Ion-optical calculation with realistic three-dimensional field mapping for the BigRIPS fragment separator

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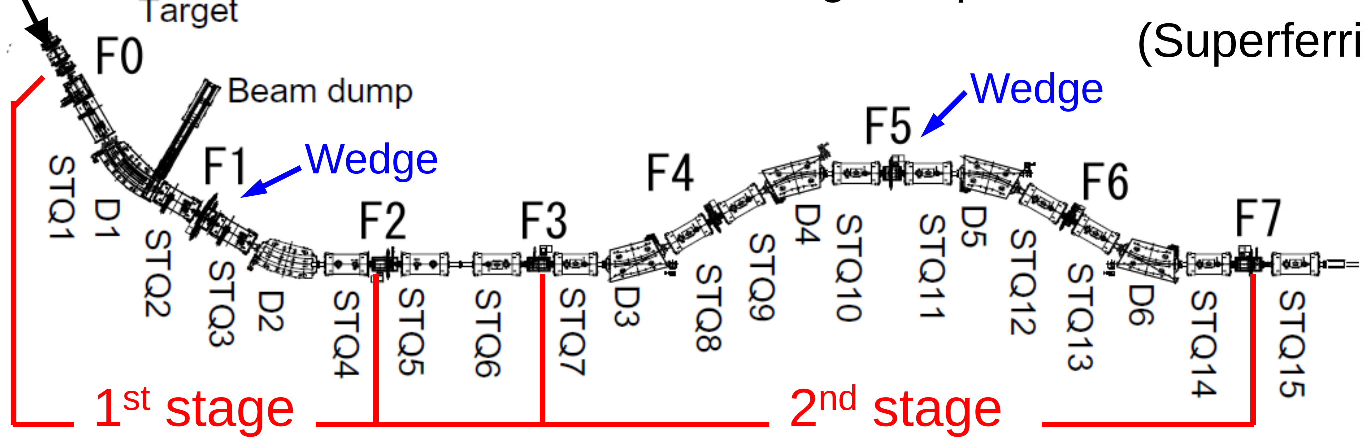
Features of BigRIPS separator

- 1) Large acceptances
  - Comparable with angular / momentum spreads of in-flight fission at RIBF energy (+/-50 mrad, +/-5%)
- 2) Superconducting quads with a large aperture, and strong field
  - Pole tip radius: 170 mm
  - Max. pole tip field: 2.4 T
- 3) Two-stage separator scheme
  - 1^{st} stage : 2 bend, p/Δp=1260
  - 2^{nd} stage : 4 bend, mirror sym. @ F5, p/Δp= 3420
  - Better resolution at 2^{nd} stage for particle ID

Parameters:
Δa = +/-40 mrad
Δb = +/-50 mrad
Δp/p = +/-3 %
Bρ = 9 Tm
L ~ 78 m

STQ
(Superferric Q)



Production & separation

Matching section

Particle identification & two-stage separation

STQ1-14: Superconducting quad. triplets

D1-6: Room temp. dipoles (30 deg)

F1-F7: focuses

Superconducting Triplet Quadrupole (STQ)

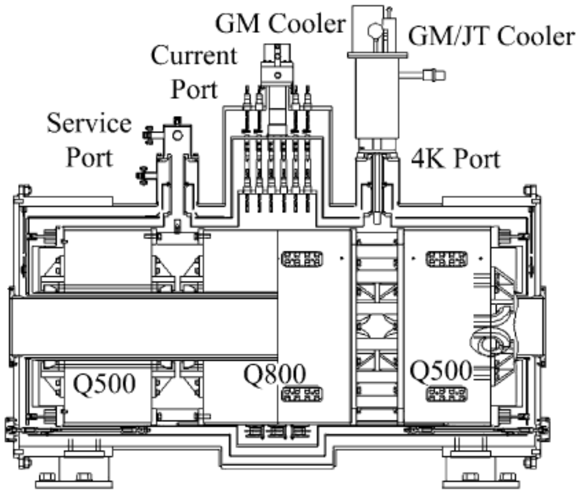


Fig. 6. A schematic diagram of the prototype quadrupole triplet with small cryocoolers.

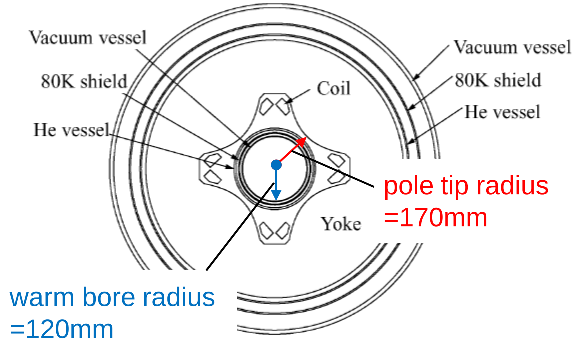
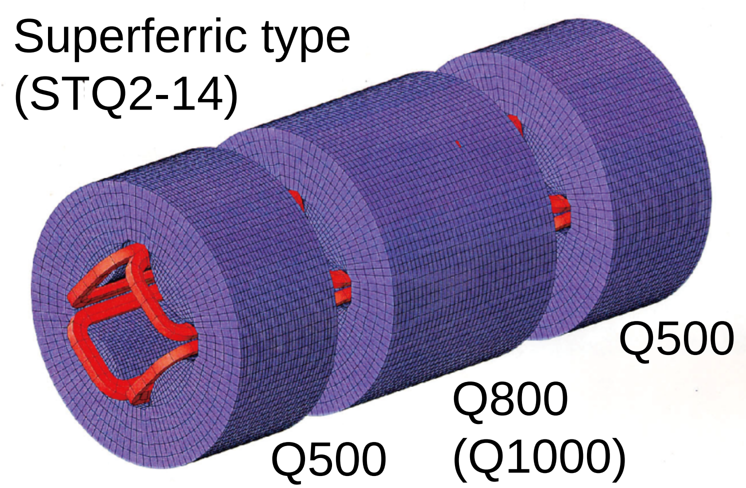
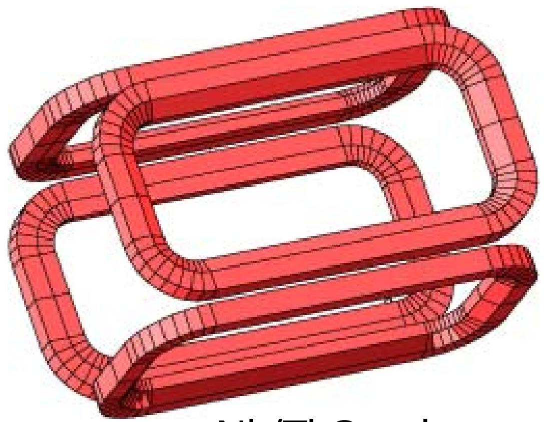


Fig. 2. Cross-sectional view of the prototype quadrupole.

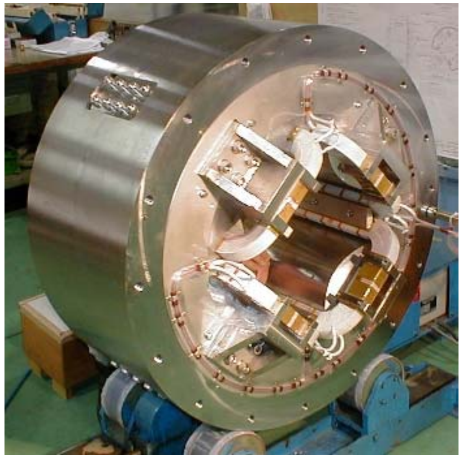


Sextupole magnet is superimposed
on one of the Q500 magnet

Racetrack Coils



Nb/Ti Conductor

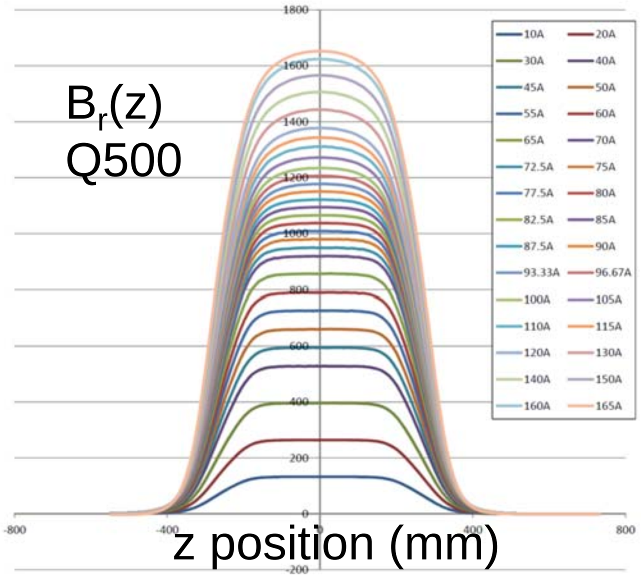


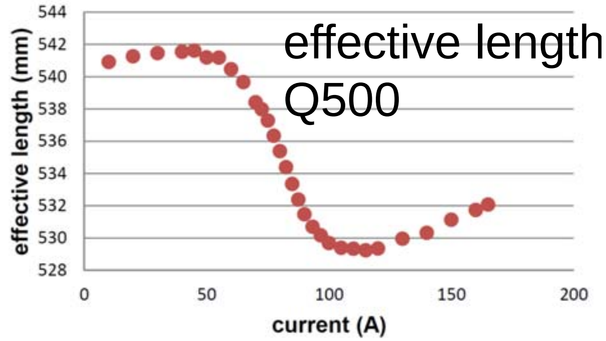
Our goal: accurate ion-optical setting *without any tuning*

We have to overcome various problems concerning short-length, large-aperture, and strong field magnets.

- **Large fringing field region**
  - Entire region must be treated as fringe.
- **Large saturation effect**
  - Shape and effective length vary drastically with the magnet excitation.

→ The effects of the varying field maps should be included in the simulation.

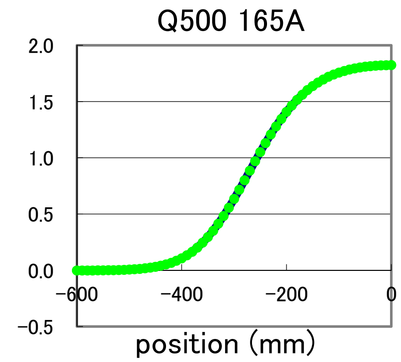




Procedure of field map analysis and ion-optical calculation

- Measure detailed 3D-field maps as a function of magnet current I.
- Deduce first-order distribution $b_{n,0}(z,I)$ from the measured field map.
- Fit $b_{n,0}$ distribution by Enge function.
  Its Enge coefficients are the function of magnet current I.
- Make detailed ion-optical calculation using the deduced Enge coefficients with COSY INFINITY code.
- Search magnet current setting, which satisfies the desired ion-optical setting.

$$F(z) = \frac{1}{1 + \exp\left[a_1 + a_2 \left(z/D \right) + \cdots + a_6 (z/D)^5 \right]}$$



Multipole analysis of 3D magnetic field

in cylindrical coordinate

(without skew terms for simplicity)

measurement

Fourier analysis

$$\begin{cases} B_r(r,\theta,z) &= \sum_{n=1}^{\infty} B_{r,n}(r,z)\sin n\theta, \\ B_\theta(r,\theta,z) &= \sum_{n=1}^{\infty} B_{\theta,n}(r,z)\cos n\theta, \\ B_z(r,\theta,z) &= \sum_{n=1}^{\infty} B_{z,n}(r,z)\sin n\theta. \end{cases}$$

n=1: dipole
n=2: quadrupole
n=3: sextupole
…

Extraction of $b_{n,0}$

$$\begin{cases} B_{r,n}(r,z) &\equiv \left(\frac{r}{r_0}\right)^{n-1} \sum_{m=0}^{\infty} b_{n,m}(z)\left(\frac{r}{r_0}\right)^{2m}, \\ B_{\theta,n}(r,z) &\equiv \left(\frac{r}{r_0}\right)^{n-1} \sum_{m=0}^{\infty} \frac{n}{n+2m} b_{n,m}(z)\left(\frac{r}{r_0}\right)^{2m}, \\ B_{z,n}(r,z) &\equiv \left(\frac{r}{r_0}\right)^{n} \sum_{m=0}^{\infty} \frac{r_0}{n+2m}\frac{\partial}{\partial z} b_{n,m}(z)\left(\frac{r}{r_0}\right)^{2m}. \end{cases}$$

3D-magnetic field (B_r, B_θ, $B_z(r,\theta,z)$) is expressed by $b_{n,0}(z)$ only.

$$b_{n,m}(z) = -\frac{r_0^2}{4m(n+m)}\frac{n+2m}{n+2(m-1)}\frac{\partial^2}{\partial z^2} b_{n,m-1}(z).$$

Procedure to deduce $b_{n,0}$ from $B_{r(\theta),n}$

(originally performed by H. Suzuki)

Differential equation for $b_{n,m}$:

$$b_{n,m}(z) = -\frac{r_0^2}{4m(n+m)}\frac{n+2m}{n+2(m-1)}\frac{\partial^2}{\partial z^2}b_{n,m-1}(z).$$

(m>0)

Fourier transform

$$\tilde{b}_{n,m}(k) = \int_{-\infty}^{\infty} b_{n,m}(z)e^{-ikz}\,dz$$

$$\frac{\partial}{\partial z} \to -ik$$

z derivative can be translated into simple algebraic calculation by FT

$$\begin{aligned}
\tilde{b}_{n,m}(k) &= -\frac{r_0^2}{4m(n+m)}\frac{n+2m}{n+2(m-1)}(-ik)^2\,\tilde{b}_{n,m-1}(k) \\
&= \frac{(r_0k)^2}{4m(n+m)}\frac{n+2m}{n+2(m-1)}\tilde{b}_{n,m-1}(k) \\
&= q_m\tilde{b}_{n,m-1}(k) \\
&= q_mq_{m-1}\tilde{b}_{n,m-2}(k) \\
&\quad\vdots \\
&= q_mq_{m-1}\cdots q_1\tilde{b}_{n,0}(k) \\
&= p_m\tilde{b}_{n,0}(k) \quad \left(p_m \equiv \prod_{i=1}^{m} q_i\right)
\end{aligned}$$

q_m

Procedure to deduce $b_{n,0}$ from $B_{r,n}$

$$B_{r,n}(r,z) = \left(\frac{r}{r_0}\right)^{n-1} \sum_{m=0}^{\infty} b_{n,m}(z) \left(\frac{r}{r_0}\right)^{2m}$$

$$B_{r,n}(r=r_0,z) = \sum_{m=0}^{\infty} b_{n,m}(z)$$

decomposed from measured data

Fourier tr.

$$\tilde{B}_{r,n}(k) = \int_{-\infty}^{\infty} \boxed{B_{r,n}(r=r_0,z)} e^{-ikz}\, dz$$

$$\tilde{B}_{r,n}(k) = \sum_{m=0}^{\infty} \tilde{b}_{n,m}(k) = \sum_{m=0}^{\infty} p_m \tilde{b}_{n,0}(k)$$

$$\tilde{b}_{n,0}(k) = \tilde{B}_{r,n}(k) \Big/ \sum_{m=0}^{\infty} p_m$$

Inv. Fourier tr.

$$b_{n,0}(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{b}_{n,0}(k) e^{+ikz}\, dk$$

$b_{n,0}(z)$ is obtained without solving high-order differential equation

Procedure to deduce $b_{n,0}$ from $B_{\theta,n}$

$$B_{\theta,n}(r,z) = \left(\frac{r}{r_0}\right)^{n-1} \sum_{m=0}^{\infty} \frac{n}{n+2m} b_{n,m}(z) \left(\frac{r}{r_0}\right)^{2m}$$

$$B_{\theta,n}(r=r_0,z) = \sum_{m=0}^{\infty} \frac{n}{n+2m} b_{n,m}(z)$$

decomposed from measured data

Fourier tr.

$$\tilde{B}_{\theta,n}(k) = \int_{-\infty}^{\infty} B_{\theta,n}(r=r_0,z) e^{-ikz}\, dz$$

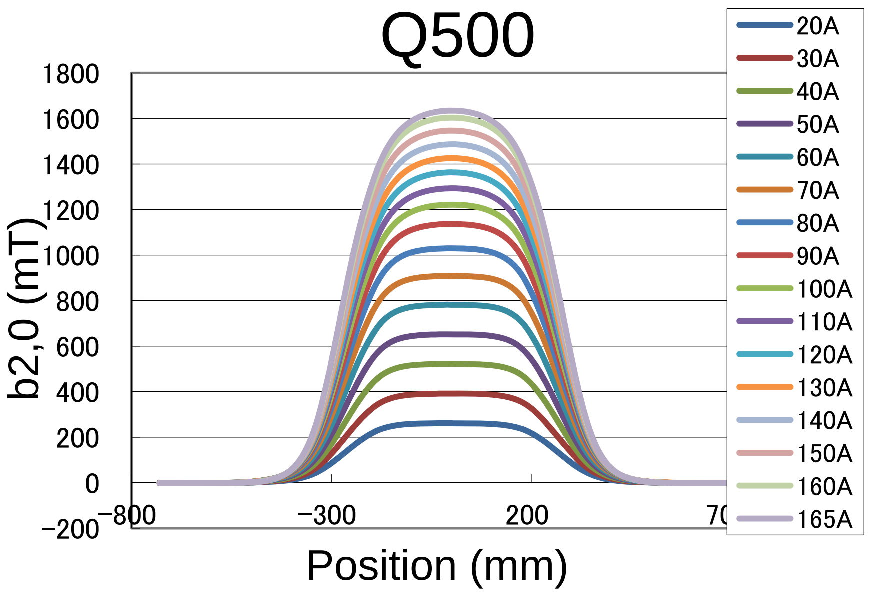
$$\tilde{B}_{\theta,n}(k) = \sum_{m=0}^{\infty} \frac{n}{n+2m} \tilde{b}_{n,m}(k) = \sum_{m=0}^{\infty} \frac{n}{n+2m} p_m \tilde{b}_{n,0}(k)$$

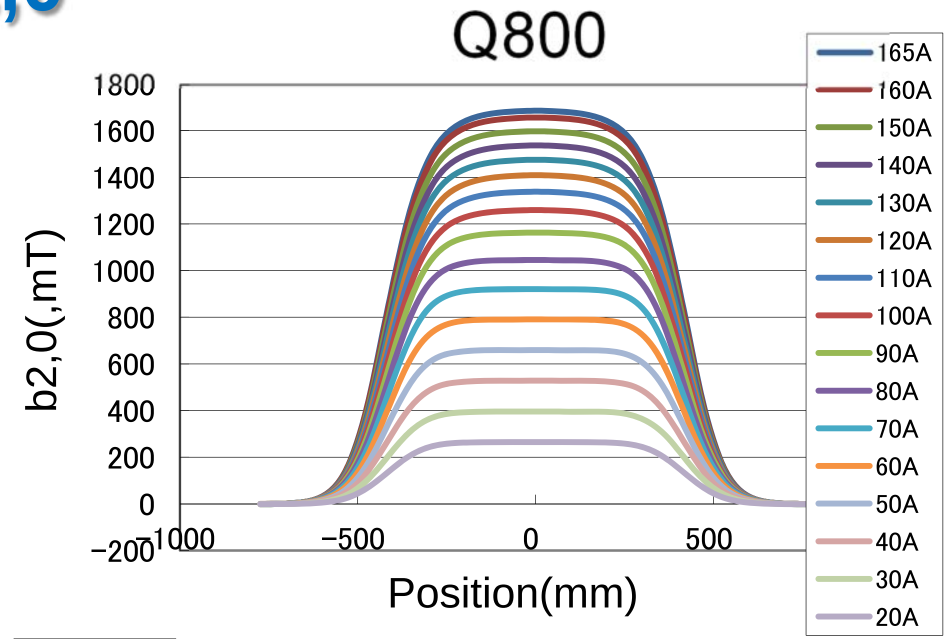
$$\tilde{b}_{n,0}(k) = \tilde{B}_{\theta,n}(k) \Bigg/ \sum_{m=0}^{\infty} \frac{n\, p_m}{n+2m}$$

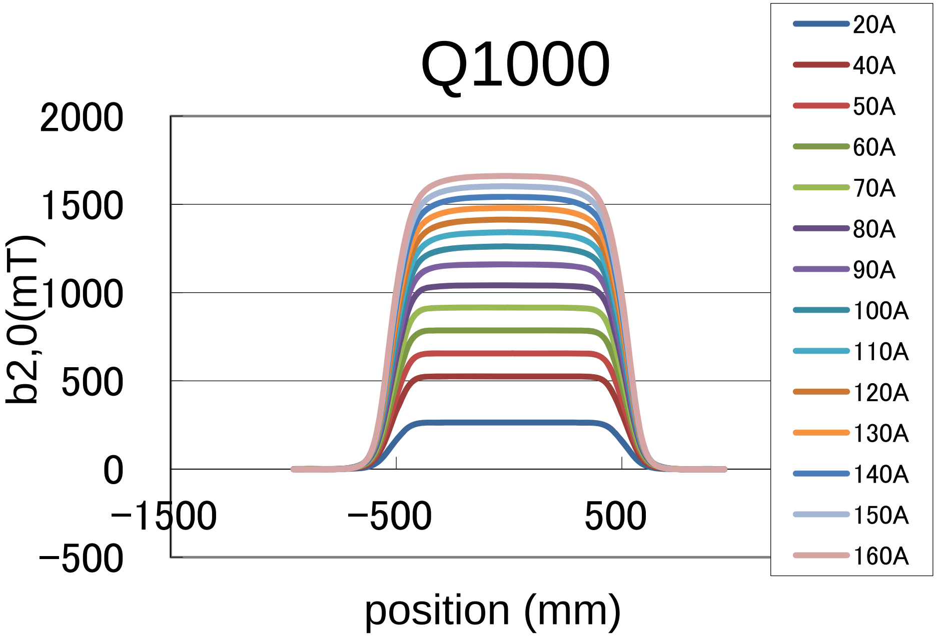
Inv. Fourier tr.

$$b_{n,0}(z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{b}_{n,0}(k) e^{+ikz}\, dk$$

Extracted $b_{2,0}$ distributions



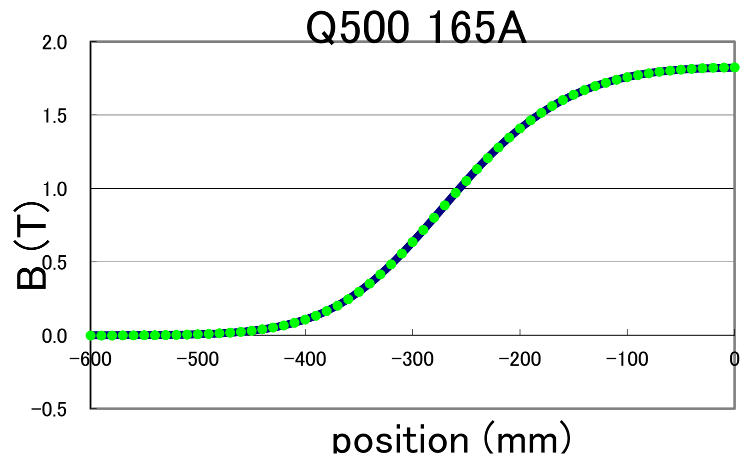


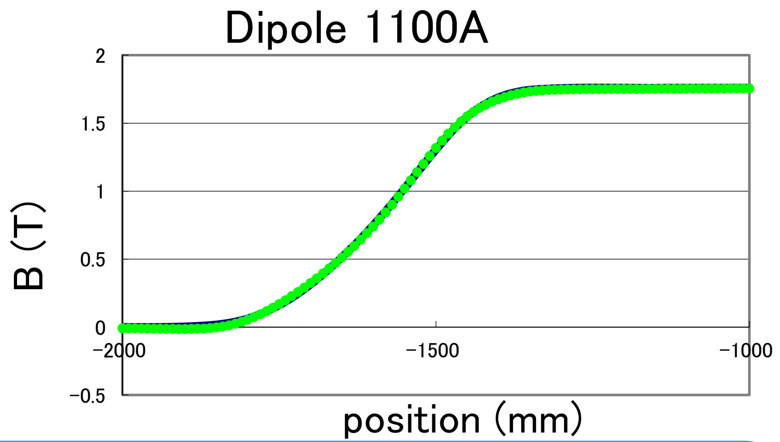


- Large fringe region
- The shape of the distribution varies much according to the excitation current.

Fringing field fitting

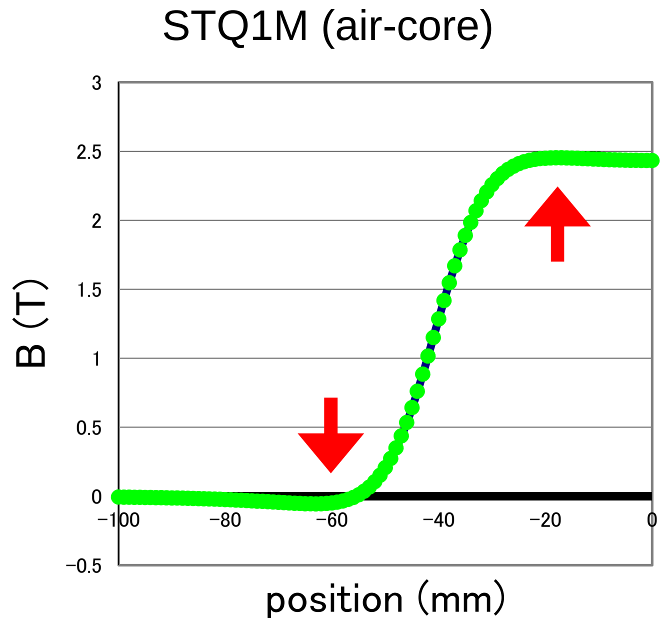
Enge coefficients a_i are freely searched to minimize $\Sigma_i\,[b_{2,0}(z_i)\text{-}Enge(z_i)]^2$.





$$F(z) = \frac{1}{1+\exp\left[a_1 + a_2(z/D) + \cdots + a_6(z/D)^5\right]}$$

(D : full aperture)

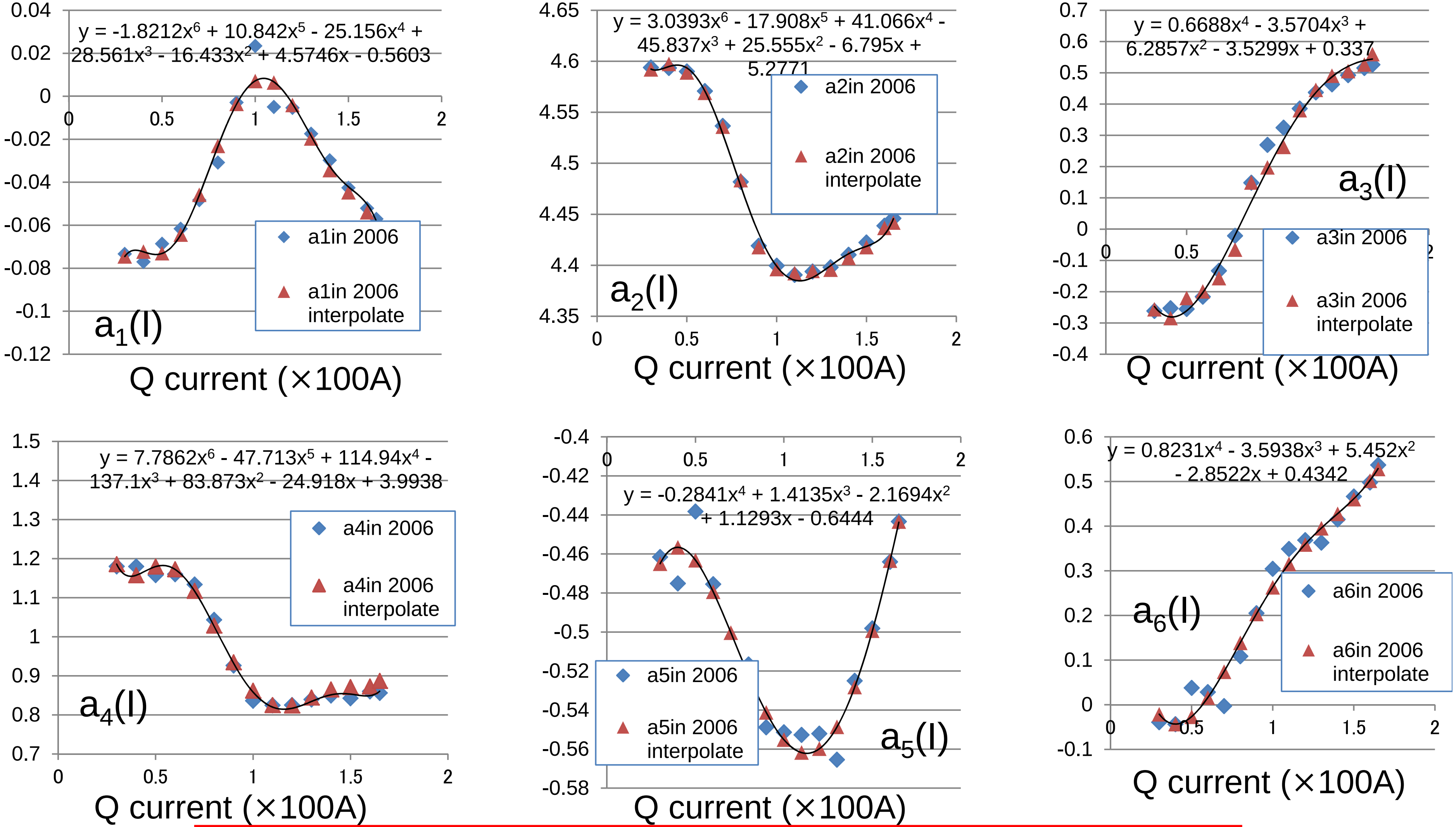


$$F(z) = \frac{1}{1+\exp\left[a_1 + a_2(z/D) + \cdots + a_6(z/D)^5\right]} + a_7 \tanh\left(a_8 + a_9(z/D)\right)\cdot\exp\left[-\left(\frac{z/D + a_{10}}{a_{11}}\right)^2\right]$$

Second term is introduced to express under- & overshooting shaped fields.

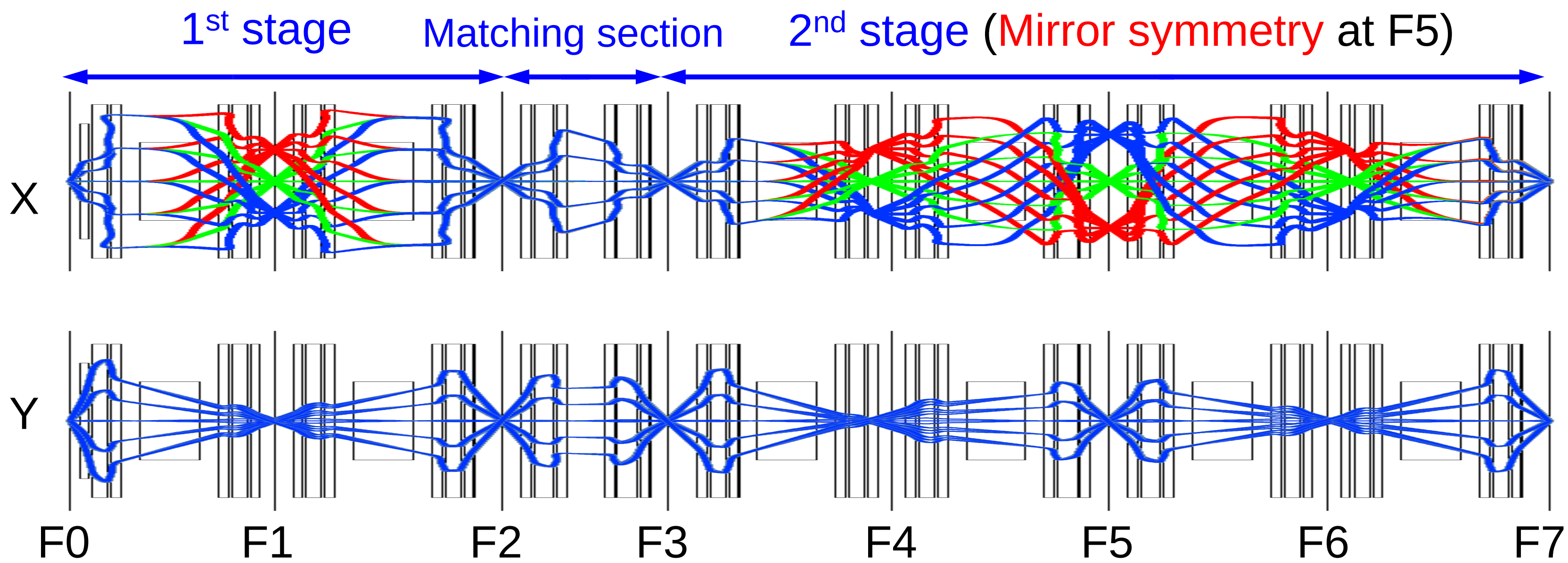
Enge coefficients

As a function of magnet current (Q500, inner side)



Enge coefficients are fitted with polynominal function.
→ Fitted Enge coefficients are used in our optics calculation.

Ion-optical setting of BigRIPS



F0F1

(x|x)=-1.7, (x|a)=0,
(y|y)=-5.0, (y|b)=0,
(x|δ)=-21.4 mm/%, (a|δ)=0

F0F2

(x|x)=2.0, (x|a)=0,
(y|y)=1.6, (y|b)=0,
(x|δ)=0 mm/%, (a|δ)=0

F2F3

(x|x)=-1.08, (y|y)=-1.18,
(x|a)=(y|b)=(a|x)=(b|y)=0

F3F4

(x|x)=-1.40, (x|a)=-0.791,
(y|y)=-3.45, (y|b)=-0.272,
(x|δ)=-22.1 mm/%, (a|δ)=0

F3F5

(x|x)=0.92, (x|a)=0,
(y|y)=1.06, (y|b)=0,
(x|δ)=31.7 mm/%, (a|δ)=0

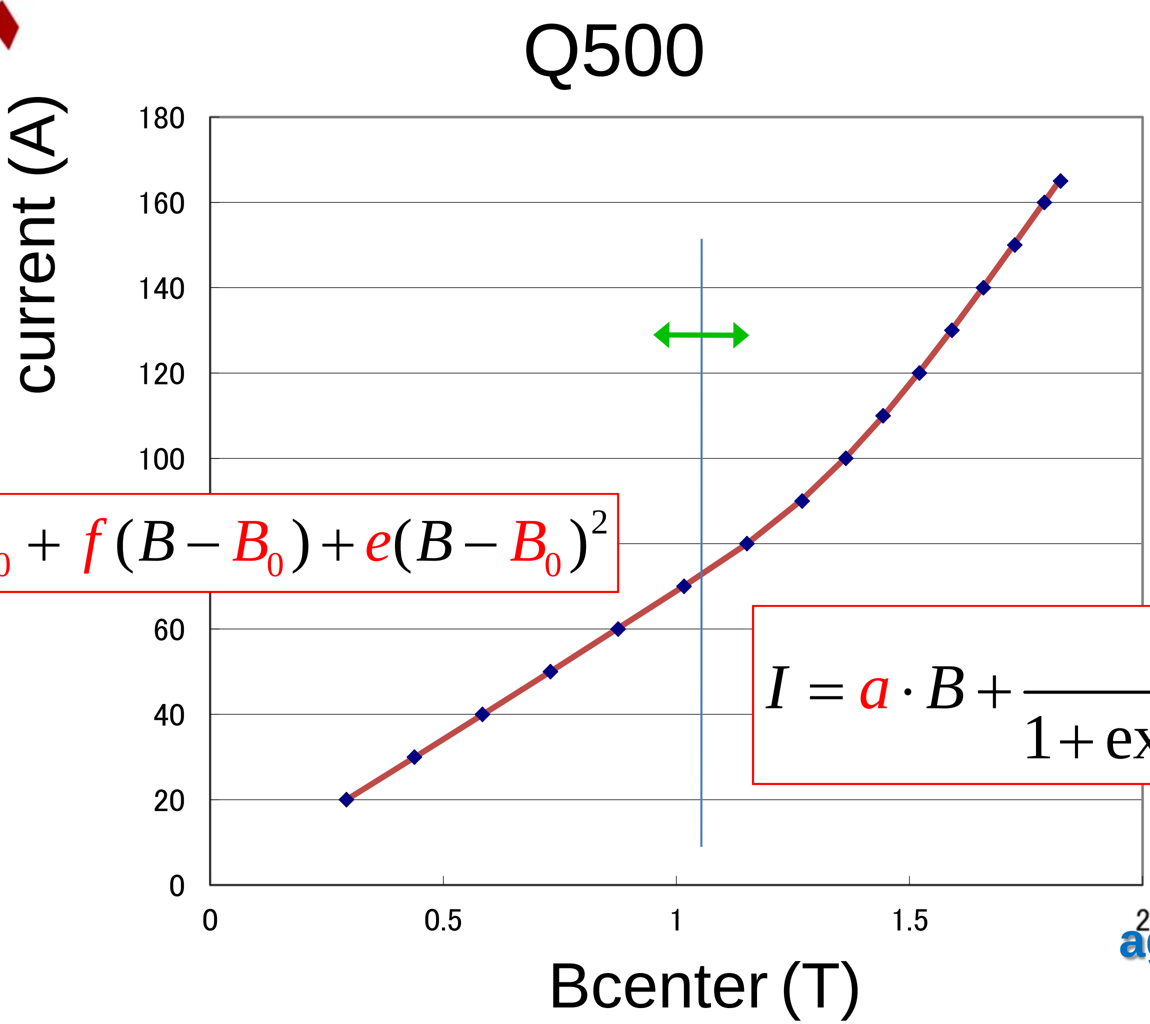
P/ΔP =
1260 (1^{st} stage)
3420 (2^{nd} stage)

Mirror symmetry of F3-F5

F7F6

F7F5

B-I curve



$$I = I_0 + f(B - B_0) + e(B - B_0)^2$$

$$I = a \cdot B + \frac{b \cdot (B - c)}{1 + \exp[-d \cdot (B - c)]}$$

agree within 0.1% order

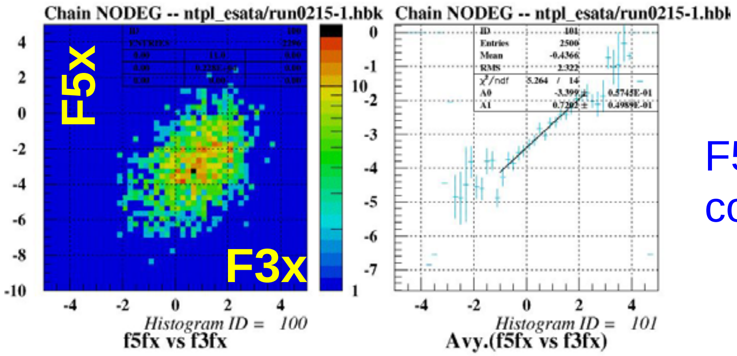
- same for other super-ferric quadrupoles and dipoles
- linear functions are used for air-core quads

Measurement of matrix elements with secondary beam

1st order matrix elements from F3 to F5

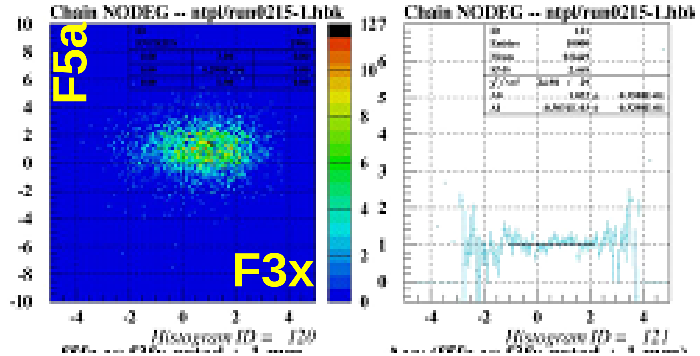
(x|x)

F5x vs F3x correlation



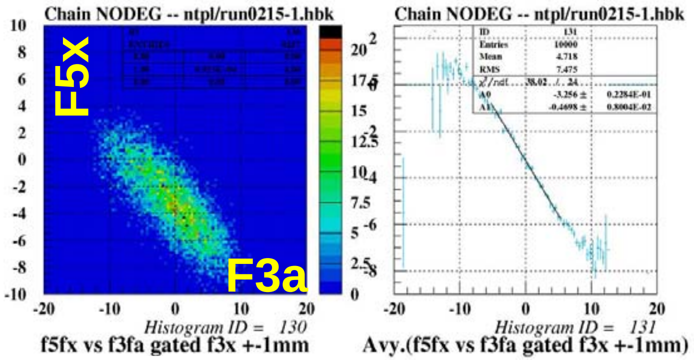
(a|x)

F5a vs F3x correlation



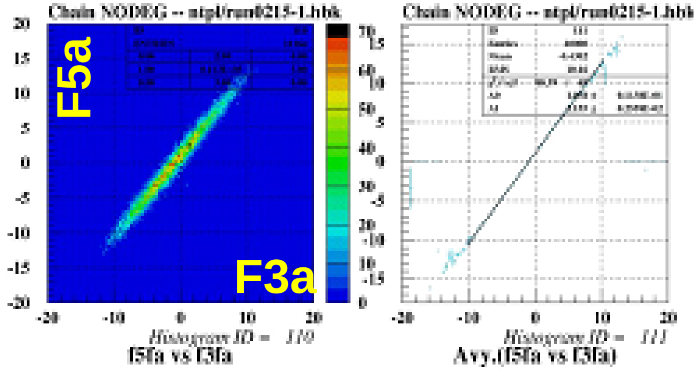
(x|a)

F5x vs F3a correlation



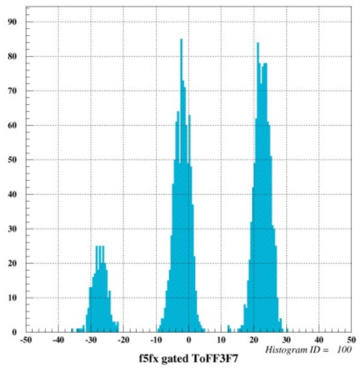
(a|a)

F5a vs F3a correlation



(x|δ)

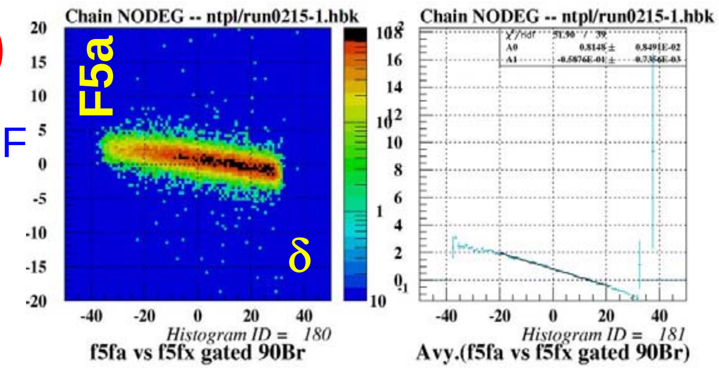
F5x vs TOF correlation



^{90}Br is selected (produced by in-fight fission).

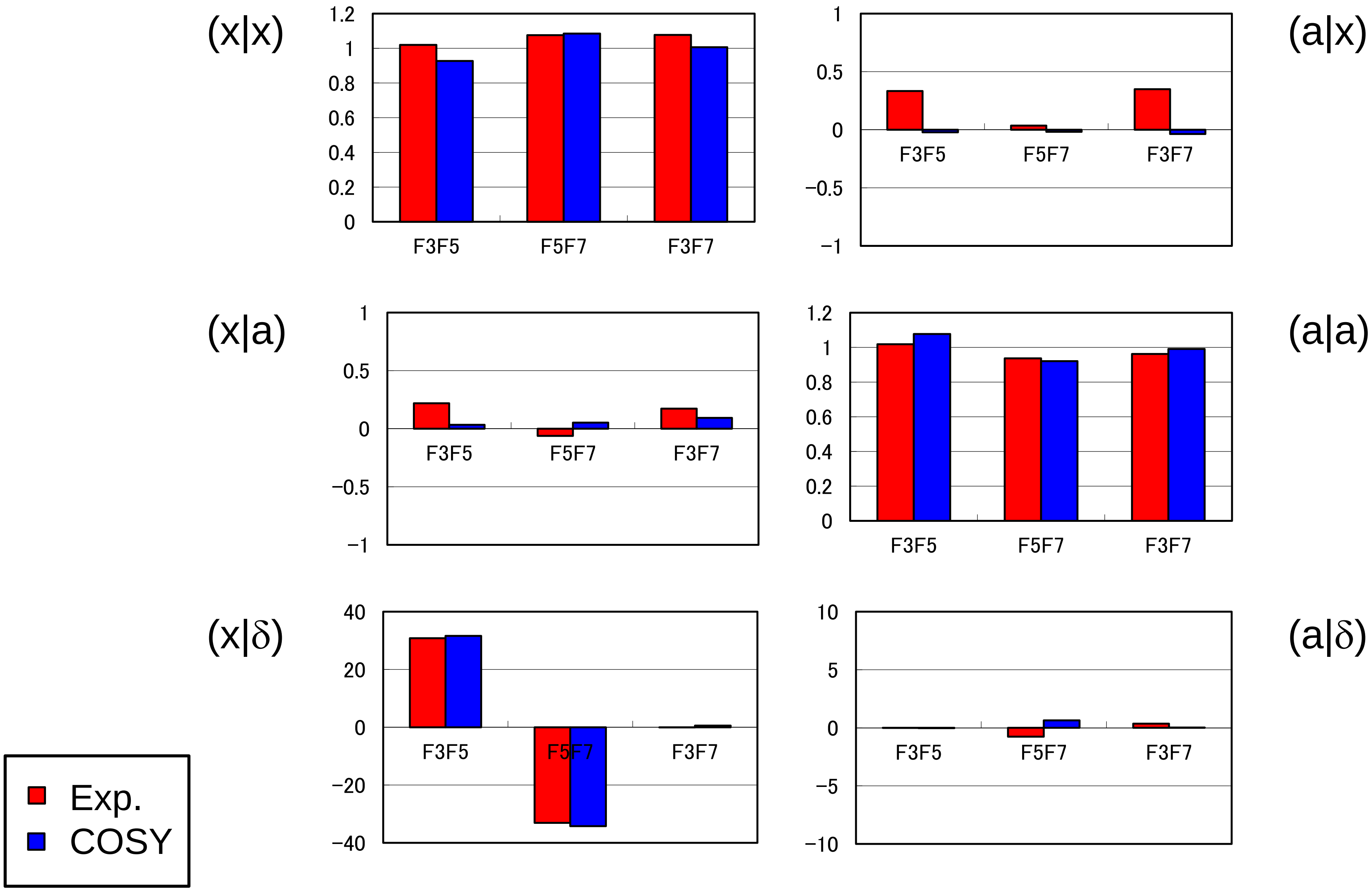
(a|δ)

F5a vs TOF correlation



F3x: ±1mm, F3a: ±1mrad gates are applied. δ is gated with TOF37: ±1ns.

Comparison of the matrix elements



G4top setting from 345 MeV/u U beam

Summary & issues

- **Short-length, large-aperture and strong field superconducting magnets are used in the BigRIPS separator for ^{238}U fission fragments.**

  → **Large fringe region with varying field distribution**
- **We are aiming at precise ion-optical setting without any tuning. Ion-optical calculation based on varying field maps is indispensable, otherwise even the first-order setting is not fulfilled.**
- **Procedures of 3D-field map analysis and ion-optical calculation are shown. New approach using the Fourier transform is applied to extract $b_{n,0}(z)$.**
- **$b_{n,0}$ distribution is fitted by Enge function and used in COSY INFINITY for ion-optical calculation.**
- **Transfer matrix elements are well reproduced by the COSY, except for the focusing term (x|a), which is very sensitive to strength of magnets. There is still room for improvement toward ion-optical setting without tuning.**
- **Application**
  - **Various optical system design and analysis are achieved in spite of the varying fringing fields.**
  - **A/Q resolution improvement → N. Fukuda's talk (yesterday)**
    - **efficient track reconstruction without using experimentally-determined first- and higher-order transfer matrices (in progress…)**

➢ Issues

- **COSY predictability improvement (first order)**
  - **measurement**
    - **improvement of field-map measurement and analysis**
    - **origin of errors**
  - **quality of parameterization**
    - **Fitting $b_{2,0}(z)$ distribution with Enge function**
    - **Fitting Enge coefficients with a function of excitation current I**
  - **B-I curve quality**
  - **…**
  - **Bρ scan quality**
  - **take care of interference**
    - **not only Q→SX but also SX→Q**
- **aberration study (higher order)**
  - **phase space, profile**
- **transmission study**
  - **MC**
- **…**