perp^2$$



"Impact factor"



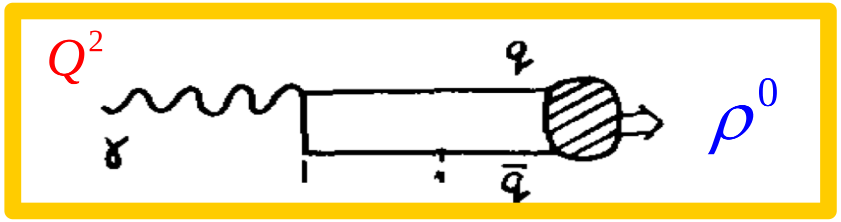
$$+\ (k,\ \mu \to k-\Delta,\ \nu) \qquad \otimes \Phi$$

$$t = \Delta^2 \simeq -\Delta_\perp^2$$

$$k_0^2 - k_3^2 \ll k_\perp^2$$

$$s \gg -t \gg \Lambda_{\mathrm{QCD}}^2$$

light-cone expansion



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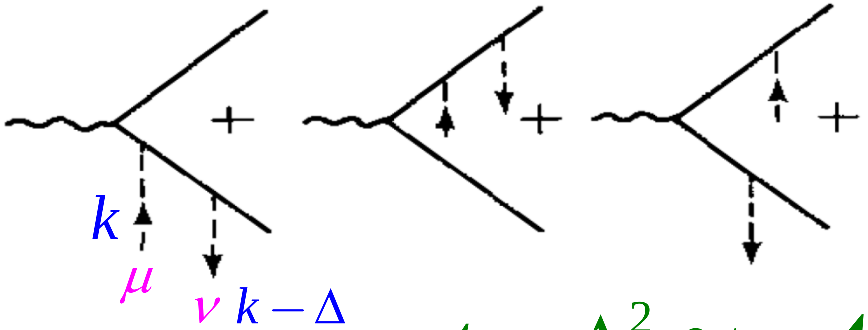
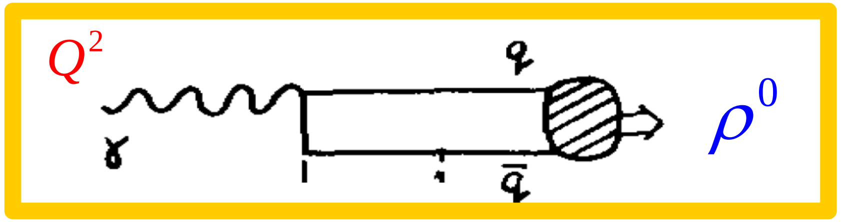
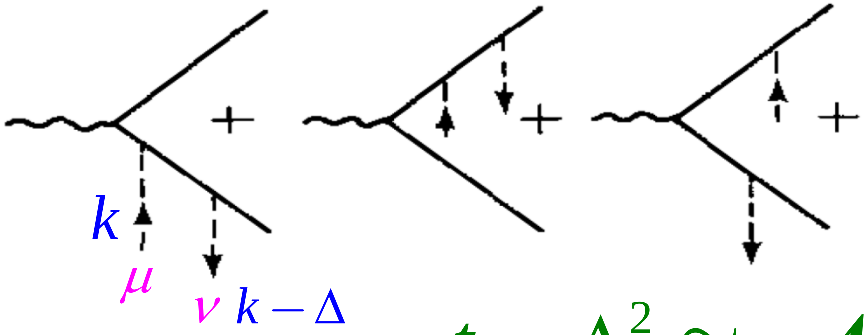


$$(k,\ \mu \rightarrow k-\Delta,\ \nu) \otimes \begin{cases} \phi_L(u) \\ g_T(u) \end{cases}$$

$$t = \Delta^2 \simeq -\Delta_\perp^2$$



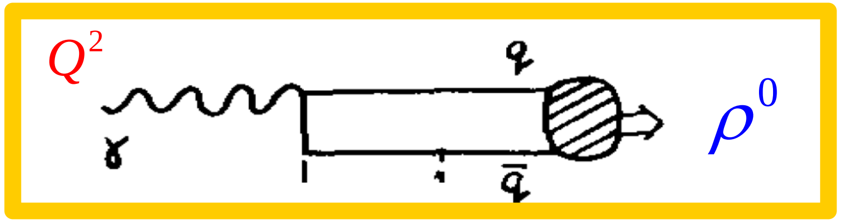
"Impact factor"



$$(k,\ \mu \to k-\Delta,\ \nu) \otimes \begin{cases} \phi_L(u) \\ g_T(u) \end{cases}$$

$$t = \Delta^2 \simeq -\boldsymbol{\Delta}_\perp^2$$

$$\sim \log \frac{Q^2}{-t} \ \text{for}\ \rho_T^0$$



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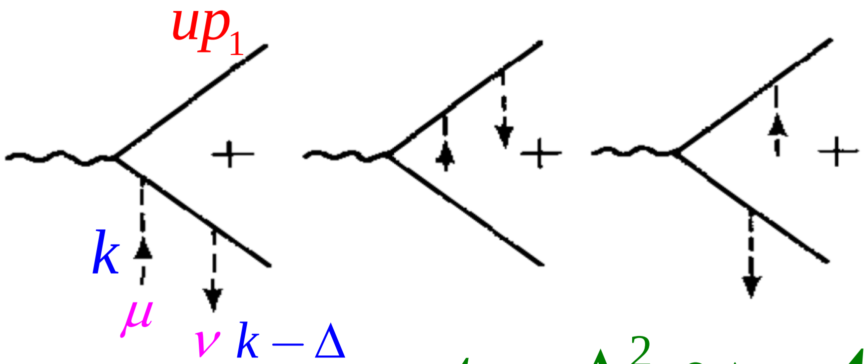

$$+ \; (k,\ \mu \to k-\Delta,\ \nu) \otimes \begin{cases} \phi_L(u) \\ g_T(u) \end{cases}$$

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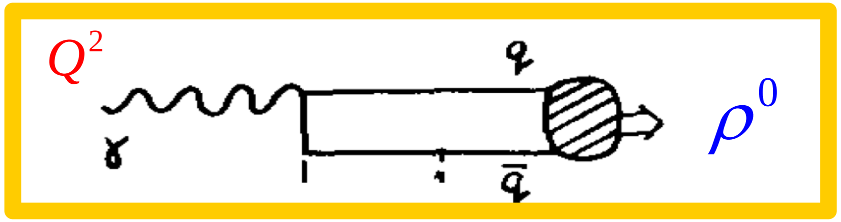

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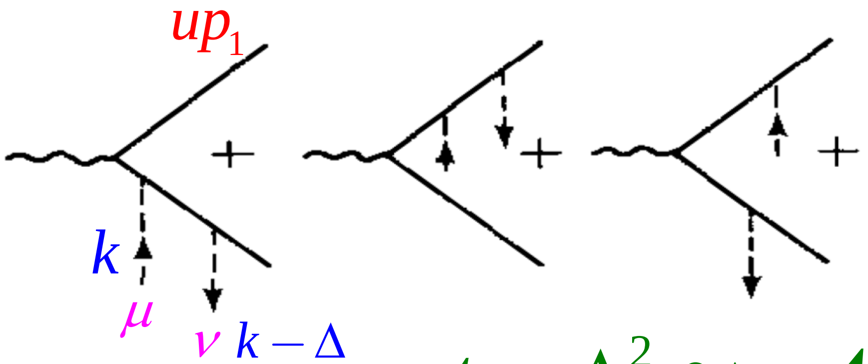


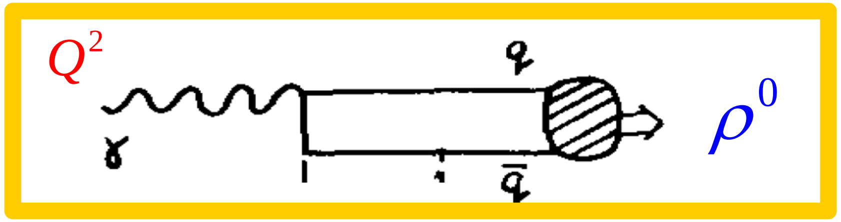
$$+ \left(k,\ \mu \to k-\Delta,\ \nu\right) \otimes \begin{cases} \phi_L(u) \\ g_T(u) \end{cases}$$

$$t = \Delta^2 \simeq -\Delta_\perp^2$$

$$\sim \log \frac{Q^2}{-t} \quad \text{for } \rho_T^0$$

end-point $\left(u \to 0,\ 1\right)$ **behaviors:**

$$\phi_L(u) \sim u(1-u) \qquad g_T(u) \sim 1$$



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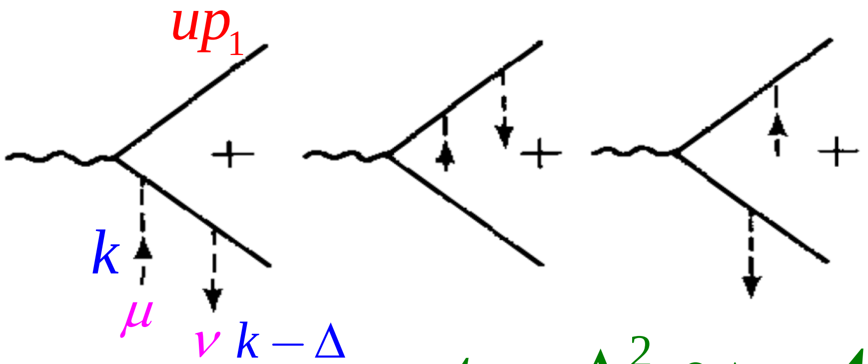


$(k,\ \mu \to k-\Delta,\ \nu) \otimes \begin{cases} \phi_L(u) \\ g_T(u) \end{cases}$

$t = \Delta^2 \simeq -\Delta_\perp^2$

IR divergent! $\sim \log \dfrac{Q^2}{-t}$

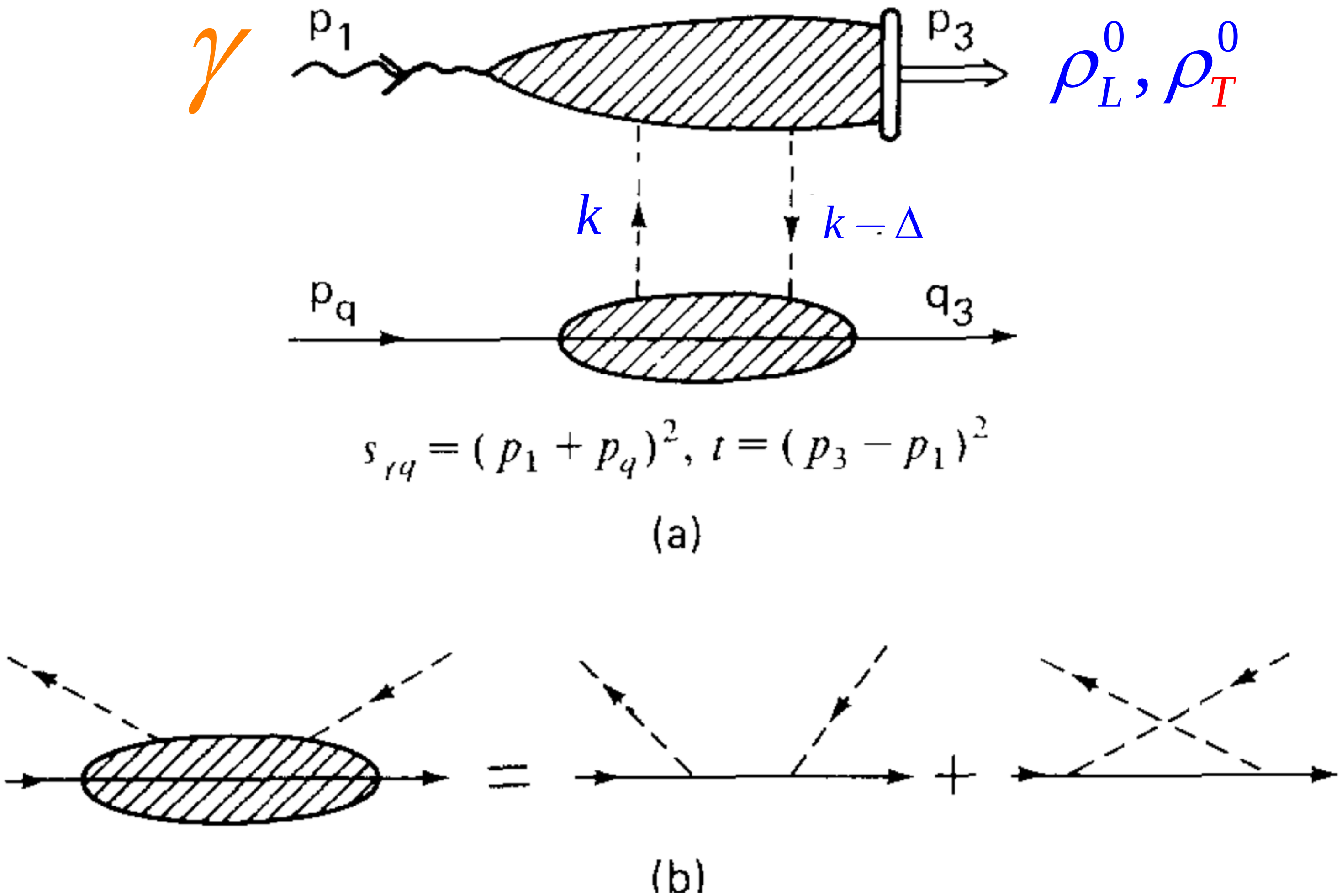
nonfactorizable for ρ_T^0

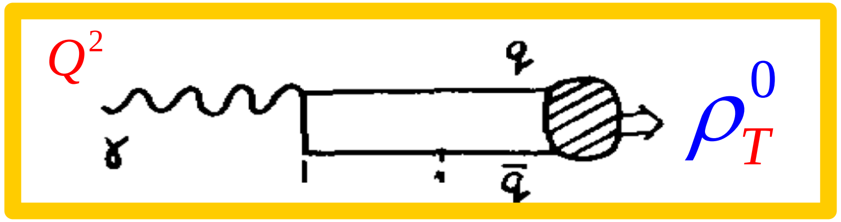
end-point $(u \to 0,\ 1)$ **behaviors:**

$$\phi_L(u) \sim u(1-u) \qquad g_T(u) \sim 1$$

$$s \gg -t \gg \Lambda^2_{\mathrm{QCD}}$$

$$t = -\frac{s}{2}(1-\cos\theta)$$





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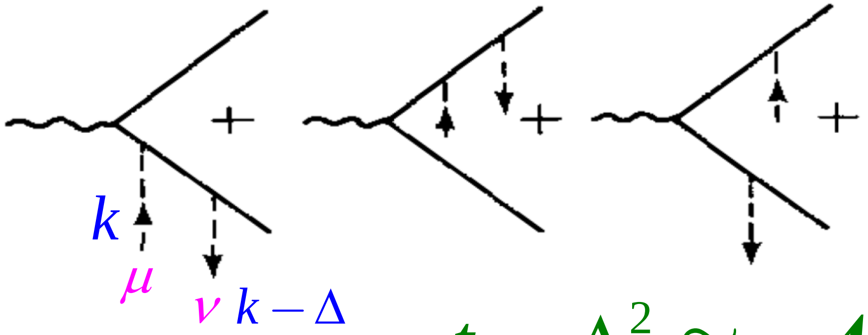

$$+\ (k,\ \mu \to k-\Delta,\ \nu) \otimes \begin{cases} \phi_L(u) \\ g_T(u) \end{cases}$$

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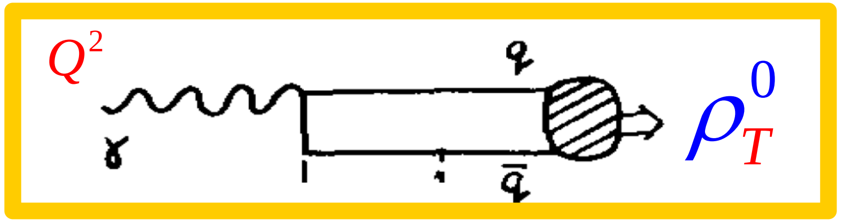
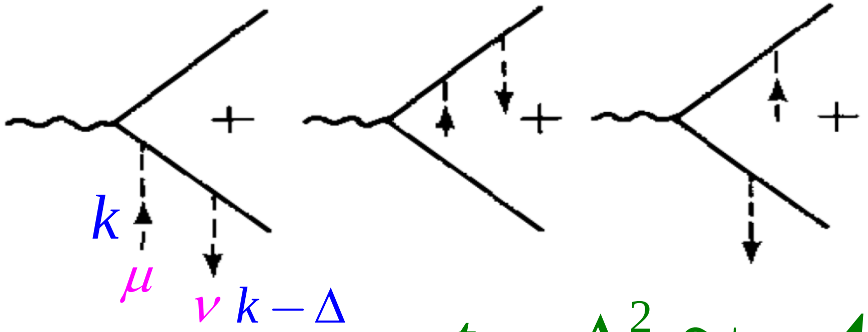
IR divergent!

$$\sim \log \frac{Q^2}{-t} \text{ for } \rho_T^0$$



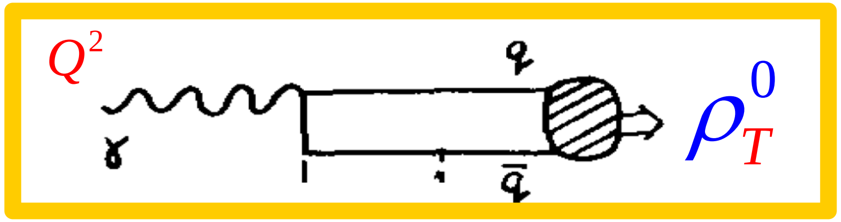
"Impact factor"



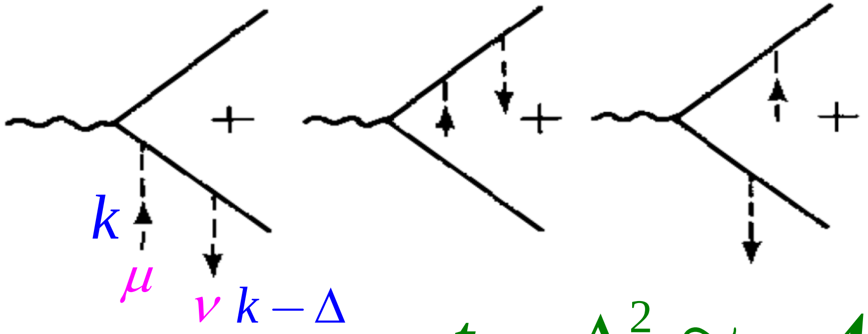
$$(k,\ \mu \rightarrow k-\Delta,\ \nu) \quad \otimes\, g_T(u)$$

$$t = \Delta^2 \simeq -\Delta_\perp^2$$

$$Q^2 \gg \Lambda_{\mathrm{QCD}}^2$$



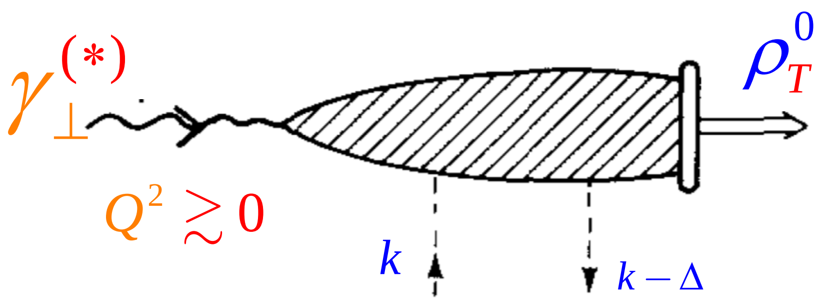
"Impact factor"



$$(k,\ \mu \to k - \Delta,\ \nu) \otimes g_T(u)$$

$$t = \Delta^2 \simeq -\Delta_\perp^2$$

$$Q^2 \gg \Lambda_{\rm QCD}^2$$



$$\int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2 + m^2}$$

$$= \frac{a}{Q^2 + m_V^2} + \int_{m_{th}^2}^\infty dm^2 \frac{\chi(m^2)}{Q^2 + m^2}$$

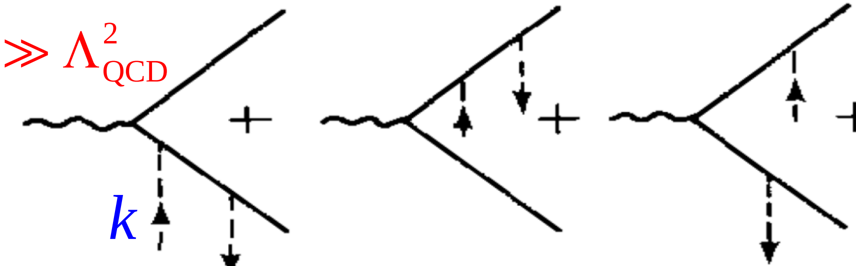
$Q^2 \gg \Lambda^2_{\rm QCD}$

ρ_T^0

"Impact factor"

$$Q^2 \gg \Lambda^2_{\rm QCD}$$

$Q^2 \gg \Lambda^2_{\rm QCD}$

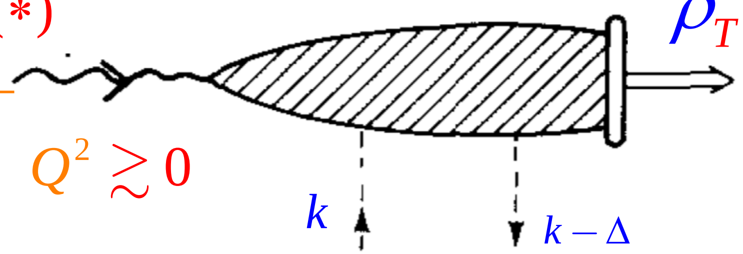


$$+ \; + \; + \;(k,\ \mu \to k-\Delta,\ \nu) \quad \otimes g_T(u)$$

k μ ν $k-\Delta$

$$a \propto e^{\frac{m_V^2}{M_B^2}} \int_{u_0}^{1} du \left(2 g_T^{(v)}(u) - \frac{1}{2} \frac{\partial g_T^{(a)}(u)}{\partial u} \right) e^{-\frac{(1-u)\Delta_\perp^2}{u M_B^2}}$$

LCSR (quark-hadron duality)



$\gamma_\perp^{(*)}$ ρ_T^0

$Q^2 \gtrsim 0$ k $k-\Delta$

$$\int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$

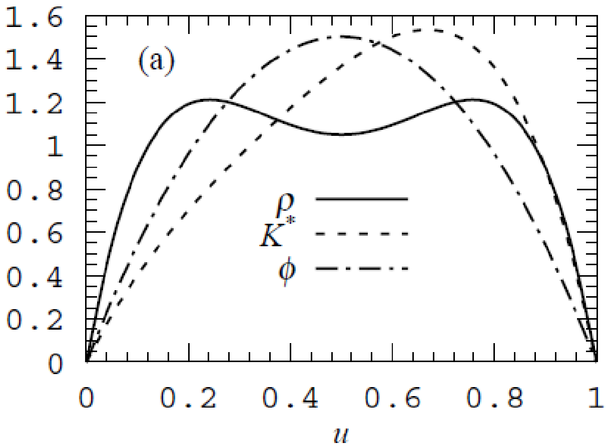
$$= \frac{a}{Q^2+m_V^2} + \int_{m_{th}^2}^{\infty} dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$

Ball, Braun, Koike, KT ('98)

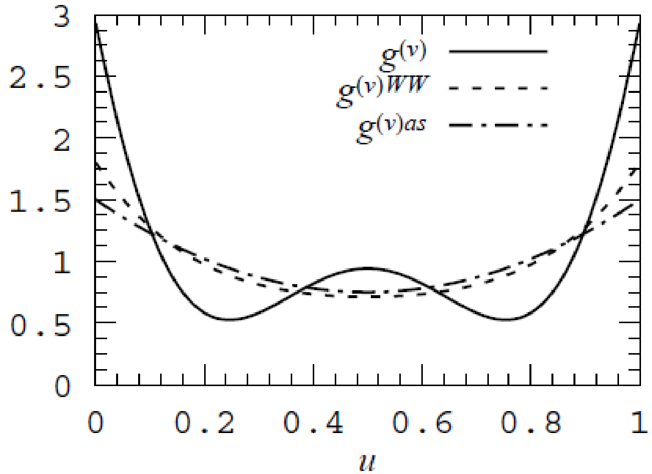
ρ meson WFs

$$\xi = u - (1-u) = 2u - 1$$

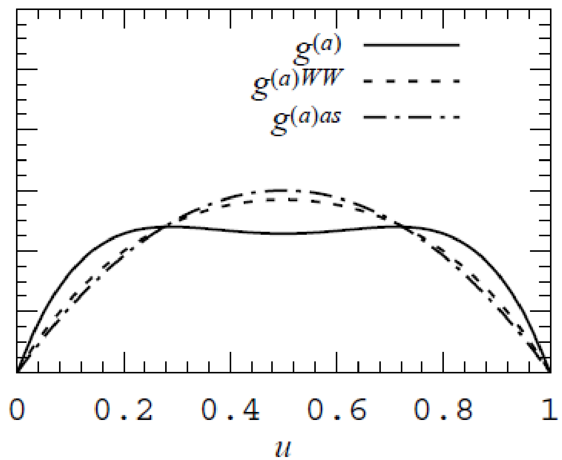
$\phi_L(u)$



$g_T^{(v)}(u)$



$g_T^{(a)}(u)$



$$\phi_L(u) = 6u(1-u) \sum_{n=0,2,4,\cdots} a_n C_n^{3/2}(2u-1) = 6u(1-u)\left(1 + a_2 \frac{3}{2}(5\xi^2 - 1) + \cdots\right)$$

$$a_2 = 0.18 \pm 0.10$$

$$g_T^{(v)}(u) = \sum_{n=0,2,4,\cdots} \left(G_n - G_{n-1}\right) C_n^{1/2}(2u-1)$$

$$g_T^{(a)}(u) = 8u(1-u) \sum_{n=0,2,4,\cdots} \frac{G_n - G_{n+1}}{(n+1)(n+2)} C_n^{3/2}(2u-1)$$

ρ meson WFs $z^2 = 0$ $\xi = u - (1-u) = 2u - 1$

Ball, Braun, Koike, KT ('98)

$$\langle 0 | \bar{q}(z)\gamma_\mu q(-z) | \rho^0(p,e)\rangle = f_\rho m_\rho\, p_\mu \frac{e\cdot z}{p\cdot z}\int_0^1 du\, e^{i\xi p\cdot z}\phi_L(u)$$

$$\mathrm{P}\exp\left[ig\int_{-1}^{1} dt z_\mu A^\mu(tz)\right]$$

$$+ f_\rho m_\rho e_{T\mu}\int_0^1 du\, e^{i\xi p\cdot z} g_T^{(v)}(u)$$

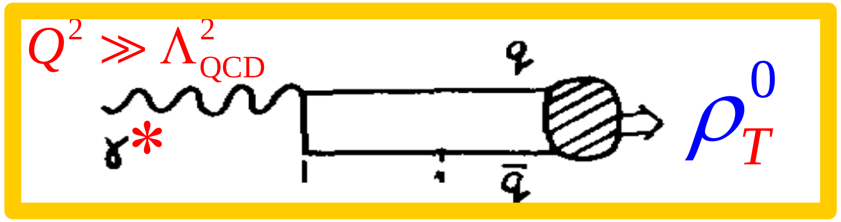
$$\langle 0 | \bar{q}(z)\gamma_\mu\gamma_5 q(-z) | \rho^0(p,e)\rangle = \frac{1}{2} f_\rho m_\rho \epsilon_{\mu\nu\alpha\beta} e_T^\nu p^\alpha z^\beta \int_0^1 du\, e^{i\xi p\cdot z} g_T^{(a)}(u)$$

$$\phi_L(u) = 6u(1-u)\left[1 + \sum_{n=1}^{\infty} a_{2n} C_{2n}^{3/2}(2u-1)\right] = 6u(1-u)\left[1 + a_2\frac{3}{2}(5\xi^2 - 1) + \cdots\right]$$

$$g_T^{(v)}(u) = \frac{3}{4}\left(1+\xi^2\right) + a_2\frac{3}{7}(3\xi^2 - 1) + \cdots$$

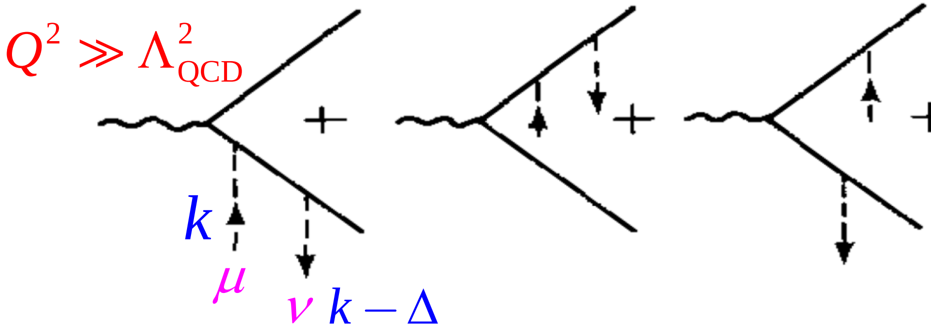
$$g_T^{(a)}(u) = 6u(1-u)\left[1 + a_2\frac{1}{4}(5\xi^2 - 1) + \cdots\right]$$

$$a_2 = 0.18 \pm 0.10$$



"Impact factor"

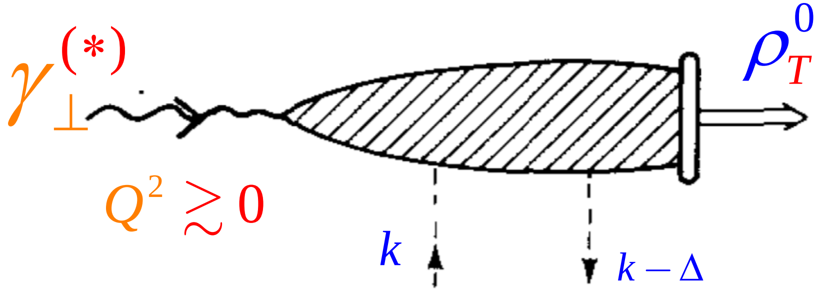
$$Q^2 \gg \Lambda_{QCD}^2$$



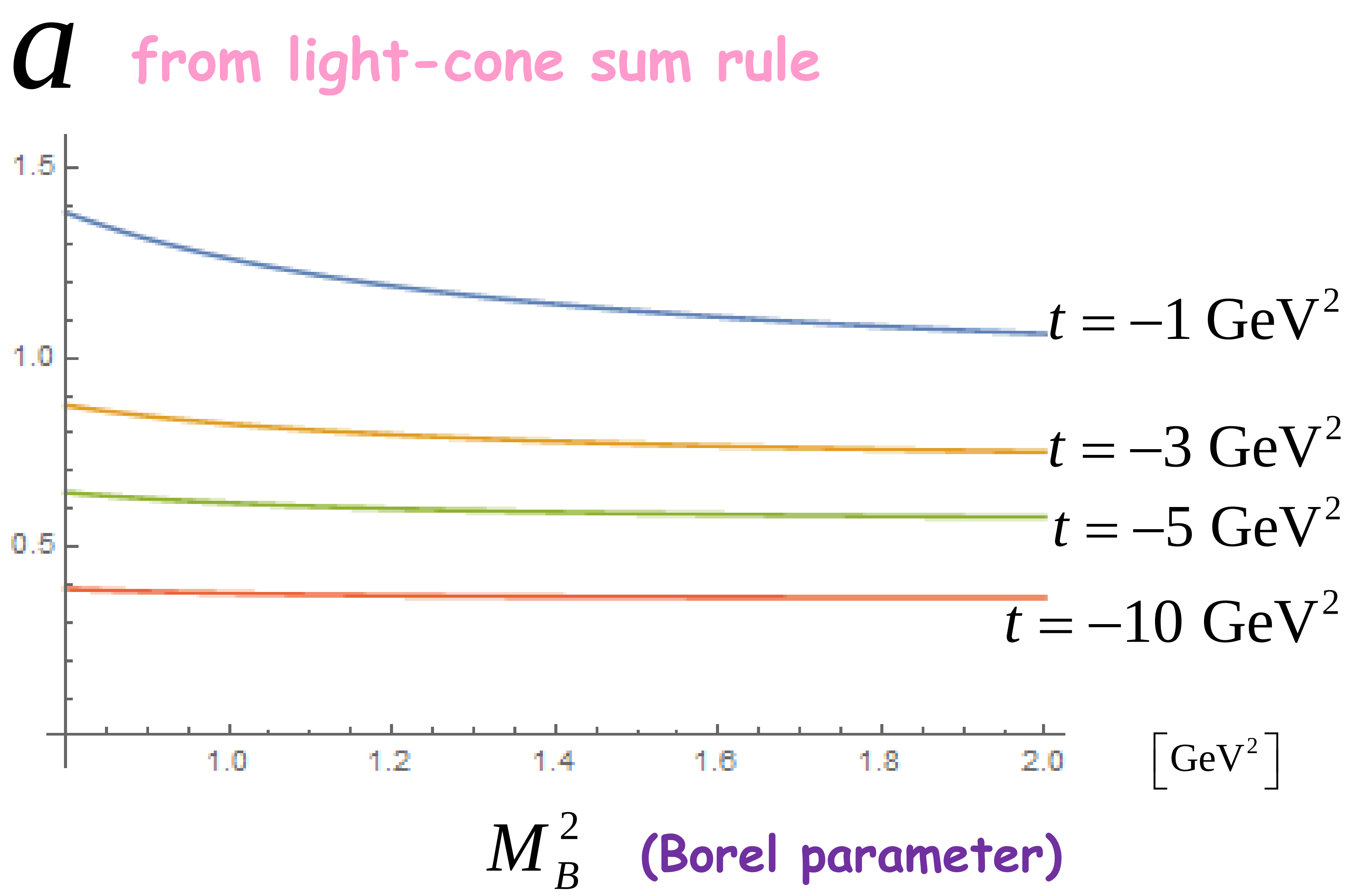
$$+ \left(k,\ \mu \to k-\Delta,\ \nu\right) \otimes g_T(u)$$

$$a \propto e^{\frac{m_V^2}{M_B^2}} \int_{u_0}^{1} du \left(2 g_T^{(v)}(u) - \frac{1}{2} \frac{\partial g_T^{(a)}(u)}{\partial u}\right) e^{-\frac{(1-u)\Delta_\perp^2}{u M_B^2}}$$

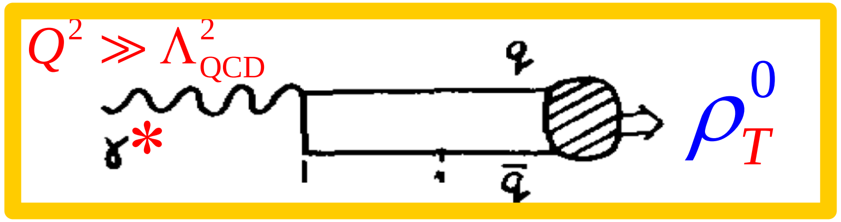
LCSR (quark-hadron duality)



$$\int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2} = \frac{a}{Q^2+m_V^2} + \int_{m_{th}^2}^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$

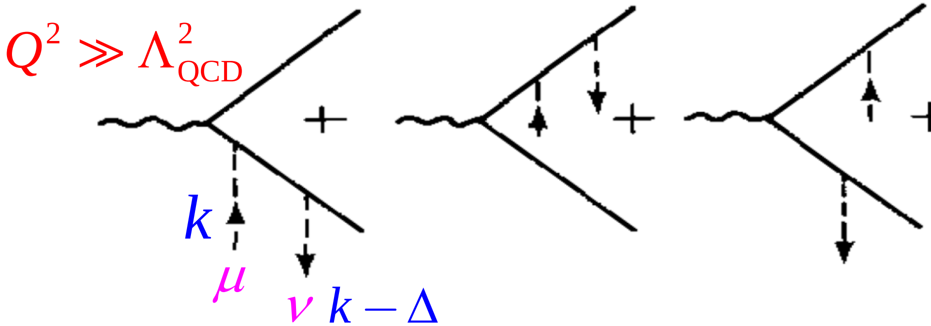


a from light-cone sum rule
1.5
1.0
0.5
t = −1 GeV²
t = −3 GeV²
t = −5 GeV²
t = −10 GeV²
1.0
1.2
1.4
1.6
1.8
2.0
[GeV²]
M_B² (Borel parameter)



"Impact factor"

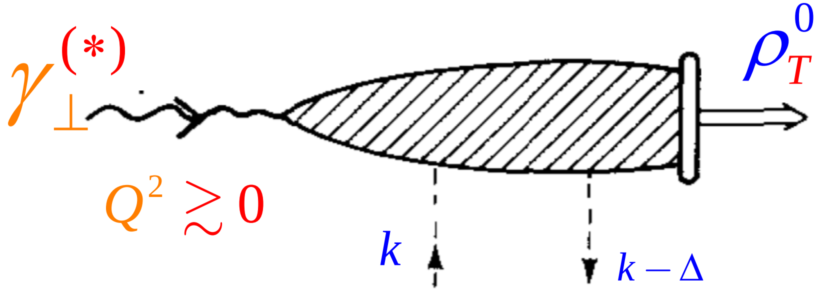
$$Q^2 \gg \Lambda^2_{QCD}$$



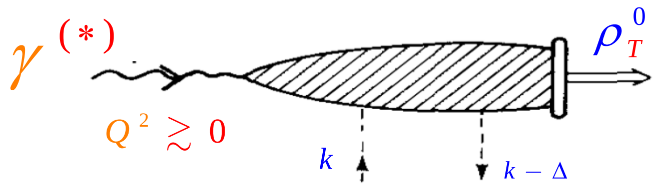
$$+ \left(k,\ \mu \to k-\Delta,\ \nu\right) \otimes g_T(u)$$

$$a \propto e^{\frac{m_V^2}{M_B^2}} \int_{u_0}^{1} du \left(2 g_T^{(v)}(u) - \frac{1}{2} \frac{\partial g_T^{(a)}(u)}{\partial u} \right) e^{-\frac{(1-u)\Delta_\perp^2}{u M_B^2}}$$

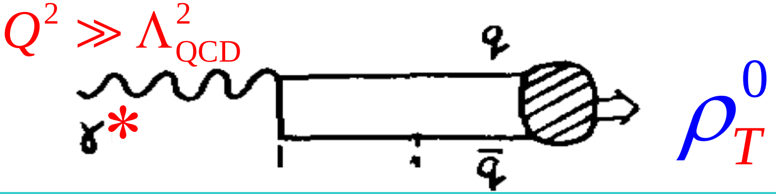
LCSR (quark-hadron duality)

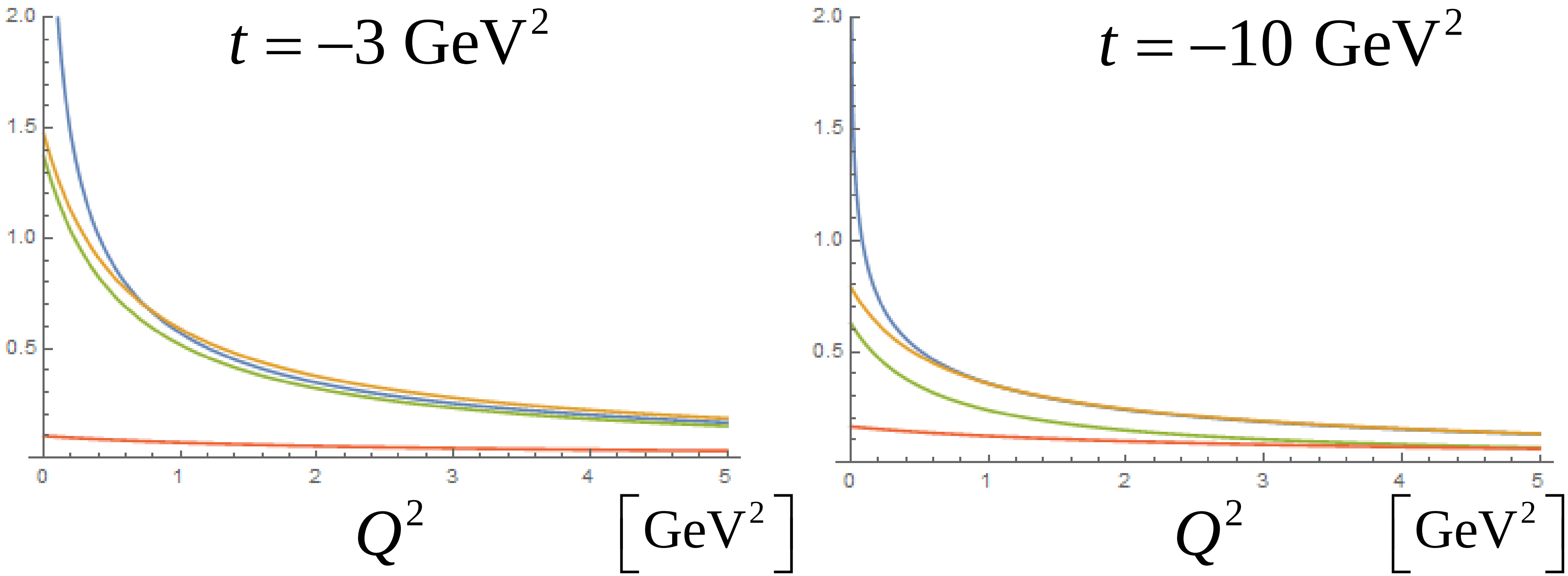


$$\int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2} = \frac{a}{Q^2+m_V^2} + \int_{m_{th}^2}^{\infty} dm^2 \frac{\chi(m^2)}{Q^2+m^2}$$



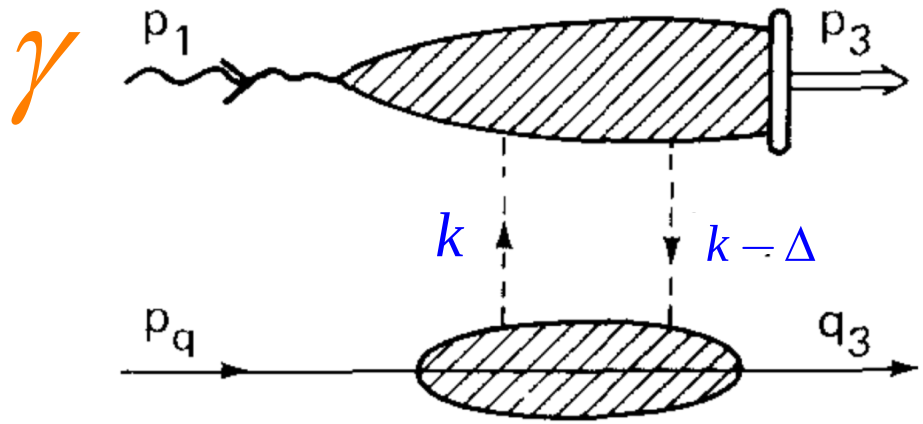
$$= \frac{a}{Q^2 + m_V^2} + \int_{m_{th}^2}^{\infty} dm^2 \, \frac{\chi(m^2)}{Q^2 + m^2}$$





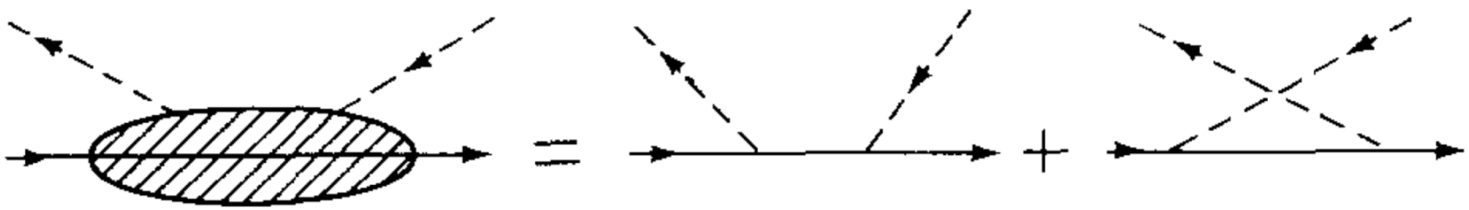
$$s \gg -t \gg \Lambda^2_{\mathrm{QCD}}$$

$$t = -\frac{s}{2}(1 - \cos\theta)$$



$$s_{\gamma q} = (p_1 + p_q)^2,\ t = (p_3 - p_1)^2$$

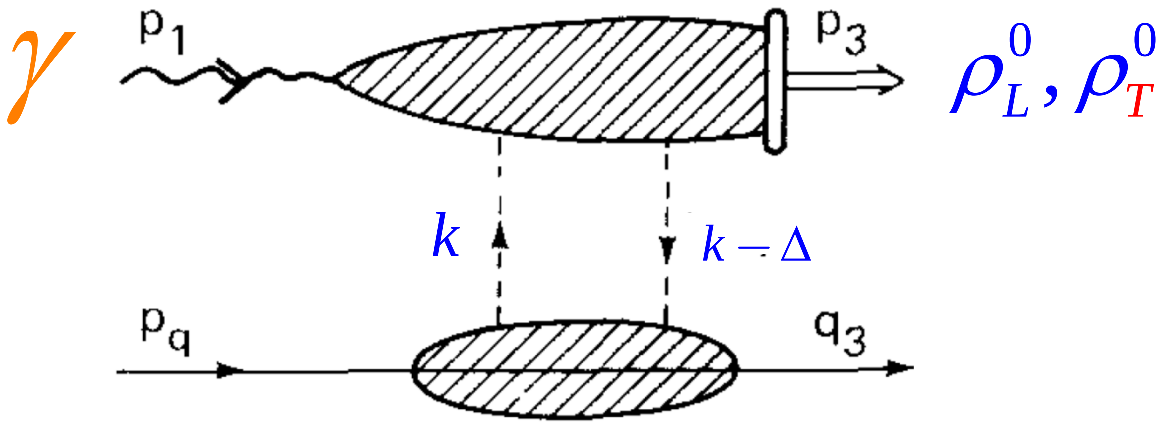
(a)



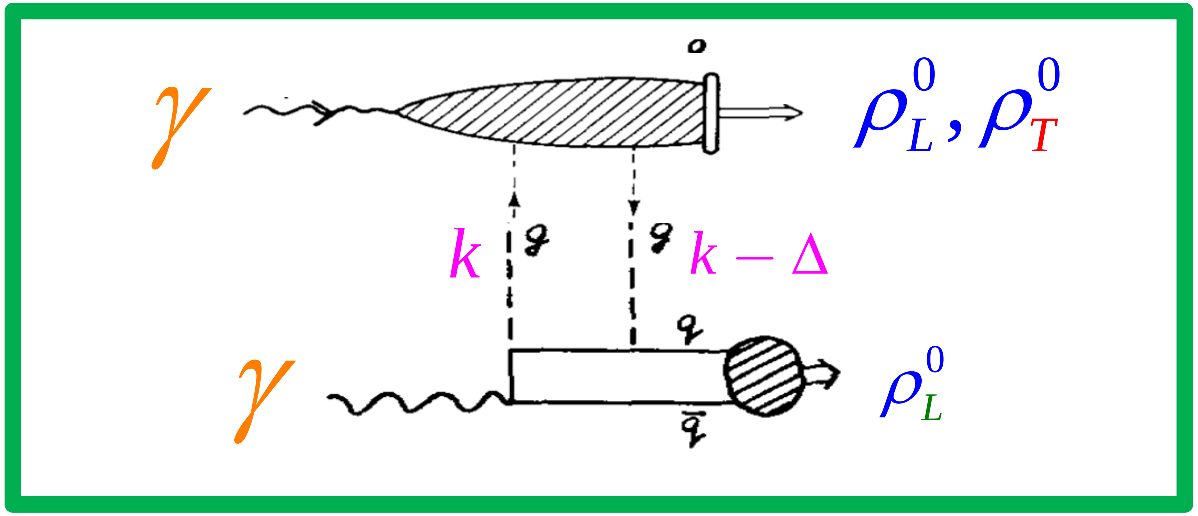
(b)

$$s \gg -t \gg \Lambda^2_{\text{QCD}}$$

$$t = -\frac{s}{2}(1 - \cos\theta)$$

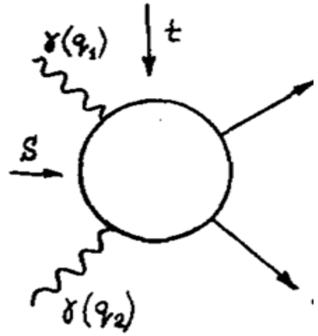


$$s_{\gamma q} = (p_1 + p_q)^2, \; t = (p_3 - p_1)^2$$

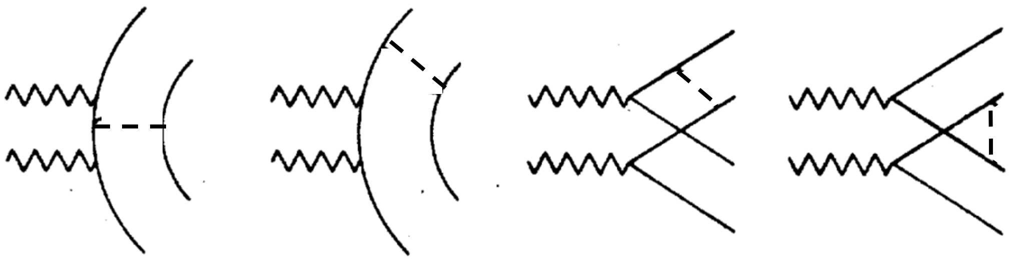


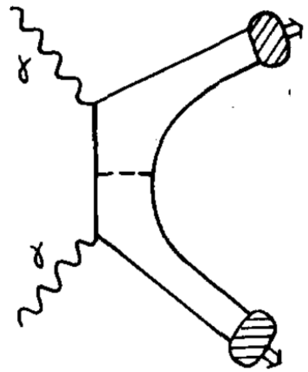
two types of pQCD mechanisms

$$t = -\frac{s}{2}(1-\cos\theta)$$



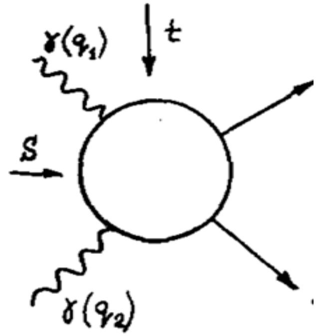
$q\overline{q}$ **exchange** Brodsky, Lepage ('81)



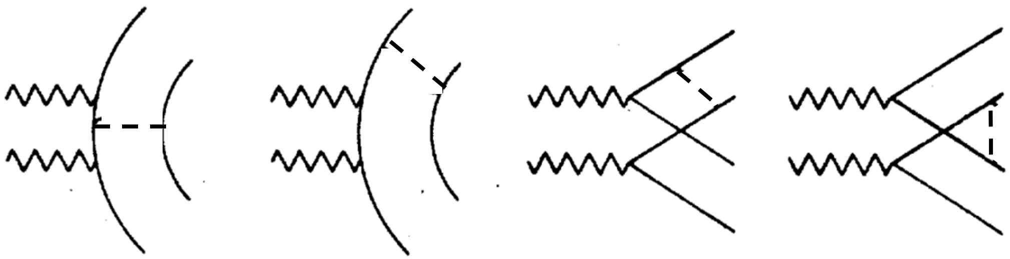


two types of pQCD mechanisms

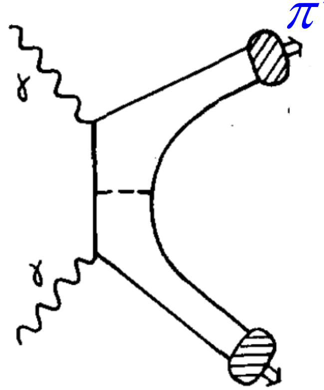
$$t = -\frac{s}{2}(1-\cos\theta)$$



$q\bar{q}$ **exchange** Brodsky, Lepage ('81)



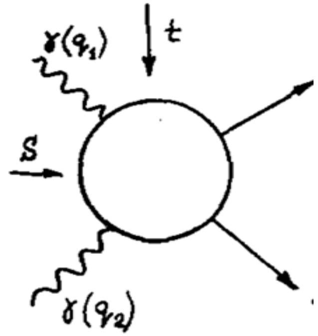
$\pi^{\pm,0}, \rho_L^{\pm,0}, \rho_T^{\pm,0}$



$\pi^{\mp,0}, \rho_L^{\mp,0}, \rho_T^{\mp,0}$

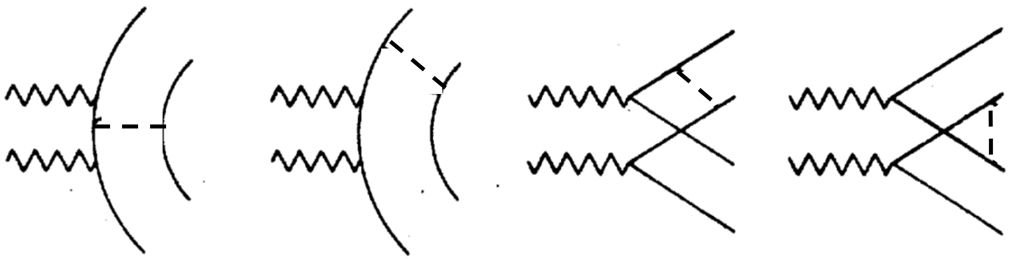
two types of pQCD mechanisms

$$t = -\frac{s}{2}(1-\cos\theta)$$

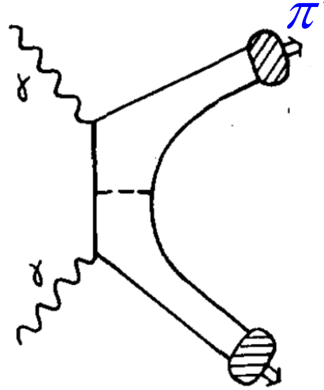


$q\bar{q}$ **exchange** Brodsky, Lepage ('81)

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^2}{s^4}$$



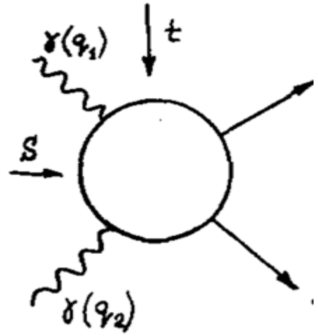
$\pi^{\pm,0}, \rho_L^{\pm,0}, \rho_T^{\pm,0}$



$\pi^{\mp,0}, \rho_L^{\mp,0}, \rho_T^{\mp,0}$

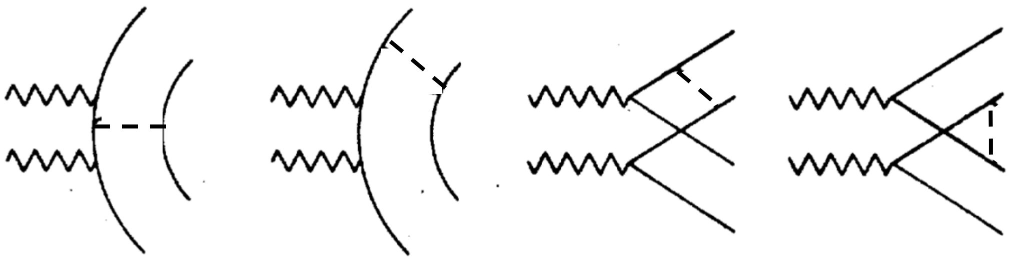
two types of pQCD mechanisms

$$t = -\frac{s}{2}(1-\cos\theta)$$

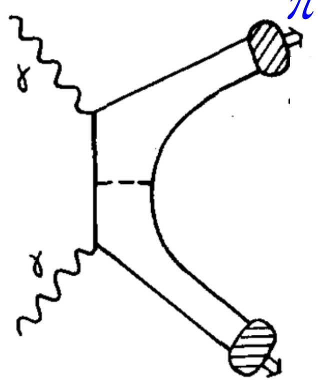


$q\bar{q}$ exchange   Brodsky, Lepage ('81)

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^2}{s^4}$$



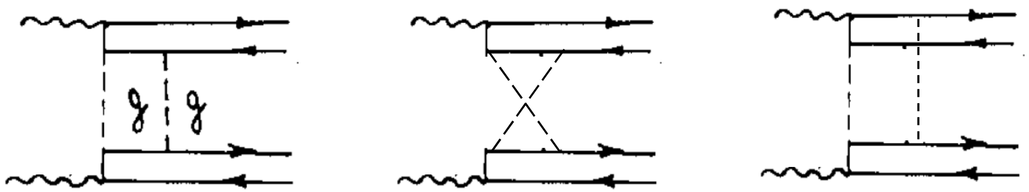
$\pi^{\pm,0}, \rho_L^{\pm,0}, \rho_T^{\pm,0}$

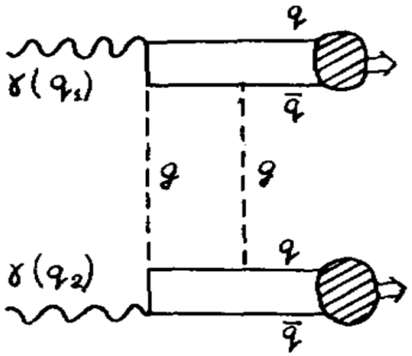


$\pi^{\mp,0}, \rho_L^{\mp,0}, \rho_T^{\mp,0}$

gg exchange   Chernyak, Zhitnitsky ('83)

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^4}{s^0}$$



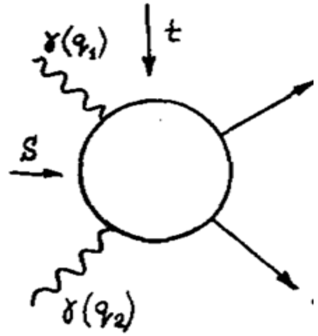


ρ^0

ρ^0, ω, ϕ

two types of pQCD mechanisms

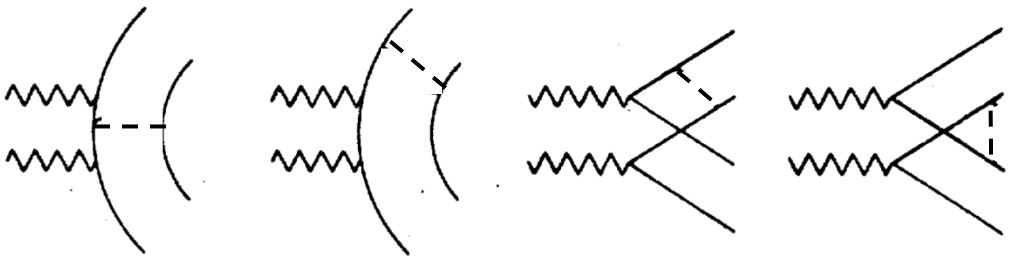
$$t = -\frac{s}{2}(1-\cos\theta)$$

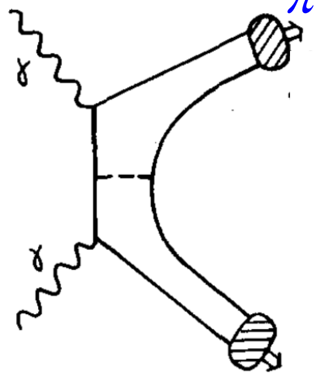


$q\bar{q}$ exchange   Brodsky, Lepage ('81)

$$s \sim -t \gg \Lambda^2_{\mathrm{QCD}}$$

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^2}{s^4}$$





$\pi^{\pm,0}, \rho_L^{\pm,0}, \rho_T^{\pm,0}$

$\pi^{\mp,0}, \rho_L^{\mp,0}, \rho_T^{\mp,0}$

gg exchange   Chernyak, Zhitnitsky ('83)

$$s \gg -t \gg \Lambda^2_{\mathrm{QCD}}$$

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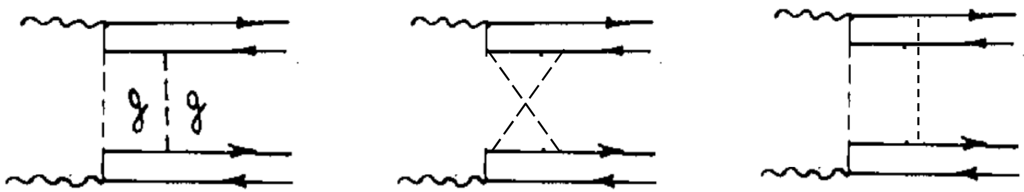
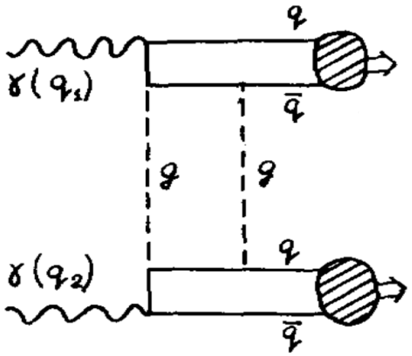
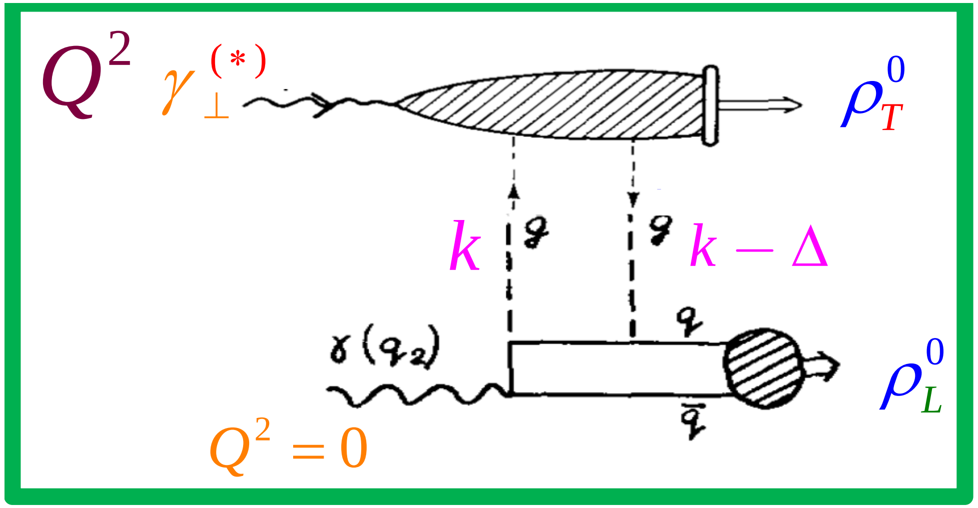




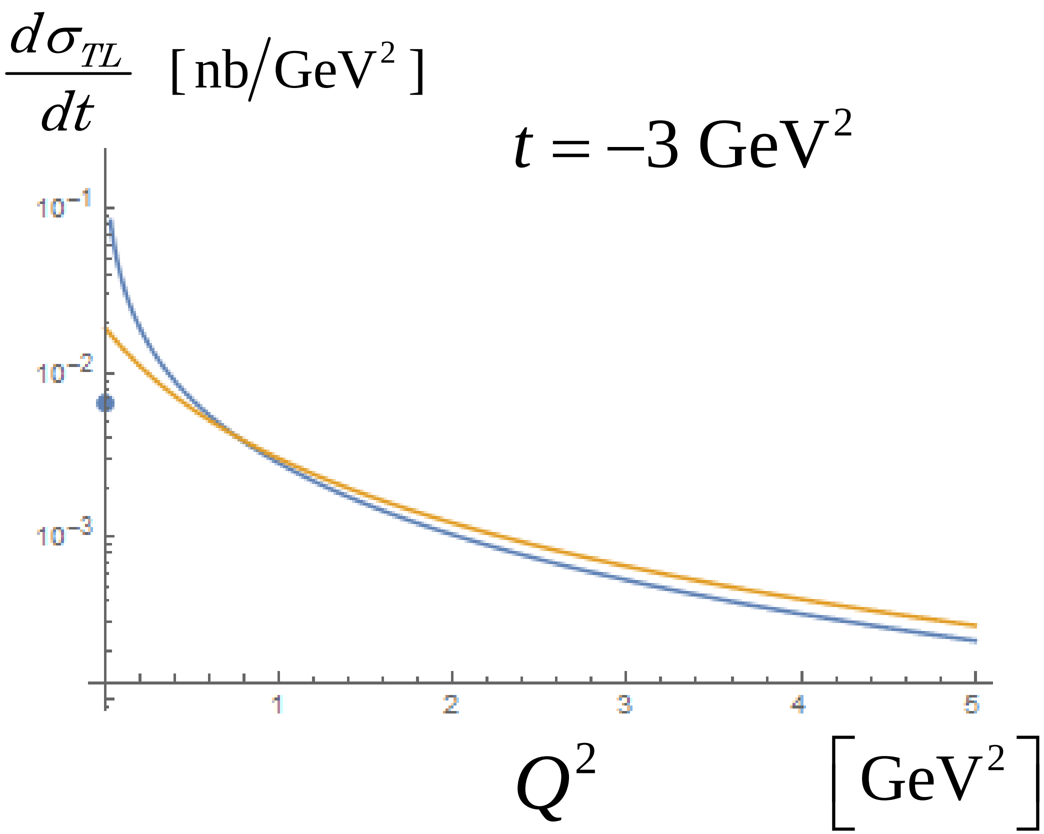
ρ^0

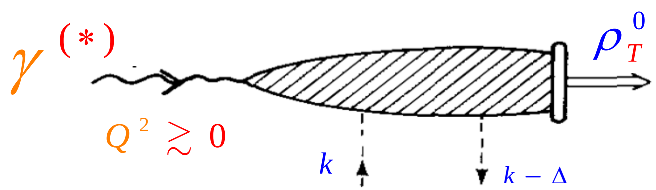
ρ^0, ω, ϕ



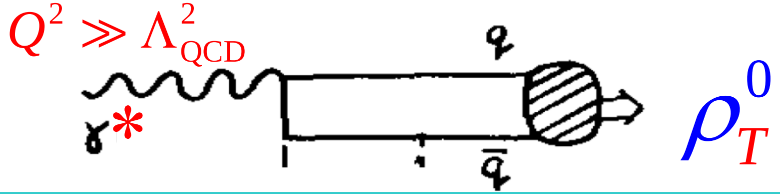
$$s \gg -t \gg \Lambda^2_{\text{QCD}}$$

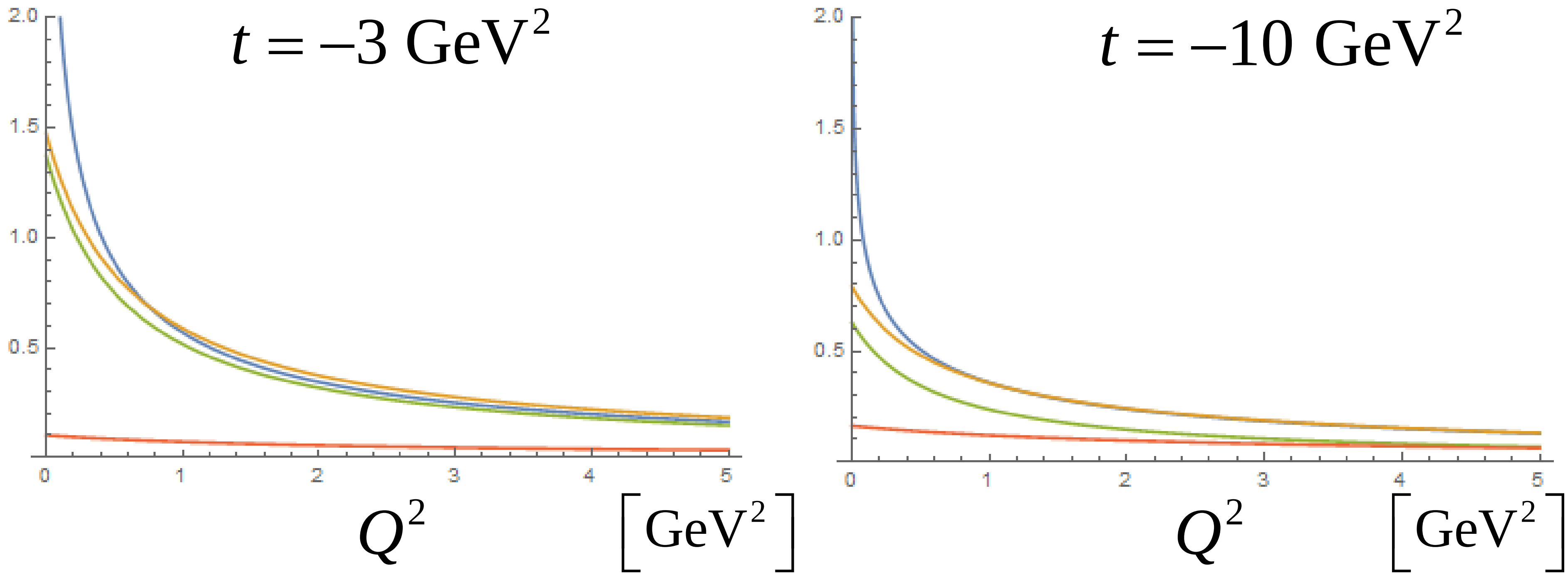
$$s = 100\ \text{GeV}^2$$

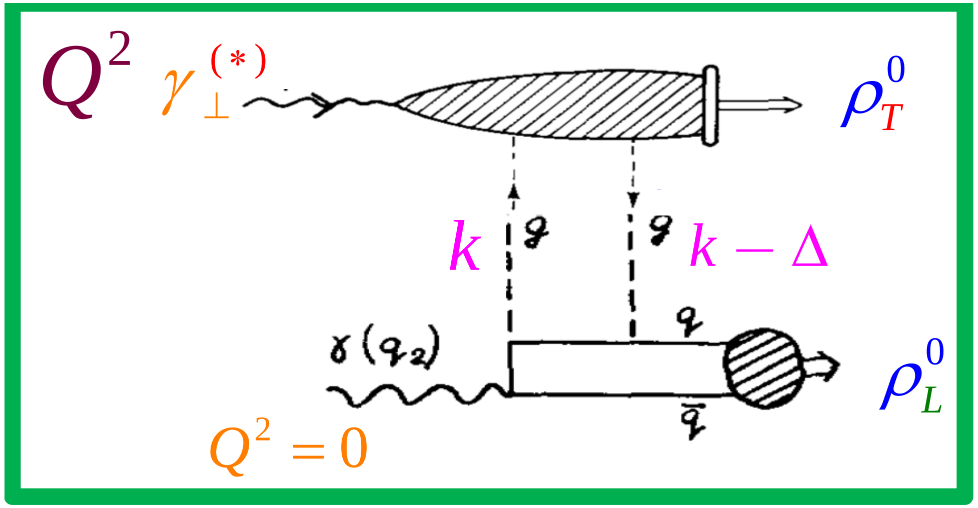




$$= \frac{a}{Q^2 + m_V^2} + \int_{m_{th}^2}^{\infty} dm^2 \frac{\chi(m^2)}{Q^2 + m^2}$$

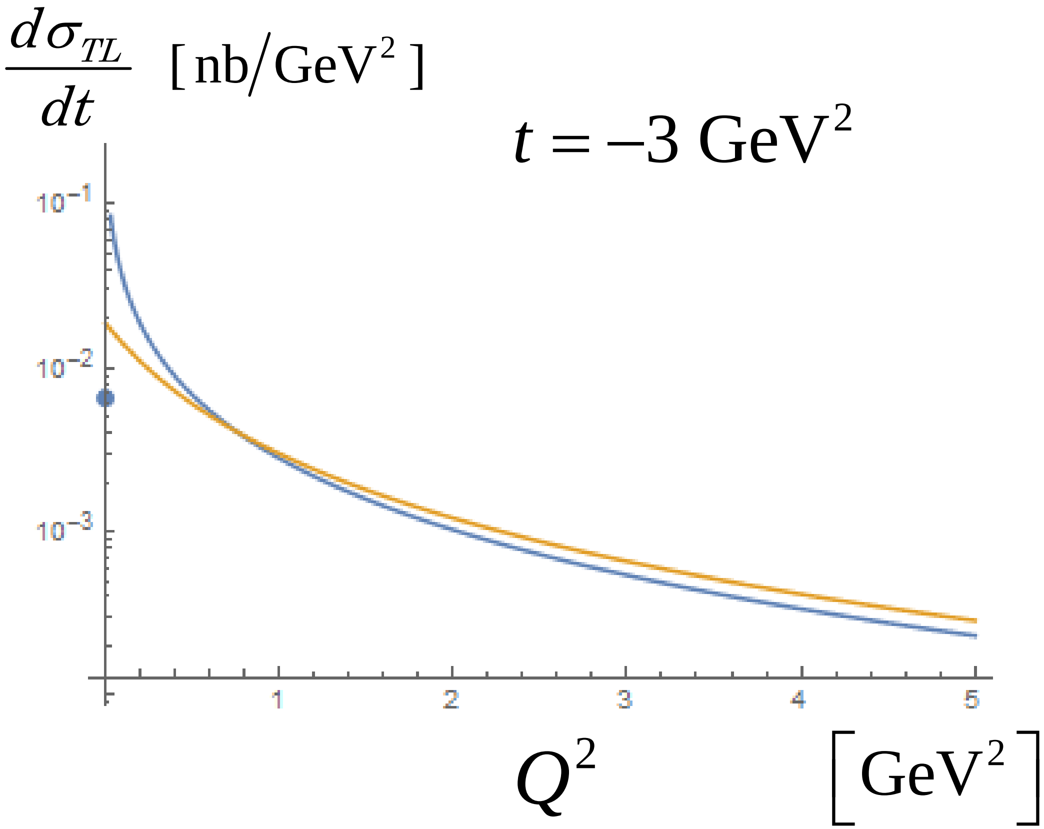


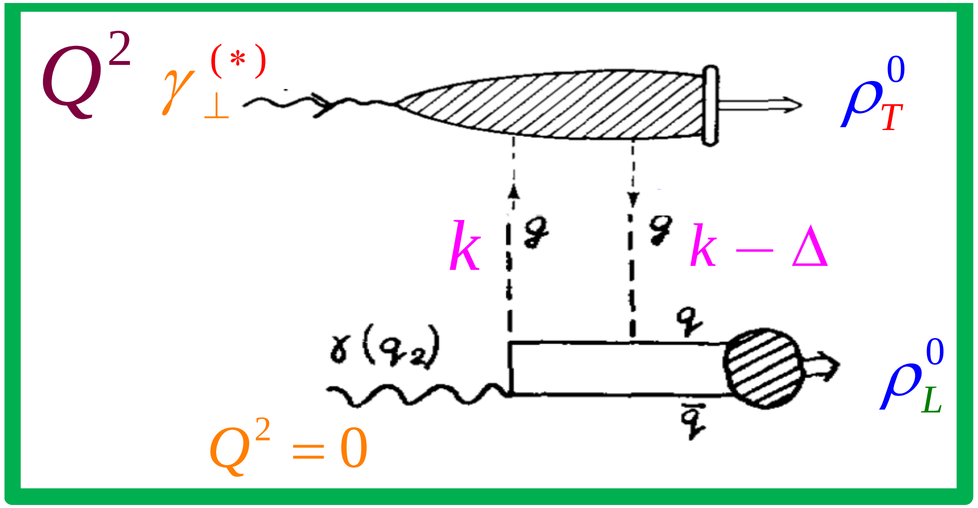




$s \gg -t \gg \Lambda^2_{\text{QCD}}$

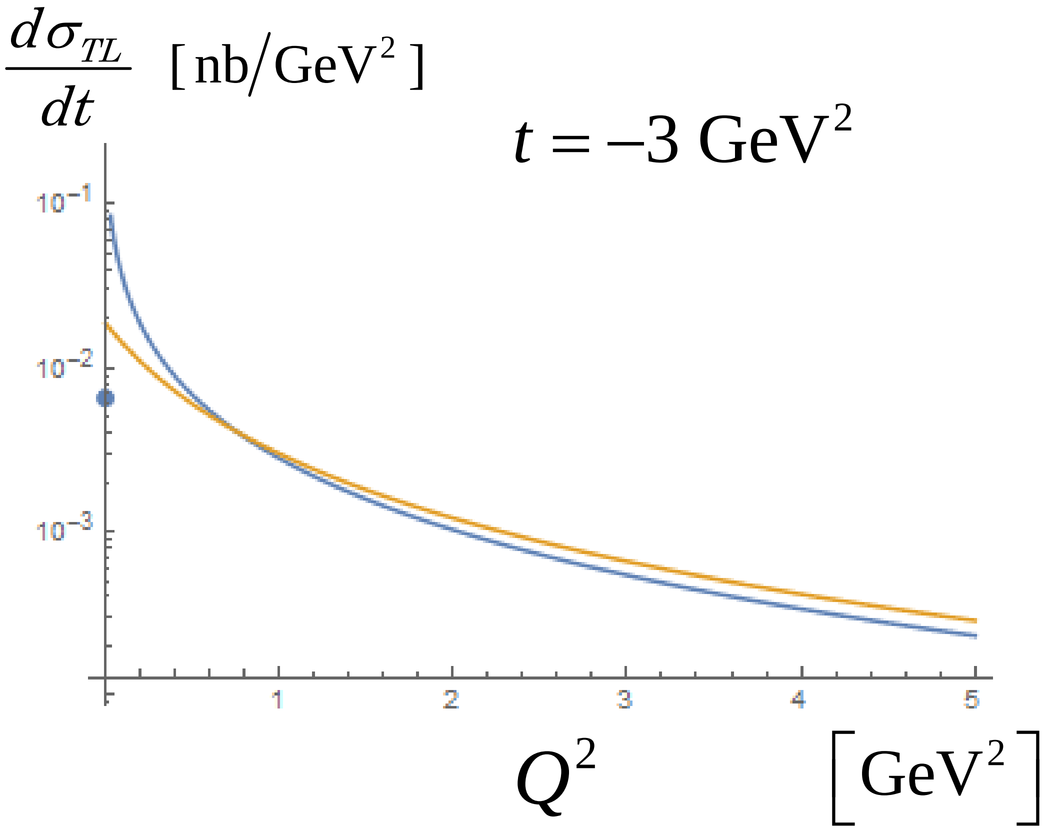
$s = 100\ \text{GeV}^2$

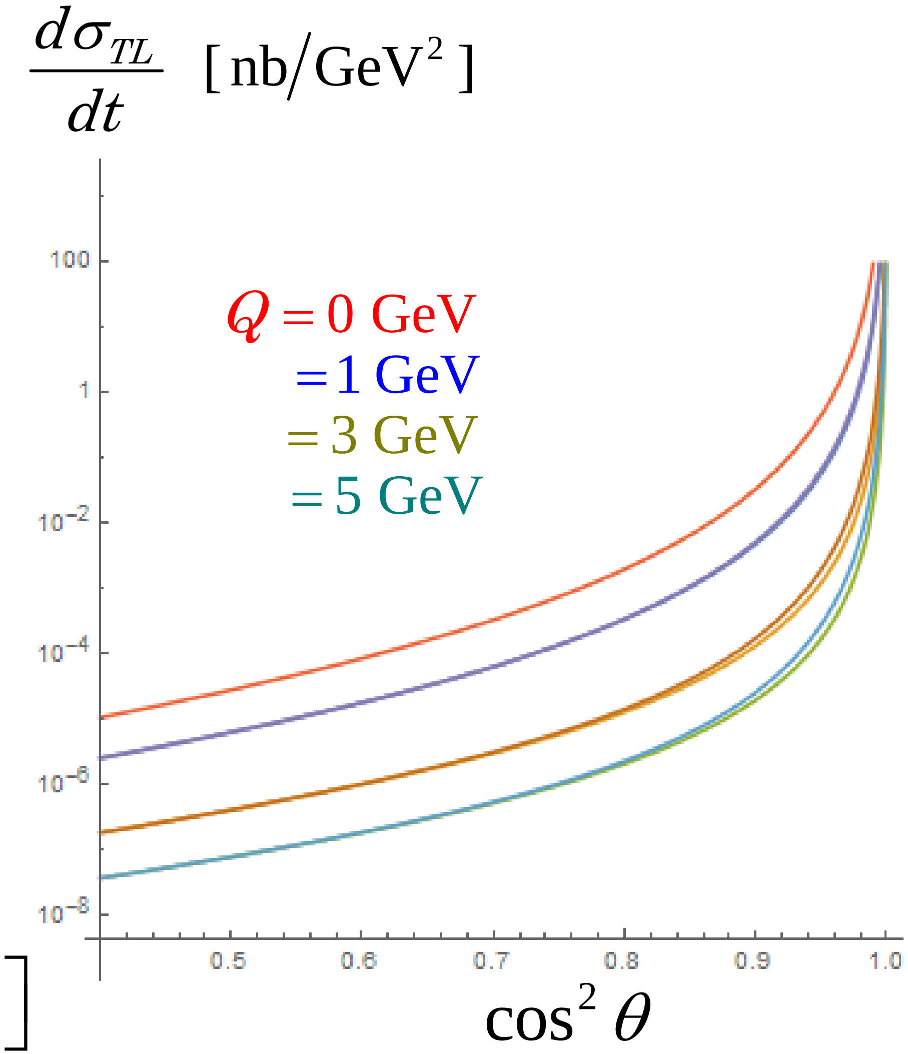


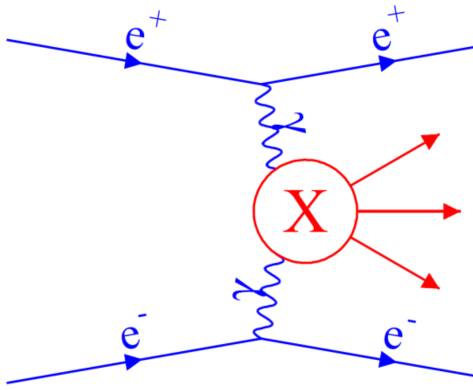


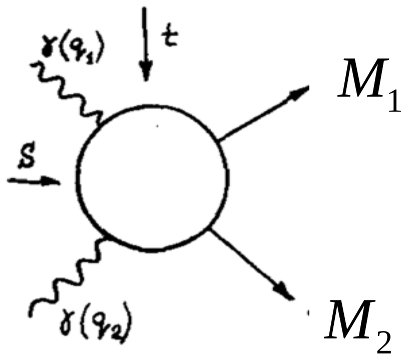
$$s \gg -t \gg \Lambda^2_{\mathrm{QCD}}$$

$$s = 100\ \mathrm{GeV}^2$$



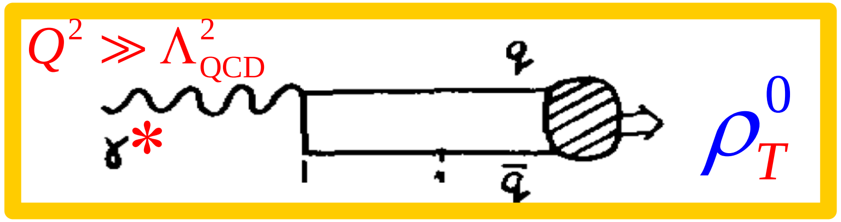


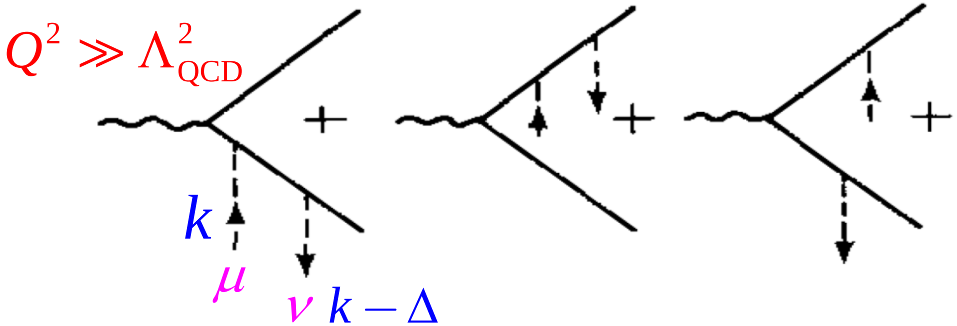




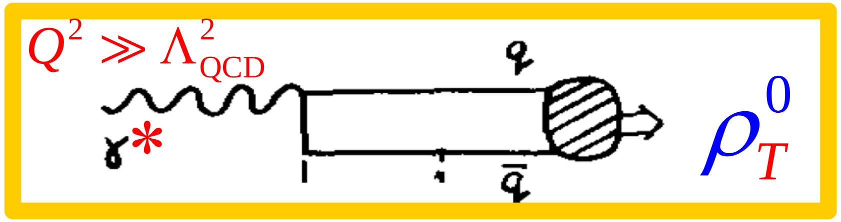
"$\gamma\gamma \to M_1 M_2$"

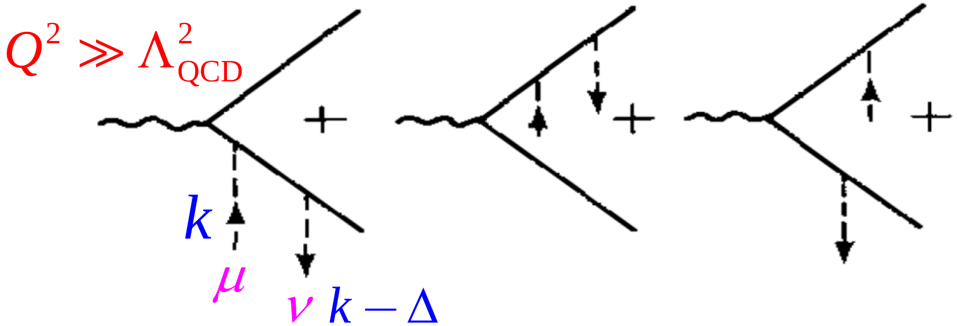
First QCD prediction for $\gamma\gamma \to \rho_L^0 \rho_T^0$





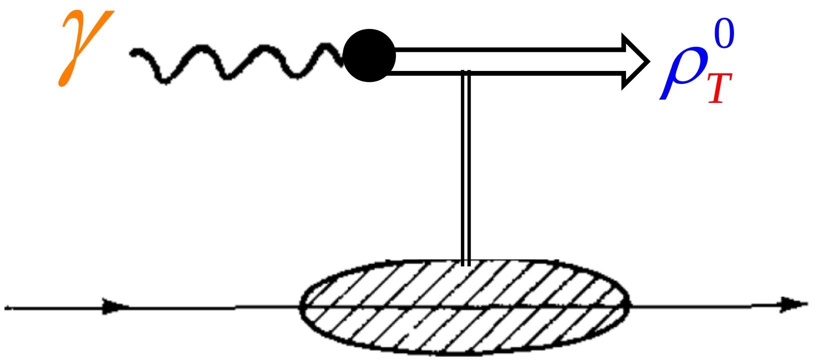
$$\left(k,\ \mu \rightarrow k-\Delta,\ \nu\right) \quad \otimes g_T(u)$$

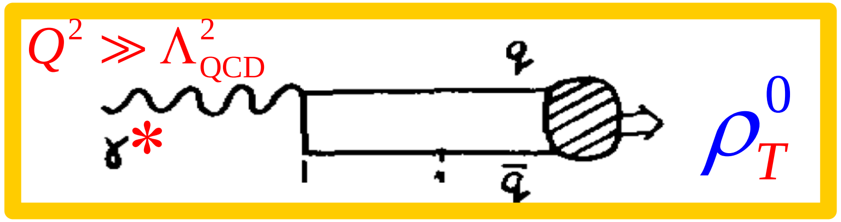


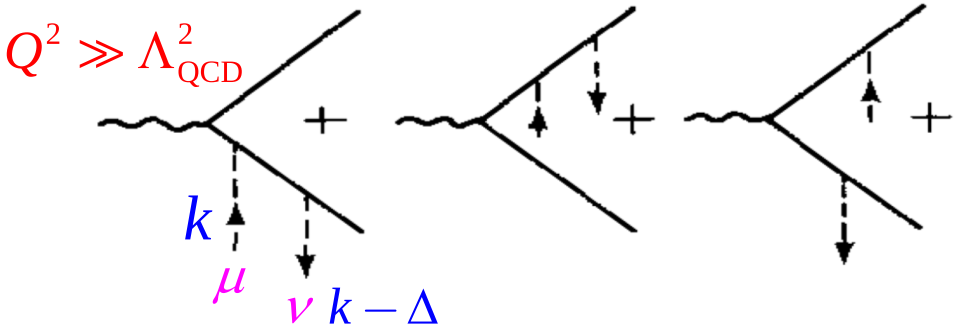


$$(k,\ \mu \rightarrow k-\Delta,\ \nu) \quad \otimes g_T(u)$$

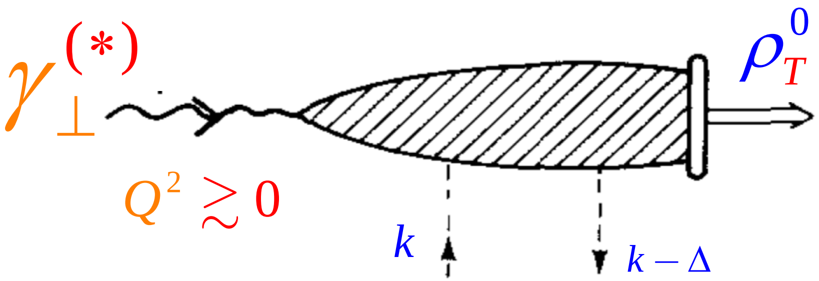
$Q^2 \sim 0$ VMD $\oplus$ pomeron



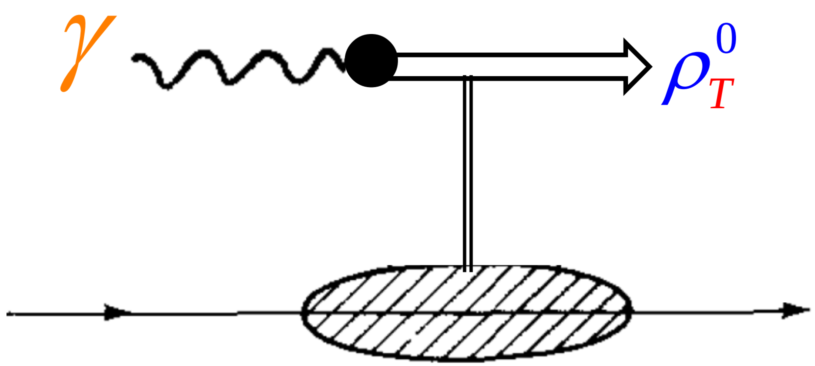




$$\left(k,\ \mu \to k-\Delta,\ \nu\right) \quad \otimes g_T(u)$$

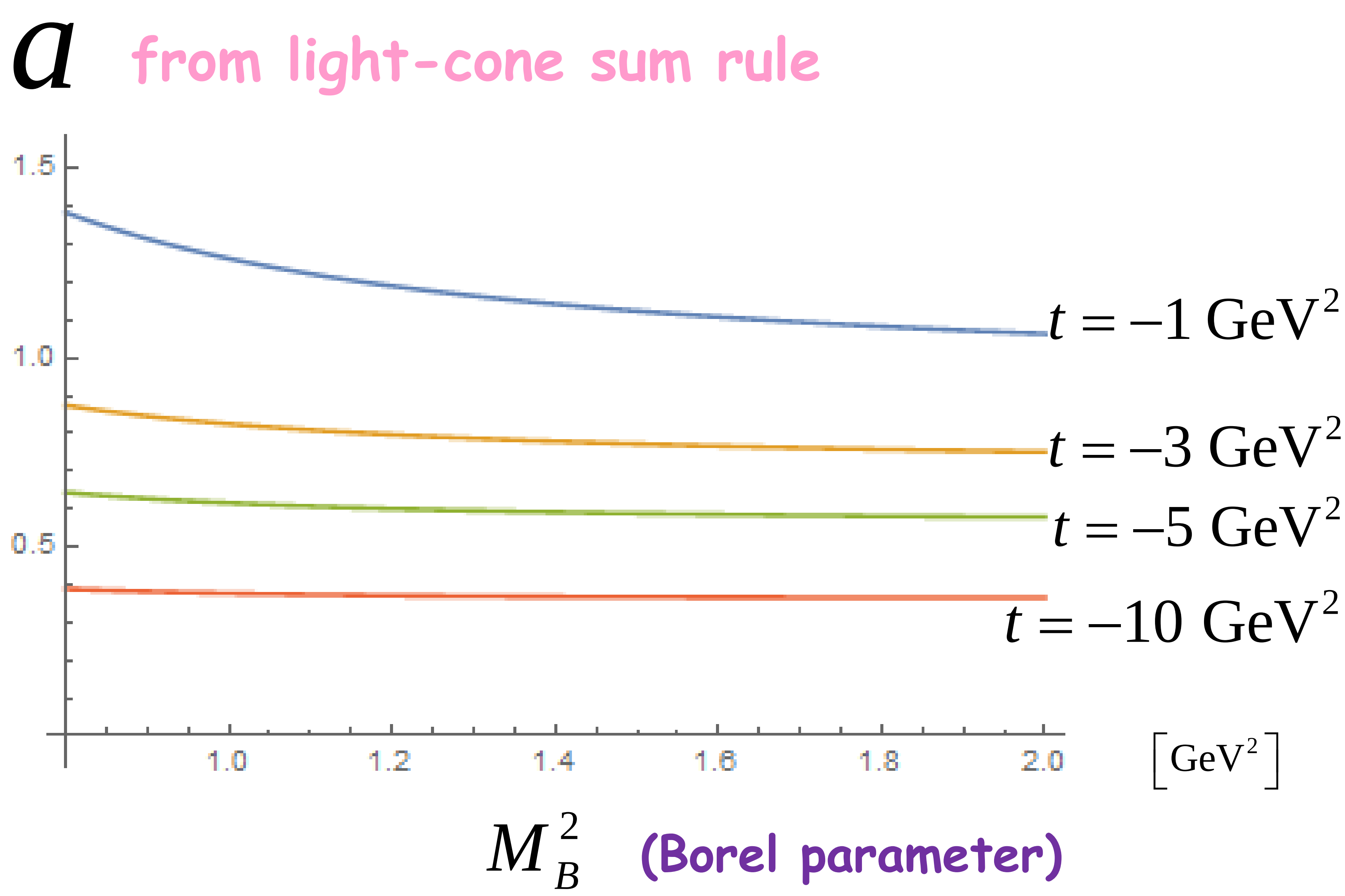


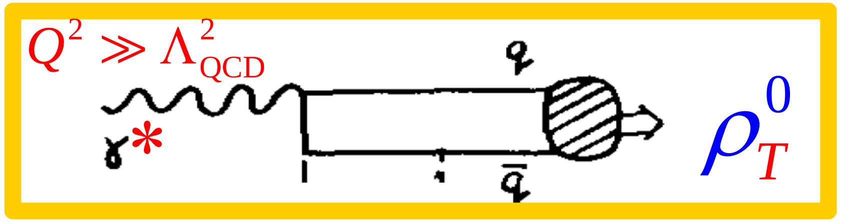
$Q^2 \sim 0$ VMD $\oplus$ pomeron



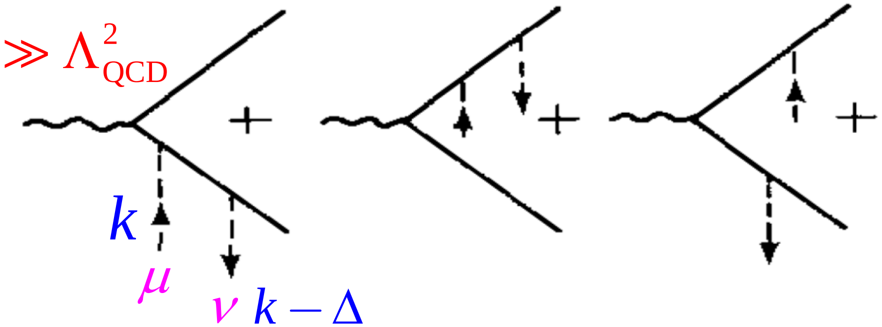
$$= \frac{a}{Q^2 + m_V^2} + \int_{m_{th}^2}^{\infty} dm^2 \, \frac{\chi(m^2)}{Q^2 + m^2}$$

$$a \propto e^{\frac{m_V^2}{M^2}} \int_{u_0}^{1} du \left(2 g_T^{(v)}(u) - \frac{1}{2} \frac{\partial g_T^{(a)}(u)}{\partial u} \right) e^{-\frac{(1-u)\Delta_\perp^2}{u M^2}}$$

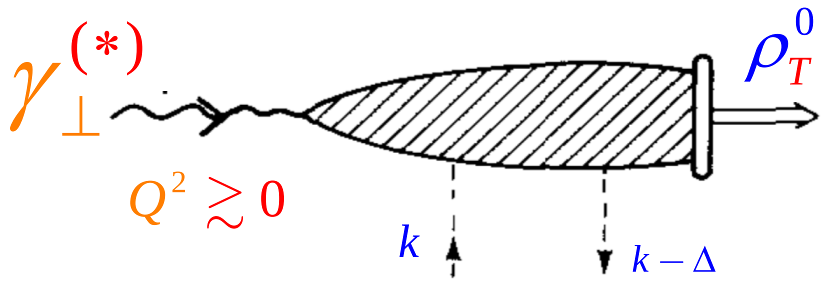

a from light-cone sum rule
1.5
1.0
0.5
t = −1 GeV²
t = −3 GeV²
t = −5 GeV²
t = −10 GeV²
1.0
1.2
1.4
1.6
1.8
2.0
[GeV²]
M_B² (Borel parameter)



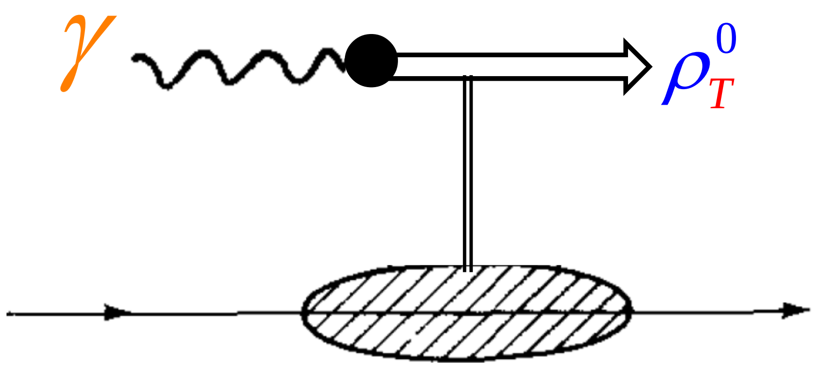
$Q^2 \gg \Lambda^2_{QCD}$



$$\left(k,\ \mu \to k-\Delta,\ \nu\right) \quad \otimes g_T(u)$$



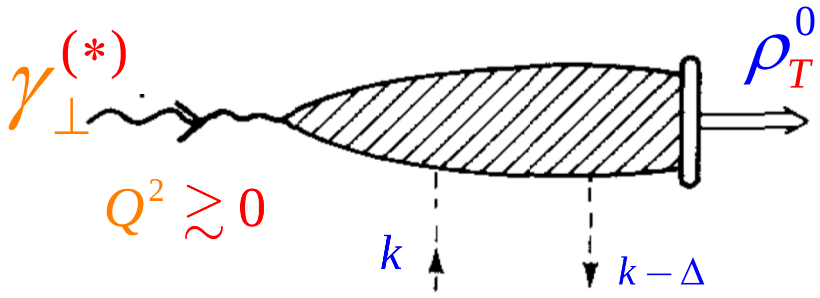
$Q^2 \sim 0$ VMD $\oplus$ pomeron



$$= \frac{a}{Q^2+m_V^2} + \int_{m_{th}^2}^{\infty} dm^2 \, \frac{\chi(m^2)}{Q^2+m^2}$$

$$a \propto e^{\frac{m_V^2}{M^2}} \int_{u_0}^{1} du \left(2 g_T^{(v)}(u) - \frac{1}{2} \frac{\partial g_T^{(a)}(u)}{\partial u} \right) e^{-\frac{(1-u)\Delta_\perp^2}{uM^2}}$$

Summary:



QCD calculation for the $\gamma \to \rho^0$ impact factor

allows us to obtain ``interpolating formula"
between pQCD for $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$ **and** $\mathrm{VMD} \oplus \mathrm{pomeron}$ for $Q^2 \sim 0$

first QCD prediction for $\dfrac{d\sigma_{TL}}{dt}$ $\gamma\gamma \longrightarrow \rho_L^0 \rho_T^0$