Basics of Pomerons and Reggeons

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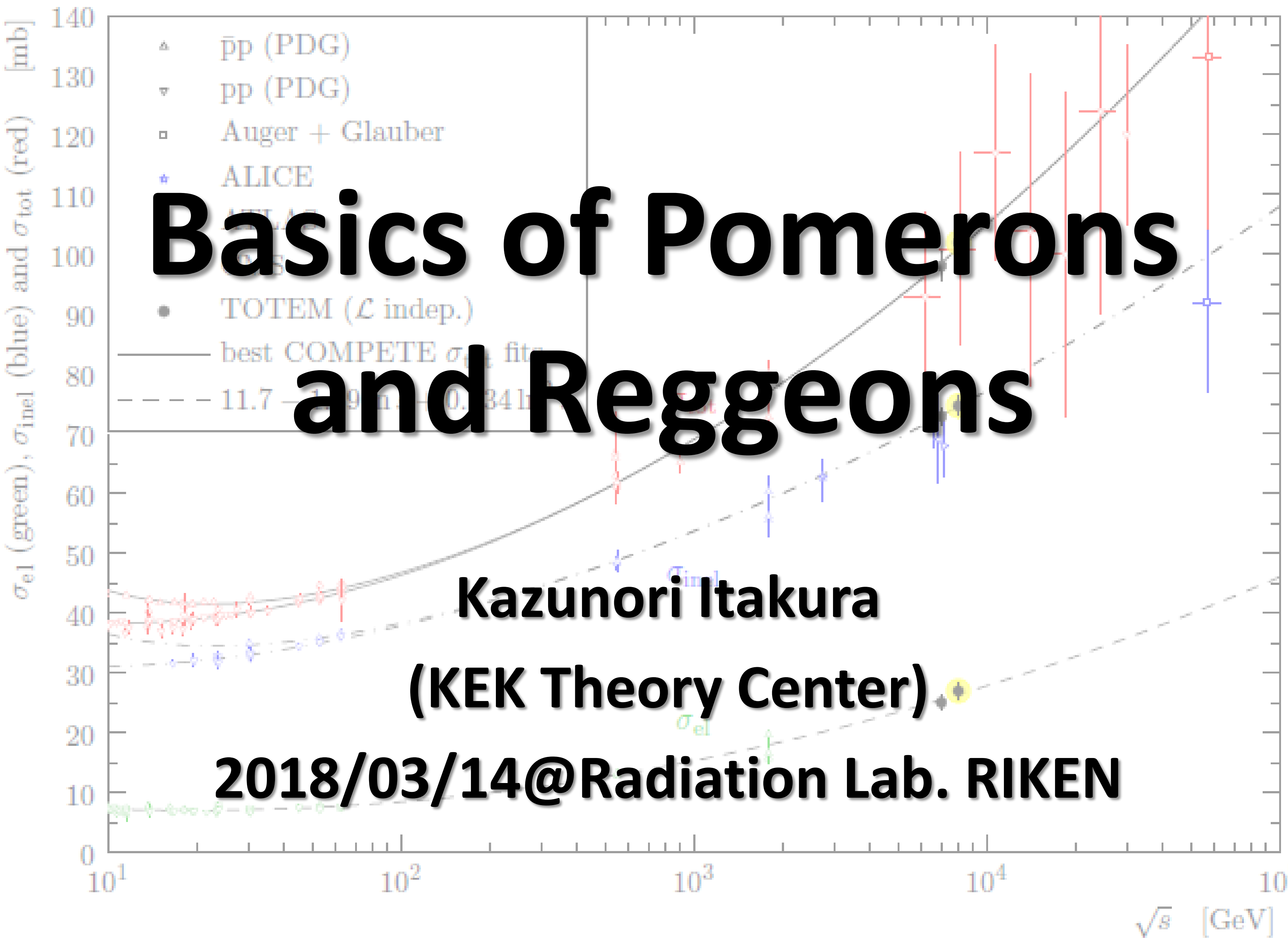


First of all …

- This talk aims at giving a rough picture of Pomerons and Reggeons to non-experts. (But I am not an expert, either.)
- Regge theory was developed long long time ago, and is not studied in standard Phd courses. But it is not a out-of-date framework, but is still useful in describing high-energy scattering in nonperturbative regime where perturbative QCD is hopeless.
- Mathematical structure of scattering amplitudes in the Regge theory is of course correct, and is sometimes discussed also in other theories (such as string theory, conformal field theory, etc.)
- Recently, “Regge-like theory” in QCD has been discussed.

Plan

- Pomeron as seen from experimental data
- Properties of S-matrix and scattering amplitude (Cutkosky rule and Optical theorem)
- Partial wave expansion and complex angular momentum
- Regge pole

break

- Reggeon and Pomeron
- Froissart bound
- Beyond 1 Pomeron : revisiting experiments
- Pomerons in deep inelastic scattering (DIS)

Reference

- P.D.B.Collins, “**An Introduction to Regge Theory and high energy physics**” (Cambridge, 1977) everything is written

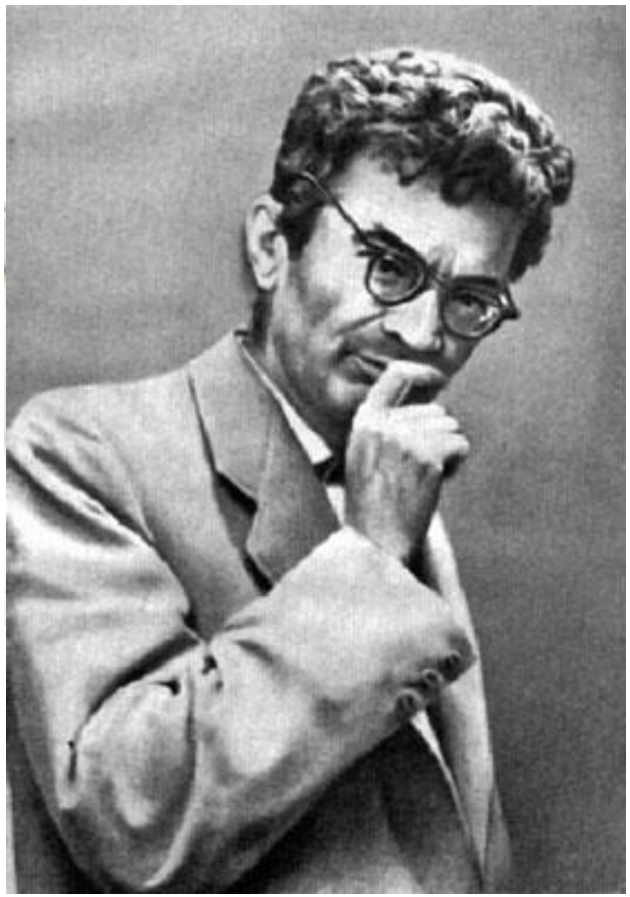
- J.R.Forshaw, D.A.Ross, “**Quantum Chromodynamics and the Pomeron**” (Cambridge, 1997) very nice summary in the first section

- V.Barone, E. Predazzi, “**High Energy Particle Diffraction**” (Springer 2002) very good with full of examples

- 小平治郎『**場の理論としての量子色力学**』 1998
  Chapter 11 “Regge理論” 未公開講義ノートだが、大変良い

- 板倉数記 原子核研究 59巻 No.1（2014年9月号）キーワード解説
  『**レッジェ理論とレッジェ極**』、『**ポメロン**』は、本講演を4ページにまとめたもの

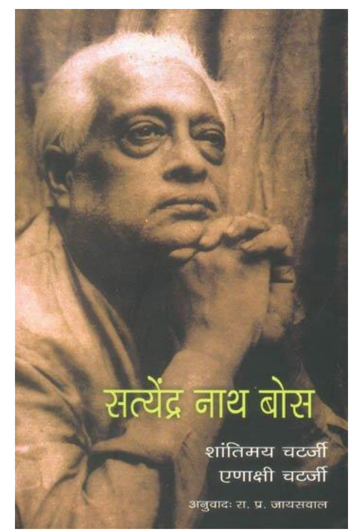
Pomer on



Isaak Pomeranchuk (1913-1966)

“on” for particles

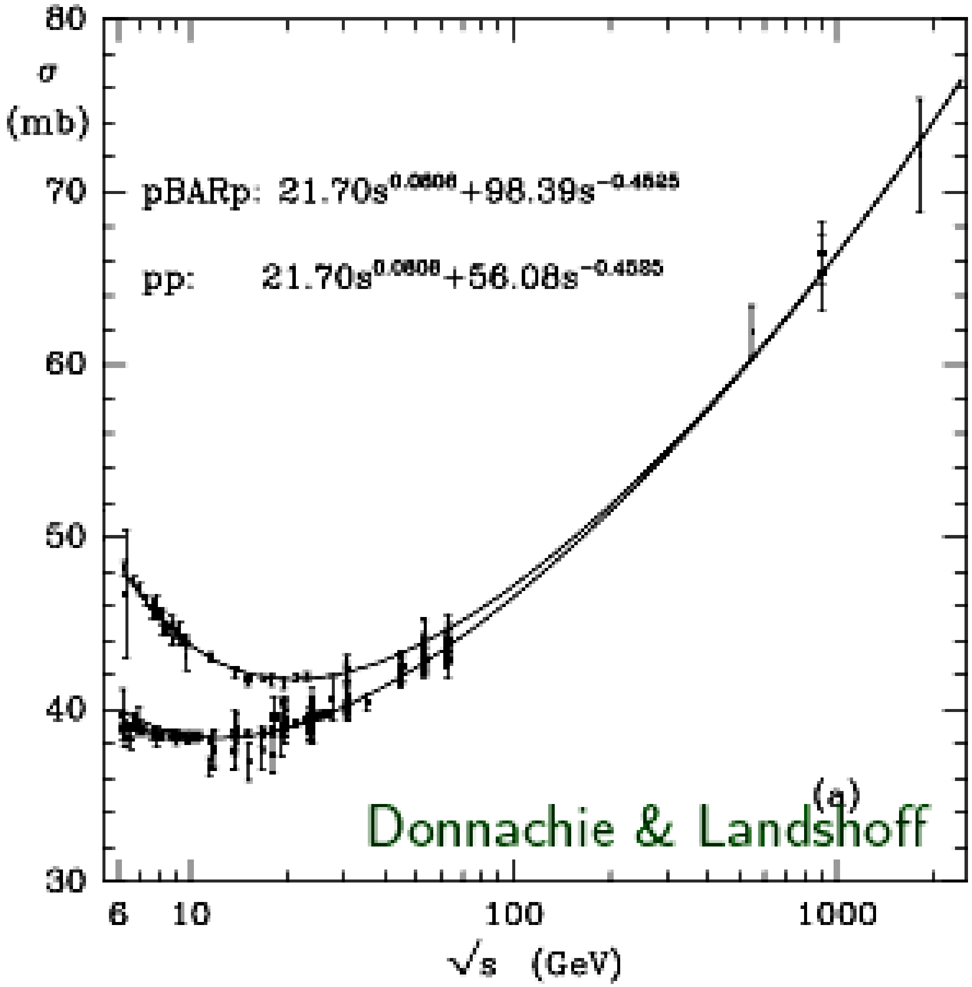
Same as Fermion, Boson



Where is Pomeron?

Total cross section in hadron scattering

→ does not show "resonance" peak in high energy region



INCREASE of total hadronic cross section

← due to Pomeron exchange

$$\sigma_{tot} \sim s^{\alpha_P(0)-1}$$

Donnachie-Landshoff, 1992

Fit to data below 100GeV can be represented by

Pomeron + Reggeon

$\alpha_P(0)=1.08 > 1$, $\alpha_R(0)=0.55 < 1$

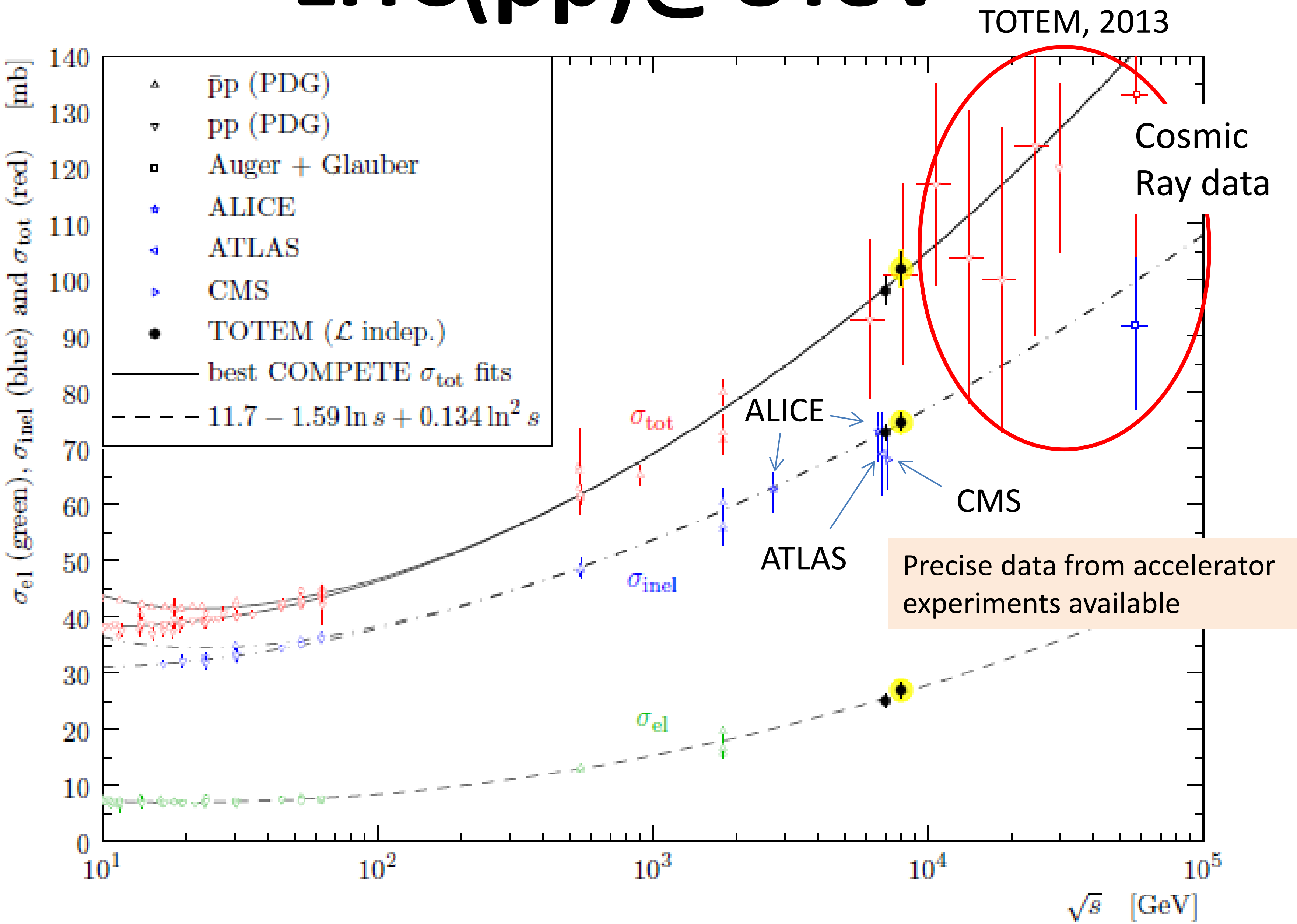
Pomeron→ leading term, same for pp, ppbar

Reggeon → subleading term

$\alpha_R(0)=0.55$ coincides with Regge slope

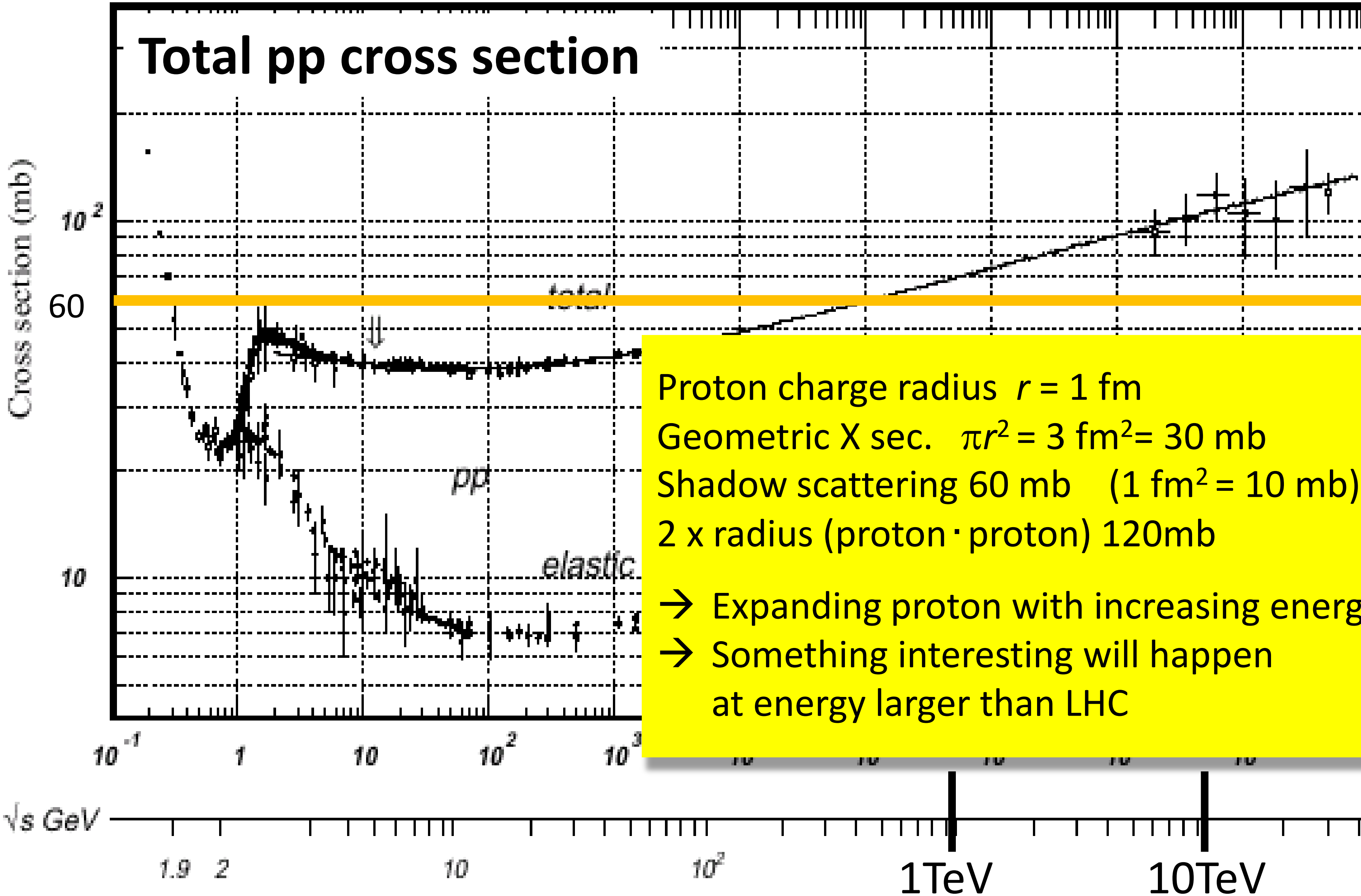
Can equally describe π^+p, π^-p, γ p scatterings

LHC(pp)@8TeV



TOTEM, 2013
p̄p (PDG)
pp (PDG)
Auger + Glauber
ALICE
ATLAS
CMS
TOTEM ($\mathcal{L}$ indep.)
best COMPETE σ_{tot} fits
$11.7 - 1.59 \ln s + 0.134 \ln^2 s$
σ_{el} (green), σ_{inel} (blue) and σ_{tot} (red) [mb]
σ_{tot}
σ_{inel}
σ_{el}
ALICE
ATLAS
CMS
Cosmic
Ray data
Precise data from accelerator experiments available
$\sqrt{s}$ [GeV]

What is interesting?



Proton charge radius r = 1 fm
Geometric X sec. πr^2 = 3 fm^2= 30 mb
Shadow scattering 60 mb (1 fm^2 = 10 mb)
2 x radius (proton･proton) 120mb

→ Expanding proton with increasing energy?
→ Something interesting will happen at energy larger than LHC

What is pomeron?

Answer at this point

Something like particle which is responsible for describing increasing total cross section of hadron-hadron scattering with increasing energy

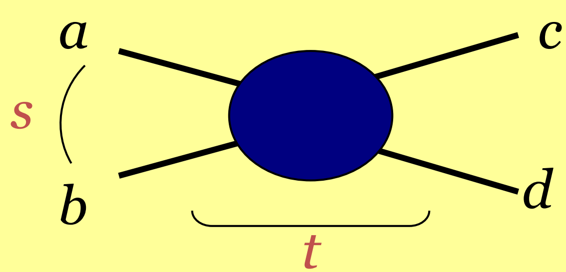
High-energy limit

Or Regge limit

\`\`total scattering energy" $\gg$ \`\`typical momentum scale in reaction"

(\`\`total scattering energy" $\gg$ \`\`particle masses" is implicit)

Hadron-hadron scattering



Square of total scatt. Energy in CM frame

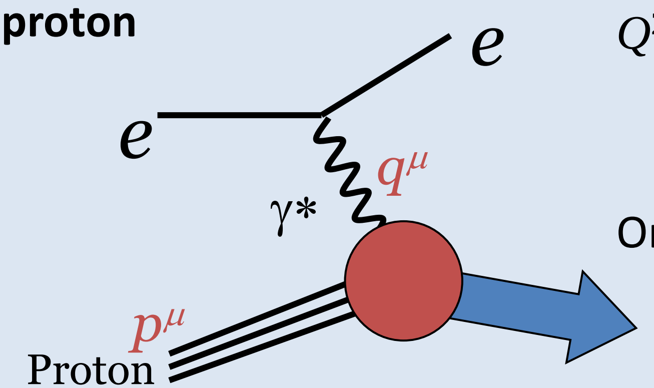
$$s=(p_a+p_b)^2$$

Square of momentum transfer

$$t=(p_a-p_c)^2$$

$$s >> |t|$$

Deep inelastic scattering of proton



$W^2=(p+q)^2$ Square of total scatt. Energy of γ-p

$Q^2=-q^2$ virtuality of photon

$$W^2 >> Q^2$$

Or, $x \sim Q^2/(W^2+Q^2) \to 0$

Cf) Bjorken limit: $x=Q^2/2pq$ fixed

$Q^2 \to \infty$, $2p^\mu q_\mu \sim W^2+Q^2 \to \infty$

Soft vs Hard

Comparison between typical momentum scale μ and Λ_{QCD}

Λ_{QCD} : momentum scale where QCD coupling diverges ~ 200-400MeV

ex) 1 loop

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{QCD}{}^2)}, \qquad \beta_0 = 11 - \frac{2}{3} n_f$$

hard (perturbative)	$\mu \gg \Lambda_{QCD}$
soft (nonperturbative)	$\mu \lesssim \Lambda_{QCD}$

ex)

- total cross section is non-perturbative (Optical theorem relates it to forward amplitude t=0)
- DIS cross section

large Q^2 → can be computed perturbatively (factorization: separation btw soft and hard)
small Q^2 → soft nonperturbative scattering btw a γ (or vector meson) and a proton

History of high energy scattering

Pre-QCD

1943 Heisenberg proposal of S-matrix theory
1956 Pomeranchuk Pomeranchuk theorem
1958 Mandelstam relativistic S-matrix theory (Mandelstam variable)
1959 Regge proposal of Regge pole in Quantum Mechanics
1961 Chew-Frautschi relativistic Regge theory completed → soft Pomeron
→ later, dual resonance model, Veneziano amp, string theory

After-QCD

linear
- 1970's QCD is established
- 1976-78 BFKL high energy scattering in QCD (LO-BFKL equation)
  → "hard Pomeron" (← measured at HERA around 1993)
- ~2000 NLO-BFKL completed → later its resummation

nonlinear
- 1983 GLR(Gribov-Levin-Ryskin) first discussion about saturation. Modification of BFKL
- 1986 Mueller-Qiu nonlinear correction to DLA(small-x limit of DGLAP)
- 1994 McLerran-Venugopalan model: effective theory for fast moving nucleus
- 2000 Iancu, McLerran, etc. Reformulation of GLR from MV model and beyond
  → JIMWLK equation, BK equation(LO) renormalization group
  → **Color Glass Condensate** （2001 Geometric scaling at HERA）
  (2004 RHIC forward dAu)

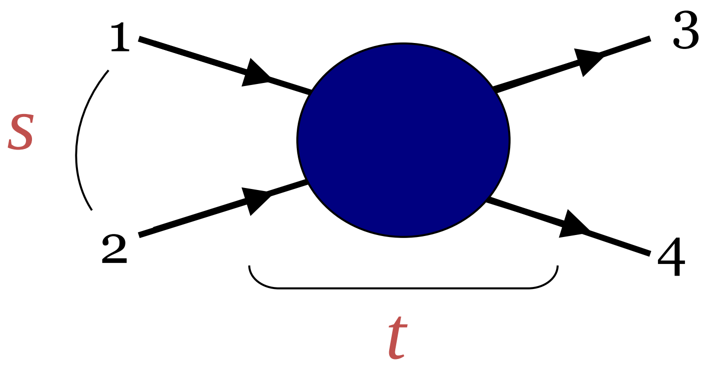
We need "Regge theory"

- Pomeron is a special case of Reggeons that are described by Regge theory
- Not based on quantum field theory (perturbative description abandoned)
- Based on S-matrix for hadronic degrees of freedom
- Constrain the possible form of scattering amplitudes by imposing several postulates on S-matrix
- Pre-QCD physics. But must be explained by QCD in future

Kinematics

Consider 2→2 scattering

of particles with masses m_i , momenta p_i^μ



Mandelstam variables ; Lorentz inv.

$$s = (p_1 + p_2)^2$$

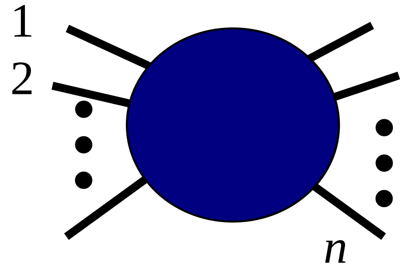
$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$

only two of them are independent

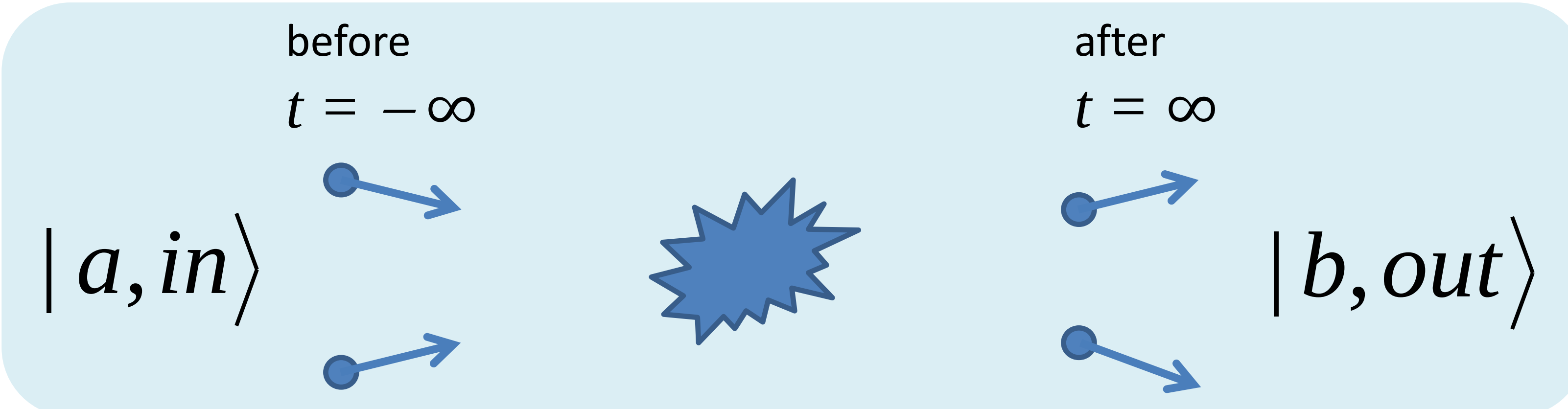
→ Represent S-matrix and scattering ampulitudes in terms of s and t : $S(s,t)$, $A(s,t)$



Can be generalized to case involving n particles

S matrix

Fundamental physical objects carrying the information of scattering
Can be related to cross section



$|a,in\rangle$ ($|b,out\rangle$) is an asymptotic state at $t \to -\infty$ ($t \to \infty$) and respectively form complete sets. Describe **on-shell** free hadrons

S-matrix is a matrix whose elements are defined below
and contains all the information of the scattering process

$$S_{ba} \equiv \langle b, out | a, in \rangle$$

One can introduce an operator S by representing outstate $|b, out\rangle$ in terms of bases of instate. $\langle b, out | a, in \rangle = \langle b, in | S | a, in \rangle$

Three postulates on S matrix

requirements/conditions

(I) S matrix is Lorentz invariant.

S matrix is a function of Lorentz invariant variables (Mandelstam variables) : $S(s, t)$ for 2-to-2 scattering

(II) S matrix is unitary. $S^{+} S = S S^{+} = 1$

(conservation of probability: $\mathrm{P}(a \rightarrow \text{anything})=1$ **)**

Cutkosky rule for scattering amp. → Optical th. (total X sec)

(III) S matrix is an analytic function of complexified Lorentz invariants and has singularity structure allowed by unitarity.

S matrix has a structure with simple poles + cuts .

Unitarity of S matrix

Transition probability from a state a to b

$$P_{a\to b} \equiv \left|S_{ba}\right|^2 = S^+_{ab} S_{ba}$$

(no summation over repeated indices)

Representing the unitarity condition $S^+ S = S S^+ = 1$ by elements

$$\sum_b S^+_{cb} S_{ba} = \sum_b \langle c, in \mid b, out\rangle \langle b, out \mid a, in\rangle = \delta_{ca}$$

$S^+ S = 1$ → probability of going to any state from state a is unity

$$\sum_b S_{cb} S^+_{ba} = \sum_b \langle c, in \mid b, in\rangle \langle b, in \mid a, out\rangle = \delta_{ca}$$

$S S^+ = 1$ → probability of getting a final state a which came from an initial state b yields unity if one sums up all the possible initial state b.

States are supposed to form orthogonal normalized complete sets

$$\sum_b | b, in\rangle \langle b, in | = \sum_b | b, out\rangle \langle b, out | = 1$$

Consequence of postulate (II): Cutkosky rule

Scattering amplitude $A(s,t)$

$$S_{ba} \equiv \delta_{ba} + i(2\pi)^4 \delta^4 \left(\sum_b p_b - \sum_a p_a \right) A_{ba}$$

final — initial

Nothing happens — due to interaction

$$S = 1 + iT$$

$$S^+ S = (1 - iT^+)(1 + iT) = 1$$

$$\therefore \; i(T^+ - T) = T^+ T$$

Sandwiching this eq. by $\langle f \mid$ and $\mid i \rangle$ and using $S_{ab}=S_{ba}$ (valid for PT symmetric system)

$$i\langle f | (T^+ - T) | i \rangle = \sum_n \langle f | T^+ | n \rangle \langle n | T \; | i \rangle$$

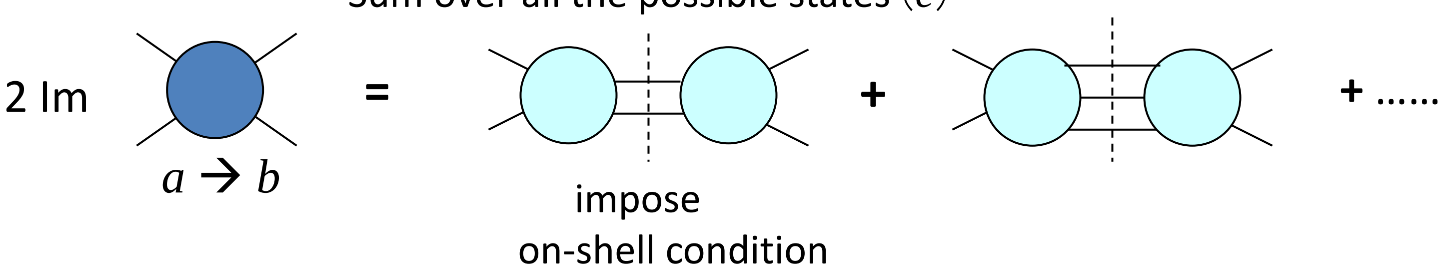
$$2\,\mathrm{Im}\, T_{fi} = \sum_n T^+_{fn} T_{ni}$$

Consequence of postulate (II): Cutkosky rule

Cutkosky rule

Summation over possible states

$$2\,\mathrm{Im}\,A_{ba} = (2\pi)^4 \sum_c \delta\left(\sum_c p_c - \sum_a p_a\right) A_{bc}{}^{+} A_{ca}$$



Optical Theorem

Put $a = b$ in Cutkosky rule

$$2\,\mathrm{Im}\,A_{aa} = (2\pi)^4 \sum_c \delta\left(\sum_c p_c - \sum_a p_a\right) |A_{ca}|^2$$

Probability of a state *a* going to any state ← total X sec

a=b ZERO momentum transfer → $t = 0$ forward scattering

$$\sigma_{total} = \frac{1}{2|p_1|\sqrt{s}}\,\mathrm{Im}\,A(s, t=0)$$

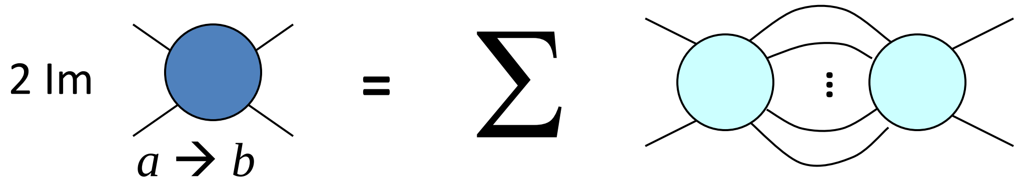
p_1 : momentum of projectile In the COM frame $p_1 = (s/4 - m^2)^{1/2}$

High energy limit $s >> m^2$ → $p_1 \sim s^{1/2}$

$$\sigma_{total} \sim \frac{1}{s}\,\mathrm{Im}\,A(s, t=0)$$

Postulate(III)：Singularity structure

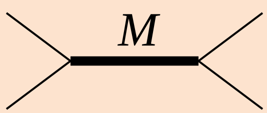
Cutkosky rule: n particles in intermediate states → evaluate each contribution



All the particles have the same *m*

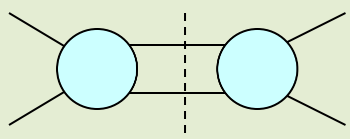
Assume particles flow in the s-channel

1particle state



Bound state with mass $M < 2m \to$ simple pole $\frac{1}{s - M^2}$, $\quad s = (p_1 + p_2)^2$

2 particle state

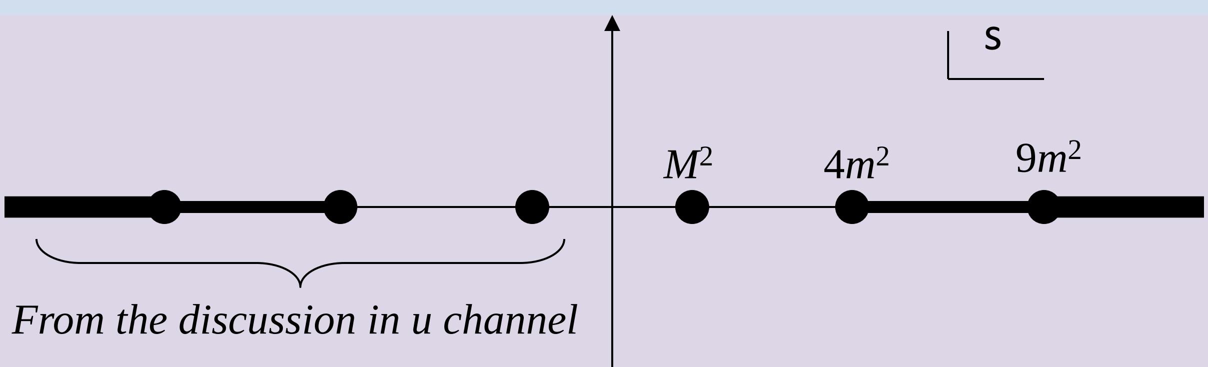


$$\mathrm{Im}\, A^{(2)}(s,t) \propto \sqrt{s - 4m^2}$$

Continuous for $s > (2m)^2$ and there is a cut with $s = 4m^2$ being the branch point

n particle state

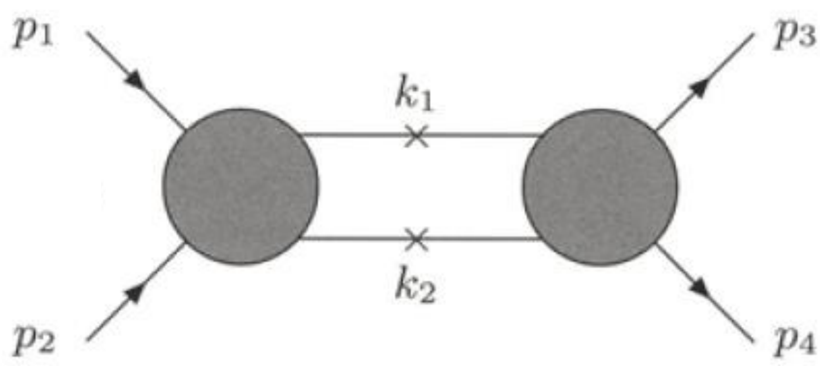
Continuous for $s > (nm)^2$ and there is a cut with $s = (nm)^2$ being the branch point



Typical singularity structure on complex s plane

Contribution of 2 particle state (1/2)

2 particle state is possible when the total energy S satisfies $(2m)^2 < s < (3m)^2$
(we impose energy conservation to the intermediate state)



Momenta of internal state

$$k_1 = (\varepsilon_1, \boldsymbol{k}_1) \qquad k_2 = (\varepsilon_2, \boldsymbol{k}_2)$$

on-shell $k_1^2 = k_2^2 = m^2$

$$t_1 = (p_1 - k_1)^2$$

$$t_2 = (p_1' - k_1)^2$$

$$\mathrm{Im}\, A_{\mathrm{el}}(s,t) = \frac{1}{2} \int \frac{\mathrm{d}^3\boldsymbol{k}_1}{(2\pi)^3\, 2\varepsilon_1} \int \frac{\mathrm{d}^3\boldsymbol{k}_2}{(2\pi)^3\, 2\varepsilon_2} \times (2\pi)^4\, \delta^4(p_1 + p_2 - k_1 - k_2)\, A_{\mathrm{el}}(s,t_1)\, A_{\mathrm{el}}^*(s,t_2)$$

$\boldsymbol{k}_2$ integral is straightforward with $\delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_2 - \boldsymbol{k}_1 - \boldsymbol{k}_2)$

$$\mathrm{Im}\, A_{\mathrm{el}}(s,t) = \frac{1}{2\,(2\pi)^2} \int \frac{|\boldsymbol{k}_1|^2\, \mathrm{d}|\boldsymbol{k}_1|\, \mathrm{d}\Omega_1}{2\varepsilon_1\, 2\varepsilon_2} \times \delta(E_1 + E_2 - \varepsilon_1 - \varepsilon_2)\, A_{\mathrm{el}}(s,t_1)\, A_{\mathrm{el}}^*(s,t_2)$$

E_1, E_2 are energies of p_1, p_2

Contribution of 2 particle state (2/2)

Using the explicit form in COM frame,

$$E_1 + E_2 = \sqrt{s}\ ,$$
$$\varepsilon_1 = \varepsilon_2 \ = \sqrt{\boldsymbol{k}_1^2 + m^2}$$

The integral over the momentum yields ($k=|\boldsymbol{k}_1|$)

$$\int \frac{k^2 dk}{k^2 + m^2} \delta\left(\sqrt{s} - 2\sqrt{k^2 + m^2}\right) = \frac{1}{\sqrt{s}} \sqrt{\frac{s}{4} - m^2}$$

Therefore, contribution of 2 particle state is

$$\mathrm{Im}\, A_{\mathrm{el}}(s,t) = \frac{1}{32\pi^2 \sqrt{s}} \sqrt{\frac{s}{4} - m^2} \int \mathrm{d}\Omega_1\, A_{\mathrm{el}}(s,t_1)\, A_{\mathrm{el}}^*(s,t_2)$$

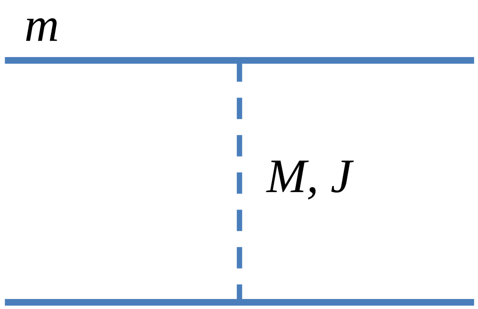
$$\sqrt{s - 4m^2}$$

This structure comes from energy conservation

→ $s > (nm)^2$ for n particle state

Contribution of meson exchange

A meson with mass *M*, spin *J* is exchanged in *t* channel



$$A_{meson}(s,t) \sim A_J(t) P_J(\cos\theta_t)$$

$$\cos\theta_t = 1 + \frac{2s}{t - 4m^2}$$

Intuitively, contribution of angular momentum *J* is Introduced by exchange of spin-J particle

For large s, using the asymptotic form of Legendre function

$$A_{meson}(s,t) \sim s^J$$

This is the amplitude when a particle with spin *J* is exchanged in *t* channel.

On the other hand, the amplitude should have the form of propagator in t-channel

$$A_J(t) \sim \frac{1}{t - M^2}$$

Legendre polynomial and function

Legendre polynomial

$$P_n(z) \equiv \frac{1}{2^n n!} \frac{d^n}{dz^n} \left(z^2 - 1\right)^n$$ is a polynomial with highest degree n

$$P_0(z) = 1, \quad P_1(z) = z, \quad P_2(z) = \frac{1}{2}\left(3z^2 - 1\right), \ldots$$

Orthogonality: $-1 < z < 1$ $$\int_{-1}^{1} dz\, P_n(z) P_m(z) = \frac{2}{2n+1} \delta_{nm}$$

However, we can consider the region $|z| > 1$, and non-integer n (even complex)

Legendre function

$$P_\nu(z) \equiv F\left(-\nu, \nu+1, 1; \frac{1-z}{2}\right)$$

Extension to non-integer by Hypergeometric function

Asymptotic form in the limit $|z| \to \infty$

$$P_\nu(z) \sim \frac{1}{\sqrt{\pi}} \frac{\Gamma(\nu + 1/2)}{\Gamma(\nu+1)} (2z)^\nu$$

ν can be complex number

Complex angular momentum

- **Partial wave expansion**

Scatt. amp. $f(\theta)$ in nonrel-QM can be expanded wrt eigenstates of ang. mom.

$$f(\theta) = \sum_{l=0}^{\infty} (2l+1) f_l(k) P_l(\cos\theta)$$

Partial wave amplitude Legendre polynomial (k is wave number of projectile)

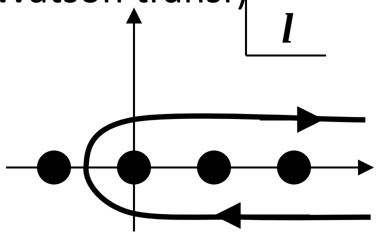
Similarly* (**consider the *t* channel**. Regard s as a function of t and θ)

*Attention! Mind the gap here.

$$A(s(z_t),t) = \sum_{l=0}^{\infty} (2l+1) a_l(t) P_l(z_t), \quad z_t = \cos\theta_t = 1 + \frac{2s}{t}$$

- **Complexification of angular momenta** (Sommerfeld-Watson transf)

$$A(s,t) = -\frac{1}{2i} \oint_C dl (2l+1) \frac{a(l,t)}{\sin \pi l} (-1)^l P_l(z_t)$$



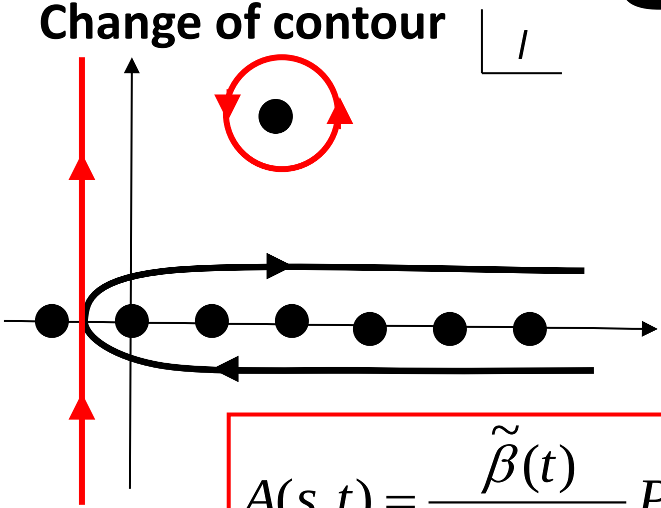
In fact, $a(l,t)$ is not uniquely determined. But unique extension is possible for even/odd angular momentum → "signature" $\eta = +,-$

$$A(s,t) = -\frac{1}{2i} \oint_C dl \frac{2l+1}{\sin \pi l} \left[a^{(+)}(l,t) - a^{(-)}(l,t) \right] P_l(z_t)$$

Introduce two analytic function depending on even (+)/odd(-) angular momenta

Regge pole

Change of contour



Assume that partial wave amplitude has a pole in Re $l > 0$

$$a(l,t) \sim \frac{\beta(t)}{l-\alpha(t)}$$ **"Regge pole"**

Amplitude will have t dependence
Ignore signature for simplicity

$$A(s,t) = \frac{\tilde{\beta}(t)}{\sin \pi\alpha(t)} P_\alpha(z_t) - \frac{1}{2i}\int_{-1/2-i\infty}^{-1/2+i\infty} dl \frac{2l+1}{\sin \pi l} a(l,t) P_l(z_t)$$

Ignore the contribution at infinity

Regge pole governs high energy behavior of scatt. amplitude.

s dependence enters only through $z_t=1+2s/t$ in $P_\alpha(z_t)$.
In Regge limit $s/|t| \to \infty$, line integral behaves as $1/s^{1/2}$ and can be ignored.

$$P_\alpha(1+2s/t) \sim \left(\frac{s}{|t|}\right)^\alpha, \quad s/|t| \to \infty$$

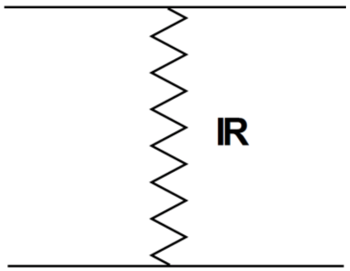
Picking up one Regge pole having the largest Re α, one finds

$$A(s,t) \to \beta(t)\, s^{\alpha(t)}, \quad s/|t| \to \infty$$ **"Reggeon"**

can be viewed as exchange of spin α particle in t-channel.

break

Regge pole



If the Reggeon exchanged in t channel is a physical particle with

spin J **mass** M

the following must hold for the angular momentum $\alpha(t)$.

$$\alpha(t = M^2) = J$$

on-shell condition
In t-channel

Regarding the pole in complex angular momentum space as the pole in complex t plane

$$\alpha(t) = \alpha(0) + \alpha' t$$

Giving a relation btw J and M

$$\alpha(M^2) = \alpha(0) + \alpha' M^2 = J$$

"Regge trajectory"

$\alpha(0) = 0.55 \quad < 1$ ← intercept

$\alpha' = 0.86\ \mathrm{GeV}^{-2}$ ← slope

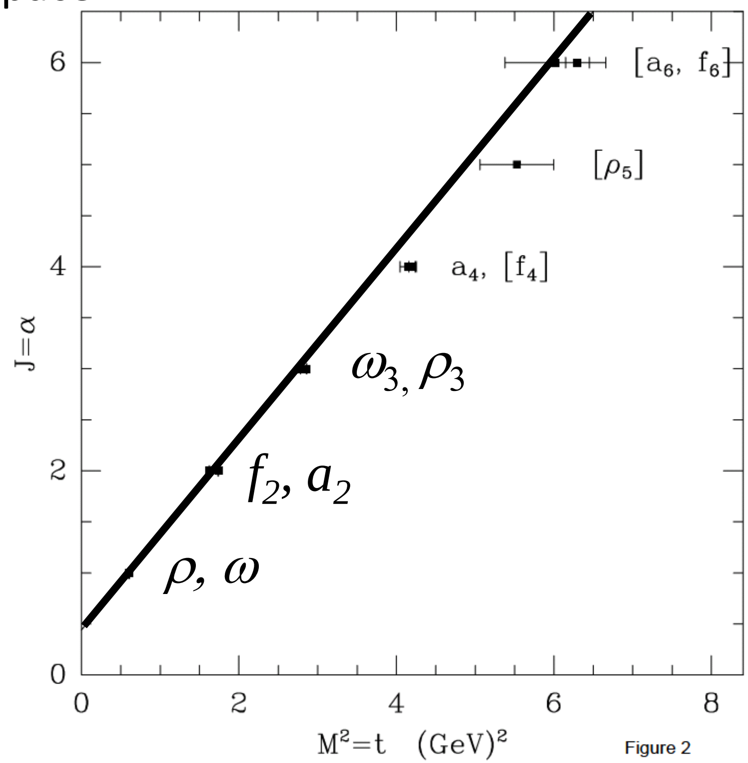
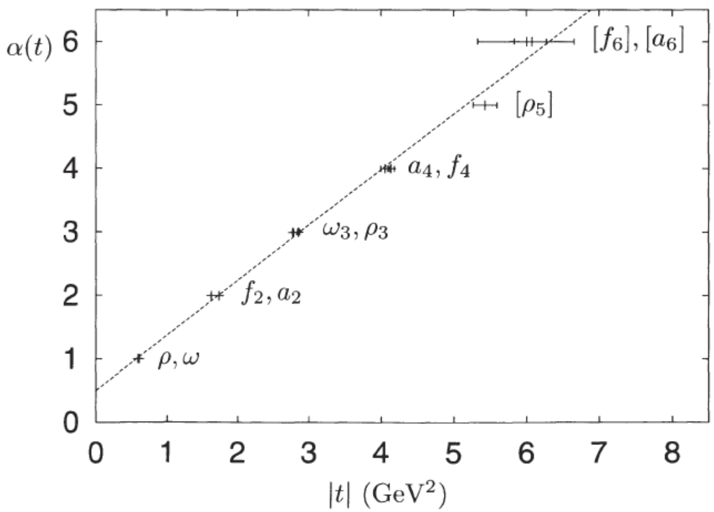


Figure 2

More about Regge trajectories

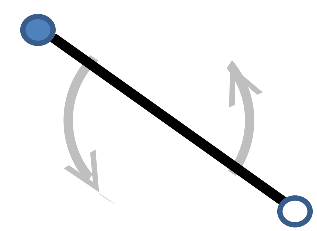


Mesons having different quantum numbers show
The same trajectory

$$f_2 : \quad P = +1,\ C = +1,\ G = +1,\ I = 0,\ \xi = +1\ ,$$
$$\rho : \quad P = -1,\ C = -1,\ G = +1,\ I = 1,\ \xi = -1\ ,$$
$$\omega : \quad P = -1,\ C = -1,\ G = -1,\ I = 0,\ \xi = -1\ ,$$
$$a_2 : \quad P = +1,\ C = +1,\ G = -1,\ I = 1,\ \xi = +1\ .$$

"string model" of hadrons

A string with length $2R$ and tension σ connecting massless quark/antiquark is rotating with angular momentum $J = 2pR$



Centrifugal force $$pv/R = Jv/(2R^2)$$ ⟺ Attractive force σ

$$R = \sqrt{Jc/(2\sigma)} \qquad Mc^2 = 2E + 2\sigma R = 2\sqrt{2\sigma Jc}$$

$$J = (c^3/8\sigma)M^2$$

Universal behavior of slope α' implies universal
Picture of "string tension"

Baryonic Regge trajectories

baryon exchange diagram

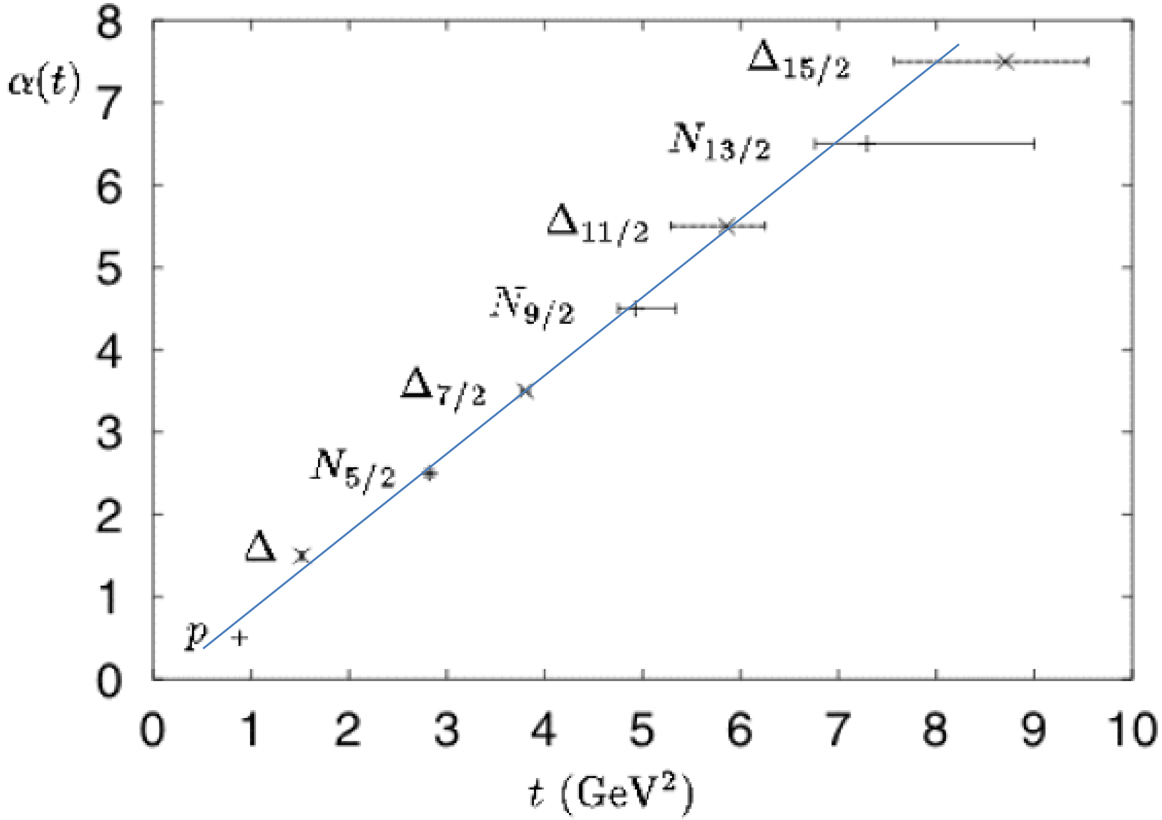


Figure 2.9. N and Δ trajectories

Slope is similar to mesons
Intercept seems negative?

Regge phenomenology

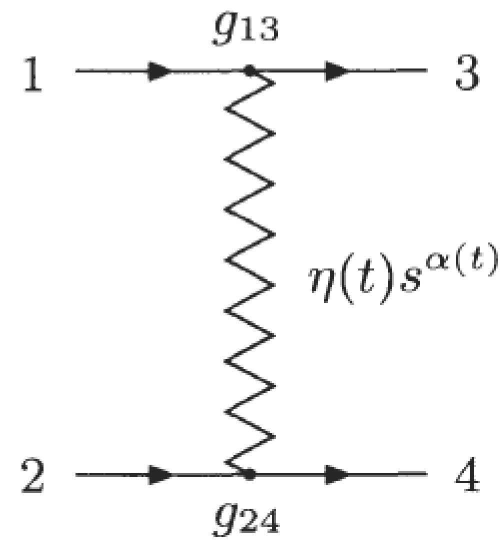
Contribution of Reggeon to the scattering amplitude

$$A(s,t) = \beta(t)\,\eta(t)\,s^{\alpha(t)}$$

where

$$\eta(t) = -\frac{1+\xi\,\mathrm{e}^{-\mathrm{i}\,\pi\,\alpha(t)}}{\sin\pi\alpha(t)}$$

$$\beta(t) = \beta(0)\,\mathrm{e}^{B_0\,t/2}$$



By using these, total cross section and elastic cross section are given as

$$\sigma_{\mathrm{tot}} \sim \sum_i A_i\, s^{\alpha_i(0)-1} \qquad \frac{\mathrm{d}\sigma_{\mathrm{el}}}{\mathrm{d}t} = F(t)\, s^{2\alpha(t)-2}$$

Pomeron

Total cross section

$$\sigma_{tot} \sim \frac{1}{s} \operatorname{Im} A(s, t=0) \sim s^{\alpha(0)-1}$$

If the intercept is smaller than 1 as that for Reggeon, the total X sec decreases with $s \to \infty$
(forward neutron production in pp collision is suppressed)

Pomeranchuk theorem (1956)

For the same target, the total cross sections of “particle”projectile and of “antiparticle” projectile are the same in the limit of high scattering energy.

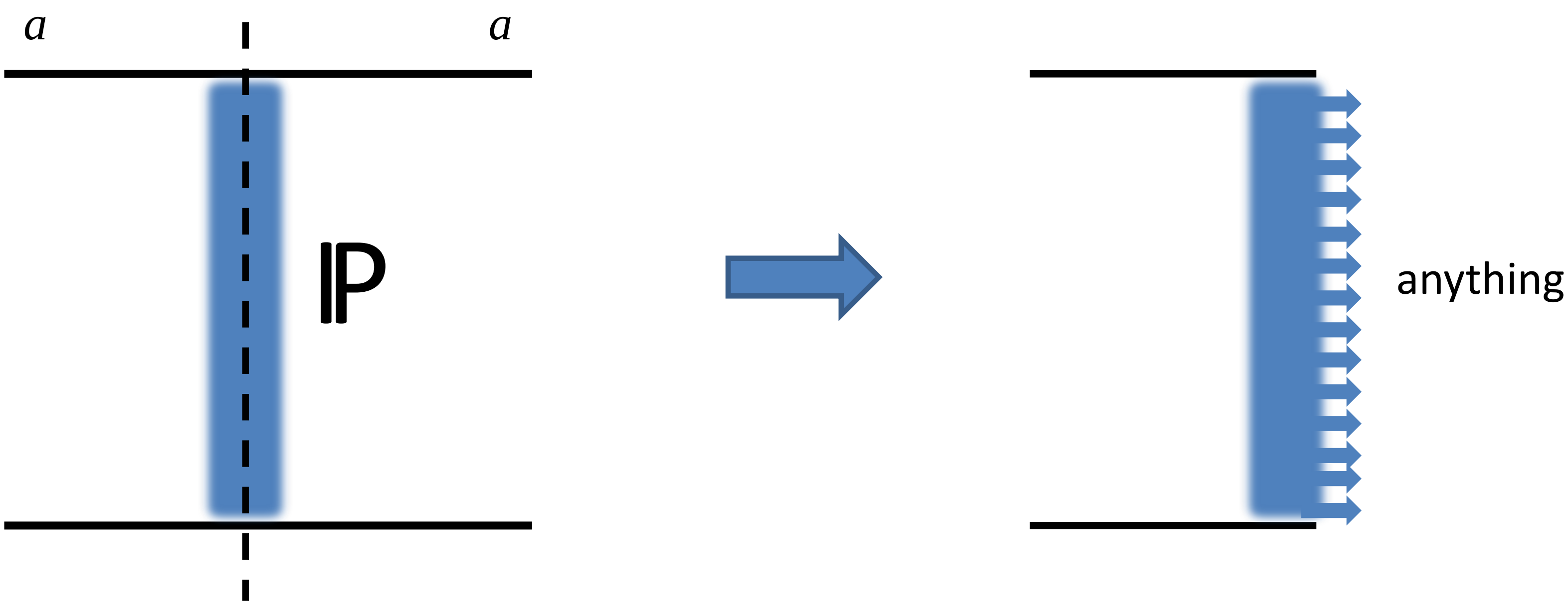
Foldy-Peierls (1963)

If the cross section does not decrease in the limit $s \to \infty$, then the scattering process is given by an exchange of an object having the same quantum numbers as the vacuum (isospin 0, charge conjugation even (charge 0))

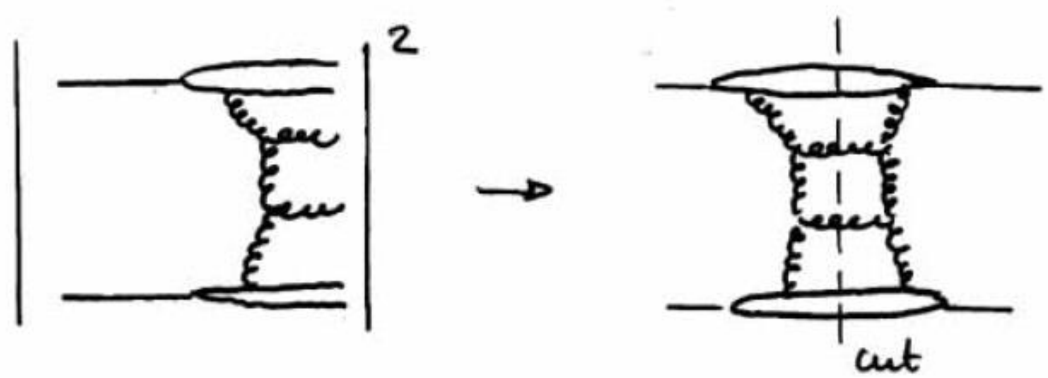
Experimentally, increase of hadron-hadron total cross sections are measured
→ Other trajectory different from the Reggeon ($\alpha(0) < 1$) would exist!!!

This is called **Pomeron** ! ($\alpha(0) > 1$)

Pomeron exchange and total cross section

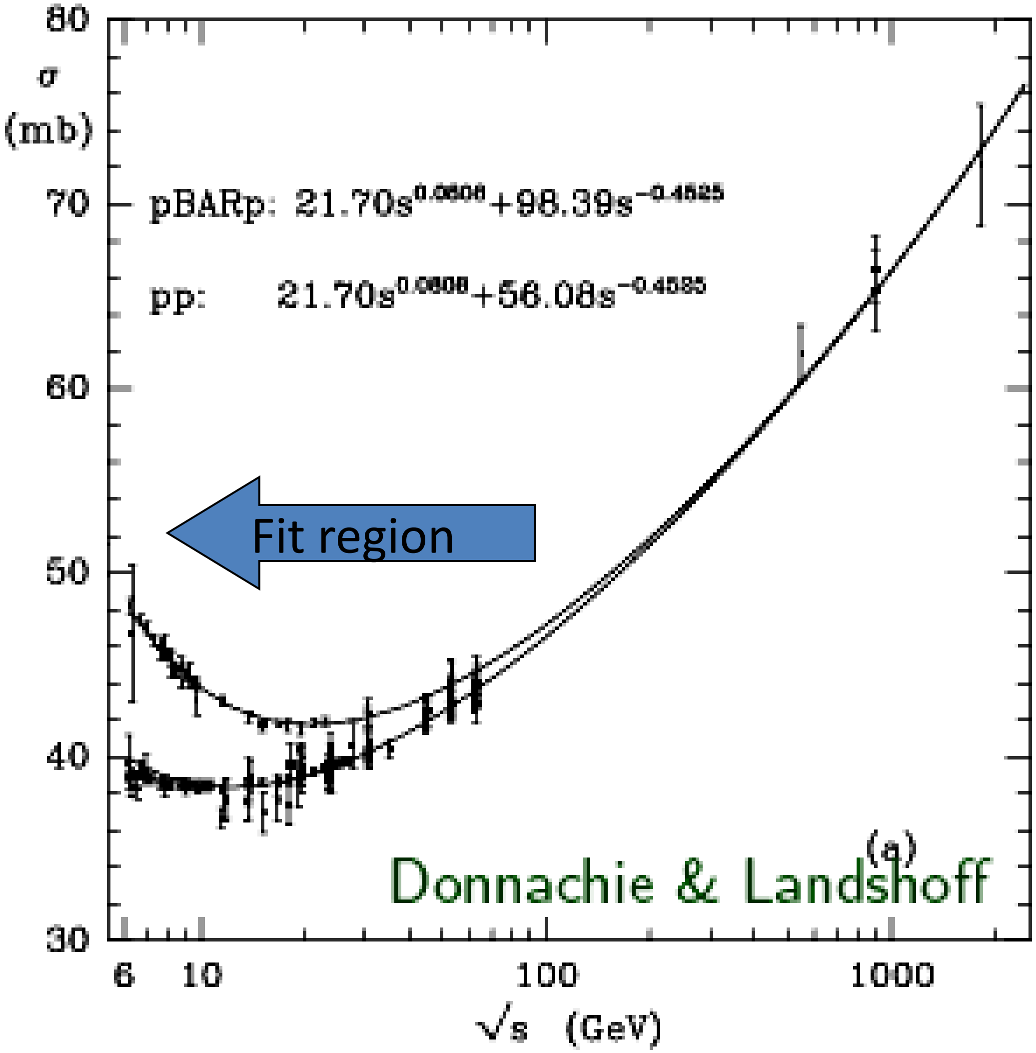


Related to multiple gluon production in BFKL Pomeron（Reggeized two gluon exchange）



Pomeron vs exp. data (standard picture)

pp/ ppbar cross sections



Donnachie-Landshoff, 1992

energy dependence can be represented by Pomeron + Reggeon

$\alpha_P(0)=1.08 > 1$, $\alpha_R(0)=0.55 < 1$

Pomeron term is the leading contribution and common for pp and ppbar.

$\alpha_R(0)=0.55$ of the Reggeon is the same as that of Regge trajectory

Can also describe π^+p, π^-p, γ p scatterings

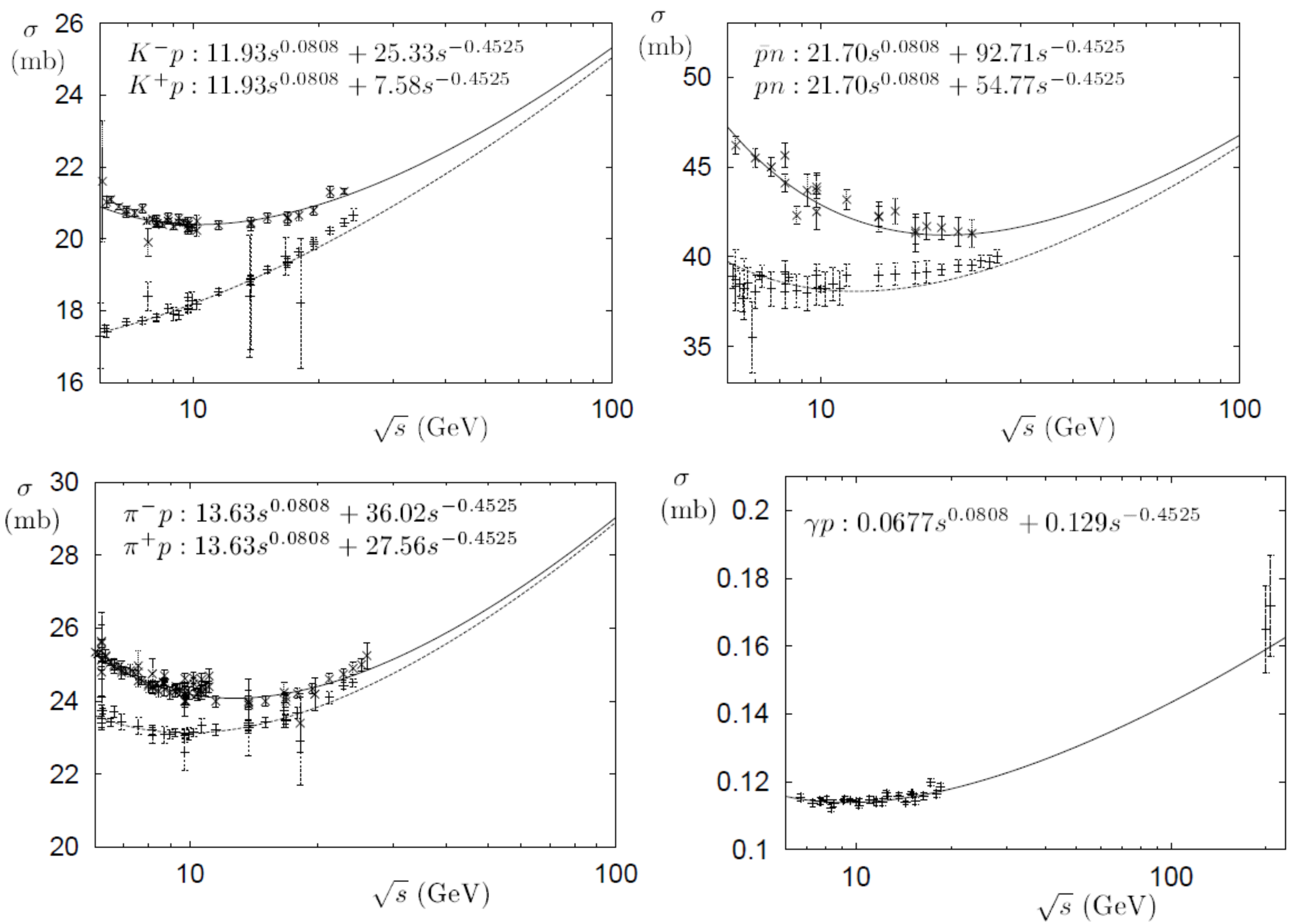
The exchange having $\alpha_P(0)=1.08$ is called ``**soft Pomeron**"

From the elastic differential cross section of pp and ppbar

$\alpha'_P = 0.25\ \mathrm{GeV}^{-2}$

Universal picture?

The Pomeron picture with the exchange having the same quantum numbers as the vacuum MUST equally apply to other hadron scattering processes.



Pomeron as a physical particle?

Experimental measurements suggest a picture that something with the same quantum numbers as the vacuum is exchanged.

Pomeron trajectory

from experiments

$$\alpha_P(t) = \alpha_P(0) + \alpha_P{}' t = 1.08 + 0.25t$$

If it is really a physical particle, it should satisfy

$$\alpha_P(t = M^2) = J \quad \Rightarrow \quad M = \sqrt{\frac{J - 1.08}{0.25}} = 1.9\,\text{GeV}\ (J = 2)$$

particle with Spin 2 , M=1.9GeV ････ $f_2(1950)$, $J^{PC}=2^{++}$??

or unknown glueball?

But it is not clear if we can regard Pomeron itself as a real particle.

(Pomeron should appear in the kinematical region far away from the on-shell region)

Unitarity violation of ``1 Pomeron" picture

Froissart bound

The power increase of total X sec due to 1 Pomeron exchange is TOO FAST and eventually violates unitarity of the scattering amplitude.

In fact, from the unitarity of the partial wave amplitudes, the following bound can be derived

$$\sigma_{tot}(s) < \frac{\pi}{m_\pi^2} \ln^2 \frac{s}{s_0}$$

Froissart 1961, Martin 1966

(s_0 is just a parameter)

The picture with 1 Pomeron exchange must be modified

→ In fact, multiple Pomeron exchange gives the same s dependence as the Froissart bound. (sometimes called Froissaron)

(Note) Since s_0 is unknown, we cannot compare the bound with experimental data. Still, if we take a typical hadronic scale $s_0 \sim 1\text{GeV}^2$, the bound gives extremely large values. (typically total pp X sec is about 100 mb even at Cosmic Ray energy)

$$\frac{\pi}{m_\pi^2} \ln^2 \frac{s}{s_0} = \begin{cases} 10\,\text{barn} \quad \text{at} \quad \sqrt{s} = 1.8\,\text{TeV (Tevatron)} \\ 25\,\text{barn} \quad \text{at} \quad \sqrt{s} = 14\,\text{TeV} \quad \text{(LHC)} \end{cases}$$

Intuitive picture of Froissart bound

Heisenberg (1952) described high-energy nucleon-nucleon scattering as a collision of two shock waves of surrounding meson cloud!!

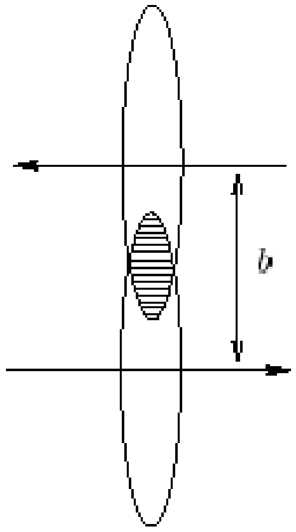
Reaction occurs when the energy density of overlapping region exceeds the threshold of two pion creation.

$$\kappa\, e^{-m_\pi b}\sqrt{s} \geq k_0$$

: Threshold energy (averaged pion energy)

Distribution of pion cloud ($\kappa\, e^{-m_\pi b}$)

Total energy ($\sqrt{s}$)



Can define maximum impact parameter $b_{\max}$

$$b_{\max} = \frac{1}{m_\pi}\ln\frac{\kappa\sqrt{s}}{k_0}$$

Leading to naïve geometric cross section (assuming satuation)

$$\sigma \sim \pi\, b_{\max}^2 = \frac{\pi}{m_\pi^2}\ln^2\frac{\kappa\sqrt{s}}{k_0} \propto \ln^2 s \quad (s \to \infty)$$

Slow growth of cross section is due to increase of effective radius

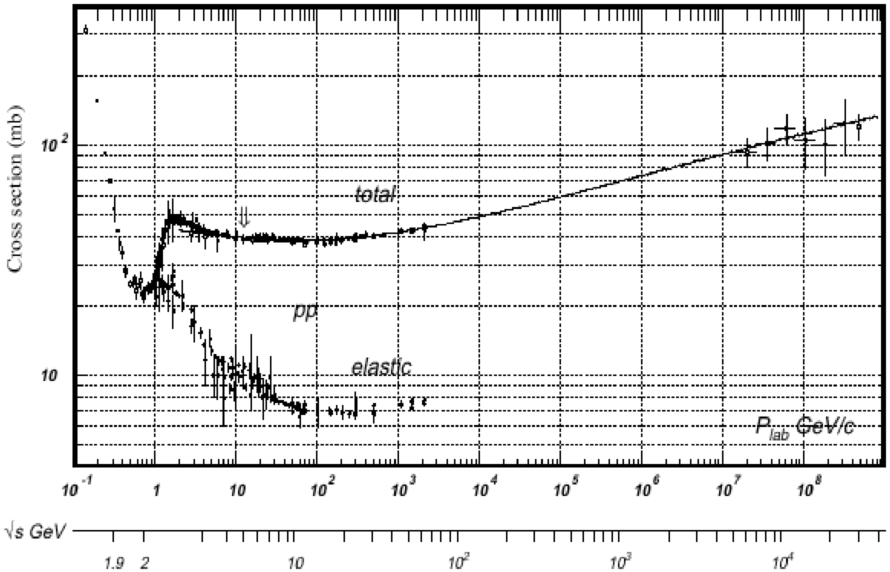
Revisiting experimental data

COMPETE Collab.

Compared ln s, ln^2 s (Froissart bound), s^{λ} (λ=0.08) (1 Pomeron)

ln^2 s is the best fit (adopted by PDG) favored with data larger than 4GeV

$$\sigma^{ab} = Z^{ab} + \underline{B \log^2(s/s_0)} + Y_1^{ab}(s_1/s)^{\eta_1} - Y_2^{ab}(s_1/s)^{\eta_2}$$



B is independent of process

$$\chi^2/dof = 0.971, \qquad \underline{B = 0.308(10)\ \text{mb}},$$
$$\eta_1 = 0.458(17), \qquad \eta_2 = 0.545(7)$$
$$\delta = 0.00308(2), \qquad \sqrt{s_0} = 5.38(50)\ \text{GeV}$$

η_1 is the value of Reggeon

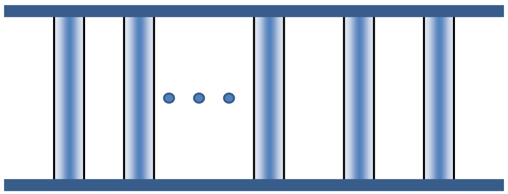
(NOTE) B is much smaller than that of the Froissart bound $\pi/m_{\pi}^2 = 62$ mb
Thus this log^2 behavior should not be identified with the unitarity effect.

Beyond 1 Pomeron exchange

- 1 Pomeron picture will violate unitarity and must break down at some large energy. So far, there is no problem.
- Still, it makes sense to evaluate the effects beyond the 1 Pomeron exchange which should exist even though the energy is not very high.
- Effects beyond 1 Pomeron exchange

Reggeon exchange ・・・・・・ Not important in the high energy limit, but necessary at experimentally accessible energy

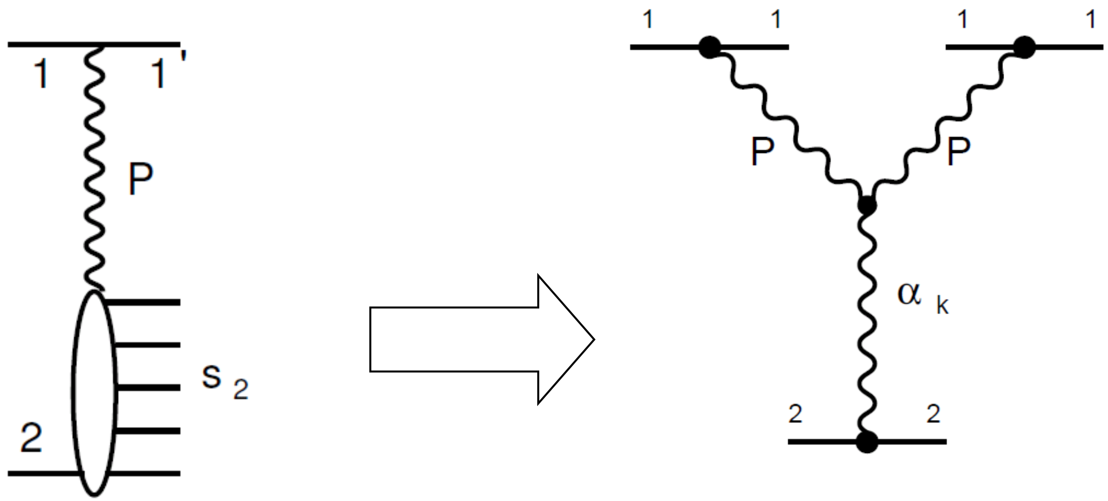
Multiple Pomeron exchange ・・・energy dependence similar to Froissart bound



Pomeron interaction ・・・ important for diffractive scattering

Pomeron interaction

- Single diffractive event

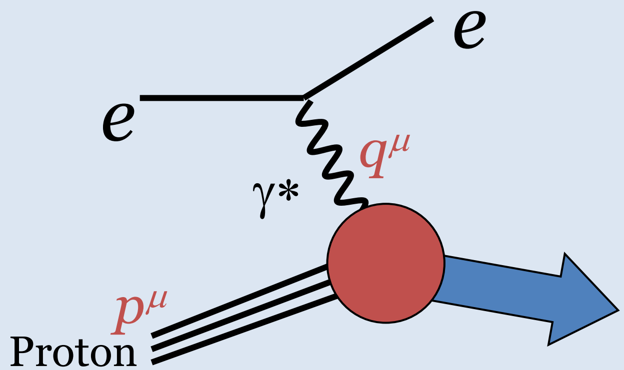


Can determine triple Reggeon vertex from diffractive data

→ At higher energies, multiple Pomeron exchange and Pomeron interaction become important and modify the simple 1 Pomeron exchange picture.
(Reggeon Field Theory)

Deep inelastic scattering of proton

- Kinematics



$W^2 = (p+q)^2$

square of total energy of γ-p system

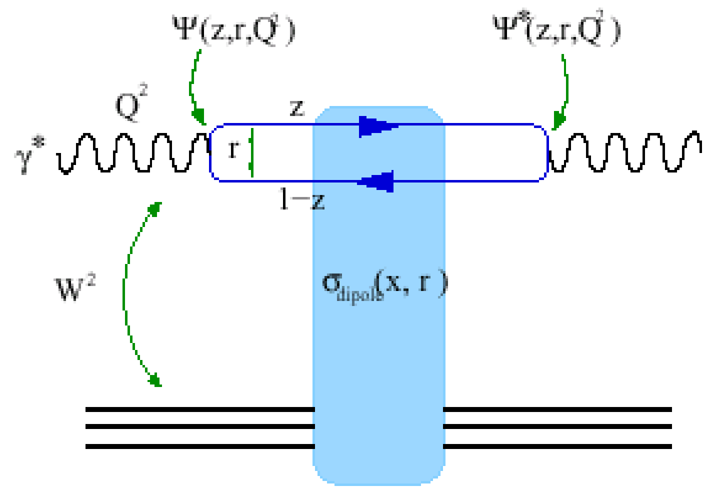
$Q^2 = -q^2$ virtuality of photon

$W^2 >> Q^2$

- F_2 structure function

$$\sigma_{\rm tot}^{\gamma^* p}(x,Q^2) = \sum_{T,L} \int_0^1 dz \int d^2 r_\perp \left|\Psi_{T,L}(z,r_\perp,Q^2)\right|^2 \sigma_{\rm dipole}(x,r_\perp)$$

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\rm EM}} \sigma_{\rm tot}^{\gamma^* p}(x,Q^2)$$



Regge limit in DIS: Small-x physics

$$s = \frac{Q^2(1-x)}{x}$$

x : Bjorken variable

$$s >> Q^2$$

$x \rightarrow 0$ "small-x physics"

Let us apply the 1 Pomeron exchange in the γ*-p total X sec in the small x limit

$$\sigma_{tot}^{\gamma^* p} \sim s^{\alpha(0)-1} \quad \Rightarrow \quad F_2(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha(0)-1} = x^{-0.08}$$

$F_2(x,Q^2)$ shows slow increase with decreasing x ?

→ Can be confirmed in experimental data

NO!!!!

$$F_2(x,Q^2) \sim x^{-0.3}$$

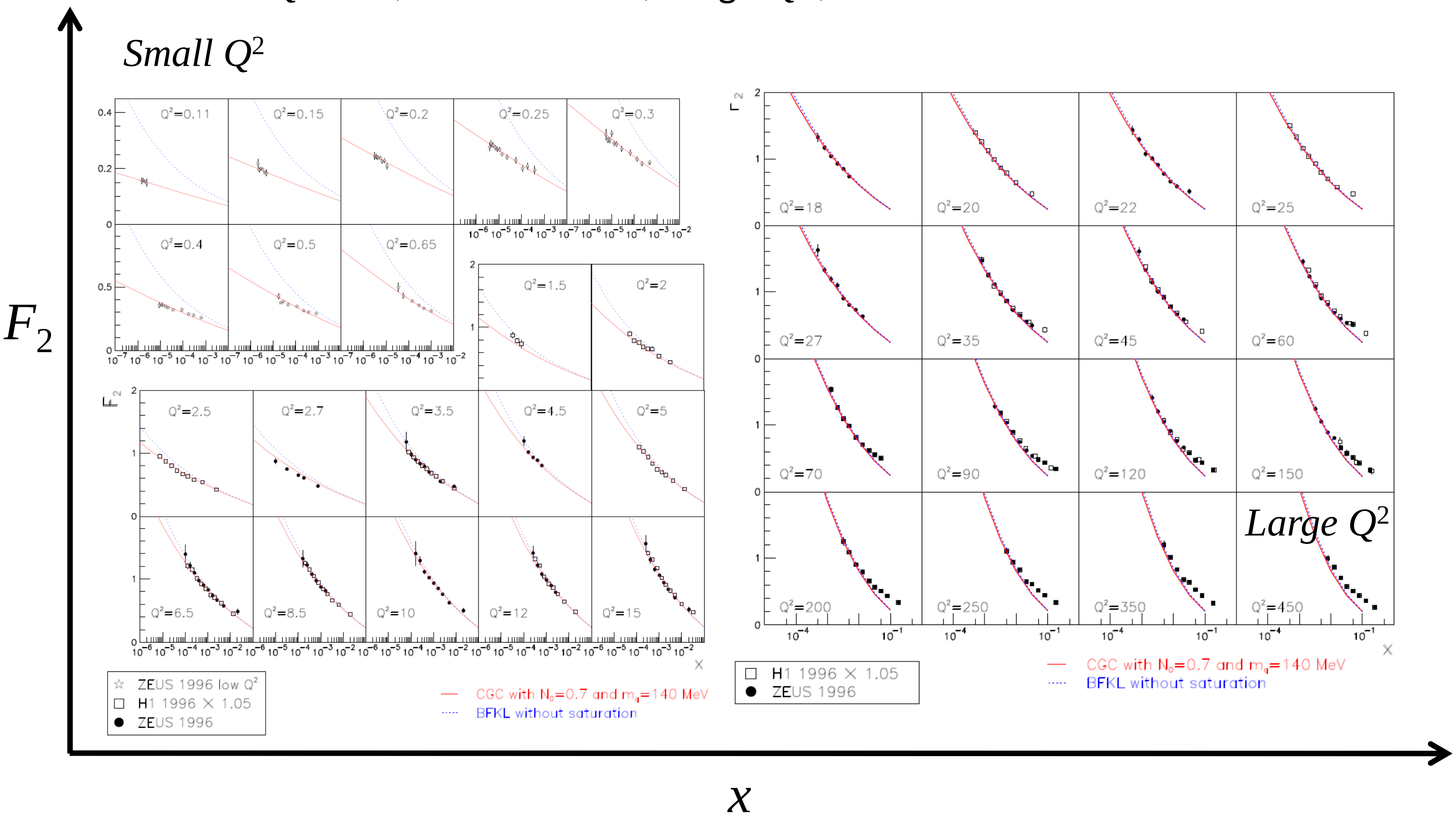
steep rise!

BFKL Pomeron (hard Pomeron), CGC

Soft Pomeron vs hard Pomeron

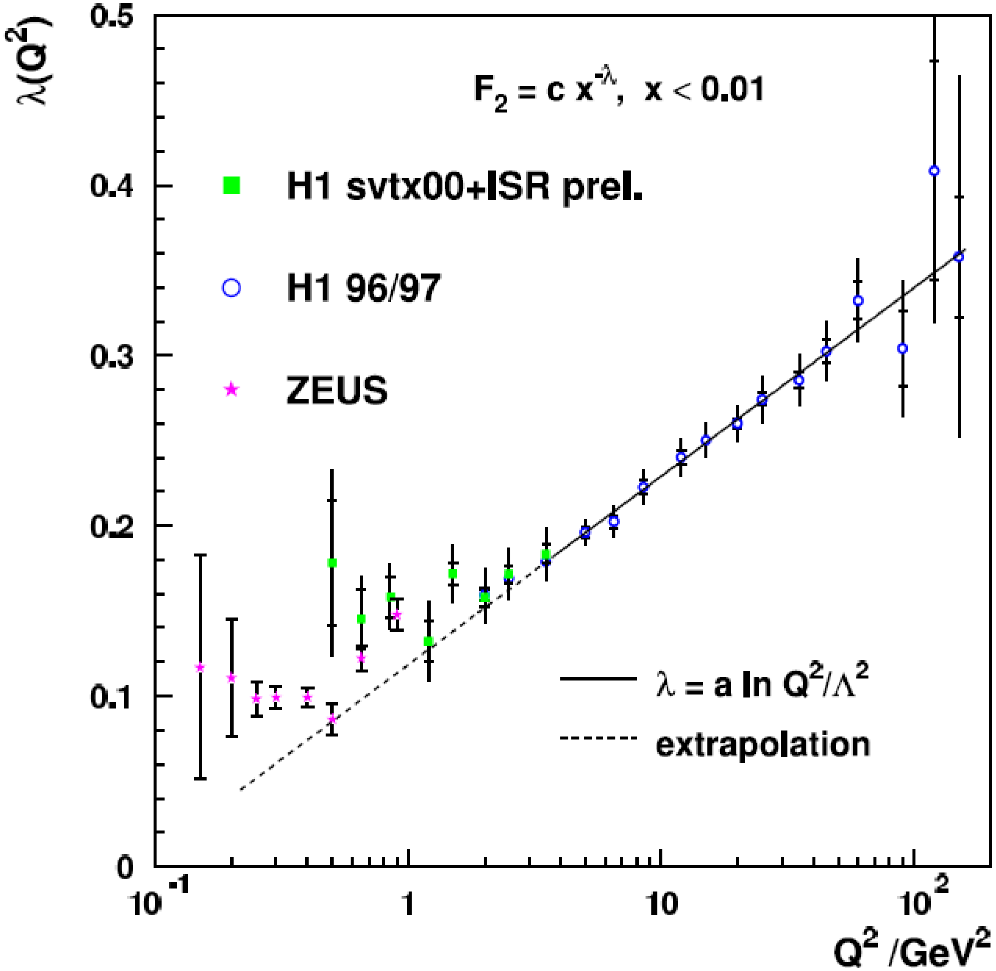
A closer look finds Q^2 dependent exponent

$Q^2 \to 0$, $\alpha - 1 \to 0.08$, large Q^2, $\alpha - 1$ increases



Q^2-dependence of exponent

$$F_2(x, Q^2) \sim x^{-\lambda(Q^2)}$$



Soft Pomeron → hard Pomeron

Instead of summary

``what is pomeron?"

Something like a particle which is responsible for describing increasing total cross section of hadron-hadron scattering.

↓

- Total cross sections of hadron-hadron scattering slowly increase with increasing scattering energy.
- In the Regge theory (a mathematical framework of relativistic S-matrix), high-energy behavior of the scattering amplitude is determined by a pole in the complex angular momentum plane. This looks like an exchange of a particle-like object in t-channel.
- In particular, an object having the same quantum numbers as the vacuum is called Pomeron and governs the high energy behavior of the total X section. Pomeron is used as a phenomenological description of the cross section.
- A close inspection of the experimental data suggests deviation from the 1 Pomeron exchange.
- In deep inelastic scattering, different properties of Pomeron is measured, and is understood from QCD (hard Pomeron, QCD Pomeron).