Generalized distribution amplitudes and two-photon processes

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https://indico2.riken.jp/indico/getFile.py/access?contribId=0&resId=0&materialId=slides&confId=2614
https://www.dropbox.com/sh/pi3vhxme7fsgjiz/AACtN2Swdx76mo5ZWa8jfjoWa?dl=0

October 16, 2017

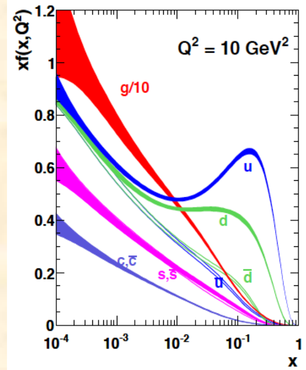
Wigner distribution and various structure functions

Form factor

$\int dx\, d^2k_T$

PDF (Parton Distribution Function)

$\int d^2k_T,\ \Delta \to 0$



GTMD $W(x, \vec{k}_T, \Delta) \xrightarrow[\Delta^+ \to 0]{}$ **Wigner** $W(x, \vec{k}_T, \vec{r}_T)$

3D world

$\Delta = p' - p$

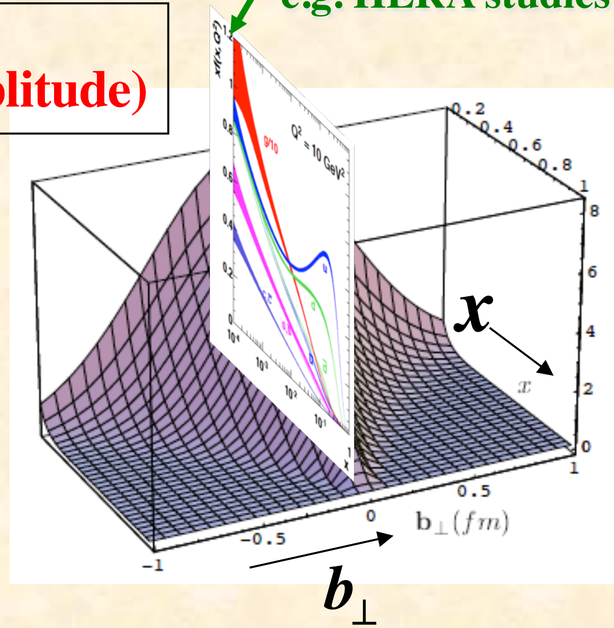
$\int d^2k_T$

GPD (Generalized Parton Distribution)

$\xrightarrow[s \leftrightarrow t]{}$ **GDA (Generalized Distribution Amplitude)**

e.g. HERA studies

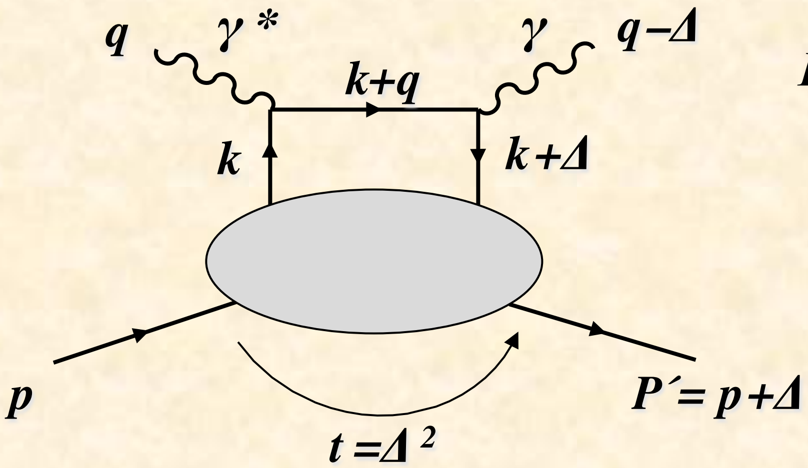
$\Delta \to 0$



TMD (Transverse Momentum Dependent) parton distribution

Our work by the two-photon process $\gamma\gamma \to h\bar{h}$.

Generalized Parton Distributions (GPDs)



$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \dfrac{Q^2}{2p\cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewdness parameter $\xi = \dfrac{p^+ - p'^+}{p^+ + p'^+} = -\dfrac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p)\right]$$

Forward limit: PDFs

$$H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2

$$\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$$

Axial and Pseudoscalar form factors G_A, G_P

$$\int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$$

Second moments: Angular momenta

Sum rule: $J_q = \dfrac{1}{2}\displaystyle\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right], \quad J_q = \dfrac{1}{2}\Delta q + L_q$

Generalized Distribution Amplitudes (GDAs)

and KEKB/ILC project

H. Kawamura and S. Kumano,
Phys. Rev. D 89 (2014) 054007.

S. Kumano, Q.-T. Song, O. Teryaev,
KEK-TH-1959, J-PARC-TH-0086.

GPD $H_q^h(x,\xi,t)$ and GDA $\Phi_q^{hh}(z,\zeta,W^2)$

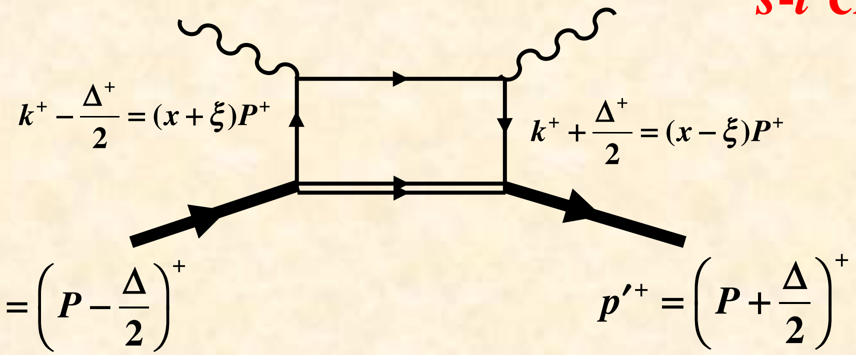
GPD: $H_q(x,\xi,t)=\int\frac{dy^-}{4\pi}e^{ixP^+y^-}\langle h(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|h(p)\rangle\Big|_{y^+=0,\vec y_\perp=0}$, $\quad P^+=\frac{(p+p')^+}{2}$

GDA: $\Phi_q(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle h(p)\bar h(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|0\rangle\Big|_{y^+=0,\vec y_\perp=0}$

DA: $\Phi_q^\pi(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle \pi(p)|\bar\psi(-y/2)\gamma^+\gamma_5\psi(y/2)|0\rangle\Big|_{y^+=0,\vec y_\perp=0}$

$H_q^h(x,\xi,t)$ $\quad\longleftrightarrow\quad$ $\Phi_q^{hh}(z,\zeta,W^2)$

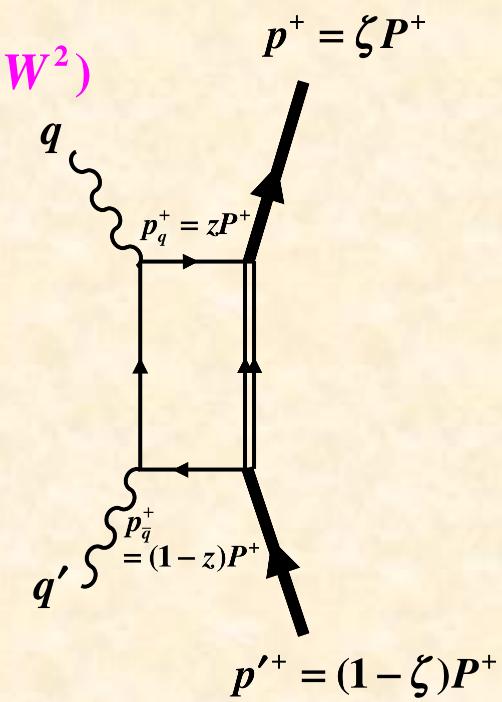
s-t crossing



$z\Leftrightarrow\frac{1-x/\xi}{2}$

$\zeta\Leftrightarrow\frac{1-1/\xi}{2}$

$W^2\Leftrightarrow t$



$P=\frac{p+p'}{2},\ \Delta=p'-p$

Bjorken variable: $\quad x=\frac{Q^2}{2p\cdot q}$

Momentum transfer squared: $\quad t=\Delta^2$

Skewdness parameter: $\quad \xi=\frac{p^+-p'^+}{p^++p'^+}=-\frac{\Delta^+}{2P^+}$

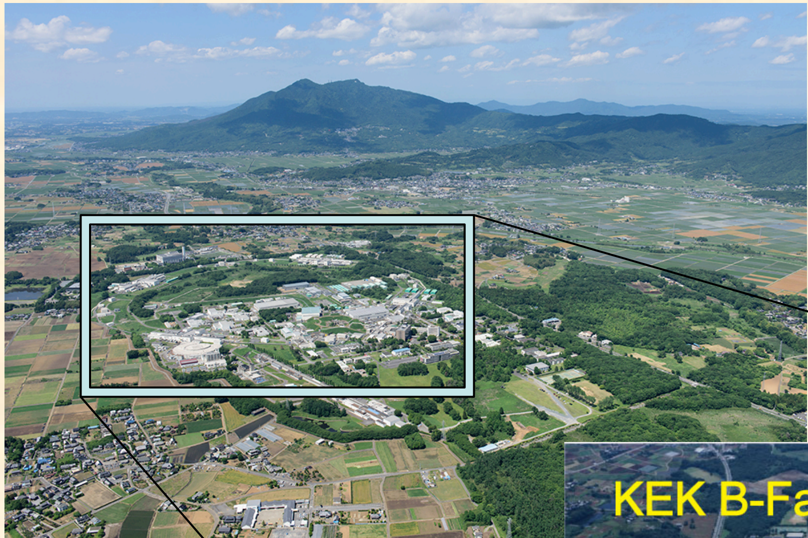
Bjorken variable for $\gamma\gamma^*$: $z=\frac{Q^2}{2q\cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar h$: $\zeta=\frac{p^+}{P^+}=\frac{1+\beta\cos\theta}{2}$

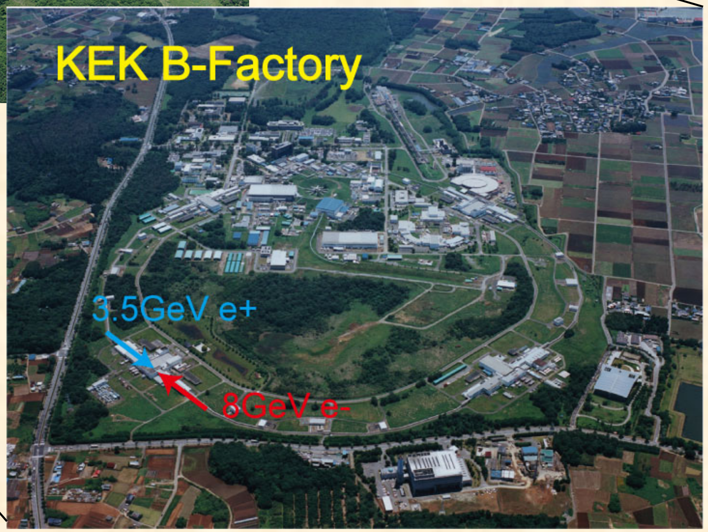
Invariant mass of $h\bar h$: $W^2=(p+p')^2$

Experimental studies of GDAs in future

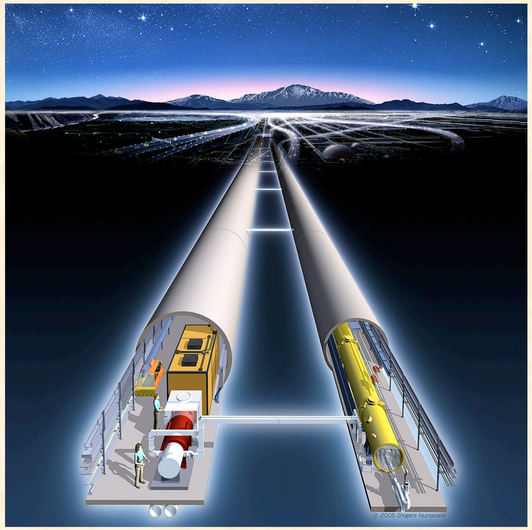
$\gamma\gamma \to h\bar{h}$ for internal structure of exotic hadron candidate h



KEK B-factory

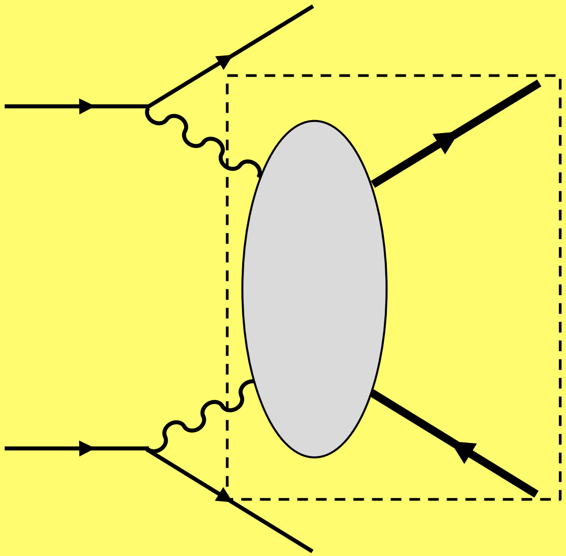


Linear Collider ?



Generalized Distribution Amplitudes (GDAs) for pion

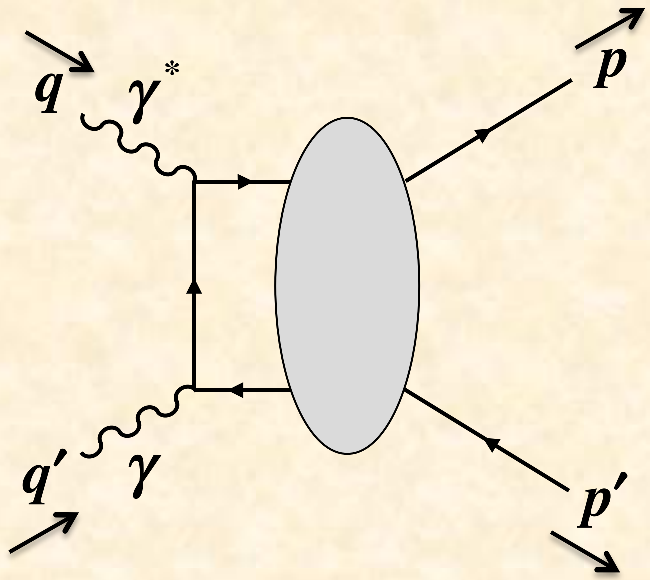
from KEKB measurements



$$\gamma\gamma \to h\bar{h}$$

SK, Q.-T. Song, O. Teryaev,
KEK-TH-1959, J-PARC-TH-0086

Cross section for $\gamma\gamma^* \to \pi^0\pi^0$



$$d\sigma = \frac{1}{4\sqrt{(q\cdot q')^2 - q^2 q'^2}} (2\pi)^4 \delta^4(q+q'-p-p') \overline{\sum_{\lambda,\lambda'}} |\mathcal{M}|^2 \frac{d^3p}{(2\pi)^3 2E} \frac{d^3p'}{(2\pi)^3 2E'}$$

$$q = (q^0,\, 0,\, 0,\, |\vec{q}|),\;\; q' = (|\vec{q}|,\, 0,\, 0,\, -|\vec{q}|),\;\; q'^2 = 0 \text{ (real photon)}$$

$$p = (p^0,\, |\vec{p}|\sin\theta,\, 0,\, |\vec{p}|\cos\theta),\;\; p = (p^0,\, -|\vec{p}|\sin\theta,\, 0,\, -|\vec{p}|\cos\theta)$$

$$\beta = \frac{|\vec{p}|}{p^0} = \sqrt{1 - \frac{4m_\pi^2}{W^2}}$$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} \overline{\sum_{\lambda,\lambda'}} |\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q') T^{\mu\nu},\quad T^{\mu\nu} = i\int d^4\xi\, e^{-i\xi\cdot q} \left\langle \pi(p)\pi(p') \right| T J_{em}^\mu(\xi) J_{em}^\nu(0) \left| 0 \right\rangle$$

$$\mathcal{M} = e^2 A_{\lambda\lambda'} = 4\pi\alpha A_{\lambda\lambda'}$$

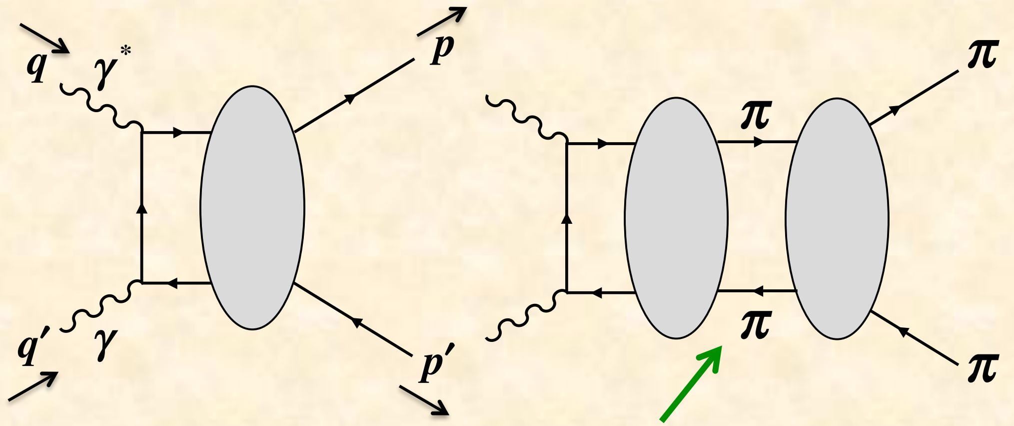
$$A_{\lambda\lambda'} = \frac{1}{e^2}\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z,\zeta,W^2)$$

$$\text{GDA:}\quad \Phi_q^{\pi\pi}(z,\zeta,s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \left\langle \pi(p)\pi(p') \right| \overline{\psi}(-y/2)\gamma^+\psi(y/2) \left| 0 \right\rangle \Big|_{y^+=0,\vec{y}_\perp=0}$$

$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z,\zeta,W^2),\quad \varepsilon_\mu^+(q)\varepsilon_\nu^+(q') g_T^{\mu\nu} = -1$$

$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} |A_{++}|^2$$

GDA parametrization for pion



Including intermediate resonance contributions

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m^2}{s}}\,|A_{++}|^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\,\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

- **Continuum: GDAs without intermediate-resonance contribution**

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha (1-z)^\beta (2z-1)\zeta(1-\zeta)F_q^\pi(s)$$

- **Resonances: Tthere exist resonance contributions to the cross section.**

$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18 N_f z^\alpha (1-z)^\alpha (2z-1)\left[\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)\right]$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$$\tilde{B}_{10}(W) = \text{resonance}\left[f_0(500)\equiv\sigma,\ f_0(980)\equiv f_0\right] + \text{continuum}$$

$$\tilde{B}_{12}(W) = \text{resonance}\left[f_2(1270)\right] + \text{continuum}$$

$f_0(500)$ or σ [g]
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400–550)$ MeV
Full width $\Gamma = (400–700)$ MeV

$f_0(980)$ [j]

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 10$ to 100 MeV

$f_2(1270)$

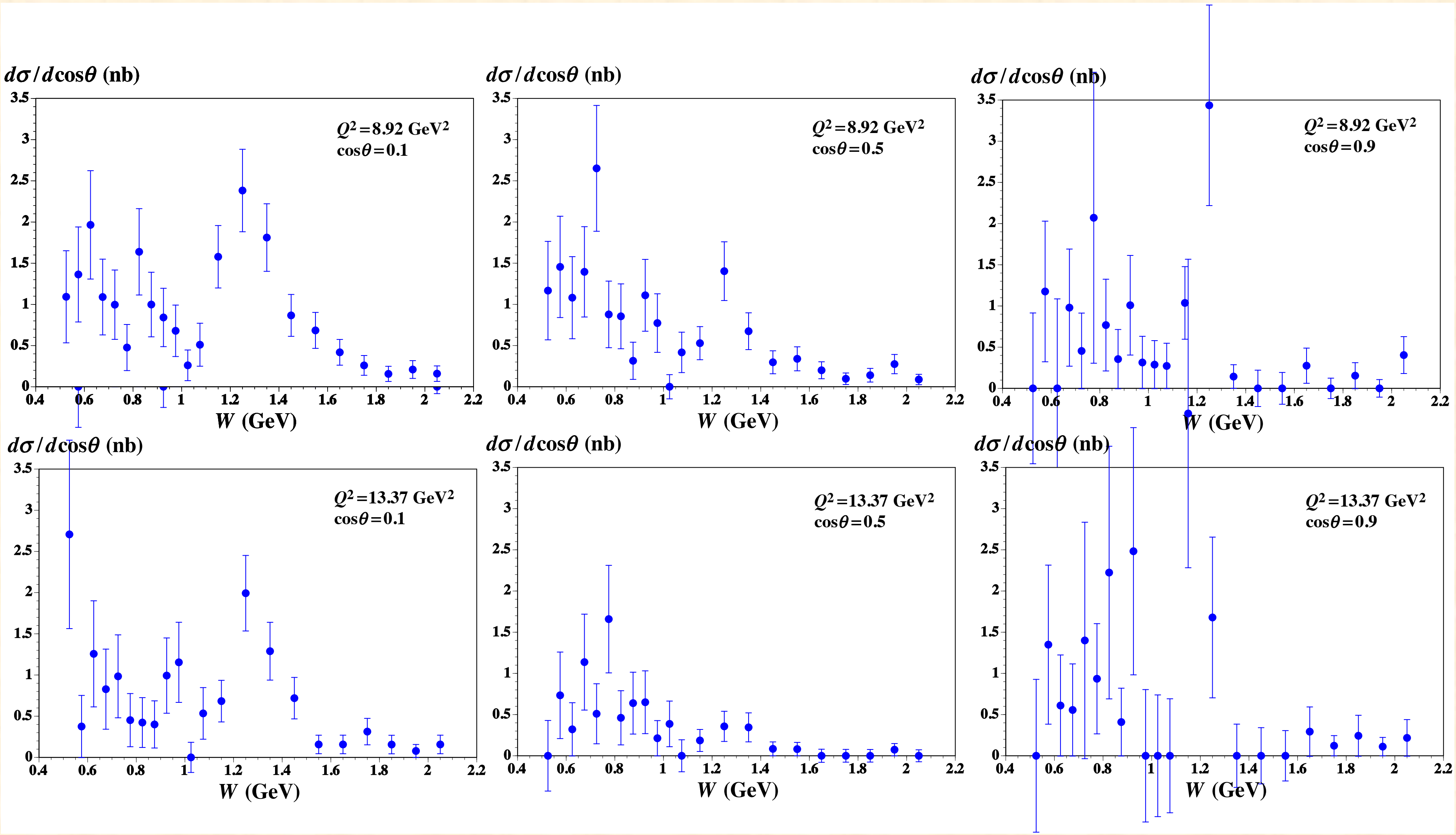
$I^G(J^{PC}) = 0^+(2^{++})$

Mass $m = 1275.5 \pm 0.8$ MeV
Full width $\Gamma = 186.7^{+2.2}_{-2.5}$ MeV (S = 1.4)

Analysis of Belle data on $\gamma\gamma^* \to \pi^0\pi^0$

$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$

Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.



Generalized Distribution Amplitudes (GDAs) and gravitational radius for pion

S. Kumano, Q.-T. Song, O. Teryaev,

KEK-TH-1959, J-PARC-TH-0086,

to be submitted for publication.

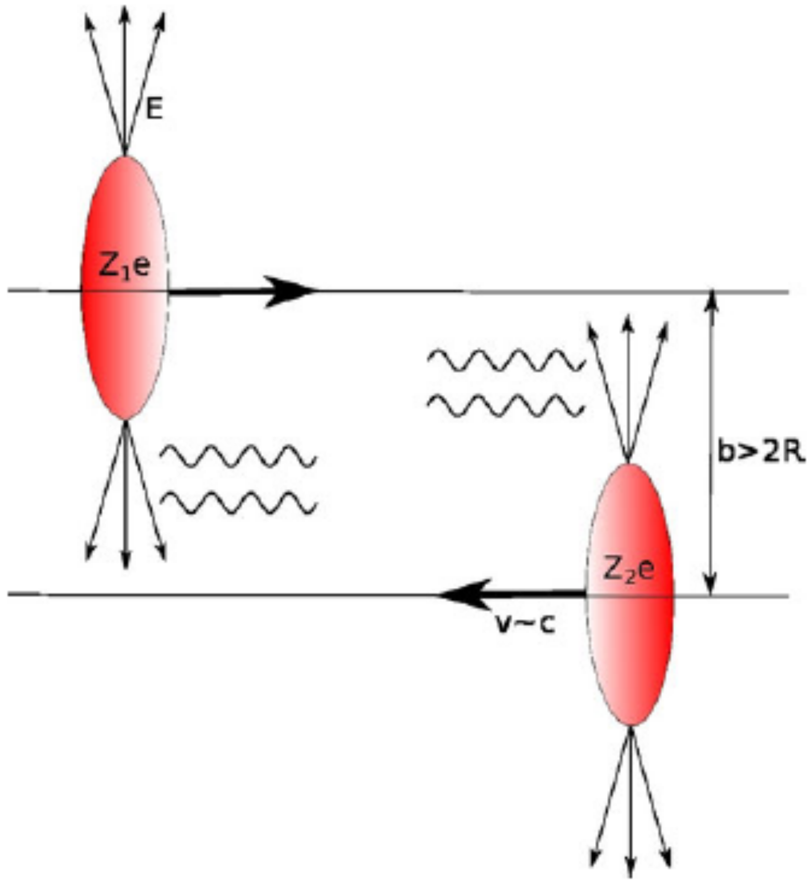
Prospects

Ultra-Peripheral Collision (UPC) @ LHC/RHIC

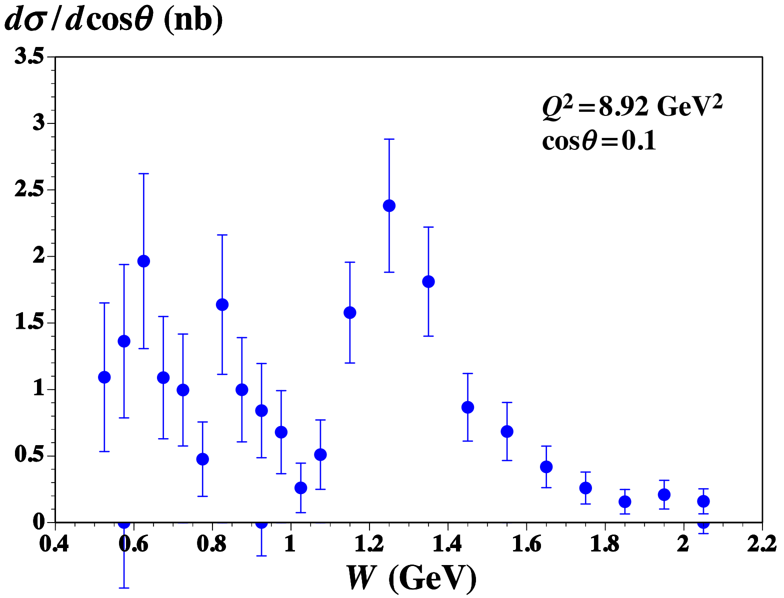
INT Workshop INT-17-65W

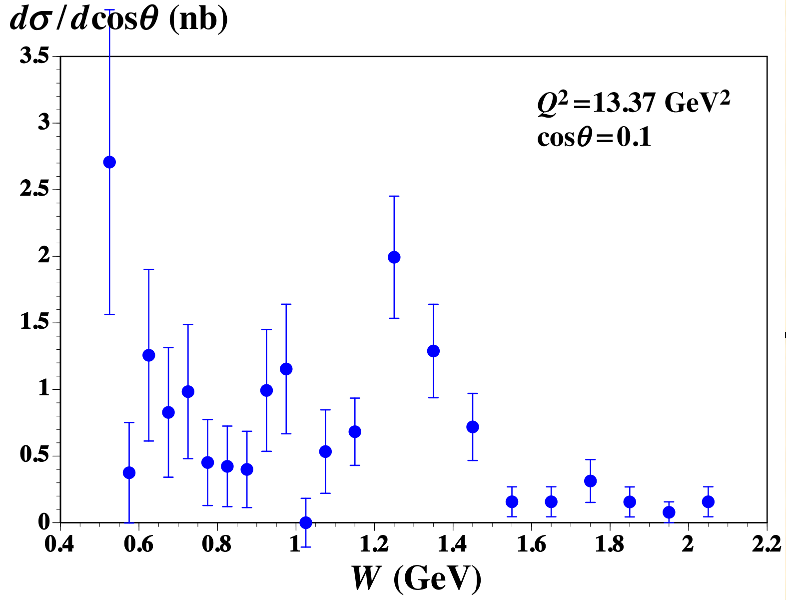
Probing QCD in Photon-Nucleus Interactions at RHIC and LHC: the Path to EIC

February 13 - 17, 2017

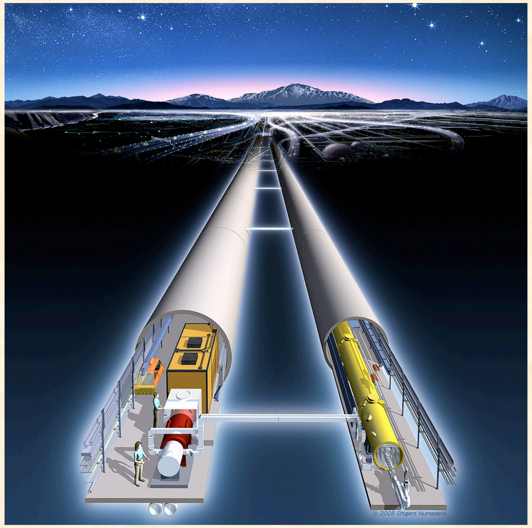


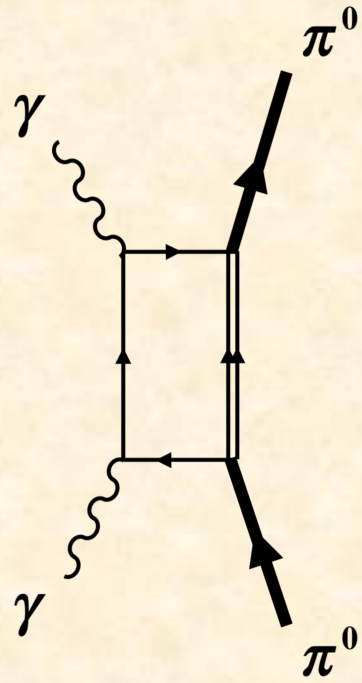
From KEKB to ILC





Linear Collider ?





- Very Large Q^2
- Large W^2

for extracting GDAs

ILC

The End

The End