XXVI International Workshop on
Deep Inelastic Scattering and
Related Subjects


# Spin Physics
# at
# an Electron-Ion Collider (EIC)

**Jianwei Qiu**

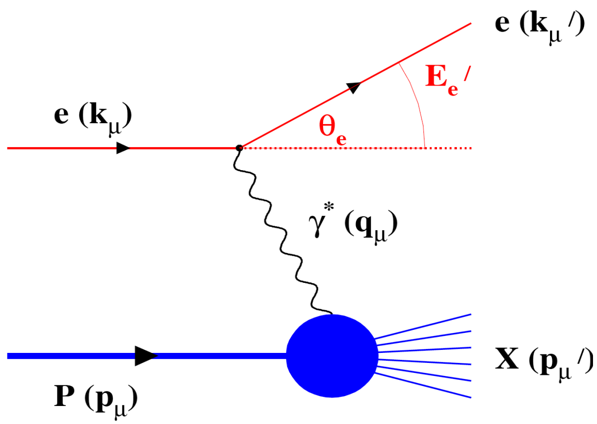
*Theory Center, Jefferson Lab*

*Pre-Workshop at DIS 2018, April, 15, 2018*

Acknowledgement: Much of the physics presented here are based on the work of EIC White Paper Writing Committee put together by BNL and JLab managements, …

# Why a lepton-hadron facility, like EIC, is special?

- **Many complementary probes at one facility:**




$Q^2$ → Measure of resolution

$y$ → Measure of inelasticity

$x$ → Measure of momentum fraction of the struck quark in a proton

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**<u>Inclusive events</u>:** e+p/A → e'+X

**Detect only the scattered lepton in the detector**

**(Modern Rutherford experiment!)**

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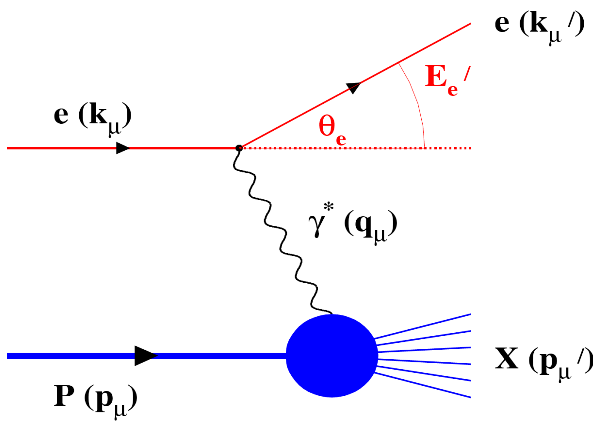


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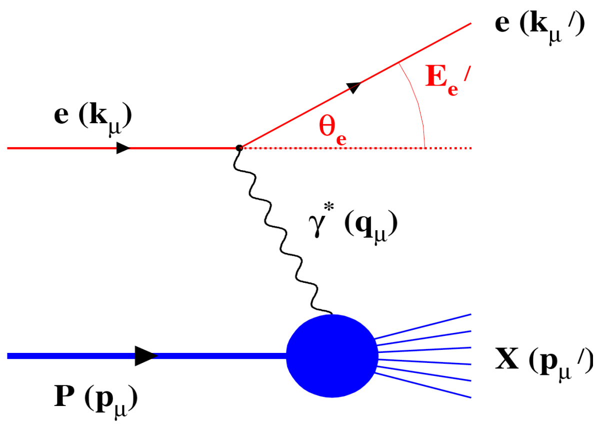


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**Exclusive events:** e+p/A → e'+ p'/A'+ h(π,K,p,jet)

Detect every things including scattered proton/nucleus (or its fragments)

(Initial hadron is NOT broken – tomography! – almost impossible for h-h collisions)

# Next frontier of QCD & hadron physics, …

- How did hadrons, the building blocks of visible world, emerge from quarks and gluons?

  Necessary knowledge for understanding
  where and how did we come from following the "Big Bang?

- What is the internal structure of hadrons, and the dynamics behind the structure?

  Necessary knowledge for understanding
  what are we made of, and
  what hold us together, as well as
  how do we improve and move forward – femtotechnology?

- What is the key for understanding color confinement?

  Necessary knowledge for understanding
  what is the mother nature of the nonlinear,
  strongly interacting dynamics of the color force?

# Dual roles of the EIC spin program

- **Understand the origin of hadron spin:**
  - Hadron, such as proton, is a composite particle of quarks and gluons
  - Spin = Angular momentum of the particle when it is at the rest
  - QCD angular momentum density in terms of energy-momentum tensor

$$M^{\alpha\mu\nu} = T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu} \qquad\qquad J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}$$

  - Proton spin:

$$S(\mu) = \sum_f \langle P, S|\hat{J}_f^z(\mu)|P, S\rangle = \frac{1}{2}$$

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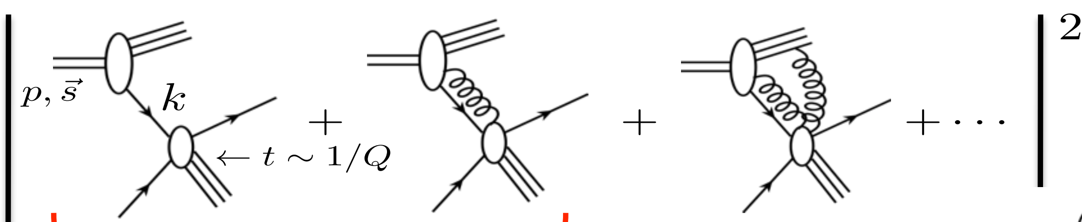
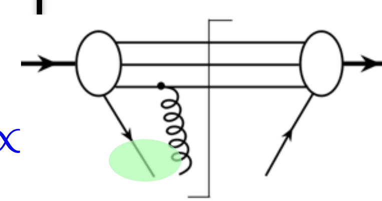
❑ **Use the spin as a tool to access QCD quantum effect:**

- ✧ Cross section is a probability – classically measured

$\sigma(Q, \vec{s}) \propto$ 




$\sigma(s) - \sigma(-s)$ ➡ **Quantum interference** ➡ $T^{(3)}(x, x) \propto$



*Q: How does hadron spin influence its internal structure - dynamics?*

# Polarization and spin asymmetry

*Explore new QCD dynamics – vary the spin orientation*

- **Cross section:**

  Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

- **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$

– Positive definite

- **Asymmetries or difference of cross sections:**

  – Not necessary positive!

  - both beams polarized $A_{LL}, A_{TT}, A_{LT}$

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1, s_2)$$

  - one beam polarized $A_L, A_N$

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

*Chance to see symmetry breaking & quantum interference directly!*

# Intellectual challenge!

- **The challenge:**

  **No modern detector has been able to see quarks and gluons in isolation!**

- **Answer to the challenge:**

  **Theory advances:** **QCD factorization**

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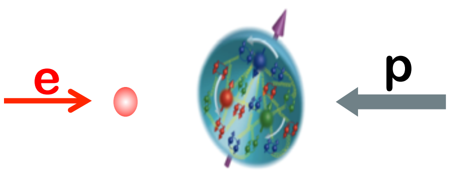
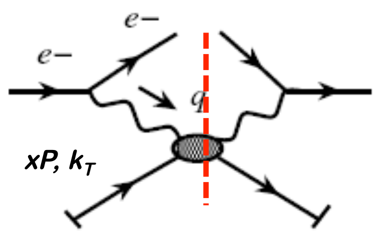
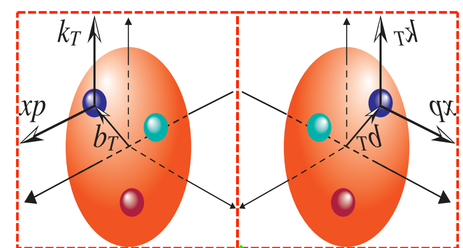



Color entanglement
Approximation

$$\sigma_{\text{tot}}^{\text{DIS}} :$$




$$\otimes$$




$$+ \, O\left(\frac{1}{QR}\right)$$

Physical
Observable

Controllable
Probe

Quantum Probabilities
Structure

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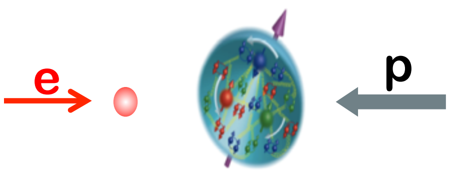


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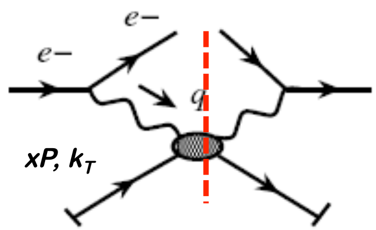


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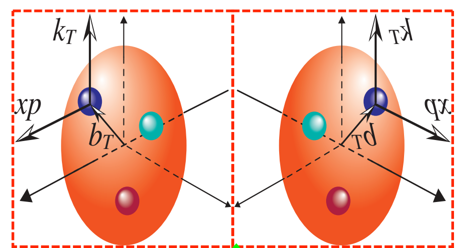


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Physical Observable

Controllable Probe

Quantum Probabilities Structure

**Experimental breakthroughs:**

**Jets** – *Footprints of energetic quarks and gluons*

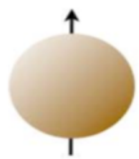
**Quarks** – *Need an EM probe to "see" their existence, …*

**Gluons** – *Varying the probe's resolution to "see" their effect, …*

*Need probes with sub-femtometer resolution – particle nature!*

# Current understanding for Proton Spin

- ❑ **The sum rule:** $$S(\mu) = \sum_f \langle P, S | \hat{J}_f^z(\mu) | P, S \rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$
  - Infinite possibilities of decompositions – connection to observables?
  - Intrinsic properties + dynamical motion and interactions
- ❑ **An incomplete story:**



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Jaffe-Manohar, 90
Ji, 96, …

Proton Spin

Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)$$
$$\sim 30\%$$

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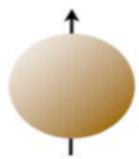

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**Gluon helicity**
**Start to know**

$$\Delta G = \int dx \Delta g(x)$$
$$\sim 20\% (\text{with RHIC data})$$

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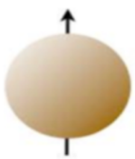

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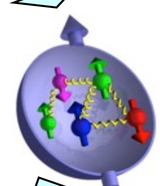

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**Orbital Angular Momentum**
**of quarks and gluons**
**Little known**

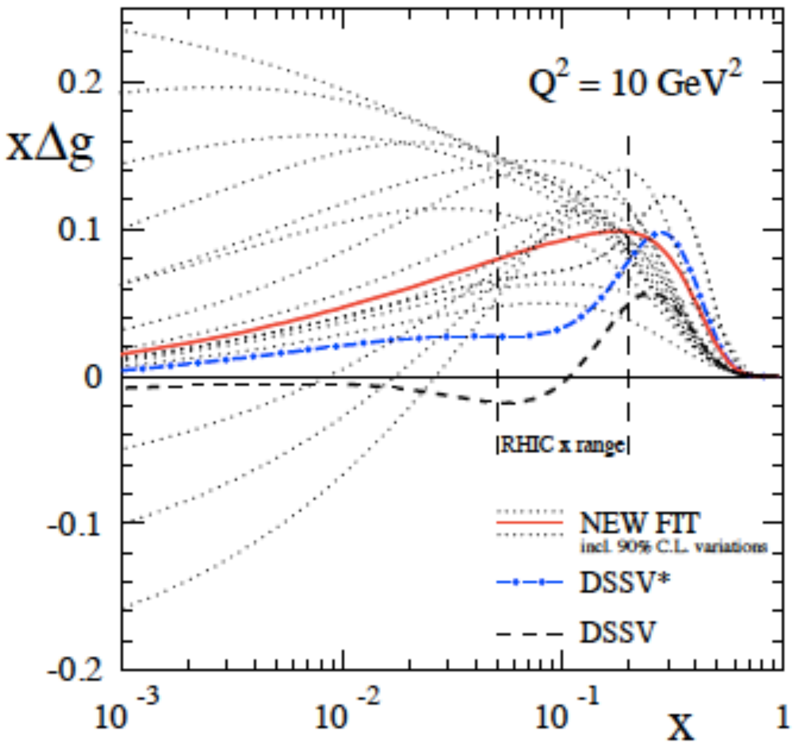
***Net effect of partons' transverse motion?***

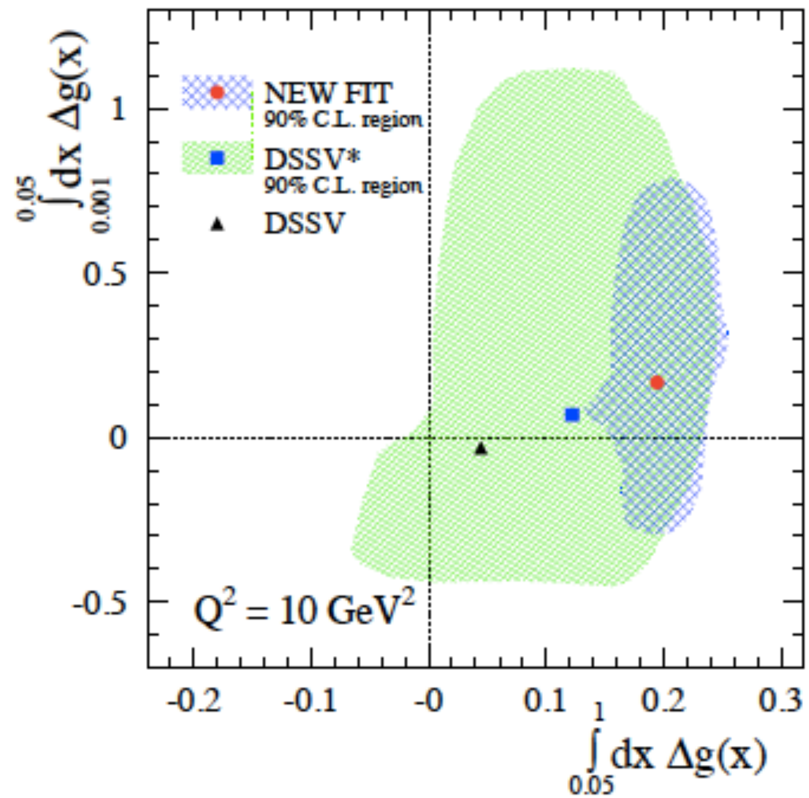
# Global QCD analysis of helicity PDFs

D. de Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014) 012001

results featured in Sci. Am., Phys. World, ...

❑ Impact on gluon helicity:



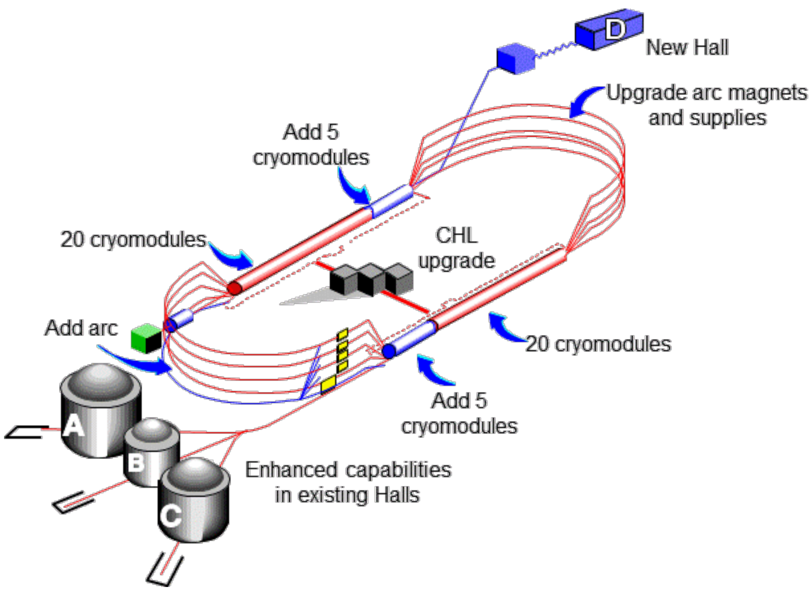

✧ Red line is the new fit
✧ Dotted lines = other fits with 90% C.L.

✧ 90% C.L. areas
✧ Leads $\Delta$G to a positive #

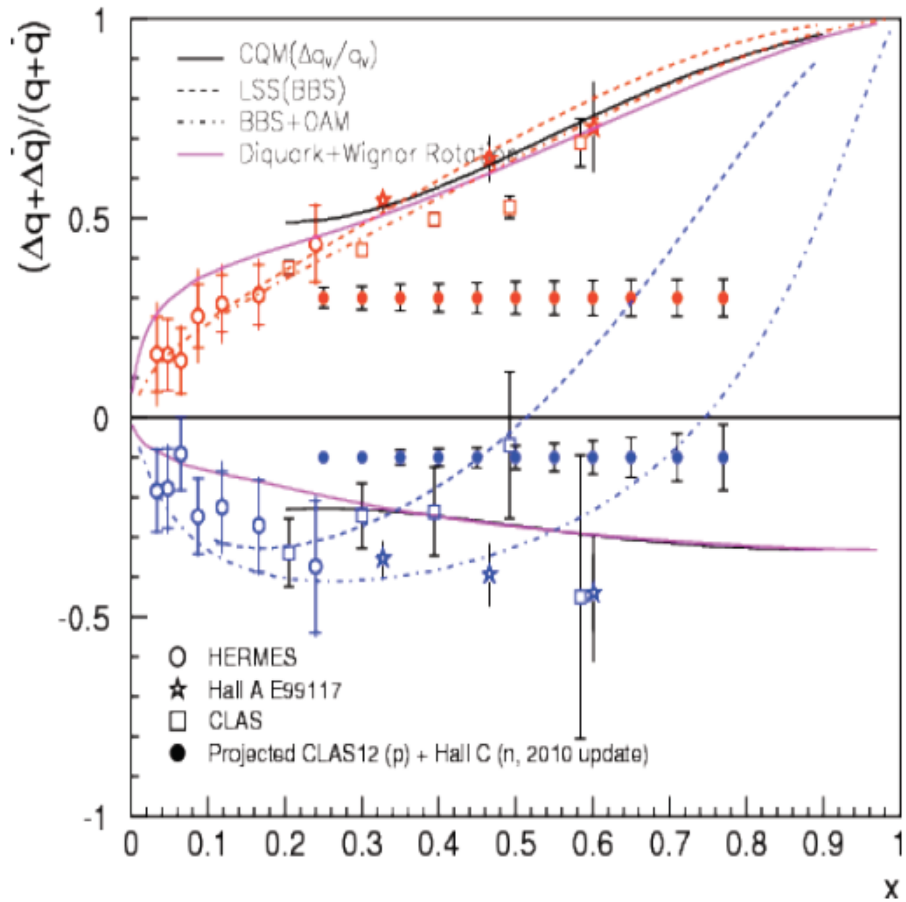
# What is next?

- **JLab 12GeV – upgrade project just completed:**

**12 GeV CEBAF Upgrade Project is just complete, and all 4-Halls are taking data**



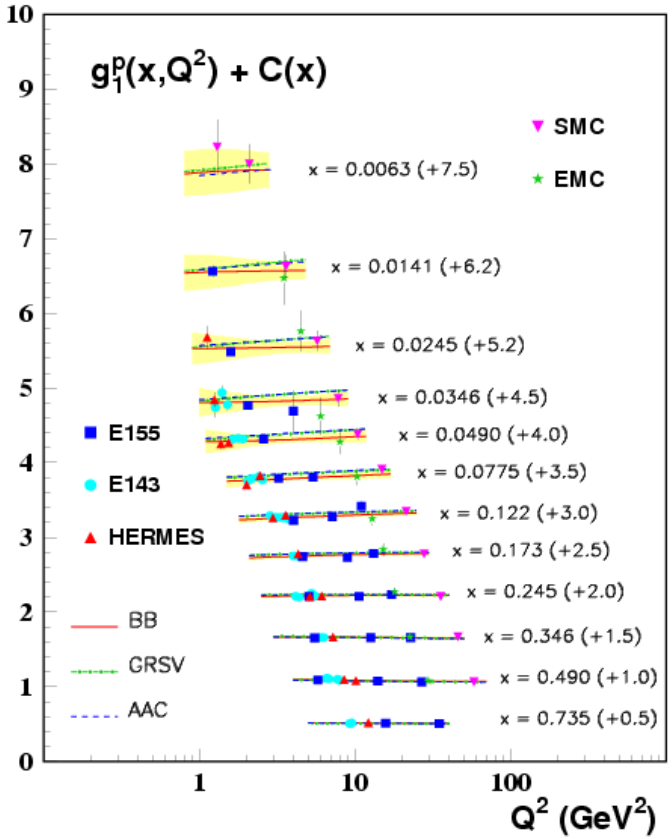

**CLAS12**



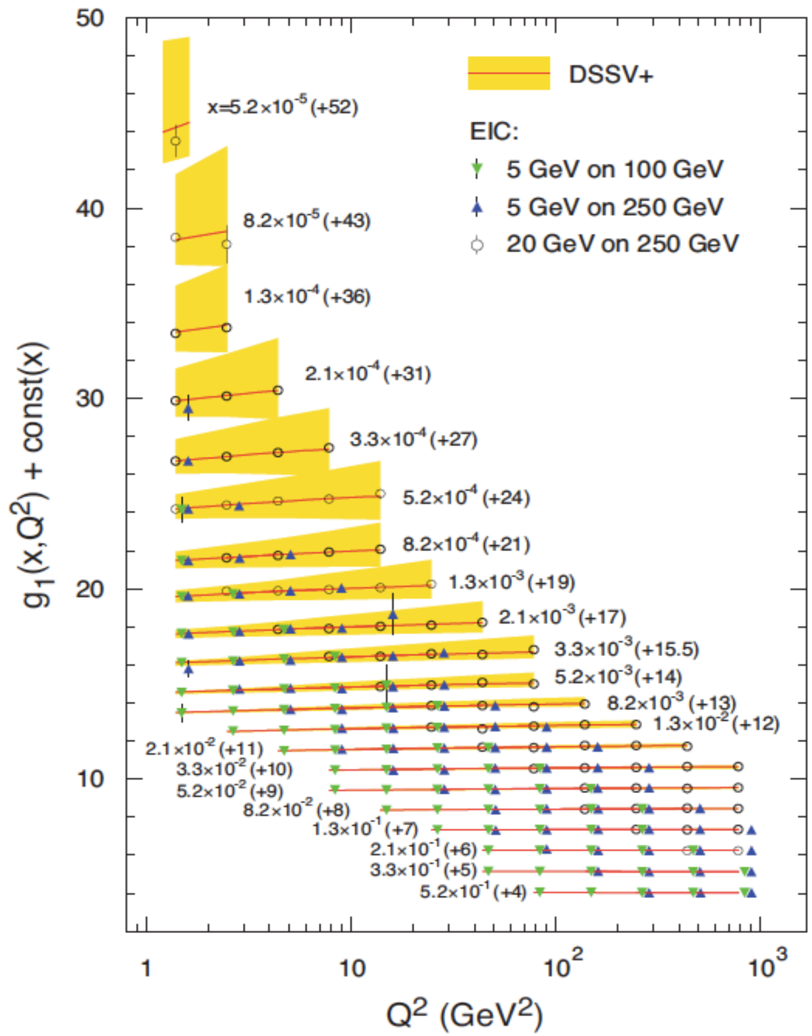

**Plus many more JLab experiments, COMPASS, Fermilab-fixed target expts …**

# The future – what the EIC can do?

- The power & precision of EIC:




at EIC



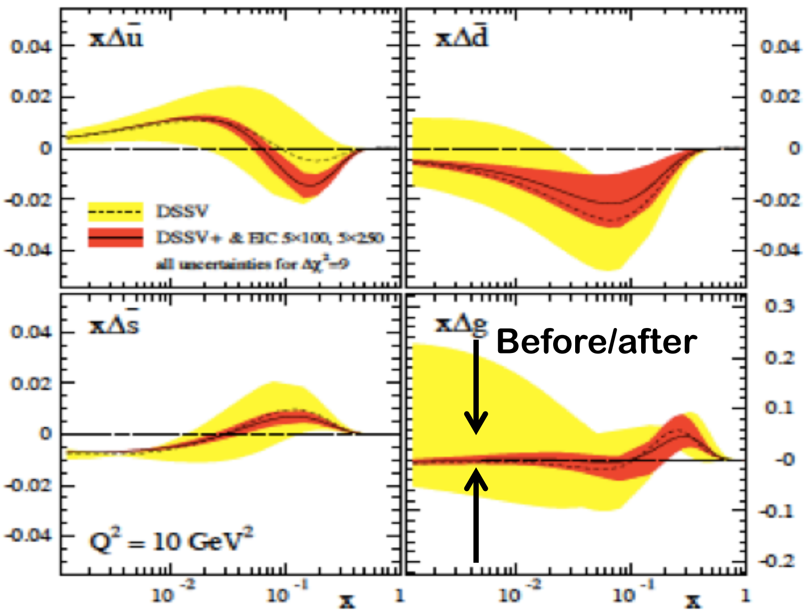

- Reach out the glue:

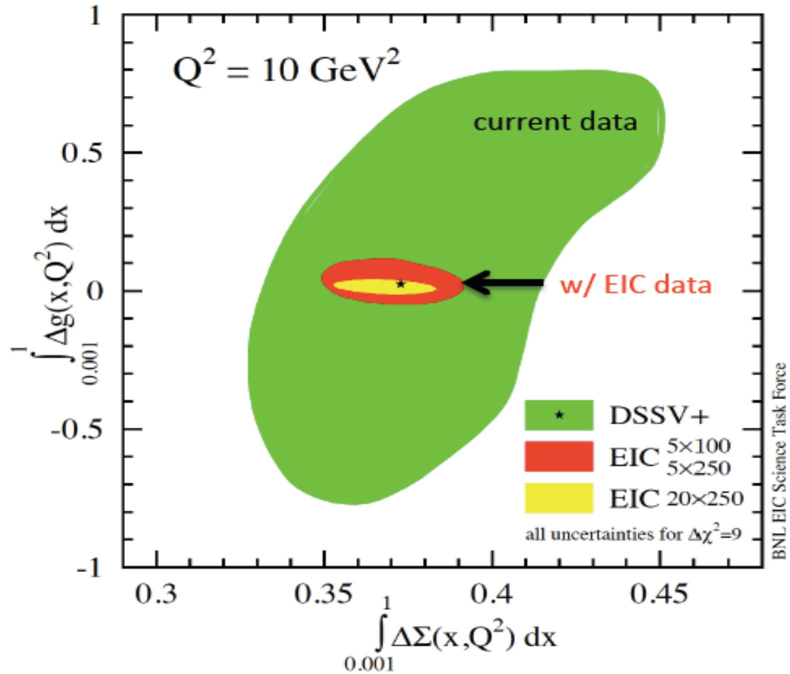
$$\frac{dg_1(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g(x,Q^2) + \cdots$$

# The future – what the EIC can do?

❑ **One-year of running at EIC:**

**Wider $Q^2$ and x range including low x at EIC!**

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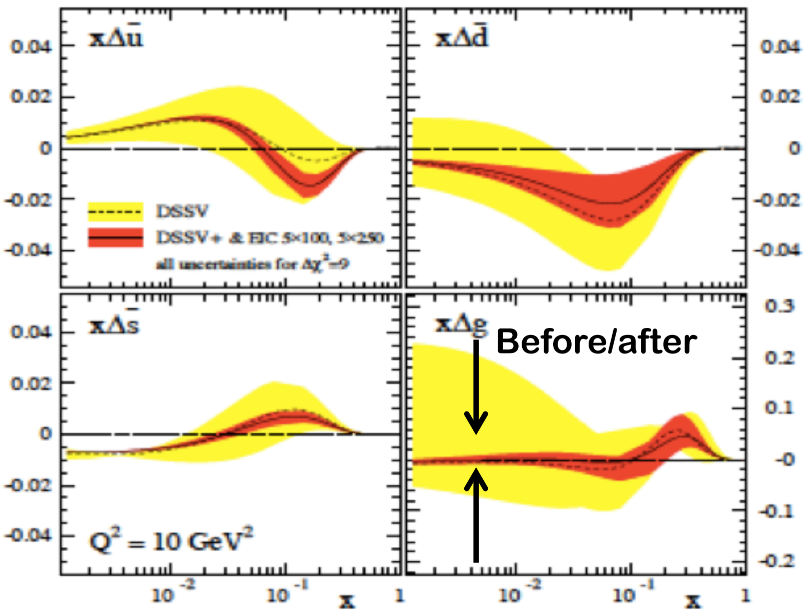


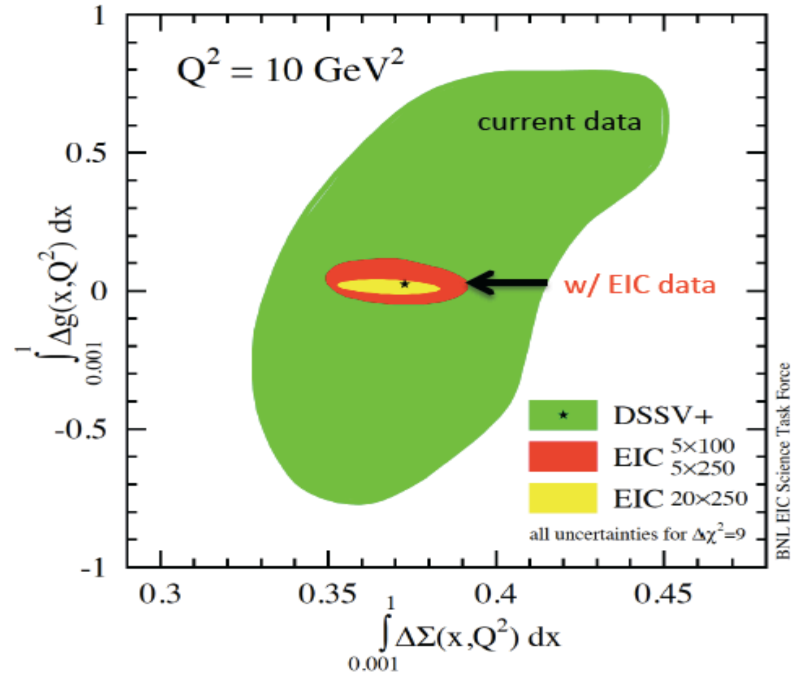


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## ❑ Ultimate solution to the proton spin puzzle:

✧ ***Precision measurement of $\Delta g(x)$ – extend to smaller x regime***

✧ ***Orbital angular momentum contribution – measurement of TMDs & GPDs!***

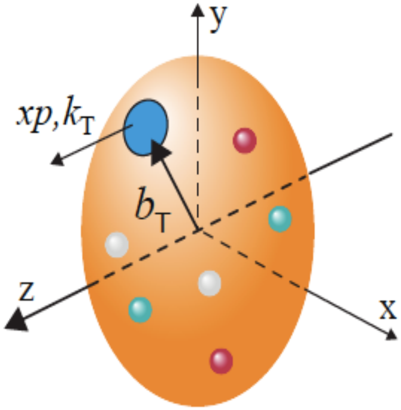
# Paradigm shift: 3D structure of hadrons

❑ **Cross sections with two-momentum scales observed:**

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

✧ **Hard scale:** $Q_1$ **localizes the probe to see the quark or gluon d.o.f.**

✧ **"Soft" scale:** $Q_2$ **could be more sensitive to hadron structure, e.g., confined motion**



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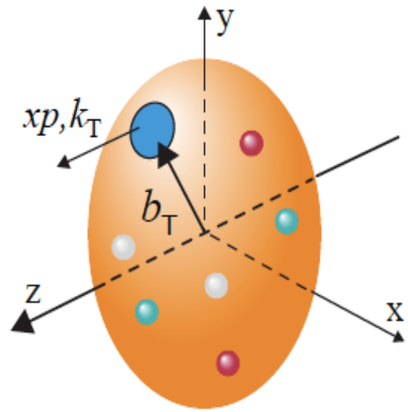


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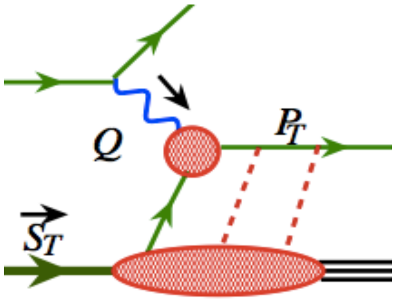
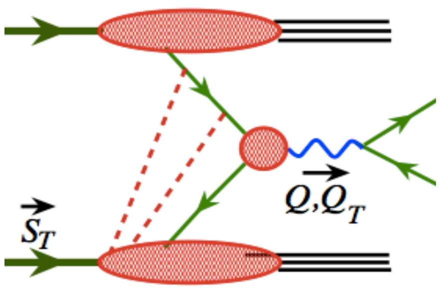



**+**




**+** **Two-jet momentum imbalance in SIDIS, …**

**SIDIS: Q>>$P_T$** **DY: Q>>$P_T$**

➡

- ✧ **Natural observables with TWO very different scales**
- ✧ **TMD factorization: partons' confined motion is encoded into TMDs**

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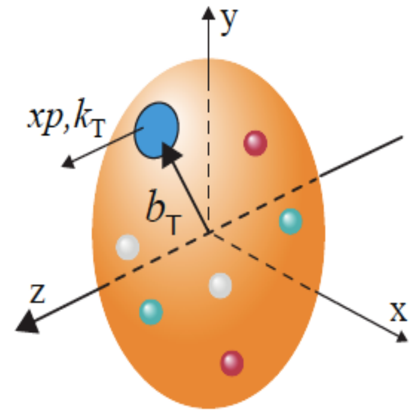


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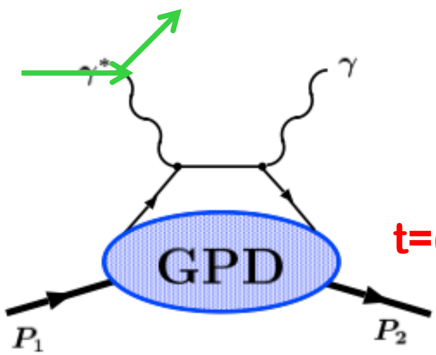
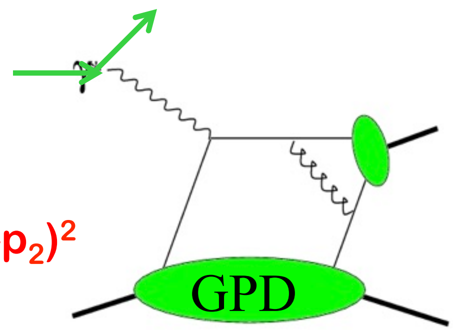



DVCS: $Q^2 >> |t|$

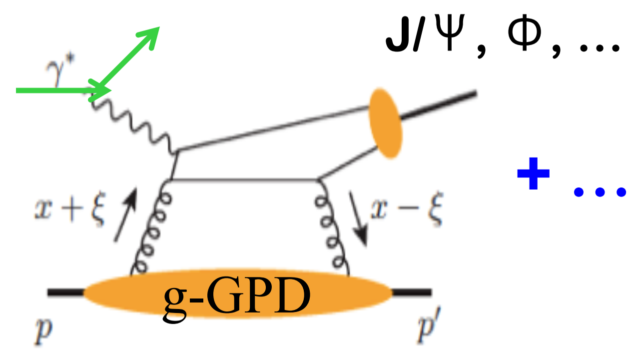
\+

$t=(p_1-p_2)^2$




DVEM: $Q^2 >> |t|$

\+




EHMP: $Q^2 >> |t|$

\+ ...

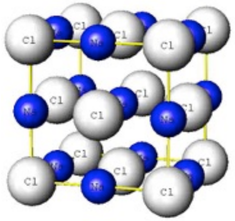
✧ Natural observables with TWO very different scales

✧ GPDs: Fourier Transform of t-dependence gives spatial $b_T$-dependence

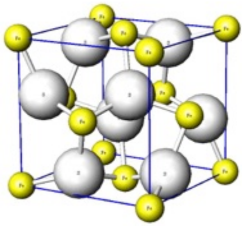
# Hadron's partonic structure in QCD

- **Structure – "a still picture"**

Crystal Structure:

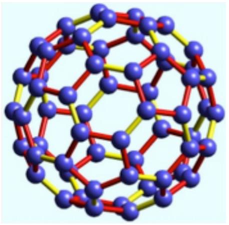


NaCl,
B1 type structure



FeS2,
C2, pyrite type structure

Nano-material:



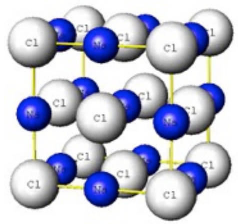
Fullerene, C60

***Motion of nuclei is much slower than the speed of light!***

# Hadron's partonic structure in QCD

- **Structure – "a still picture"**

**Crystal Structure:**



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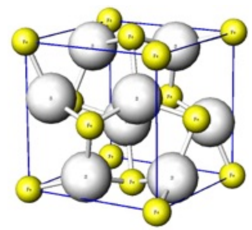

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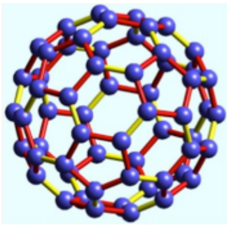
C2, pyrite type structure

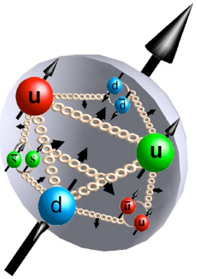
**Nano-material:**



Fullerene, C60

***Motion of nuclei is much slower than the speed of light!***

- **No "still picture" for hadron's partonic structure!**



***Motion of quarks/gluons is relativistic!***

**Partonic Structure:**

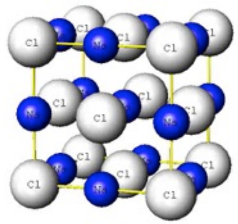
Quantum "probabilities" $\langle P, S | \mathcal{O}(\overline{\psi}, \psi, A^{\mu}) | P, S \rangle$

***None of these matrix elements is a direct physical observable in QCD – color confinement!***

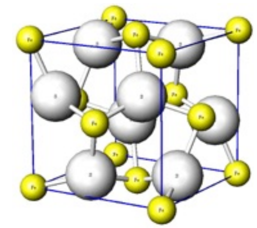
# Hadron's partonic structure in QCD

## ❑ Structure – "a still picture"

Crystal Structure:

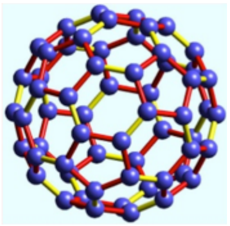


NaCl,
B1 type structure



FeS2,
C2, pyrite type structure

Nano-material:



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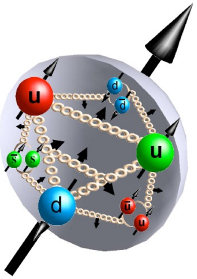


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Partonic Structure:

Quantum "probabilities" $\langle P,S|\mathcal{O}(\overline{\psi},\psi,A^{\mu})|P,S\rangle$

*None of these matrix elements is a direct physical observable in QCD – color confinement!*

## ❑ Accessible hadron's partonic structure?

= Universal matrix elements of quarks and/or gluons

1) can be related to good physical cross sections of hadron(s) *with controllable approximation*,

2) can be calculated in lattice QCD, …

# Unified description of hadron structure

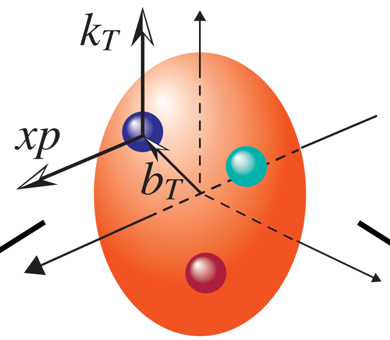
- **Wigner distributions in 5D (or GTMDs):**

*Momentum Space*

*TMDs*

Confined motion

$\int d^2 b_T$




$\int d^2 k_T$

*Coordinate Space*

*GPDs*

Spatial distribution

$f(x, k_T)$ $\boxed{\text{Two-scales observables}}$ $f(x, b_T)$

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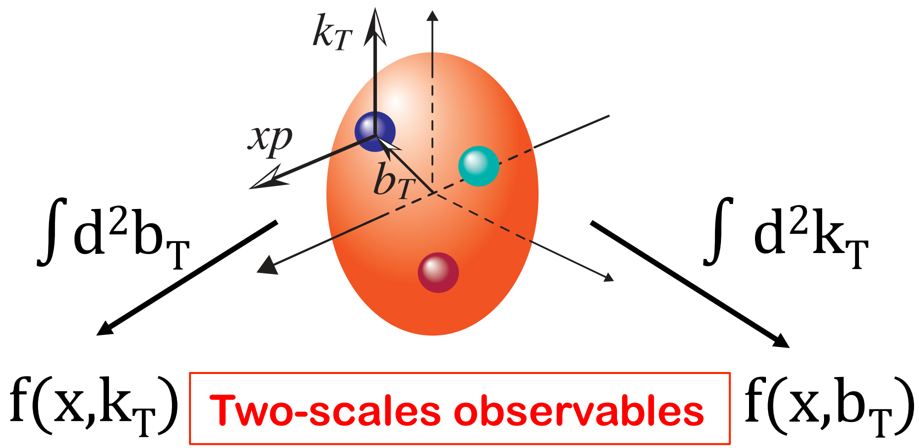



*Coordinate Space*

*GPDs*

Spatial distribution

## ❑ Theory is solid – *TMDs & SIDIS as an example*:

✧ **Low $P_{hT}$ ($P_{hT}$ << Q) – TMD factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f(x, k_\perp) \otimes \mathcal{D}_{f\to h}(z, p_\perp) \otimes \mathcal{S}(k_{s\perp}) + \mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

✧ **High $P_{hT}$ ($P_{hT}$ ~ Q) – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, P_{h\perp}, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)$$

✧ **$P_{hT}$ Integrated - Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, x_B, z_h) = \tilde{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f\to h} + \mathcal{O}\left(\frac{1}{Q}\right)$$

✧ **Very high $P_{hT}$ >> Q – Collinear factorization:**

$$\sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \sum_{abc} \hat{H}_{ab\to c} \otimes \phi_{\gamma\to a} \otimes \phi_b \otimes D_{c\to h} + \mathcal{O}\left(\frac{1}{Q}, \frac{Q}{P_{h\perp}}\right)$$

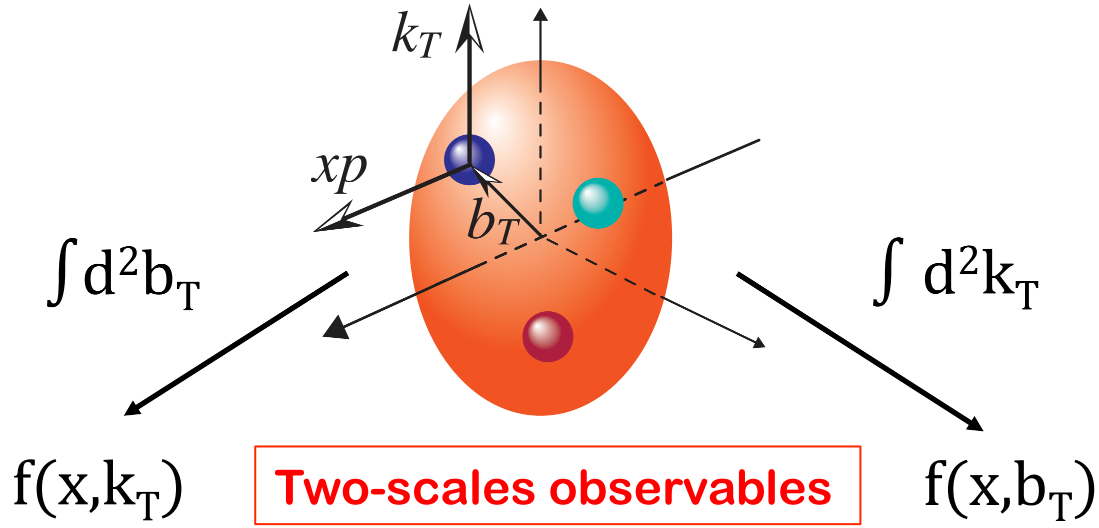
# Advantages of the lepton-hadron facilities

- **3D boosted partonic structure:**

*Momentum Space*

*TMDs*

Confined motion



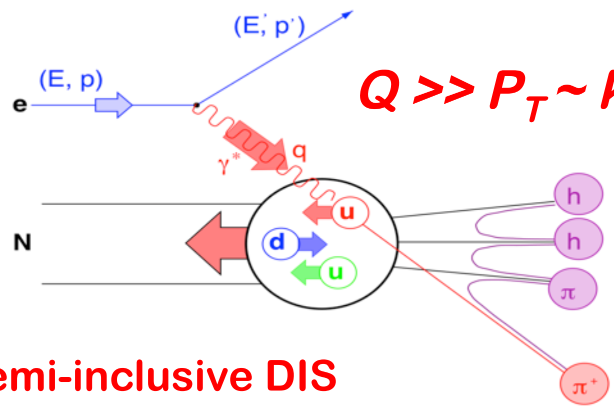

Two-scales observables

*Coordinate Space*

*GPDs*

Spatial distribution

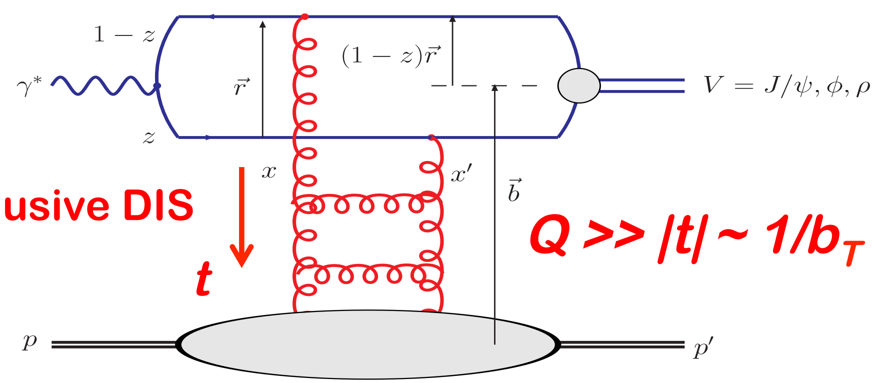
3D momentum-space images




$Q >> P_T \sim k_T$

Semi-inclusive DIS

2+1D coordinate-space images

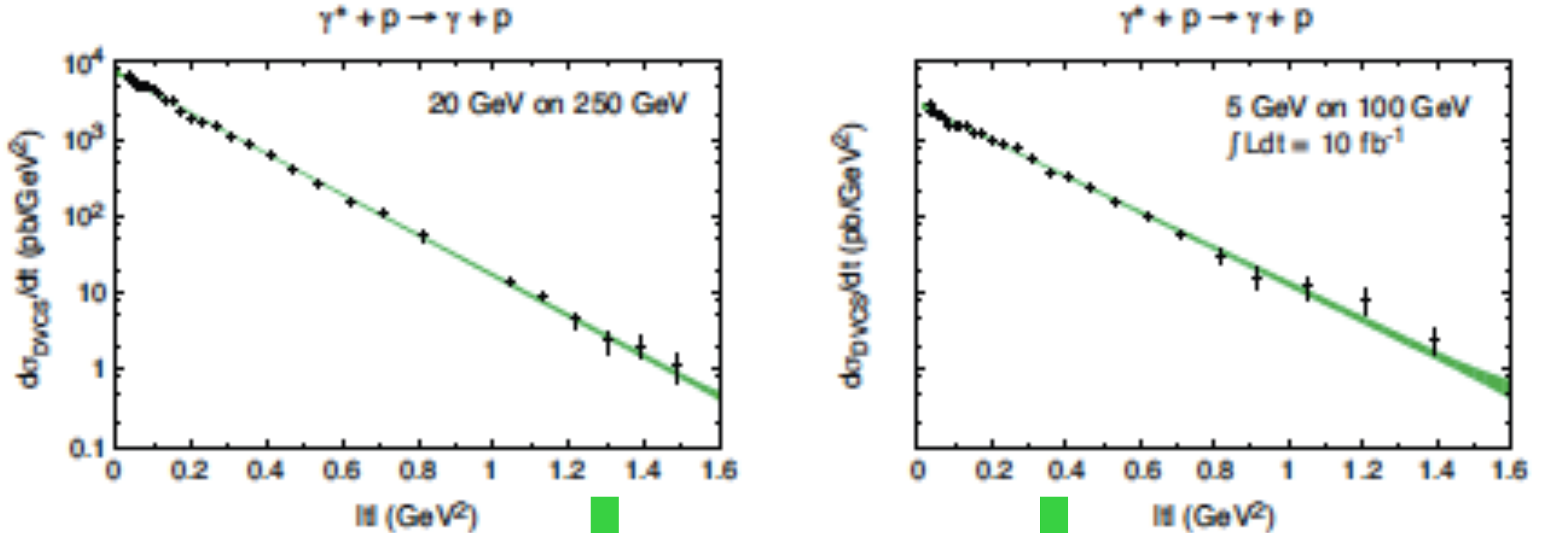



$Q >> |t| \sim 1/b_T$

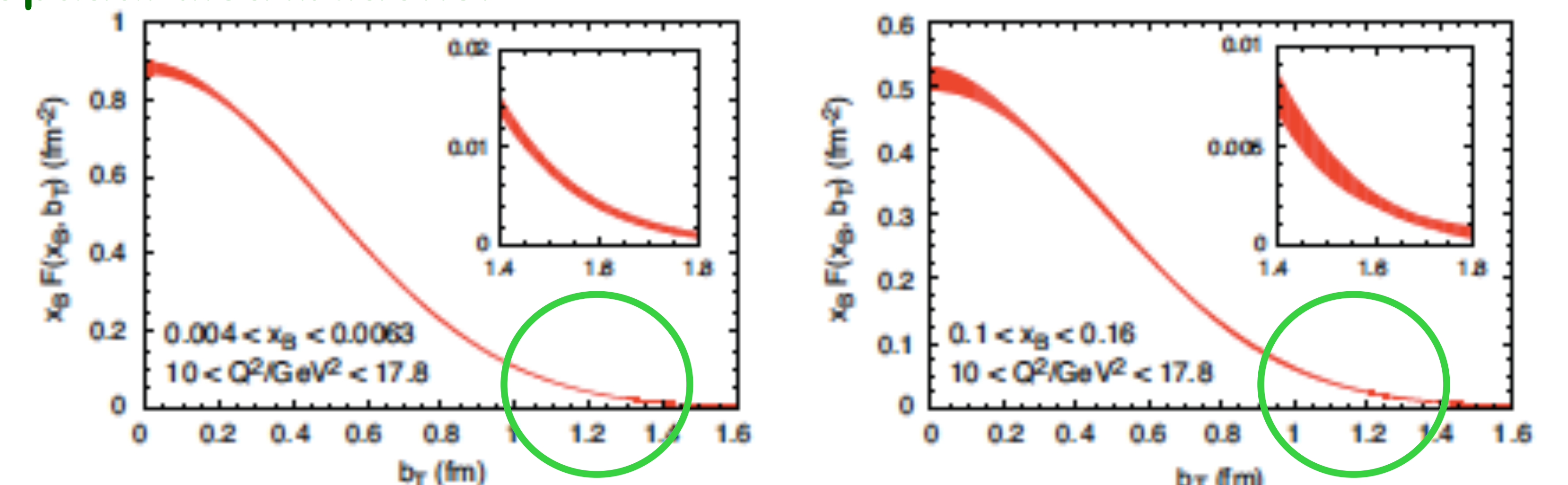
JLab12 – valence quarks, EIC – sea quarks and gluons

# DVCS @ EIC

- ❑ Cross Sections:



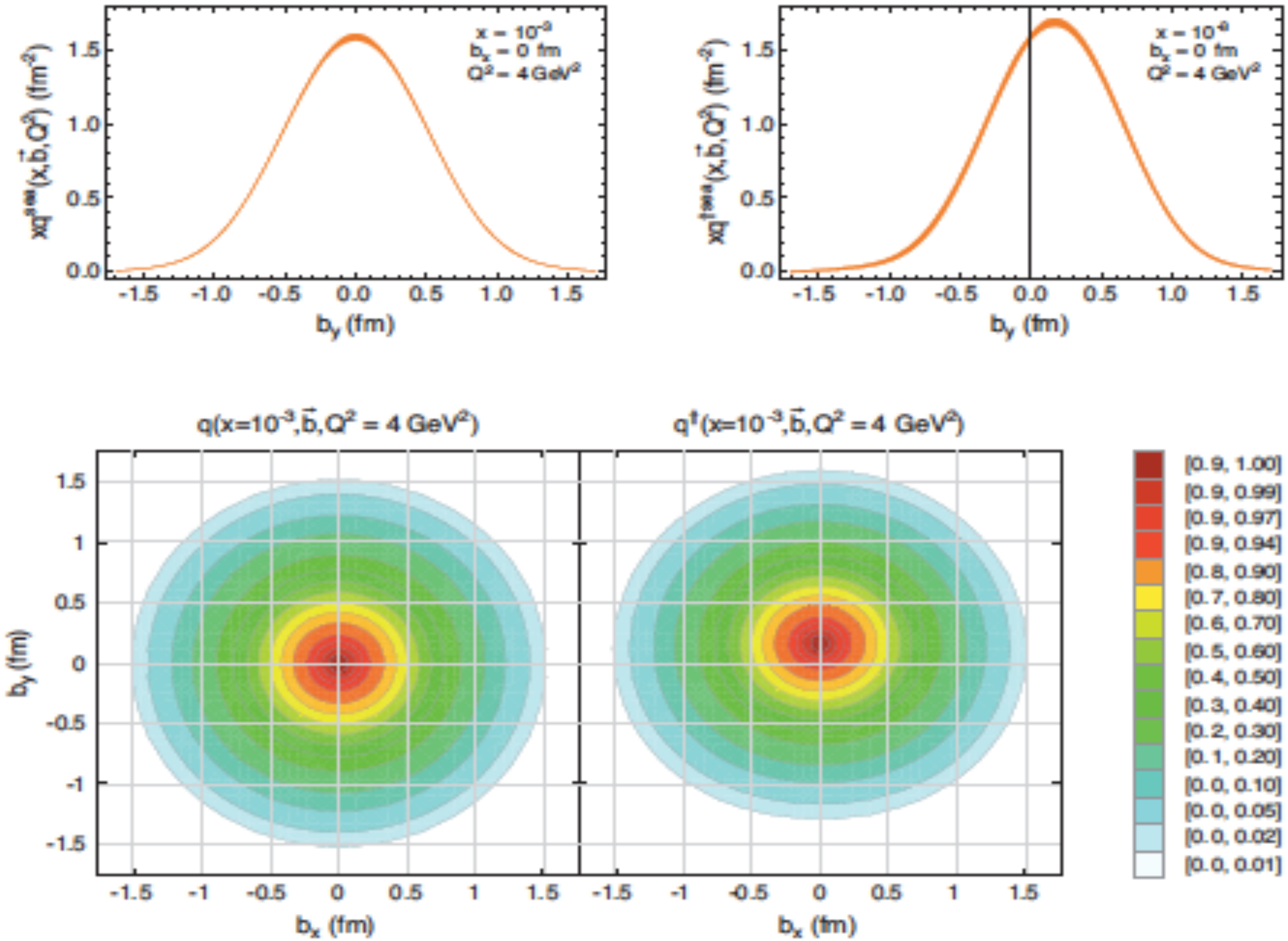

- ❑ Spatial distributions:




Quark radius (x)!

# Polarized DVCS @ EIC

- Spin-motion correlation:

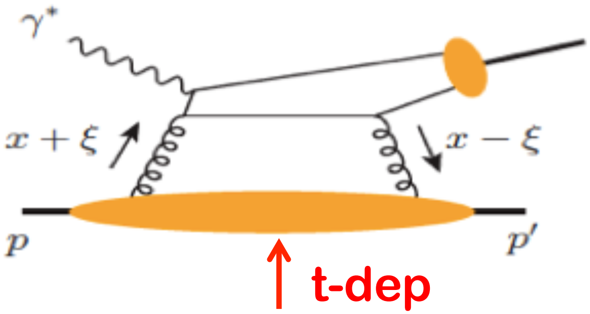


# Spatial distribution of gluons

- **Exclusive vector meson production:**

$$\frac{d\sigma}{dx_B dQ^2 dt}$$

EIC-WhitePaper



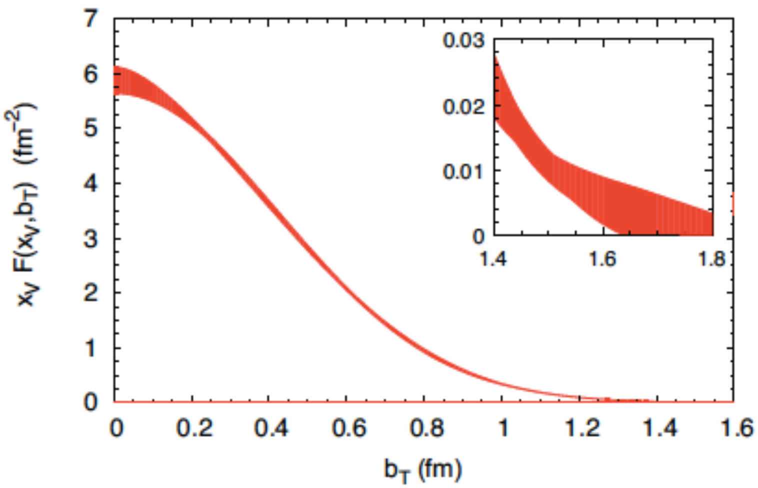

J/Ψ, Φ, …

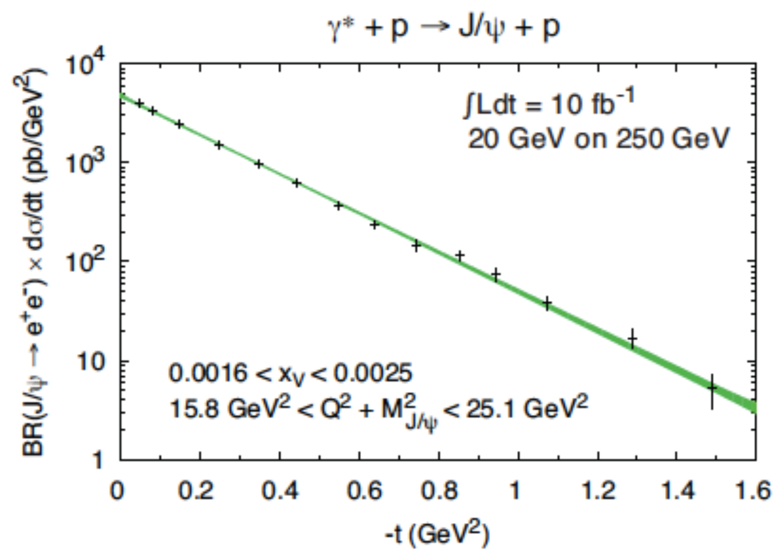
✧ Fourier transform of the t-dep

Spatial imaging of glue density

✧ Resolution ~ 1/Q or 1/$M_Q$

- **Gluon imaging from simulation:**



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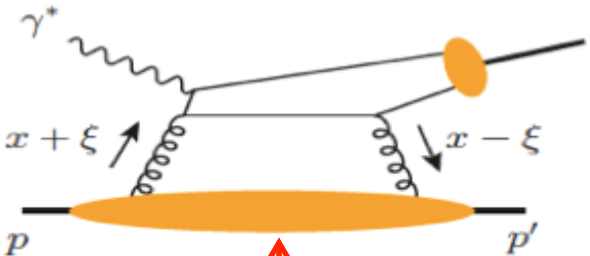


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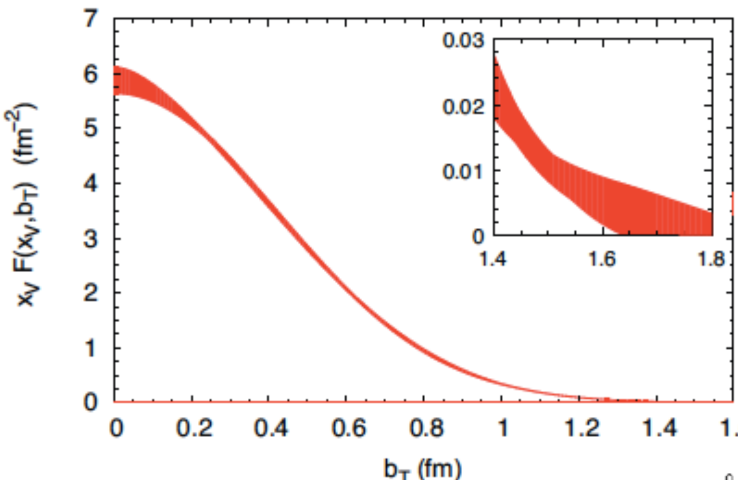
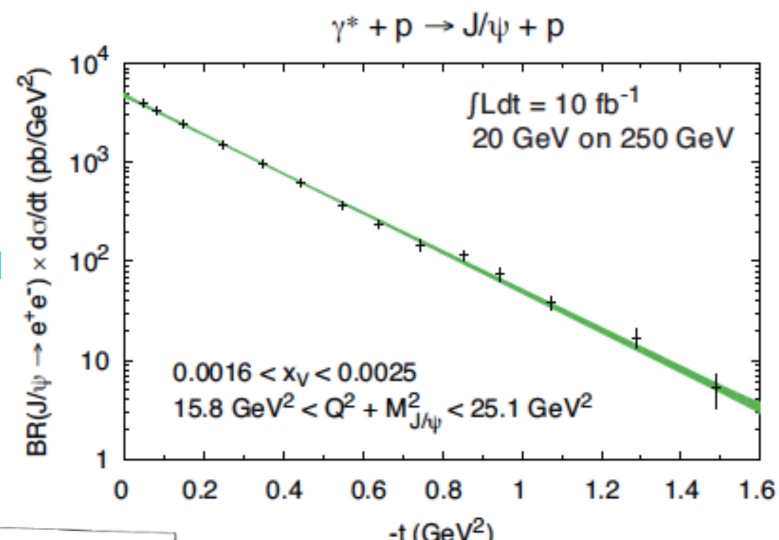
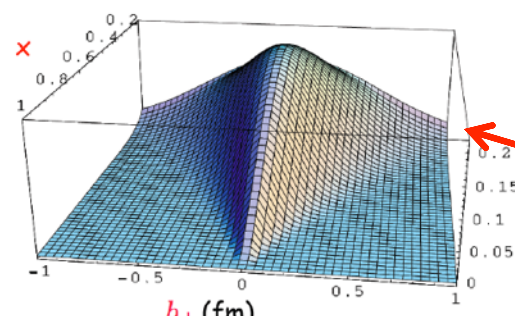



*Only possible at the EIC*

*Gluon radius?*

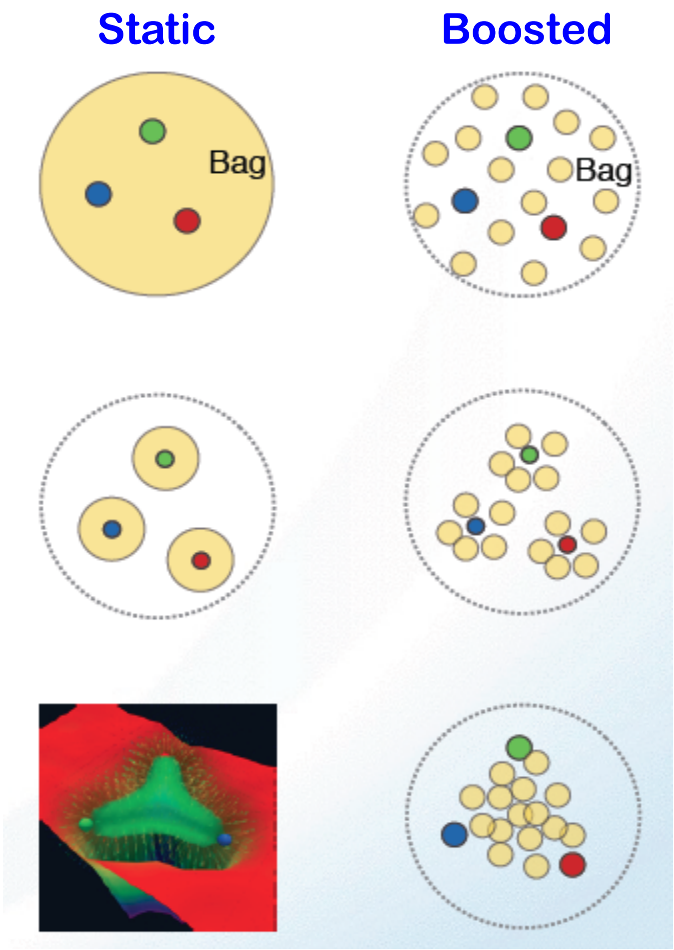
Gluon radius (x)!

How spread at small-x?

Color confinement

# Why 3D nucleon structure?

❑ **Spatial distributions of quarks and gluons:**




**Bag Model:**
Gluon field distribution is wider than the fast moving quarks.
Gluon radius > Charge Radius

**Constituent Quark Model:**
Gluons and sea quarks hide inside massive quarks.
Gluon radius ~ Charge Radius

**Lattice Gauge theory (with slow moving quarks):**
Gluons more concentrated inside the quarks
Gluon radius < Charge Radius

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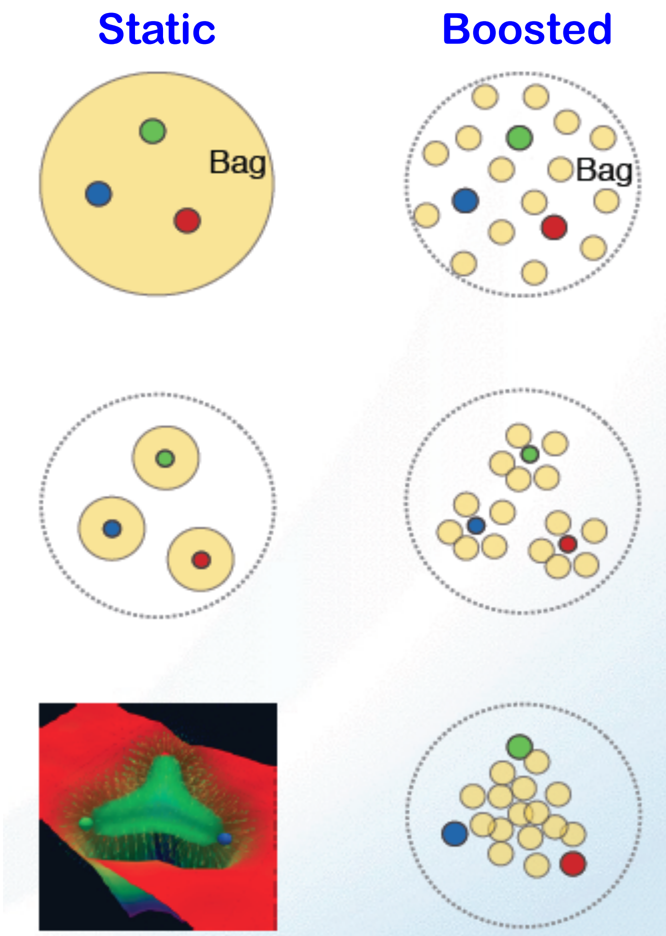


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*3D confined motion (TMDs) + spatial distribution (GPDs)*
*Hints on the color confining mechanism*
Relation between charge radius, quark radius (x), and gluon radius (x)?

# Orbital angular momentum

**OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense**

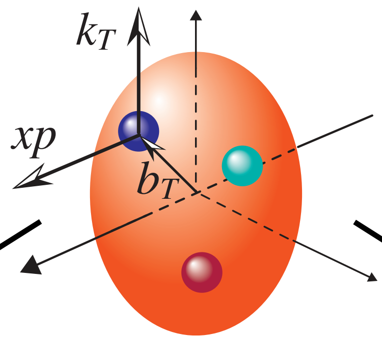
- **Wigner distribution:**

*Momentum Space*

*TMDs*

Confined motion

$\int d^2 b_T$




$\int d^2 k_T$

*Coordinate Space*

*GPDs*

Spatial distribution

$f(x,k_T)$ Two-scales observables $f(x,b_T)$

**Position *r* x Momentum *p* → Orbital Motion of Partons**

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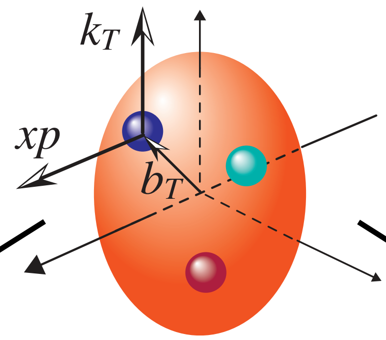


$\int d^2 k_T$

*Coordinate Space*

*GPDs*

Spatial distribution

$f(x,k_T)$ $\boxed{\text{Two-scales observables}}$ $f(x,b_T)$

- **Note:**

  ✧ Partons' confined motion and their spatial distribution are unique – the consequence of QCD

  ✧ But, the TMDs and GPDs that represent them are not unique!

  – Depending on the definition of the Wigner distribution and QCD factorization to link them to physical observables

**Position *r* x Momentum *p* → Orbital Motion of Partons**

# Orbital angular momentum

**OAM: Its definition is not unique in gauge field theory!**

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

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- **Difference between them:**

Hatta, Lorce, Pasquini, …

✧ compensated by difference between gluon OAM density

✧ represented by different choice of gauge link for OAM Wagner distribution

$$\mathcal{L}_q^3 \left\{L_q^3\right\} = \int dx\, d^2b\, d^2k_T \left[\vec{b} \times \vec{k}_T\right]^3 \mathcal{W}_q(x,\vec{b},\vec{k}_T) \left\{W_q(x,\vec{b},\vec{k}_T)\right\}$$

with

$$\mathcal{W}_q\left\{W_q\right\}(x,\vec{b},\vec{k}_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\vec{\Delta}_T\cdot\vec{b}} \int \frac{dy^- d^2y_T}{(2\pi)^3} e^{i(xP^+y^- - \vec{k}_T\cdot\vec{y}_T)}$$

$$\times \langle P'|\,\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\,\underbrace{\Phi^{\mathrm{JM}\{\mathrm{Ji}\}}(0,y)}_{\text{Gauge link}}\,\psi(y)\,|P\rangle_{y^+=0}$$

JM: "staple" gauge link

Ji: straight gauge link

between 0 and $y=(y^+=0, y^-, y_T)$

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- **Difference between them:**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

✧ **generated by a "torque" of color Lorentz force**

$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2 y_T}{(2\pi)^3} \langle P' | \overline{\psi}_q(0) \frac{\gamma^+}{2} \int_{y^-}^{\infty} dz^- \Phi(0, z^-)$$
$$\times \underbrace{\sum_{i,j=1,2} \left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{"Chromodynamic torque"}} \Phi(z^-, y)\psi(y)|P\rangle_{y^+=0}$$

**Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of $g_2$**

# Unified view of nucleon structure

- **Wigner distributions:**

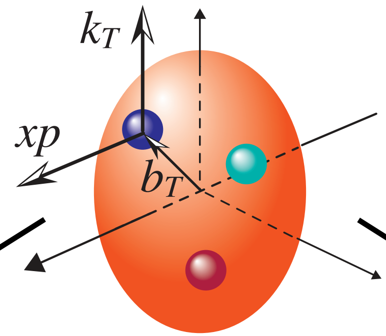
***Momentum Space***

***TMDs***

**Confined motion**

$\int d^2b_T$

$f(x,k_T)$




Two-scales observables

$\int d^2k_T$

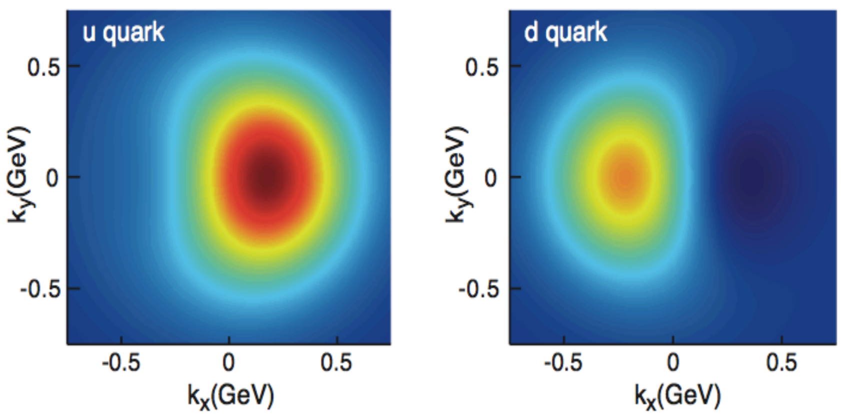
$f(x,b_T)$

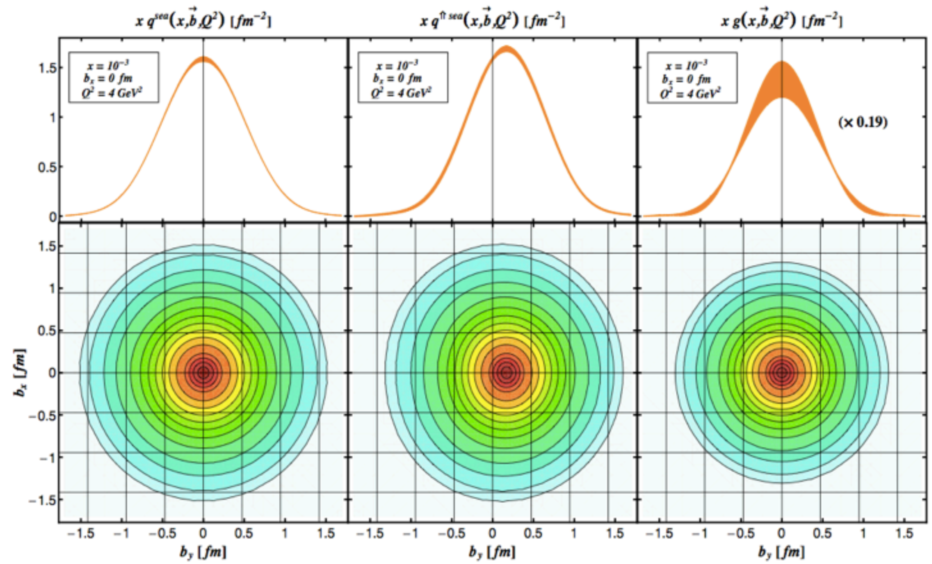
***Coordinate Space***

***GPDs***

**Spatial distribution**

**Sivers Functions**

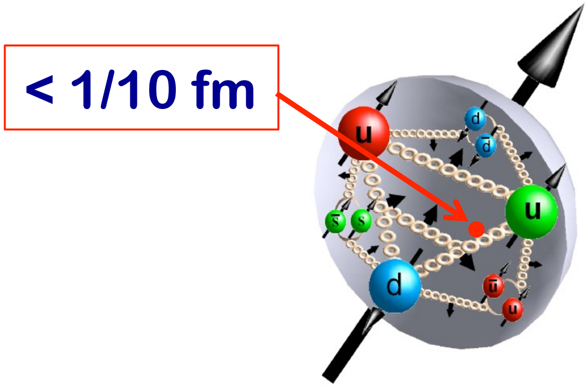



**Position** ***r*** **x Momentum** ***p*** **→ Orbital Motion of Partons**

# Summary

- QCD has been extremely successful in interpreting and predicting high energy experimental data!




- But, we still do not know much about hadron structure – a lot of work to do!

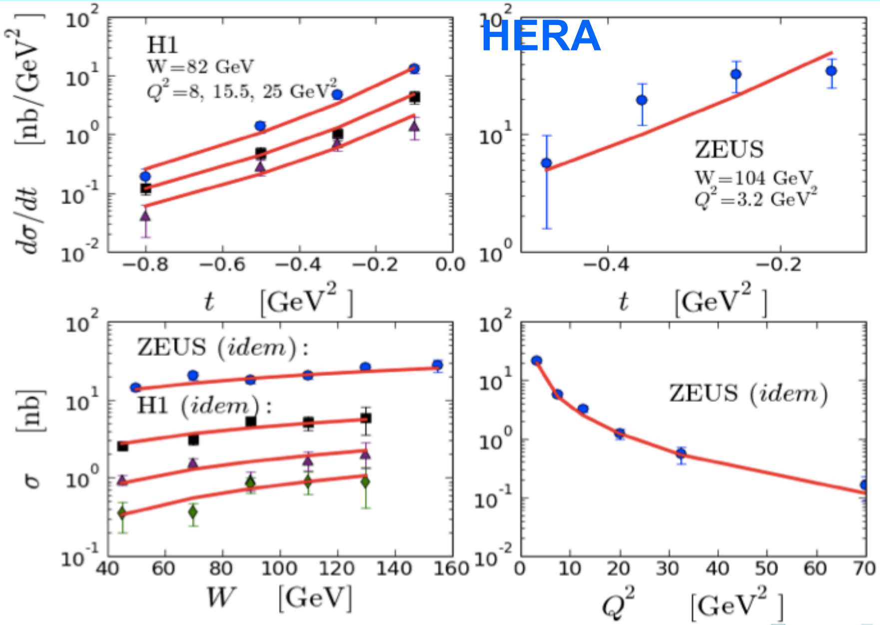
- Since the "spin crisis" in the 80th, we have learned a lot about proton spin – but, still a long way to go!

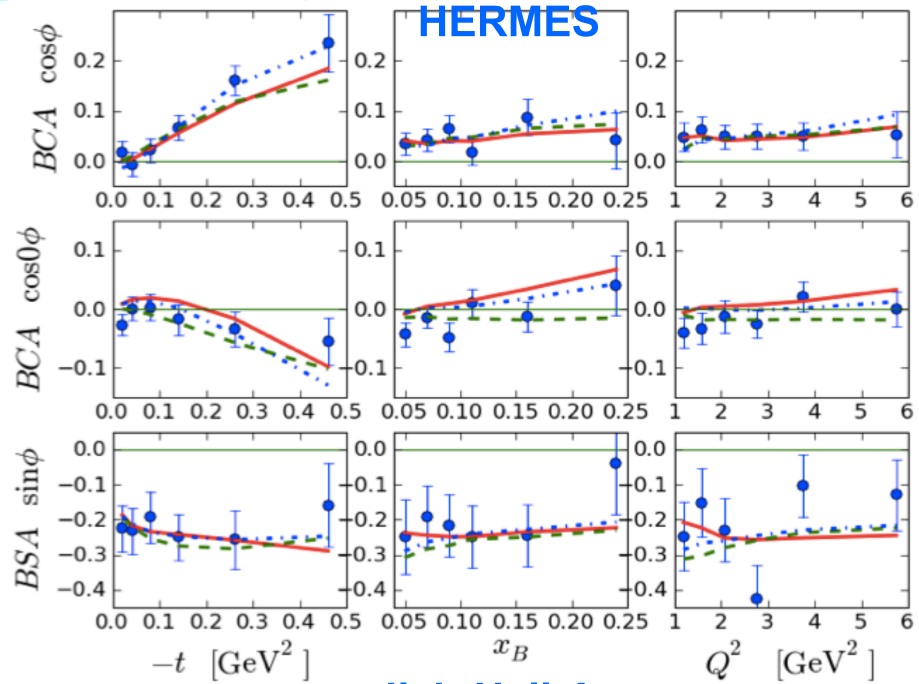
- TMDs and GPDs, accessible by high energy scattering with polarized beams at EIC, carry important information on hadron's 3D structure, and its correlation with hadron's spin!

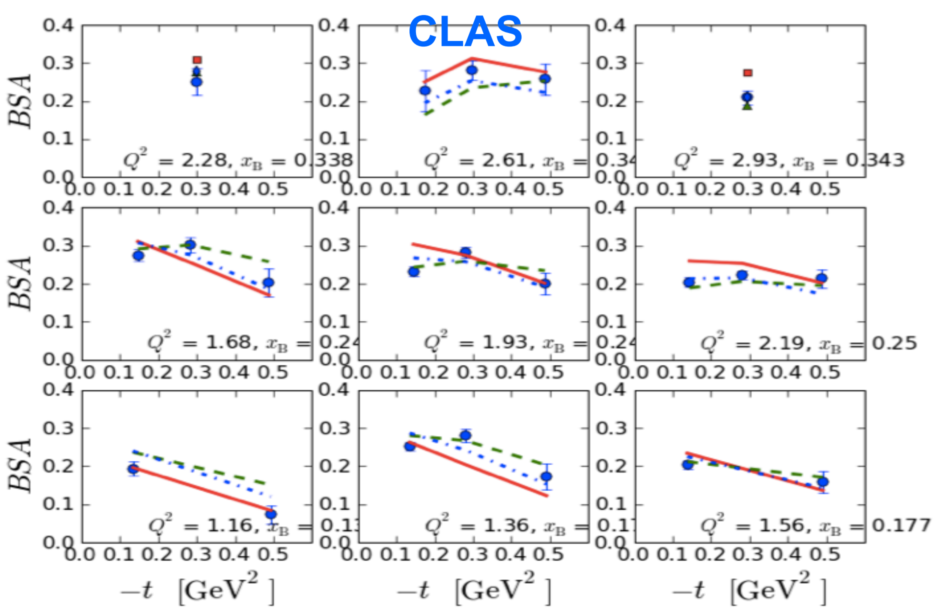
*No "still pictures", but quantum distributions, for hadron structure!*

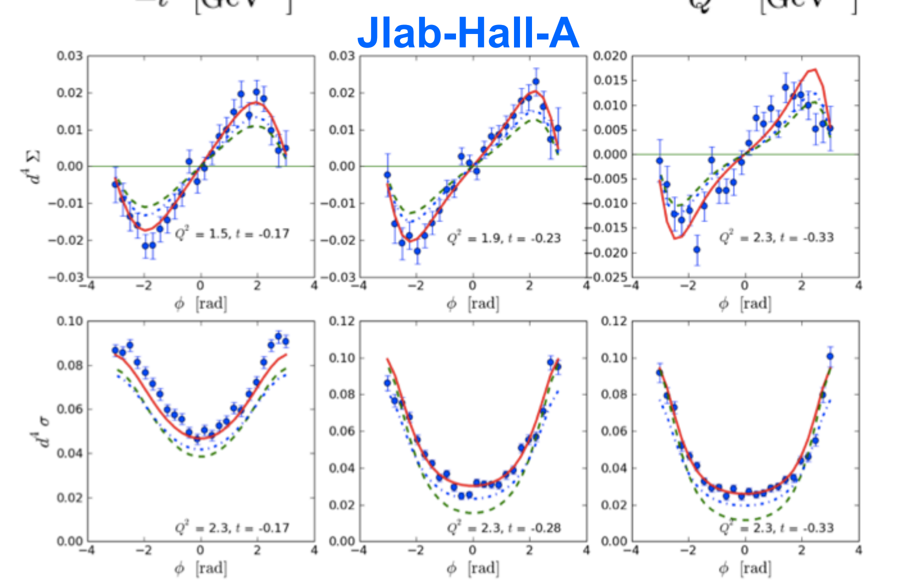
**Thank you!**

# GPDs: just the beginning



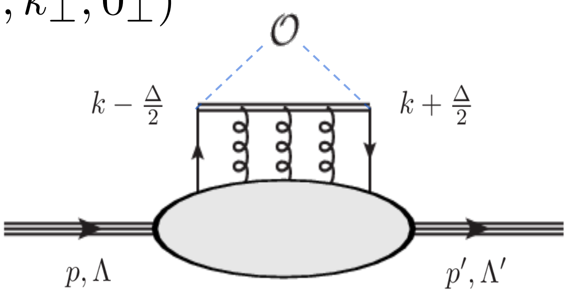
# OAM from Generalized TMDs?

## ❑ Canonical OAM:

GTMD

Meissner, et al. 2008
See talk by Metz
Also, Lorce at ECT*

$$l_q^3 = \int d^2 b_\perp \, [\vec{b}_\perp \times \langle \vec{k}_\perp \rangle (\vec{b}_\perp)] = -\int dx \, d^2 k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp)$$



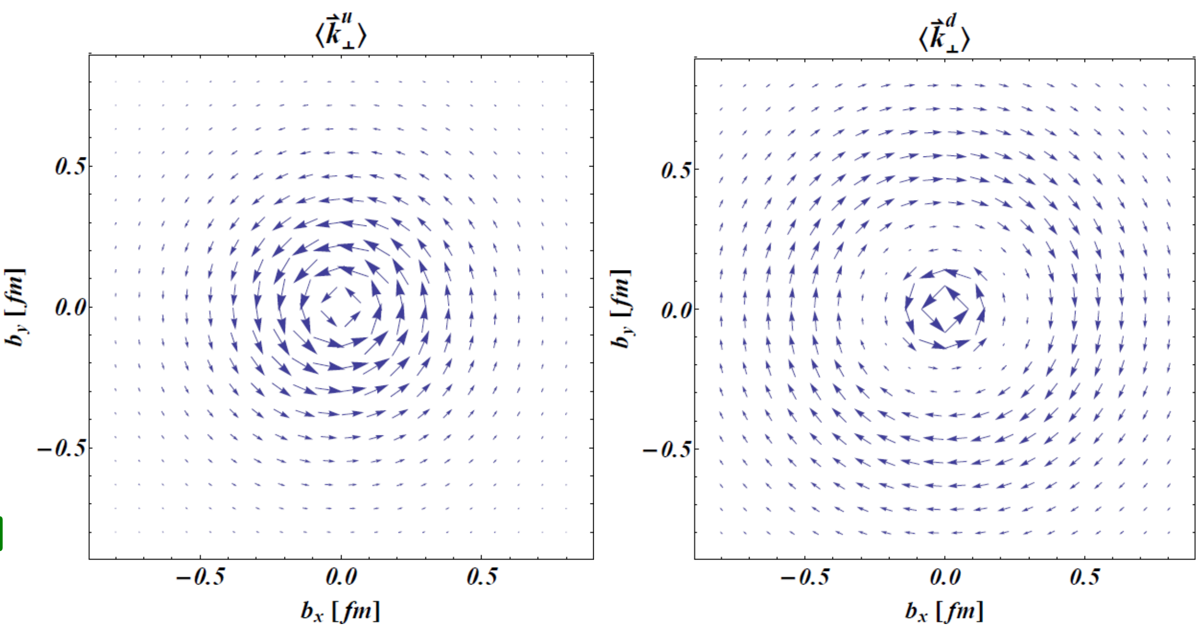

## ❑ Connection to the Wigner distribution:

2D FT of GTMDs ( $\Delta_\perp \to b_\perp$ )

$$\rho^{\mathcal{O}}_{\Lambda'\Lambda}(P, k, \vec{b}_\perp; \mathcal{W}) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} W^{\mathcal{O}}_{\Lambda'\Lambda}(P, k, \Delta; \mathcal{W})\Big|_{\Delta^+=0}$$

## ❑ Spatial distribution of $\langle \vec{k}_\perp \rangle$ :

$$\langle \vec{k}_\perp \rangle (\vec{b}_\perp) = \int d^4 k \, \vec{k}_\perp \, \rho^{\gamma^+}_{\Lambda'\Lambda}(P, k, \vec{b}_\perp; \mathcal{W})$$




[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (2012)]
[Hatta (2012)]
[Kanazawa, et al. (2014)]

# Orbital angular momentum contribution

- **The definition in terms of Wigner function:**

Ji, Xiong, Yuan, PRL, 2012
Lorce, Pasquini, PRD, 2011
Lorce, et al, PRD, 2012

✧ **Gauge invariant:**

$$L_q \equiv \frac{\langle P,S|\int d^3r\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{D}_\perp)\psi(\vec{r})|P.S\rangle}{\langle P,S|P,S\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{FS}(x,\vec{b}_\perp,\vec{k}_\perp)dx\,d^2\vec{b}_\perp d^2\vec{k}_\perp$$

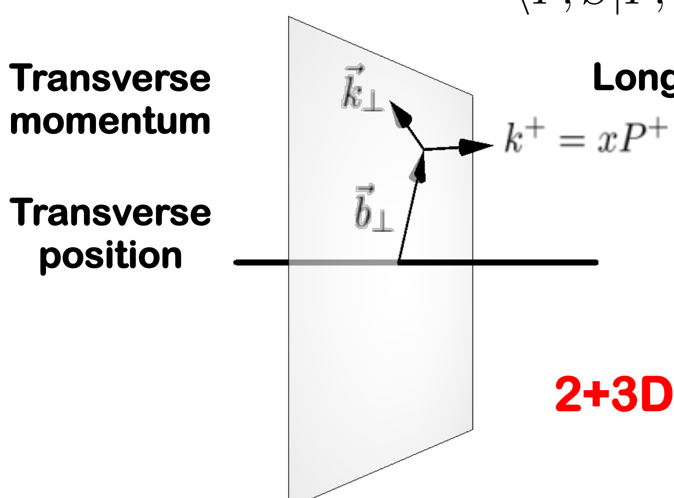
✧ **Canonical:**

$$l_q \equiv \frac{\langle P,S|\int d^3r\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times i\vec{\partial}_\perp)\psi(\vec{r})|P.S\rangle}{\langle P,S|P,S\rangle} = \int (\vec{b}_\perp \times \vec{k}_\perp) W_{LC}(x,\vec{b}_\perp,\vec{k}_\perp)dx\,d^2\vec{b}_\perp d^2\vec{k}_\perp$$

✧ **Gauge-dependent potential angular momentum – the difference:**

$$l_{q,pot} \equiv \frac{\langle P,S|\int d^3r\,\overline{\psi}(\vec{r})\gamma^+(\vec{r}_\perp \times (-g\vec{A}_\perp))\psi(\vec{r})|P.S\rangle}{\langle P,S|P,S\rangle} = L_q - l_q$$

**Quark-gluon correlation**




**2+3D**

$$\langle \mathcal{O} \rangle = \int \mathcal{O}(\vec{b}_\perp,\vec{k}_\perp)\, W_{GL}(x,\vec{b}_\perp,\vec{k}_\perp)\, dx\, d^2\vec{b}_\perp d^2\vec{k}_\perp$$

**Gauge-link dependent Wigner function**

***Same for gluon OAM***

# Orbital angular momentum contribution

Ji, Xiong, Yuan, PRL, 2012
Lorce, Pasquini, PRD, 2011
Lorce, et al, PRD, 2012

## ❑ The Wigner function:

✧ **Quark:**

$$W^q_{GL}(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int \frac{dz^- d\vec{z}_\perp}{(2\pi)^3} e^{ik\cdot z} \left\langle P+\frac{\vec{\Delta}_\perp}{2}\right| \overline{\Psi}_{GL}\left(-\frac{z}{2}\right)\gamma^+\Psi_{GL}\left(\frac{z}{2}\right)\left|P-\frac{\vec{\Delta}_\perp}{2}\right\rangle$$

GL: gauge link dependence

Gauge to remove "GL"

$$\Psi_{FS}(z) = \mathcal{P}\left[\exp\left(-ig\int_0^\infty d\lambda\, z\cdot A(\lambda z)\right)\right]\psi(z)$$

Fock-Schwinger

$$\Psi_{LC}(z) = \mathcal{P}\left[\exp\left(-ig\int_0^\infty d\lambda\, n\cdot A(\lambda n + z)\right)\right]\psi(z)$$

Light-cone

✧ **Gluon:**

$$W^g_{GL}(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int \frac{dz^- d\vec{z}_\perp}{(2\pi)^3} e^{ik\cdot z} \left\langle P+\frac{\vec{\Delta}_\perp}{2}\right| \mathbf{F}^{i+}_{GL}\left(-\frac{z}{2}\right)\mathbf{F}^{+i}_{GL}\left(\frac{z}{2}\right)\left|P-\frac{\vec{\Delta}_\perp}{2}\right\rangle$$

## ❑ Gauge-invariant extension (GIE):

$$i\vec{\partial}^\alpha_\perp = i\vec{D}^\alpha_\perp(\xi) + \int^{\xi^-} d\eta^-\, L_{[\xi^-,\eta^-]}\, gF^{+\alpha}(\eta^-,\xi_\perp)\, L_{[\eta^-,\xi^-]}$$

Twist-3 correlators

Fixed gauge local operators ⟺ gauge invariant non-local operators

*Note: the 2+3D Wigner distributions are not "physical"*
*But, their reduced distributions could be connected to observables*