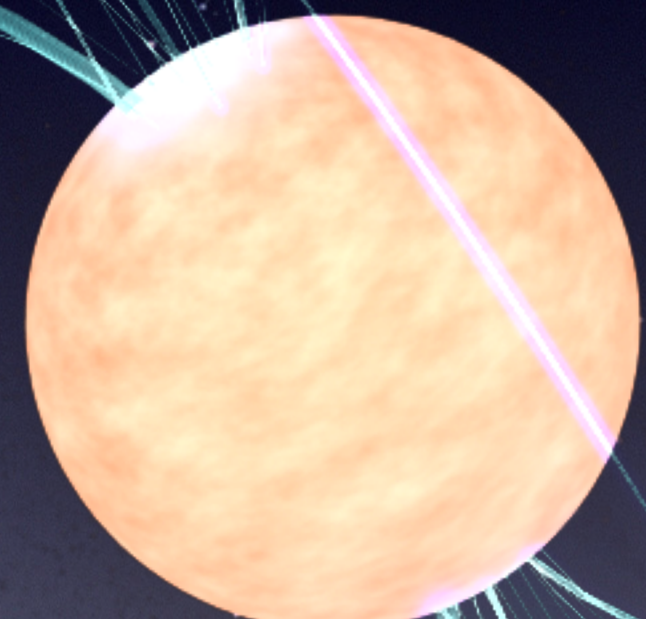
Global Current Circuit Structure in a Resistive Pulsar Magnetosphere Model

電気抵抗を含むパルサー磁気圏モデルの大域電流構造



加藤　祐悟

Yugo Kato

Publication paper

Yugo. E. Kato The Astrophysical Journal, 850:205 (2017)

arXiv paper is draft. So, please check ApJ paper

〜中性子星の観測と理論〜 研究活性化ワークショップ 2019

Outline

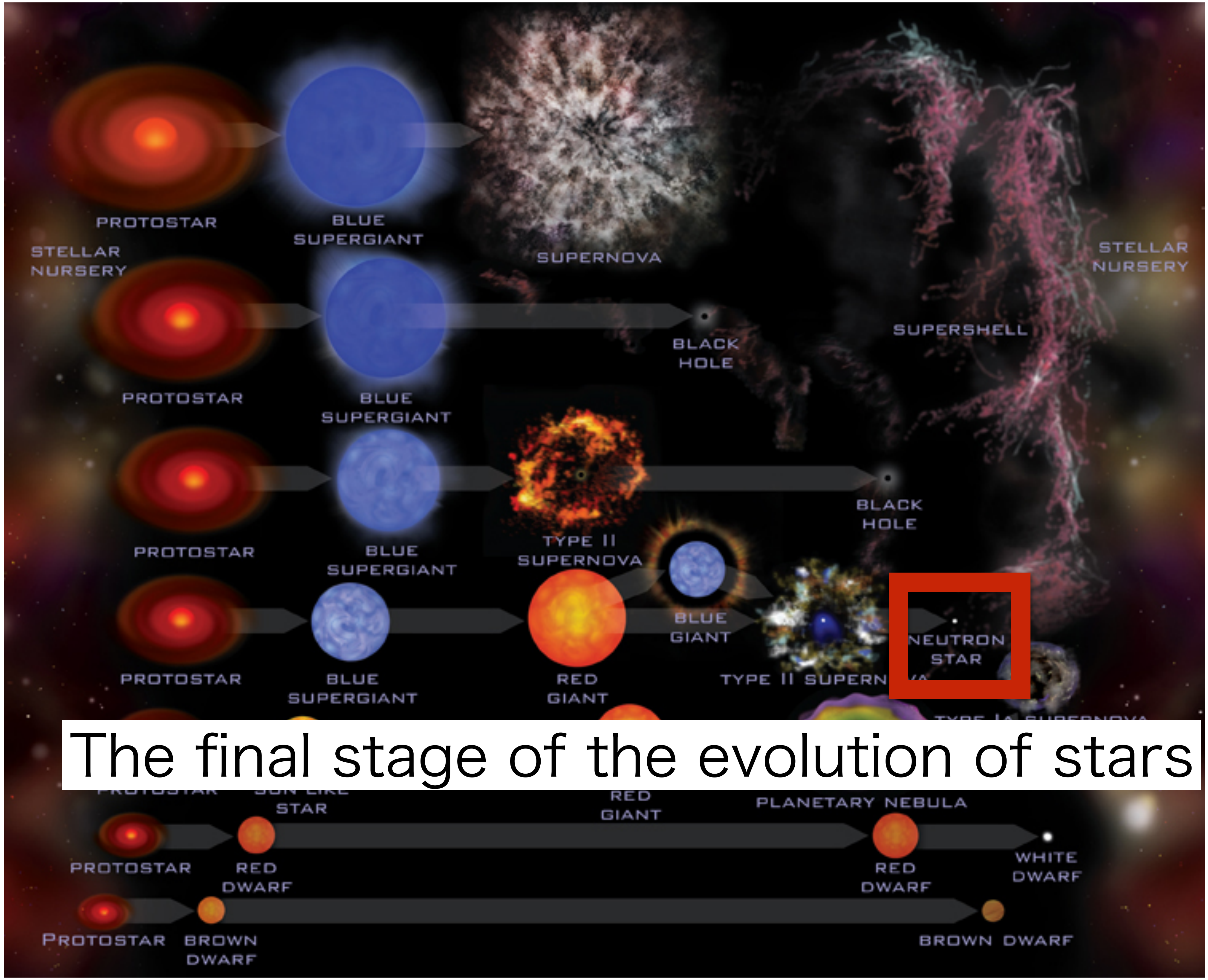
Background

- Neutron Star · Pulsar
- Force-Free pulsar magnetosphere

Contents of this research

- Pulsar magnetosphere including dissipation
- Simulation method
- Simulation results and discussion

Evolution of stars and neutron stars



The final stage of the evolution of stars

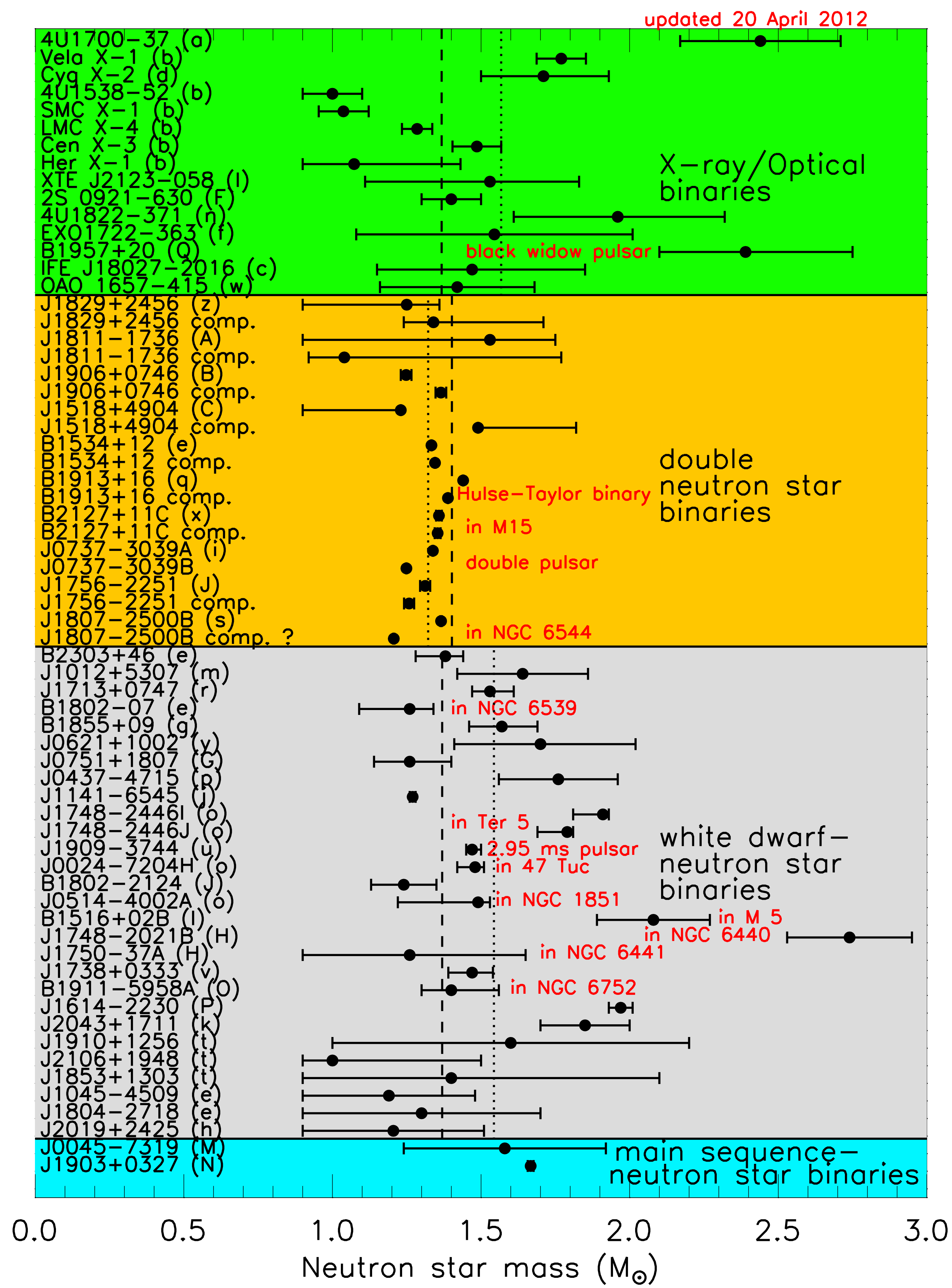
http://chandra.harvard.edu/resources/illustrations/stellar_evolution.html

Neutron star mass

1.4 Solar mass

$$M \sim 1.4 M_{\odot}$$

$$= 2.78 \times 10^{33} \mathrm{g}$$

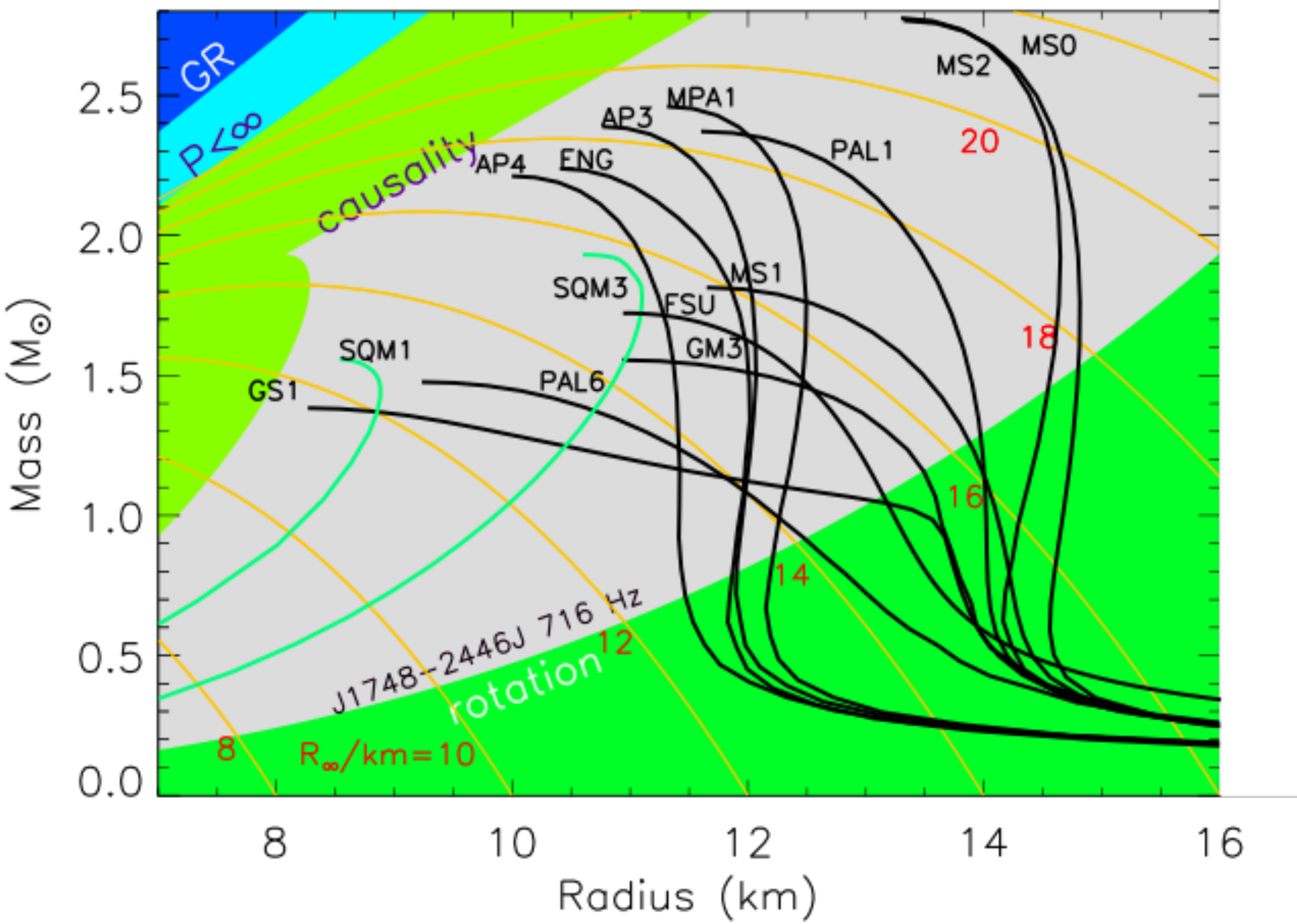


Equation of state, Radius of Neutron Stars

Neutron star radius

Lattimer JM, Prakash M. ApJ. 550:426 (2001)

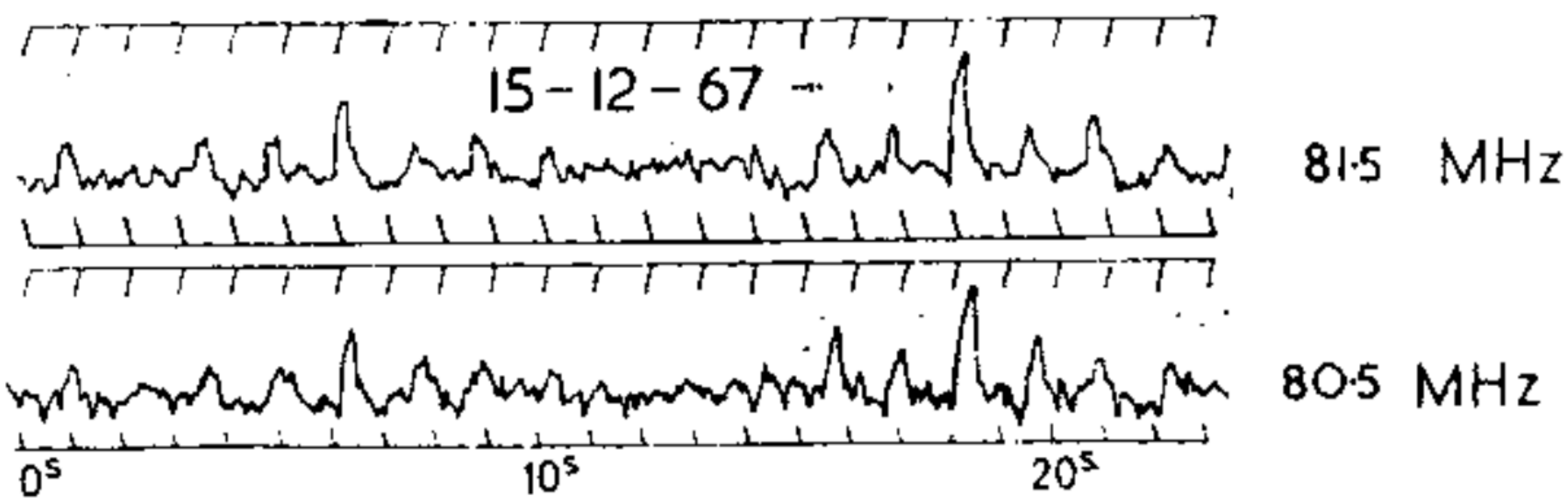
$R \sim 10\mathrm{km}$



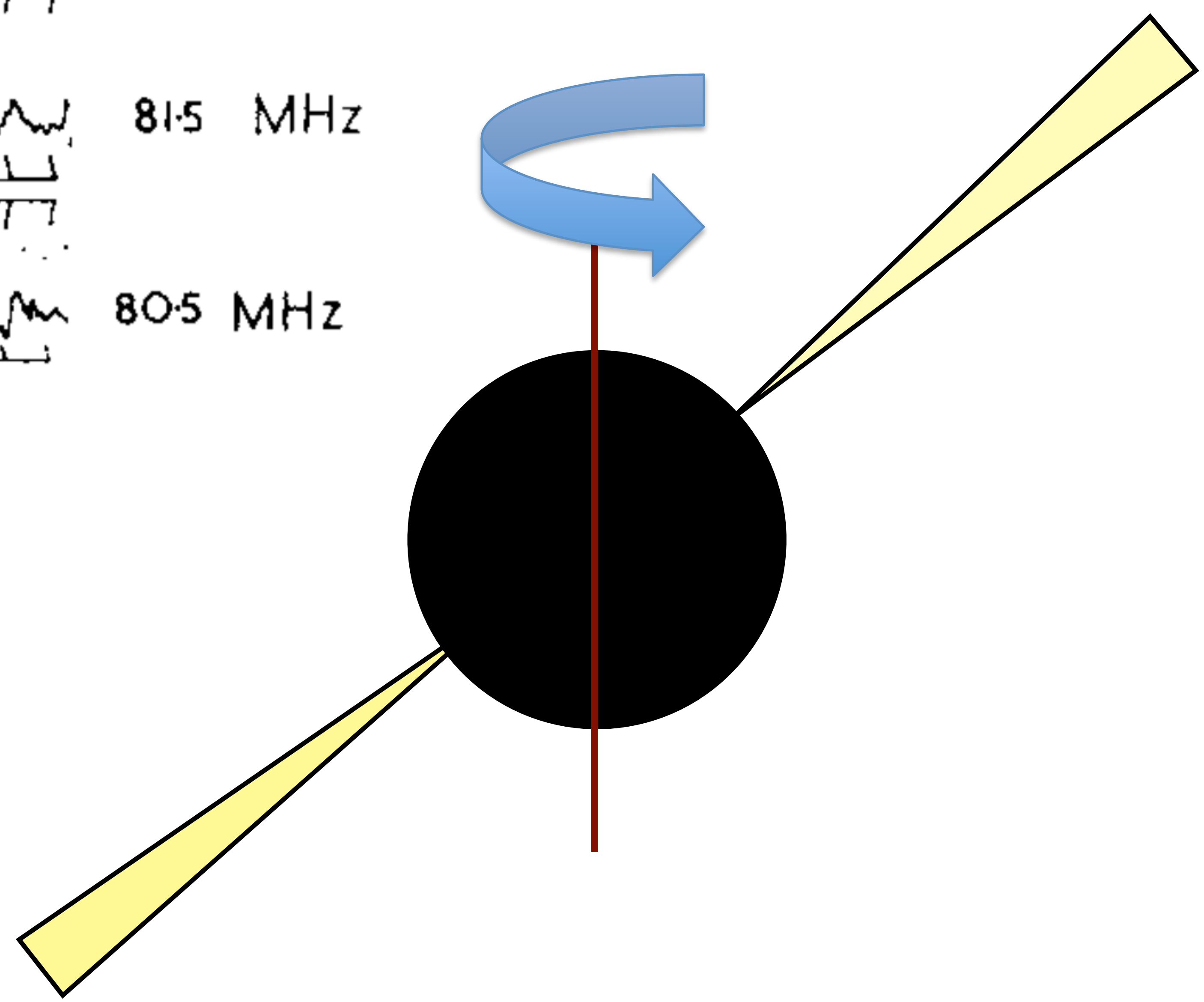
Pulsar

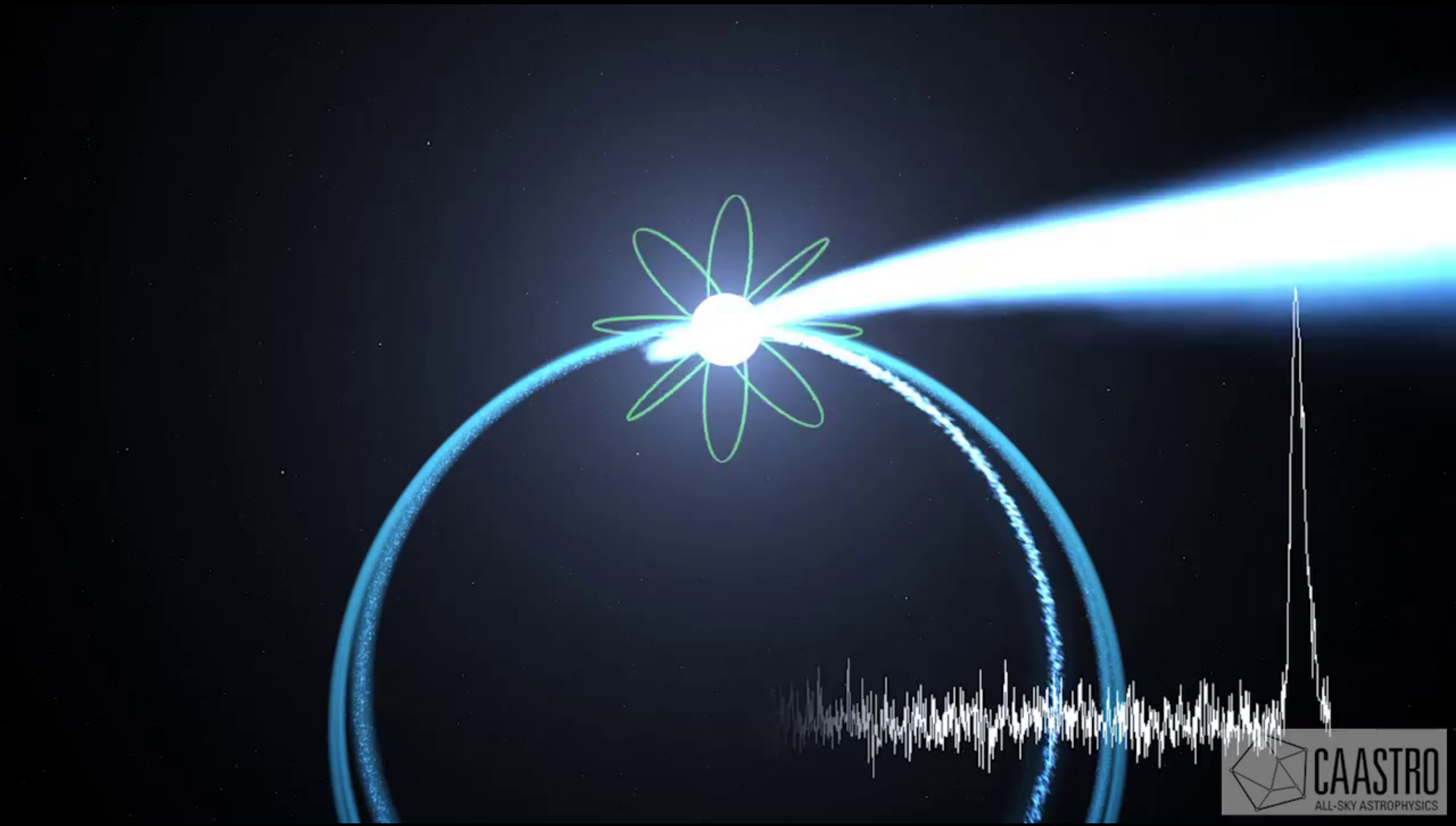
Radiating periodic pulses (neutron stars)

Stable cycle



Hewish et ai.(1968)

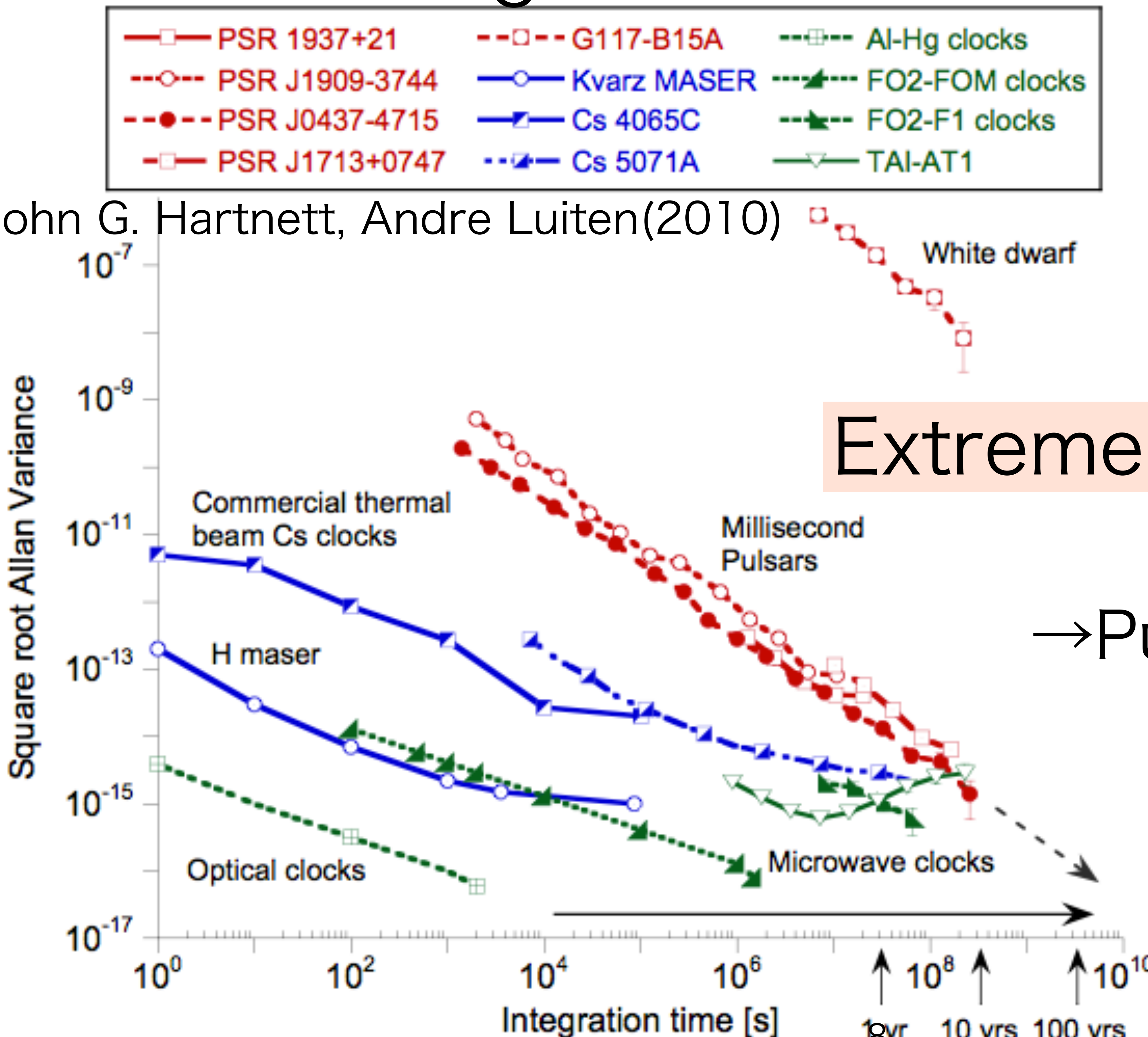




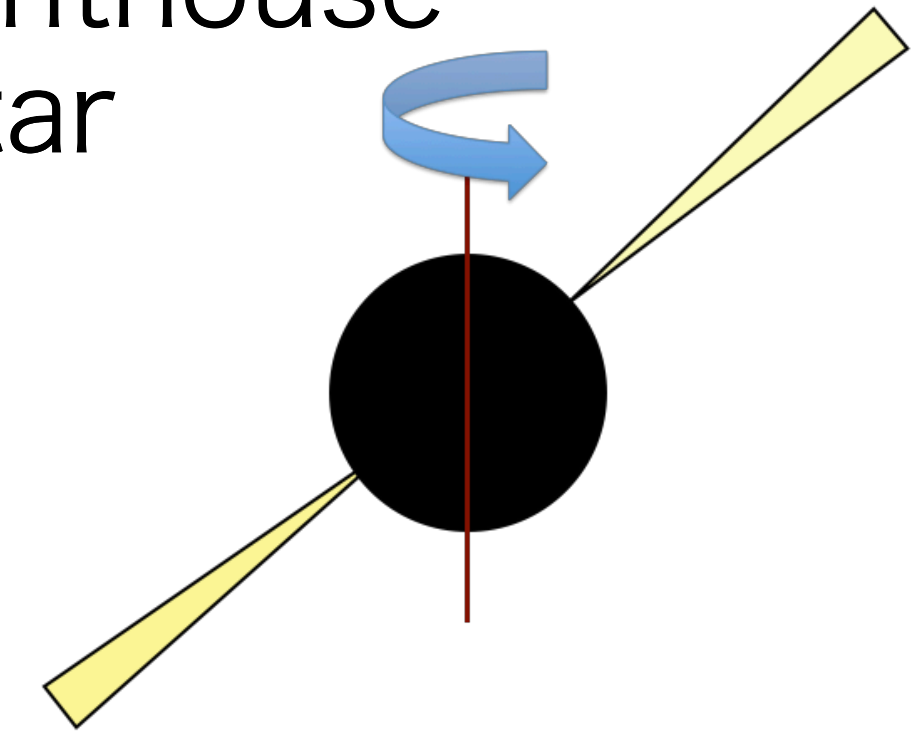
credit: ARC Centre of Excellence for All-sky Astrophysics (CAASTRO)

Pulsar

Astronomical equivalent of a lighthouse
Beam radiating from neutron star



John G. Hartnett, Andre Luiten(2010)



Extremely accurate clock

→Pulsar Timing Array

Pulsar Timing Array

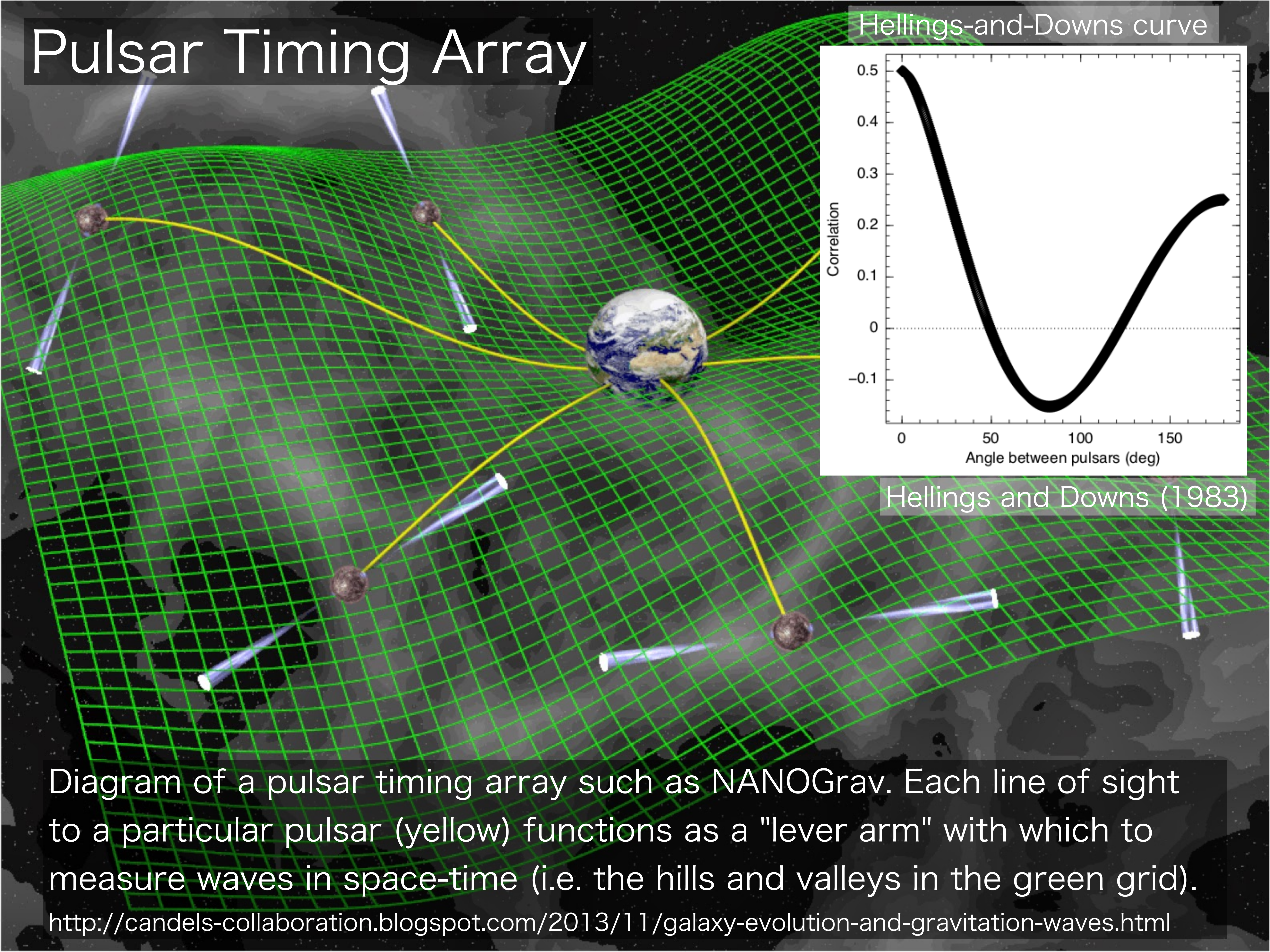


Diagram of a pulsar timing array such as NANOGrav. Each line of sight to a particular pulsar (yellow) functions as a "lever arm" with which to measure waves in space-time (i.e. the hills and valleys in the green grid).

http://candels-collaboration.blogspot.com/2013/11/galaxy-evolution-and-gravitation-waves.html

Pulsar

Assumption of rotating magnetic dipole has a constant magnetic field, and the observation of the period P and the time differential dP/dt

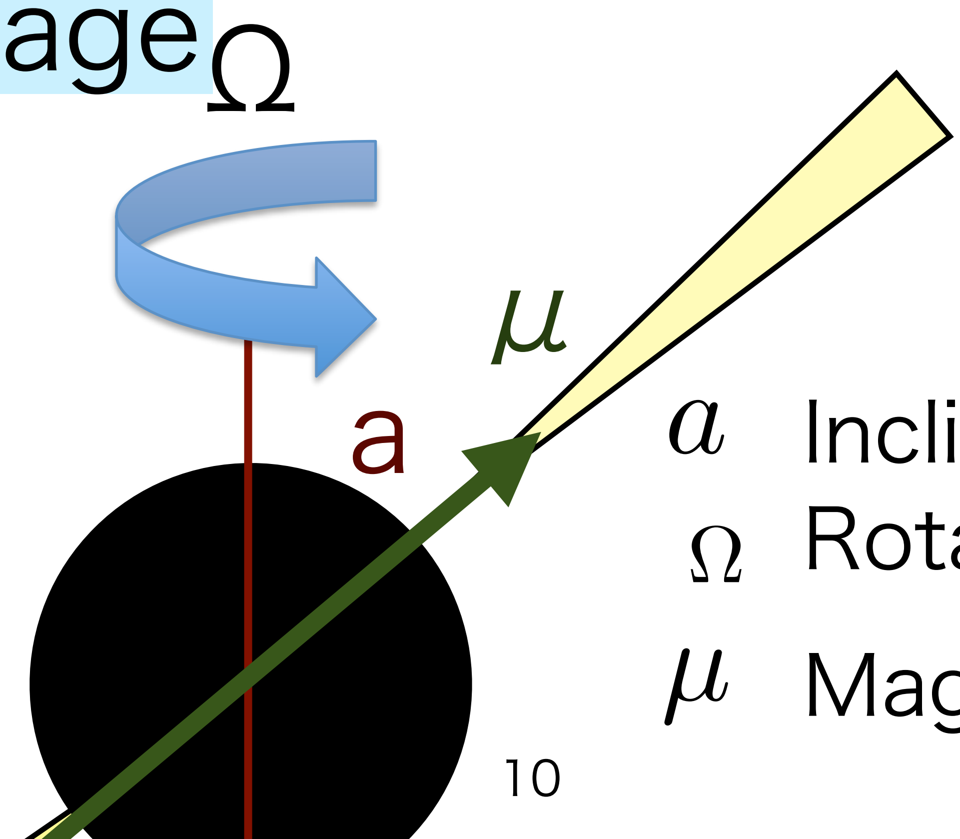
Surface Magnetic field

$$B_S = 3.2 \times 10^{19} \sqrt{P\dot{P}}\ \mathrm{G}$$

$$L_{\mathrm{Vacuum}} = \frac{2\mu^2\Omega^4}{3c^3} \sin^2 a$$

Characteristic age

$$\tau_c = \frac{P}{2\dot{P}}$$



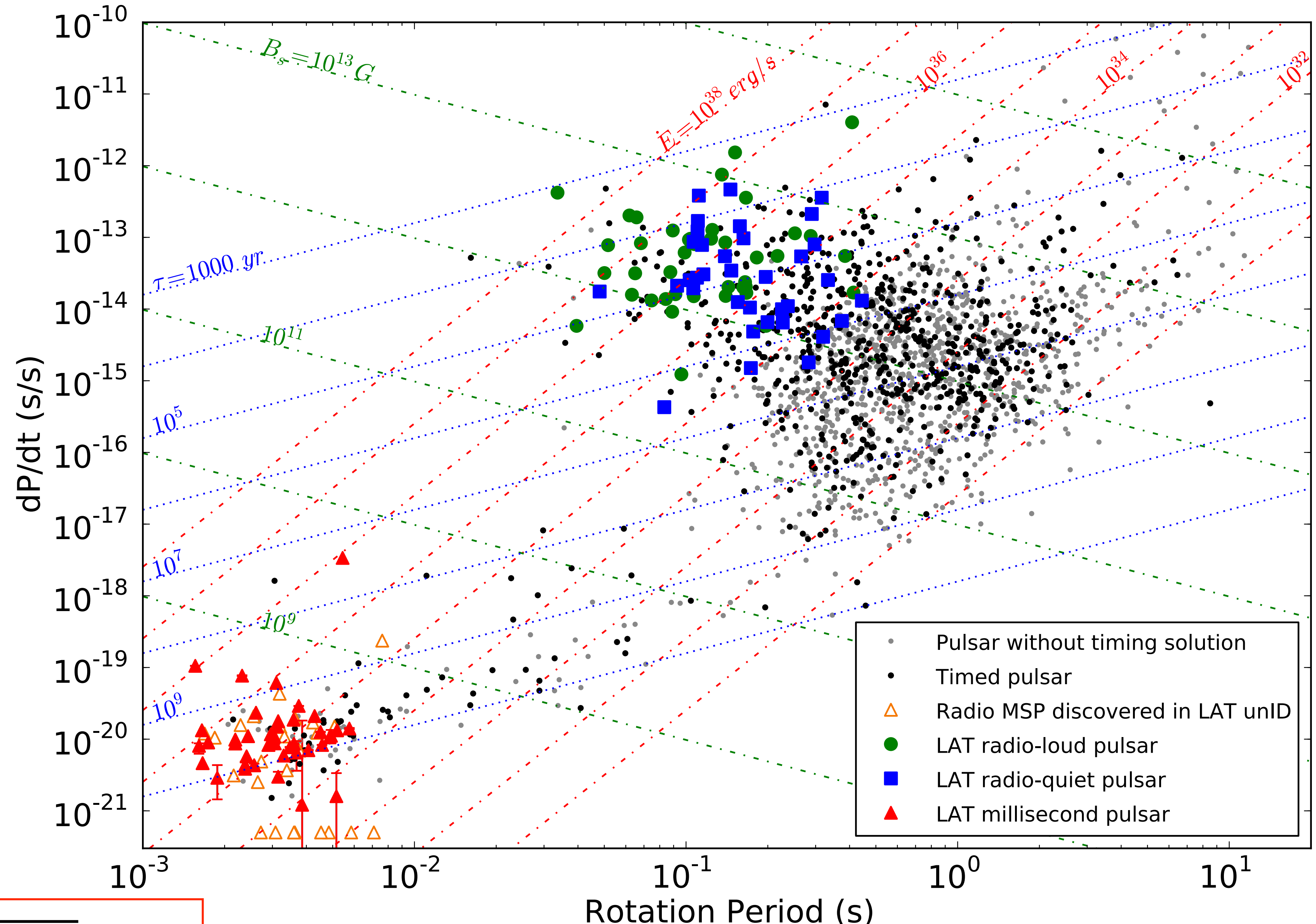
a Inclination angle

Ω Rotation angular velocity

μ Magnetic dipole moment

The observation of pulsar period

$P - \dot{P}$ diagram



$$B_S = 3.2 \times 10^{19} \sqrt{P\dot{P}}\ \mathrm{G}$$

Abdo et al.(2013)

<u>Pulsar has strong magnetic field</u>

Phenomenology ⟺ Theory

Observation

Numerical simulation

Multi-wavelength electromagnetic wave

Magnetosphere structure

Polar Cap

Outer Gap

Equation of state

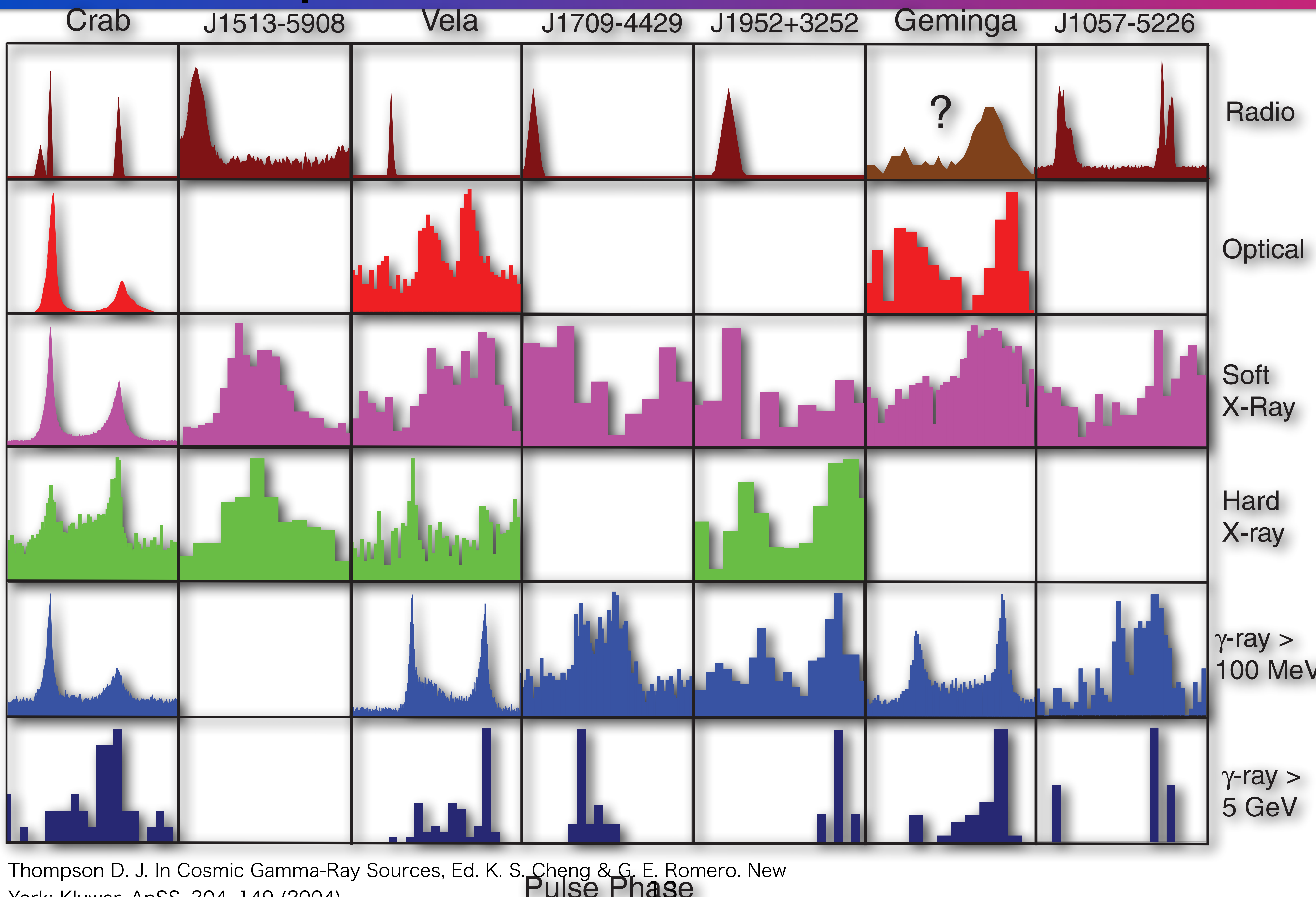
Cosmic rays

Neutron star shape

Gravitational waves

Theoretical research is more needed

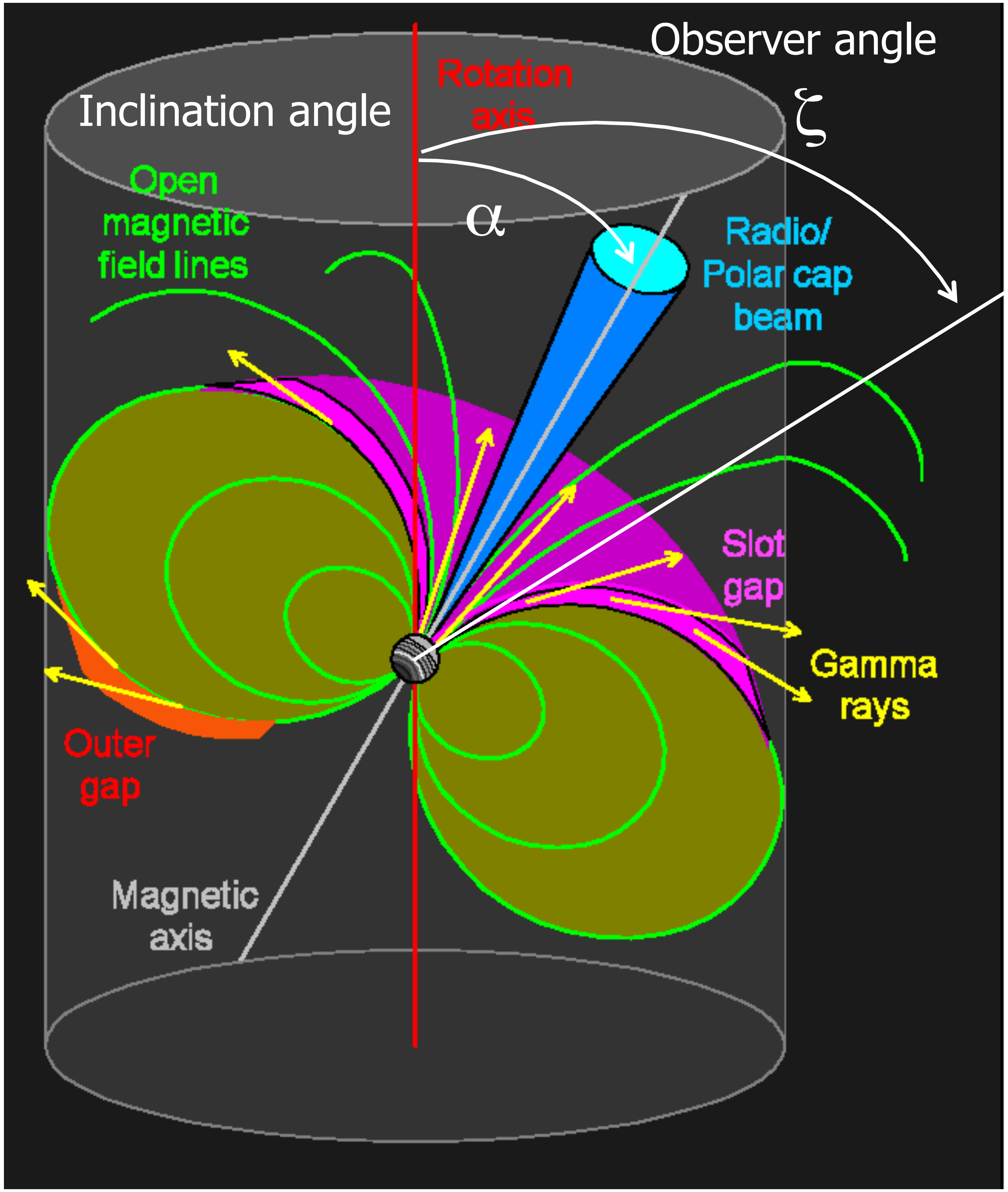
Various pulse waveforms



Thompson D. J. In Cosmic Gamma-Ray Sources, Ed. K. S. Cheng & G. E. Romero. New York: Kluwer, ApSS, 304, 149 (2004)

Geometric structure of radiation

Pulsar emission geometry



Harding

Gravitational wave detection of binary neutron coalescence

- GW170817

Chirp mass

$$M = 1.188^{+0.004}_{-0.002}$$

Neutron star mass

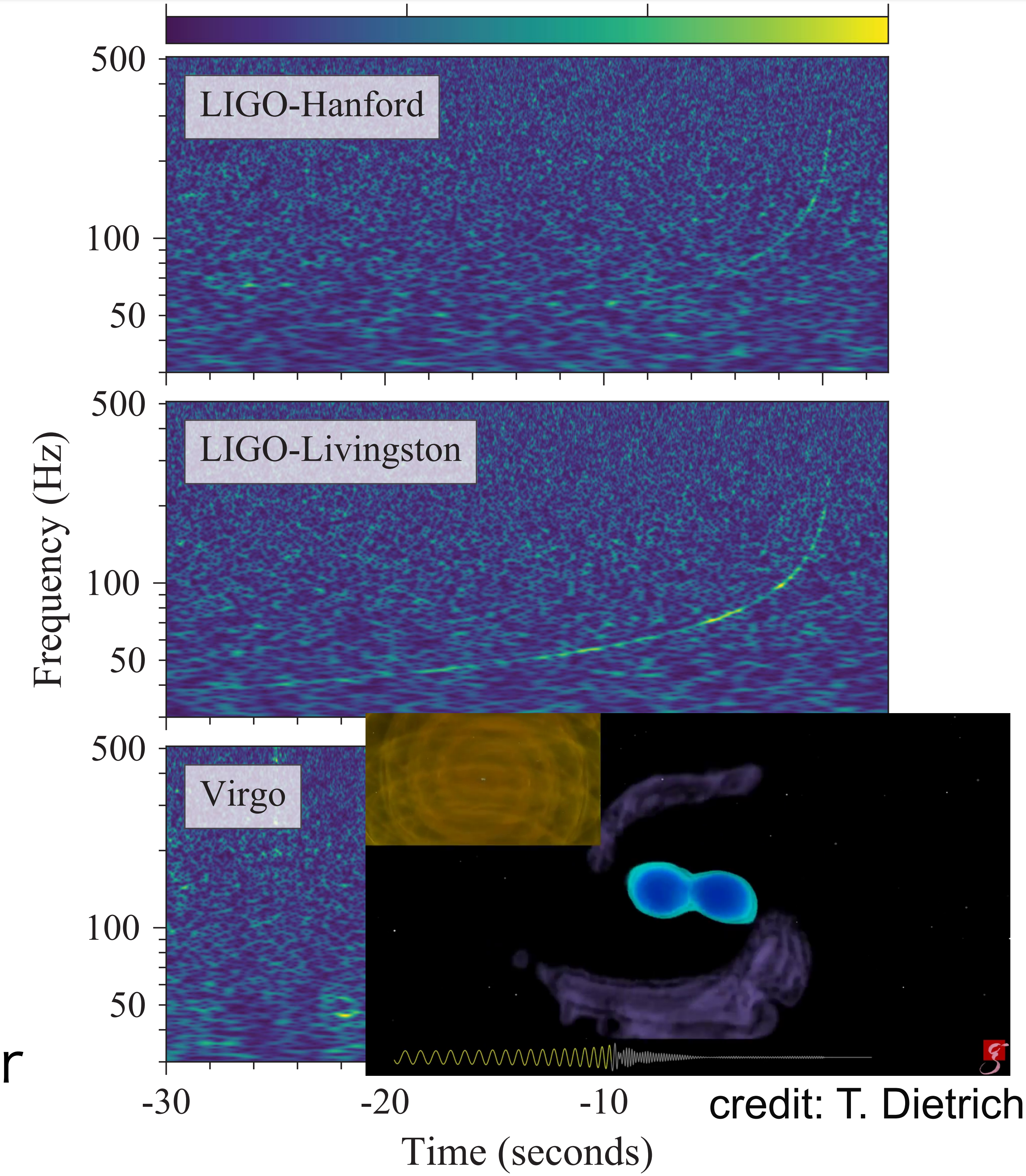
$$1.16M_\odot \le m_1 \le 1.36M_\odot$$

$$1.36M_\odot \le m_2 \le 1.60M_\odot$$

Tidal deformation rate

$$\tilde{\Lambda} = 320^{+420}_{-230}$$

The radius is over 13 km or under 9 km is not suitable



credit: T. Dietrich

Abbott et al.(2017)

Phenomenology ⇔ Theory

Observation

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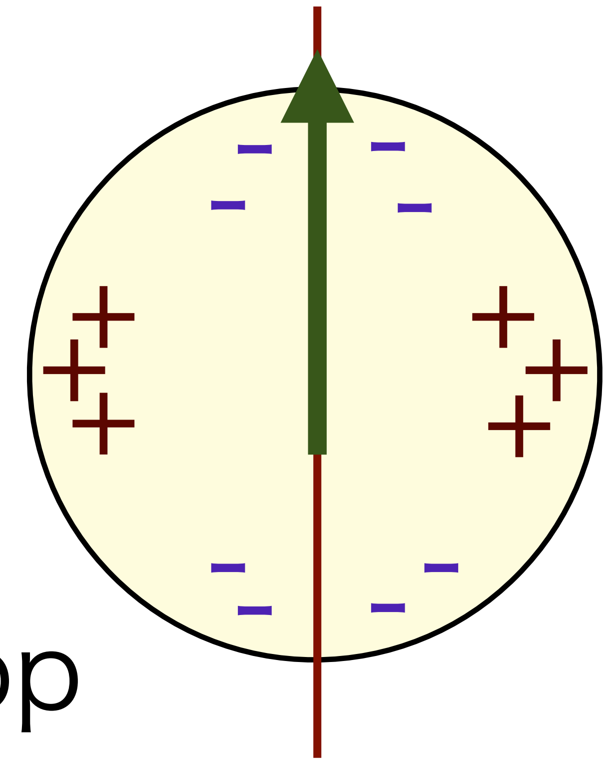
Theoretical research is more needed

Unipolar induction

When the magnetic dipole rotates, an electric field can be generated by unipolar induction

Charged particles inside the star affected by Lorentz forces and polarize



Pulsar power supply

The magnitude of the Potential drop

$$V = \int_0^{r_{\rm pc}} \Phi_{\rm surface}(r)dr = 6\times10^{12}\ [{\rm V}]\left(\frac{B_0}{10^{12}[{\rm G}]}\right)\left(\frac{P}{1[{\rm s}]}\right)^{-2}$$

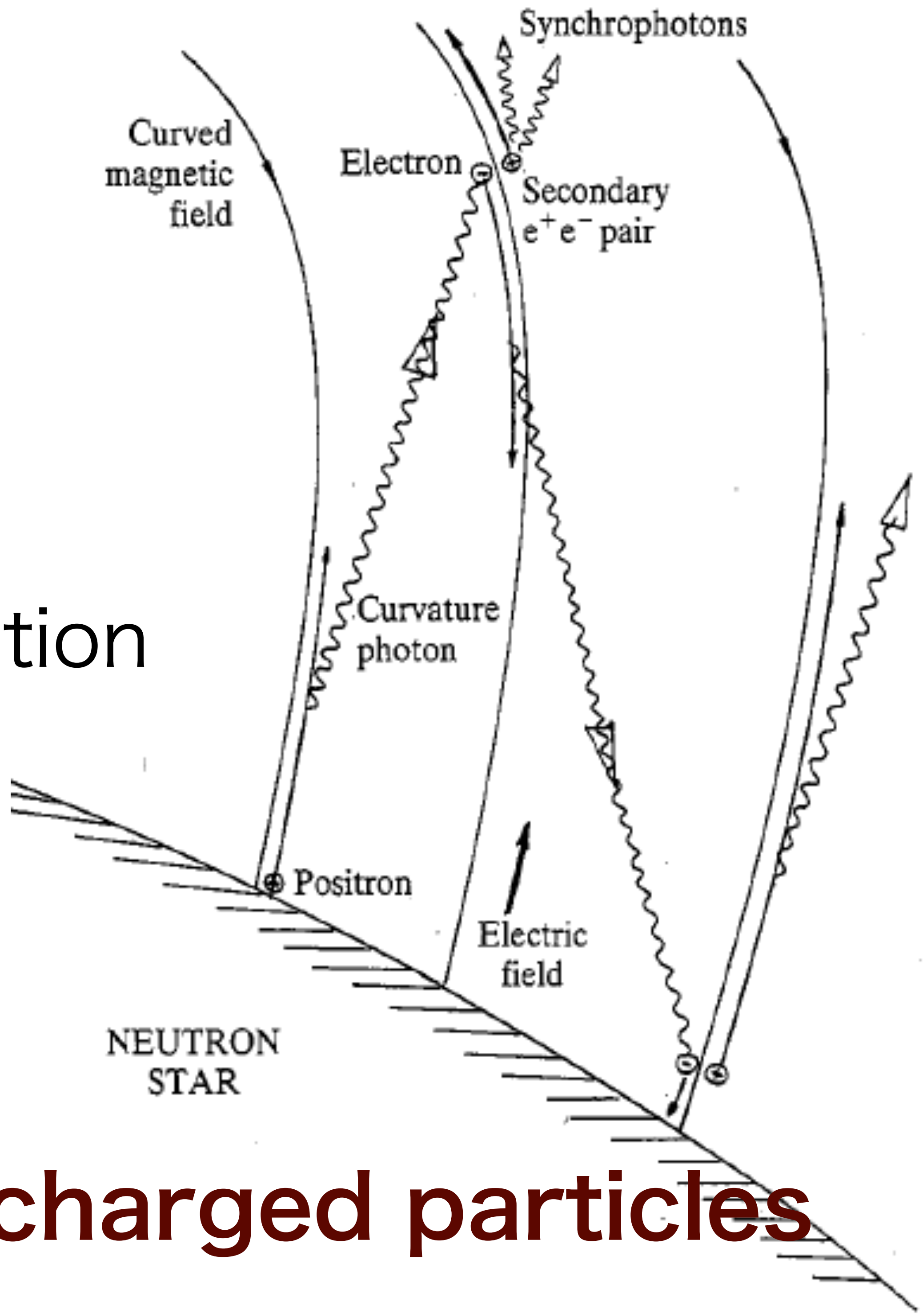
Electron positron cascade

Electrons emit high energy photons by curvature radiation

A photon generates an electron positron pair because of a strong magnetic field

Electron and positrons emit radiation

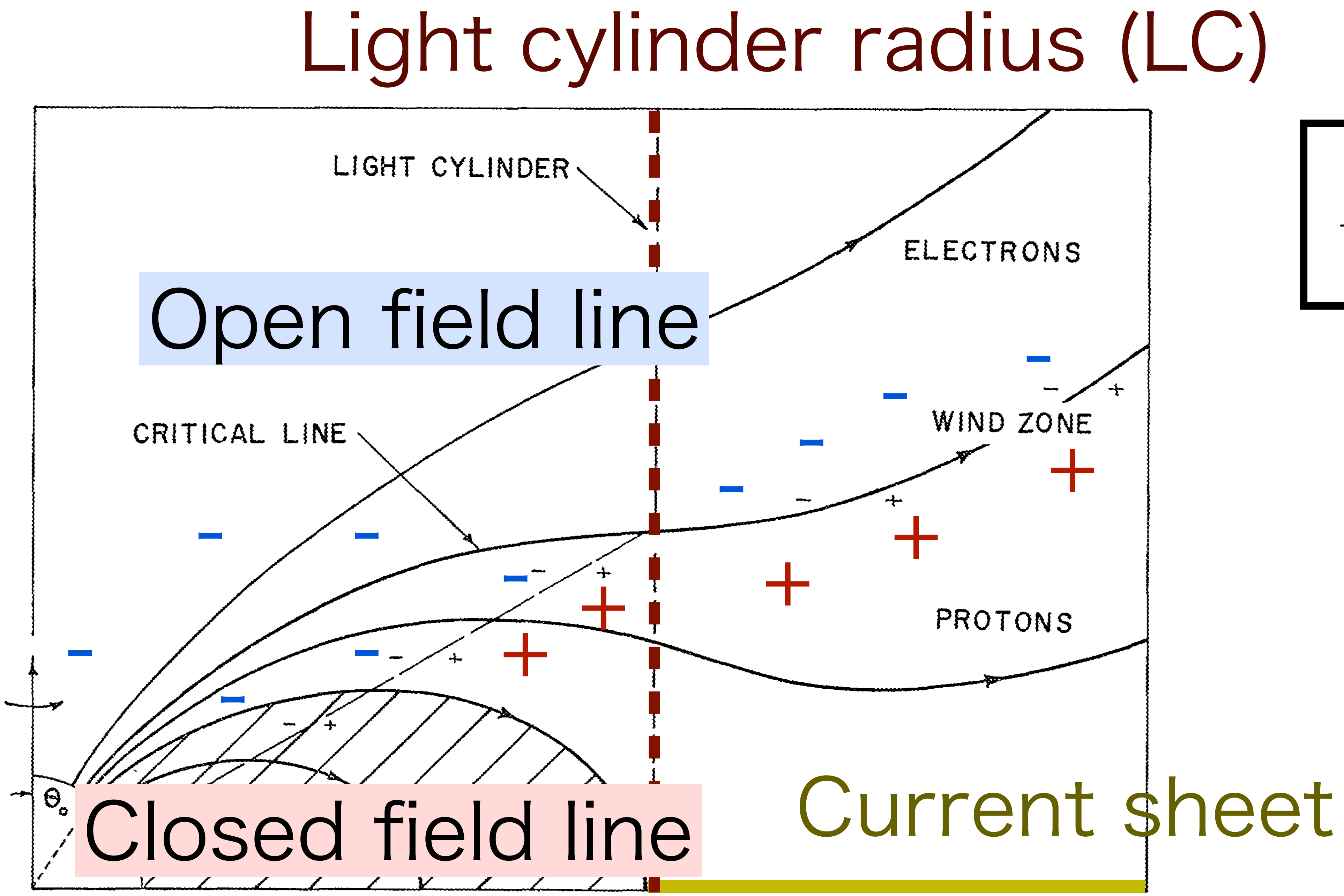
Electron and positrons are generated one after another in an avalanche manner



The whole star is filled with charged particles

Structure of pulsar magnetosphere

Open magnetic field lines and closed field lines



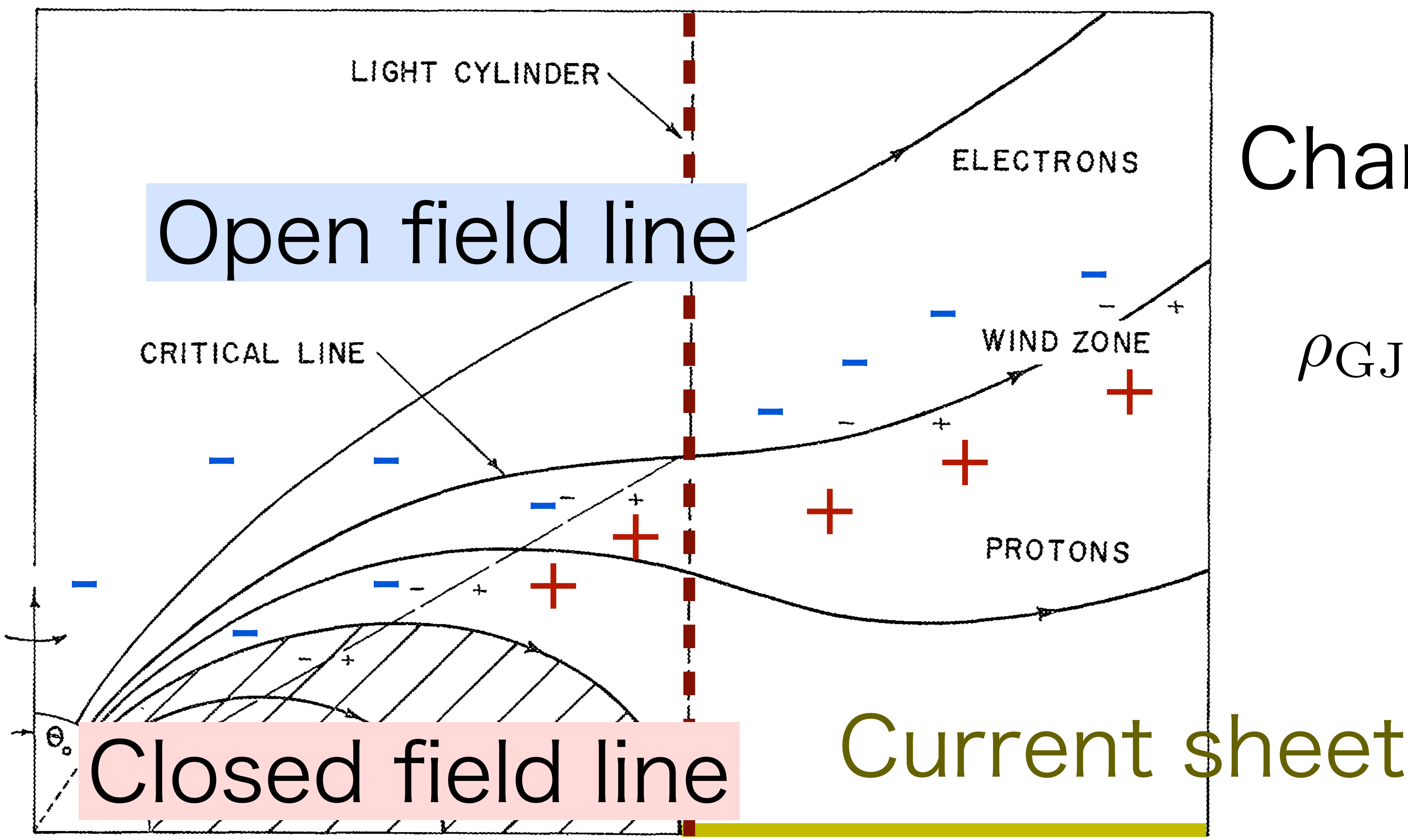
$$R_{\rm LC} = \frac{c}{\Omega}$$

Goldreich & Julian(1969)

Structure of pulsar magnetosphere

Positive and negative charged particles are polarized because the plasma co-rotates with the star

Light cylinder radius (LC)

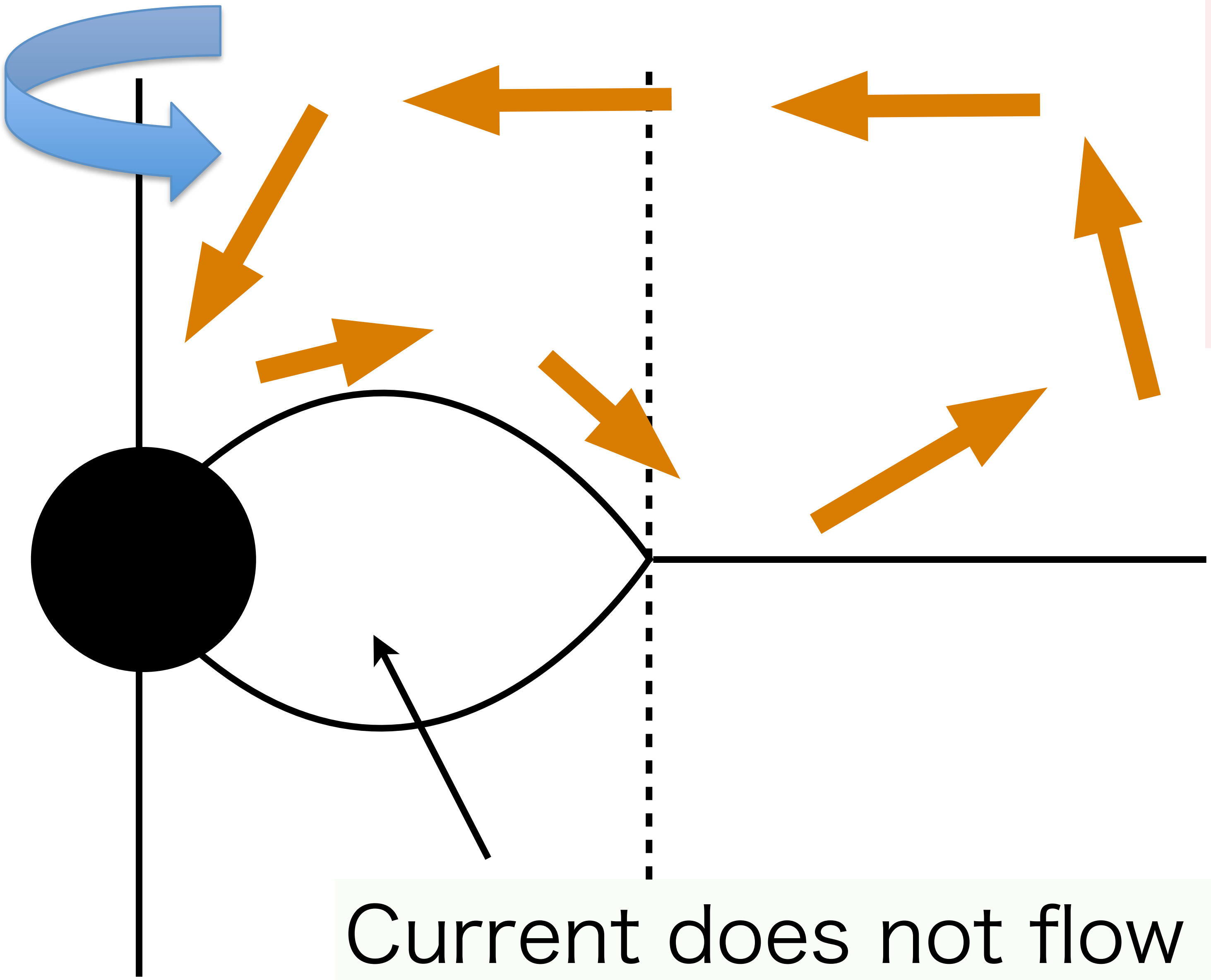


Charge density

$$\rho_{\mathrm{GJ}} \simeq -\frac{\vec{\Omega}\cdot\vec{B}}{2\pi c}$$

Goldreich & Julian(1969)

Global current structure



In the vicinity of the pole, the electrons flow from the outside

In the vicinity of the equator, positrons flow outward

Current does not flow in the closed region

Because it does not deal with dissipation in the force-free approximation There is no result about global current structure result including the outside

Approach to Resistive Pulsar Magnetosphere

	Advantage	Drawback
Force-Free approximation Spitkovsky(2006)	Calculation cost is Small	Can not answer about acceleration
PIC Chen & Beloborodov(2014) Philippov et al.(2015) Cerutti et al.(2016)	Track the movement of charged particles	Insufficient particle numbers, and pair creation assumptions
MHD (Two-field) Komissarov(2006)	Include information about the velocity of the plasma	Large calculation cost

In this paper, the Force-Free approximation is extended and the defects are improved
Introduce radial dependence of current density model

Parallel electric field to the magnetic field and Lorentz invariant

Relation between the Lorentz invariant

$$E_0^2 - B_0^2 = \mathbf{E}^2 - \mathbf{B}^2.$$

$$E_0 B_0 = \mathbf{E} \cdot \mathbf{B}.$$

$$B_0^2 = \frac{1}{2}\left(\mathbf{B}^2 - \mathbf{E}^2 + \sqrt{(\mathbf{B}^2 - \mathbf{E}^2)^2 + 4(\mathbf{E} \cdot \mathbf{B})}\right)$$

$$E_0 = \sqrt{B_0^2 - \mathbf{B}^2 + \mathbf{E}^2}$$

$$B_0 = \mathrm{sign}(\mathbf{E} \cdot \mathbf{B})\sqrt{B_0^2}$$

Current density model

σ Electrical conductivity　　Lyutikov(2003)

Ohm's law　$\mathbf{j}_{\mathrm{fluid}} \equiv \sigma \mathbf{E}_{\mathrm{fluid}}.$

$$\mathbf{j} = \frac{\rho_e c \mathbf{E} \times \mathbf{B} + \sqrt{\frac{B^2+E_0^2}{B_0^2+E_0^2}} \sigma E_0 \left(B_0 \mathbf{B} + E_0 \mathbf{E} \right)}{B^2 + E_0^2}$$

On the outer side, the electric conductivity become small

$$\sigma(r) = \frac{\sigma_0}{r^n}$$

cf. FFE regime inside light cylinder
and dissipative regime outside (FIDO)
 Inside of light cylinder $\sigma \rightarrow \infty$
 Outside of light cylinder σ High & Finite
Kalapotharakos et al.(2016)

σ_0　Electrical conductivity of surface

n　Radial dependence parameter　$n = 1, 2$

Equations

$$\mathbf{B} = \frac{1}{r\sin\theta}\nabla G \times \mathbf{e}_\phi + \left(\frac{S}{r\sin\theta}\right)\mathbf{e}_\phi$$

Time evolution of magnetic field

$$\mathcal{D} \equiv \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \mathcal{D}\right) G = \frac{4\pi}{c} j_\phi r \sin\theta$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \mathcal{D}\right) S = \frac{4\pi}{c}\left(\frac{\partial(r j_\theta)}{\partial r} - \frac{\partial j_r}{\partial\theta}\right)\sin\theta$$

Poisson equation

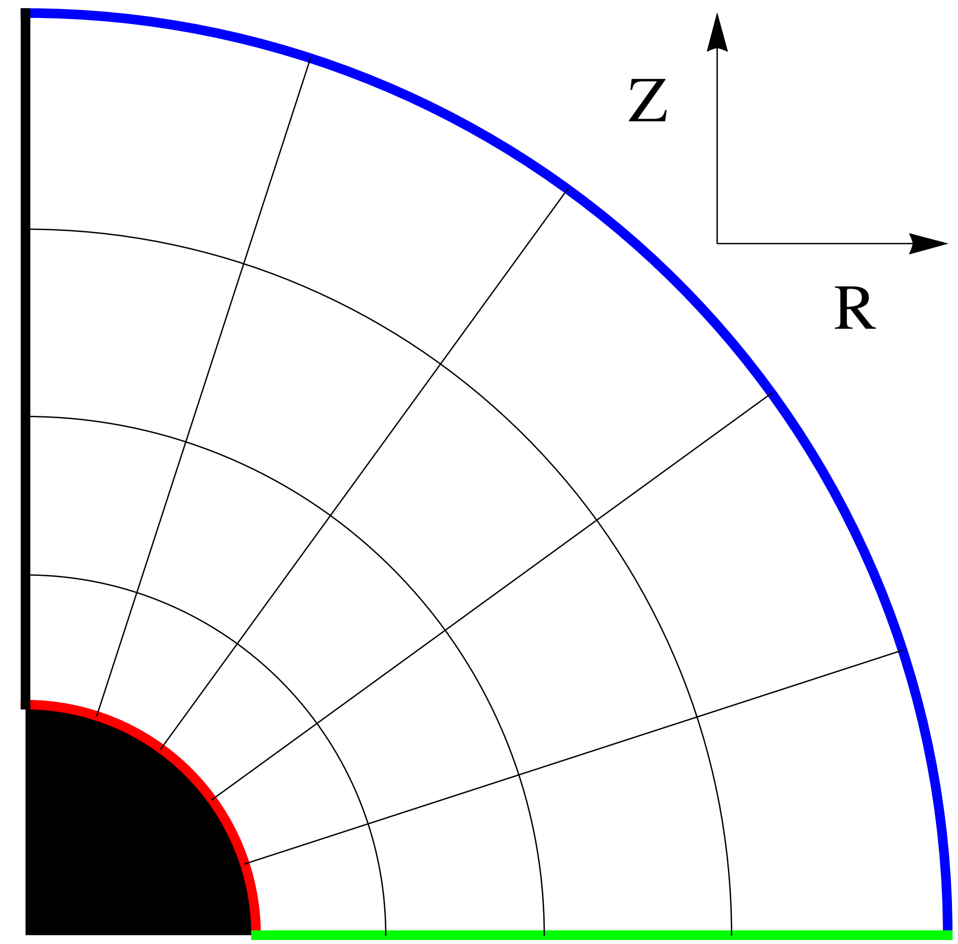
$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right)\Phi = -4\pi\rho_e$$

Current density model

$$\mathbf{j} = \frac{\rho_e c\mathbf{E}\times\mathbf{B} + \sqrt{\frac{B^2+E_0^2}{B_0^2+E_0^2}}\,\sigma E_0\left(B_0\mathbf{B} + E_0\mathbf{E}\right)}{B^2 + E_0^2}$$

Boundary condition(1/2)

Spherical coordinate system (r, θ)



Star surface

$$E_\theta = -\frac{2\mu\Omega\cos\theta\sin\theta}{r_0^2}$$

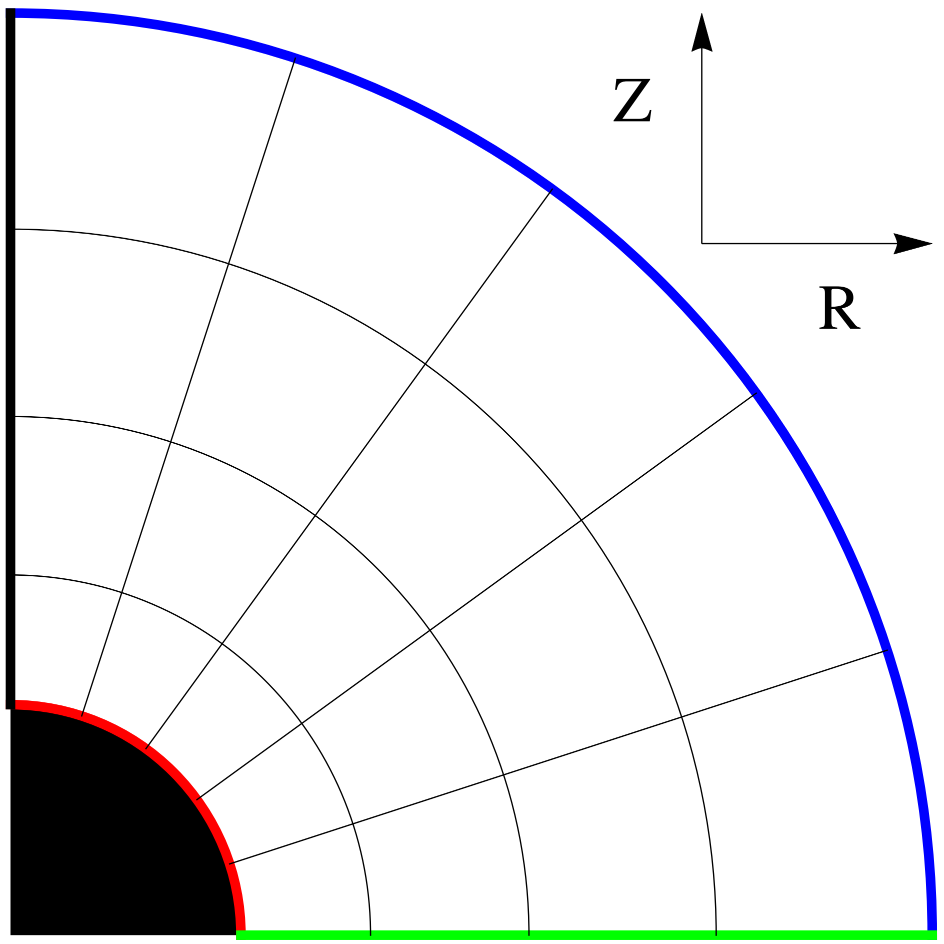
$$E_\phi = 0$$

$$B_r = -\frac{2\mu\cos\theta}{r_0^3}$$

$$\frac{R_{\rm LC}}{r_0} = \frac{1}{\Omega} = 5$$

$$B_\phi = \begin{cases} -\frac{2\alpha\mu\Omega}{r_0^2}\sin\theta\left[1-\frac{\sin^2\theta}{\sin^2\theta_p}\right] & (\theta \le \theta_p) \\ 0 & (\theta > \theta_p). \end{cases}$$

Boundary condition(2/2)



Outside

Out going condition

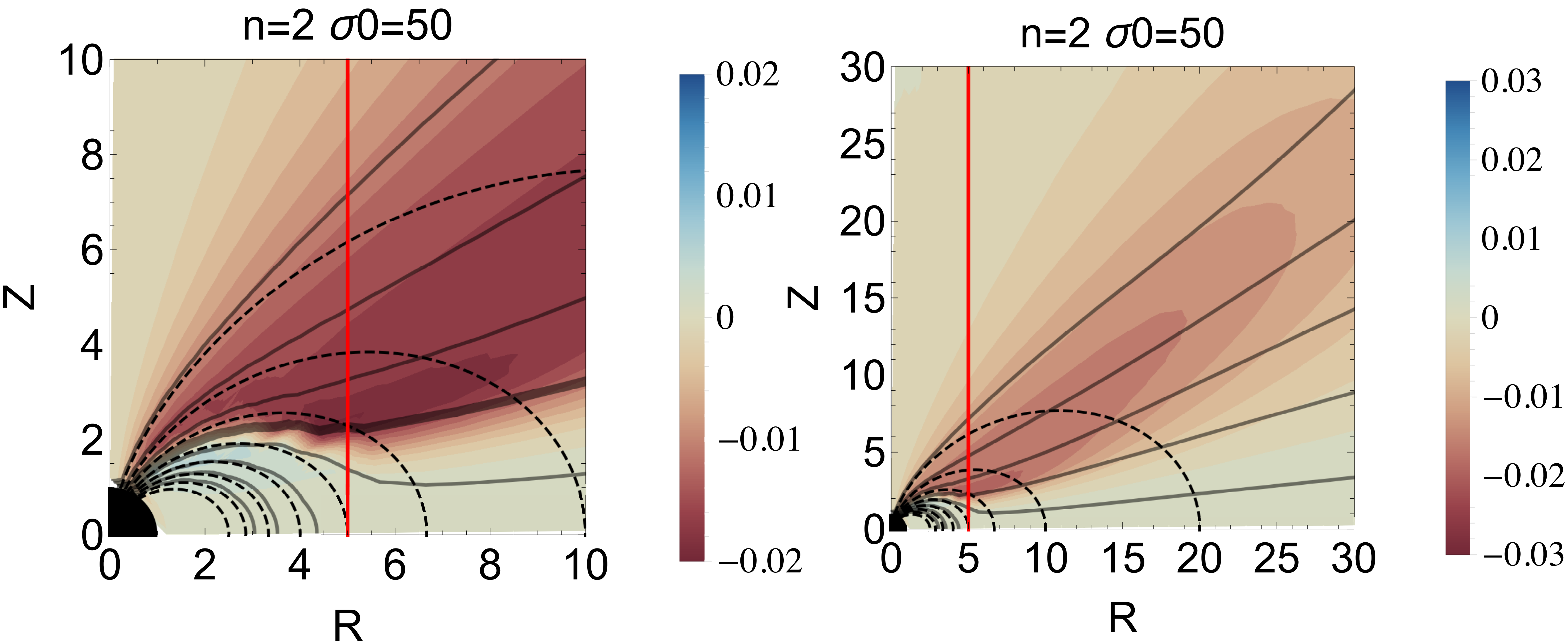
Rotation axis, magnetization axis

$$G(r,0)=0,\quad S(r,0)=0,\quad F(r,0)=0,\quad \frac{\partial\Phi(r,0)}{\partial\theta}=0$$

Equatorial plane

$$S(r,\pi/2)=0,\quad F(r,\pi/2)=0,\quad \frac{\partial\Phi(r,\pi/2)}{\partial\theta}=0$$

Time evolution of magnetic field for $n = 2, \sigma_0 = 50$

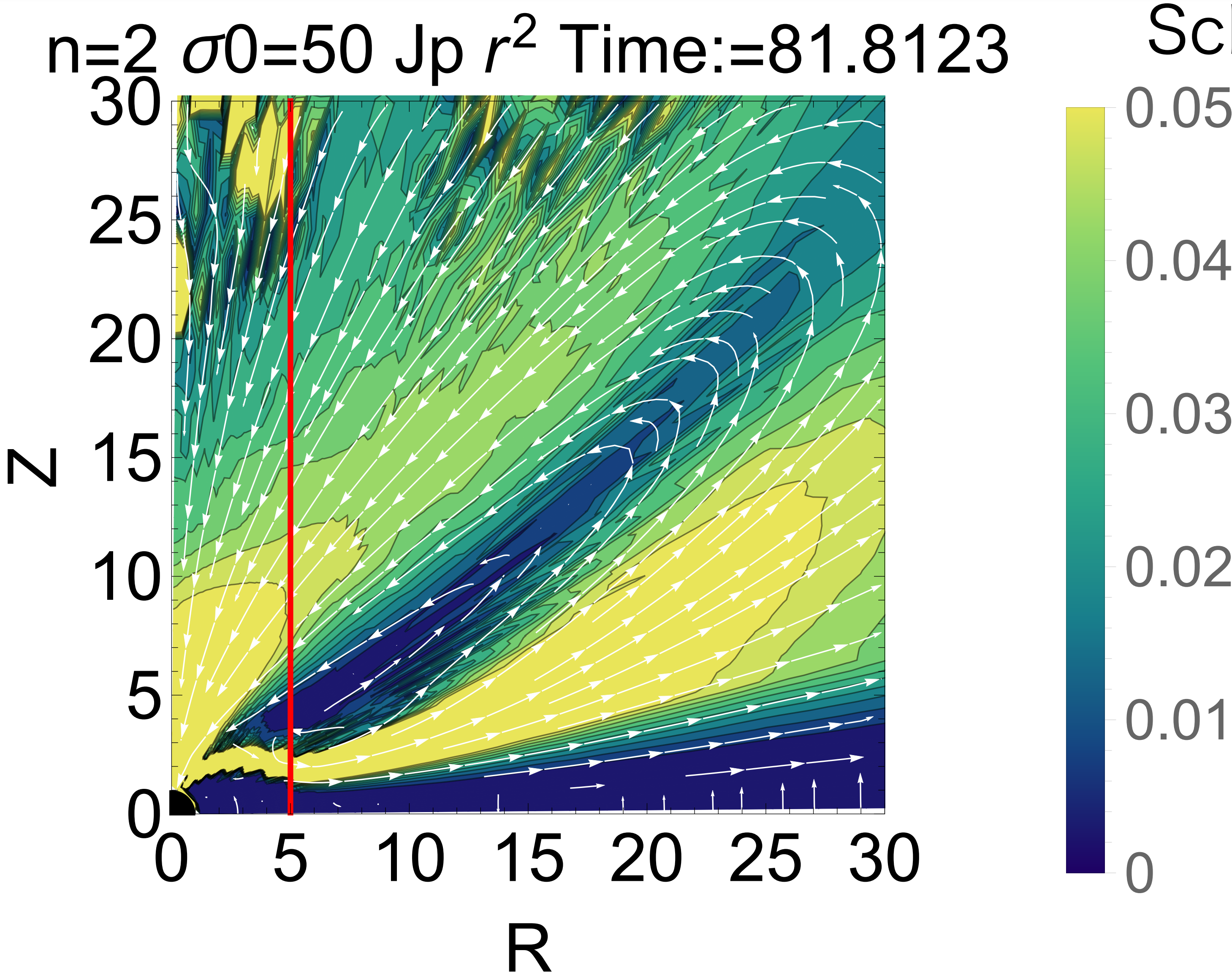


Become steady state with time

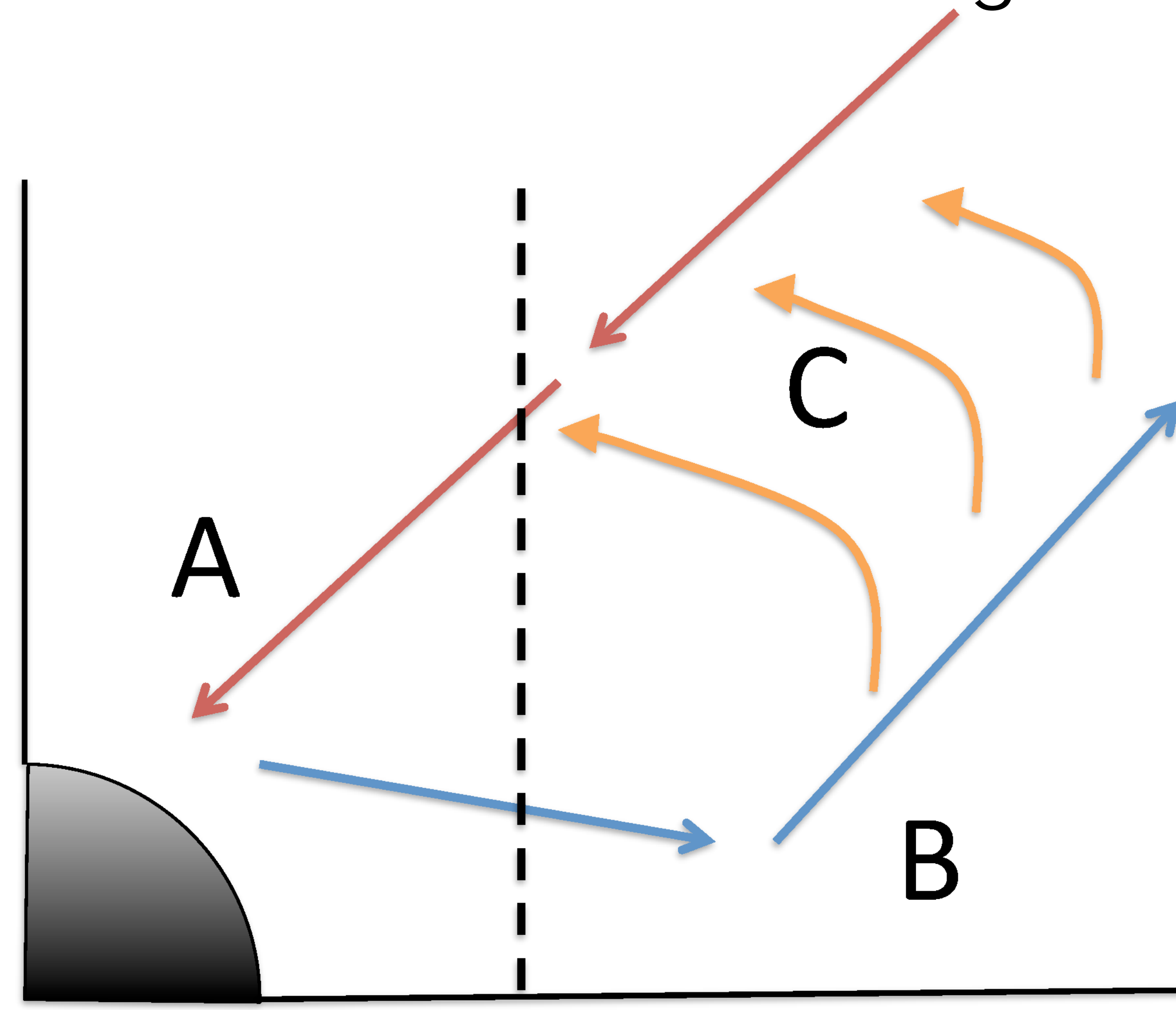
The dotted line indicates the dipole polar magnetic field G

The solid line indicates the polar magnetic field G, the color indicates the torsional magnetic field S

Current circuit structure



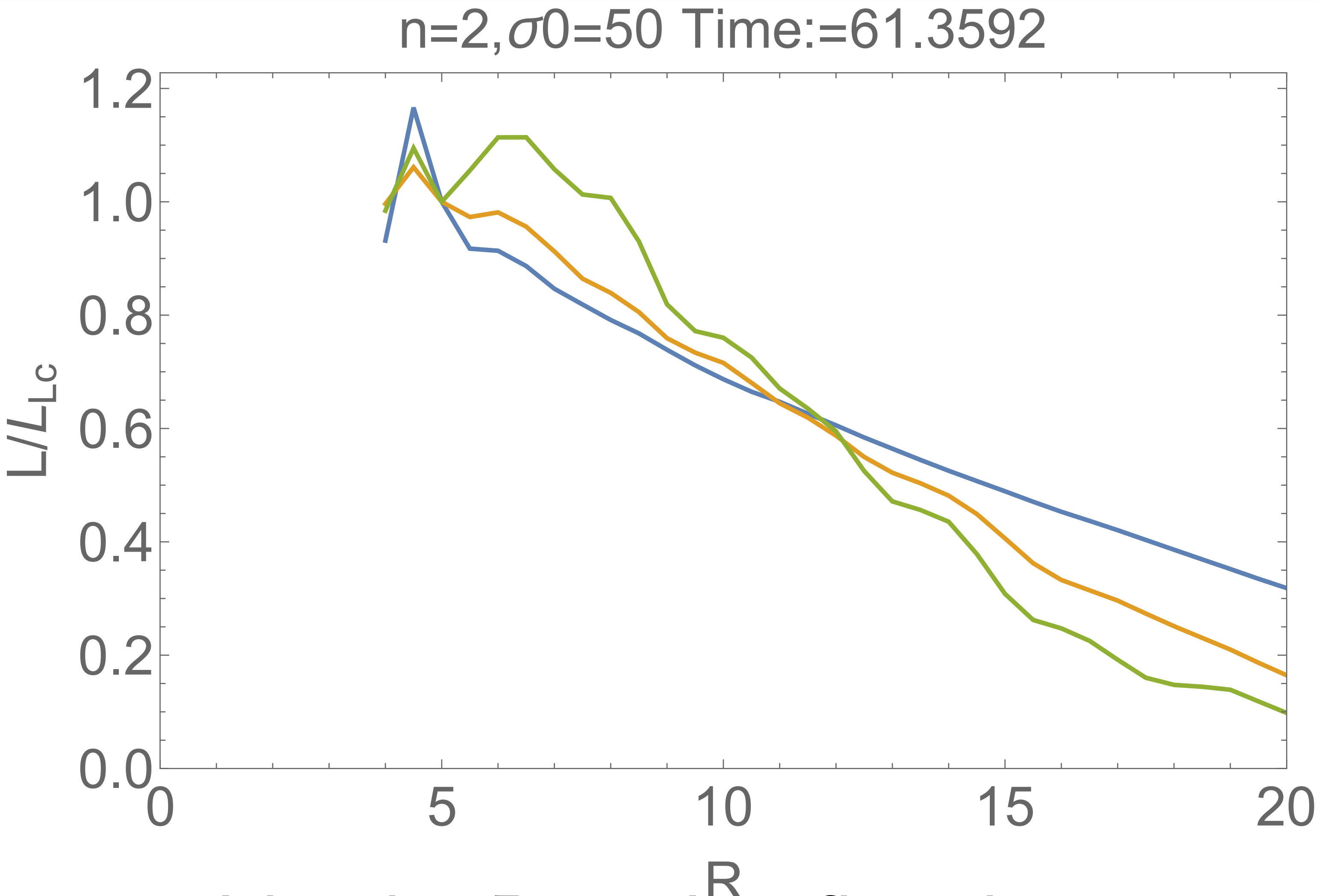
Schematic current circuit diagram



A large current circuit is formed beyond the light cylinder

Even in the area where the magnetic field lines are open, the direction of the poloidal current is opposite , and the current across the magnetic field lines in region C.

Radial dependence of Poynting flux



$\sigma_0 = 50$

$\sigma_0 = 20$

$\sigma_0 = 10$

On the outside, the Poynting flux decreases

And Poynting flux decreased more greatly when the electric conductivity σ_0 is small

→ Because, as the electric conductivity increases, the global current circuit structure expand to the outside

Conclusion

At present, the structure of the actual pulsar magnetosphere is not clear.

Since it does not include dissipation in the magnetospheric model based on the ideal MHD and Force Free approximation.

I introduce an electrical conductivity dependent upon distance from the star. A steady state is obtained by combining Maxwell equations and the boundary condition. These resistive force-free solutions show that the current has width and circuit shape. The large-scale current circuit including the outside of light cylinder is formed.

Taking into account the global pulsar magnetosphere structure is important.