Recent progress on nucleon charges from lattice QCD

Takashi Kaneko (KEK, Sokendai)

13th meeting on high energy QCD and nucleon structure

nucleon charge

$$g_{\Gamma,q} = \frac{1}{2M_N} \langle N(p,s) | \bar{q} \Gamma q | N(p,s) \rangle$$

nucleon ME of quark bi-linear operator

nucleon charge

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nucleon ME of quark bi-linear operator

- (polarized) nucleon @ rest p=0 ⇔ normalization of FF

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nucleon ME of quark bi-linear operator

- (polarized) nucleon @ rest p=0 ⇔ normalization of FF
- in this talk : scalar, axial and tensor ⊗ up, down, strange

$$S_q = \frac{1}{2M_N}\left\langle N\middle|\bar{u}u\middle|N\right\rangle$$

$$\Delta_q = \frac{1}{2M_N}\left\langle N(s_z=1/2)\middle|\bar{u}\gamma_3\gamma_5 u\middle|N(s_z=1/2)\right\rangle$$

$$\delta_q = \frac{1}{2M_N}\left\langle N(s_z=1/2)\middle|\bar{u}\sigma_{03}\gamma_5 u\middle|N(s_z=1/2)\right\rangle$$

and "$\bar{d}\Gamma d$", "$\bar{s}\Gamma s$"

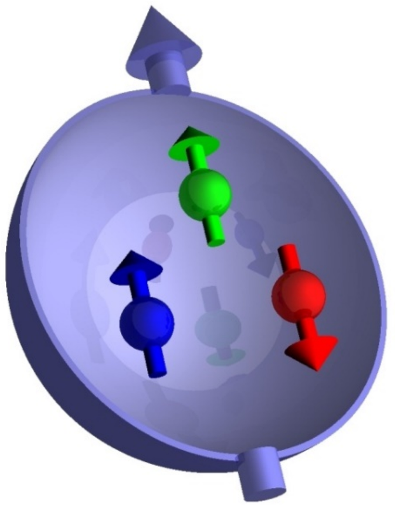
or linear combinations

for lattice, phenomenology

in nuclear physics

fundamental parameters on nucleon structure

$$S_q = \langle N | \bar{q} q | N \rangle$$



$$\Delta_q = \langle N | \bar{q} \gamma_k \gamma_5 q | N \rangle$$

in nuclear physics

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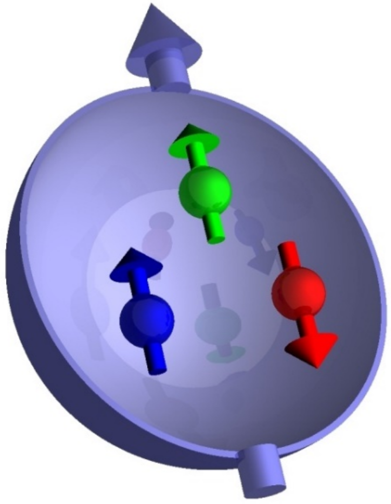
$$S_q = \langle N | \bar{q}q | N \rangle$$

$$T^\mu_\mu = \frac{\beta}{2g} G^a_{\mu\nu} G^{a\mu\nu} + \sum_q m_q \bar{q}q$$

trace anomaly of energy-mom tensor

$$2M_N^2 = 2p^2 = \langle N(p) | T^\mu_\mu | N(p) \rangle \ni m_u S_u + m_d S_d + m_s S_s$$



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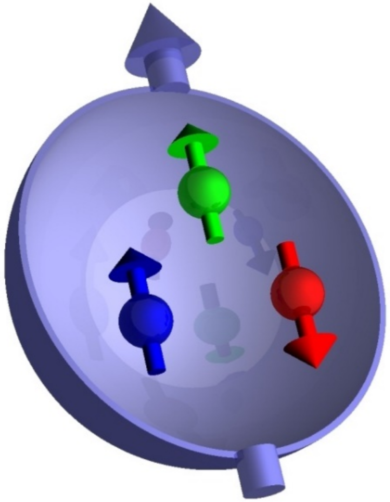
rotation → angular momentum operator

$$\boldsymbol{J}_k = \int d^3x \left\{ \bar{q}\boldsymbol{\gamma}\gamma_5 q + i q^\dagger (\boldsymbol{x} \times \boldsymbol{D}) q + \cdots \right\}$$

$$\frac{1}{2}\Sigma \simeq \frac{1}{2}(\Delta_u + \Delta_d + \Delta_s)$$



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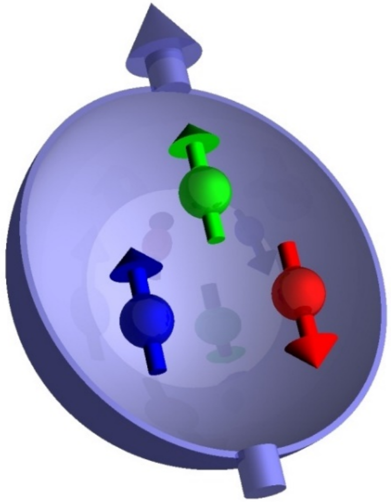

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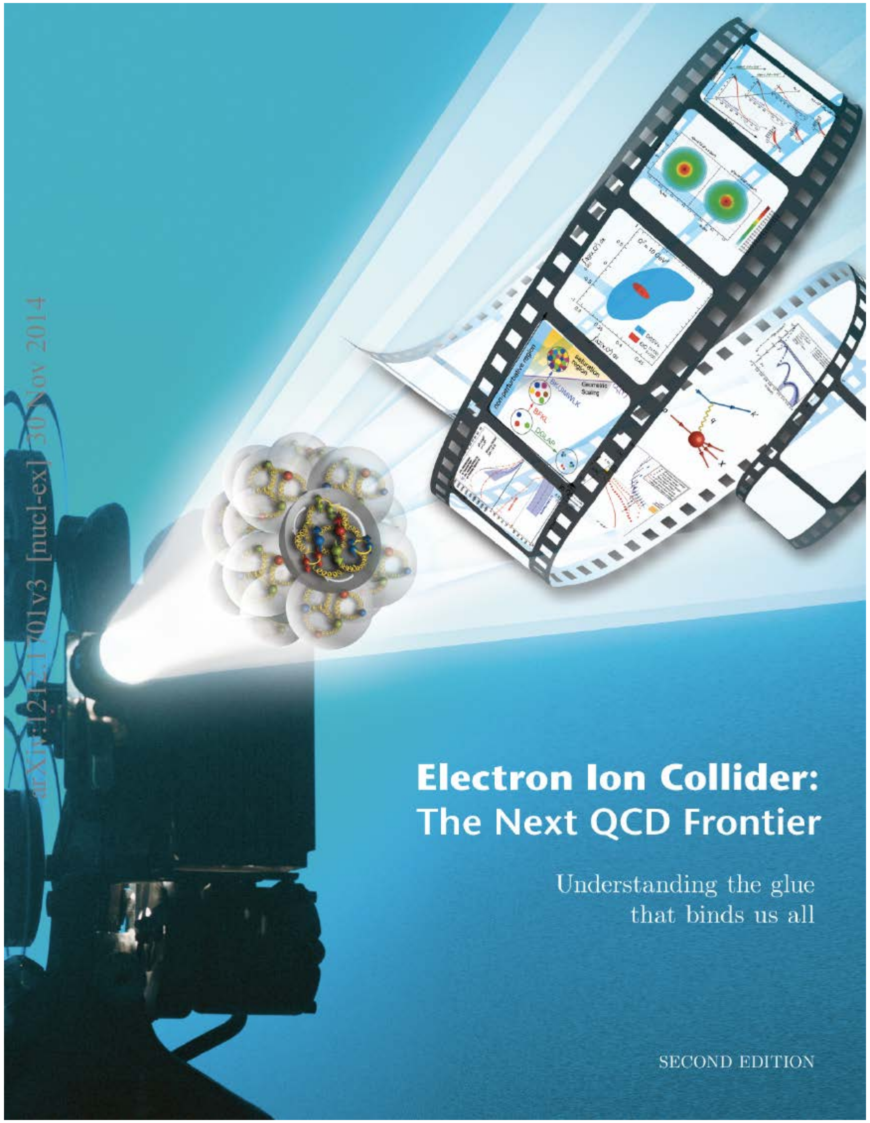
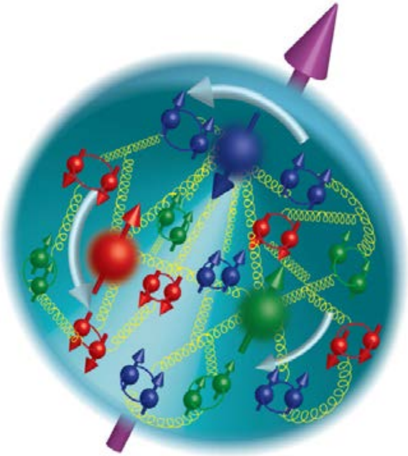
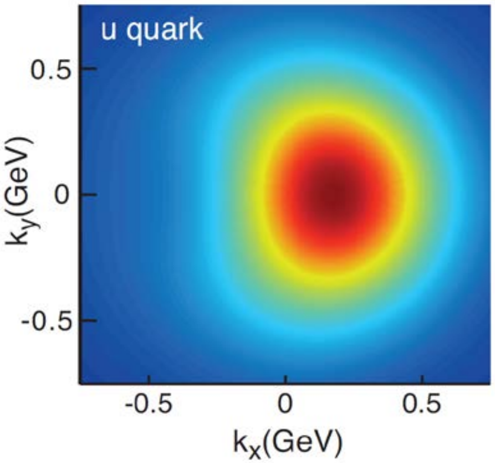
quark contribution to nucleon mass, spin

Electron-Ion Collider

arXiv: 1212.1701







spin structure

⇒ axial charges

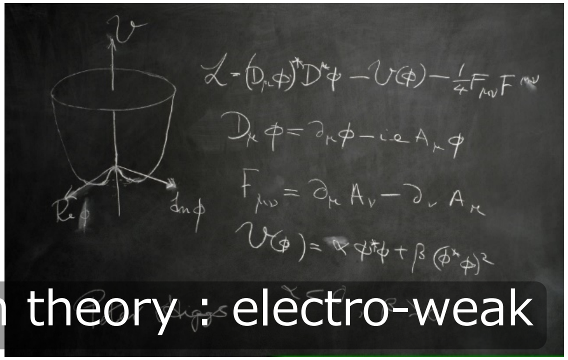
quark transversity distribution

⇒ tensor charges

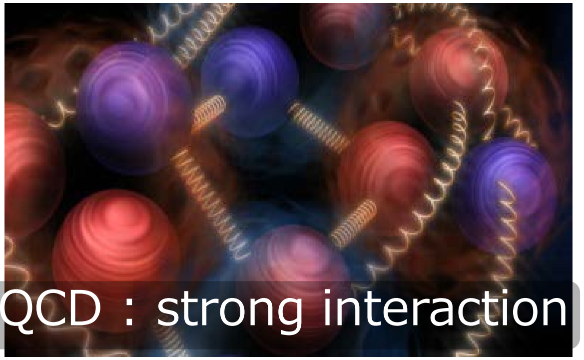
in elementary particle physics

search for new physics beyond the standard model

the SM



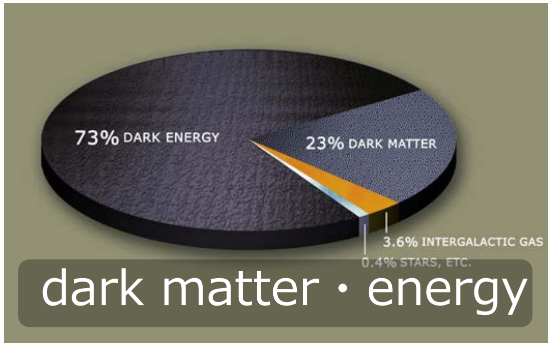
Glashow-Weinberg-Salam theory : electro-weak



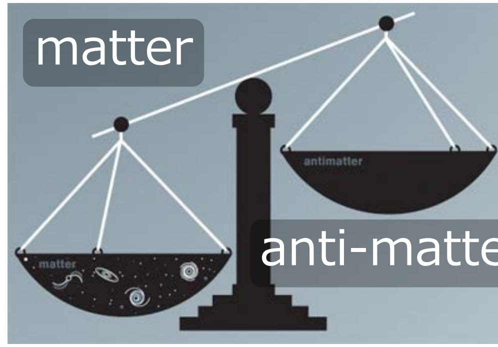
QCD : strong interaction

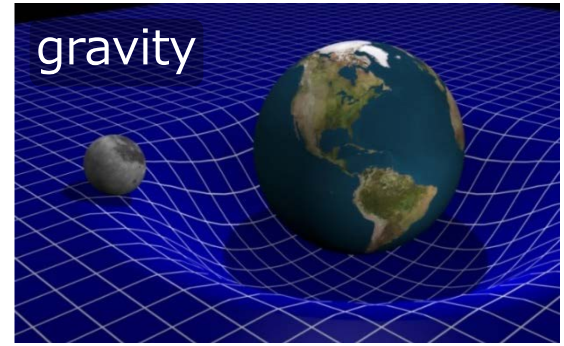
in elementary particle physics

search for new physics beyond the standard model

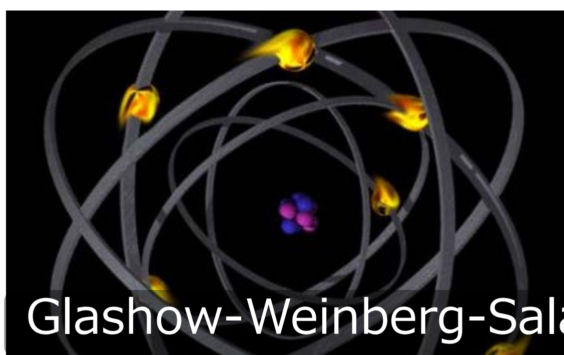


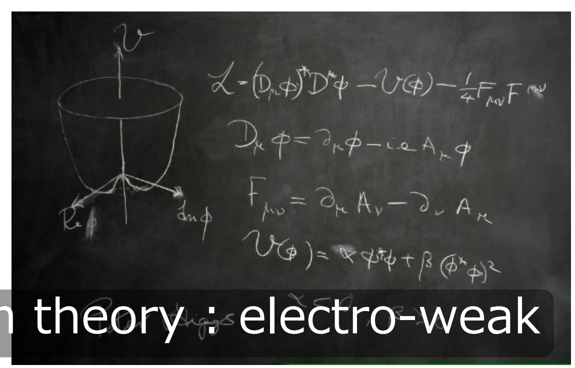
dark matter・energy





the SM





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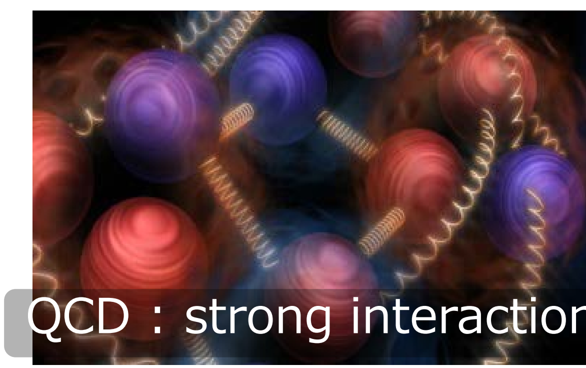

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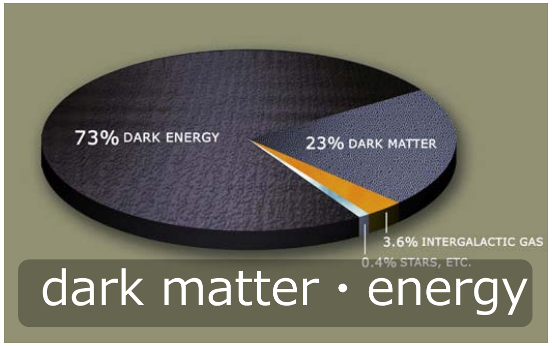
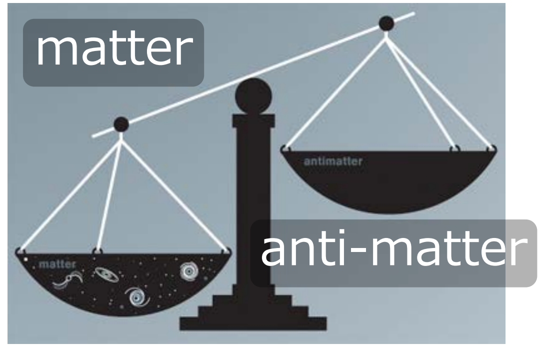
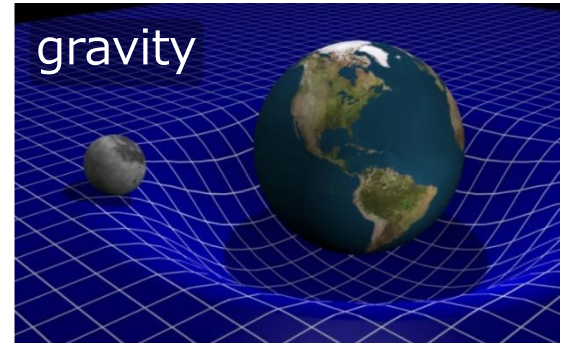
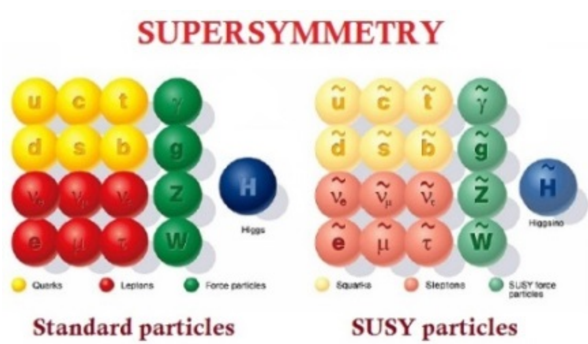
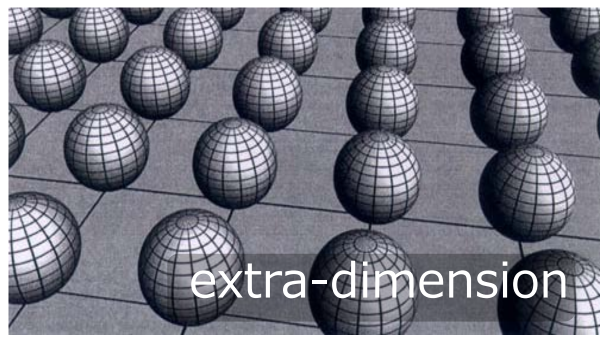




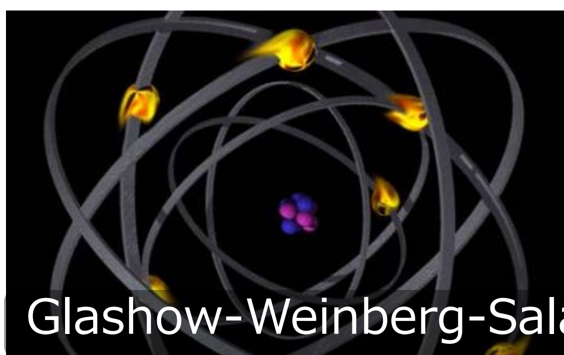


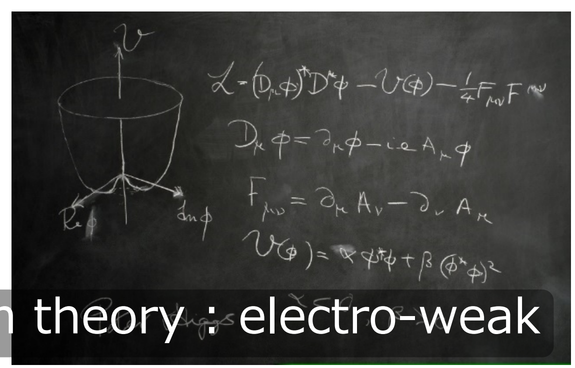


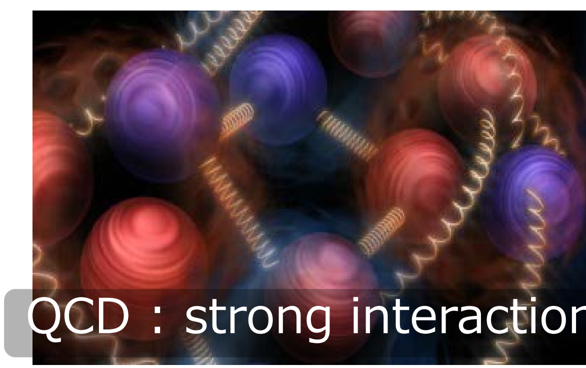


new physics

… or else







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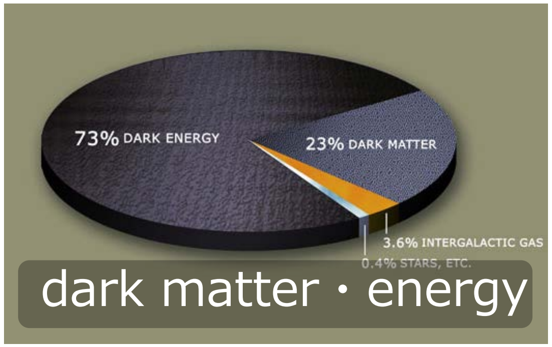
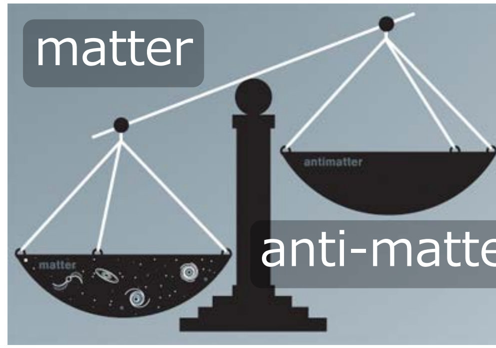
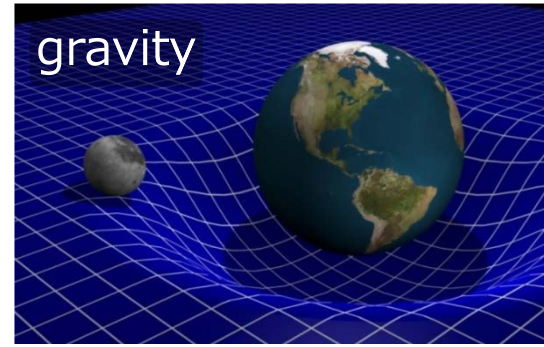
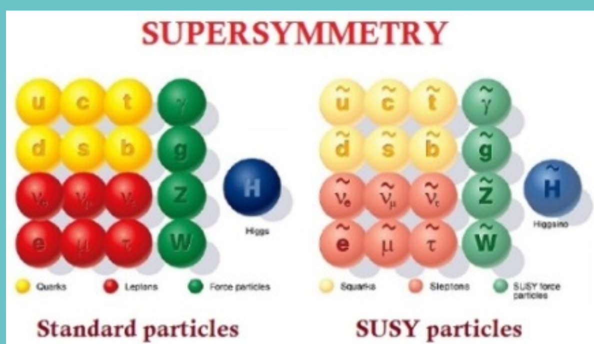
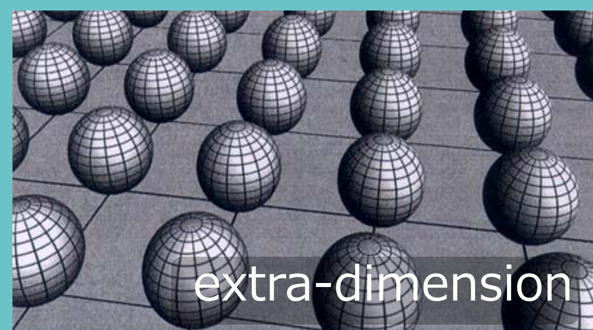




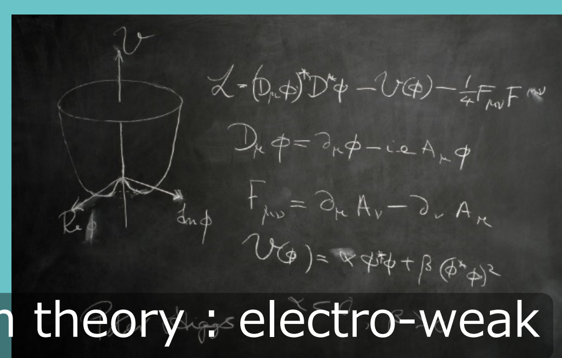


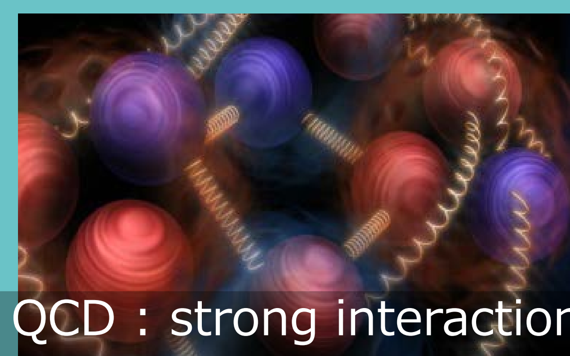




new theory
w/ new physics

… or else





search for new physics

direct search for dark matter

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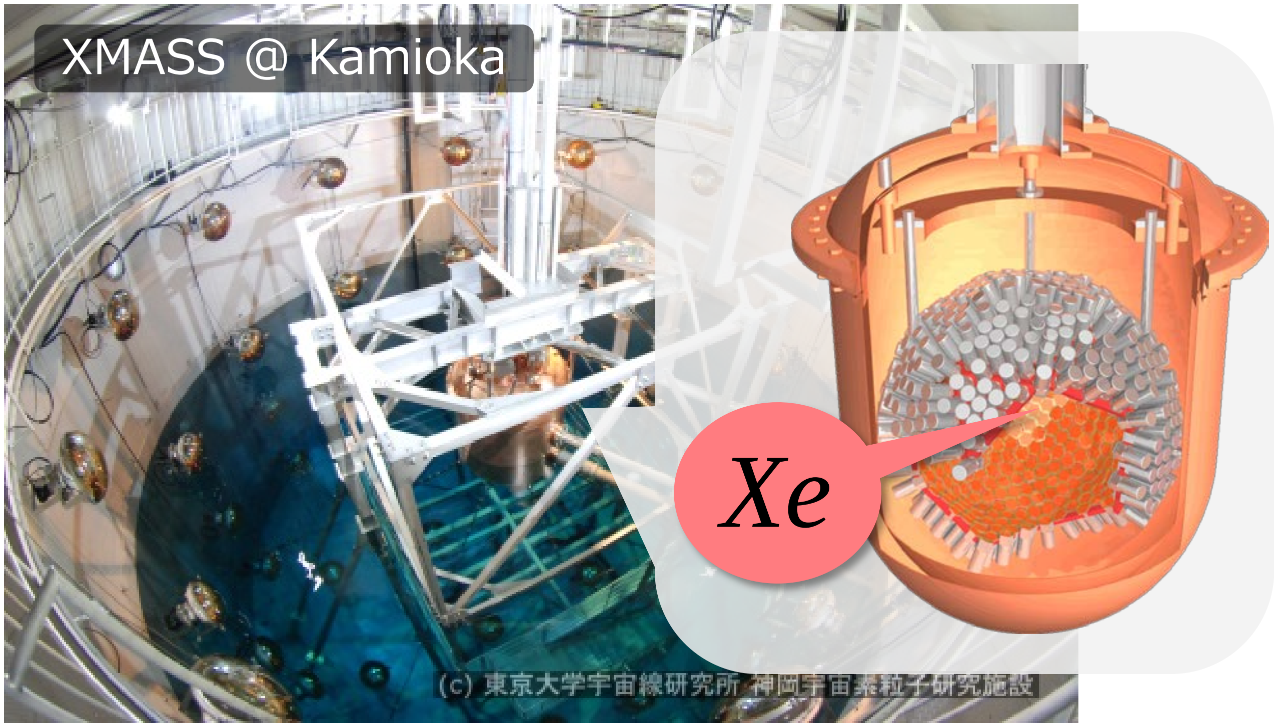

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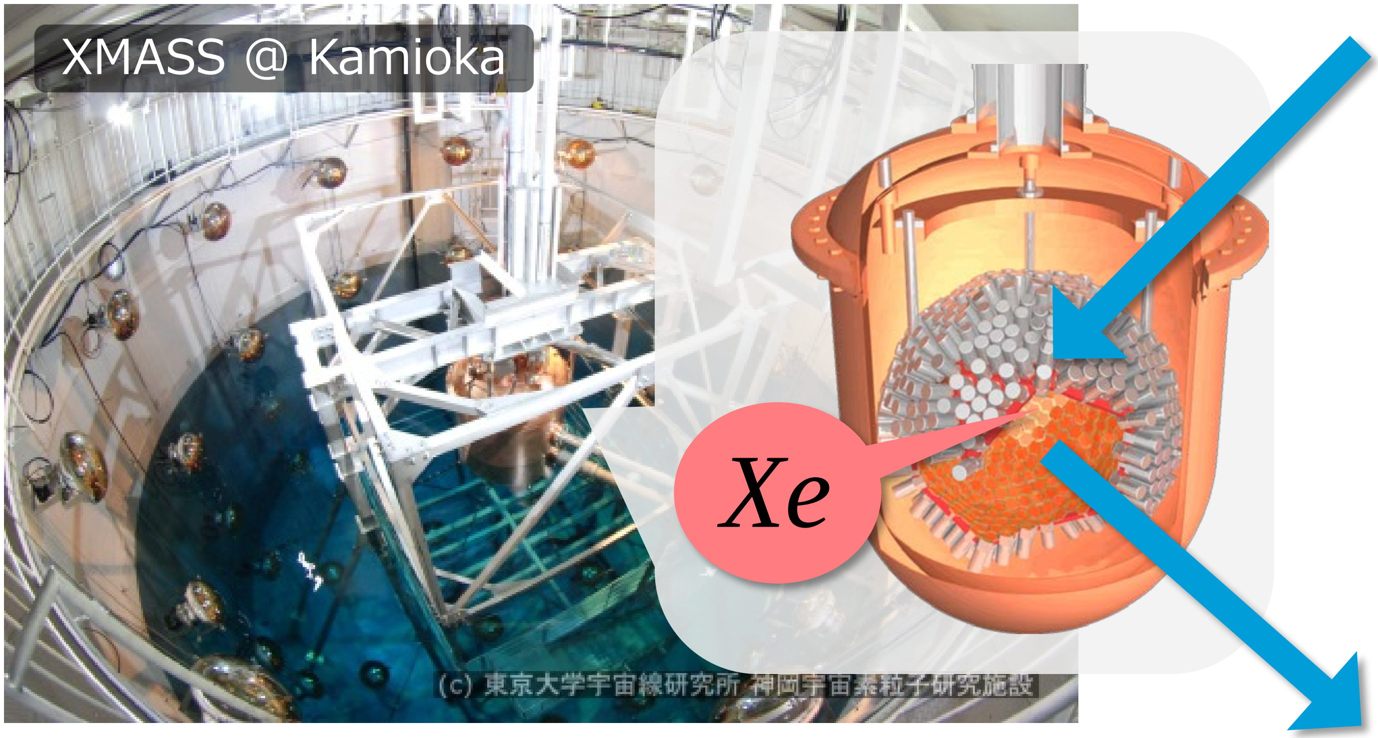


dark matter

e.g. neutralino χ
in SUSY

detect momentum
transfer to Xe

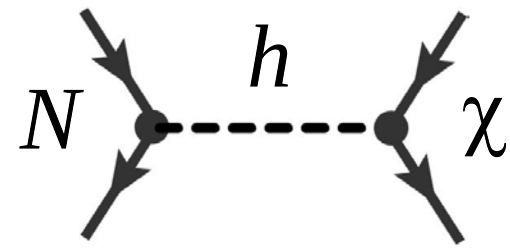
search for new physics

direct search for dark matter

dark matter

e.g. neutralino χ in SUSY

$N\chi$ scattering via Higgs



detect momentum transfer to Xe

search for new physics

direct search for dark matter

dark matter

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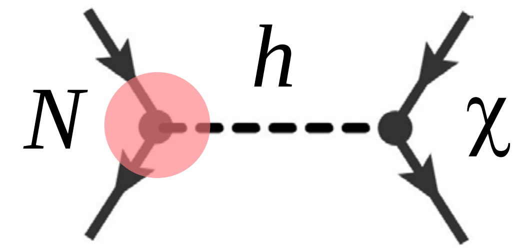
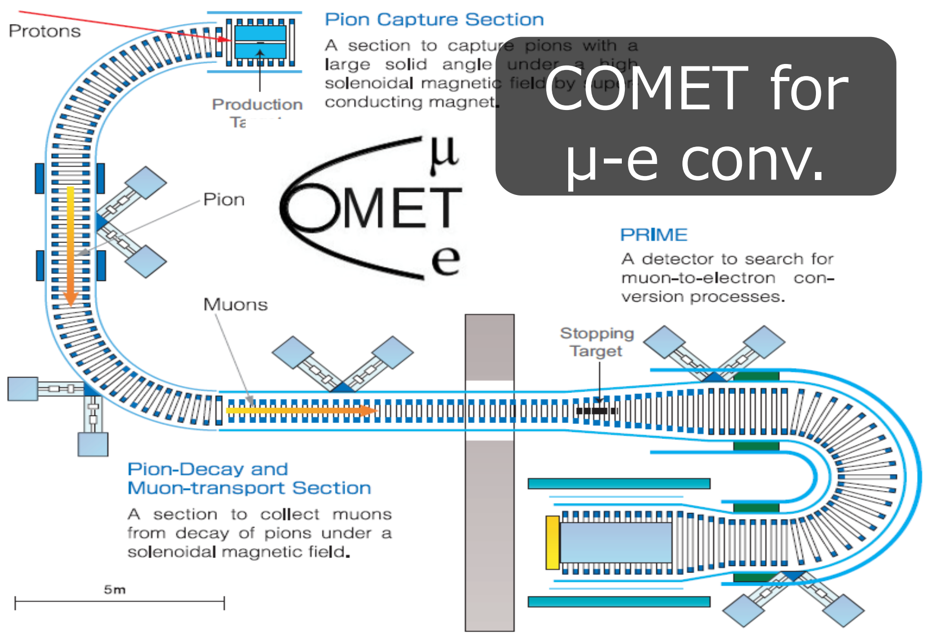


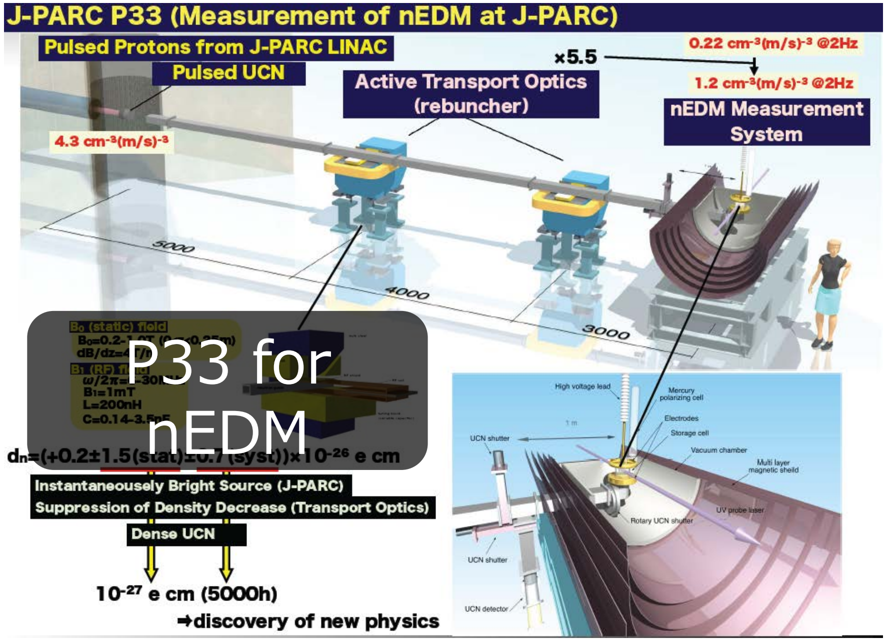
detect momentum
transfer to Xe

- scalar charges $m_q S_q = m_q \langle N|qq|N \rangle$ appear
- enhancement by m_q $\rightarrow$ strange quark charge S_s important

search @ J-PARC

μ-e conversion and nEDM





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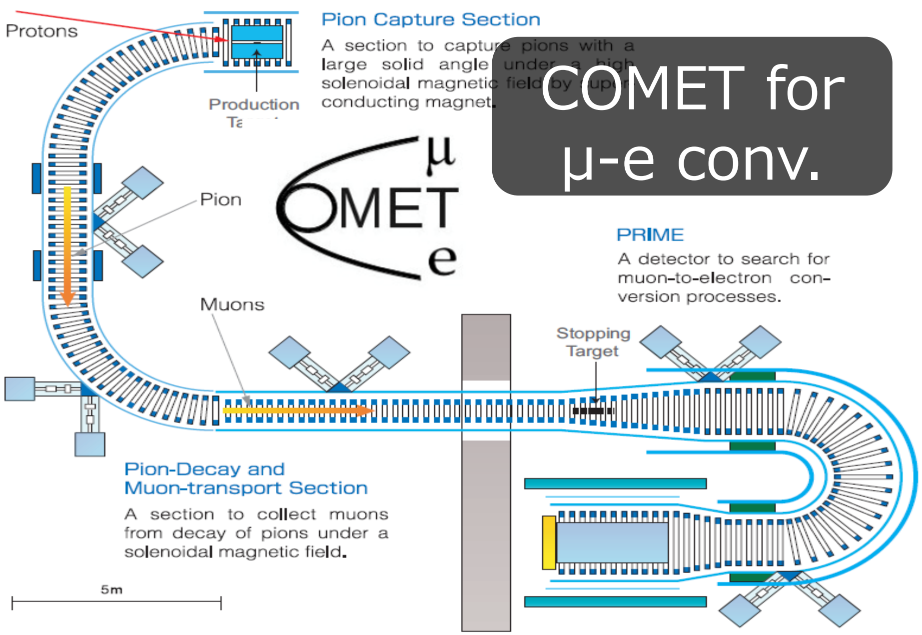

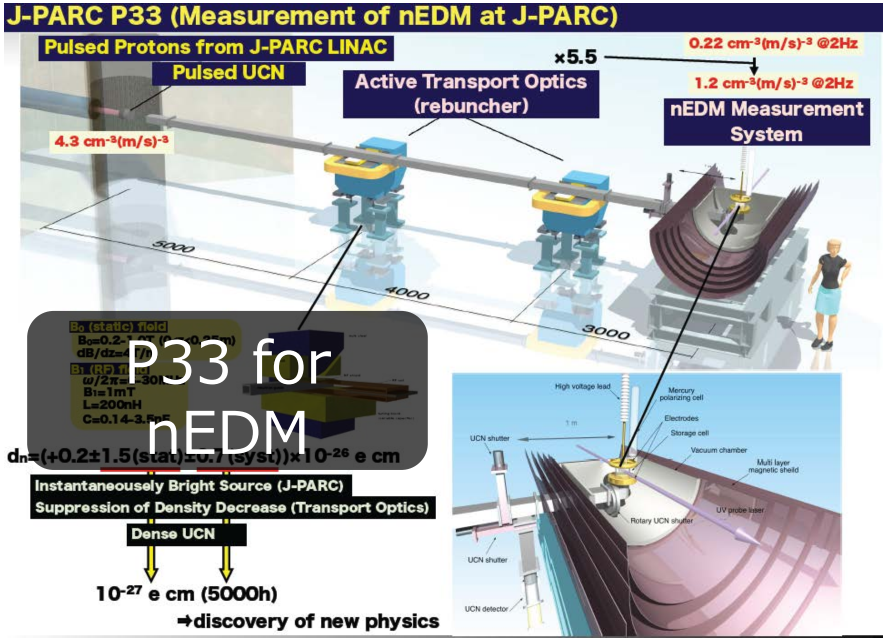

$$\mathcal{L}_{\rm int} \ni \sum_q C_{S,q}\, m_q\, \bar{q}q \cdot m_\mu \bar{e}\mu$$

$S_q \rightarrow$ amplitude in muonic atom

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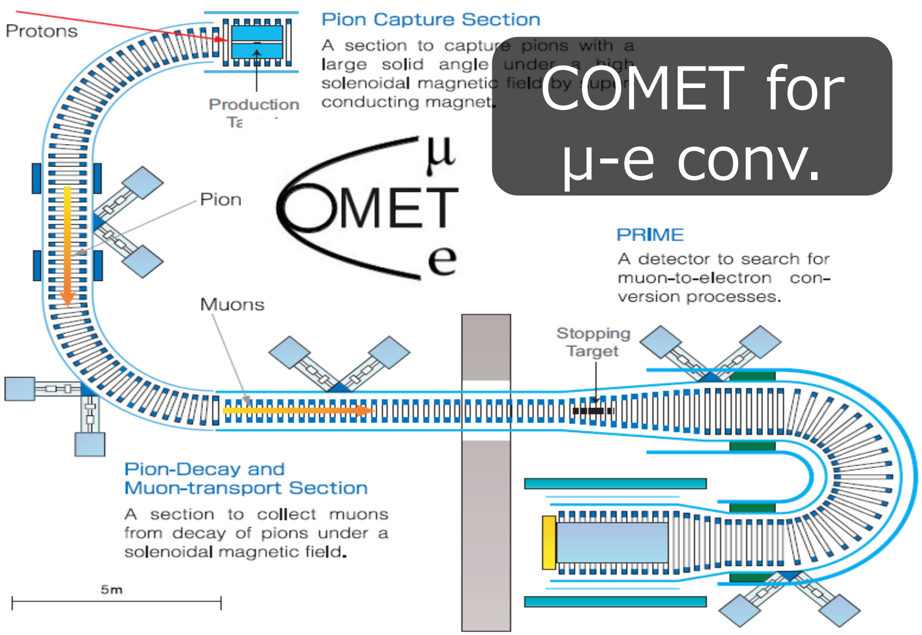
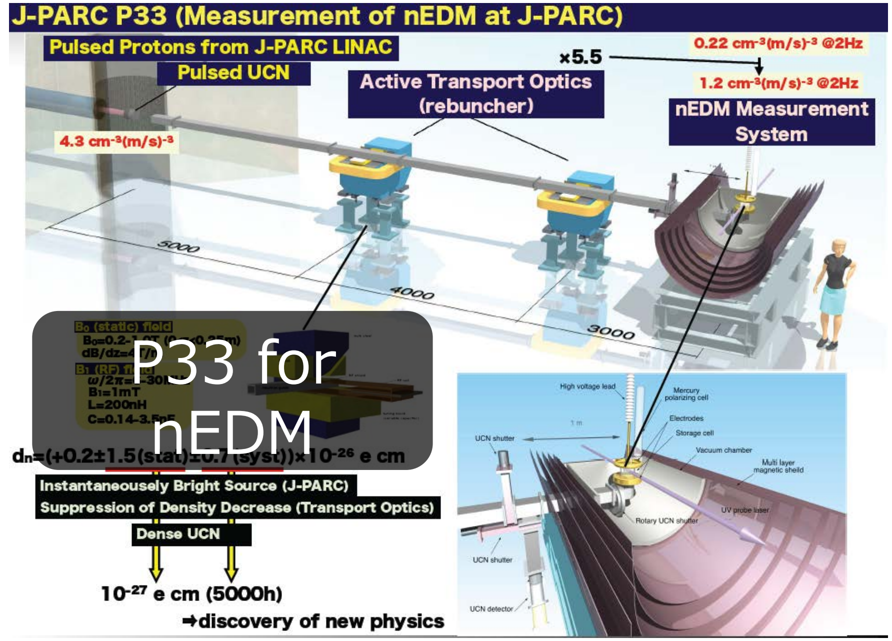
μ-e conversion and nEDM





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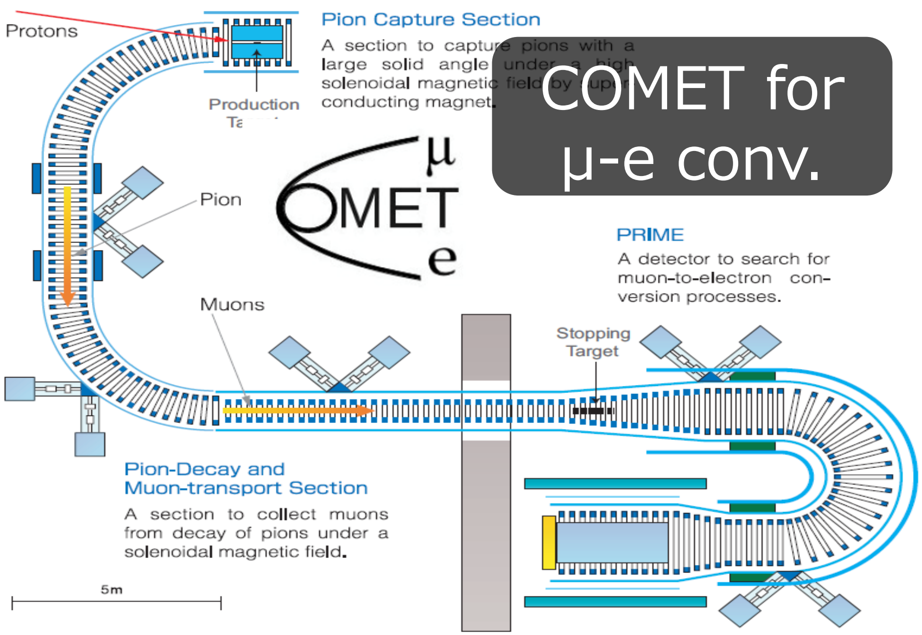
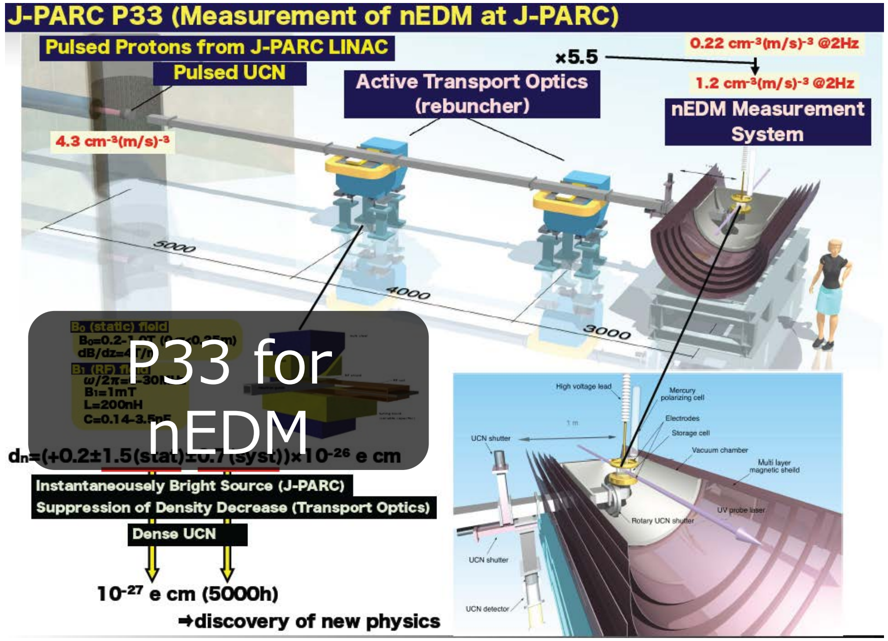
$\delta_q \to$ quark contribution to nEDM

search @ J-PARC

μ-e conversion and nEDM





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$S_q \to$ amplitude in muonic atom $\qquad\qquad \delta_q \to$ quark contribution to nEDM

nucleon charges are fundamental input in searches for NP

this talk

lattice calculation of scalar, axial and tensor charges

JLQCD - N. Yamanaka (Orsay, RIKEN) et al. – in preparation

this talk

lattice calculation of scalar, axial and tensor charges

JLQCD - N. Yamanaka (Orsay, RIKEN) et al. – in preparation

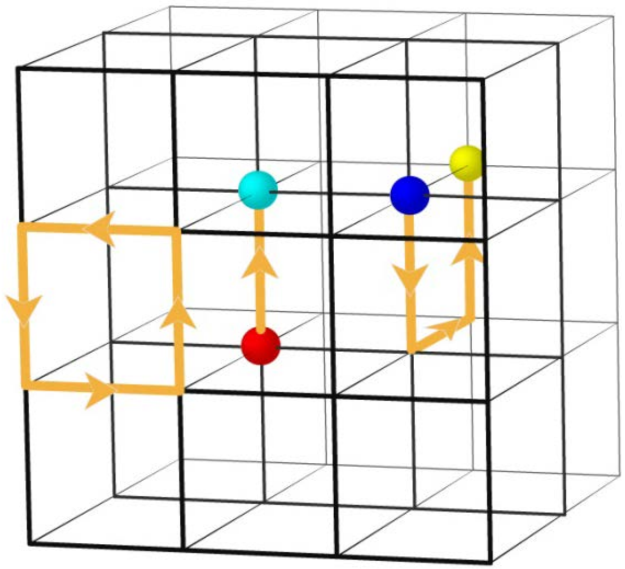
outline

- how to calculate nucleon charges on the lattice
- difficulties and challenges – isovector charge g_A -
- isoscalar charges uu+dd; each flavor uu, dd, ss
- summary + our perspective

1.
how to calculate nucleon charges on the lattice

lattice QCD (K.G.Wilson, 1974)

- QCD formulated on discrete Euclid lattice



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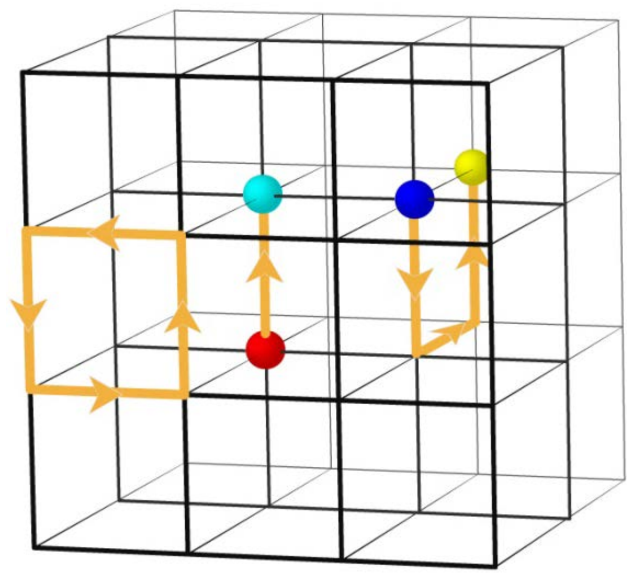
- QCD path integral and d.o.f.

$$\langle O \rangle = \int [d\bar{q}][dq][dA]\, O \exp\left[-S_{QCD} \right]$$



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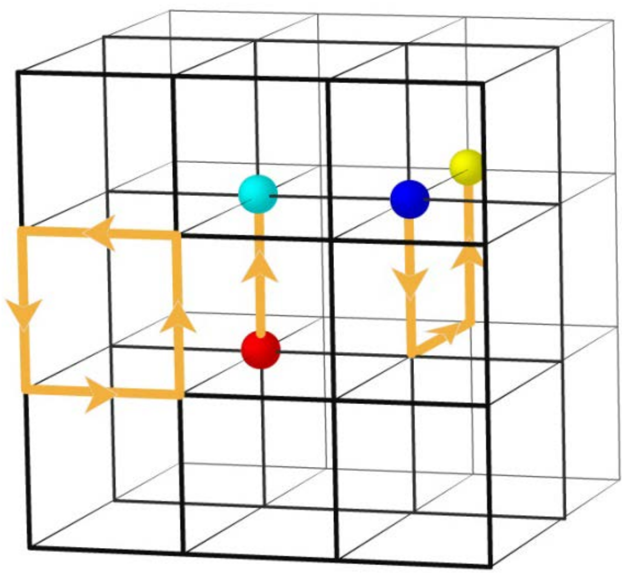

- quark : on each lattice site

  $q_{a\alpha}{}^{f}(x)$: $4 \times N_c \times N_f \times 2$ real variables

- gluon : on each link b/w 2 adjacent sites

  $A_\mu{}^a(x)$: $4 \times (N_c{}^2 - 1)$ real variables

finite volume $\Rightarrow$ #d.o.f. = $152\times$ #sites $\Rightarrow$ 23×10^6 / $(16^3\times 32)$

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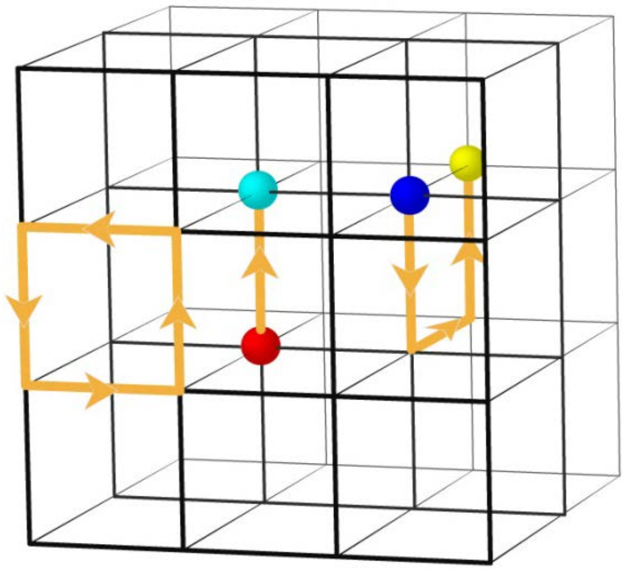

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finite volume ⇒ #d.o.f. = 152 × #sites ⇒ 23×10^6 / $(16^3 \times 32)$

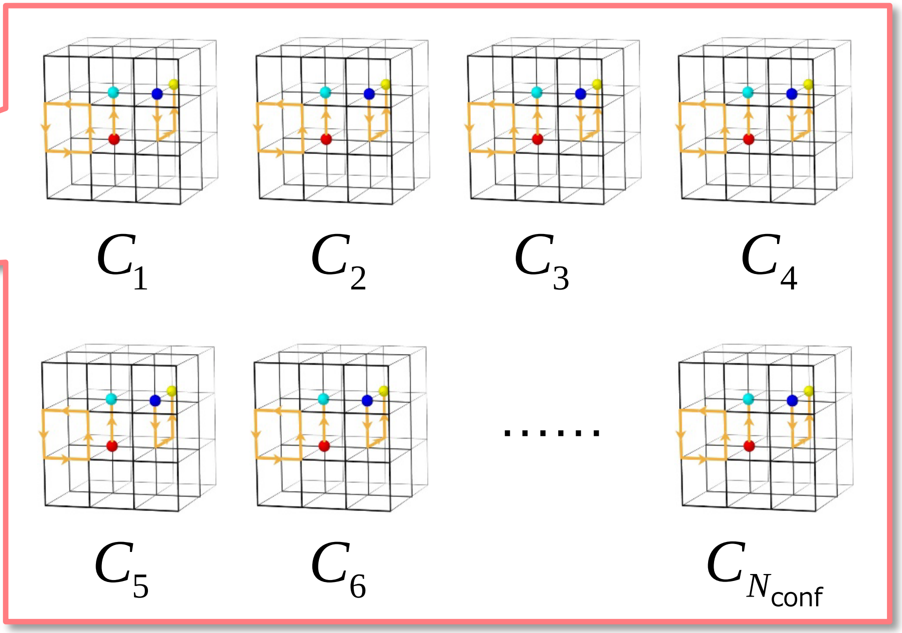
integral by hand : difficult / perturbative expansion

computer simulation

generate field configurations on powerful computer

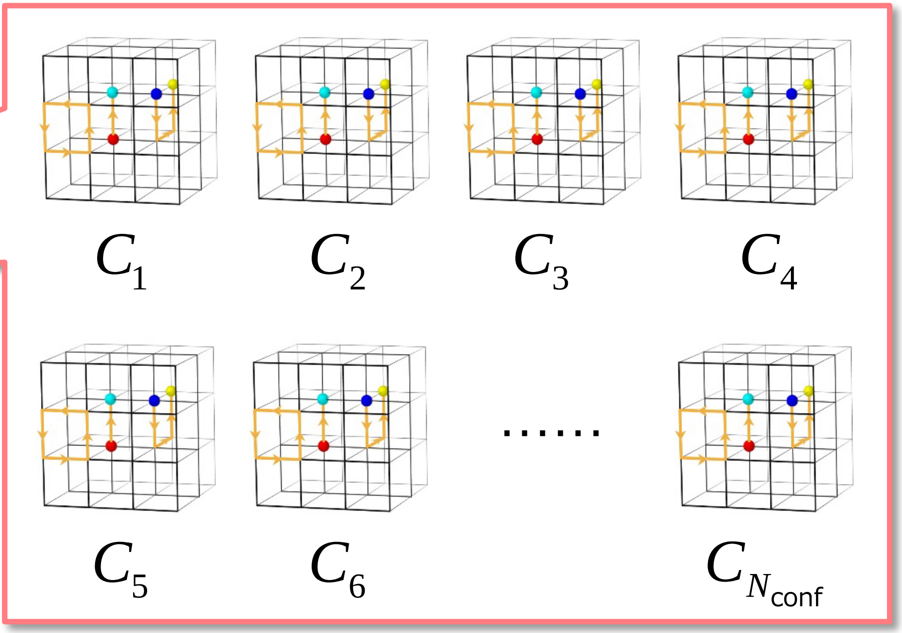
computer simulation

generate field configurations on powerful computer



computer simulation

generate field configurations on powerful computer



$$\langle O \rangle = \int [dA][d\bar{q}][dq]\, O \exp\left[-S_{\text{QCD}}\right] = \lim_{N_{\text{conf}} \to \infty} \frac{1}{N_{\text{conf}}} \sum_{i=1}^{N_{\text{conf}}} O[C_i]$$

computer simulation

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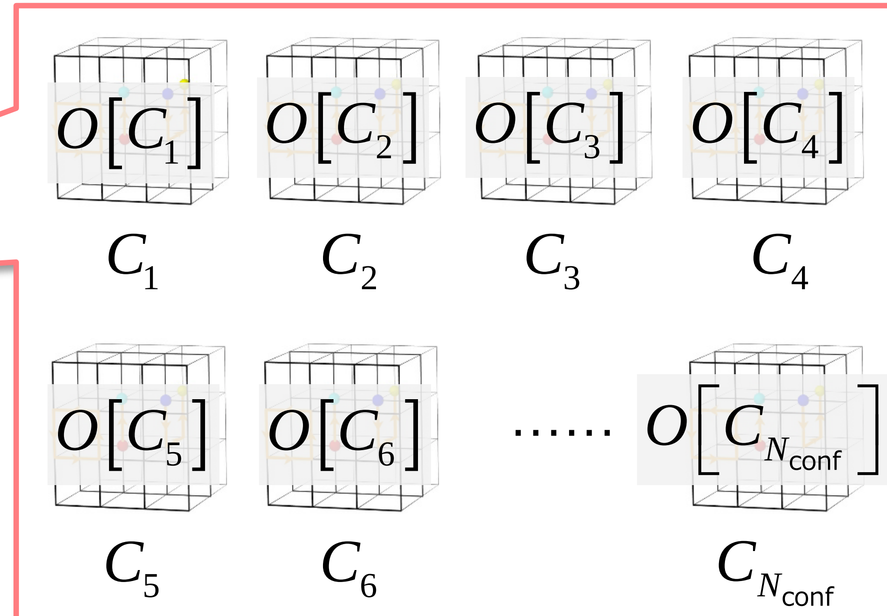
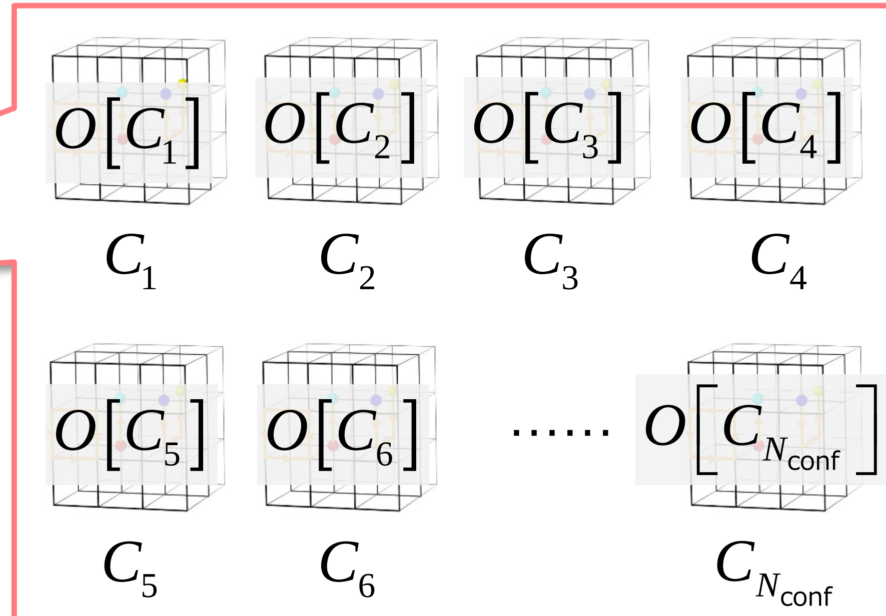


$$\langle O \rangle = \int [dA][d\bar{q}][dq]\, O \exp\left[-S_{\rm QCD}\right] = \lim_{N_{\rm conf}\to\infty} \frac{1}{N_{\rm conf}} \sum_{i=1}^{N_{\rm conf}} O[C_i]$$

computer simulation

generate field configurations on powerful computer



$$\langle O \rangle = \int [dA][d\bar{q}][dq]\, O \exp\left[-S_{\rm QCD}\right] = \lim_{N_{\rm conf}\to\infty} \frac{1}{N_{\rm conf}} \sum_{i=1}^{N_{\rm conf}} O[C_i]$$

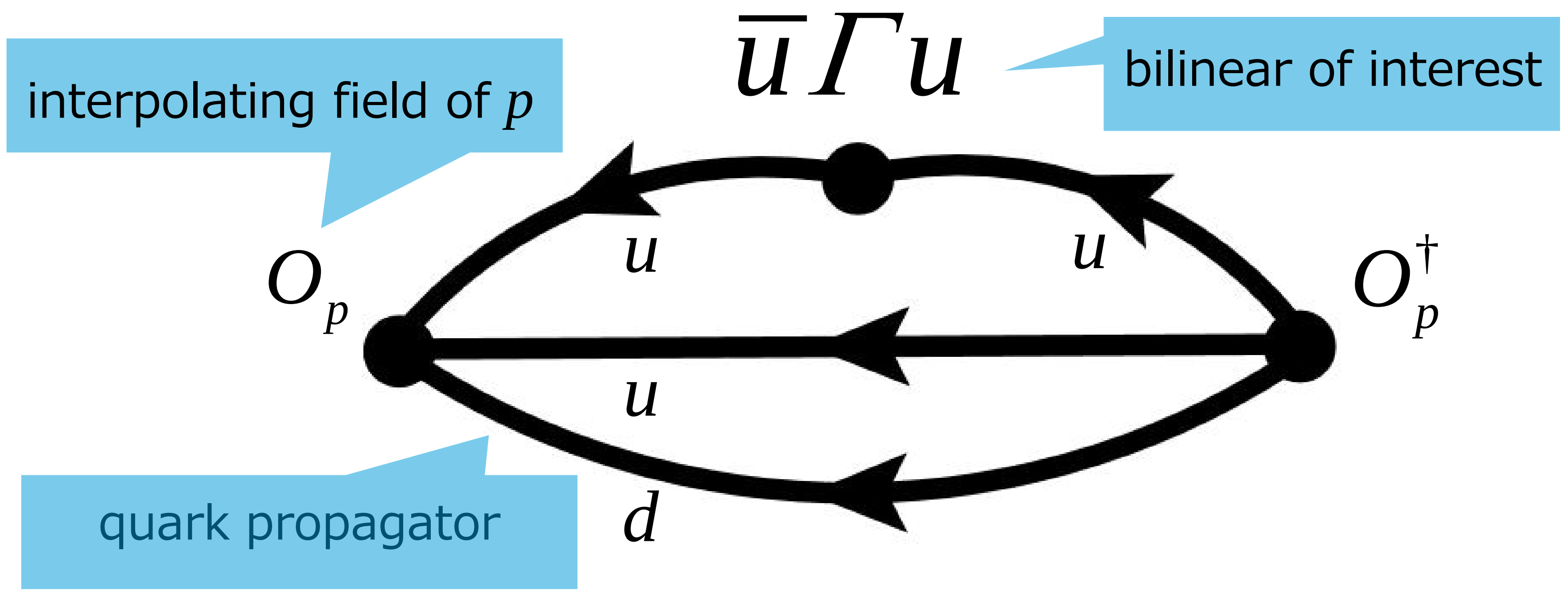
non-perturbative method to study QCD !

control of "statistical" and "systematic" uncertainties

how to calculate nucleon charges

nucleon 3-point function

e.g. up quark charge of proton



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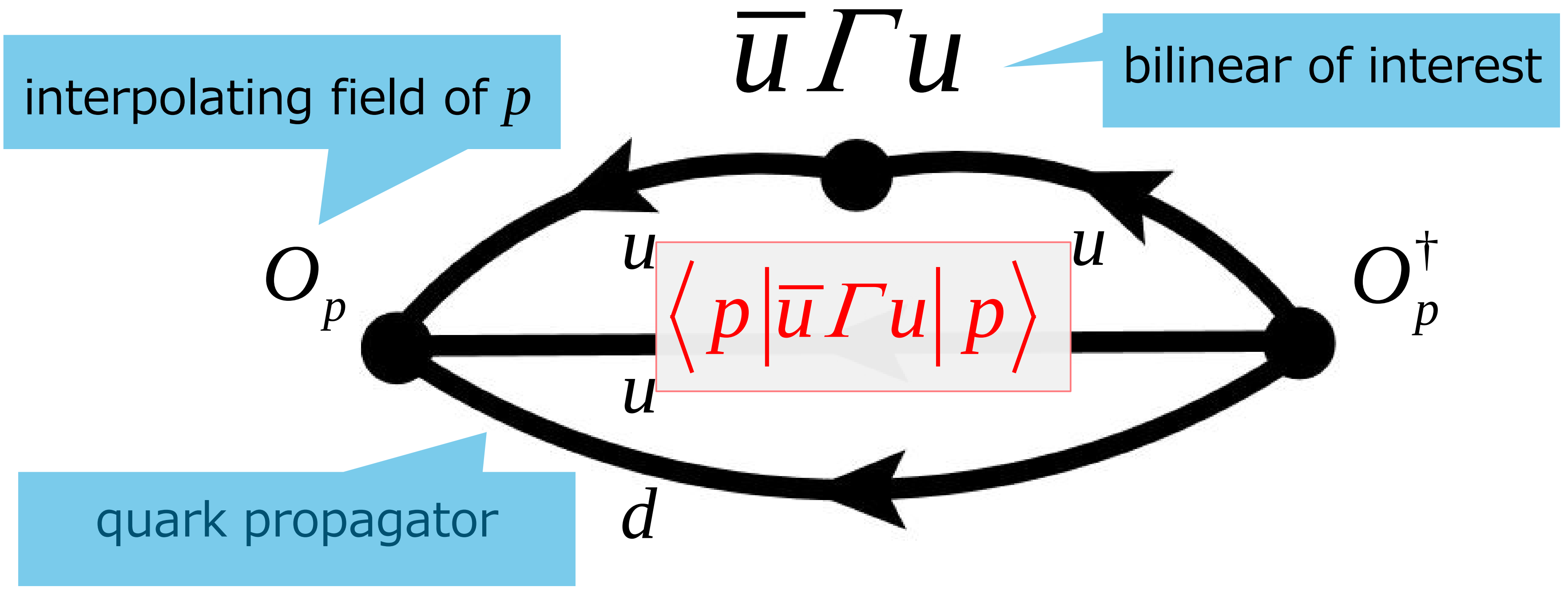

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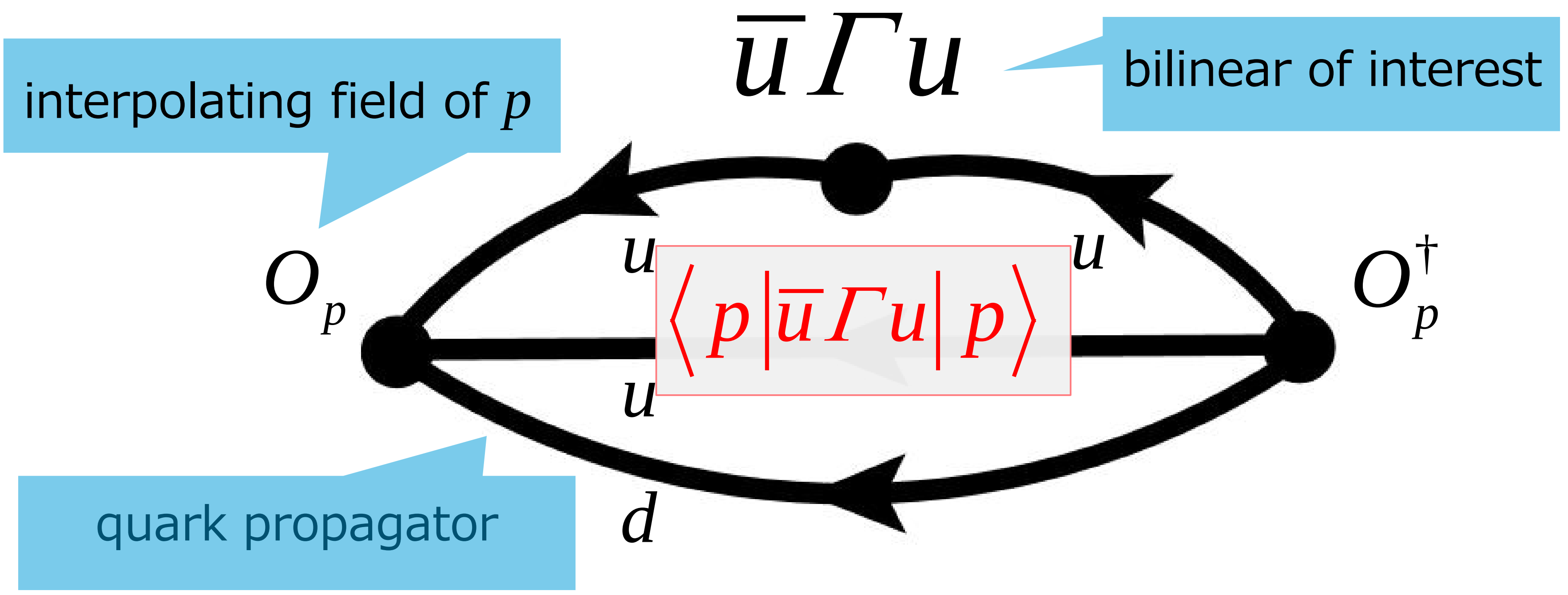


- inverse of Dirac operator D^{-1} $\quad S_q = \sum_{\text{space-time,color,spinor}} \bar{q}_X D_{XY} q_Y$
- huge matrix O(10^7) x O(10^7) $\Rightarrow$ need "super computer"
- still can calculate part of D^{-1} (discuss later)

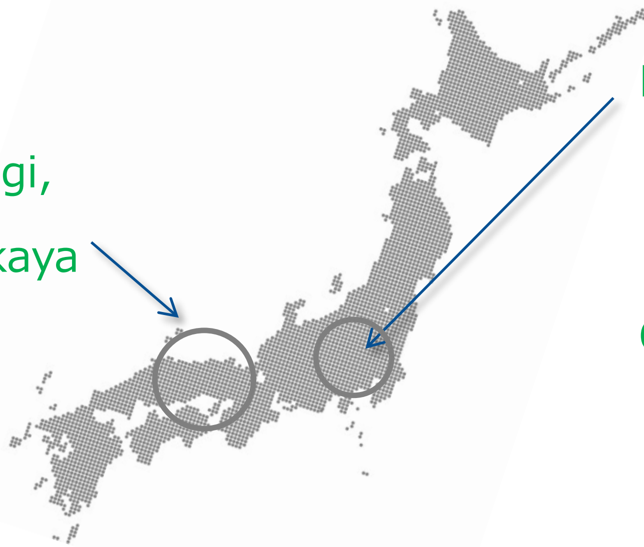
the JLQCD collaboration

lattice simulations (mainly) on computers @ KEK

YITP: Sinya Aoki

Osaka: Tetsuya Onogi,
Hidenori Fukaya

Nara: Hiroshi Ohki



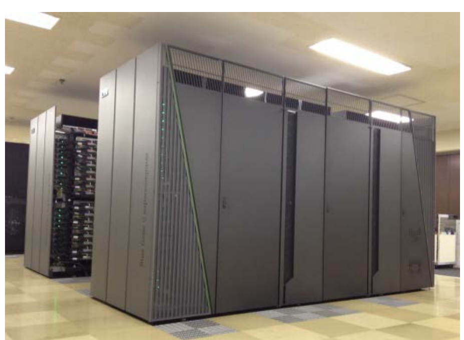
KEK: Shoji Hashimoto, Yasumichi Aoki,
TK, Brian Colquhoun, Kei Suzuki,
Katsumasa Nakayama

Oversea: Nodoka Yamanaka, Guido
Cossu, Christian Rohrhofer

for 20 years since '96

KEK system 2011-2017

Hitachi SR16000 (55 TFLOPS)



IBM BlueGene/Q (1.2 TFLOPS)

Oakforest-PACS (JCAHPC)

TOP500 9 th (25 PFLOPS)

post-K (RIKEN)

Coming
Soon
2020 -

w/ chiral symmetry

- Nielsen-Ninomiya ’82 : chiral symmetry $\Rightarrow$ *e.g.* renormalization

w/ chiral symmetry

- Nielsen-Ninomiya ’82 : chira 

metry $\Rightarrow$ $e.g.$ renormalization

w/ chiral symmetry

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- chiral fermions (domain-wall, overlap) (Kaplan, Neuberger, ···, ’92-98)

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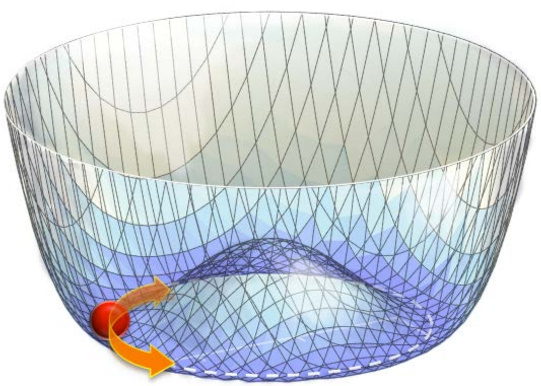
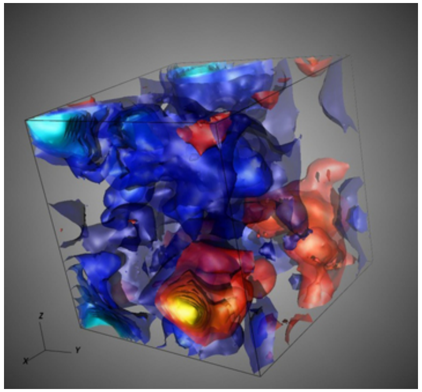
QCD vacuum





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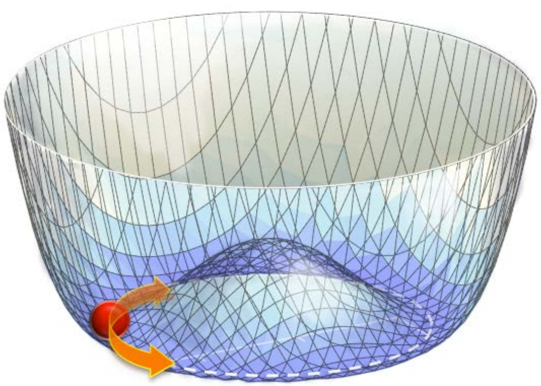
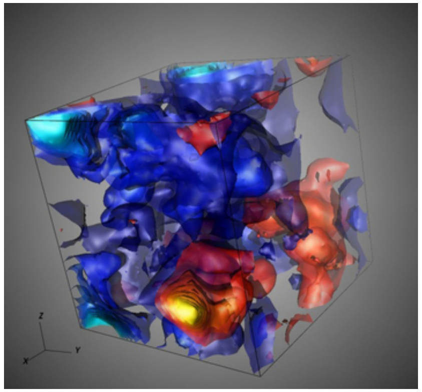
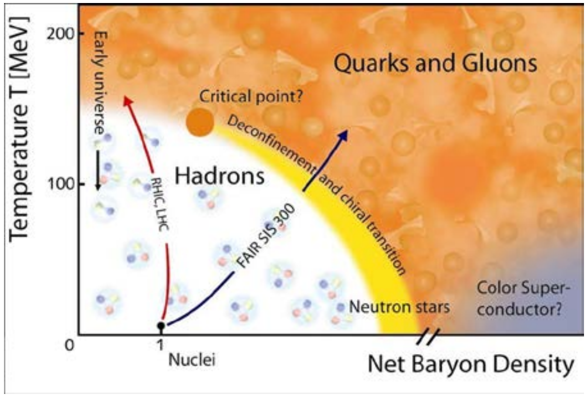




QCD phase structure



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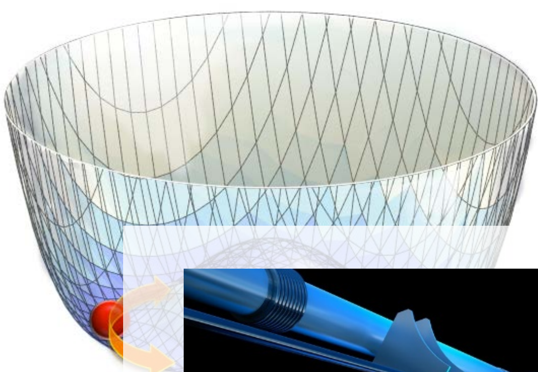

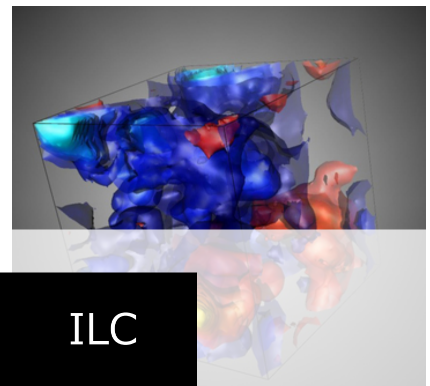

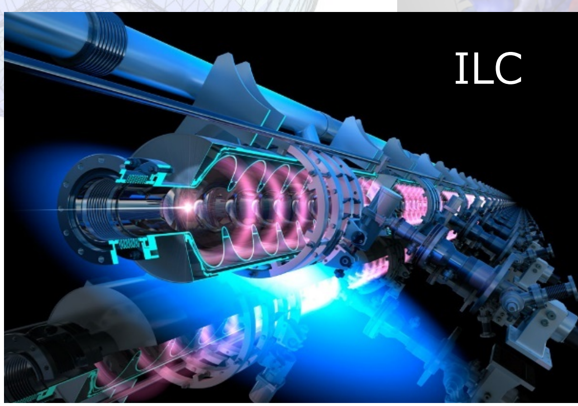

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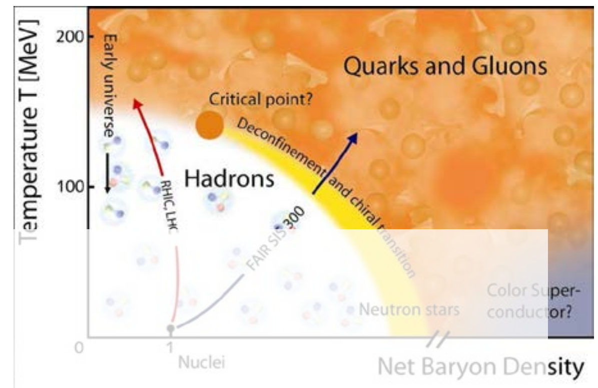

fundamental parameters α_s, m_q

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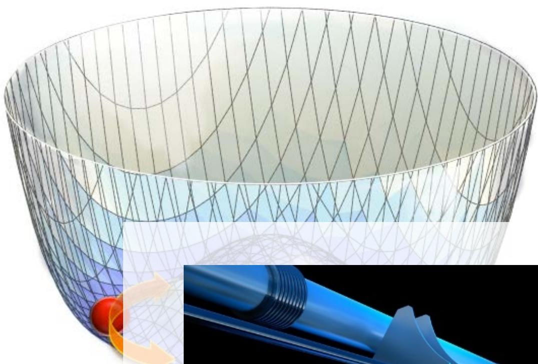

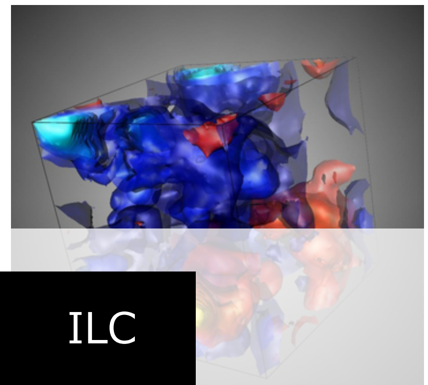

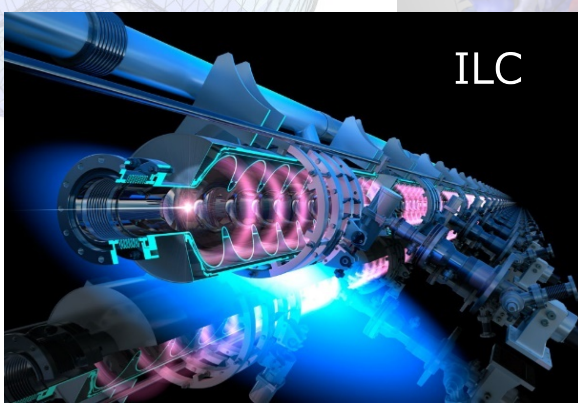

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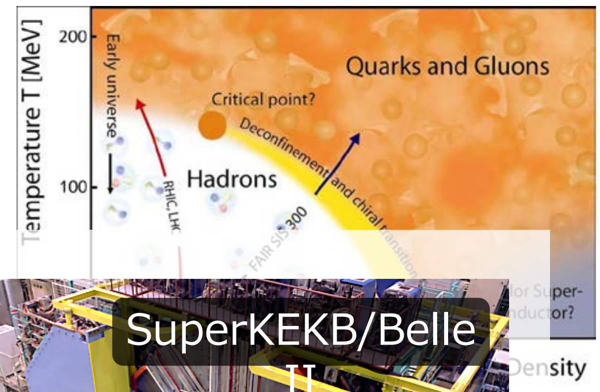

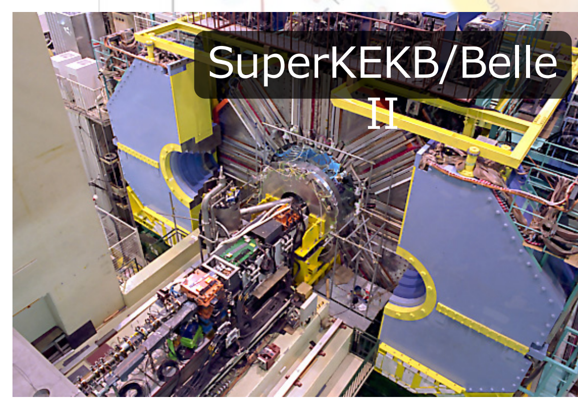

search of new physics

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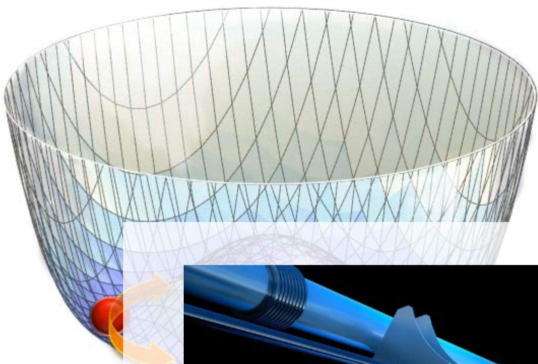

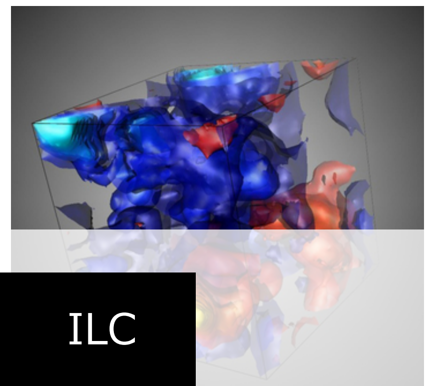

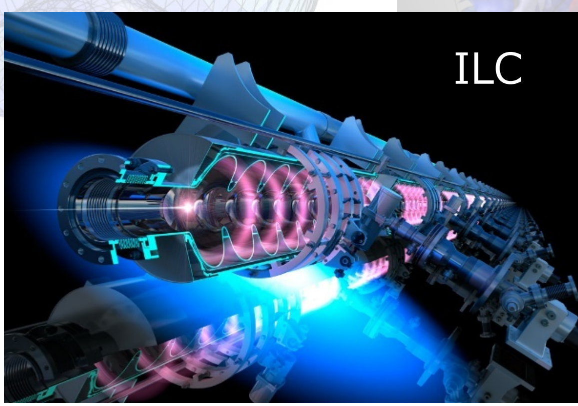

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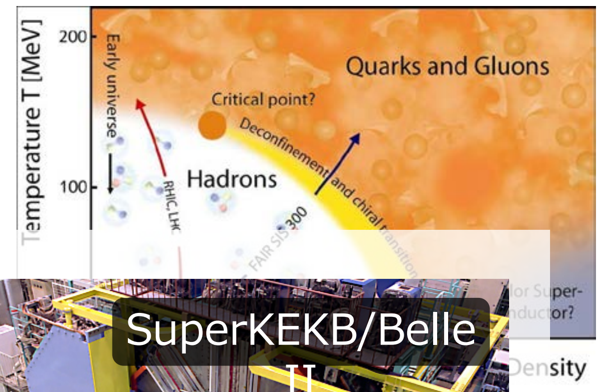
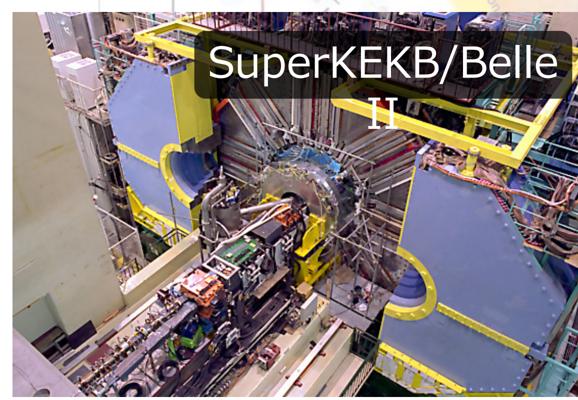




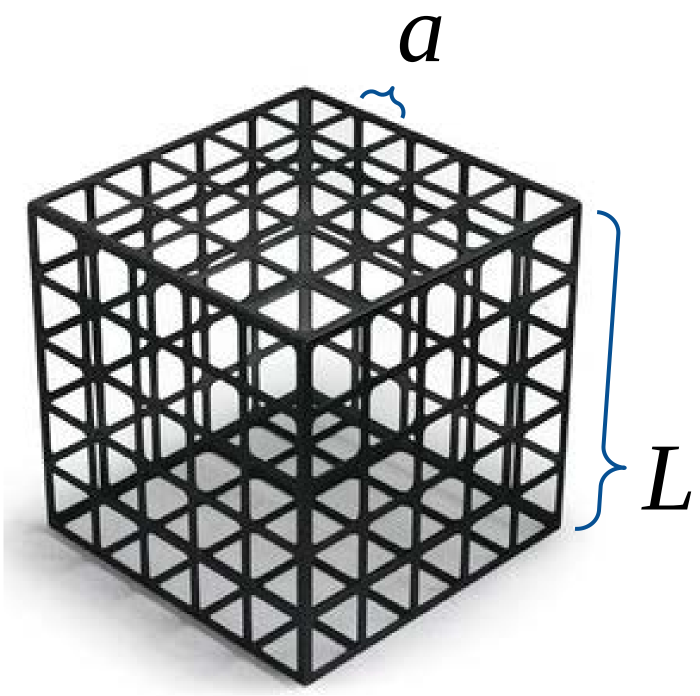
fundamental parameters α_s, m_q

search of new physics

wide applications!!

set up for nucleon charges

N_f=2+1 QCD w/ exact chiral symmetry using overlap action



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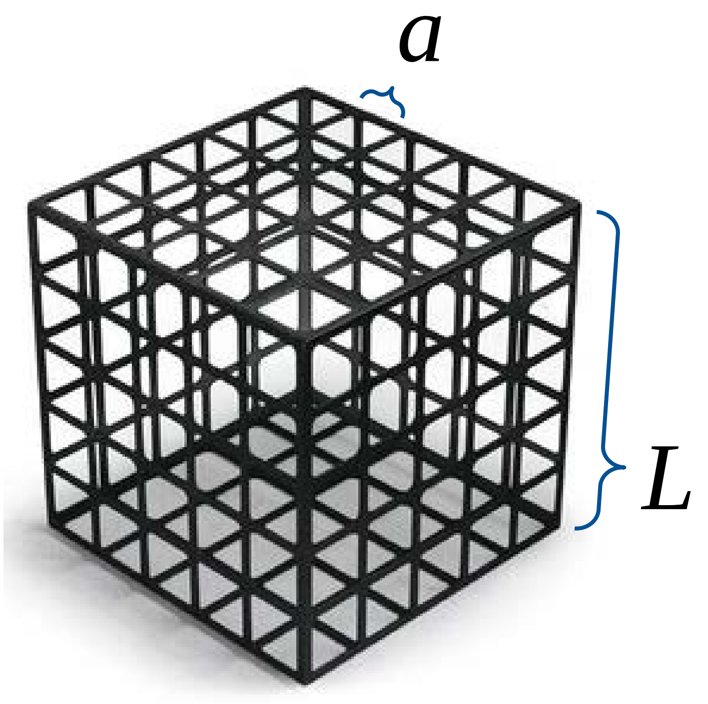
N_f=2+1 QCD w/ exact chiral symmetry using overlap action

a = 0.11(1) fm $\Rightarrow O((a\Lambda)^2) \sim$ 8% error



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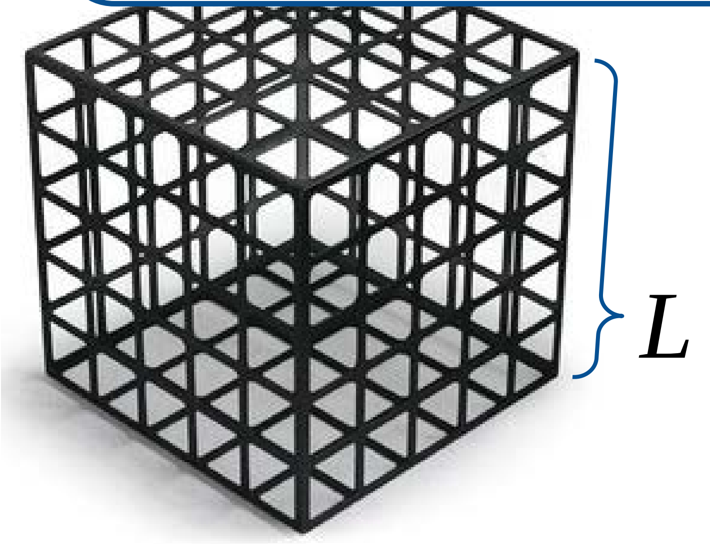
- post-diction of g_A @ 1% accuracy
- new physics search, spin puzzle



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u d

$m_u = m_d = m_{ud}$

4 values of M_π
290 – 540 MeV

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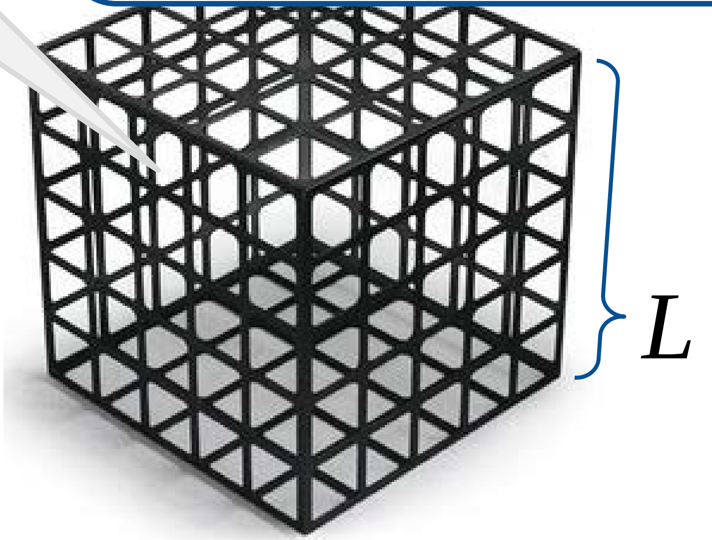

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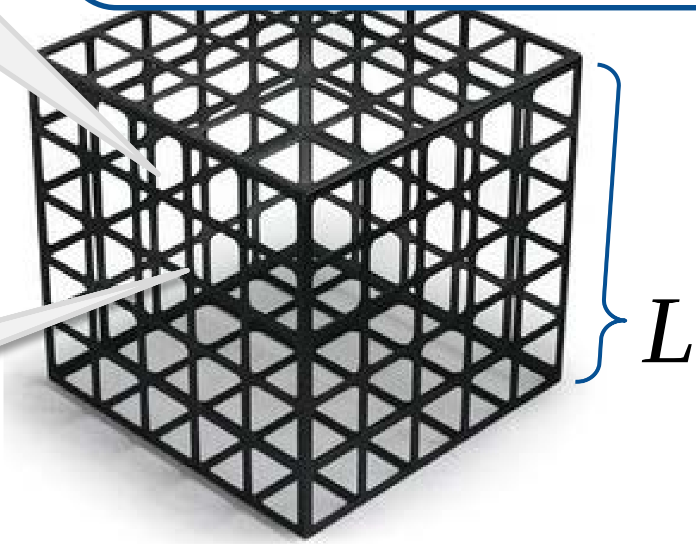

s $m_s \cong m_{s,\mathrm{phys}}$

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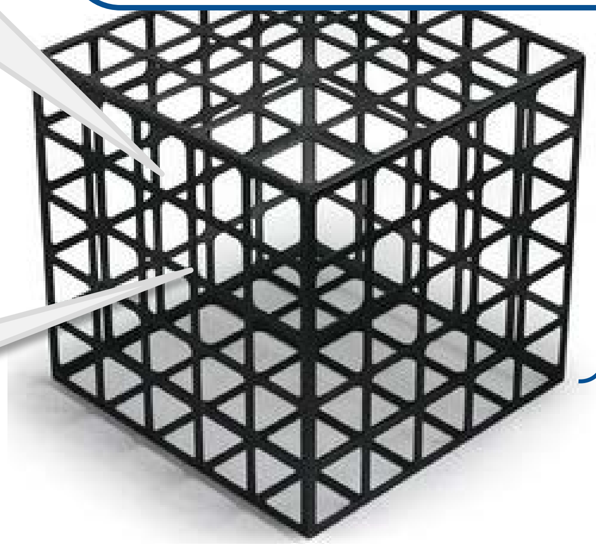

L

$16^3 \times 48$, $24^3 \times 48$

$\Rightarrow M_\pi L \geq 4$

s $m_s \cong m_{s,\mathrm{phys}}$

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a = 0.11(1) fm ⇒ $O((a\Lambda)^2)$ ~ 8% error

post-diction of g_A @ 1% accuracy

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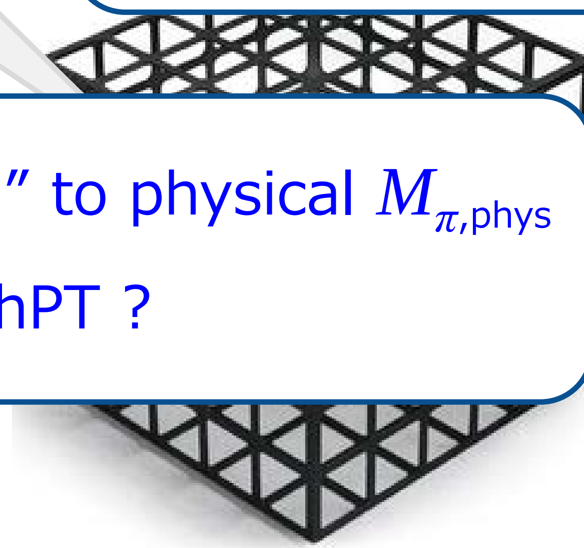

"chiral extrapolation" to physical $M_{\pi,\text{phys}}$

⇔ controllable?, ChPT ?

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290 – 540 MeV

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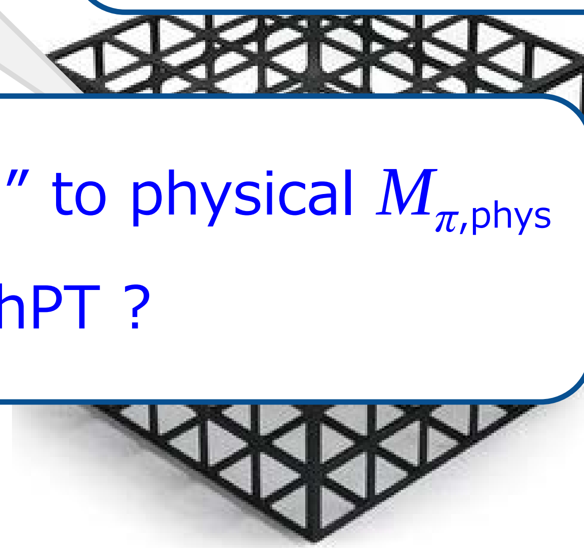
☹ post-diction of g_A @ 1% accuracy

☺ new physics search, spin puzzle



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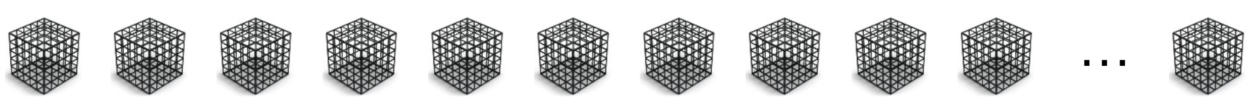

50 configurations

set up for nucleon charges

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post-diction of g_A @ 1% accuracy

new physics search, spin puzzle

u d

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4 values of M_π

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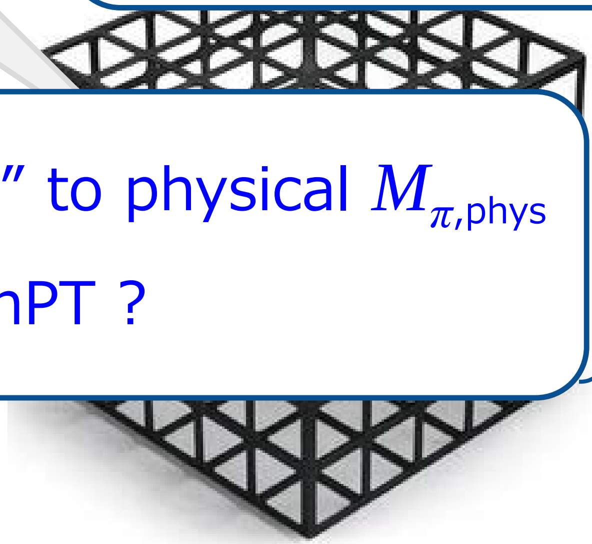


"chiral extrapolation" to physical $M_{\pi,\mathrm{phys}}$

⇔ controllable?, ChPT ?

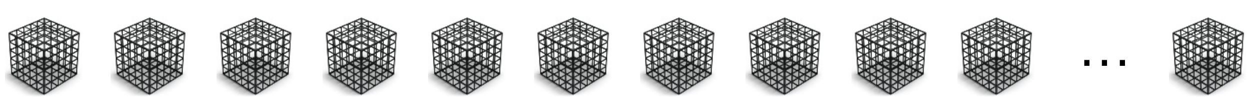
L

$16^3 \times 48$, $24^3 \times 48$

$\Rightarrow M_\pi L \geq 4$

s $m_s \cong m_{s,\mathrm{phys}}$

statistical accuracy?

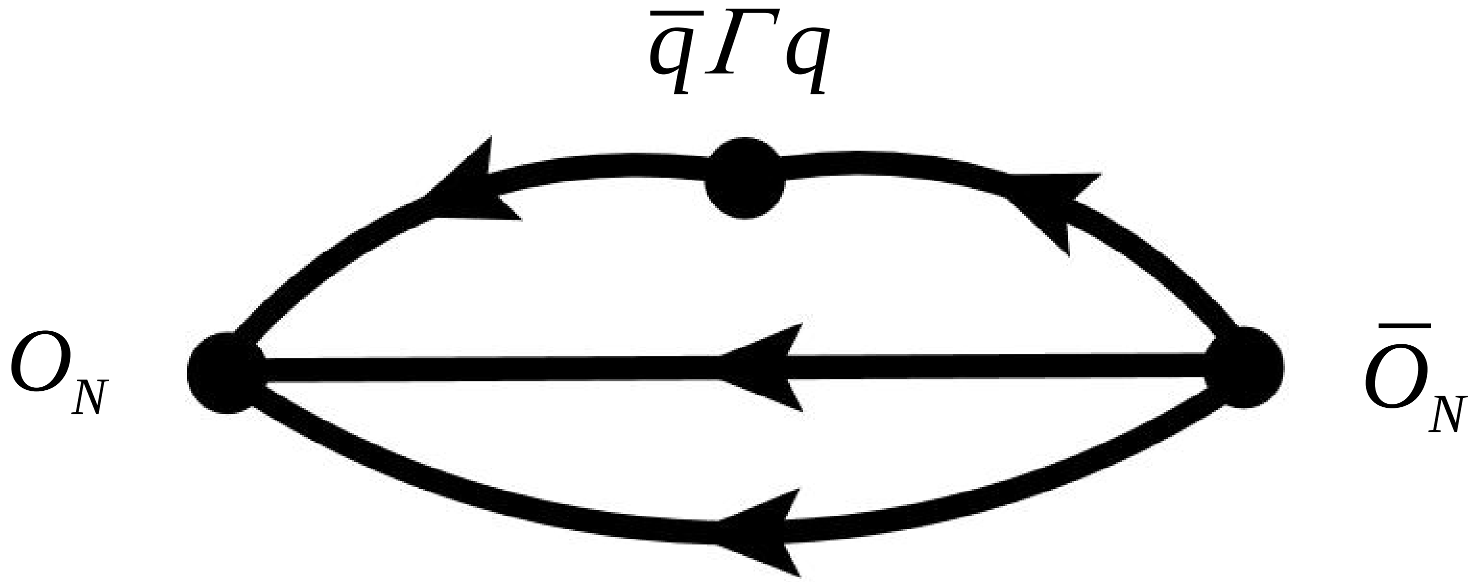


… 50 configurations

2.
difficulties and challenges
- isovector g_A -

axial charge from 3pt function

$$g_A = \left\langle N \left| \bar{u}\gamma_3\gamma_5 u - \bar{d}\gamma_3\gamma_5 d \right| N \right\rangle \Big/ 2M_N\,, \quad N = p$$



axial charge from 3pt function

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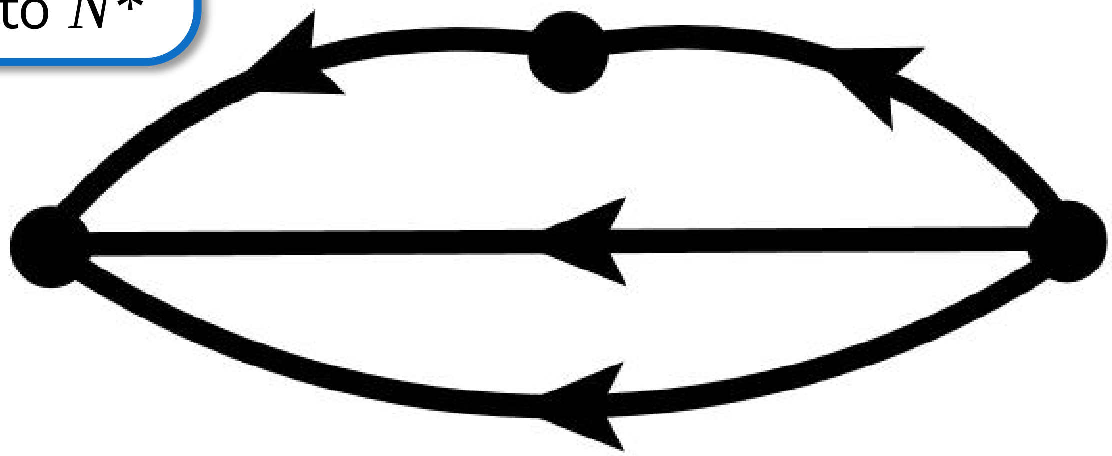
interpolating field

$\varepsilon_{abc}\left(u_a^T C\gamma_5 d_b\right)u_c$

also couples to N^*



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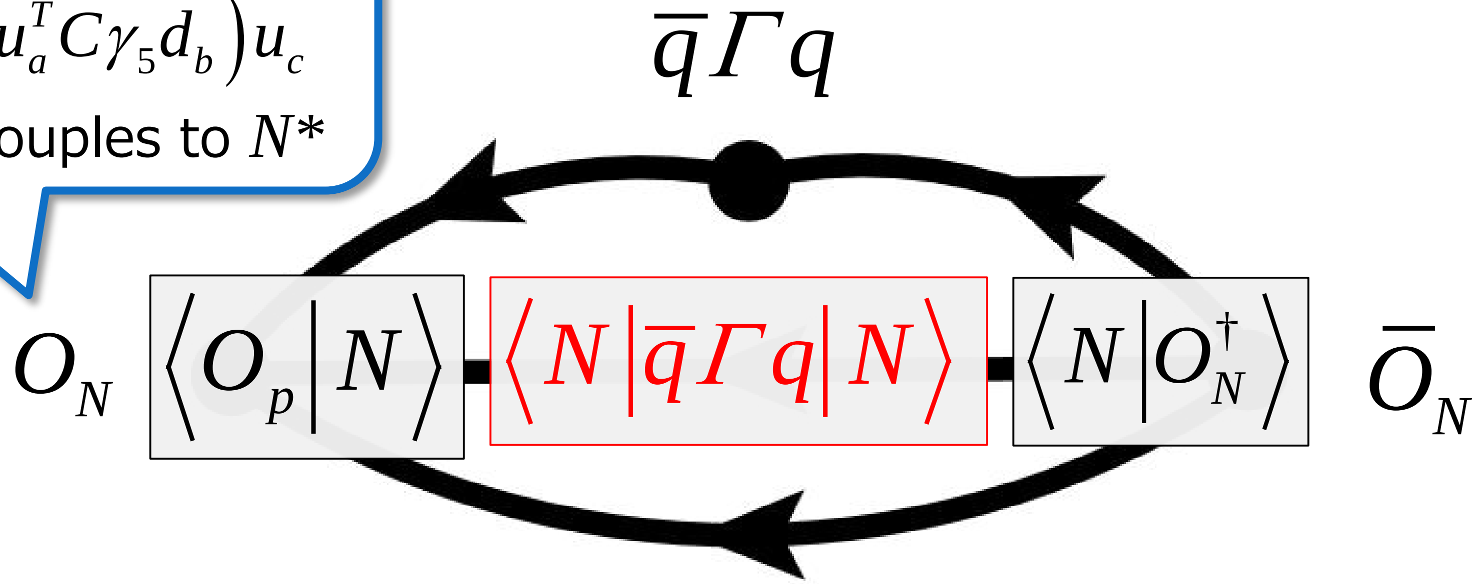

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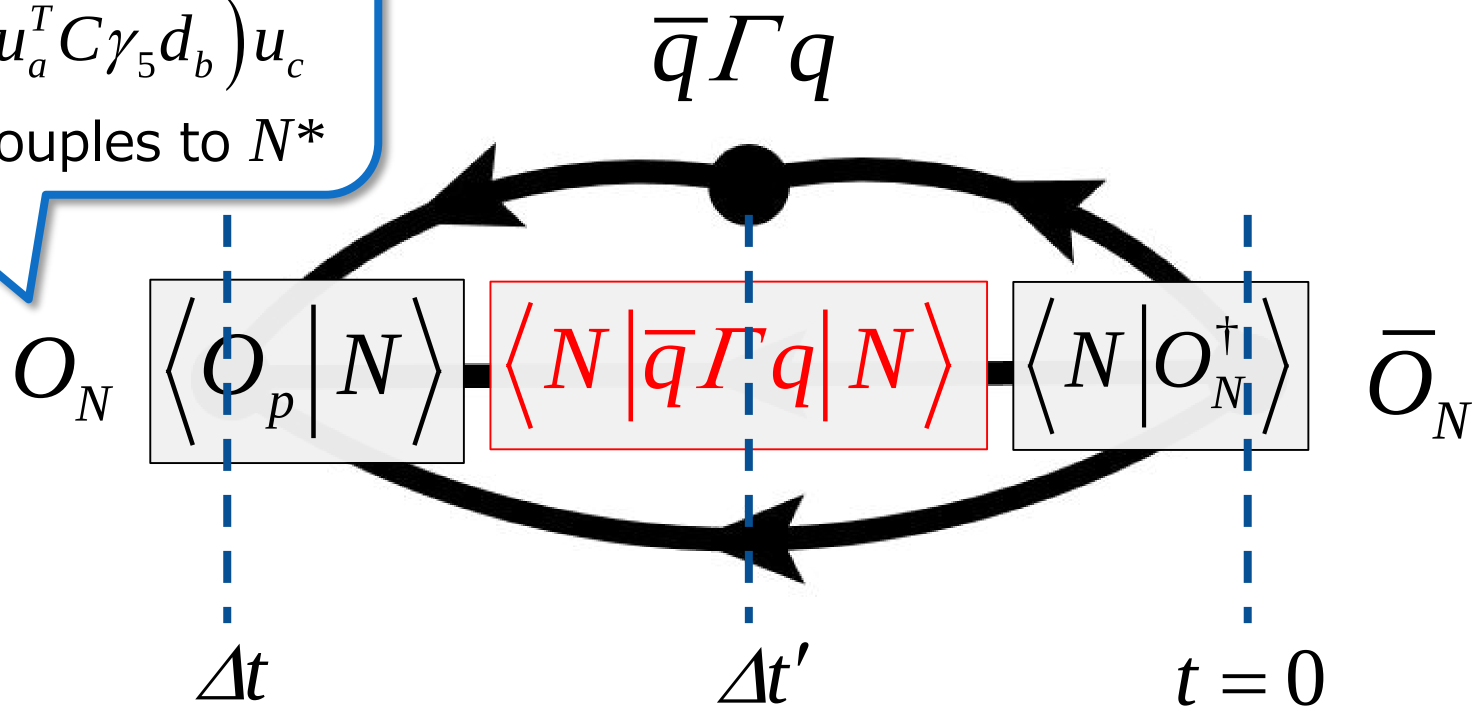

axial charge from 3pt function

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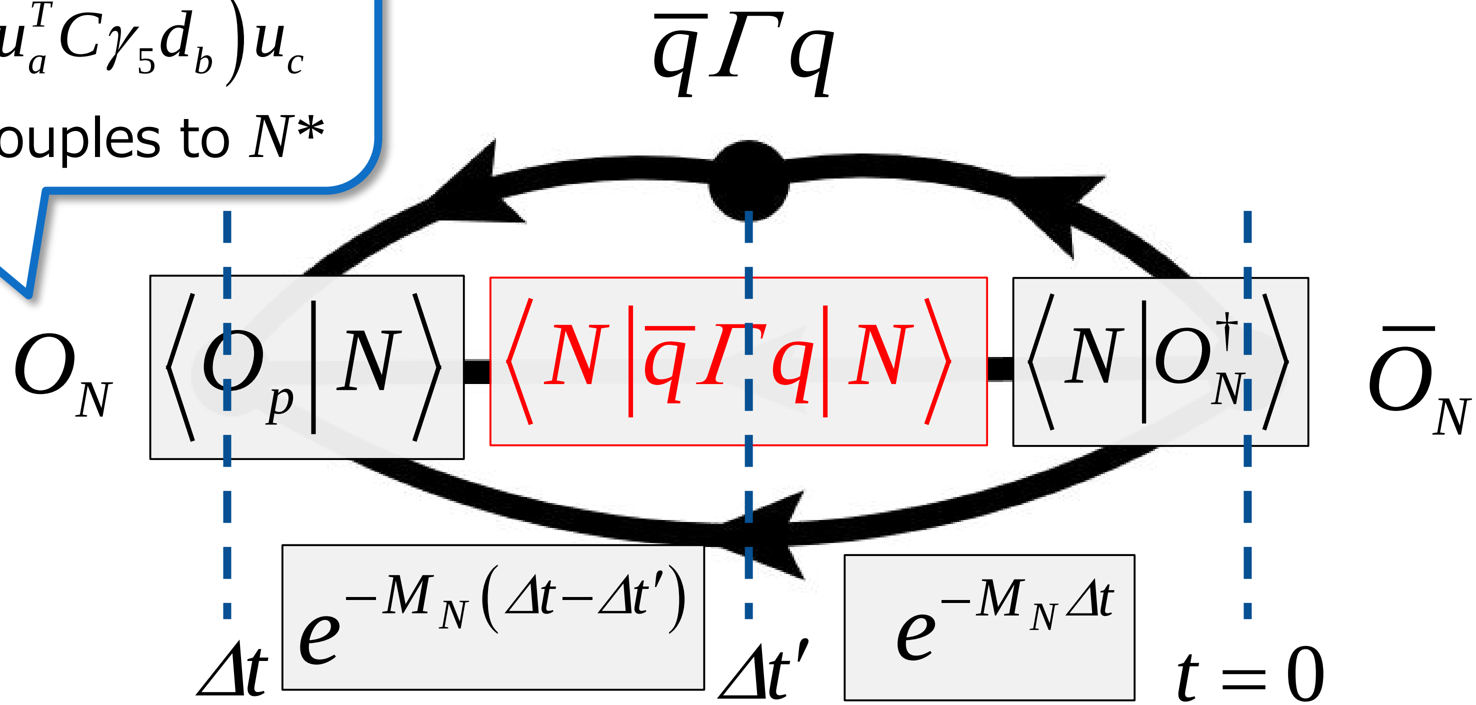

axial charge from 3pt function

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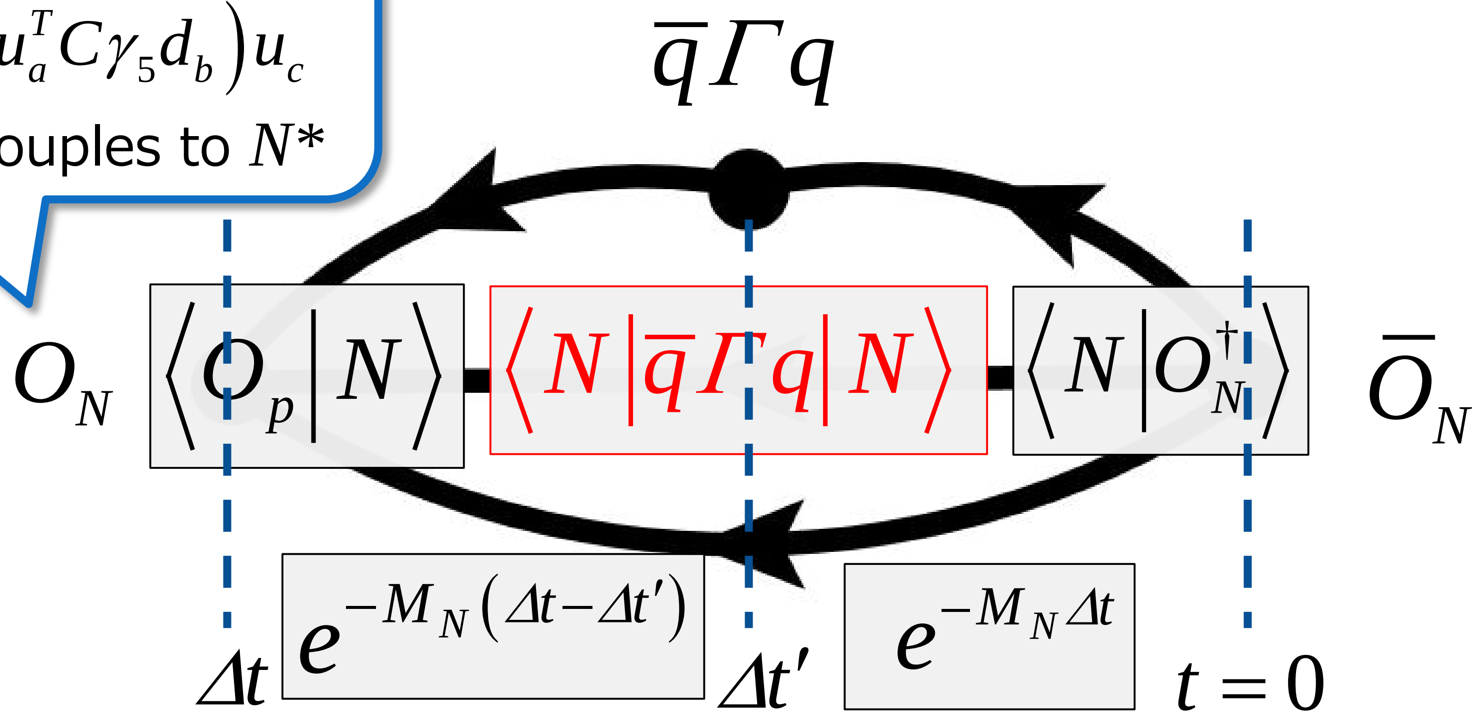

$$\rightarrow \frac{\langle O_N|N\rangle\langle N|O_N^\dagger\rangle}{4M_N^2}\langle N|\bar{q}\Gamma q|N\rangle \exp\left[-M_N\Delta t\right] + \text{"}N^*\text{"}\exp\left[-\Delta M\,\Delta t^{(\prime)}\right]$$

statistical accuracy

Lepage's argument on 2-pt function (TASI, '89)

$$N_{\mathrm{conf}}\sigma_{\mathrm{2pt}}^{2} \sim \left\langle \left(O_N(\Delta t)\bar{O}_N(0) \right)^{2} \right\rangle - \left\langle O_N(\Delta t)\bar{O}_N(0) \right\rangle^{2}$$

statistical accuracy

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$$\left\langle O_N(\Delta t)\bar{O}_N(0)\right\rangle \propto \exp\left[-M_N \Delta t\right]$$

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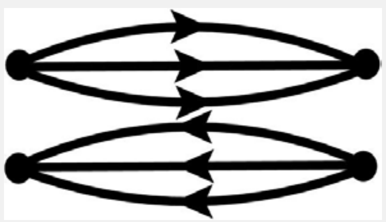
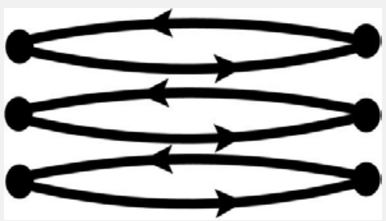


$\propto e^{-2M_N \Delta t} \ll$



$\propto e^{-3M_\pi \Delta t}$

statistical accuracy

Lepage's argument on 2-pt function (TASI, '89)

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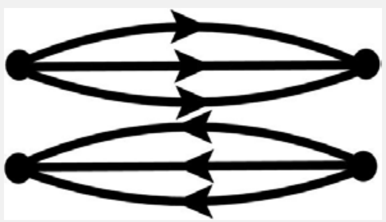

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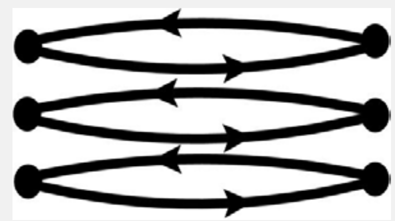


$\propto e^{-3M_\pi \Delta t}$

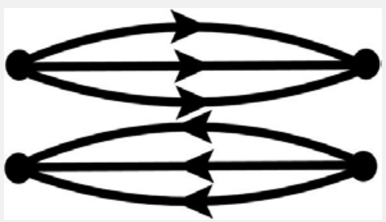
$$S/N \propto \sqrt{N_{conf}}\,\exp\left[-\left(M_N - 3M_\pi/2\right)\Delta t\right]$$

statistical accuracy

Lepage's argument on 2-pt function (TASI, '89)

$$\left\langle O_N(\Delta t)\bar{O}_N(0)\right\rangle \propto \exp\left[-M_N \Delta t\right]$$

$$N_{\text{conf}}\sigma_{\text{2pt}}^2 \sim \left\langle \left(O_N(\Delta t)\bar{O}_N(0)\right)^2 \right\rangle - \left\langle O_N(\Delta t)\bar{O}_N(0)\right\rangle^2$$



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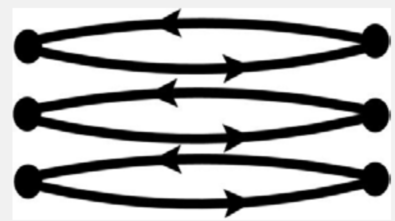
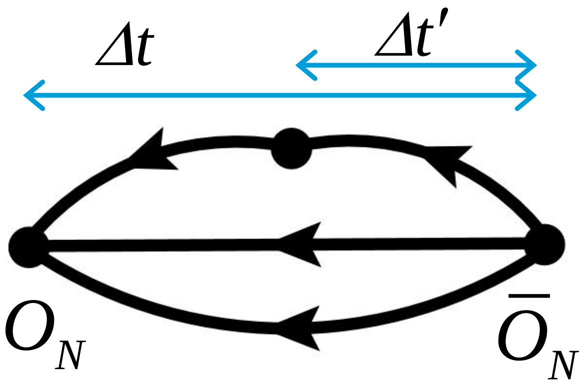


$$\propto e^{-3M_\pi \Delta t}$$

$$S/N \propto \sqrt{N_{conf}}\exp\left[-\left(M_N - 3M_\pi/2\right)\Delta t\right]$$

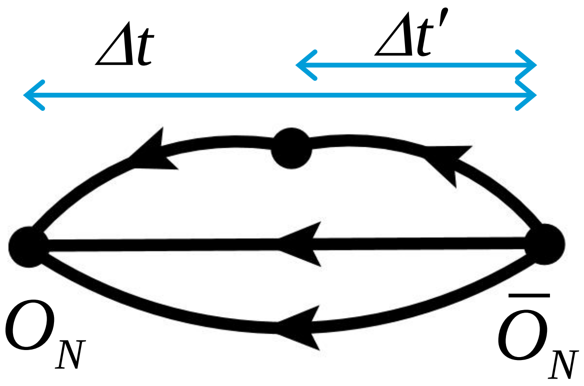
- towards smaller (physical) M_π ⇔ large M_π + chiral extrapolation
- towards larger Δt ⇔ smaller Δt+ excited state contamination

ground-state saturation



$/\exp\left[-M_N \Delta t\right]$

ground-state saturation



$$/\exp\left[-M_N \Delta t\right]$$

$$\Downarrow$$

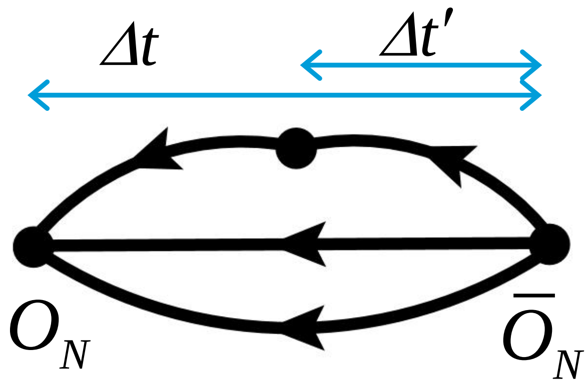
$$\langle O_N | N \rangle \langle N | \bar{q} \Gamma q | N \rangle \langle N | \bar{O}_N \rangle$$

$$+$$

$$\langle O_N | N^* \rangle \langle N^* | \bar{q} \Gamma q | N^* \rangle \langle N^* | \bar{O}_N \rangle$$

$$\times \exp\left[-\left(M_{N^*} - M_N\right) \Delta t'\right]$$

ground-state saturation



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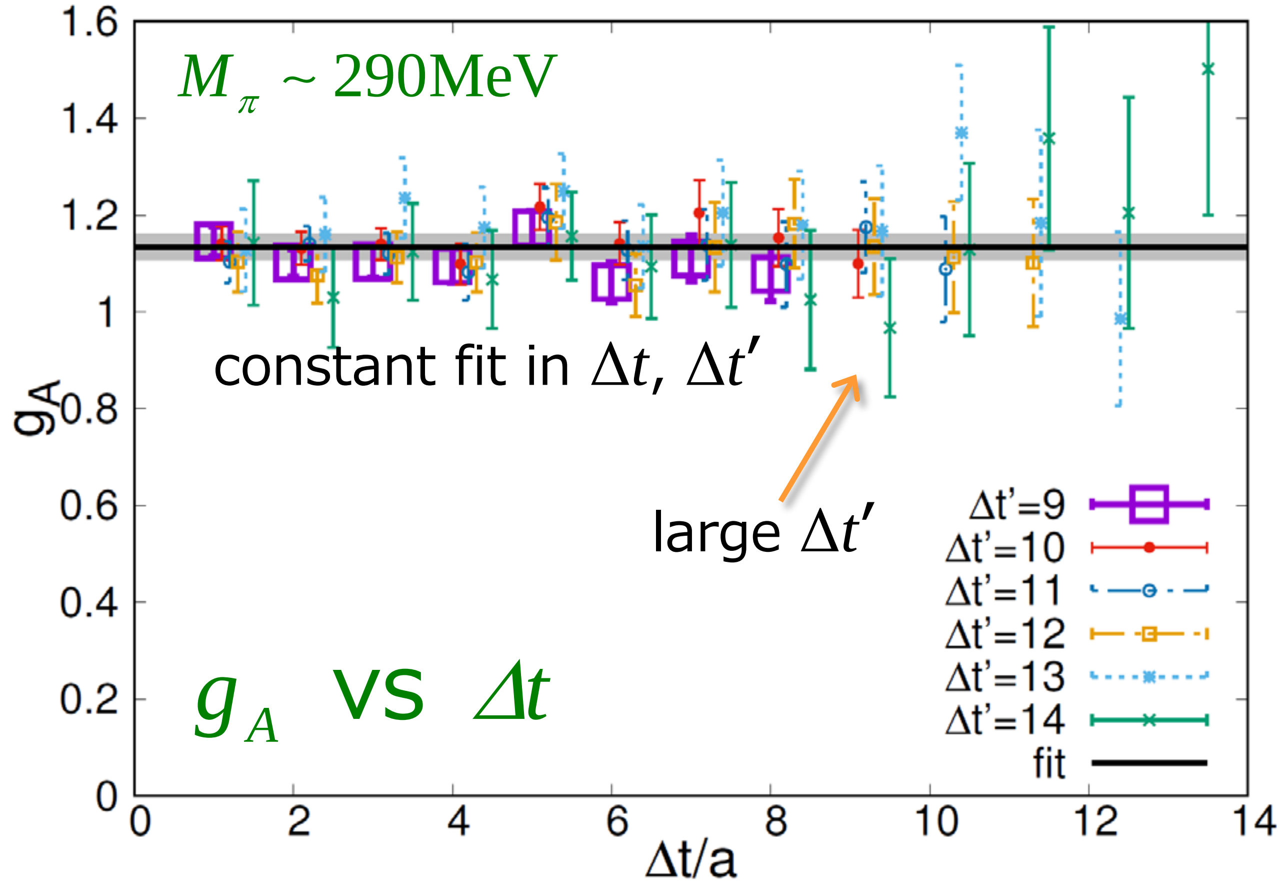
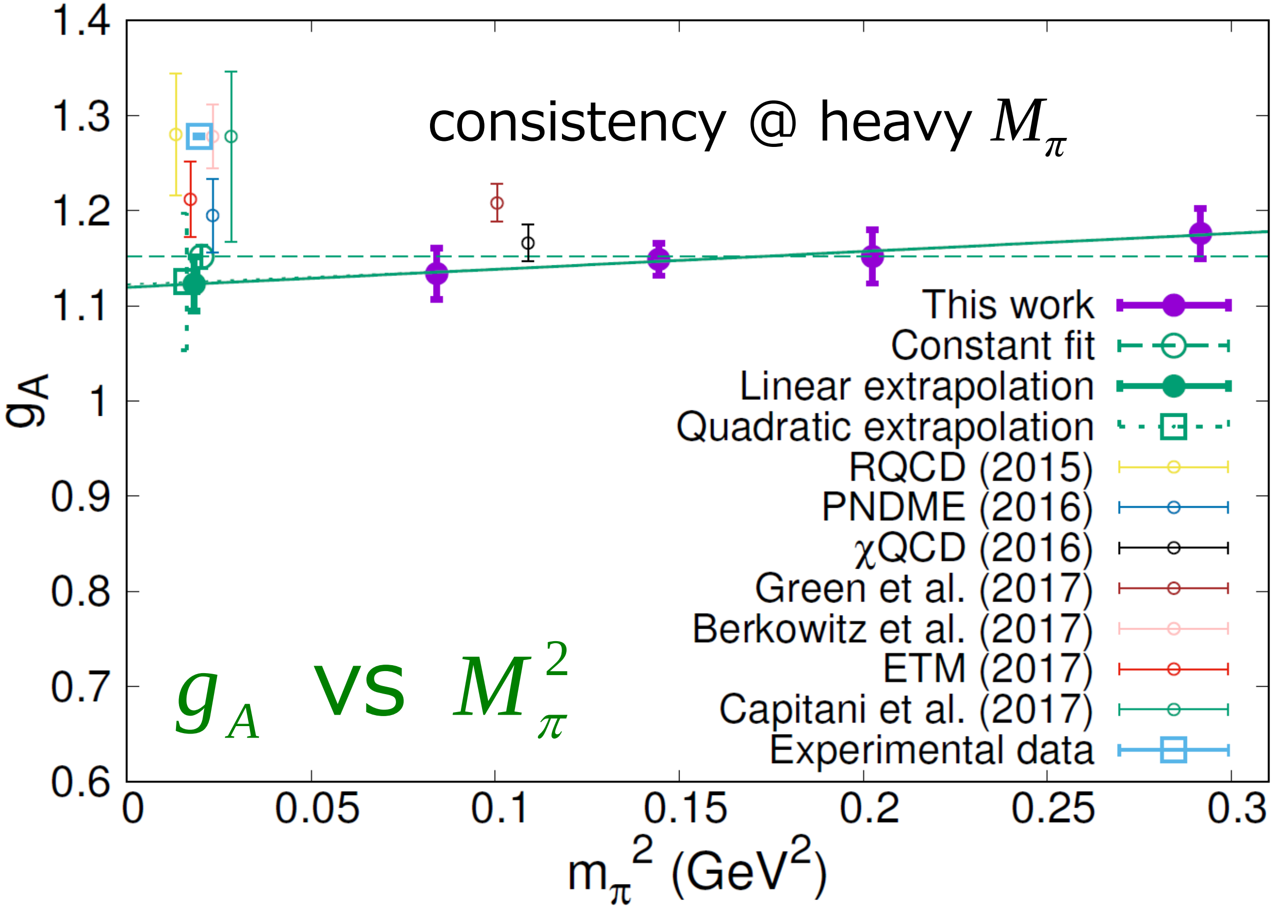
Yamanaka et al. (JLQCD) in prep.



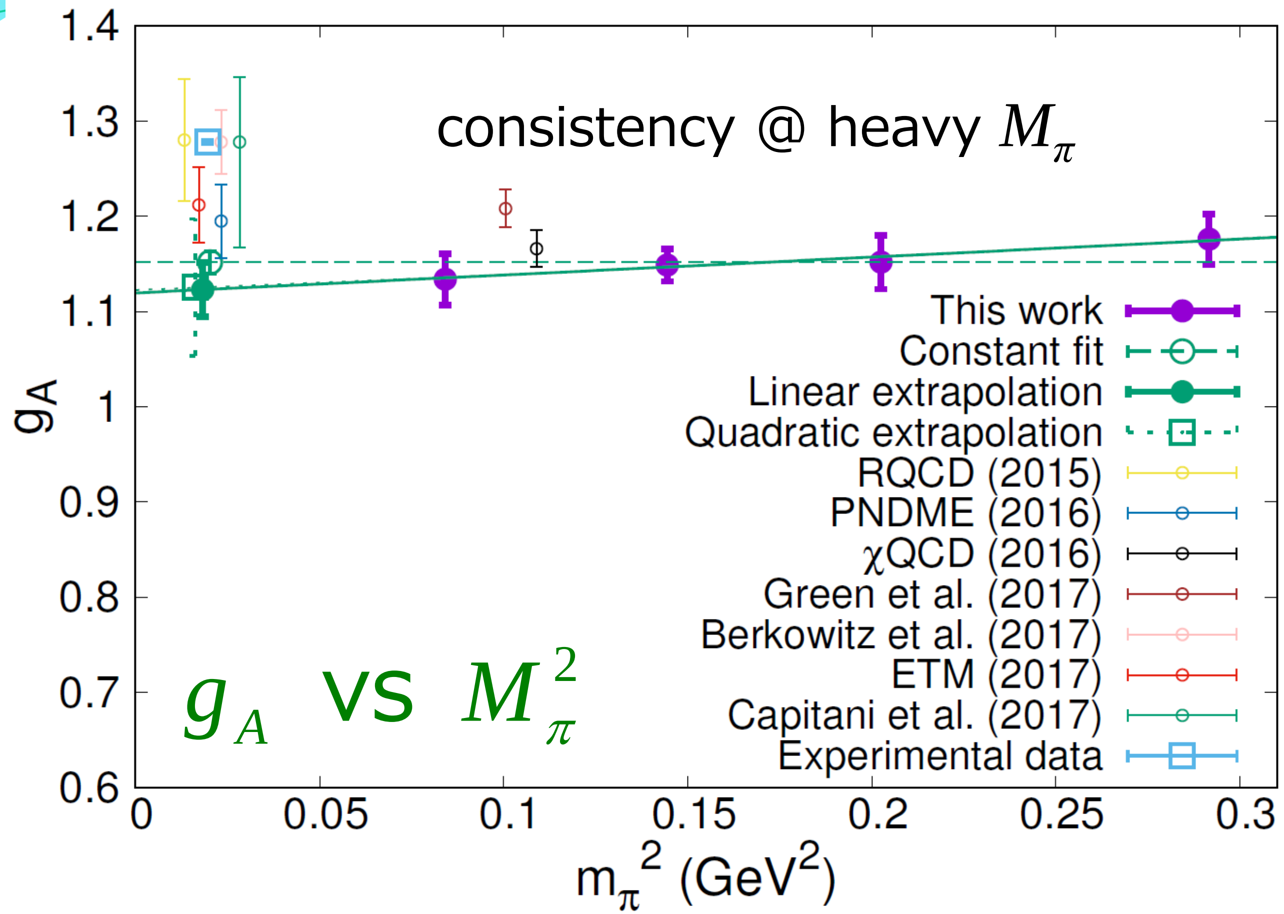
choice of $\Delta t^{(\prime)}$, O_N, fit form $\Leftrightarrow$ target accuracy of g_A

chiral extrapolation



mild M_π dependence $\Rightarrow$ test polynomial extrapolations

chiral extrapolation



constant fit

χ^2/dof $\sim$ 0.4

g_A = 1.15(1)

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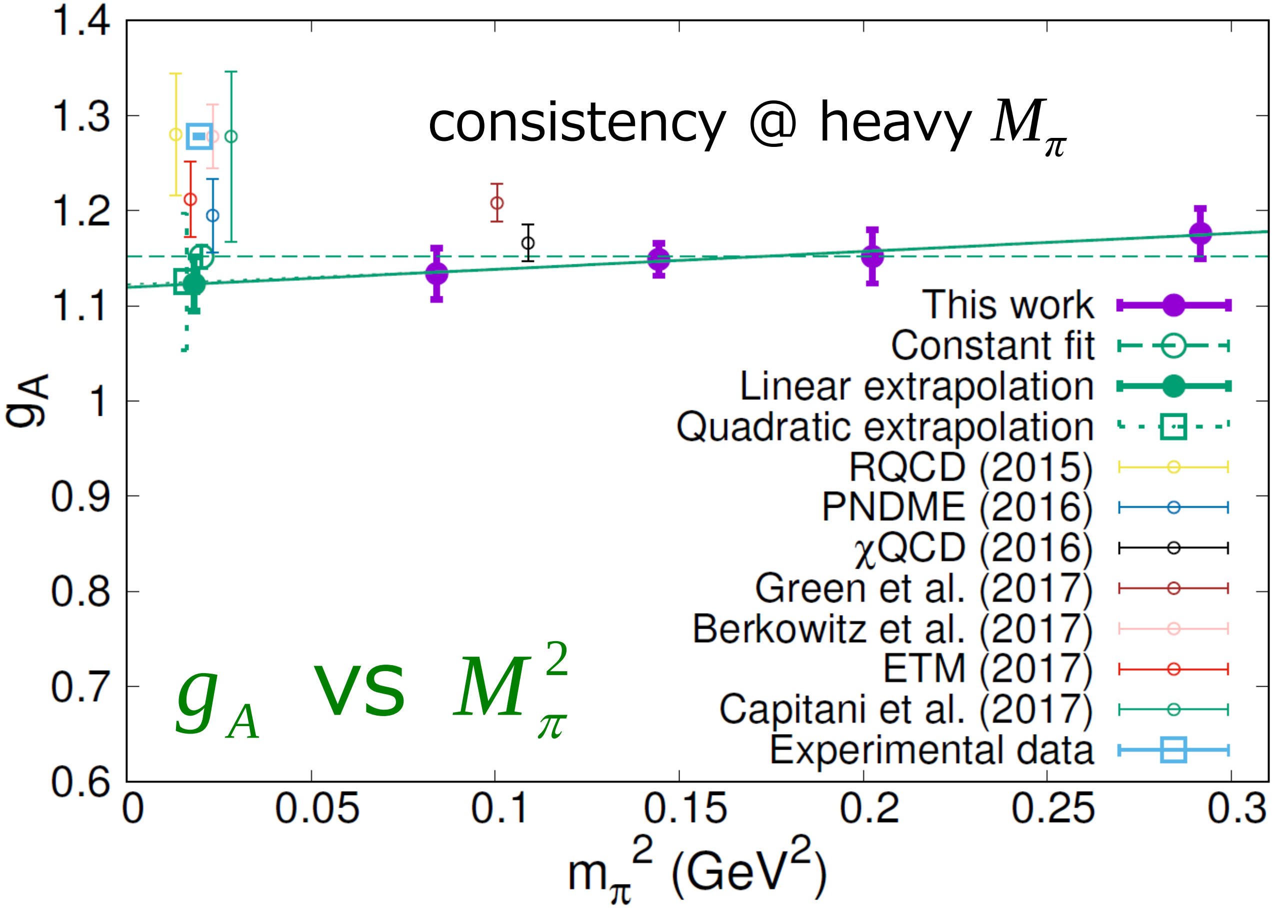

constant fit

χ^2/dof $\sim$ 0.4

g_A = 1.15(1)

linear c_1 = 0.2(0.2)

χ^2/dof $\sim$ 0.1

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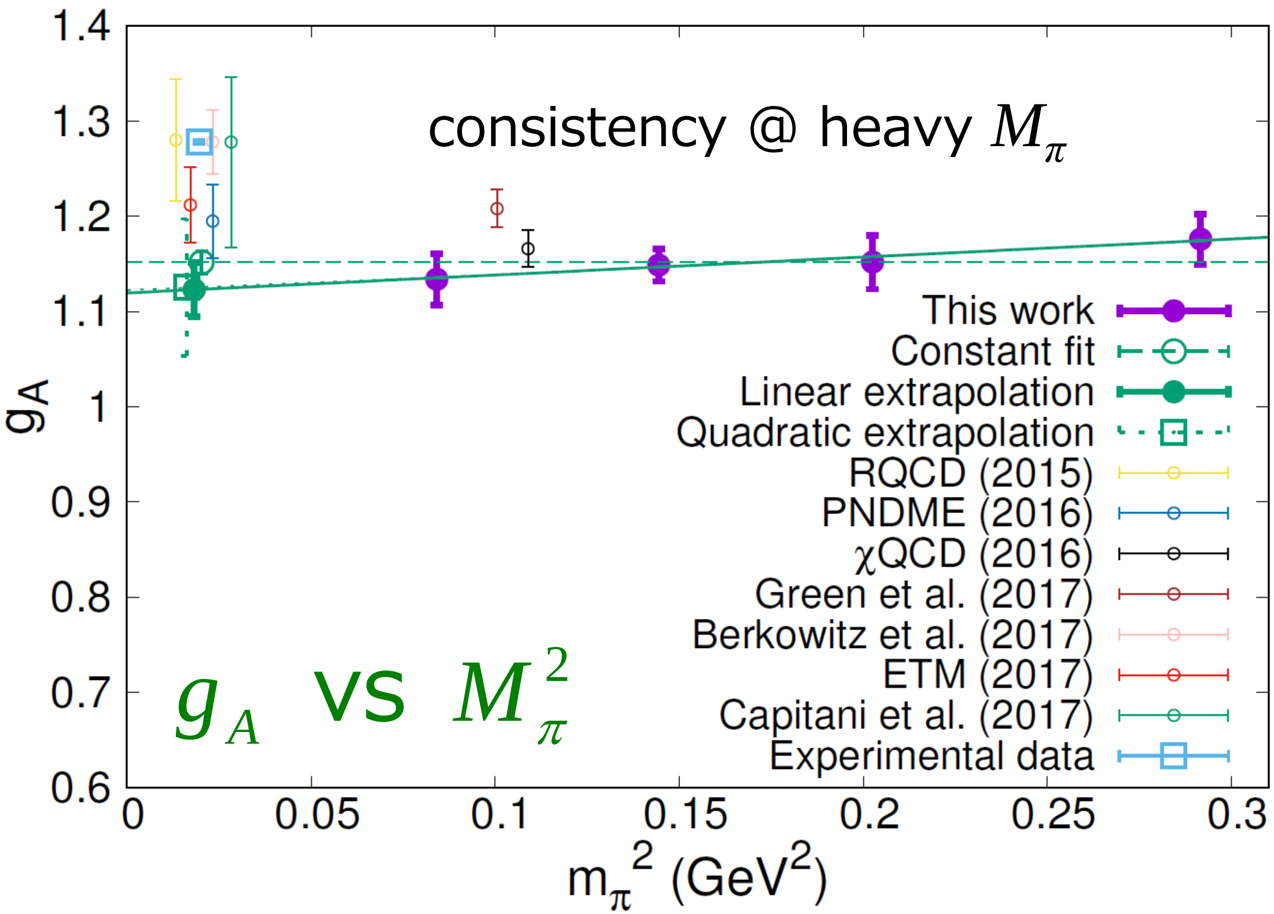

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$\chi^2/\mathrm{dof} \sim 0.1$

$g_A = 1.13(7)$

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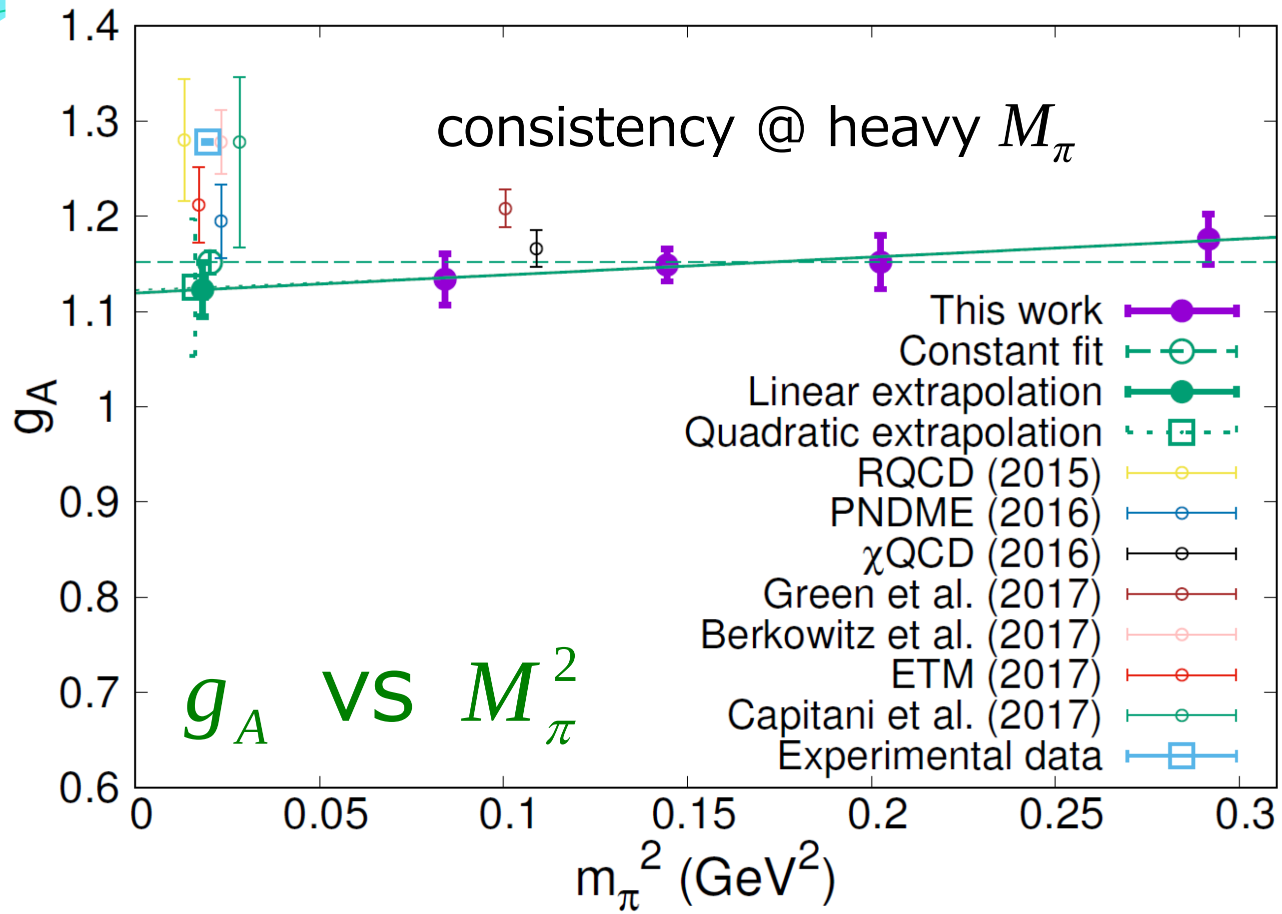

constant fit

χ^2/dof ~ 0.4

g_A = 1.15(1)

linear c_1 = 0.2(0.2)

χ^2/dof ~ 0.1

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mild M_π dependence ⇒ test polynomial extrapolations

$$g_A = 1.13(3)_{stat}\,(3)_{chiral}\,(9)_{a\neq 0} \quad \Leftrightarrow \quad 1.278(2)$$

chiral perturbation theory (ChPT)

effective theory of QCD based on chiral symmetry

- meson ChPT (Gasser-Leutwyler, '82) : work reasonably @ $M_\pi \leq$ 500MeV

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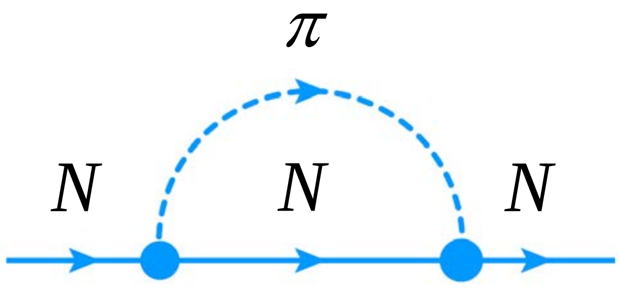
  - expansion in hadron momenta, mass

    $p \sim M_\pi, M_K \cdots$ OK, M_N ???



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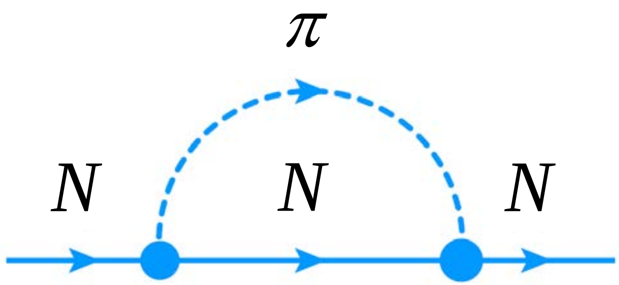

  - predict functional form w/ free parameters + chiral logarithm

$$M_N = M_{N,0}\left(1 + c_2 M_\pi^2 + c_3 M_\pi^3 + c_4 M_\pi^4 \ln\left[M_\pi^2\right] + \cdots\right)$$

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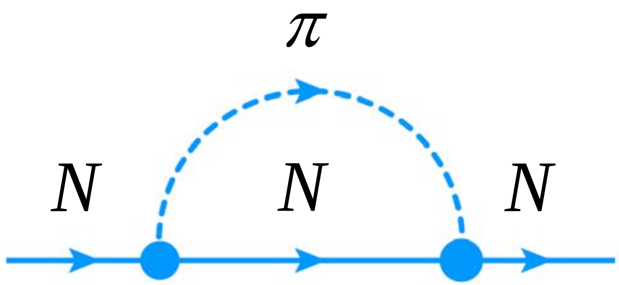

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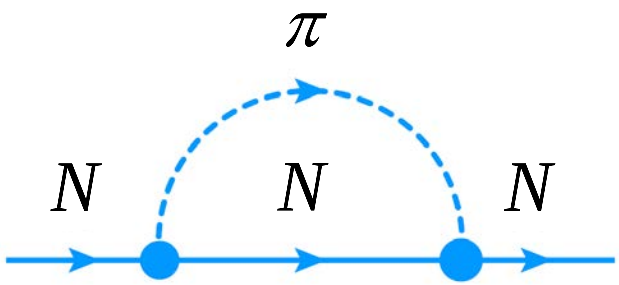

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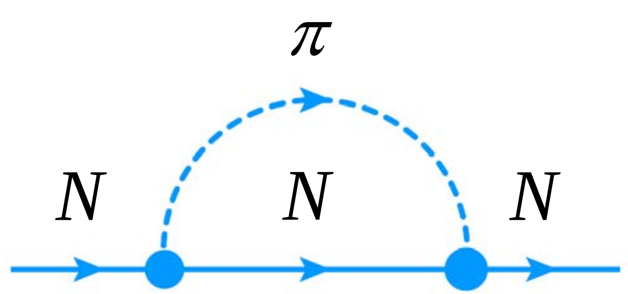

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1 + 0.34 - 0.35 + 0.24 + ⋯

- generally have slow convergence $p \ni M_N \Leftrightarrow p^2 \not\ni M_N^2$

chiral extrapolation based on ChPT

Bijnens et al. '85
Jenkins-Manohar '91
Detmold et al. '02

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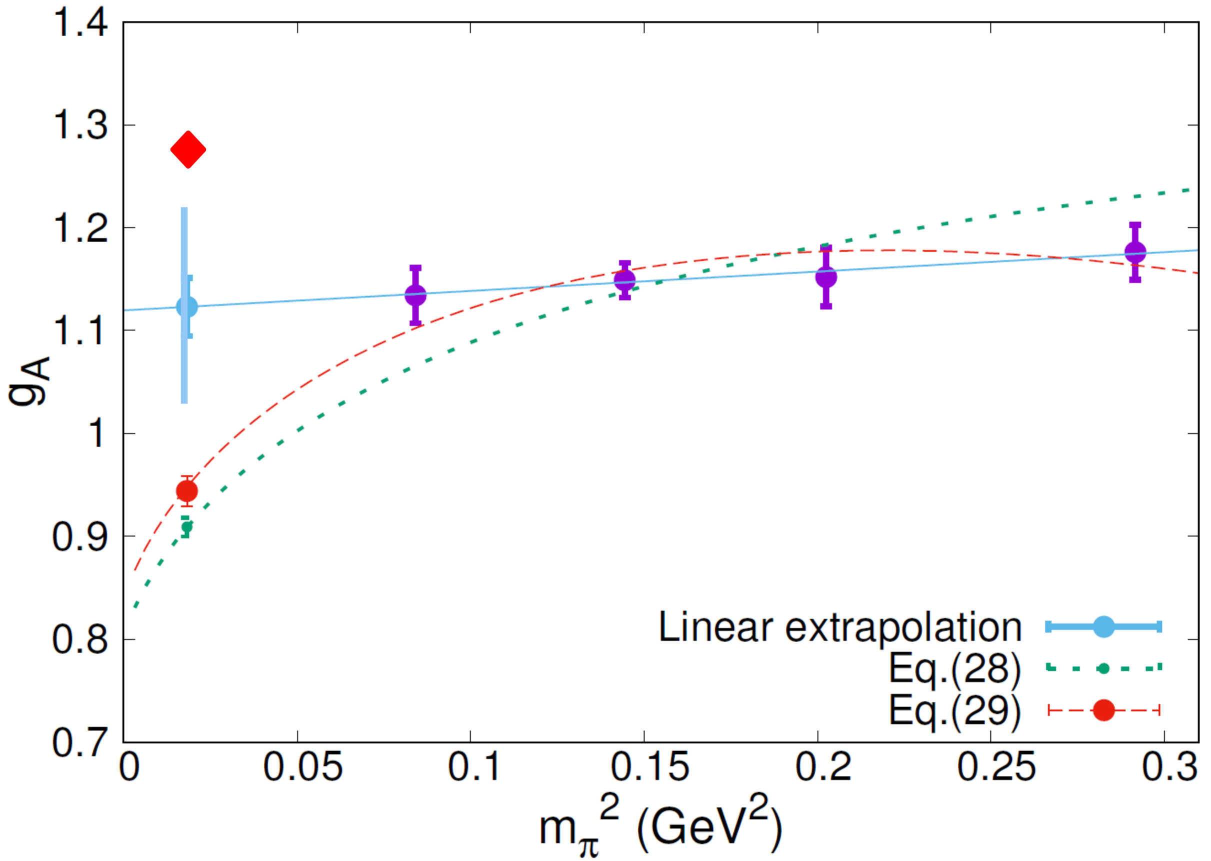

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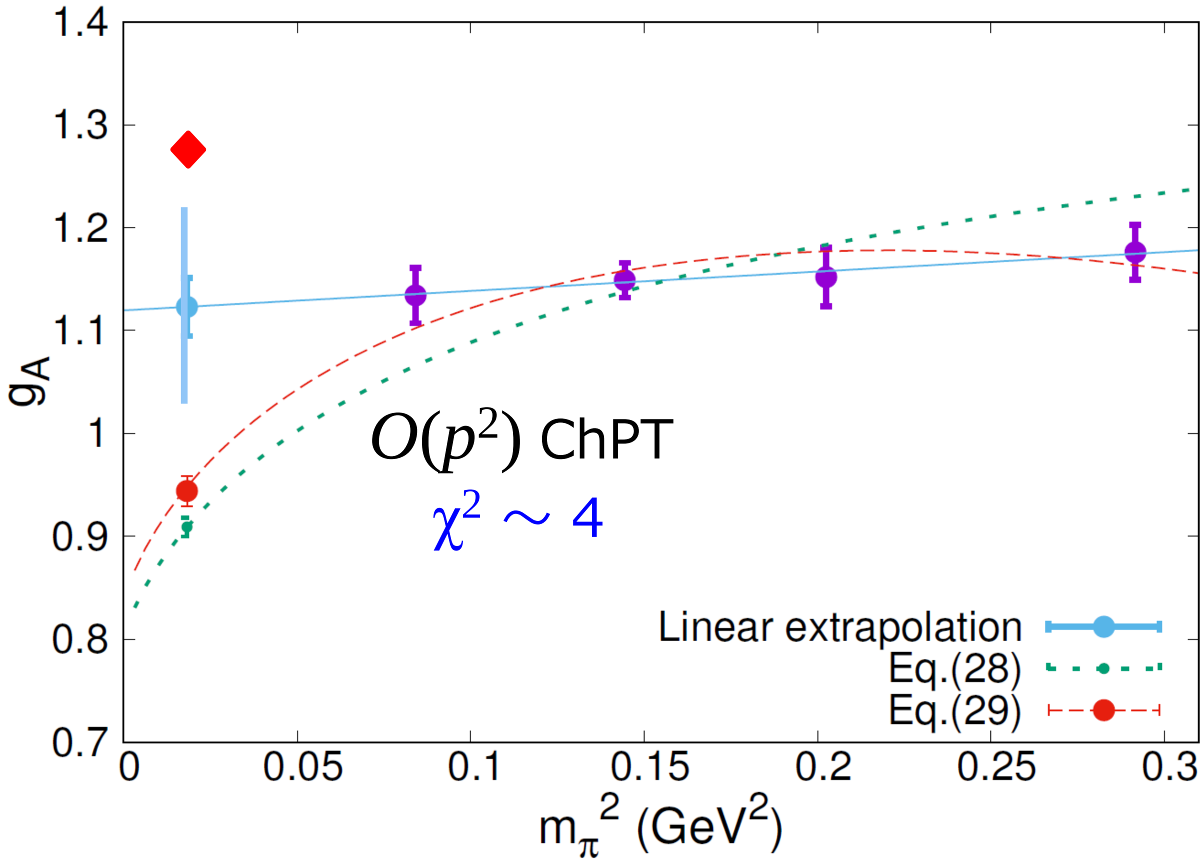

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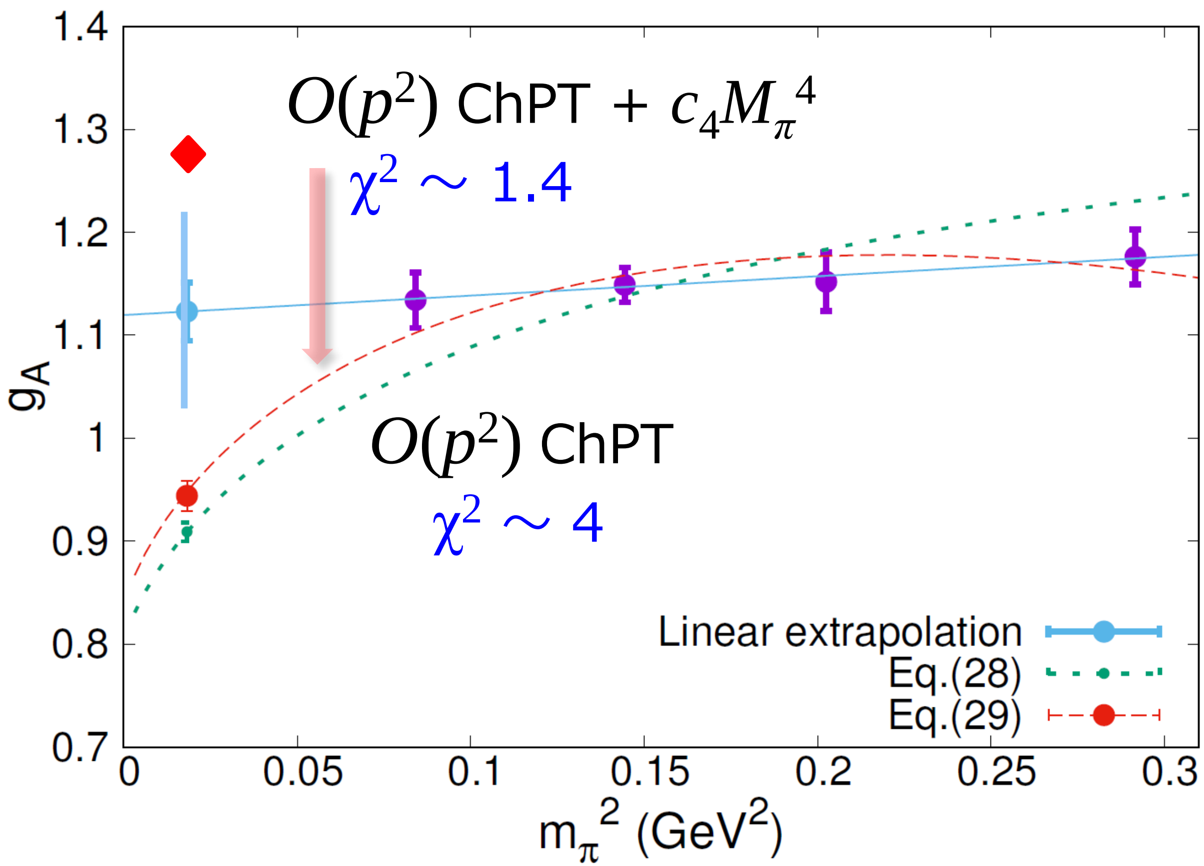

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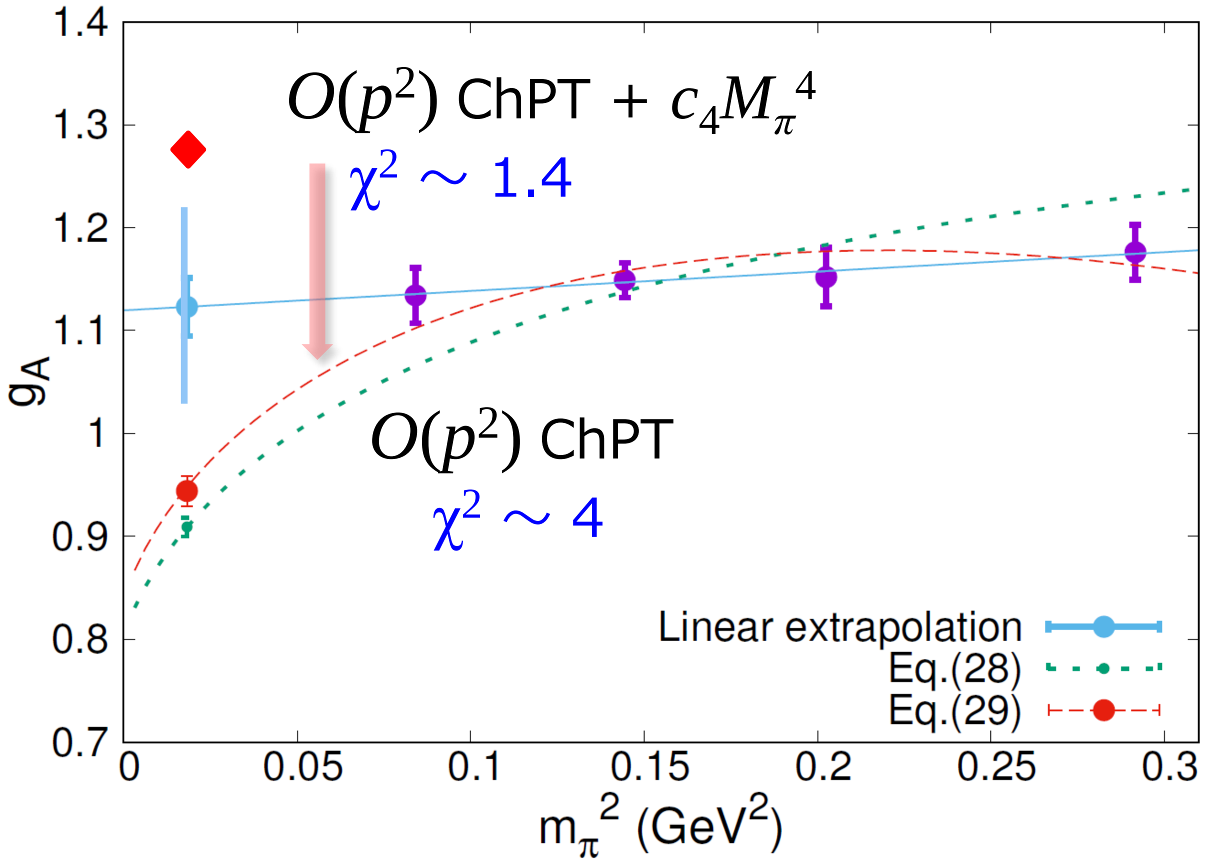

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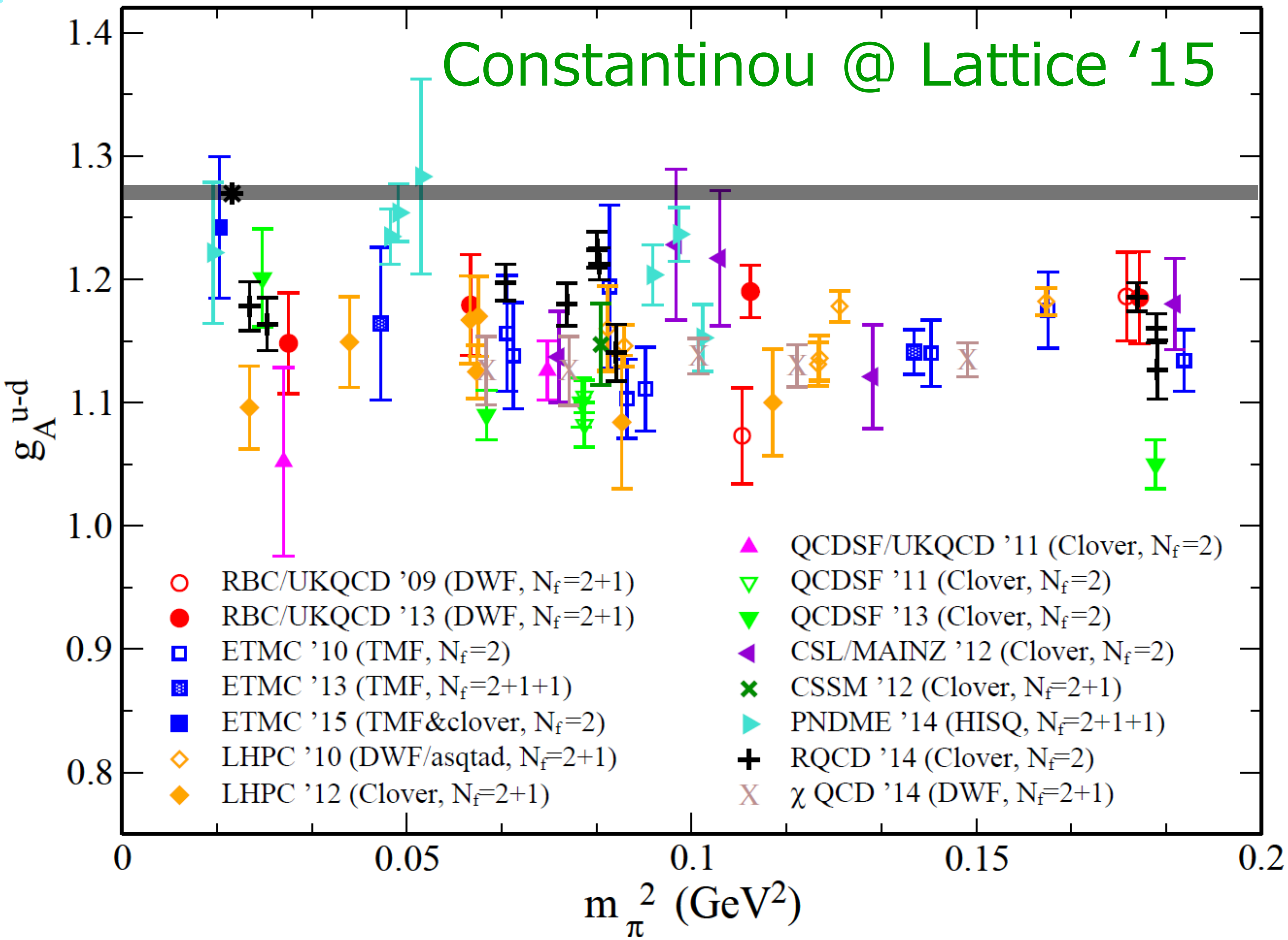
large corrections @ simulated M_π's ⇒ ~~lattice and expr't data~~

recent post-diction of g_A

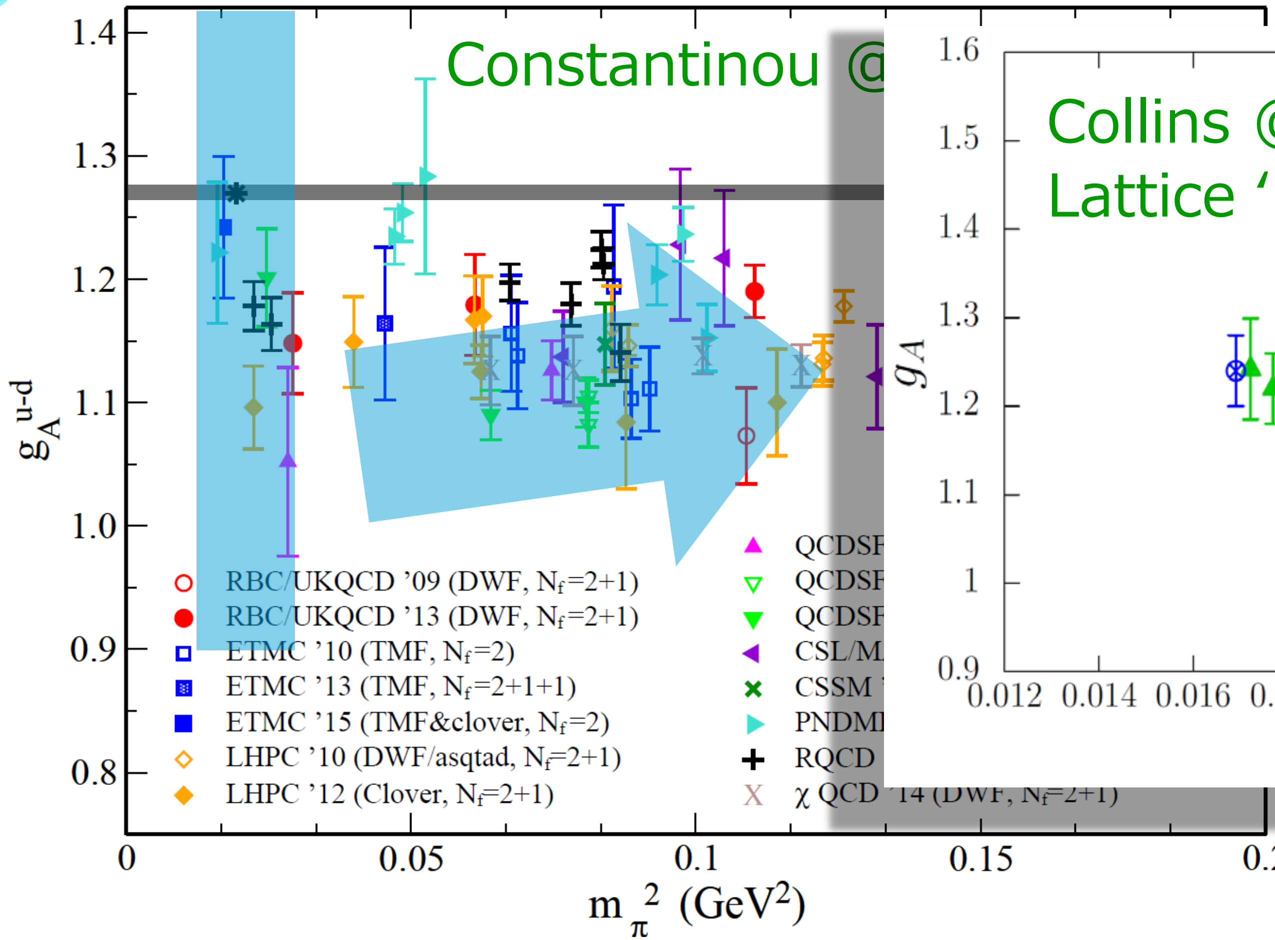


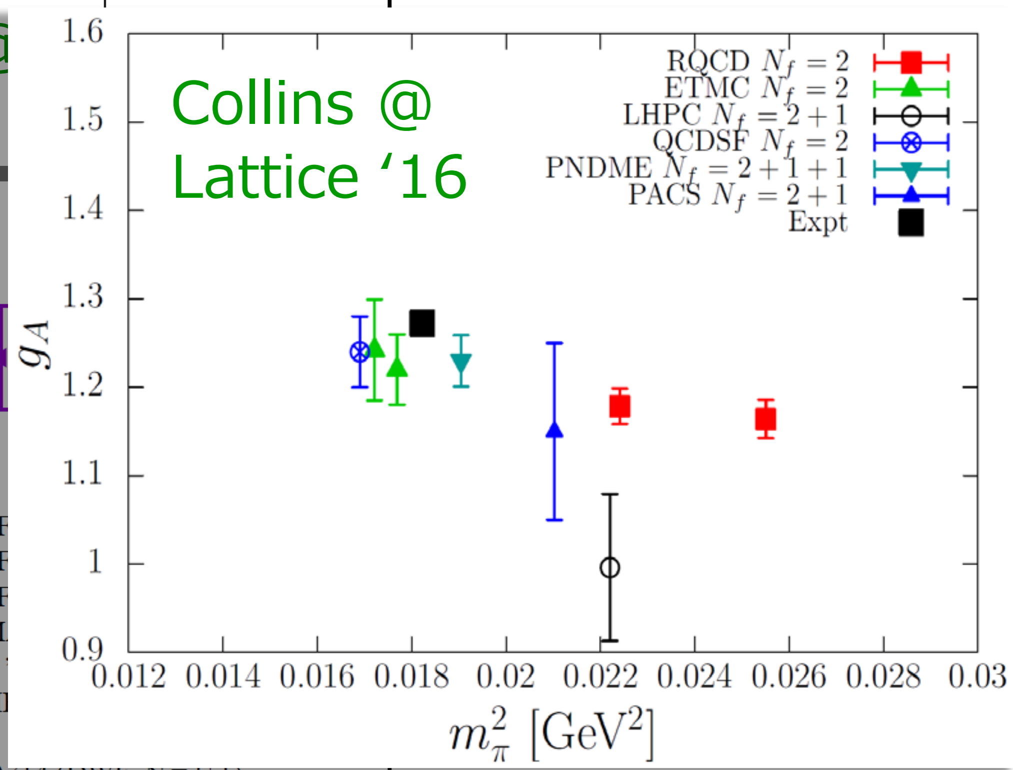
- effort over a decade

  ⇒ validation of LQCD

- systematically smaller?

recent post-diction of g_A





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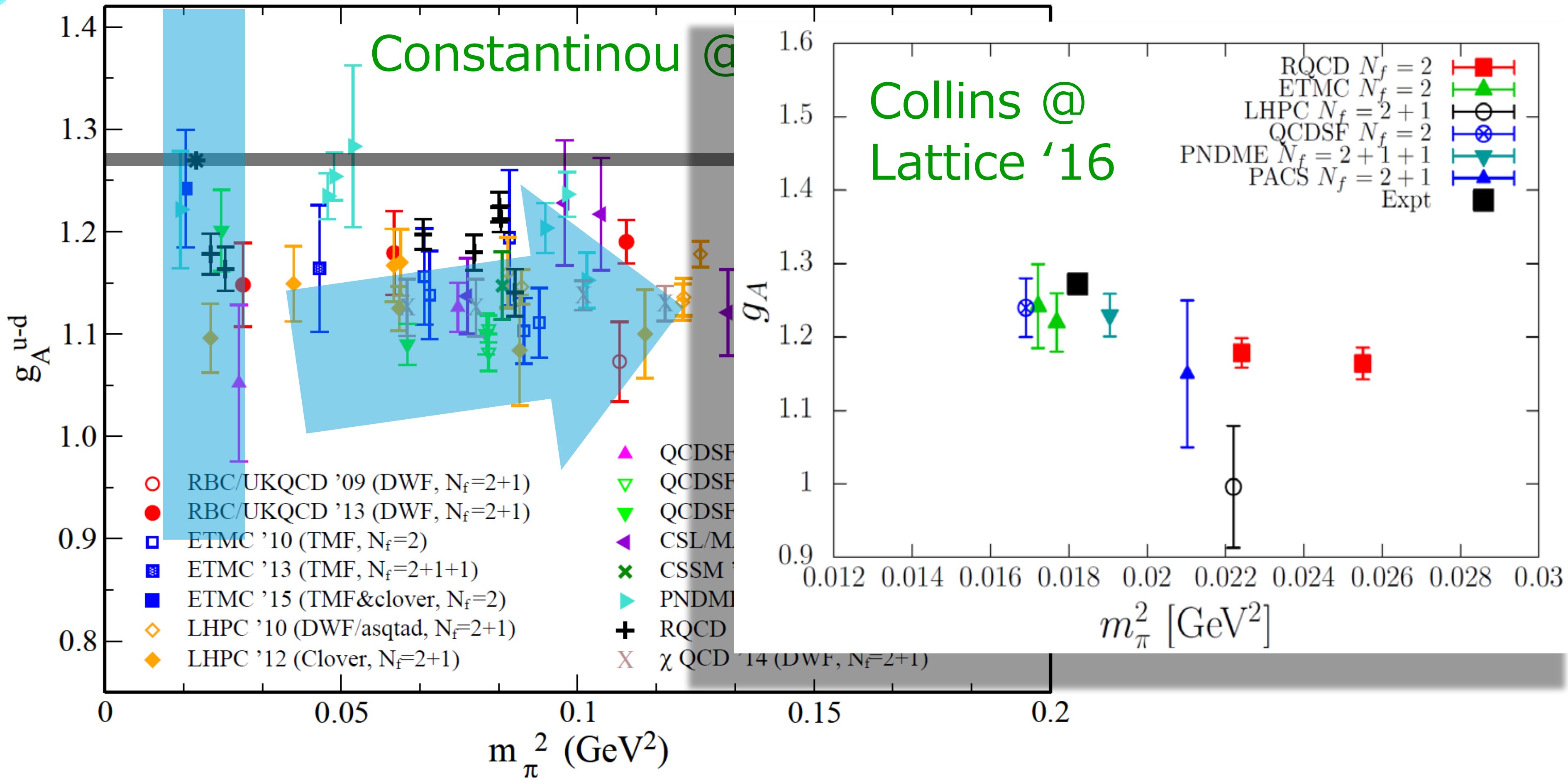

Chang et al. @ Lat'17: g_A = 1.285(17) 1% ∋ chiral, finite V 0.2%

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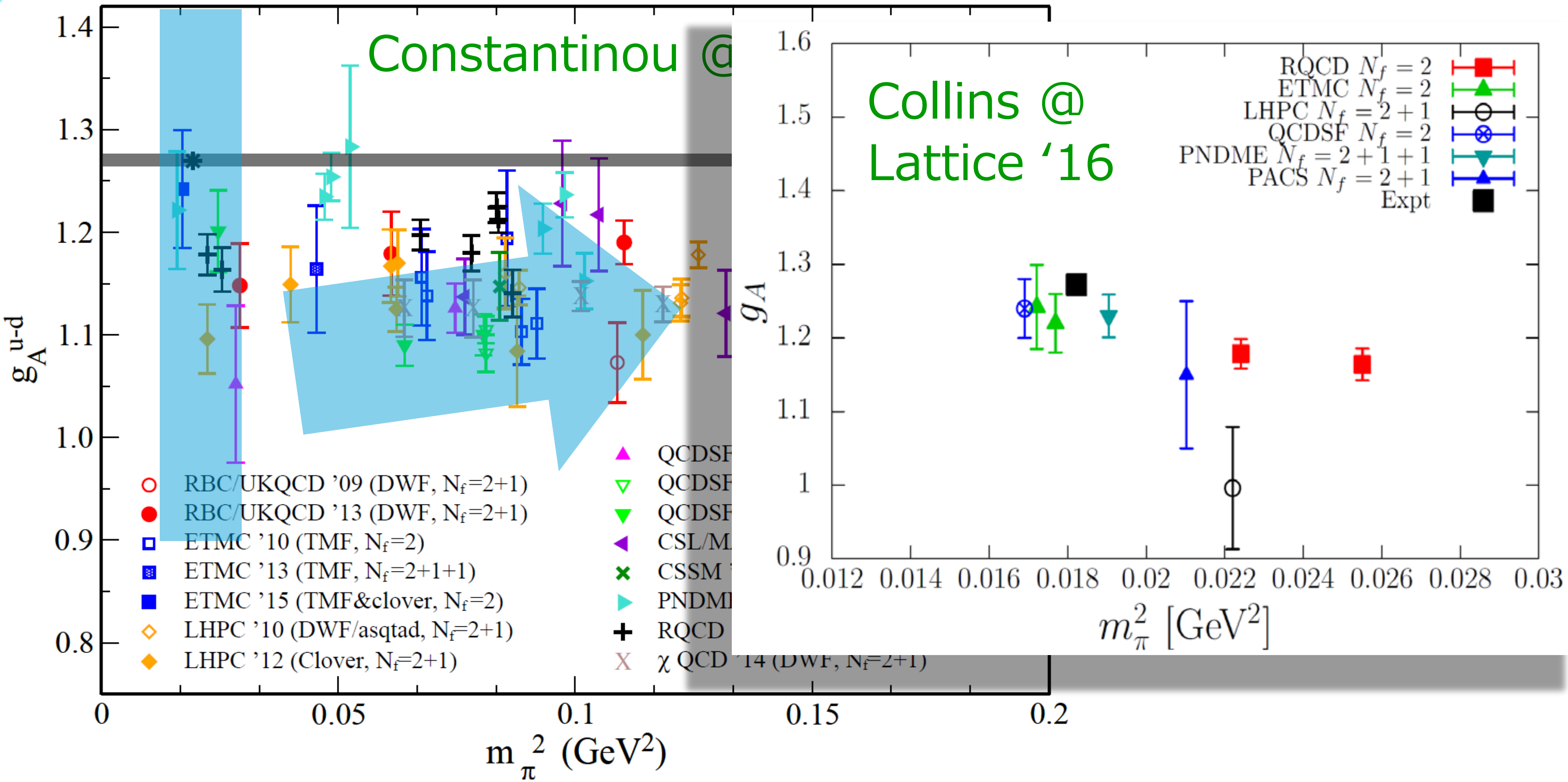

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very encouraging, systematics to be studied more extensively

isovector scalar and tensor charges

$g_s = 0.88(8)_{\text{stat}}(3)_{\text{chiral}}(7)_{a\neq 0}$ $g_T = 1.08(3)_{\text{stat}}(3)_{\text{chiral}}(9)_{a\neq 0}$

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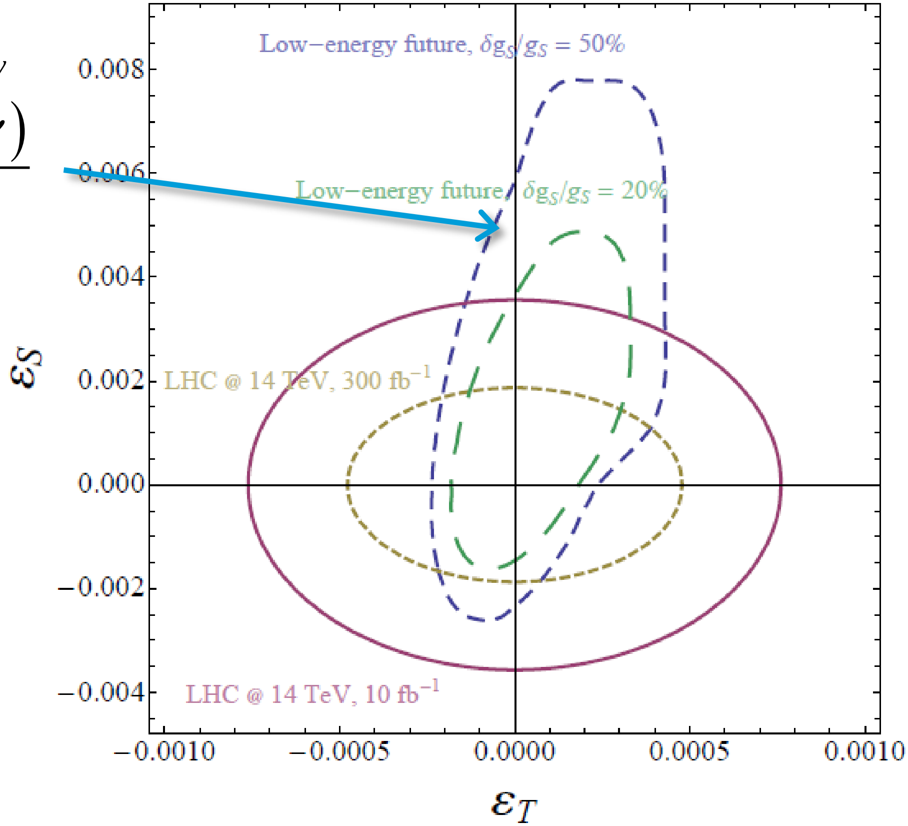
constraint from β decay of ultra cold nertron (3mK!)

0.1% for b, b_ν

$$\frac{d\Gamma(n \to pe\nu)}{dE_e d\Omega_e d\Omega_\nu}$$

Bhattacharya et al. '11



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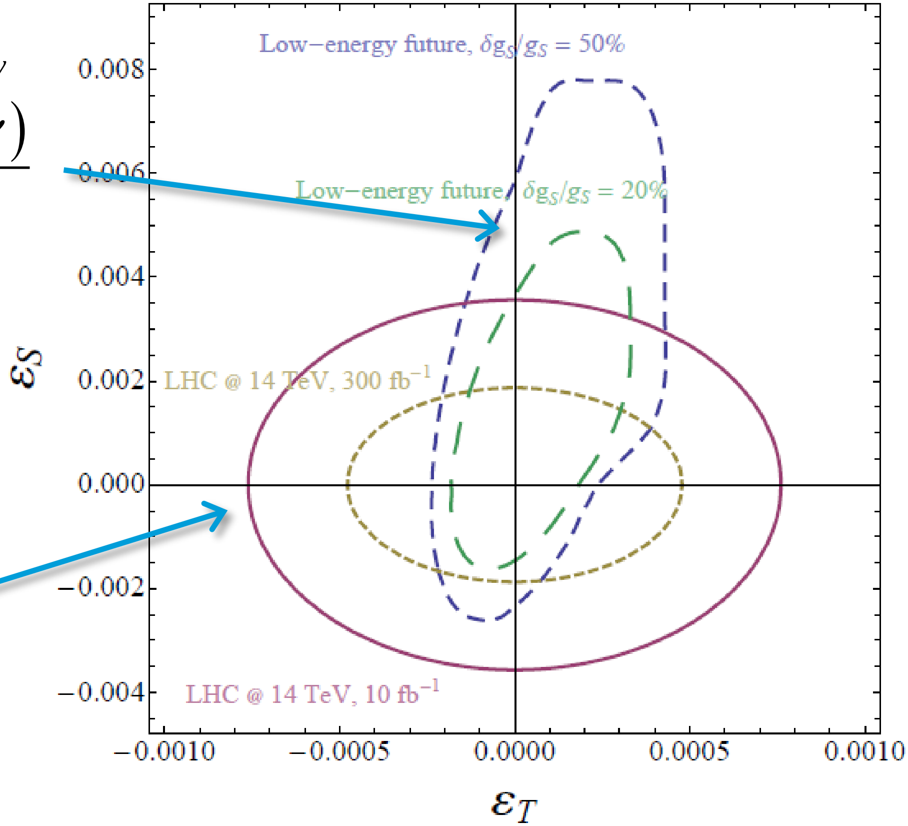
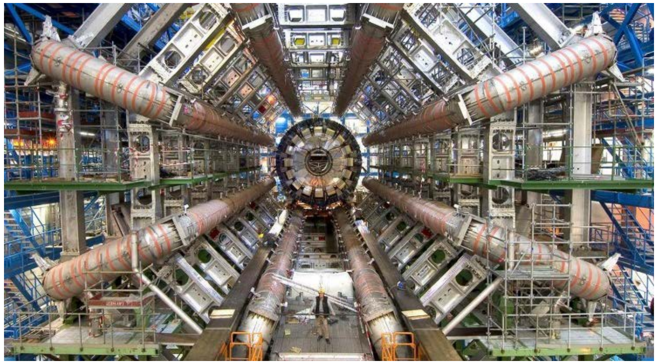


constraint from LHC (14TeV!)

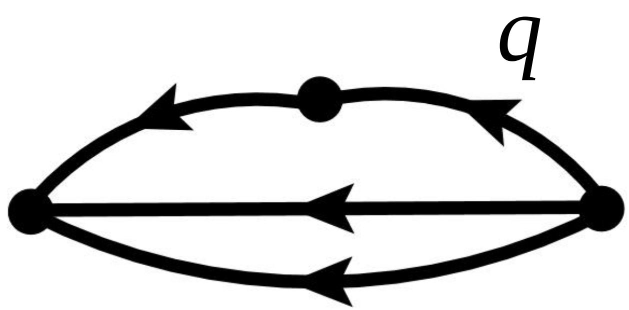


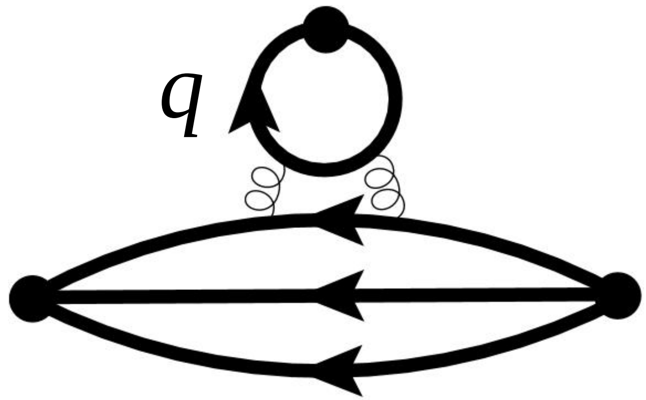
synergy of low- and high-energy experiments for NP search

3.
isoscalar uu+dd,
each flavor uu, dd, ss

disconnected diagram

e.g. light quark charge $\bar{u}u+\bar{d}d$, $\bar{u}u$, $\bar{d}d$

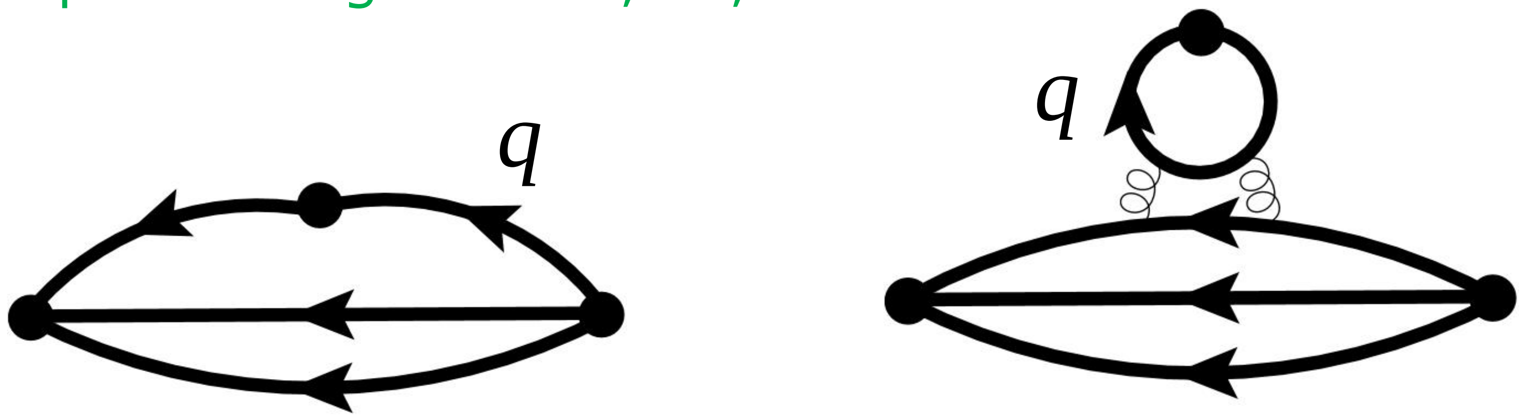




- generally present! ⇔ $\langle N | \bar{u}\Gamma u - \bar{d}\Gamma d | N \rangle$ in isospin limit
- Zweig rule suppressed : small for light-quark charges?
- strange quark charges S_s, Δ_s, δ_s : purely disconnected

disconnected diagram

e.g. light quark charge $\bar{u}u+\bar{d}d$, $\bar{u}u$, $\bar{d}d$



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- expensive to calculate $\Rightarrow$ stochastic methods ($\rightarrow$ backup slide)

  momentum projection $\mathbf{p}_N=0$ $\Rightarrow$ sum over vertex point

  $\Rightarrow$ quark loops at arbitrary lattice sites!

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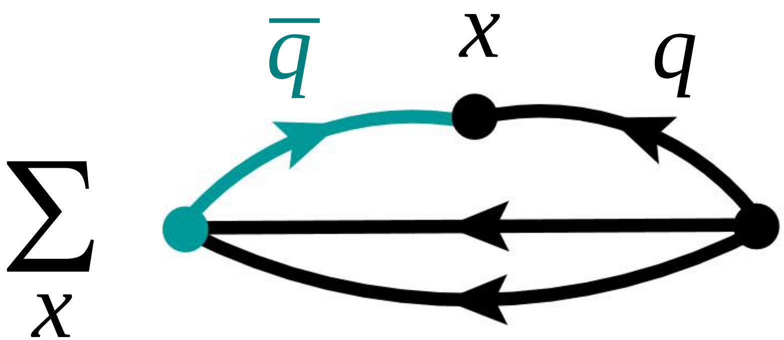

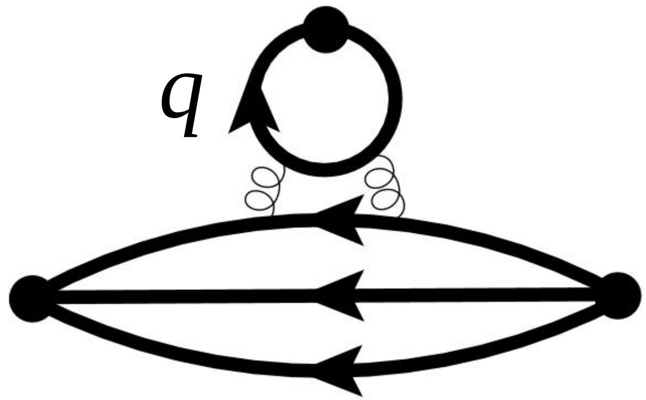

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renormalization

chiral symmetry greatly simplifies renormalization

e.g. renormalization of $\bar{s}s$

$$\left(\bar{s}s\right)_{\text{renorm}} = \frac{1}{2}\left[Z_s\left(\bar{s}s\right)_{\text{bare}} + Z_{ud}\left(\bar{u}u + \bar{d}d\right)_{\text{bare}} + \frac{Z_1}{a^3}\right]$$

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"operator mixing" w/ $\bar{u}u+\bar{d}d$

- potentially dangerous : $\bar{s}s \ll \bar{u}u+\bar{d}d$
- vanish for scalar, tensor in mass-independent scheme

$$Z_{ud} = Z_0 - Z_8, \quad \left(\bar{q}q\right)_{\text{renorm}} = Z_0\left(\bar{q}q\right)_{\text{bare}}, \quad \left(\bar{q}\lambda^8 q\right)_{\text{renorm}} = Z_8\left(\bar{q}\lambda^8 q\right)_{\text{bare}}$$

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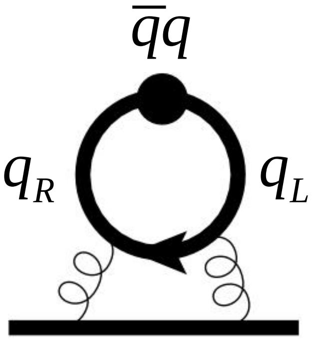

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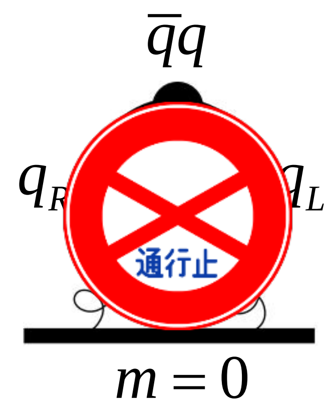
$$Z_{ud} = Z_0 - Z_8, \quad (\bar{q}q)_{\text{renorm}} = Z_0(\bar{q}q)_{\text{bare}}, \quad (\bar{q}\lambda^8 q)_{\text{renorm}} = Z_8(\bar{q}\lambda^8 q)_{\text{bare}}$$



renormalization

chiral symmetry greatly simplifies renormalization

e.g. renormalization of $\bar{s}s$

$$(\bar{s}s)_{\text{renorm}} = \frac{1}{2}\left[Z_s (\bar{s}s)_{\text{bare}} + Z_{ud} \left(\bar{u}u + \bar{d}d\right)_{\text{bare}} + \frac{Z_1}{a^3} \right]$$

"operator mixing" w/ $\bar{u}u+\bar{d}d$

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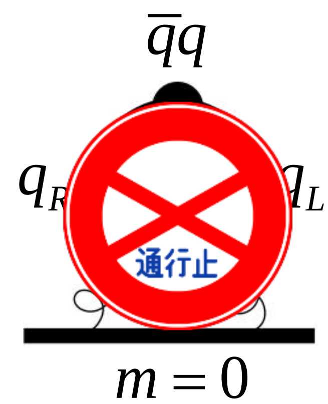

mixing w/ $1/a^3$

- only for scalar, but disaster $1/a^3 \to \infty \ (a \to 0)$
- forbidden by chiral symmetry

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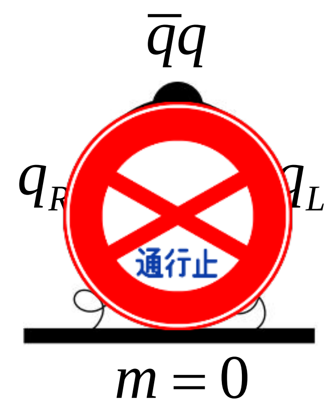

mixing w/ $1/a^3$

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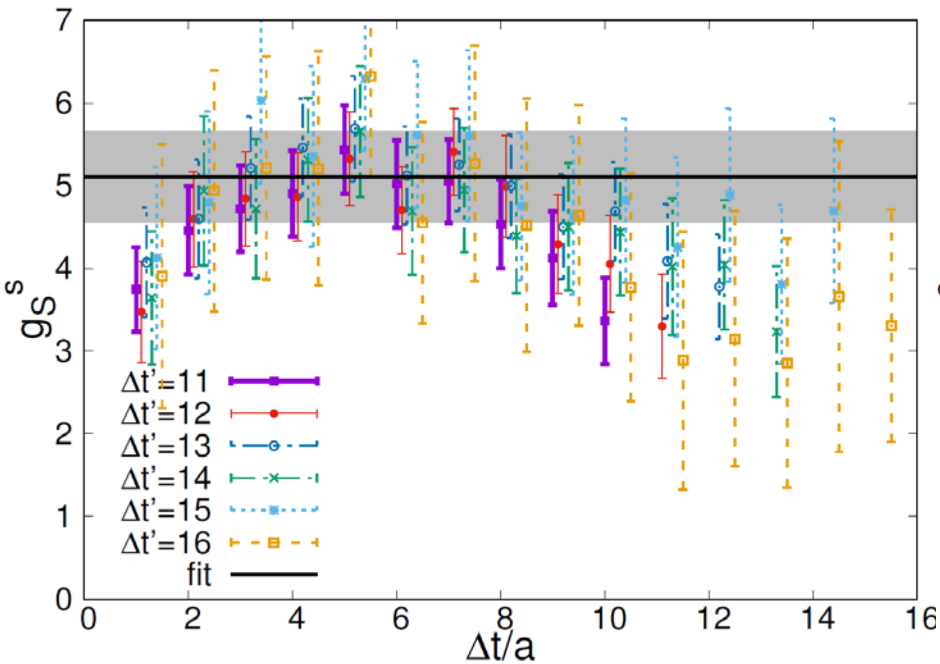
better control of renormalization

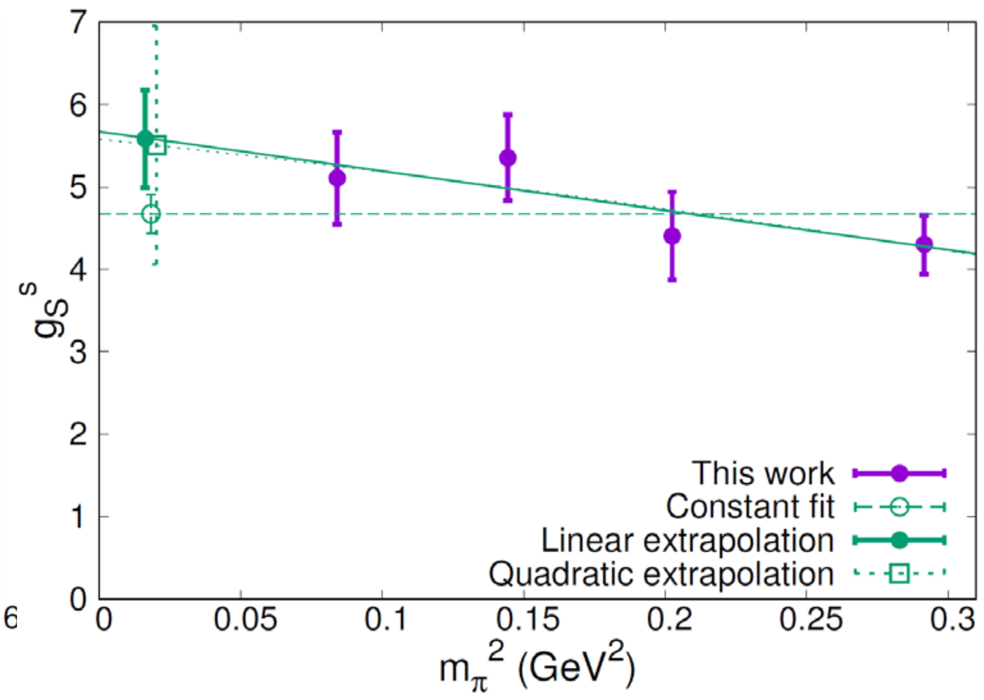
isoscalar charges, uu, dd, ss

analysis similar to g_A

3pt function → charge

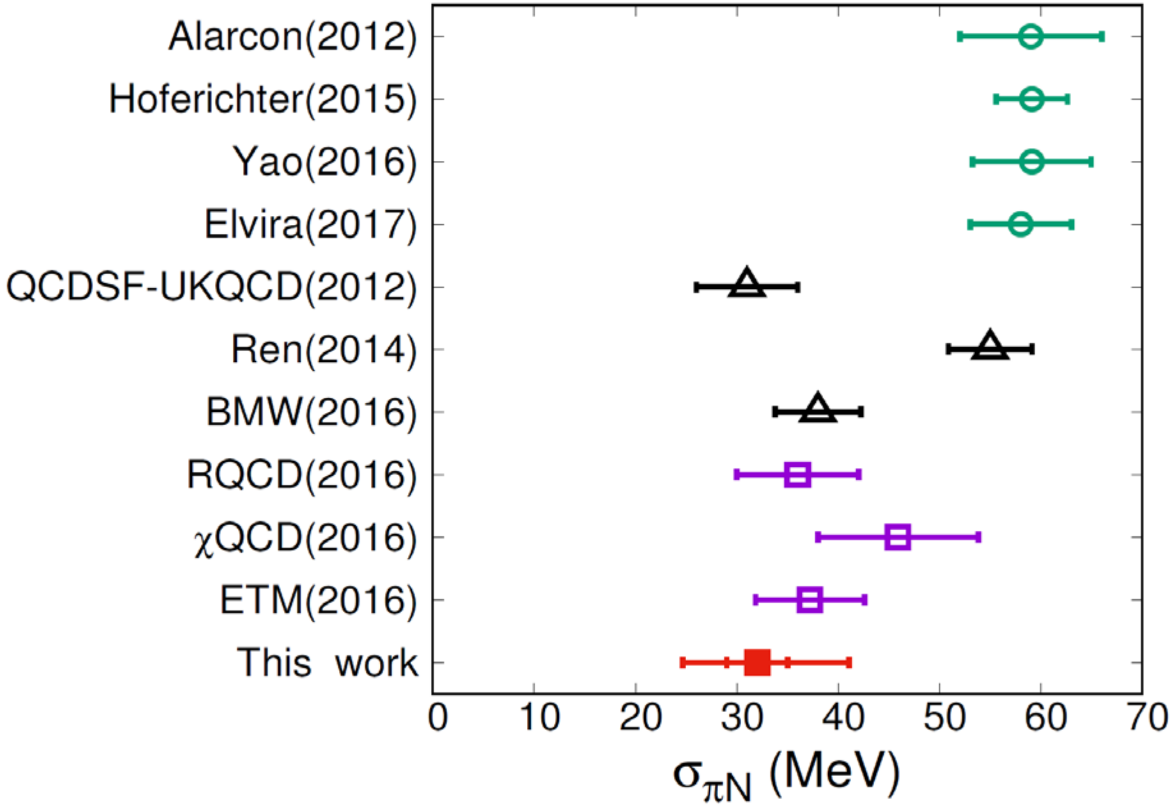


chiral extrapolation



- noisy disconnected diagrams → larger uncertainties
- mild M_π dependence ⇒ poorly described by one-loop ChPT

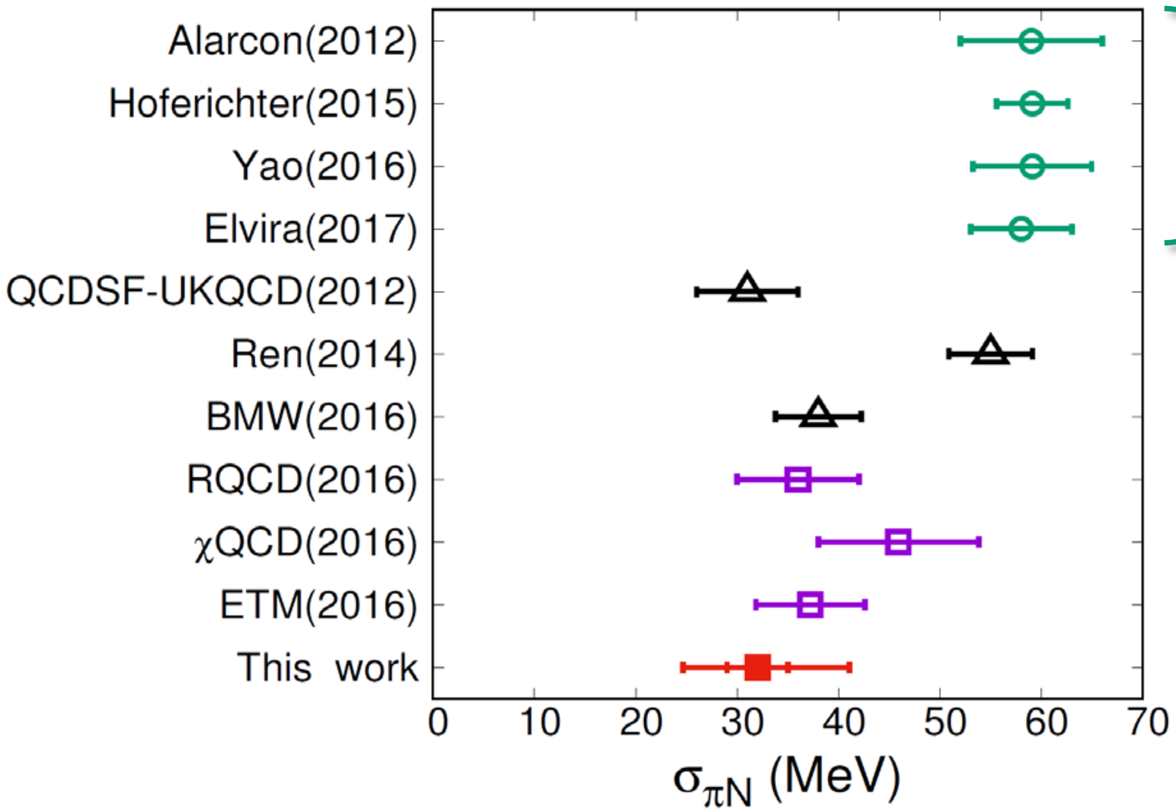
πN σ term



our result ($\overline{\mathrm{MS}}$, 2GeV)

$$\sigma_{\pi N} = m_{ud} \frac{\langle N | \bar{u}u + \bar{d}d | N \rangle}{2M_N} = 32(3)_{\mathrm{stat}} \left(^{+8}_{-6}\right)_{\mathrm{chiral}} (3)_{a\neq 0} \ \mathrm{MeV}$$

πN σ term



phenomenology

$\sigma_{\pi N} \Leftrightarrow T_{\pi N}$, +disp.+ChPT

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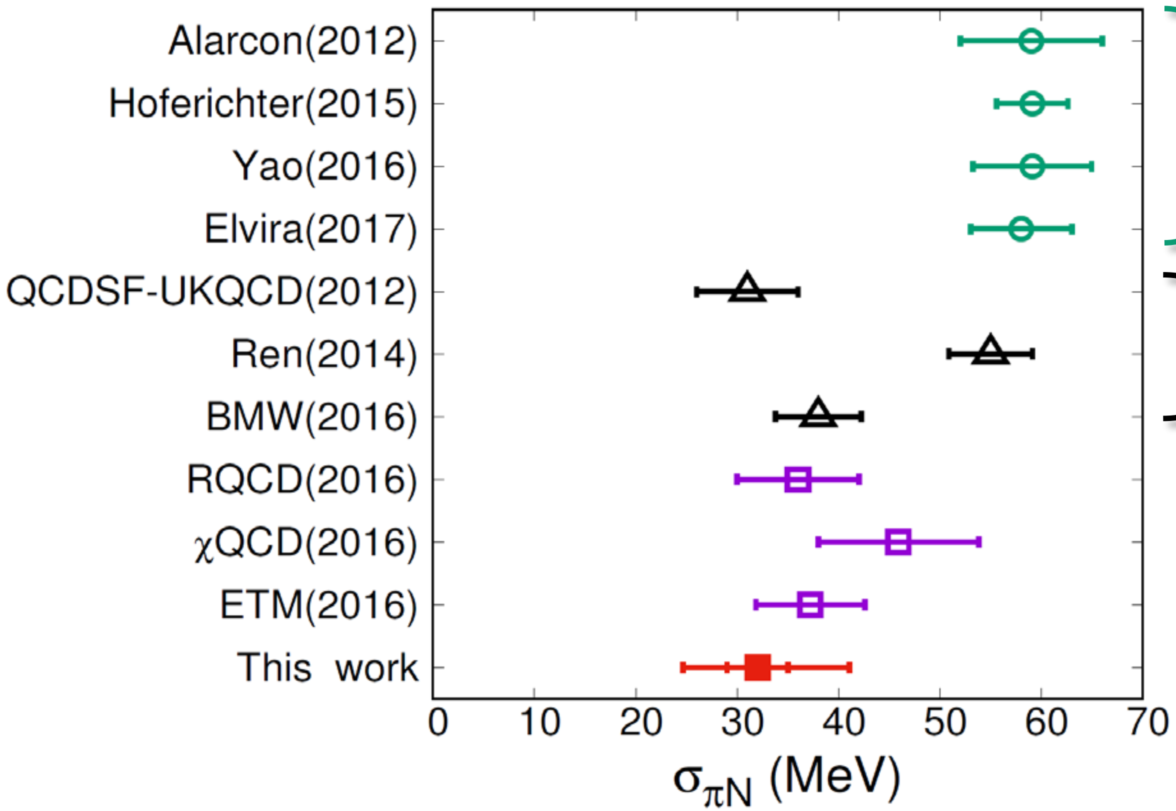

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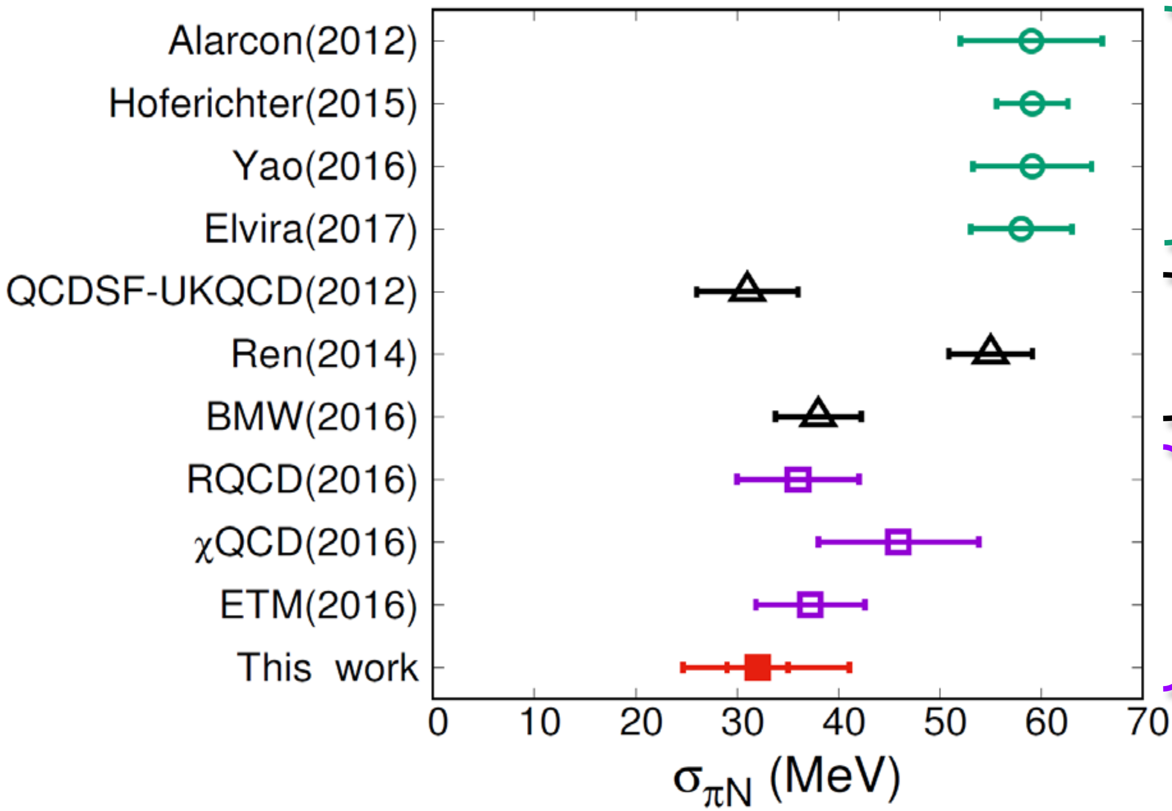

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lattice QCD, direct

realistic simulation"s"

- different S_{QCD}, a, V
- physical M_π

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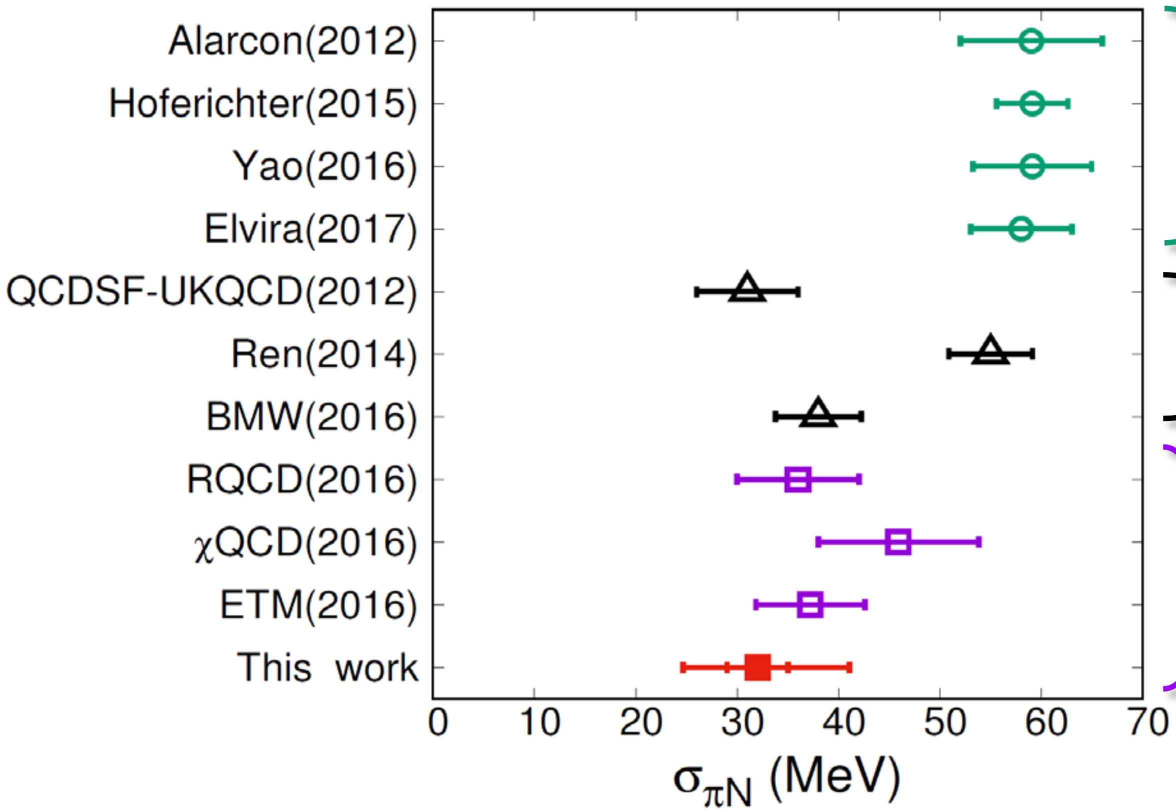

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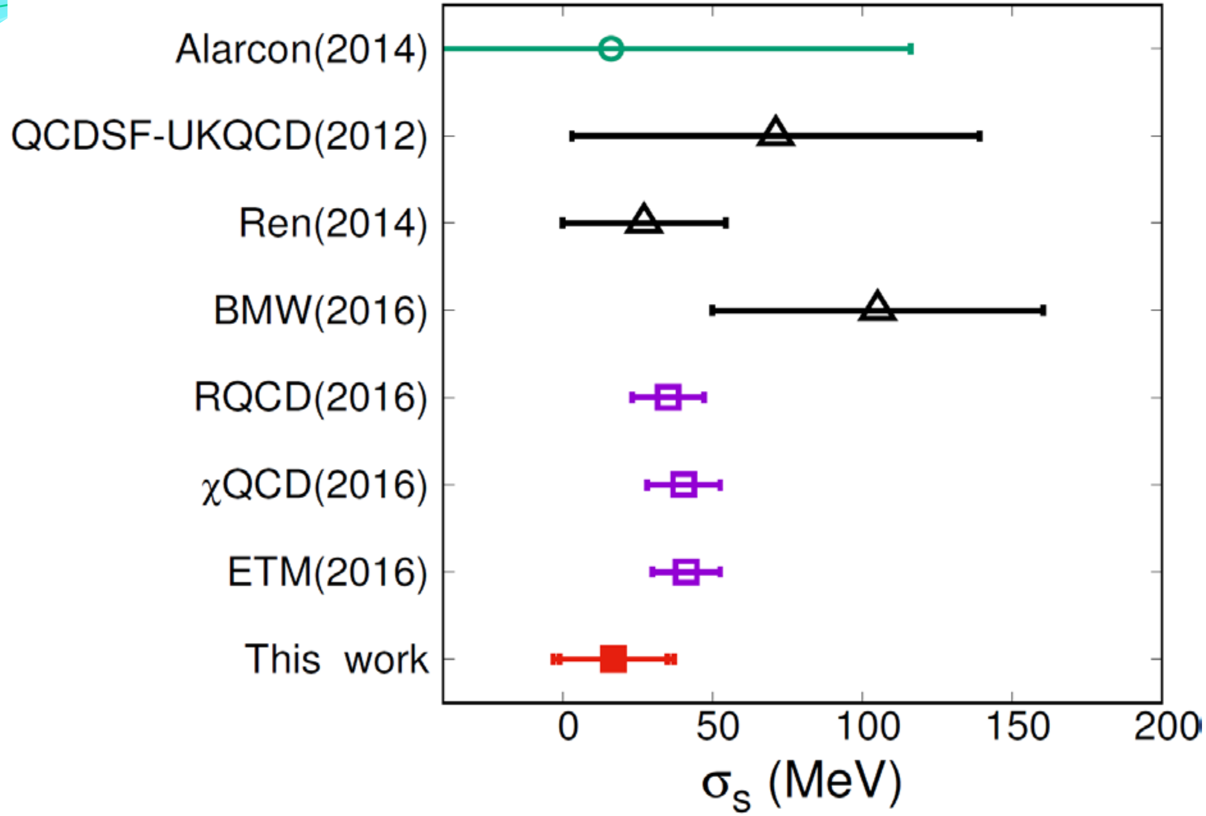
tension? ⇒ ChPT convergence ??? (BChPT, HBChPT, cBChPT)

strange quark content



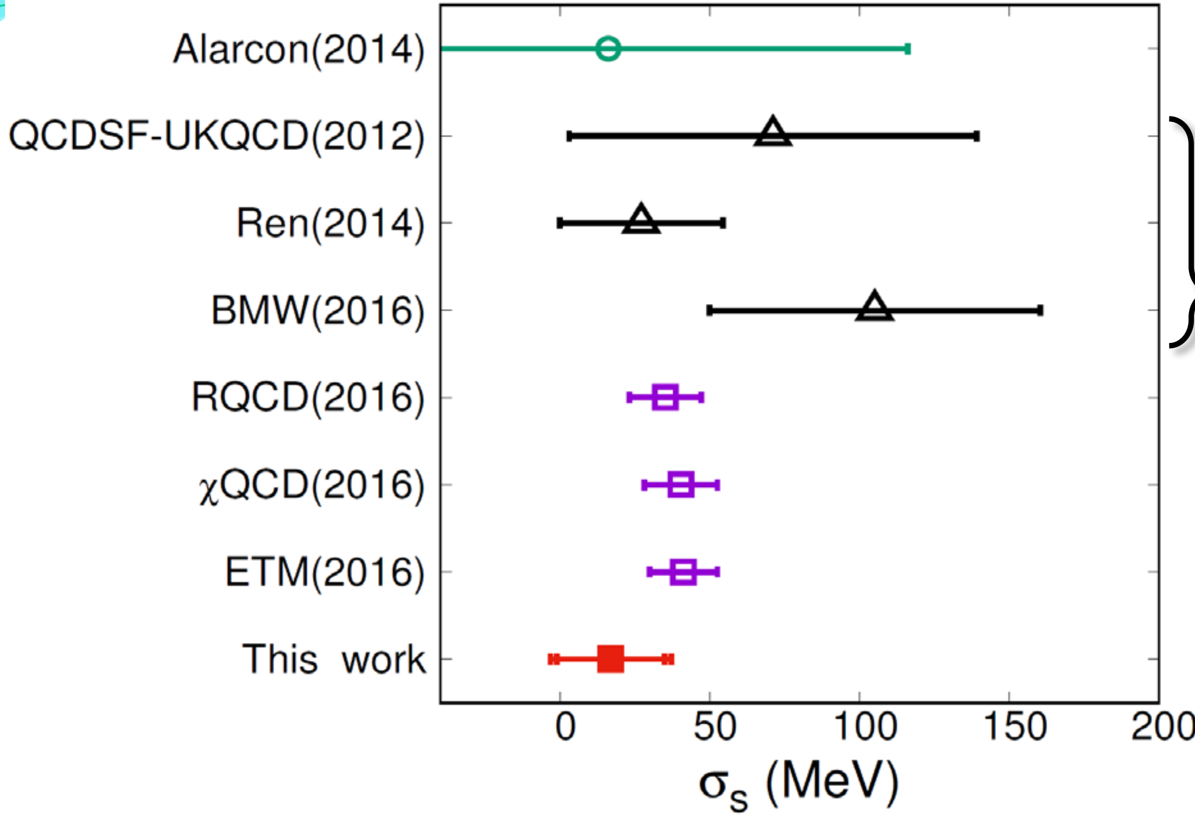
pheno. $\leftarrow \sigma_{\pi N}, \sigma_{0,\mathrm{ChPT}}$

$$\sigma_0 = \left\langle N \left| \bar{u}u + \bar{d}d - 2\bar{s}s \right| N \right\rangle$$

$$\sigma_s = m_s \frac{\langle N | \bar{s}s | N \rangle}{2M_N} = 17(18)_{\mathrm{stat}}(9)_{\mathrm{chiral}}(2)_{a\neq 0}\ \mathrm{MeV}$$

- pheno., Feynman-Hellmann suffer from large uncertainty
- $\sigma_{\pi N} \sim \sigma_s \sim 40\mathrm{Mev} \Rightarrow$ dark matter cross section

strange quark content



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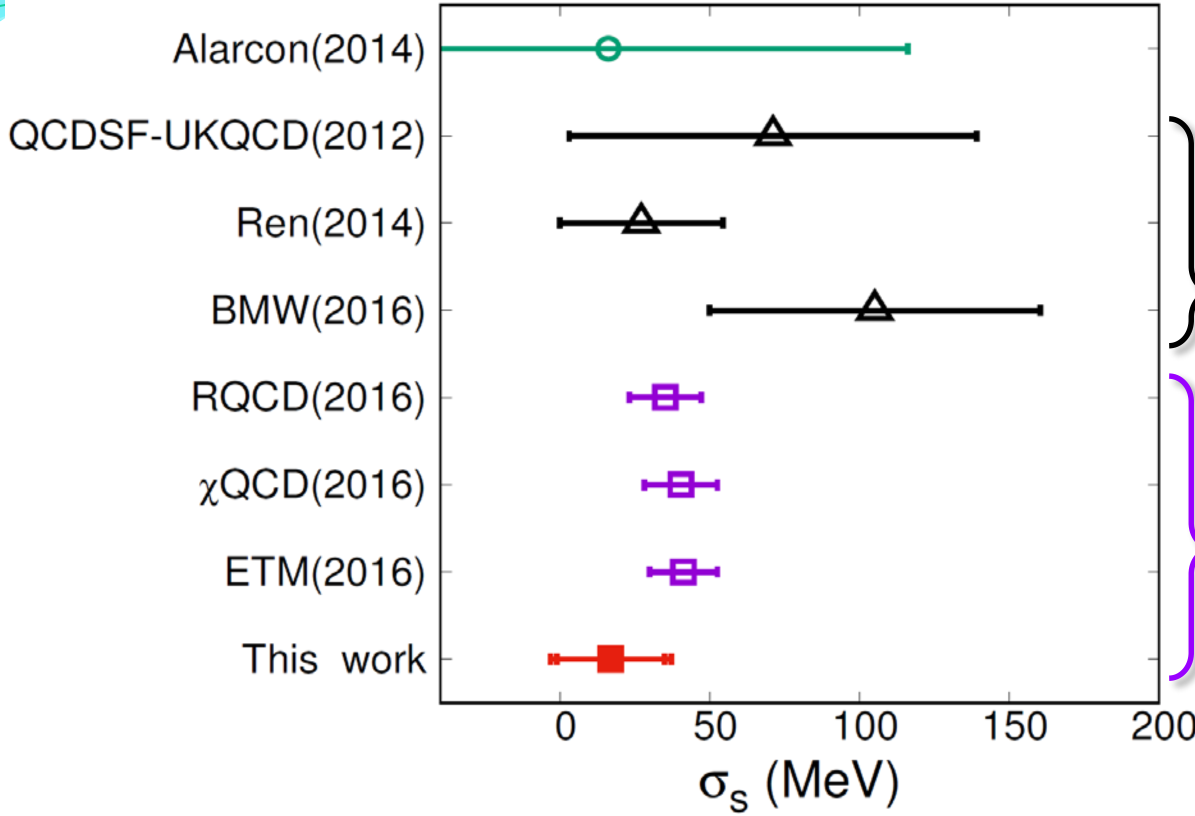

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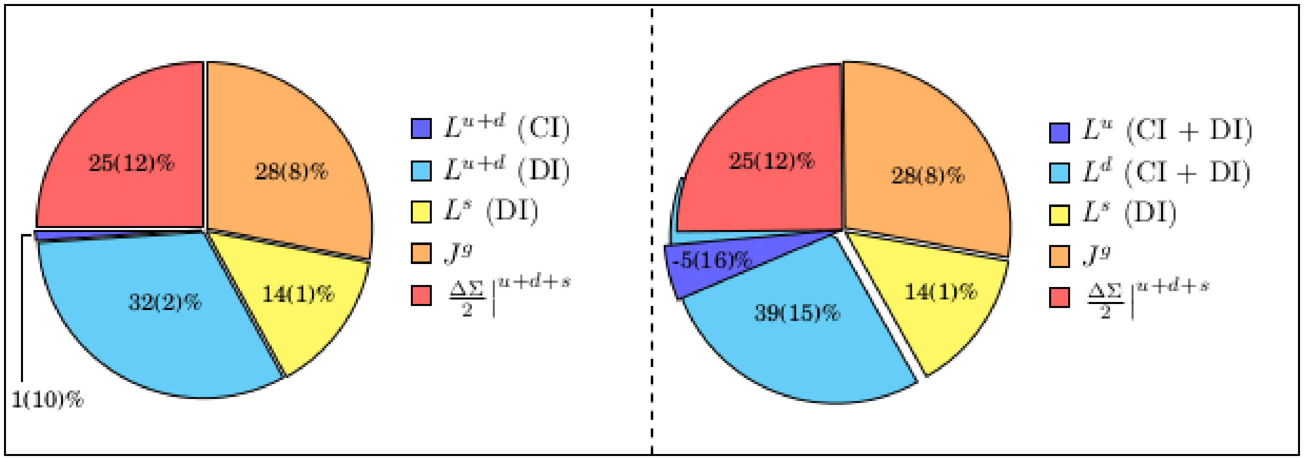
axial charges

Doi-san's talk @ 2nd meeting in 2014

χQCD, first calculation of all contributions '13

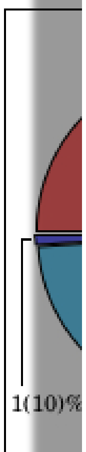


- 25% from $\Sigma_q/2$
- 45% from L_q
- 30% from gluon

axial charges

Doi-san's talk @ 2nd meeting in 2014

χQCD, first calculation of all contributions '13



Systematic errors to be explored

- Dynamical quark effect
  - This is quenched calc.
- Uncertainty in (long) chiral extrapolation
  - m(pi) = 0.48--0.65 GeV in this calc
- Contamination from excited states
  - Sys error could be large (quite common in N on lat)
- Finite volume artifact, discretization artifact
  - m(pi) L >~ 4, a = 0.11fm
- Renormalization
  - Perturbative vs. non-perturbative, etc.

% from $\Sigma_q / 2$

% from L_q

% from gluon

axial charges

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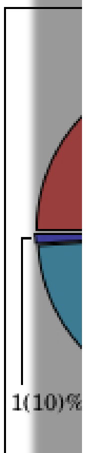

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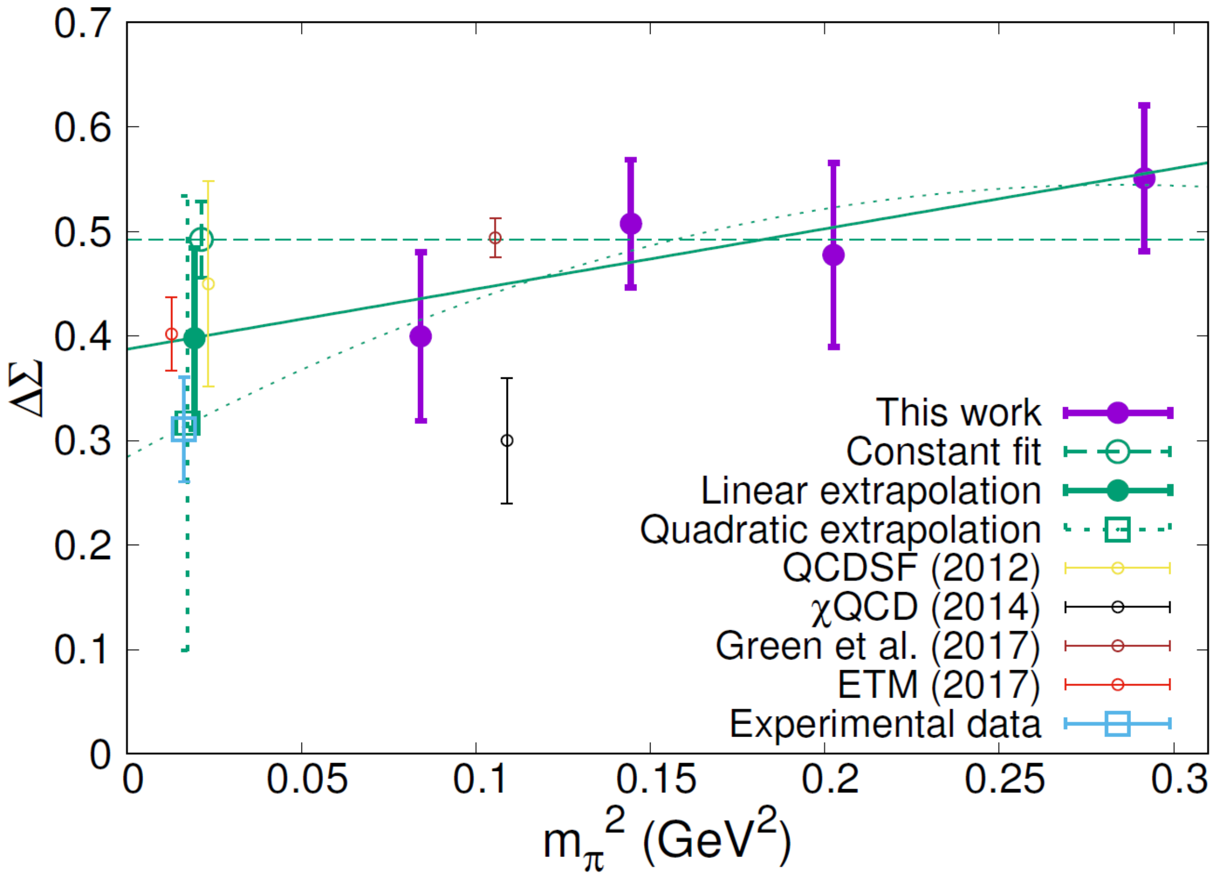


$$\Delta\Sigma = 0.40(13)$$

$$\Leftrightarrow 0.50(24)$$

$$\Delta u = 0.74(15)$$

$$\Leftrightarrow 0.79(11)$$

$$\Delta d = -0.39(15)$$

$$\Leftrightarrow -0.42(12)$$

$$\Delta s = -0.046(28)$$

$$\Leftrightarrow -0.12(1)$$

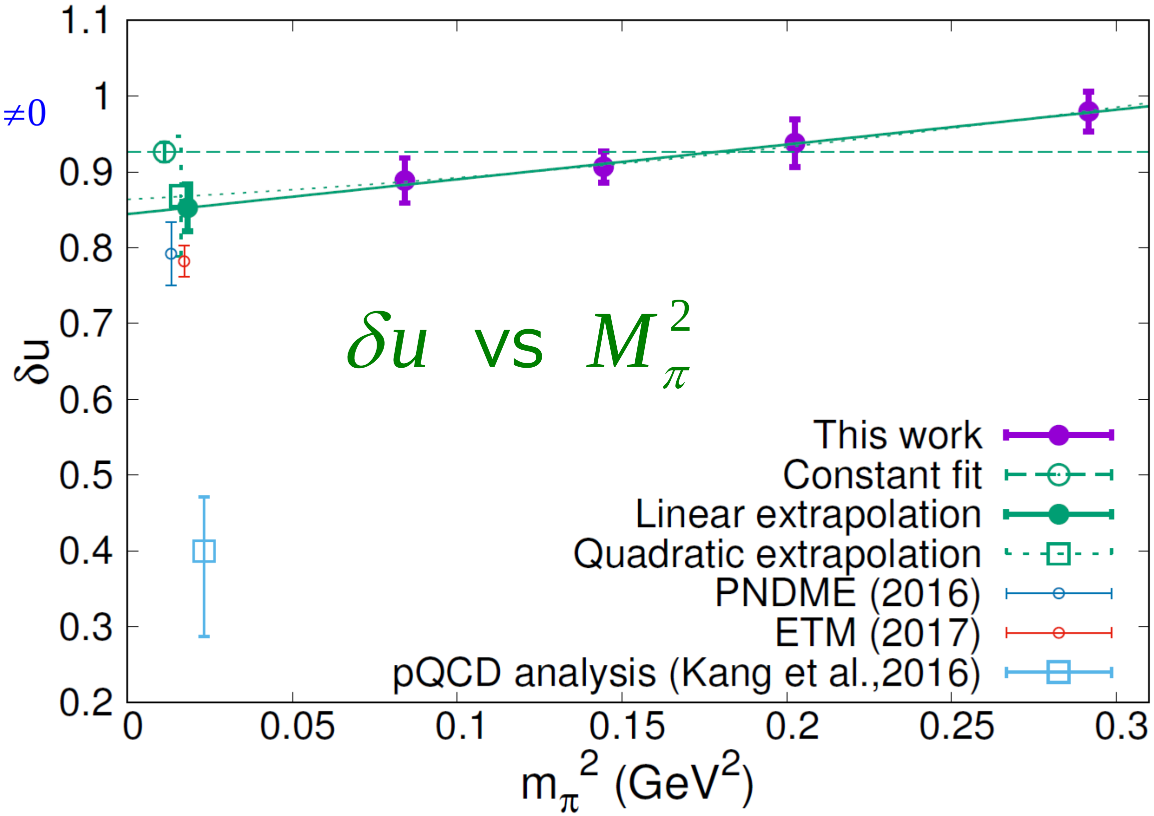
- quark spins : reasonably consistent w/ χQCD and others
- ETM '17 : J_g = 27(3)% for N_f=2 ⇔ 28(8)% N_f=0 χQCD

tensor charges

$$\delta d = -0.24(2)_{\text{stat}}(0)_{\text{chiral}}(2)_{a\neq 0} \qquad \delta s = -0.012(16)_{\text{stat}}(8)_{\text{chiral}}$$

$$\delta u = 0.85(3)_{\text{stat}}(2)_{\text{chiral}}(7)_{a\neq 0}$$

consistent w/ PNDME, ETM



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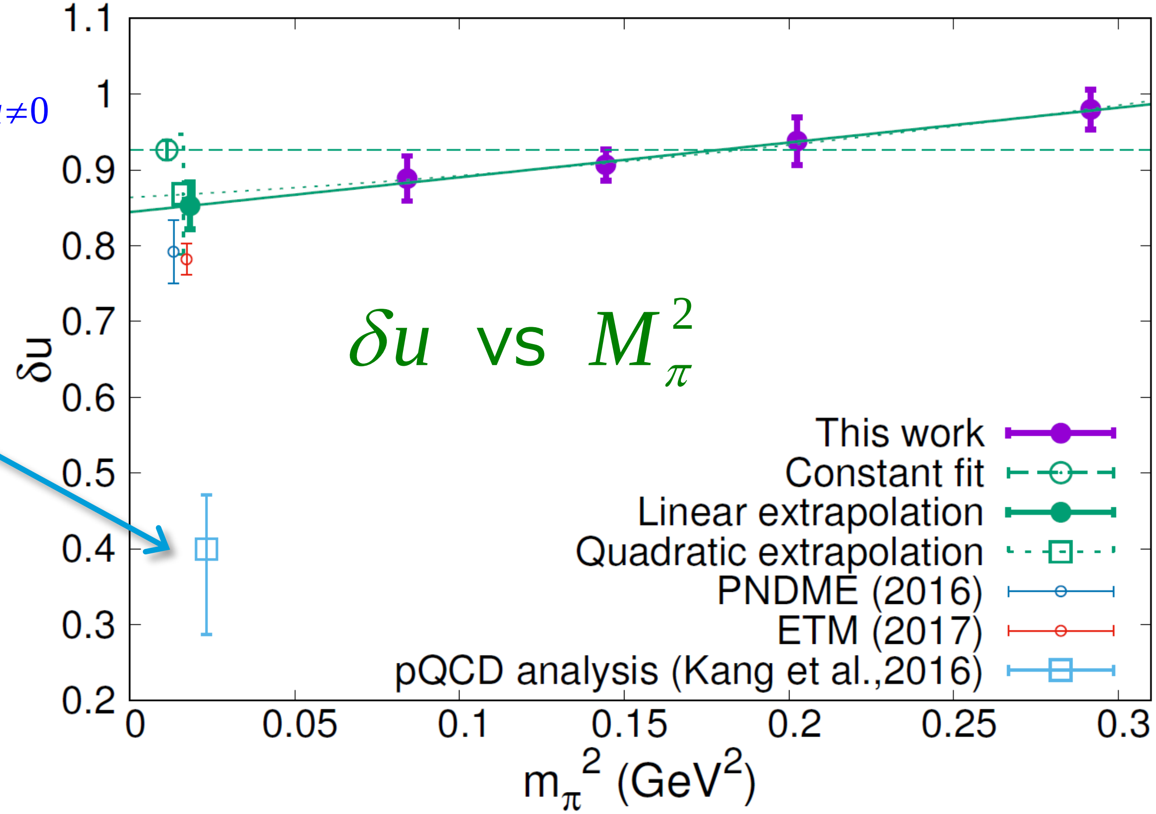


Kang et al. '16

transversity distribution

Belle, Babar (dihadron production) HERMES, COMPASS, Jlab (semi-inclusive hadron prod.)

⇔ limited x ?

improvement by future experiments, EIC

summary + perspective

recent progress on nucleon charges from lattice QCD

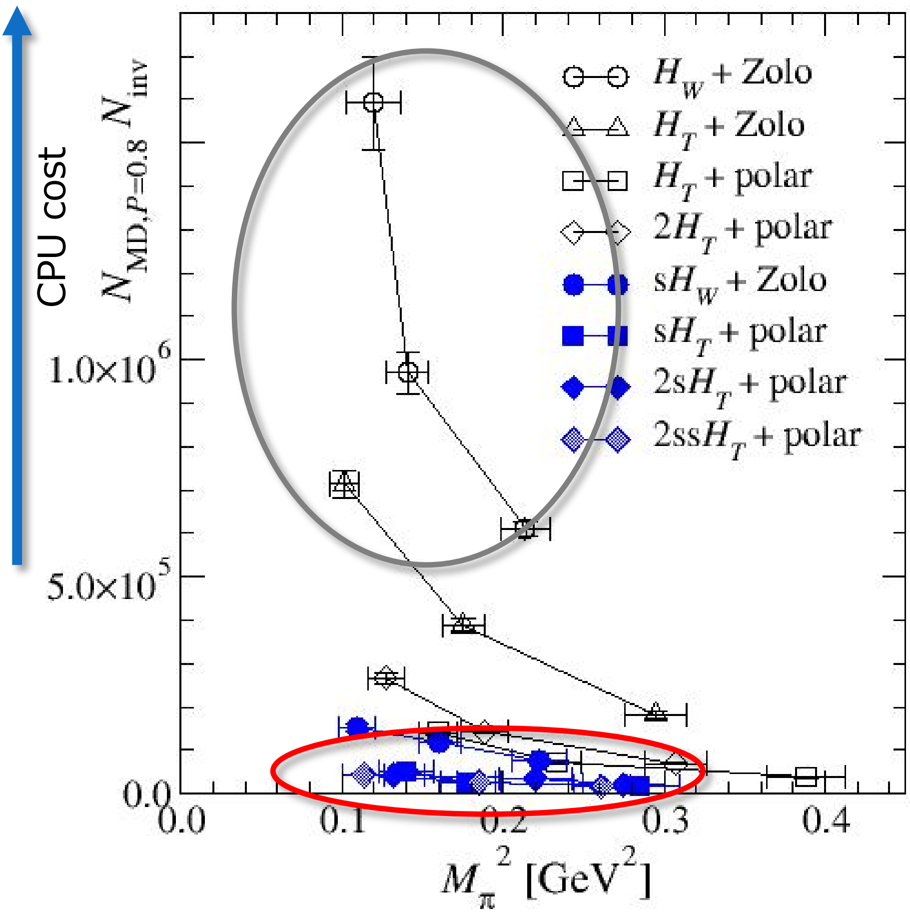
- remarkable progress in recent years
  - realistic simulations $\Rightarrow$ isovector g_A, g_S, g_T
  - improved techniques $\Rightarrow$ isoscalar, up, down, strange charges

- JLQCD's study w/ exact chiral symmetry
  - simplified renormalization, direct comparison w/ ChPT
  - accuracy reasonable for new physics search g_S, g_T, σ_s, δ_u, δ_d, $\cdots$
  - more precise calculation needs smaller a and M_π
  - more precise calculation of disconnected diagrams

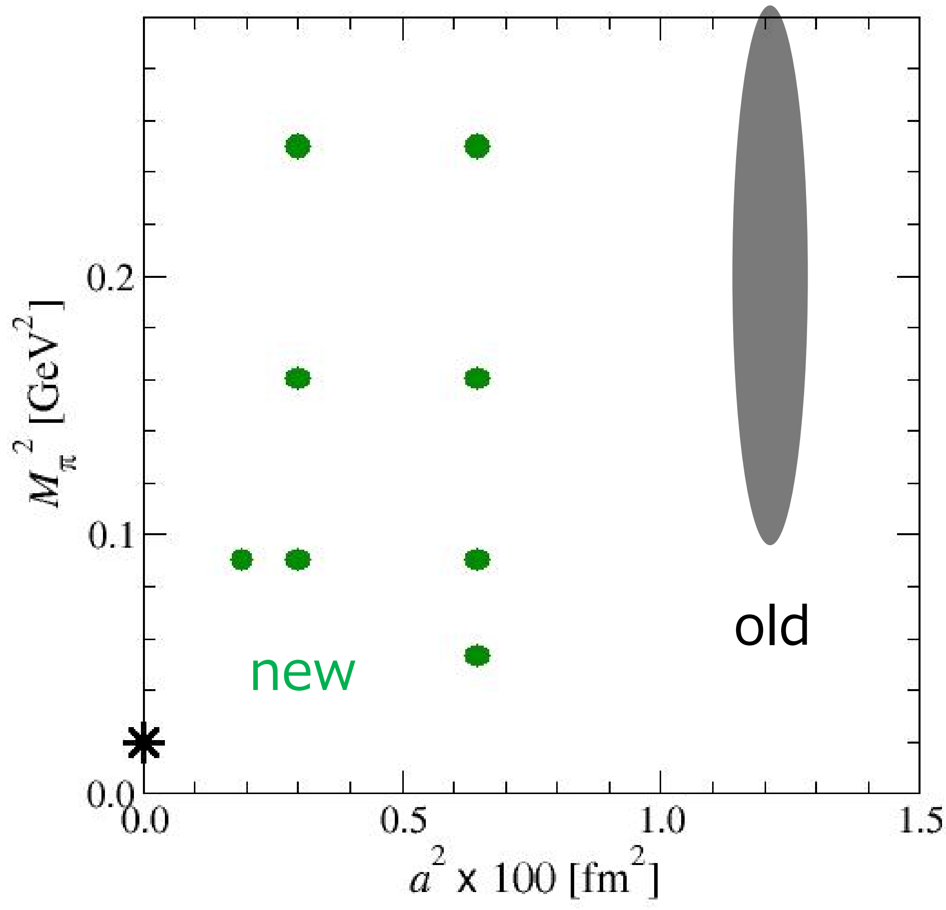
summary + perspective

toward precision calculation

faster formulation w/ chiral sym.



simulation parameters



better control of discretization error and chiral extrapolation

Backup slides

nucleon correlation functions

$$\left\langle O_N \bar{q} \Gamma q \bar{O}_N \right\rangle$$

$$= \left\langle O_N \right| \left(\sum_n \frac{|n\rangle\langle n|}{2E_n} \right) \bar{q} \Gamma q \left(\sum_m \frac{|m\rangle\langle m|}{2E_m} \right) \bar{O}_N \Big\rangle \exp\left[-E_n\left(\Delta t - \Delta t'\right)\right] \exp\left[-E_m \Delta t'\right]$$

$$= \frac{\langle O_N | N \rangle \langle N | \bar{O}_N \rangle}{4M_N^2} \left\langle N \middle| \bar{q} \Gamma q \middle| N \right\rangle \exp\left[-M_N \Delta t\right] + "N^*" \exp\left[-\Delta M \Delta t^{(\prime)}\right]$$

$$\left\langle O_N \bar{O}_N \right\rangle$$

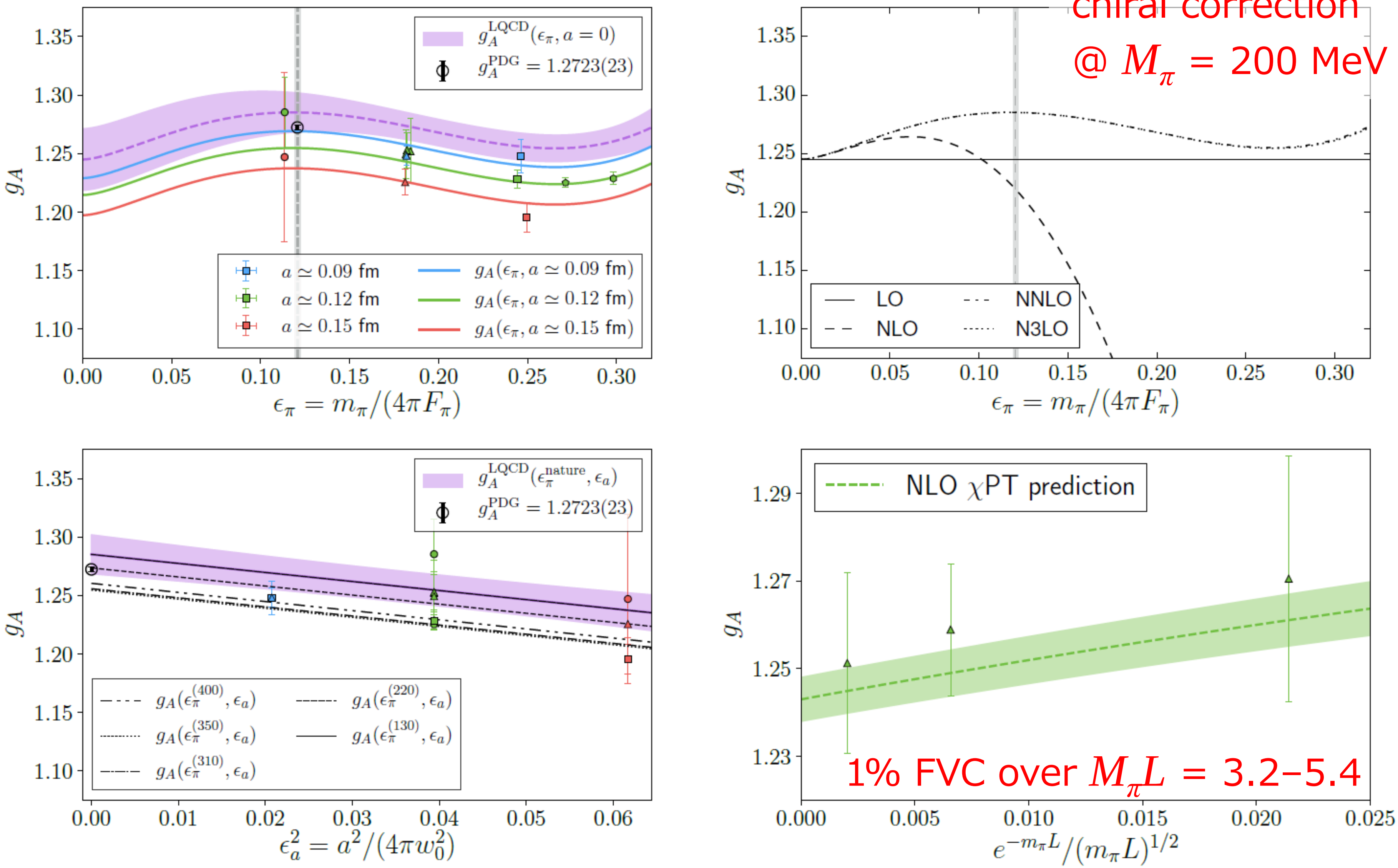
$$= \left\langle O_N \right| \left(\sum_n \frac{|n\rangle\langle n|}{2E_n} \right) \bar{O}_N \Big\rangle \exp\left[-E_n \Delta t\right]$$

$$= \frac{\langle O_N | N \rangle \langle N | \bar{O}_N \rangle}{2M_N} \exp\left[-M_N \Delta t\right] + "N^*" \exp\left[-\Delta M \Delta t\right]$$

a precise calculation of gA

Chang et al. '17

10% NNLO
chiral correction
@ M_π = 200 MeV



1% FVC over $M_\pi L$ = 3.2–5.4

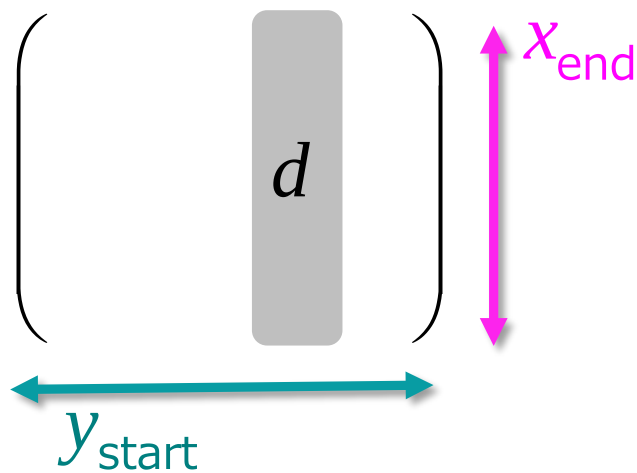
quark propagator

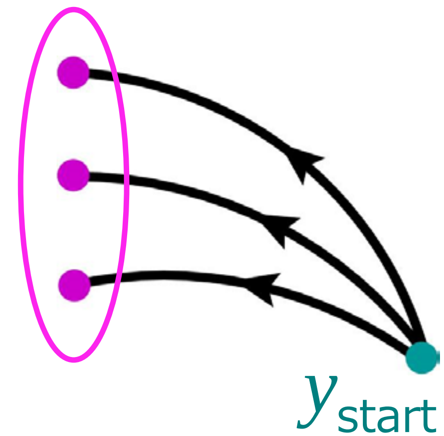
"point-to-all" propagator : standard

Krylov method : CG, GMRES, ⋯

D^{-1} : $O(10^6) \times O(10^6)$ matrix $\Rightarrow$ $Dd = e_{x_{\mathrm{end}}}$ $\Rightarrow$ a column d

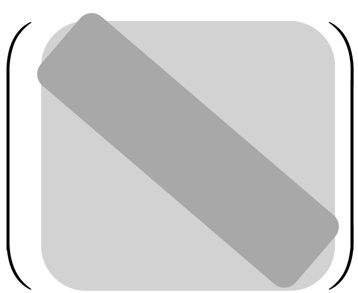
$D^{-1}(x_{\mathrm{end}}, y_{\mathrm{start}}) =$





"all-to-all" propagator : modern, improvable stochastic method

e.g. $D^{-1} =$



$= \sum_{k}^{N_{\mathrm{eigen}}} \frac{1}{\lambda_k} u_k u_k^{\dagger}$ + small correction

w/ limited# low-lying modes

stochastic method

low energy theorem for $\sigma_{\pi N}$

extension to more involved quantities

correction to theorem : small $\sim O(M_\pi^4)$, no chiral log

$$F_\pi^2 \bar{D}_+\left(\nu = 0, t = 2M_\pi^2\right) = \sigma\left(t = 2M_\pi^2\right) + \Delta_R = \sigma(0) + \Delta_\sigma + \Delta_R$$

$$\nu = \left(s - M_N^2 + t/2 - M_\pi^2\right)$$

correction to scalar FF : BChPTs

isoscalar πN scattering amplitude ("PS Born term" subtracted)

$$T_{\pi N}^{ba} = \bar{u}(p')\left[\delta^{ba}\left\{A^+(s,t) + \not{q}B^+(s,t)\right\} + i\varepsilon^{bac}\tau^c\left\{A^-(s,t) + \not{q}B^-(s,t)\right\}\right]u(p)$$

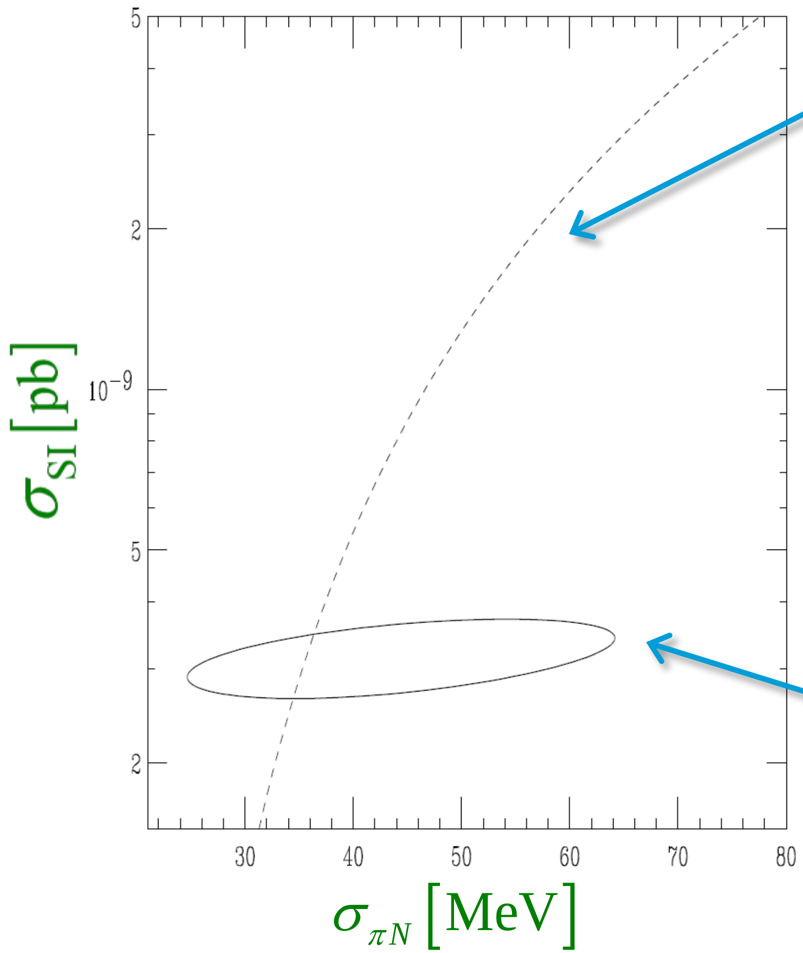
$$\bar{D}^+\left(0, 2M_\pi^2\right) = A^+\left(M_N^2, 2M_\pi^2\right) - \frac{g_{\pi N}^2}{M_N}$$

but @ unphysical Cheng-Dashen point $\nu=0$, $s=M_N^2$, $t=2M_\pi^2$

⇒ dispersive analysis

dark matter cross section

Giedt et al., ’09, also Ellis ’08-’09



conventional analysis

determine “unknown” σ_s

from $\sigma_{\pi N}$, σ_0

$$\sigma_0 = \left\langle N \left| \bar{u}u + \bar{d}d - 2\bar{s}s \right| N \right\rangle$$

$$\sigma_s = \frac{ms}{2m_{ud}}\left(\sigma_{\pi N} - \sigma_0\right)$$

w/ lattice QCD σ_s

- $\sigma_{\pi N} \sim \sigma_s \sim$ 40Mev $\Rightarrow$ better estimate of DM cross section