Nuclear Theory
原子核理論

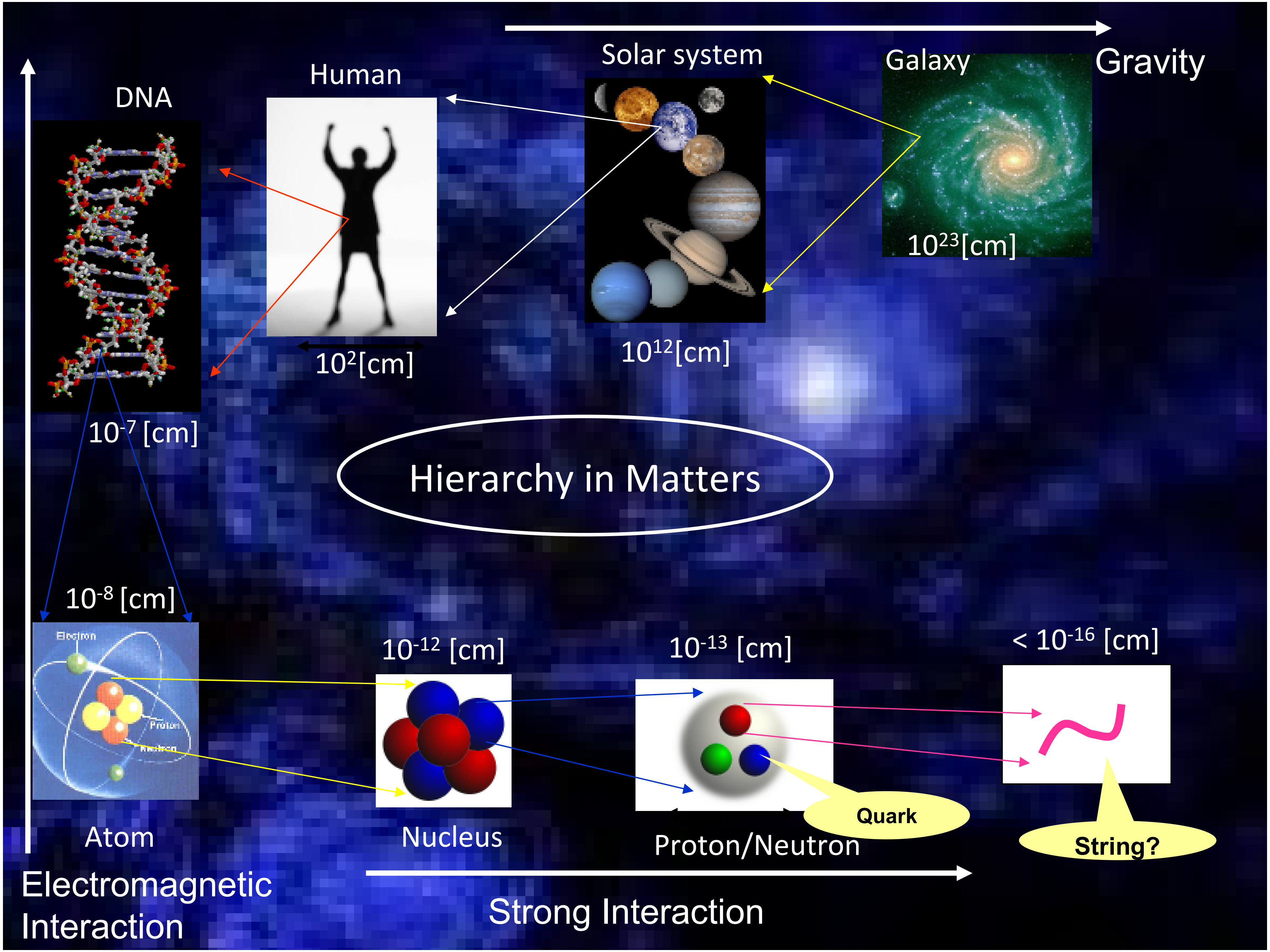
中務　孝

Takashi NAKATSUKASA

RIKEN Nishina Center

Nishina School（仁科学校）

Hierarchy in Matters



Gravity

Galaxy

10^{23}[cm]

Solar system

10^{12}[cm]

Human

10^{2}[cm]

DNA

10^{-7} [cm]

10^{-8} [cm]

Atom

10^{-12} [cm]

Nucleus

10^{-13} [cm]

Proton/Neutron

Quark

< 10^{-16} [cm]

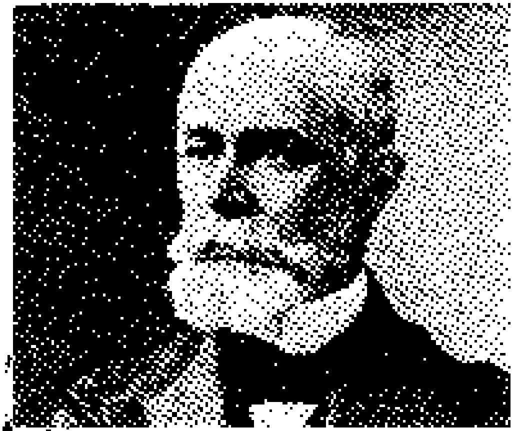
String?

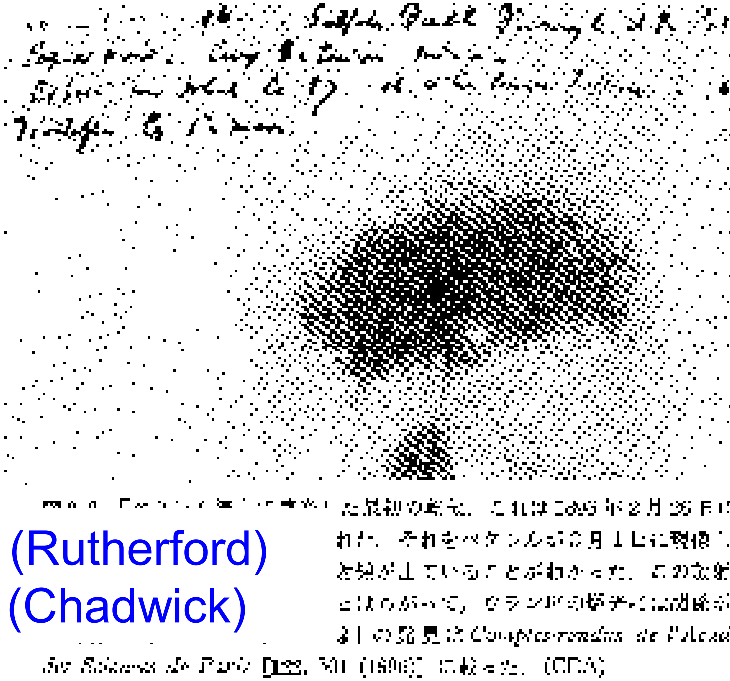
Electromagnetic Interaction

Strong Interaction

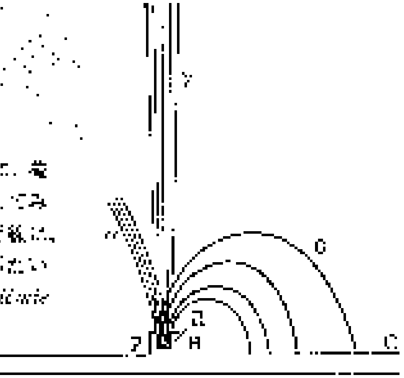
Discovery of radioactivity (End of 19th century)

1895 Discovery of X-ray (Roentgen)
1896 Natural radioactivity (Becquerel)
alpha-ray from Uranium
1897 Discovery of electron (Thomson)
1898 Polonium, Radium (Currie)
1900 alpha, beta, gamma (Rutherford)



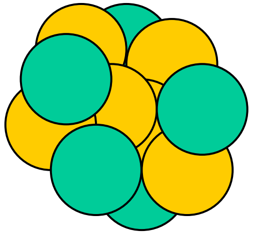


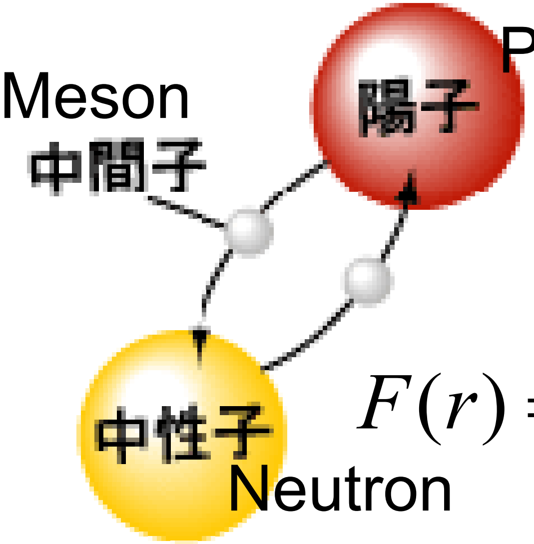
1911 Discovery of nucleus (Rutherford)
1932 Discovery of neutron (Chadwick)



Nuclear interaction:
Force to bind nucleus

Protons and neutrons bound together to form a nucleus.





$$F(r) = F_0 \frac{e^{-kr}}{r}$$

中間子をやりとりすることで
「陽子と中性子の間には力がはたらき近づいている」

1934：Meson theory
(Hideki Yukawa)

Quarks and gluons are described by the QCD.

How small is the nucleus?

Estimate the Compton wave length of pion

$$\lambda_\pi = \frac{\hbar}{m_\pi c} = ?$$

This is a typical magnitude of range of nuclear force

$$F(r) \sim F_0 \frac{\exp\left(-r/\lambda_\pi\right)}{r}$$

and the typical size of nucleus.

Nuclear Physics ~ Femto Physics

How strong is the nuclear force?

There is a deuteron (p+n) that is a bound state of a proton and a neutron.

The range of the force is roughly the pion Compton wave length.

(1) Estimate order of magnitude using the uncertainty principle

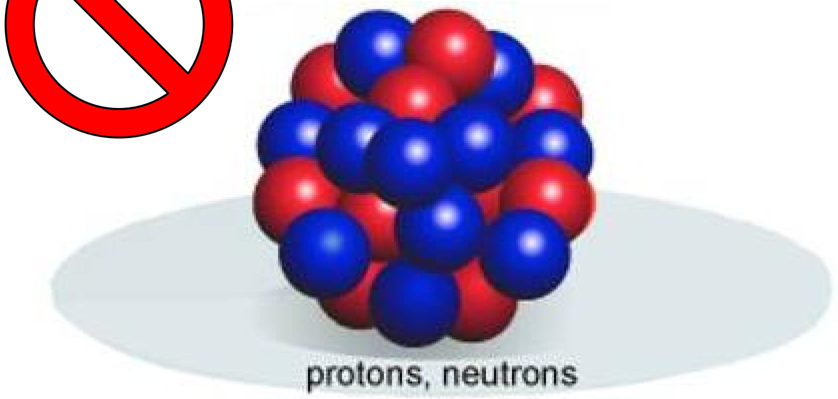
$$\Delta x \cdot \Delta p \sim \hbar$$

$$Mc^2 \sim 1\,\mathrm{GeV} = 1000\,\mathrm{MeV}$$

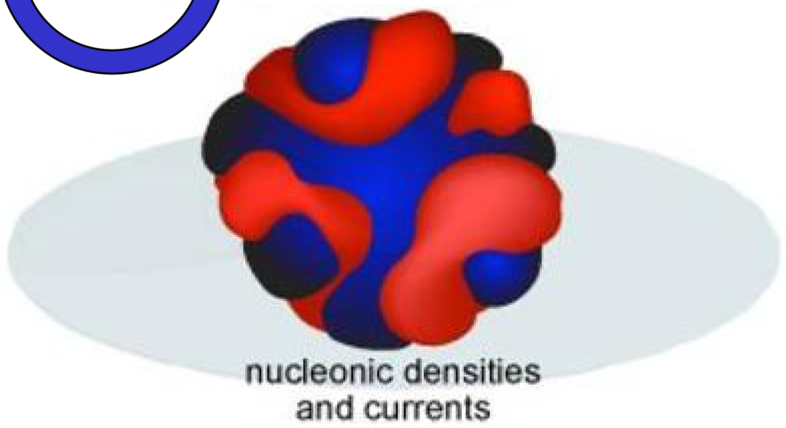
(2) Estimate order of nuclear time scale

Image of nucleus

🚫



○



Liquid-drop Model

Binding energy

$B/A \approx 8$ MeV

Density

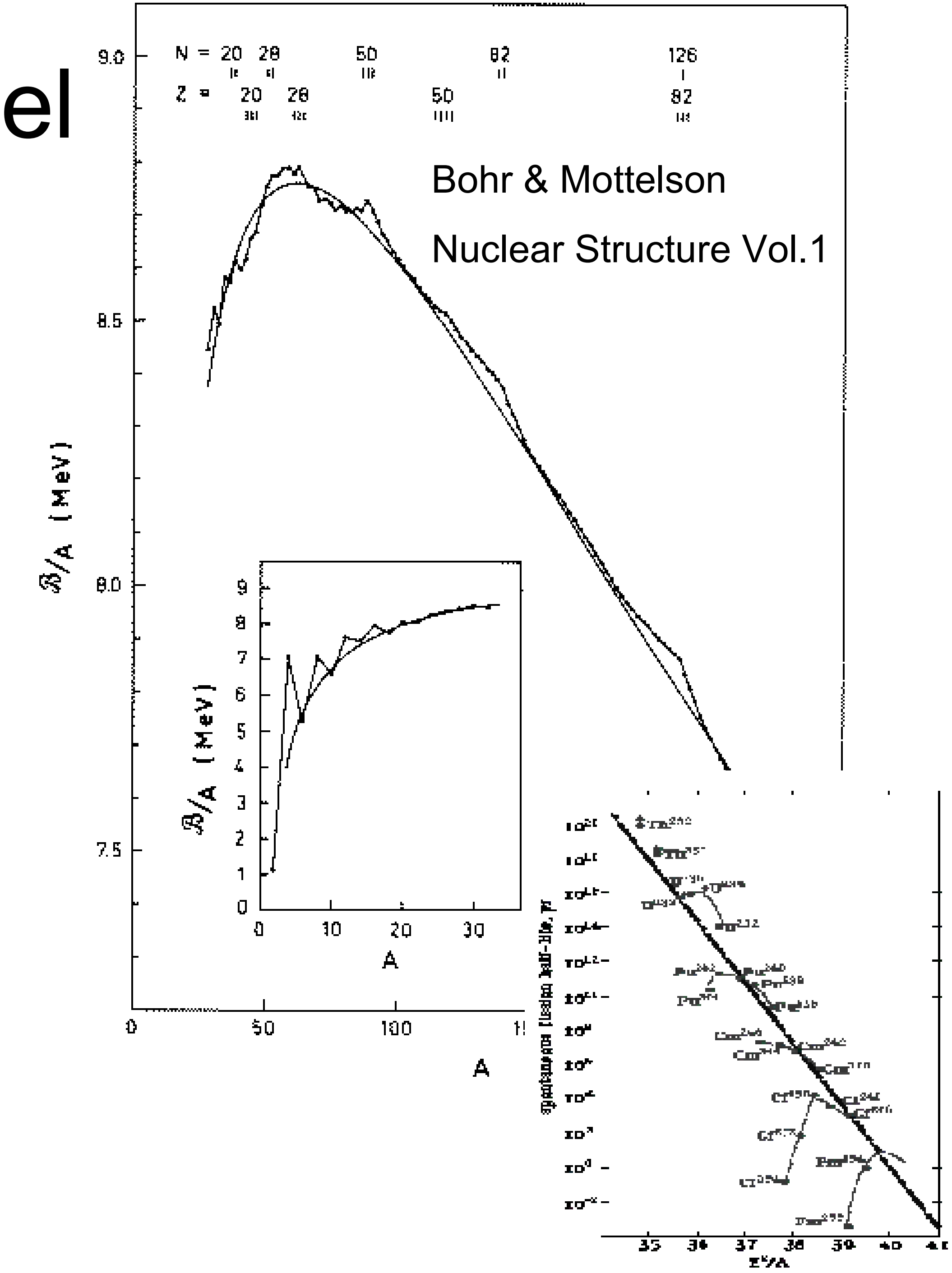
$\rho \approx 0.14\ \mathrm{fm}^{-3}$ $d \approx 2$ fm

Bethe-Weizsäcker mass formula

$$B(N,Z) = a_V A - a_S A^{2/3} - a_{sym}\frac{(N-Z)^2}{A} - a_C \frac{Z^2}{A^{1/3}} + \delta(A)$$

Nuclear fission

$$x = \frac{E_C}{2E_S} \sim \frac{Z^2}{A}$$



Bohr & Mottelson

Nuclear Structure Vol.1

Nucleus as a quantum liquid

- Classical vs Quantum
  - Strength of interaction vs Zero-point kinetic energy

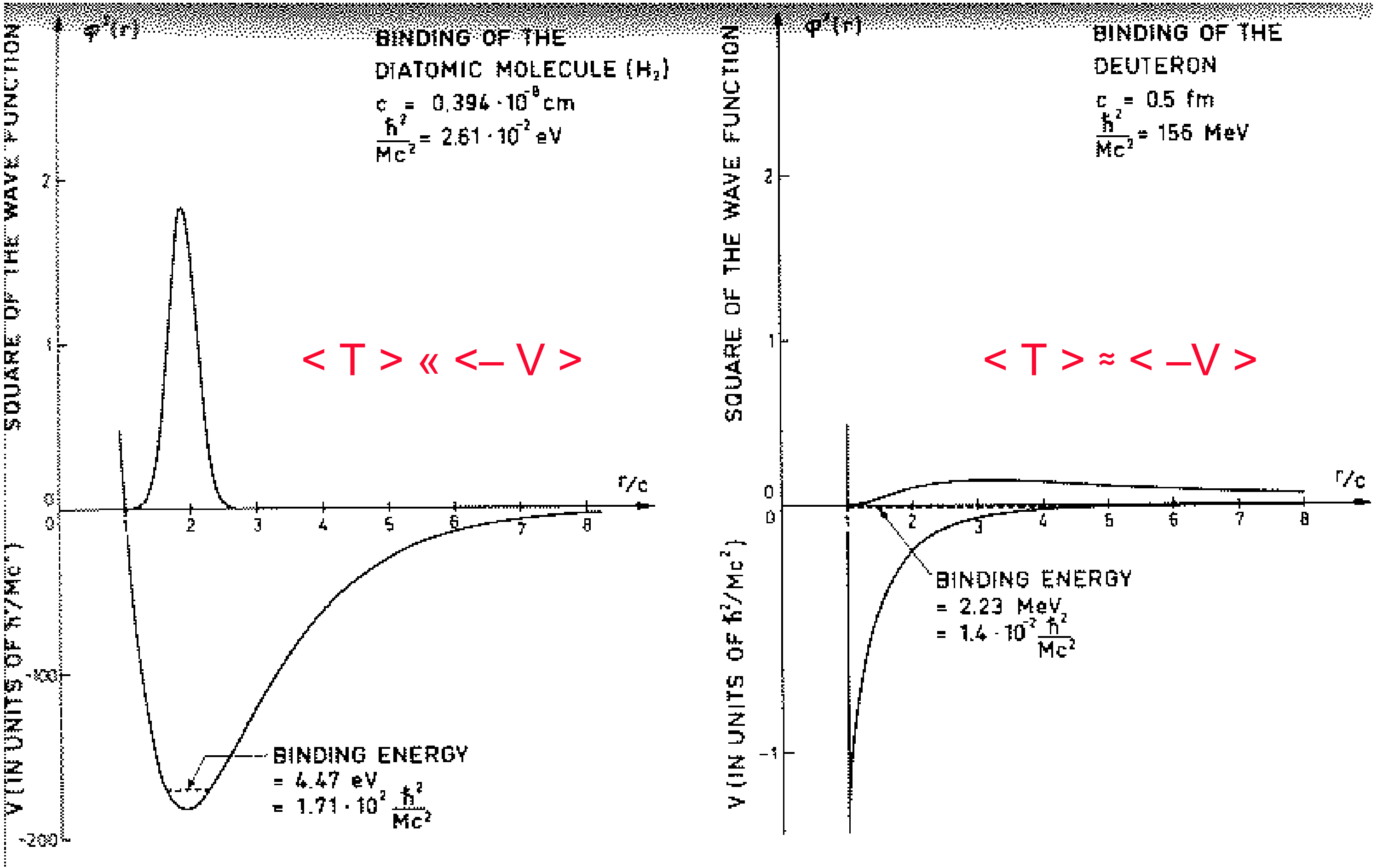
$$V_0 \quad \text{vs} \quad \hbar^2 \big/ 2Mc^2$$

$$c: \text{ Length scale of the interaction}$$

$$V_0: \text{Energy scale of the interaction}$$

Nuclear force vs molecular force

Bohr, Mottelson, Nucl. Str. Vol.1



$< T > \ll < -V >$

$< T > \approx < -V >$

Figure 2-36 The molecular interaction corresponds to a "Morse potential" [illegible] with the constants adjusted

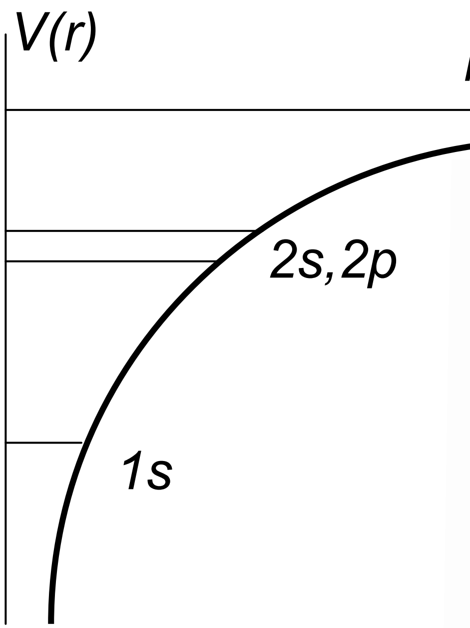
Crystallized at low temperature

Classical → MD

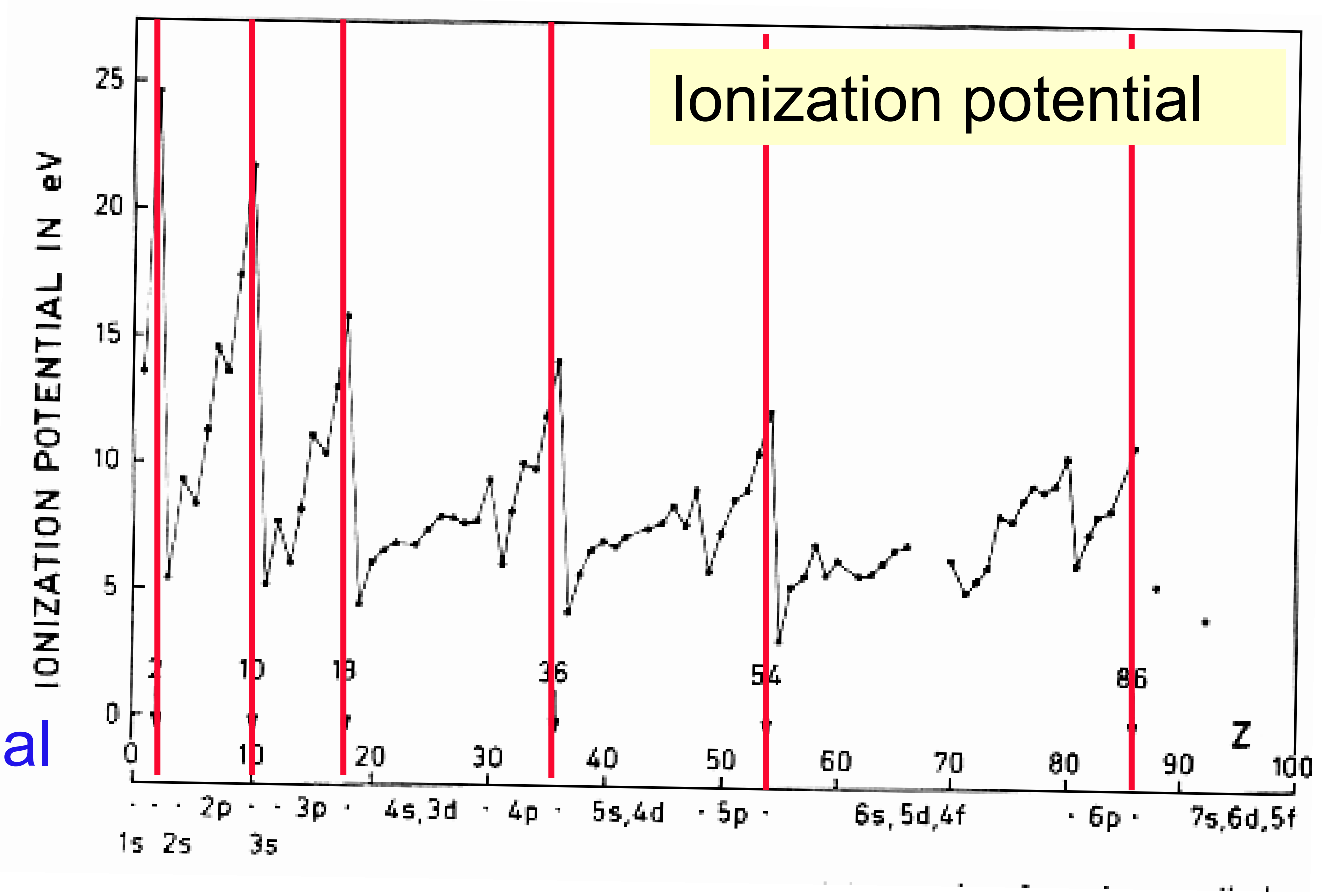
Liquid at low temperature

Quantum

Electronic single-particle motion in atoms

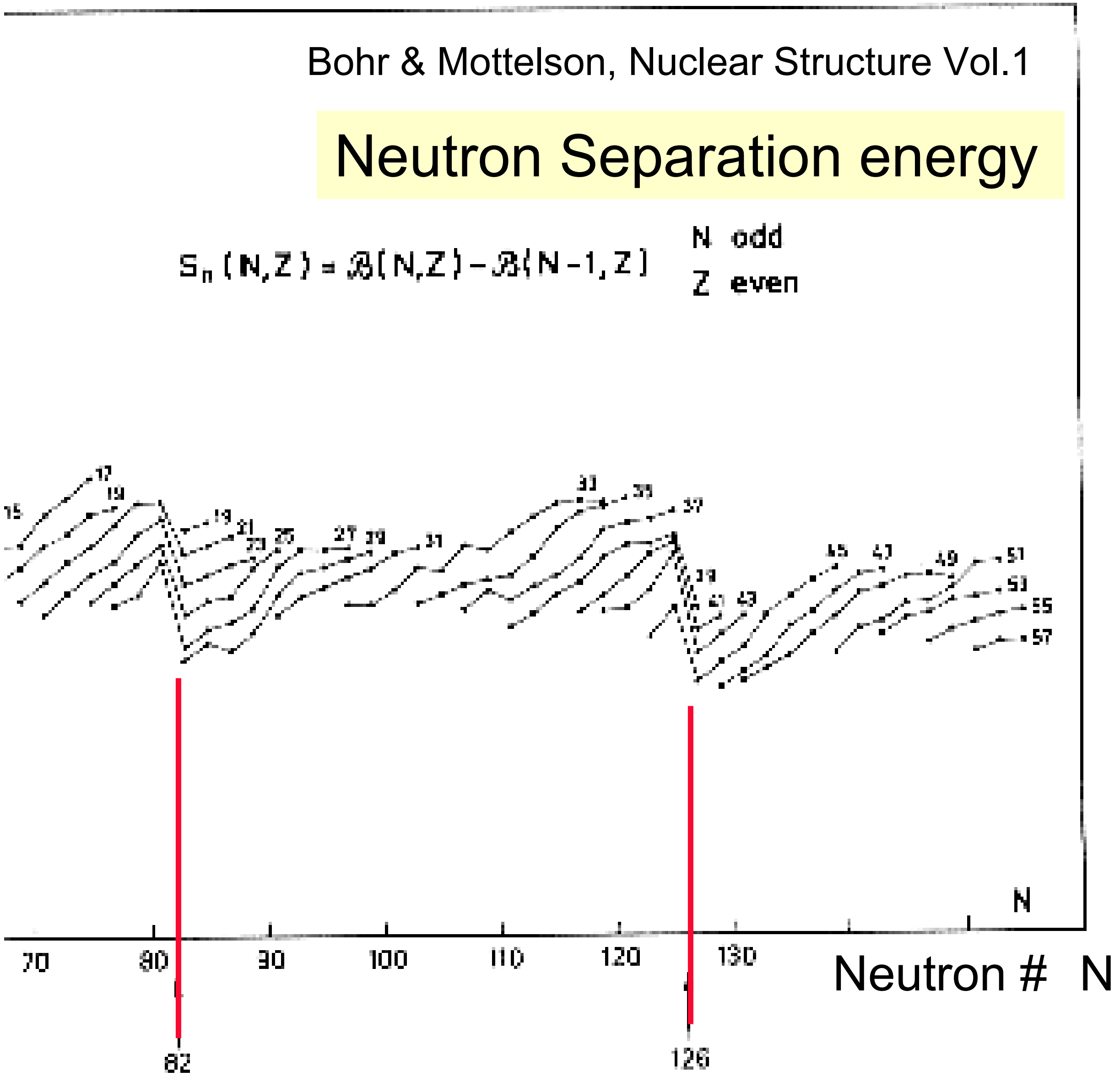


Single-particle orbitals in the Coulomb potential → Magic number


Ionization potential

Free particles in Coulomb potential

Nucleonic single-particle motion in nucleus

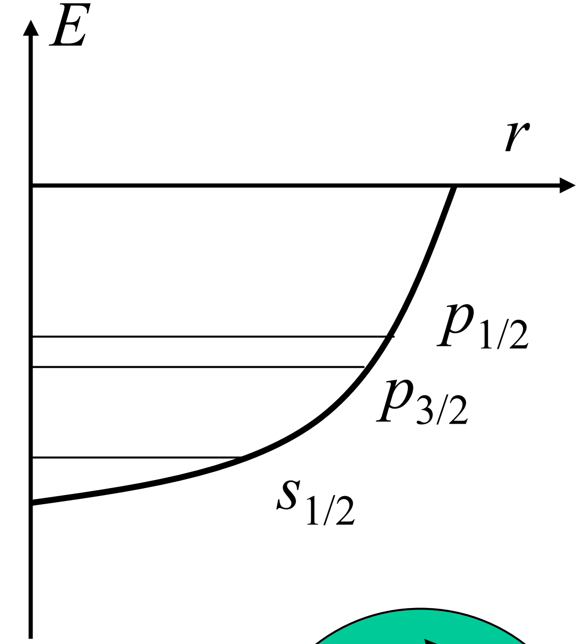


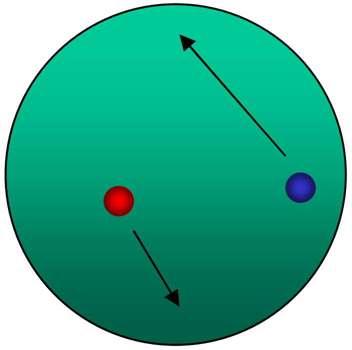
Bohr & Mottelson, Nuclear Structure Vol.1

Neutron Separation energy

Neutron # N

Shell Model
(Mayer-Jensen)



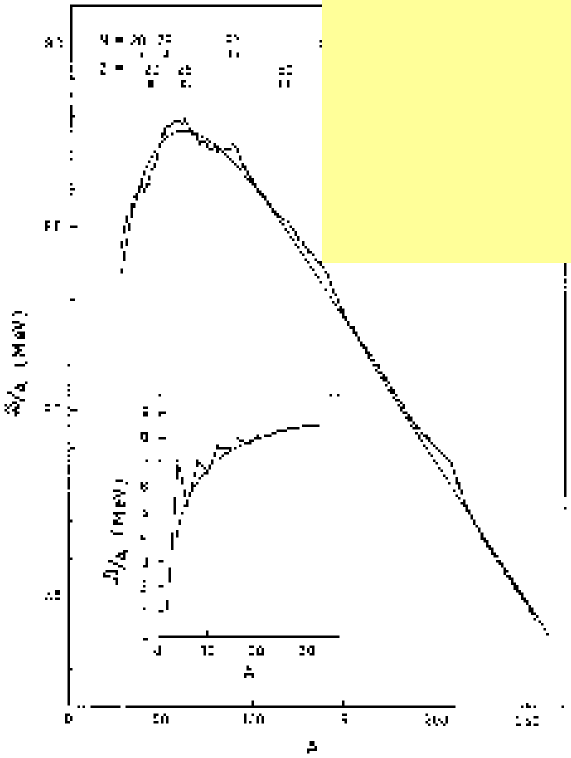


Nucleus is liquid or gas?

Liquid drop model

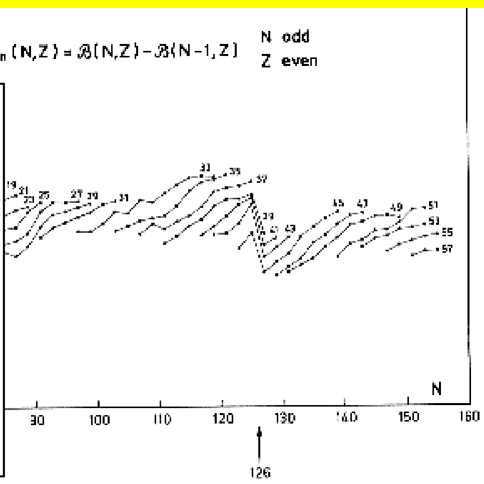
(Bethe-Weizsäcker)

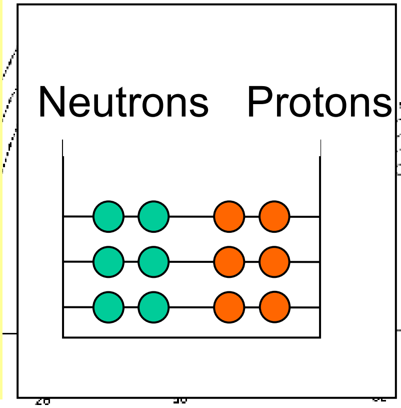
$$B(N,Z) = a_V A - a_S A^{2/3} - a_{sym}\frac{(N-Z)^2}{A} - a_C\frac{Z^2}{A^{1/3}} + \delta(A)$$



Shell model

(Mayer-Jensen)





$$V(r) = \frac{1}{2}M\omega^2 r^2 + v_{ll}\ell^2 + v_{ls}\vec{\ell}\cdot\vec{s}$$

→ Symmetry breaking in the Unified Model

(Bohr-Mottelson)

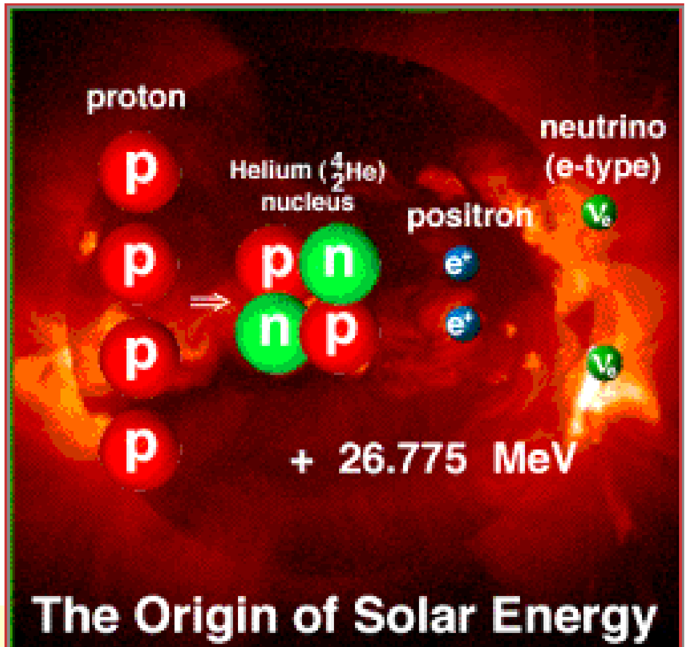
Different reaction rate

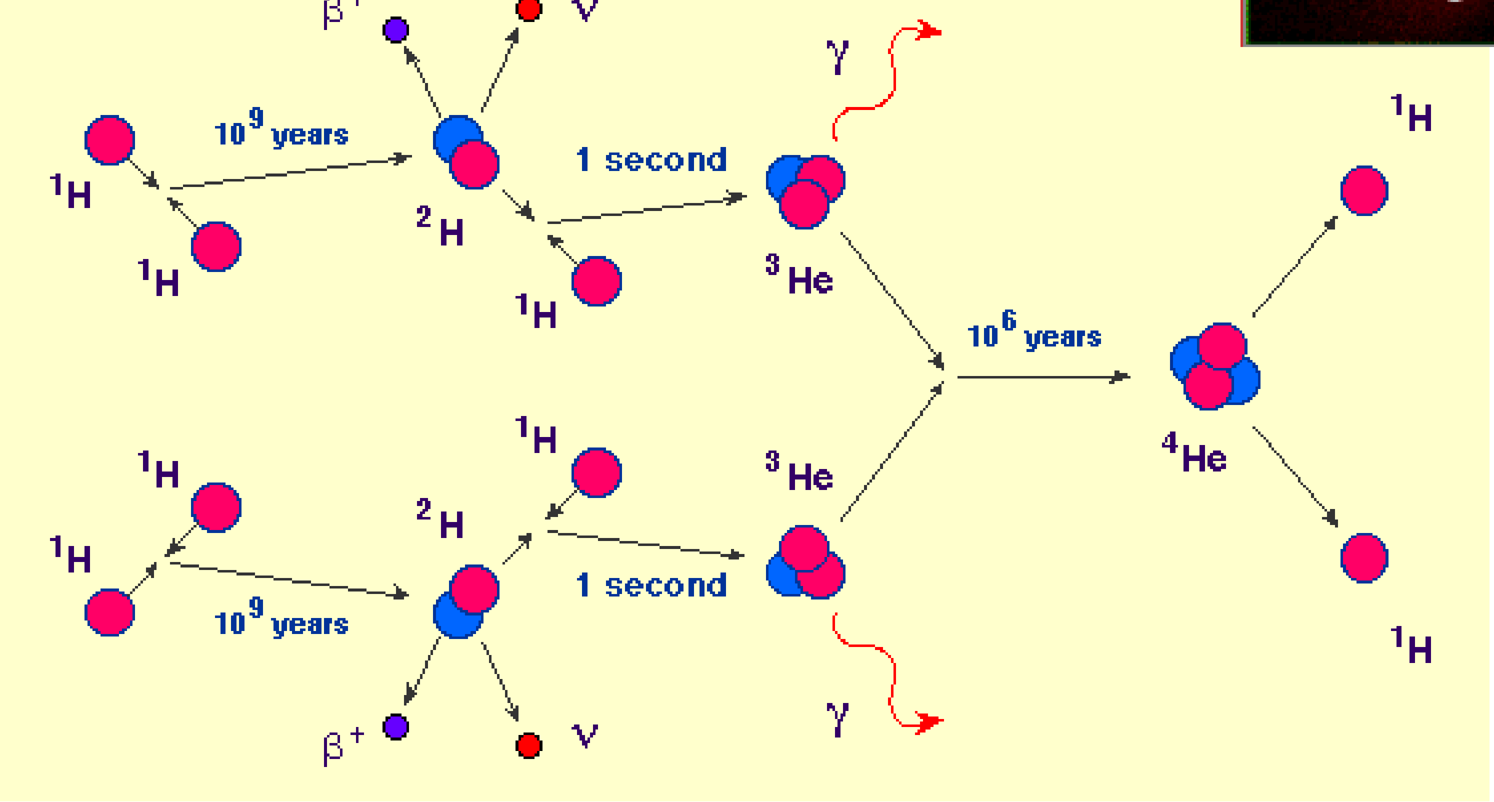
- Transfer reaction (Strong interaction)
  - $^{15}\mathrm{N}(p,\alpha)^{12}\mathrm{C}$
  - $\sigma \sim 0.5$ b (E=2 MeV)
- Capture reaction (Electromagnetic interaction)
  - $^{3}\mathrm{He}(\alpha,\gamma)^{7}\mathrm{Be}$
  - $\sigma \sim 10^{-6}$ b (E=2 MeV)
- Weak process (Weak interaction)
  - $p(p,\mathrm{e}^{+}\nu)d$
  - $\sigma \sim 10^{-20}$ b (E=2 MeV)

→ *Different time scale*

pp chain (I)

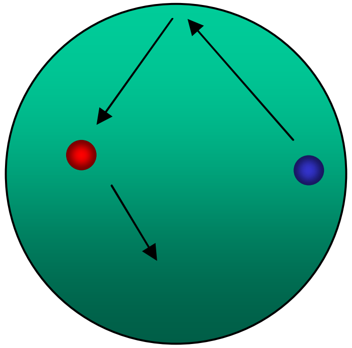
- $p(p,e^+\nu)d$ reaction determines the lifetime of the sun



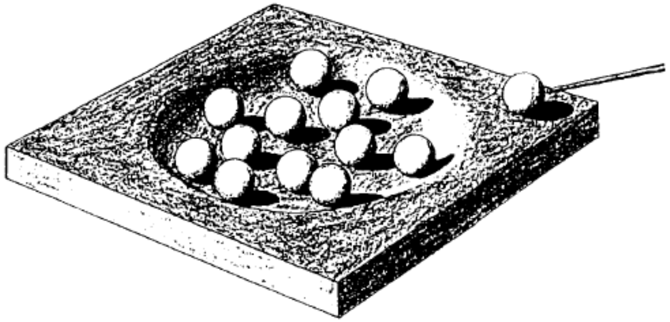


Nucleus with Different Time Scales

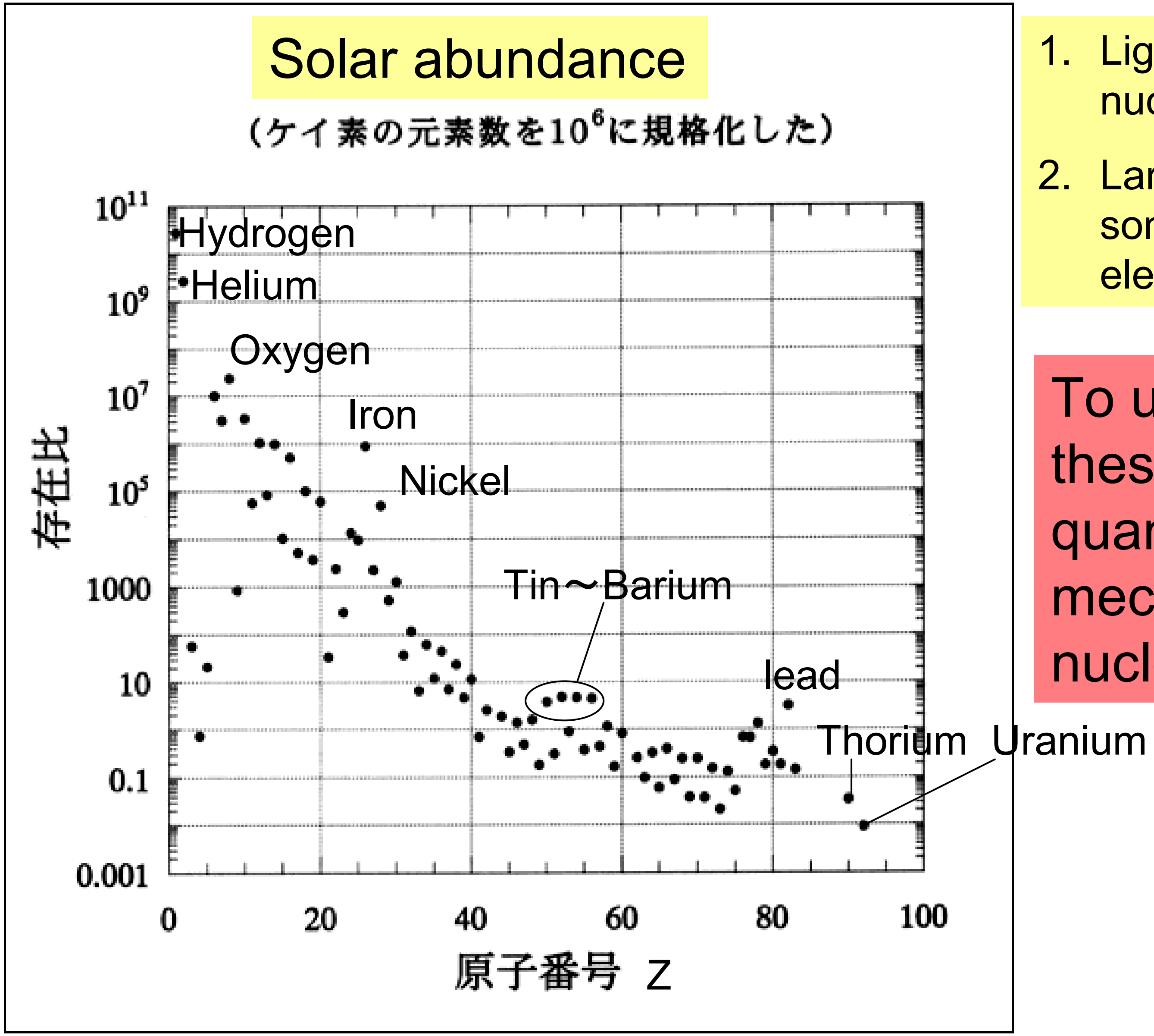
- Time period of nucleonic Fermi motion
  - $\tau_F \sim R/v_F \sim 10^{-22}$ sec
- Collision time
  - $\tau_c >> \tau_F$ (Nucleon near Fermi energy)
  - $\tau_c \geq \tau_F$ (Thermal neutron)
  - If the residence time is much larger than τ_c, "chaotic state" (compound nucleus).



Nucleus shows different faces (aspects) in different time scales.



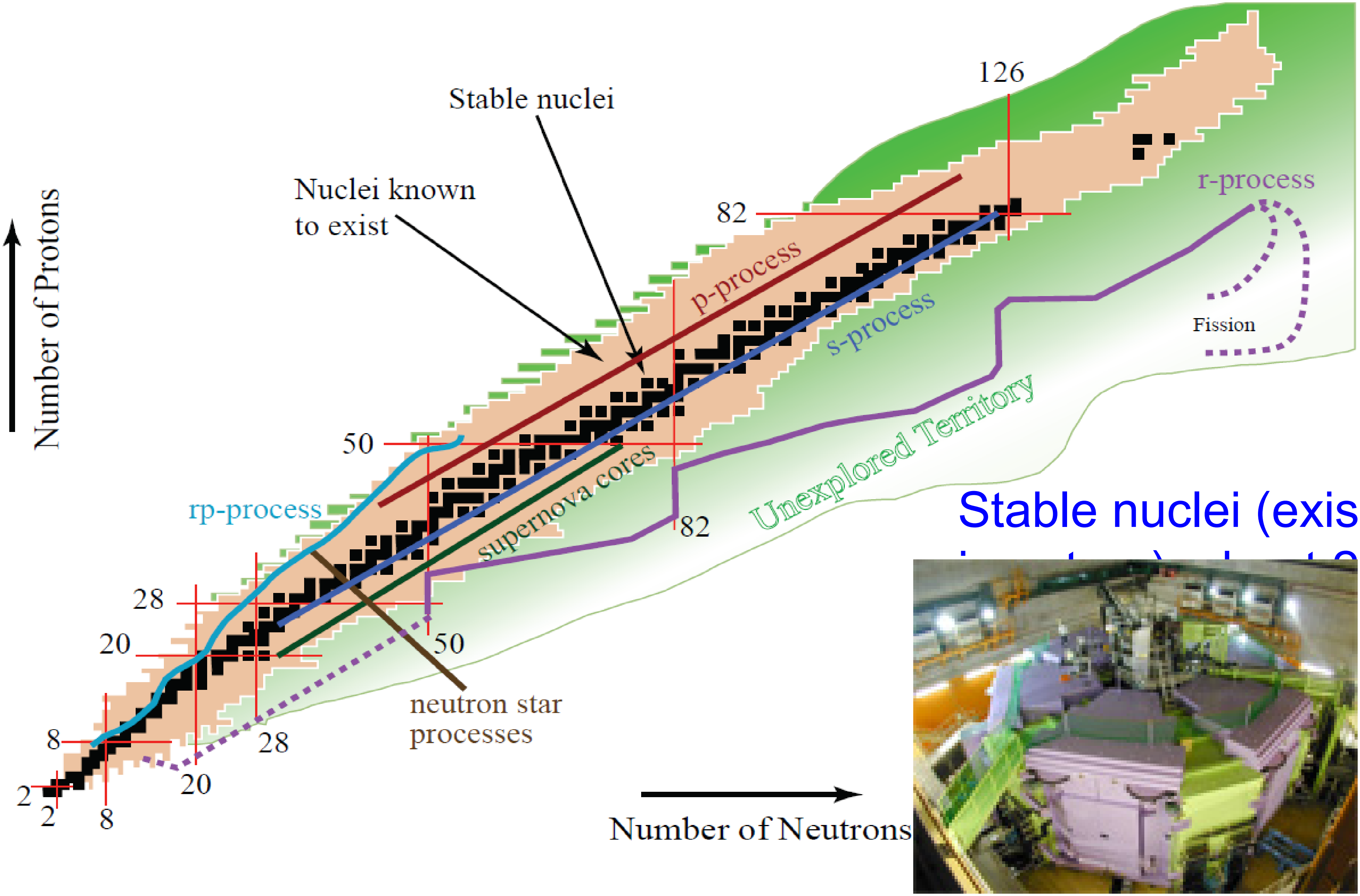
Where we came from?


Solar abundance

1. Light nuclei > Heavy nuclei
2. Large abundance for some specific elements

To understand these, we need quantum mechanics and nuclear physics

Nuclear Chart and Element Synthesis

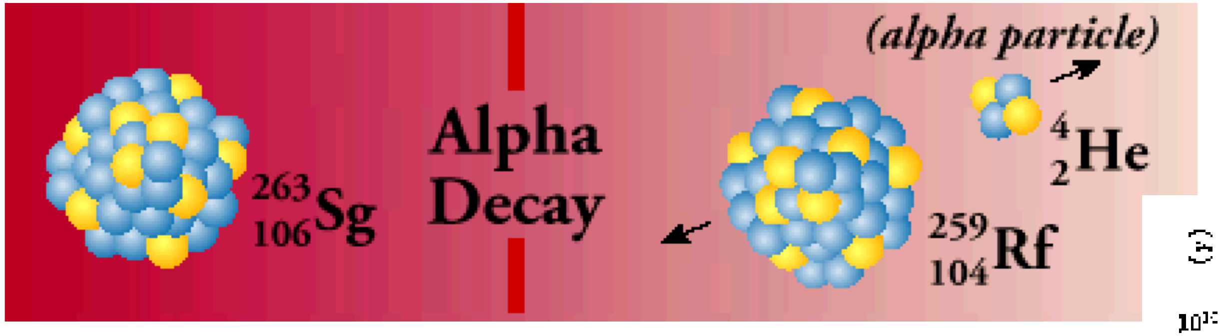


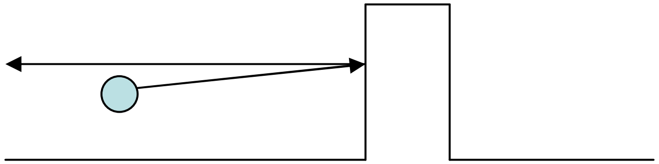
Stable nuclei (exist

70

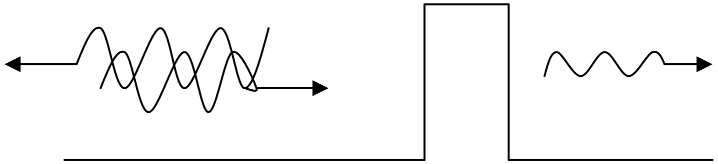
RIKEN Superconducting Cyclotron

Tunnel effect (alpha decay)





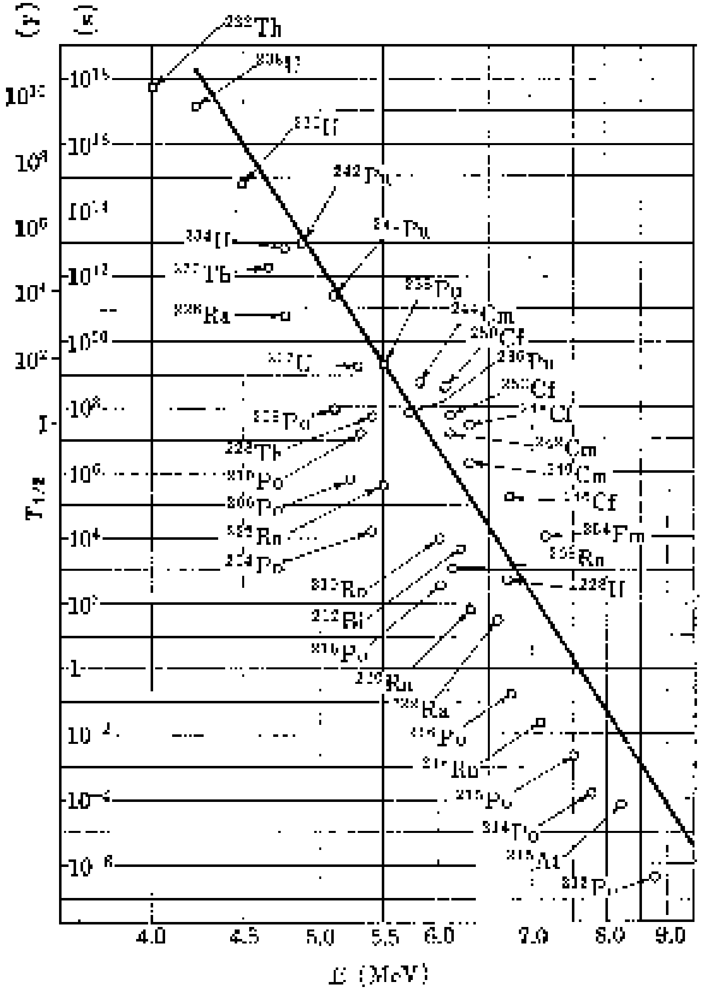
Quantum tunneling phenomena



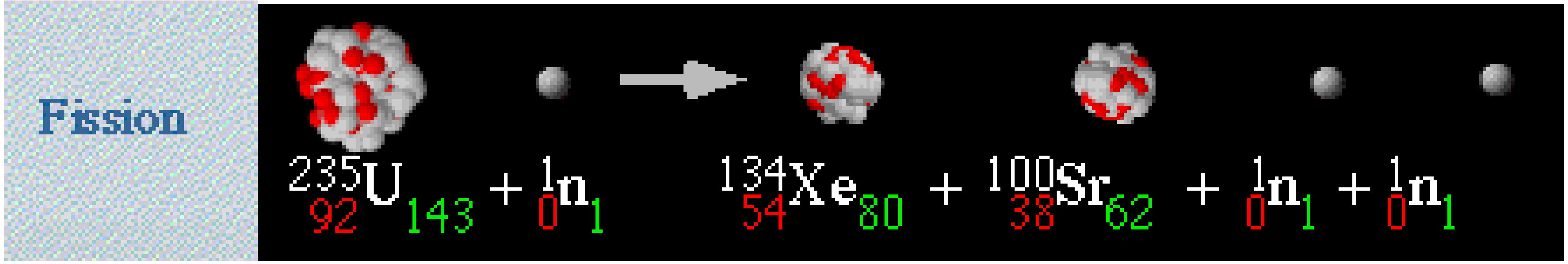
Geiger-Nuttal

$$\log T_{1/2} = \frac{a}{\sqrt{E}} + b$$

George Gamow uses the quantum mechanics to show the rule (1928).



Nuclear Power



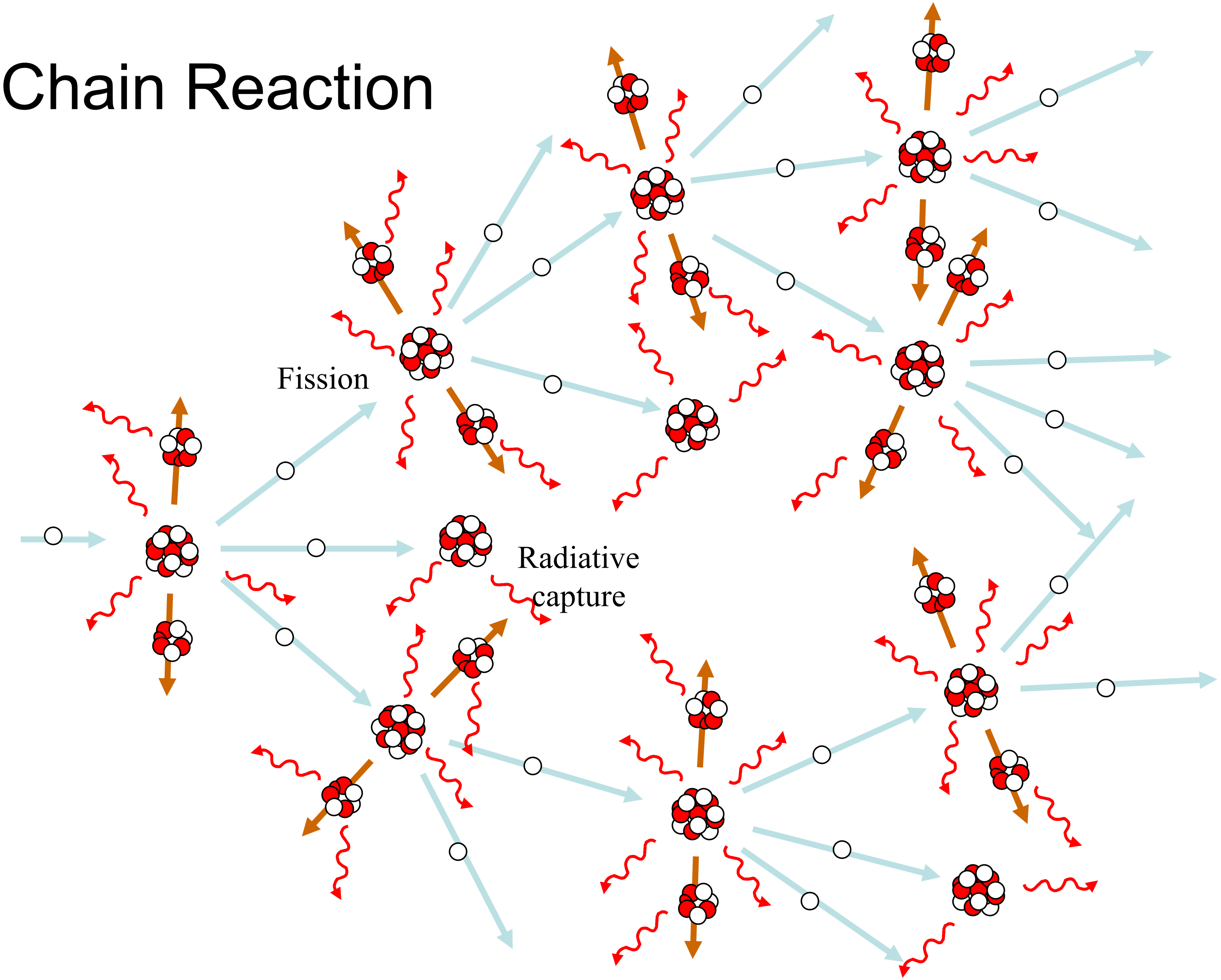
If Uranium absorbs a slow neutron, the fission occurs and neutrons are emitted. These neutrons are decelerated and collide with other Uranium.

(Chain Reaction)

The neutron absorption cross section of 235U enhances at low energy (~several thousands barns).

The chain reaction stops if the speed of neutrons are too fast.

Chain Reaction

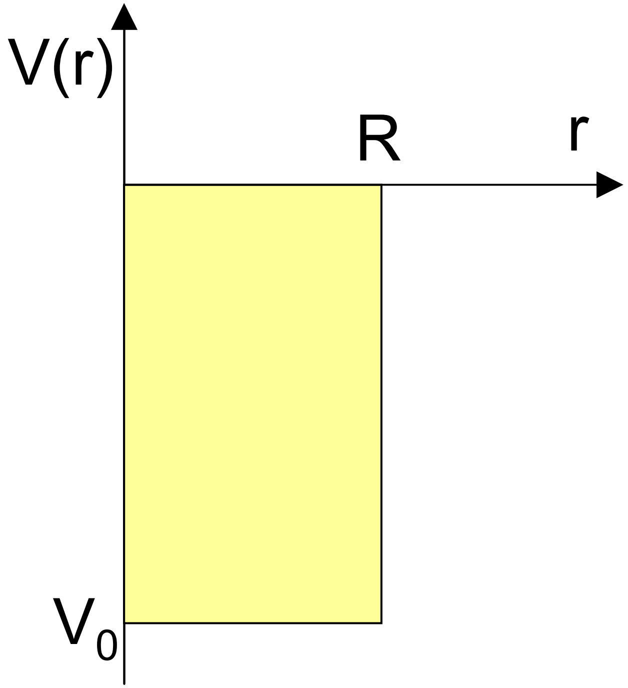


Neutron Cross Section

The radius of stable nucleus is approximately given by

$$R = r_0 A^{1/3}, \quad r_0 = 1.2 \text{ fm}, \quad A = N + Z$$

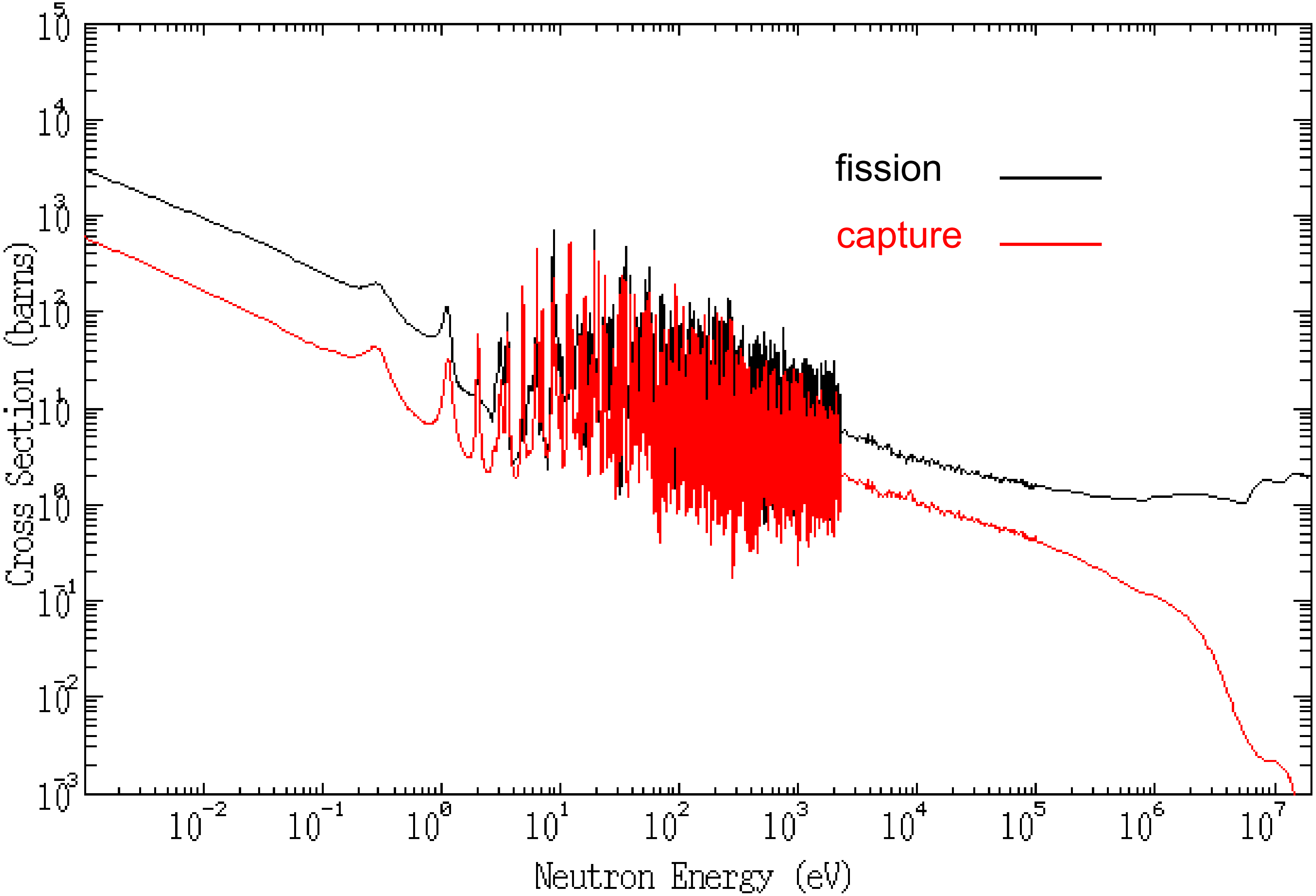
Assume a simple square-well potential model of radius R, obtain cross section in the classical mechanics?



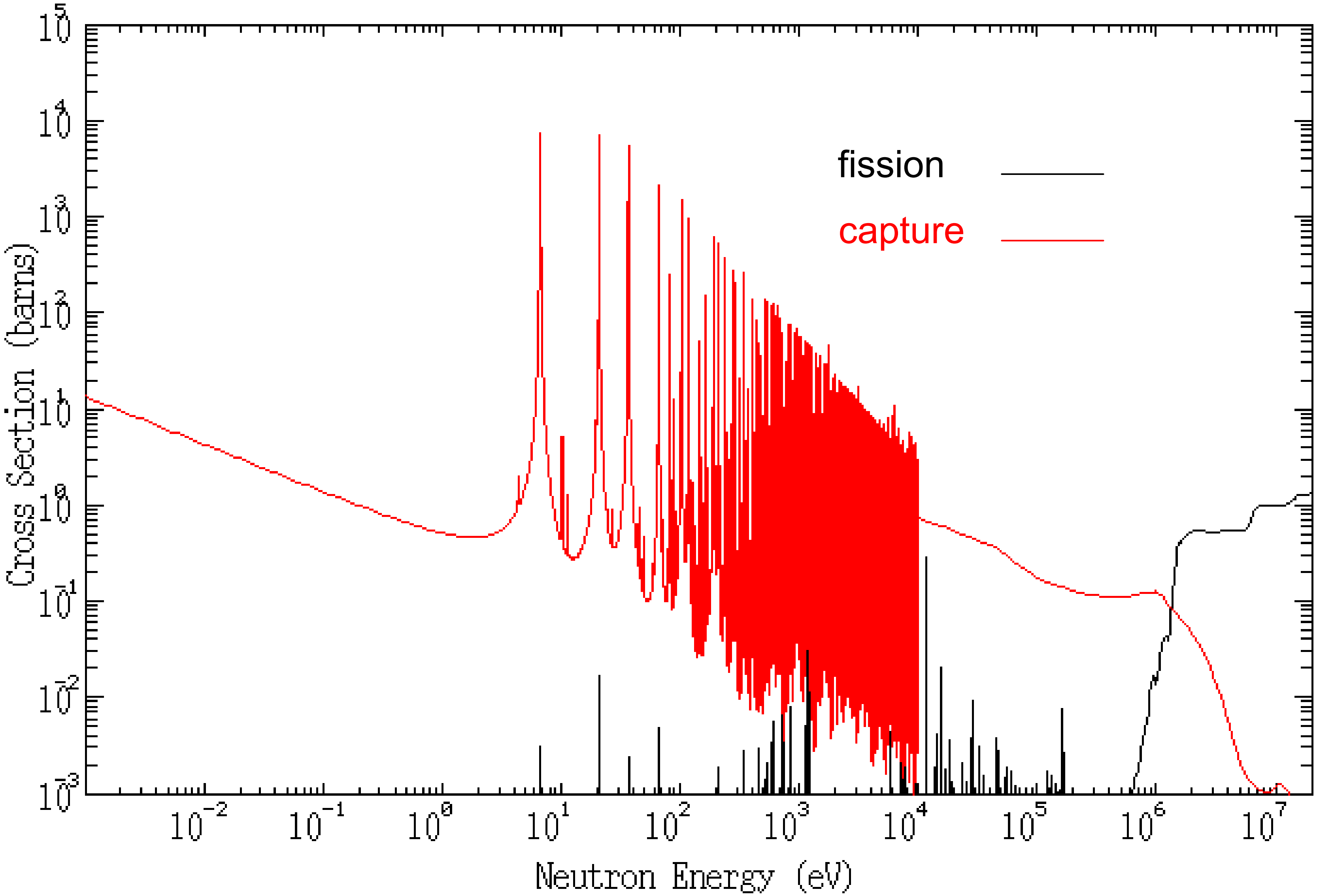
In the quantum mechanics, the cross section depends on energy.

At low-energy limit,

^{235}U Cross Sections



^{238}U Cross Sections



Nuclear Landscape

🟥 Ab initio
🟩 Configuration Interaction
🟦 Density Functional Theory