マグネターの磁場進化
-Magnetic field evolution of magnetars-

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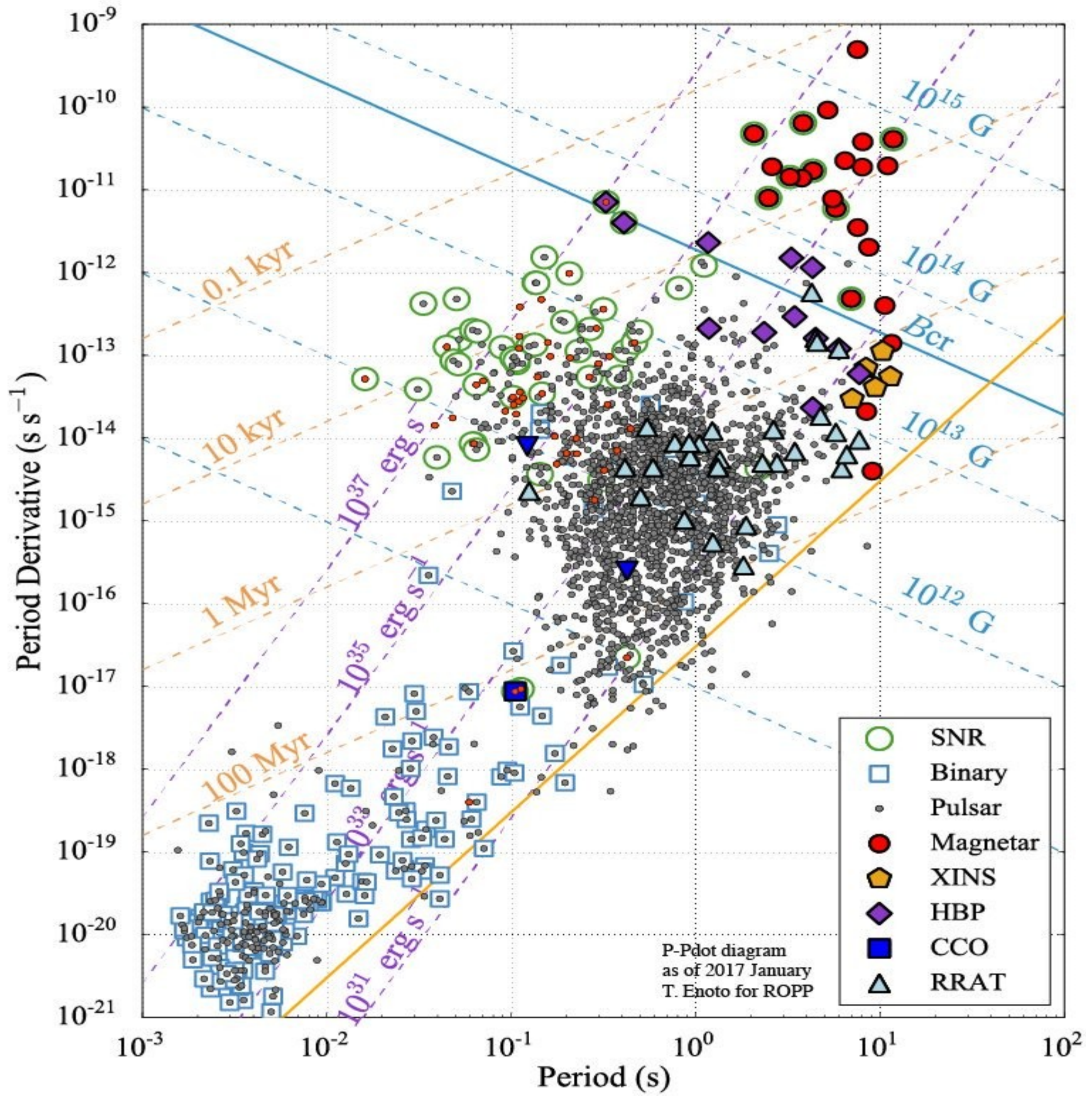
- Age vs magnetic field of magnetars
- Phenomenological models of magnetic field evolution
- Physical models of magnetic field evolution
- Discussion and summary

P-Pdot and B- τ

$\dot{P}(s/s)$



$P(s)$

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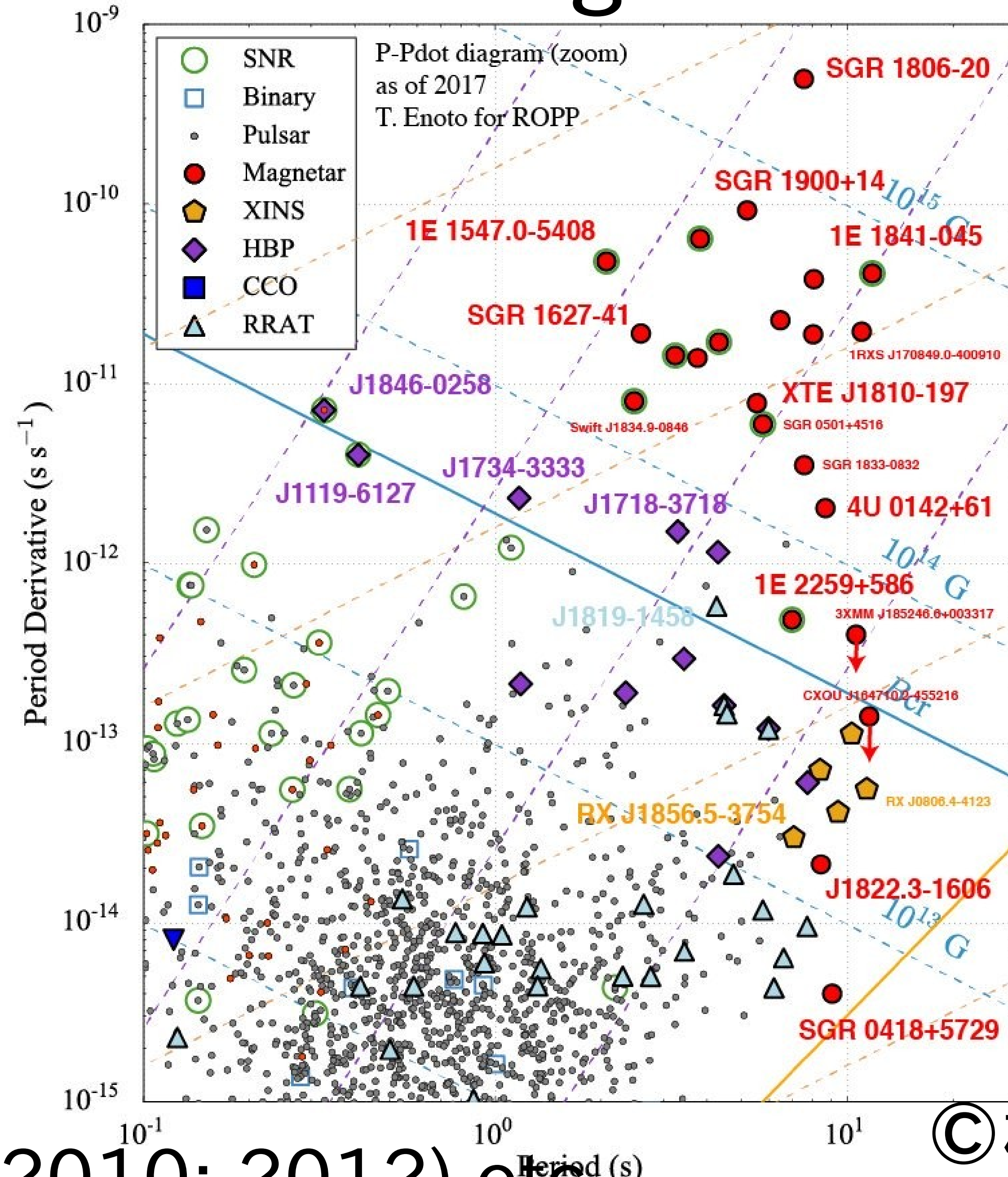
Magnetar

- Long pulse period (P ～ 2-11 sec.)
  - Rotational energy is low
- Intense persistent X-ray ($L_x \sim 10^{35}$ erg/s)
  - Larger than spindown luminosity $\dot{E}_{rot} = 10^{32-34}$ erg/s
  - Large P and Pdot

    →Dipole field on the surface $B_d = 10^{14-15}$ G
- Young characteristic age ($\tau_c = 1 - 100\, k\, yr$)

Low-field magnetars



Rea et al. (2010; 2012) etc.

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Examples of low-field magnetars

1. SGR 0418+5729 (Rea+ 2010, 2013)

   $B \sim 6 \times 10^{12}$ G、$\tau_c \sim$ 36000 kyr. P$\sim$ 9.1s

2. SWIFT J 1822.3-1606 SGR 1822-1606 (Rea+ 2012)

   $B \sim 1.4 \times 10^{13}$ G、$\tau_c \sim$ 6300 k yr. P$\sim$8.4s

3. CXOU J164710.2-455216 (An et al. 2013)

   $B < 7 \mathrm{x} 10^{13}$ G、$\tau_c \sim >$ 420 kyr. P$\sim$ 10.6s

4. 3XMM J 185246.6+003317 (Rea+ 2014)

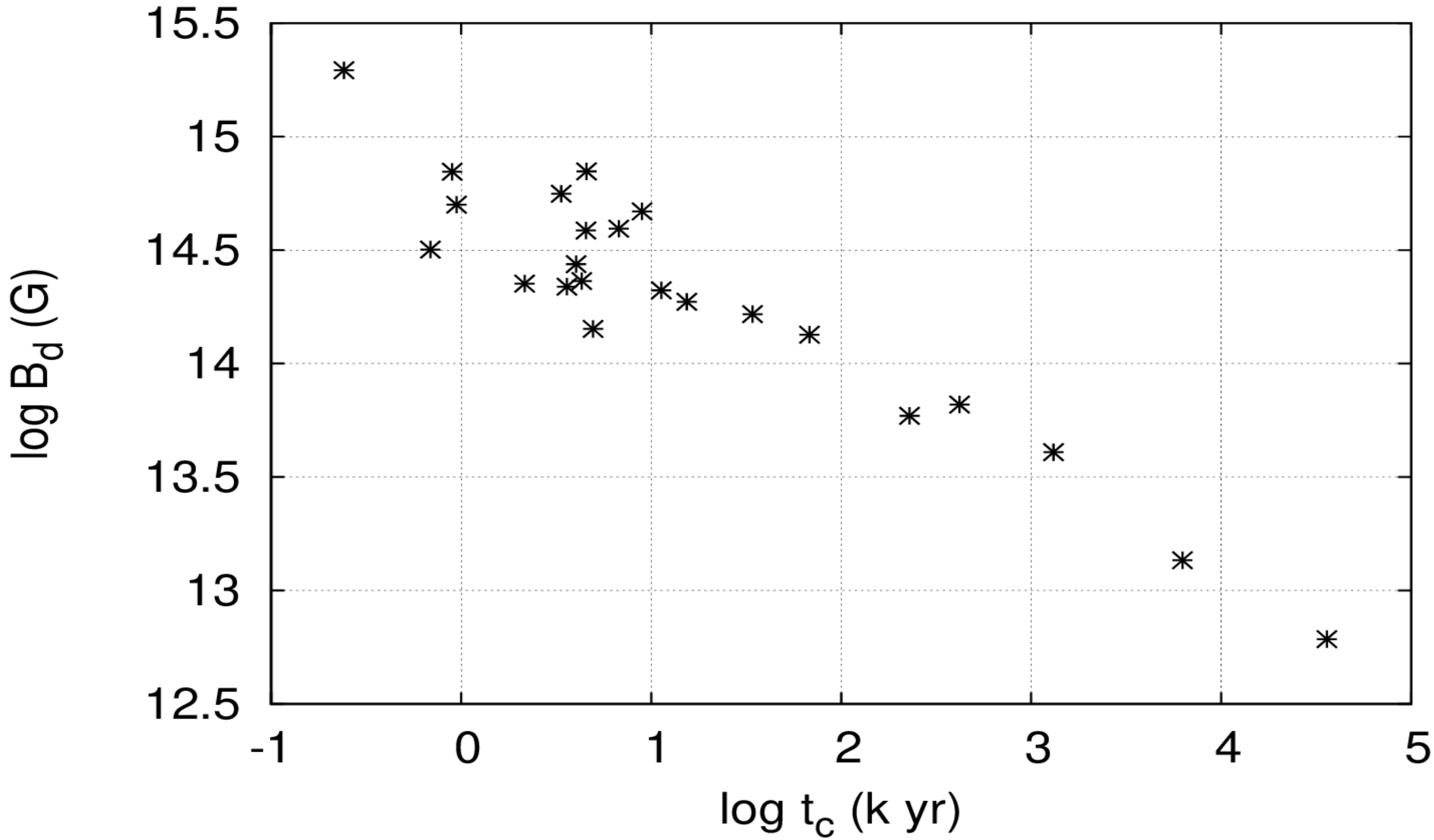
   $B < 4.1 \mathrm{x} 10^{13}$ G 、$\tau_c \sim >$ 1300 kyr. P$\sim$ 10.6s

5. 1E 2259+586 (Dib & Kaspi 2014)

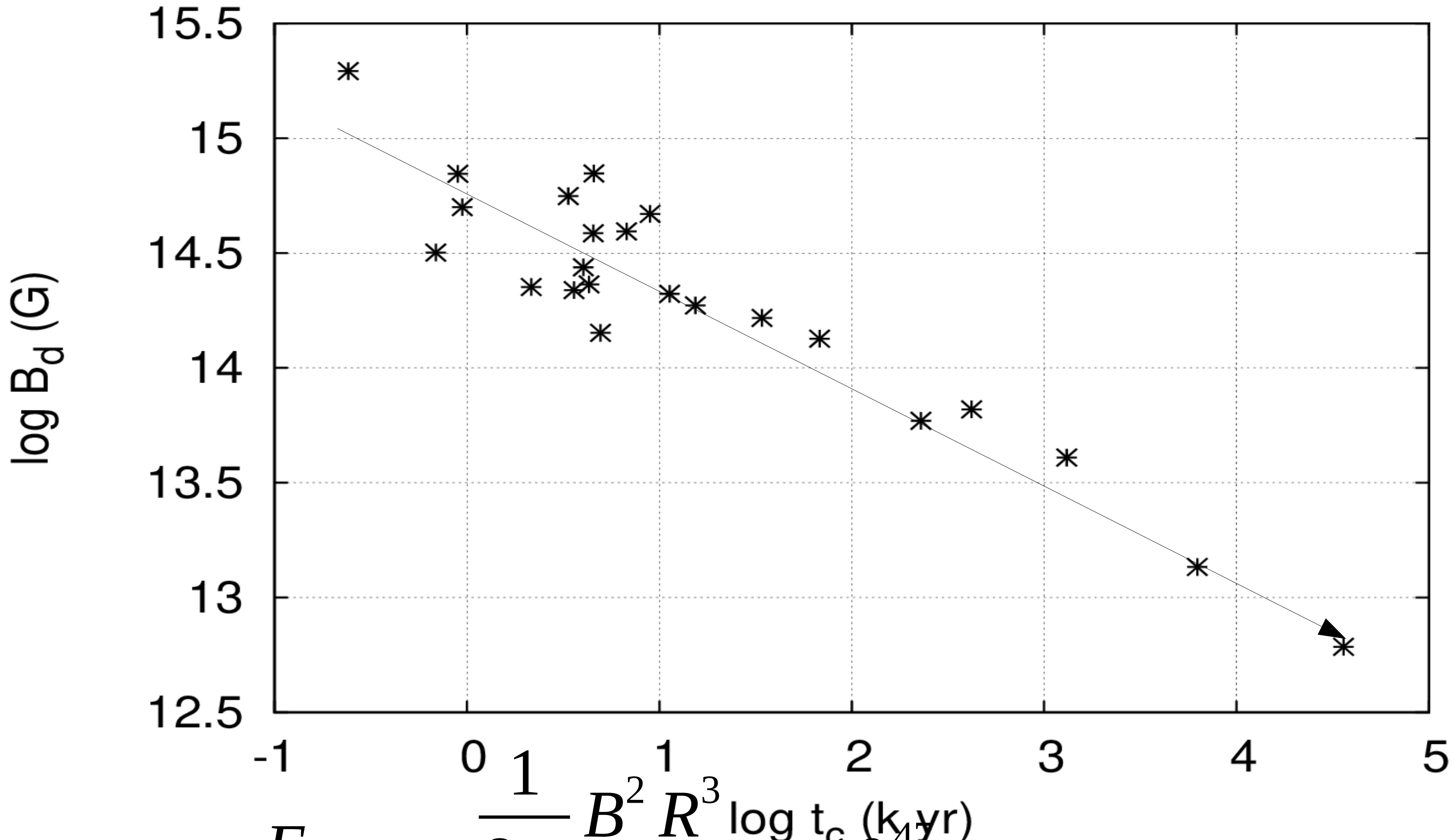
   $B \sim 5.9 \mathrm{x} 10^{13}$ G 、$\tau_c \sim$ 230 kyr. P$\sim$ 6.97s

Large τ_c and low B-field

Characteristic age vs dipole field



Characteristic age vs dipole field



$$\frac{E_{mag}}{L_x} \sim \frac{\frac{1}{8\pi} B^2 R^3}{L_x} = \frac{10^{47} erg}{10^{35} erg/s} \sim 10^4 k\ yr$$

Evolutionary models of magnetic field

- 1. Phenomenological models

  Colpi et al. (2000); Dall'Osso et al. (2012)

  Nakano et al. (2015)

  2. Physical models

  - Hall drift
  - Ambipolar diffusion

Phenomenological models

- Colpi et al. (2000); Dall'Osso et al.(2012); Nakano et al. (2015)

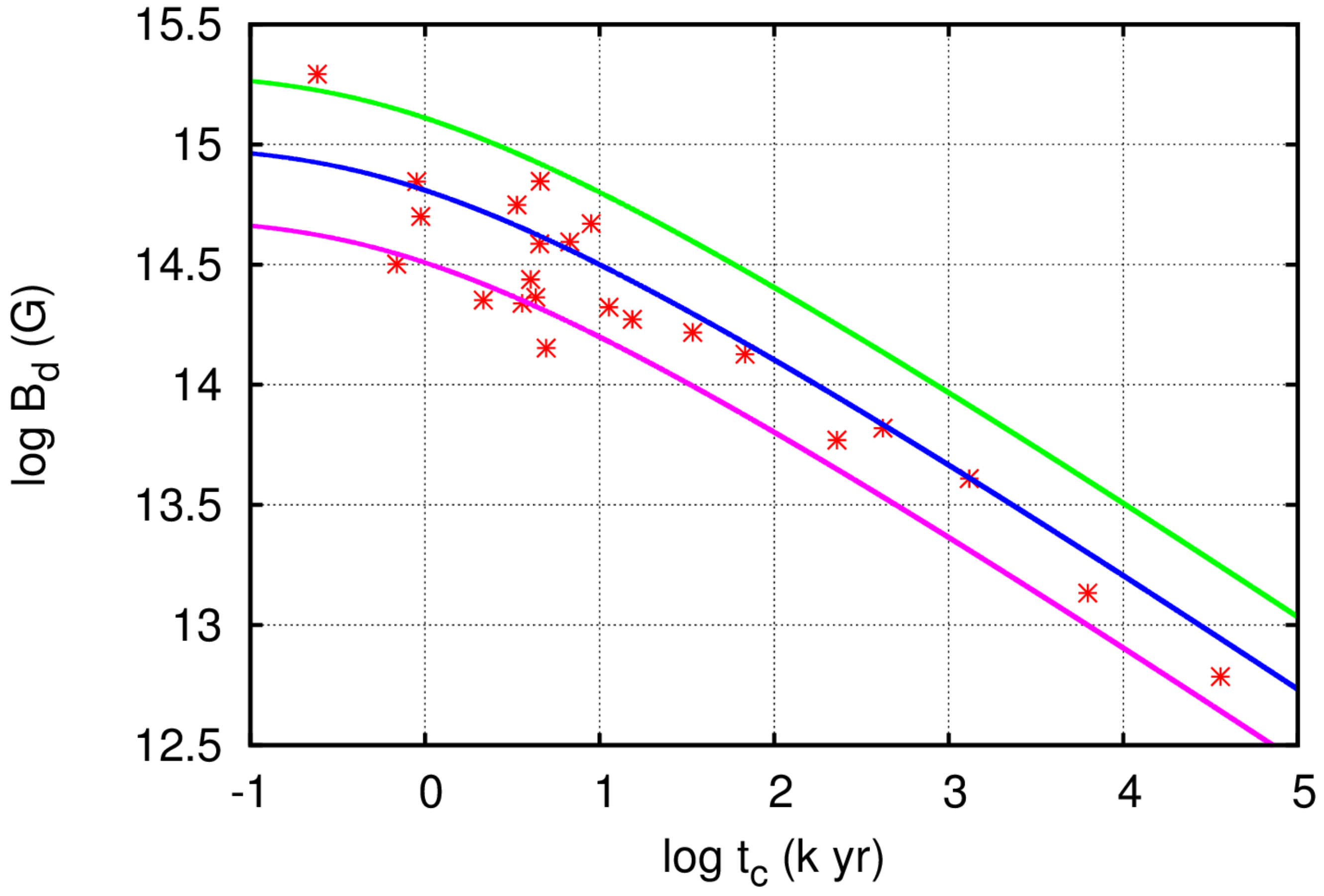
$$\frac{dB}{dt} = -A\,B^{\alpha+1}$$

$$A = \left(\tau_d \left(B^{15}\right)^{\alpha}\right)^{-1} \qquad \tau_d = \left(A\left(B^{15}\right)^{\alpha}\right)^{-1} = 1\,k\,yr\,.$$

- (Dissipation timescale is 1kyr when B = 10^{15}G)

Fitting by phenomenological models

$\alpha = 1.75$



Observational data are fitted by $1.5 < \alpha < 1.75$ models.

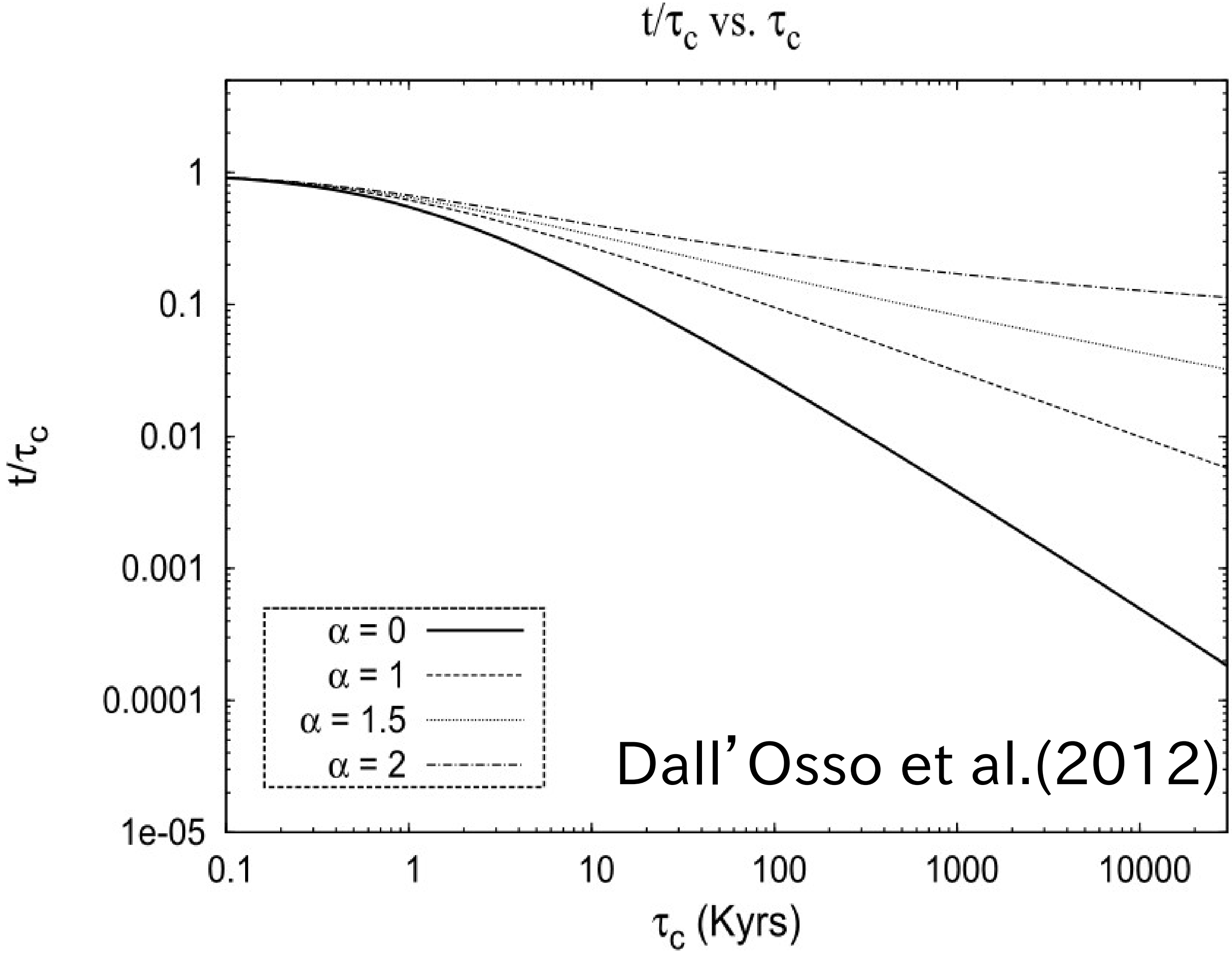
.(Dall'Osso et al.(2012); Nakano et al. (2015)

If the magnetic field is not constant,

$$\frac{dP^2}{dt} = \frac{16\pi^2 R^6 B^2}{3c^3 I} \qquad \tau_c \equiv \frac{P}{2\dot{P}}$$

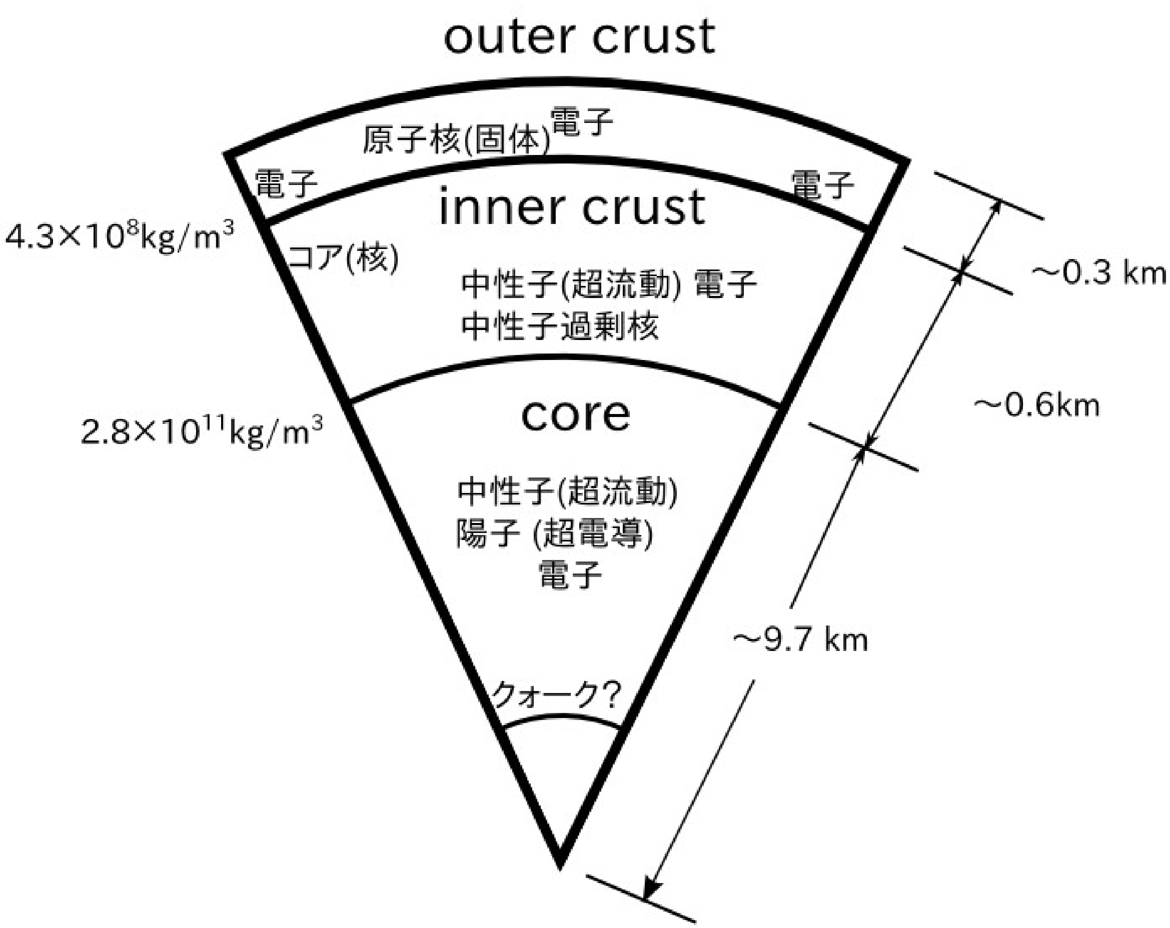
- B=const. → $\tau_c \sim t$ (true age)
- B decreases ↘ → $\tau_c > t$
- B increases ↗ → $\tau_c < t$

True age is younger than its characteristic age
(see also Nakano et al. 2015)



Physical models

Composition of magnetar interior



Crust:

- Free electron

Core:

- Neutron, proton, and electron

  (npe 3fluid)

Origin of a magnetic field

- Charged particles (electric current) in core and crust (charged particle)
  - Fossil fields
  - Dynamo, MRI, winding field
    - Thompson & Duncan (1992); Sawai & Yamada (2014)
- Ferromagnetism
  - Makishima et al. (1999); Tatsumi (2000);

    Eto, Hashimoto & Hatsuda (2013);Hashimoto (2015)

Three fluid (npe) model

Stationary and ignore inertial terms

Goldreich & Reisenegger (1992)

$$0 = -n_n \nabla \mu_n - n_n \frac{\mu_n}{c^2} \nabla \phi - \sum_{j \neq n} \gamma_{nj} n_n n_j (\boldsymbol{v}_n - \boldsymbol{v}_j),$$

$$0 = -n_c \nabla \mu_c - n_c \frac{\mu_p}{c^2} \nabla \phi + e n_p (\boldsymbol{E} + \frac{\boldsymbol{v}_p}{c} \times \boldsymbol{B}) - \sum_{j \neq p} \gamma_{pj} n_p n_j (\boldsymbol{v}_p - \boldsymbol{v}_j),$$

$$0 = -n_c \nabla \mu_e - n_c \frac{\mu_e}{c^2} \nabla \phi - e n_c (\boldsymbol{E} + \frac{\boldsymbol{v}_i}{c} \times \boldsymbol{B}) - \sum_{j \neq p} \gamma_{ej} n_e n_j (\boldsymbol{v}_e - \boldsymbol{v}_j),$$

Three fluid (npe) model

Goldreich & Reisenegger (1992)

$$\boldsymbol{E} \cong \frac{\boldsymbol{j}}{\sigma} - \frac{\boldsymbol{v}_A}{c} \times \boldsymbol{B} - \frac{\gamma_{en} - \gamma_{cn}}{\gamma_{cn}} \frac{\boldsymbol{j} \times \boldsymbol{B}}{n_c e c}$$

$$\boldsymbol{v}_H \equiv \frac{\gamma_{en} - \gamma_{pn}}{\gamma_{cn}} (\boldsymbol{v}_p - \boldsymbol{v}_e) = \frac{c(\gamma_{en} - \gamma_{pn})}{ne\gamma_{cn}} \boldsymbol{j}.$$

$$\boldsymbol{v}_A \equiv \frac{\gamma_{pn}(\boldsymbol{v}_p - \boldsymbol{v}_n) + \gamma_{en}(\boldsymbol{v}_e - \boldsymbol{v}_n)}{\gamma_{pn} + \gamma_{en}}.$$

$$\frac{\boldsymbol{j} \times \boldsymbol{B}}{n_c} - \nabla \Delta \mu = (\gamma_{pn} + \gamma_{en}) \boldsymbol{v}_A$$

Three fluid (npe) model

Goldreich & Reisenegger (1992)

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times [(\boldsymbol{v}_A + \boldsymbol{v}_H) \times \boldsymbol{B}] - \nabla \times \left(\frac{c^2 \nabla \times \boldsymbol{B}}{4\pi\sigma} \right)$$

$$\boldsymbol{v}_H \equiv \frac{\gamma_{en} - \gamma_{pn}}{\gamma_{cn}} (\boldsymbol{v}_p - \boldsymbol{v}_e) = \frac{c(\gamma_{en} - \gamma_{pn})}{ne\gamma_{cn}} \boldsymbol{j}.$$

$$\boldsymbol{v}_A \equiv \frac{\gamma_{pn}(\boldsymbol{v}_p - \boldsymbol{v}_n) + \gamma_{en}(\boldsymbol{v}_e - \boldsymbol{v}_n)}{\gamma_{pn} + \gamma_{en}}.$$

$$\frac{\boldsymbol{j} \times \boldsymbol{B}}{n_c} - \nabla \Delta\mu = (\gamma_{pn} + \gamma_{en}) \boldsymbol{v}_A$$

Magnetic field evolution

- If a magnetic field comes from the npe particles,
  - Ohm dissipation ($\sim B^0$)
  - Hall drift ($\sim B^1$)
  - Ambipolar diffusion ($\sim B^2$)

$$\frac{dB}{dt} = \nabla \times \left(D_{Ohm} \nabla \times B \right) + \nabla \times \left(D_{Hall} B \times \left(\nabla \times B \right) \right)$$
$$- \nabla \times \left(D_{Ambipolar} \left(\nabla \times B \right) \times B \times B \right)$$

- A few kyr timescale is favored from the fitting model.

Ohmic dissipation in the core

Baym et al. (1969a,b)

- Electric conductivity is very high

$$\sigma \sim 1.5 \times 10^{29} s^{-1} \left(T = 10^{8} K, \rho \quad 10^{13} g/cm^{3} \right)$$

$$t_{Ohmic} \sim \frac{4\pi\sigma L^{2}}{c^{2}} \sim 10^{13} yr.$$

→ Timescale is much longer than age.

Ohmic dissipation in the crust

$$t_{Ohmic} \sim \frac{4\pi\sigma L^2}{c^2}$$

$$t_{Ohmic} \sim 5 \times 10^3 \left(\frac{L}{10^5\, cm}\right)^2 \left(\frac{\sigma}{10^{24}\, s^{-1}}\right) k\, yr\,.$$

If the L is small (small scale magnetic field),
Ohmic dissipation might be efficient.

Hall drift in a crust

Naito & Kojima (1994)
Goldreich & Reisenegger (1992)

- If a crustal magnetic field comes from free electrons,

$$\frac{dB}{dt} = \nabla\times\left(D_{Ohm}\nabla\times B\right) + \nabla\times\left(D_{Hall}B\times\left(\nabla\times B\right)\right)$$

$$t_{Hall}\sim 5.0\times 10^{2}\left(\frac{B}{10^{15}G}\right)^{-1}\left(\frac{L}{10^{5}cm}\right)^{2}\left(\frac{\rho}{10^{14}g/cm^{3}}\right)kyr\,.$$

$$R_m = \left(\frac{t_{Ohm}}{t_{Hall}}\right) = \frac{\sigma B}{e c n_e} = 10-100\left(\frac{B}{10^{15}G}\right)\left(\frac{\sigma}{10^{24}}\right)$$

- Electrons are drifted by the Lorentz force (≠dissipation)
- Hall drift makes small scale magnetic fields.
- Toroidal/poloidal field and transformed into poloidal/toroidal field.

Ambipolar diffusion in a core

Goldreich & Reisenegger (1992)

- Diffusion due to a multi-fluid system

$$t_{ambipolar} \sim 3\left(\frac{B}{10^{15}\,G}\right)^{-2}\left(\frac{L}{10^{5}\,cm}\right)^{2}\left(\frac{T}{10^{8}\,K}\right)^{2} kyr\,.$$

- If a magnetic filed is strong, the timescale is much faster than other mechanisms.

- Toroidal/poloidal fields are not transformed.

Summary of physical models

- Evolutionary timescale with B = 10^{15} G
  - Ohmic dissipation ($\sim B^0$) :$\sim$10^13 yr(core) :$\sim$5000 k yr(crust).
  - Hall drift ($\sim B^1$) :$\sim$100 k yr (crust).
  - Ambipolar diffusion ($\sim B^2$) :$\sim$1 k yr (core).

$$\frac{dB}{dt} = \nabla \times \left(D_{Ohm} \nabla \times B\right) + \nabla \times \left(D_{Hall} B \times \left(\nabla \times B\right)\right) - \nabla \times \left(D_{Ambipolar} \left(\nabla \times B\right) \times B \times B\right)$$

$1.5 < \alpha < 1.75$ from the phenomenological fitting model.

New phenomenological model

Core field(Ambipolar diffusion) + crust field (Hall drift)

$$B_d = B_{cr} + B_{co} \qquad \tau_{Hall} = 100\,k\,yr. \;\; \tau_{Amb} = 1\,k\,yr.$$

$$\frac{dB_{cr}}{dt} = -D_{Hall} B_{cr}^2 \qquad \frac{dB_{co}}{dt} = -D_{Amb} B_{co}^3$$

Previous model

$$\frac{dB}{dt} = -A\,B^{\alpha+1}$$

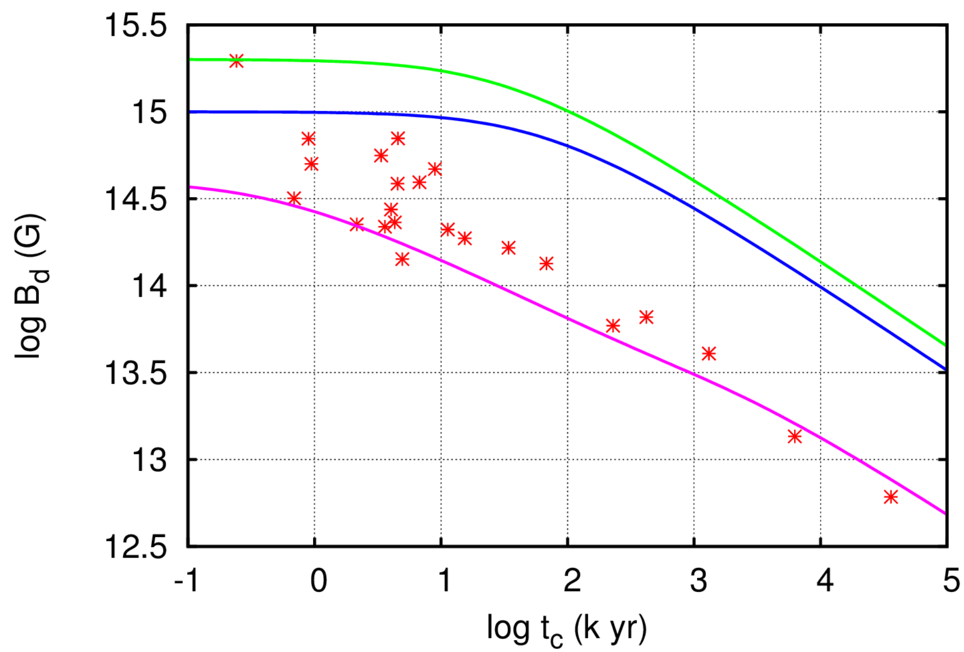
τ = 1000 yr. 1.5 < α < 1.75

(Dissipation timescale is 1kyr)

Numerical results

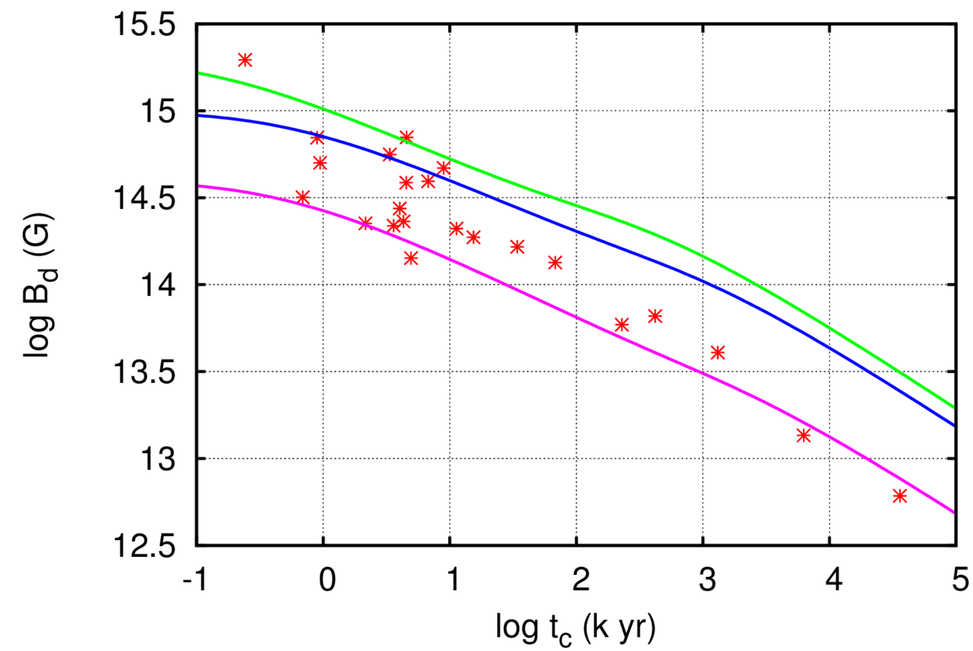
Crust field dominant

$$B_{cr} : \quad B_{co} = 90 : 10$$



core field dominant

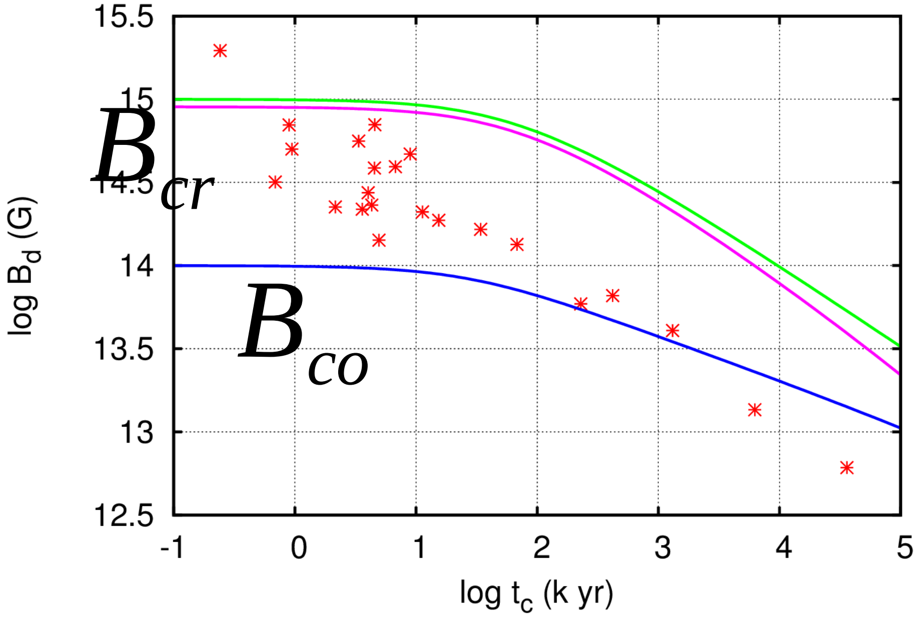
$$B_{cr} : \quad B_{co} = 10 : 90$$

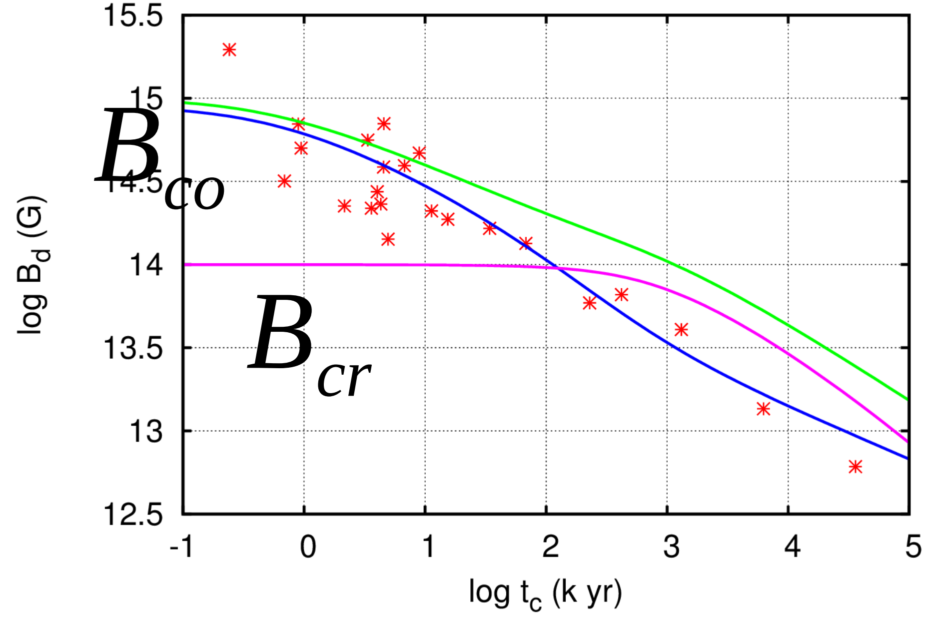


Core field dominant model is favored.

Numerical results

- Crust field dominant core field dominant





Ambipolar diffusion is effcient within 10 kyr.

Hall drift is dominant after 100kyr.

Summary of new models

- Core field dominant model is favored.
  - Fast dissipation by ambipolar diffusion (～10kyr)
  - Hall drift becomes efficient after 100kyr.
- Young magnetar
  - Core field is dissipated by ambipolar diffusion.
- Old magnetar
  - Crustal and surface field are dissipated by Hall drift.

Hall drift

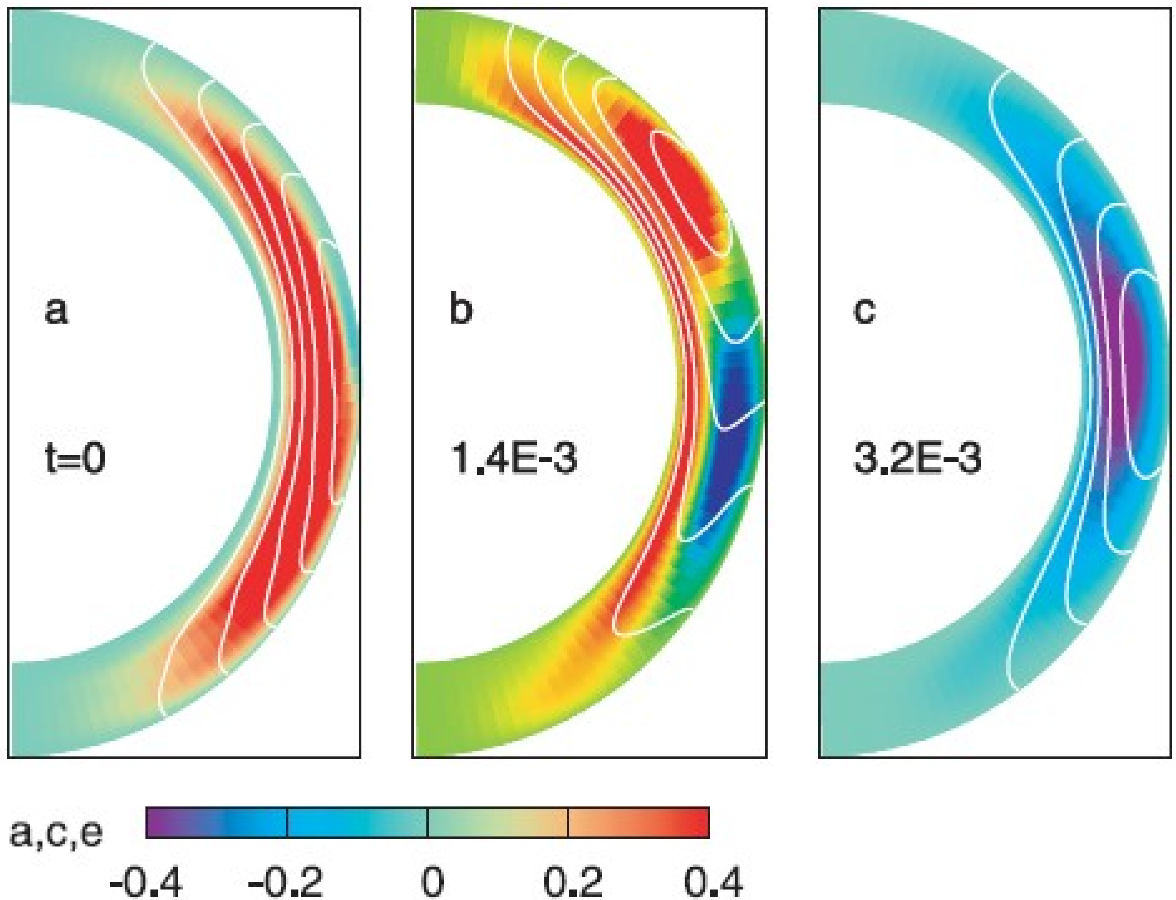
Numerical simulations of Hall drift

- 2D Hall (Kojima & Kisaka 2012)
- 2D Hall + thermal evolution

  (Vigano et al. 2013)
- 3D Hall (Gourgouliatos et al. 2016)

Hall MHD simulation

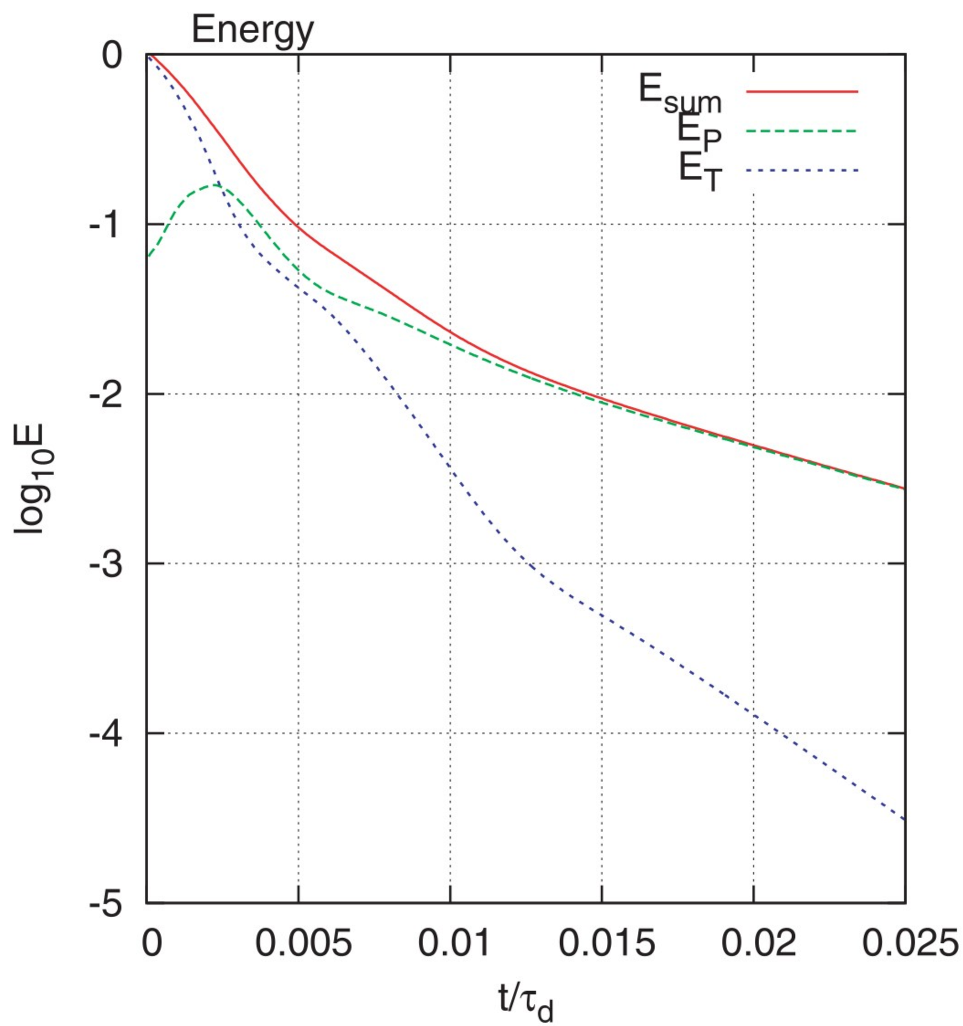
Kojima & Kisaka (2012)

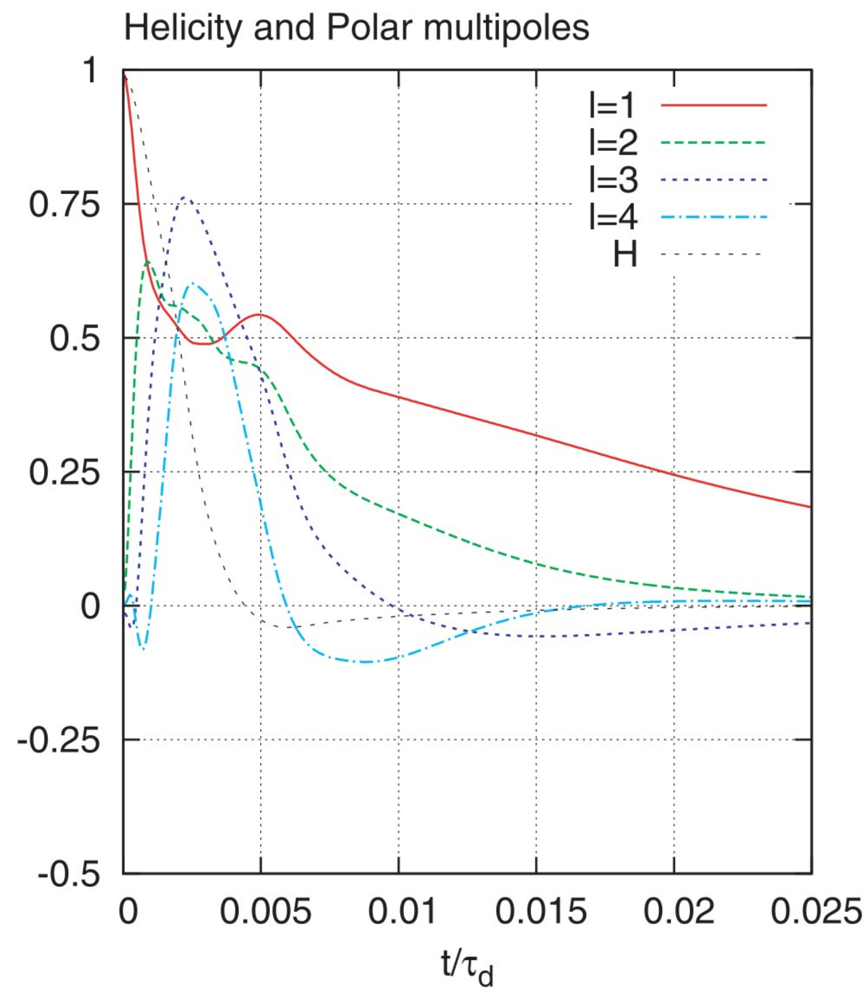


Toroidal field is transformed into the higher order poloidal fields.

Hall MHD simulation

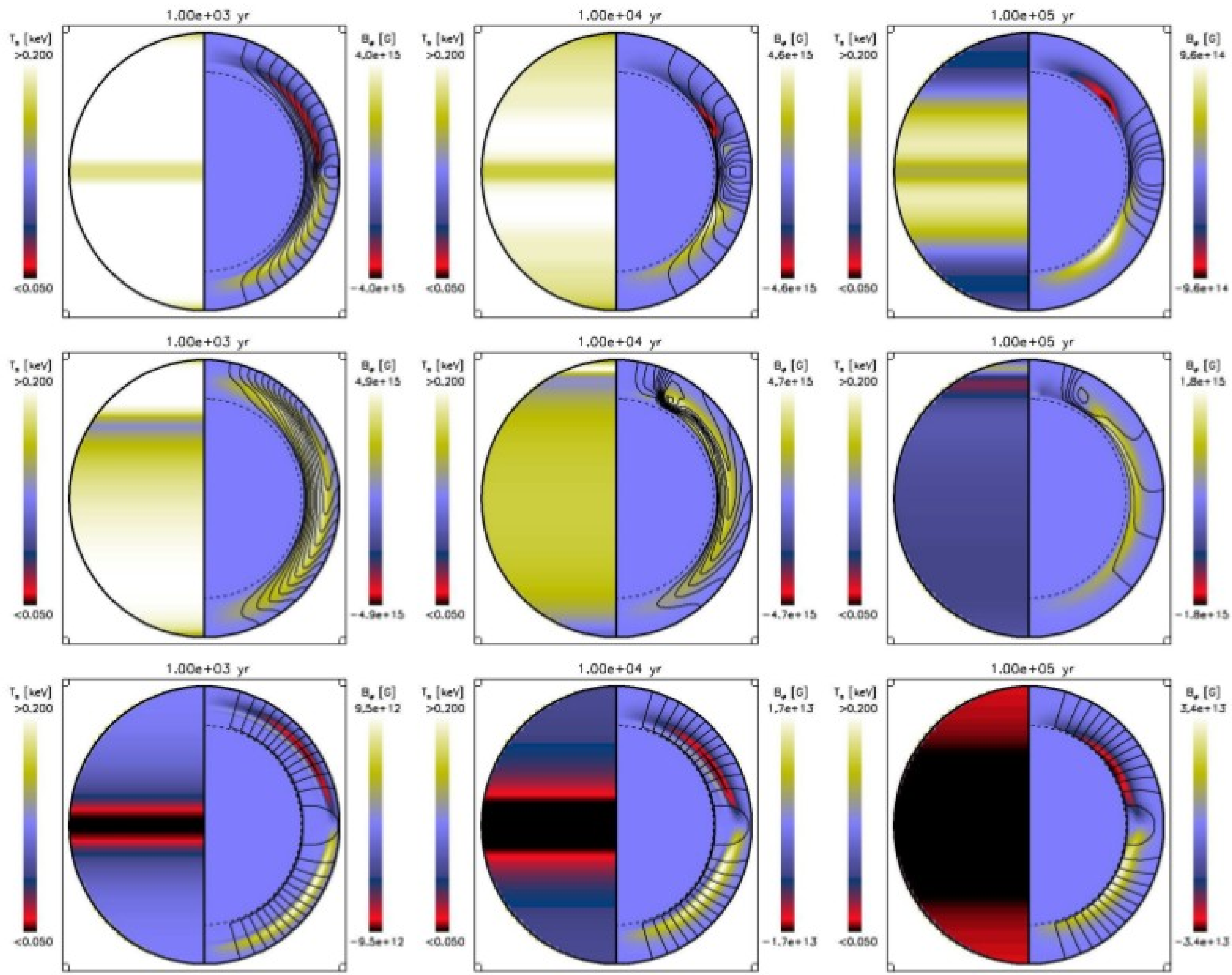
Kojima & Kisaka (2012)





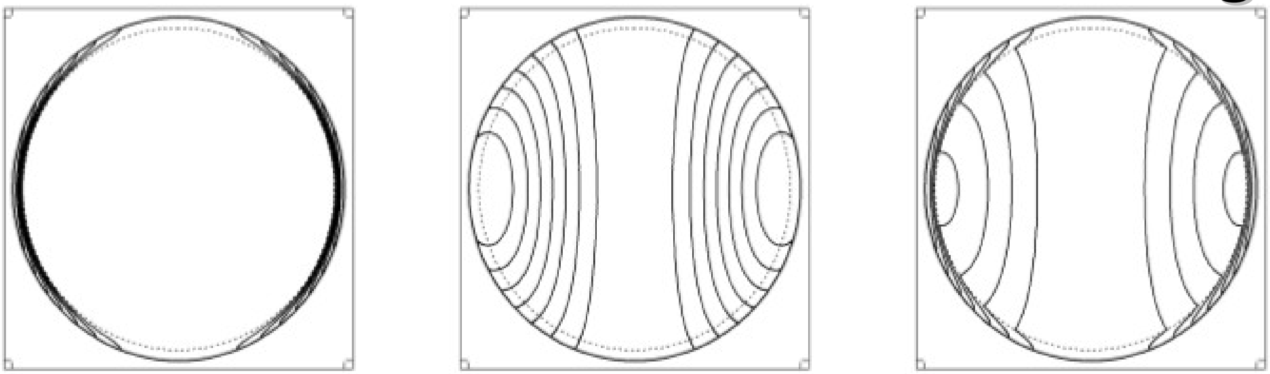
Toroidal field is transformed into the higher poloidal fields.

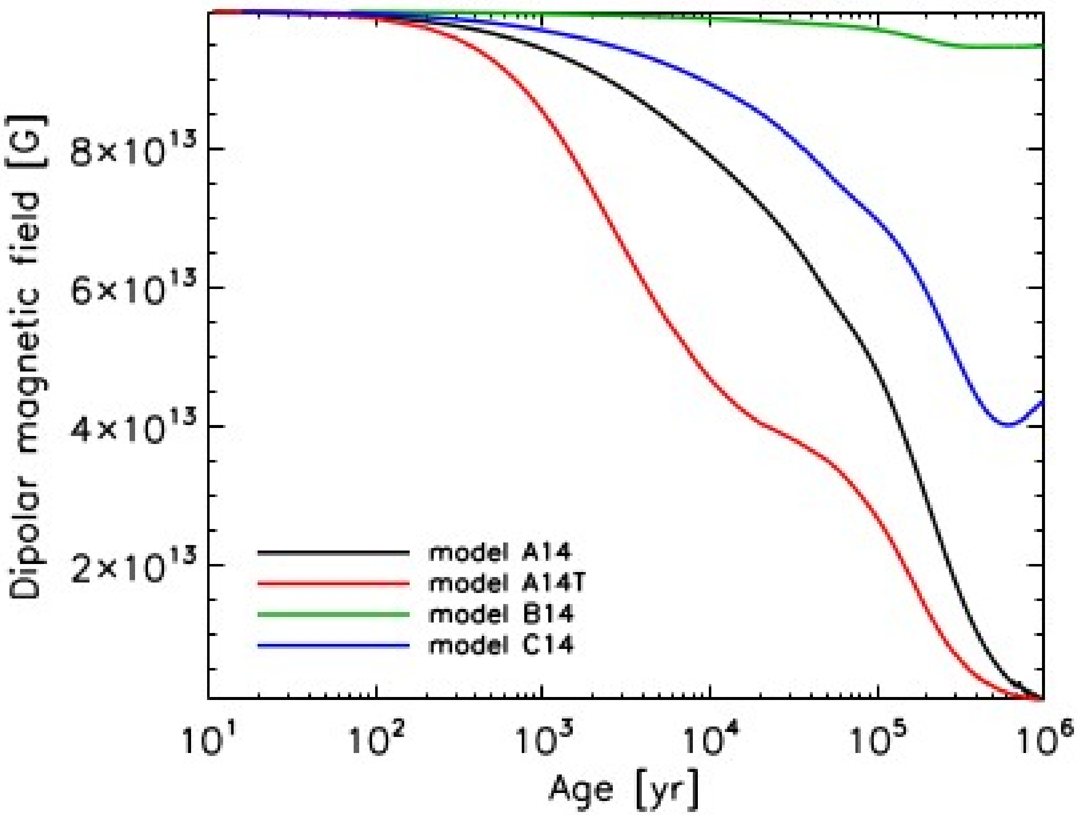
Vigano et al. (2013)



Initial conditions and evolution

Vigano et al. (2013)

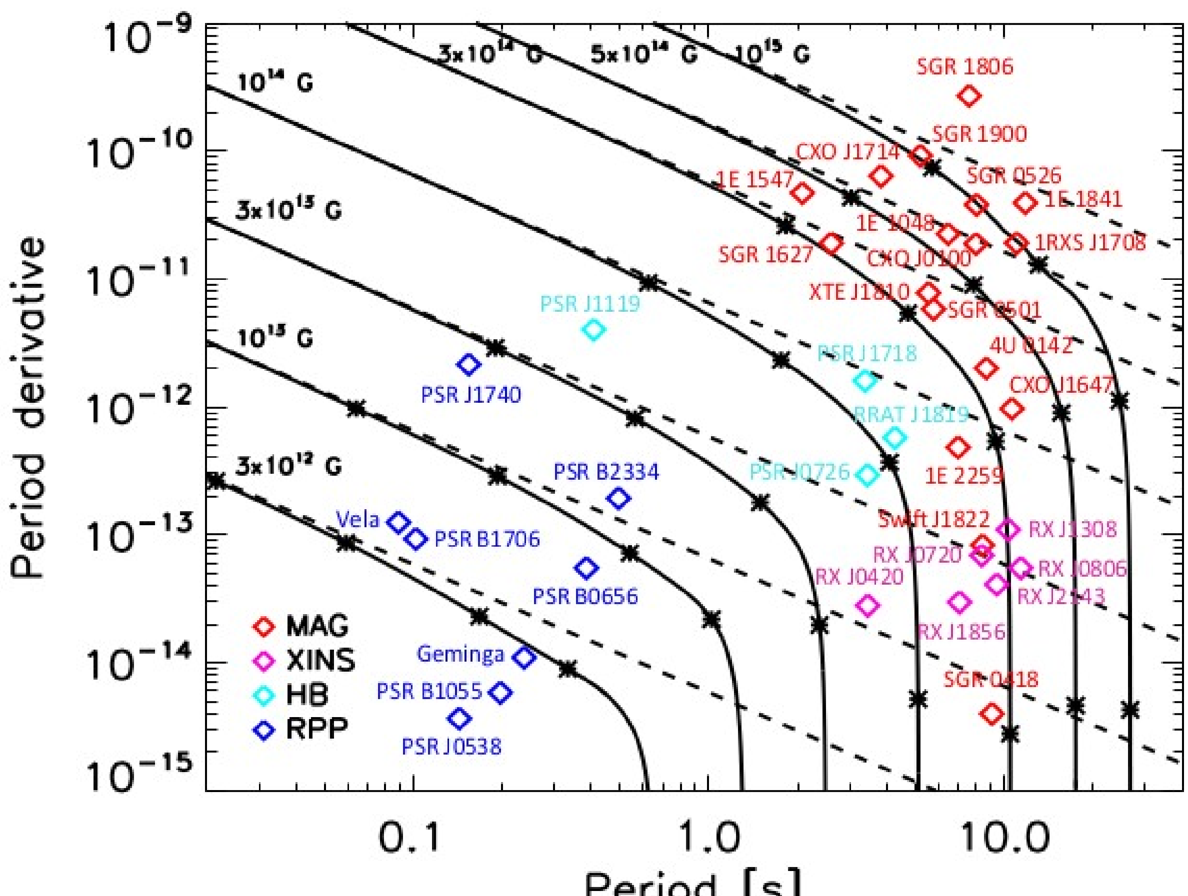


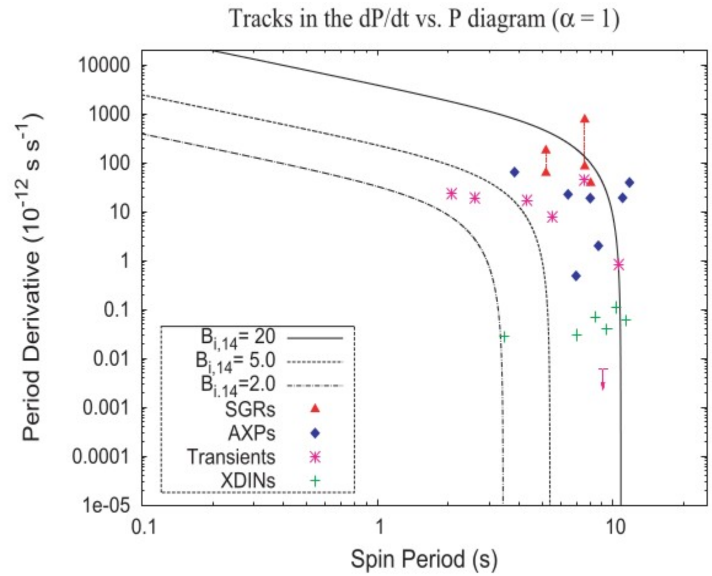


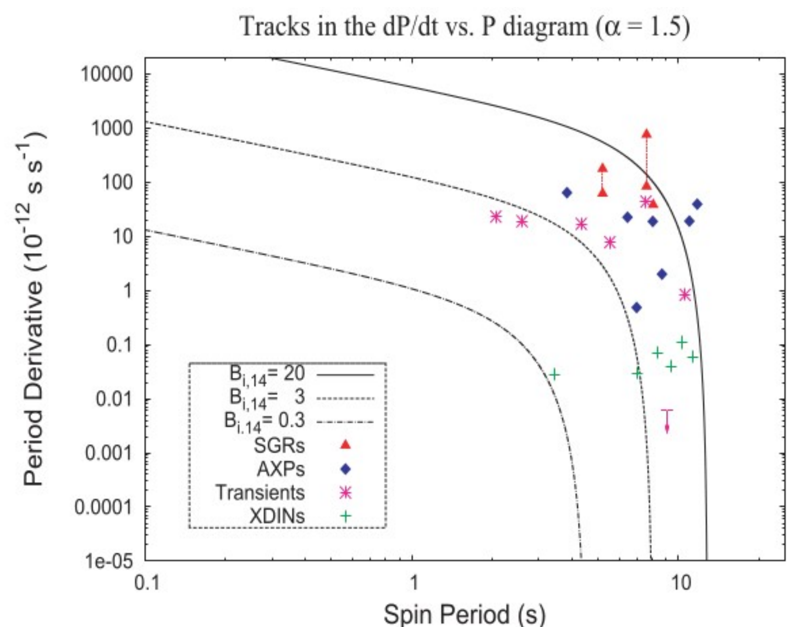
Evolutionary path of model A

Vigano et al. (2013)

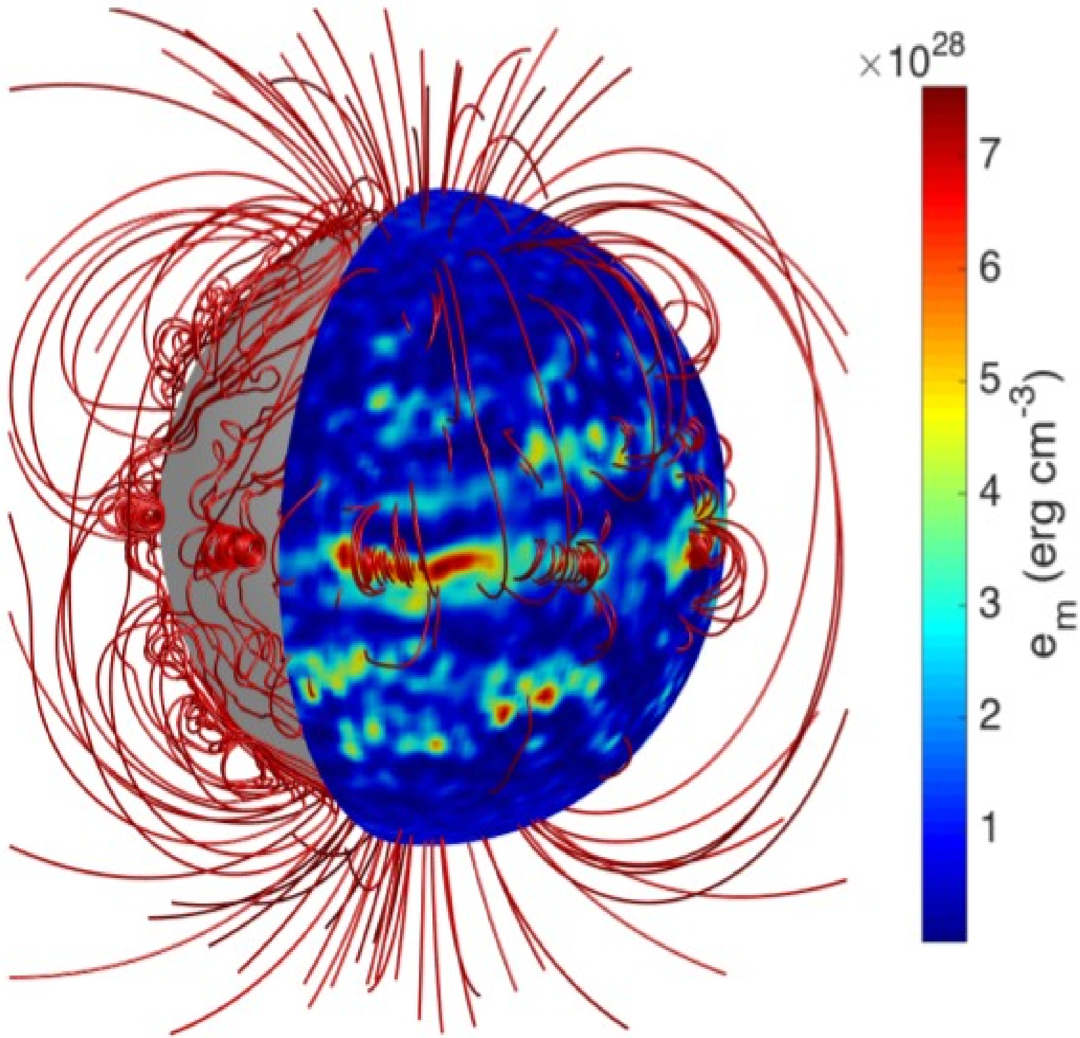
Looks like phenomenological models.



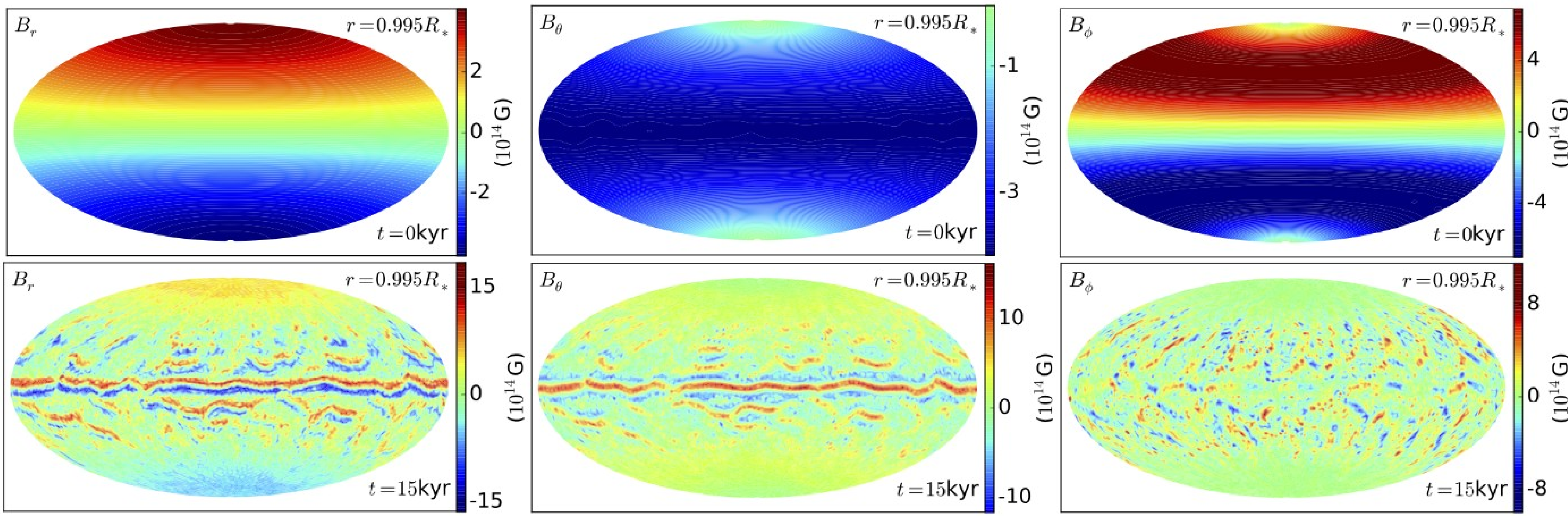




Gourgouliatos et al. (2016)



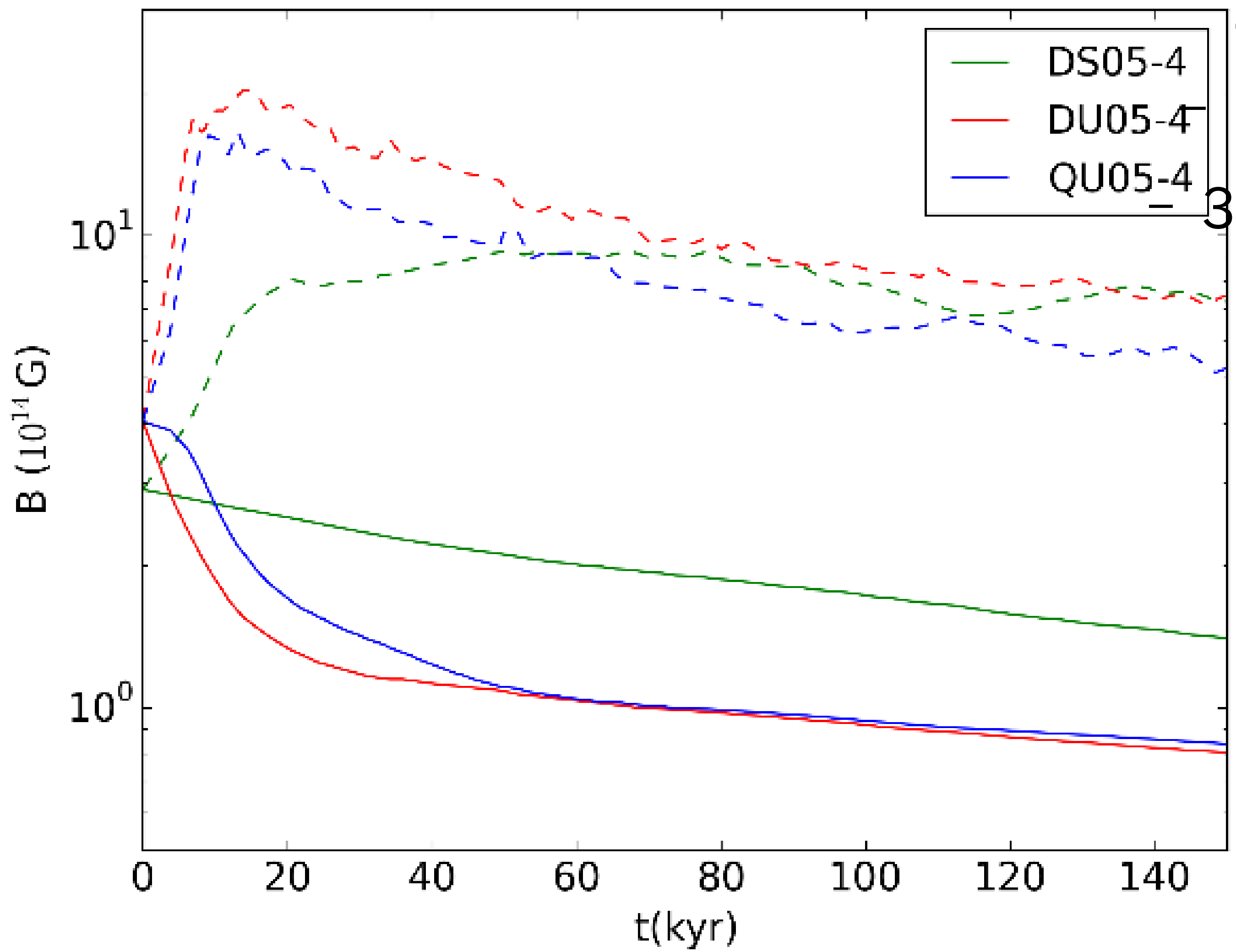
Magnetic field structures



Note:Gourgouliatos' scheme is based on a spectral method.

- Hall drift is a "burgers type (切り立つ波)" equation.
  - Infinite numbers of modes are required.
  - Numerical error?

Evolution of dipole field



– 2D(Axisymmetric) models

– 3D model

Summary of Hall drift

- 90% of dipole magnetic field
  ($\neq$ magnetic energy) is dissipated within 10 kyr
  - Evolution depends on the initial field largely.
- Non-axisymmetric effects might be important
  - Induce localized strong magnetic field.
- Small scale surface fields are formed by Hall drift
  $\rightarrow$ Surface fields of old magnetars?

Ambipolar diffusion

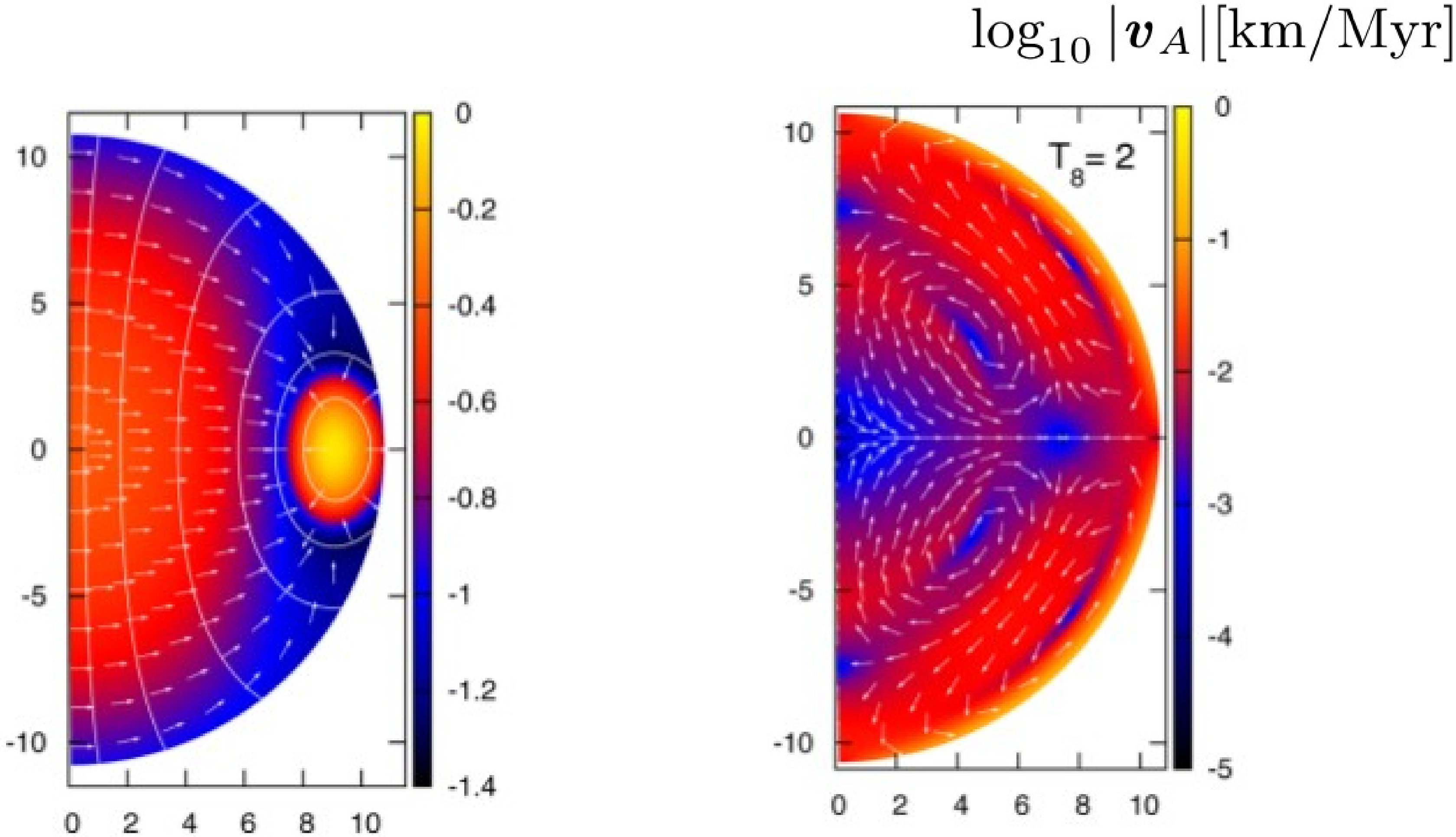
Recent studies

- Rough estimations (Goldreich & Reisenegger 1992)
- 1D evolutionary model

  (Hoyos et al. 2008;2010; Beloborodov & Li 2016)
- 2D formulation and estimations of ambipolar diffusion velocity
  - Simplified background matter (Passamonti et al. 2017a)
  - More general formulation

    (Gusakov et al. 2017; Ofengeim& Gusakov 2018)
- 2D evolutionary model
  - Immovable neutron and constant background (Castillo et al. 2017)
- 3D evolutionary model (Vigano et al. 2018)
  - Ignore micro physics. Code check and test calculations.

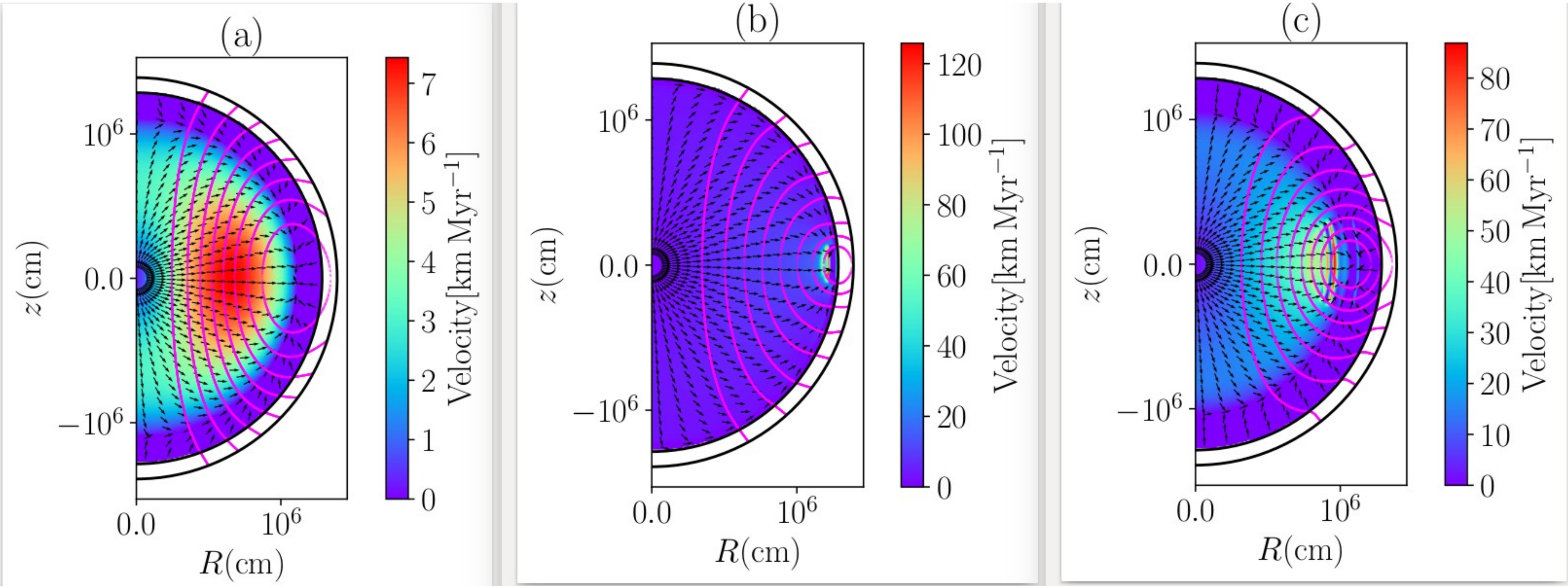
Passamonti et al. (2017)



Passamonti et al. 2017a

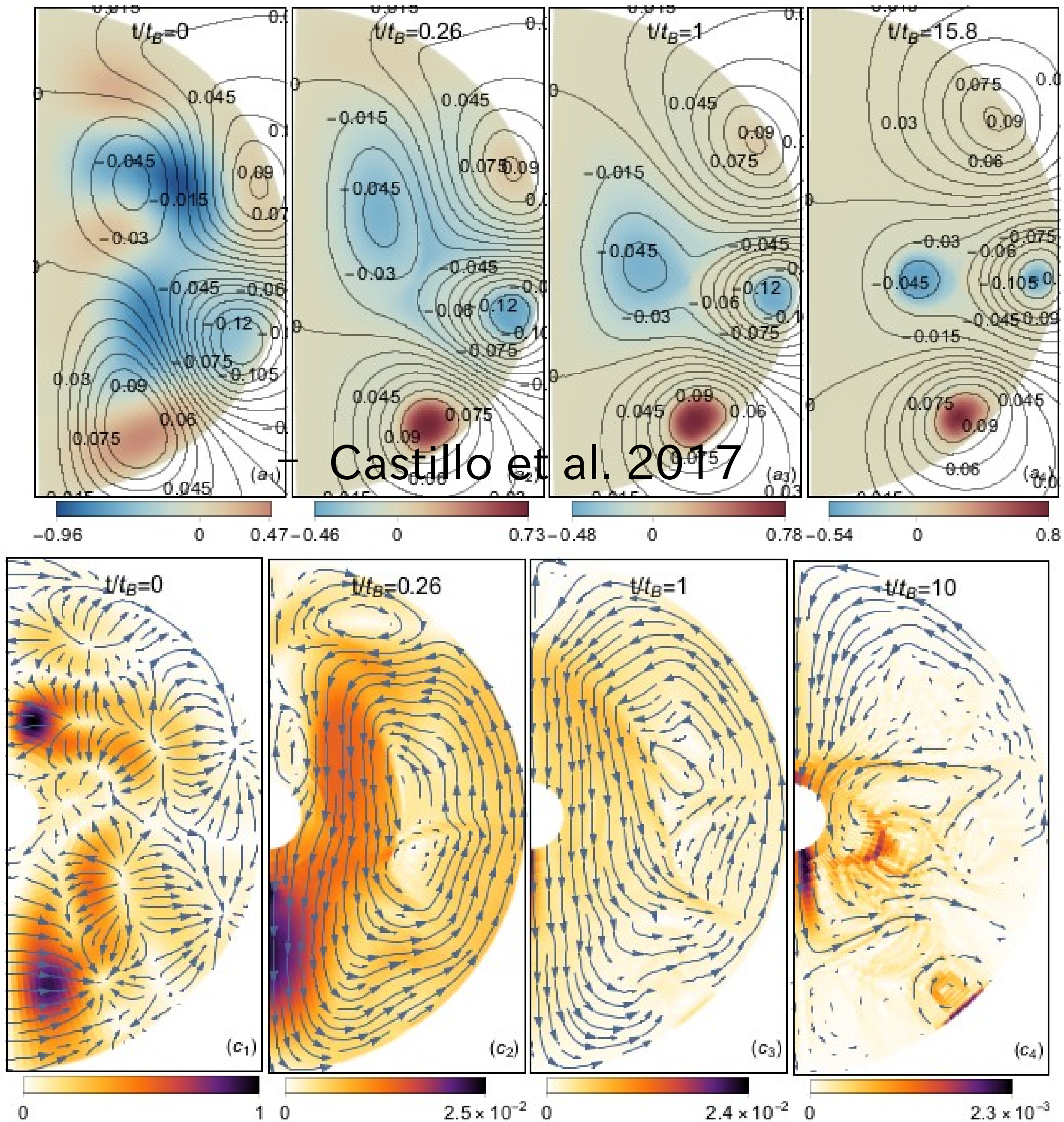
Ambipolar diffusion velocities

Fujisawa et al. (in prep)



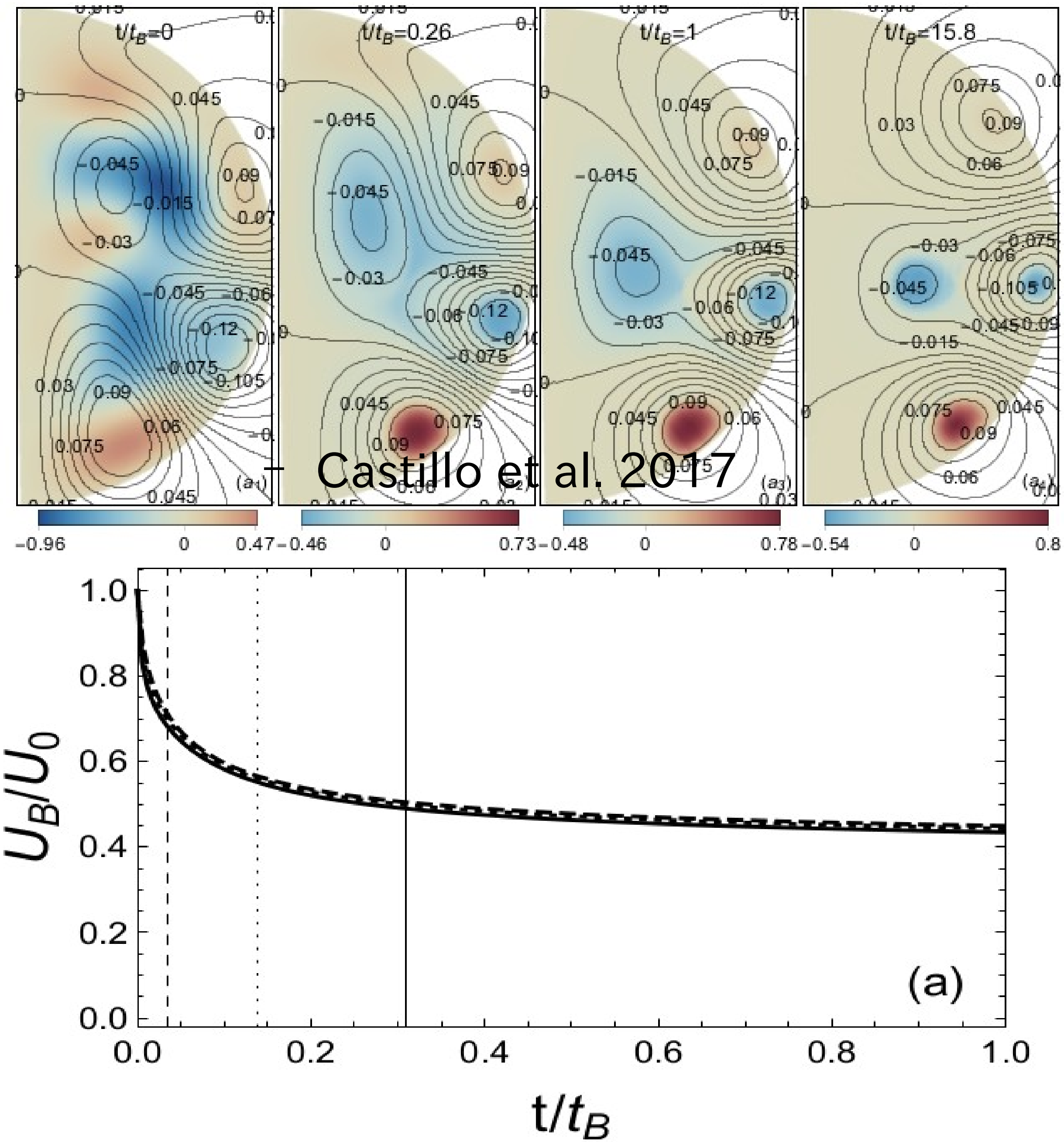
$\tau_{Amb} \sim 0.4\,k\,yr.$ $\tau_{Amb} \sim 0.069\,k\,yr.$ $\tau_{Amb} \sim 0.5\,k\,yr.$

Castillo et al. (2017)



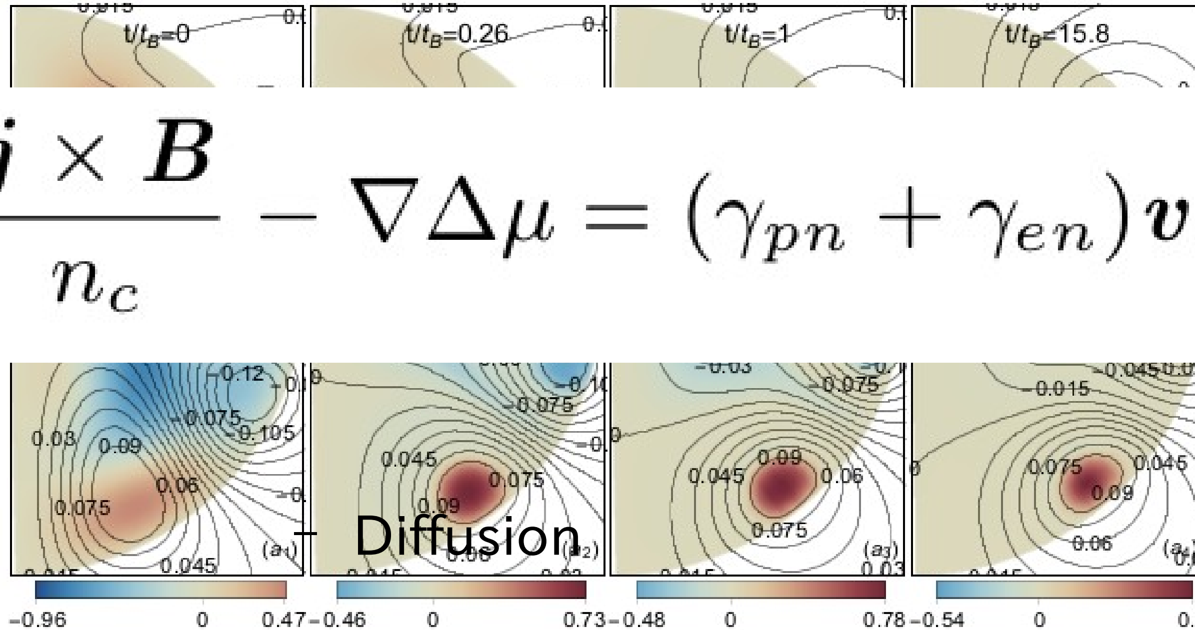
– Castillo et al. 2017

Castillo et al. (2017)



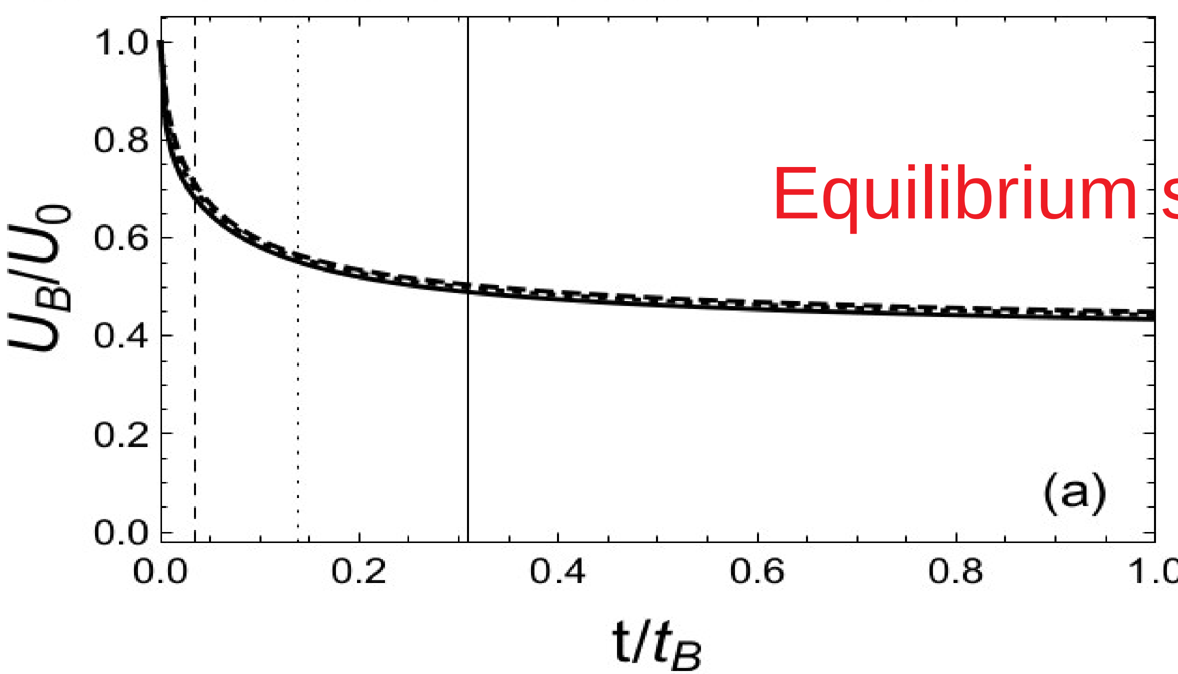
- Castillo et al. 2017

Ambipolar equilibrium



$$\frac{\boldsymbol{j}\times\boldsymbol{B}}{n_c} - \nabla\Delta\mu = (\gamma_{pn}+\gamma_{en})\boldsymbol{v}_A$$

− Diffusion

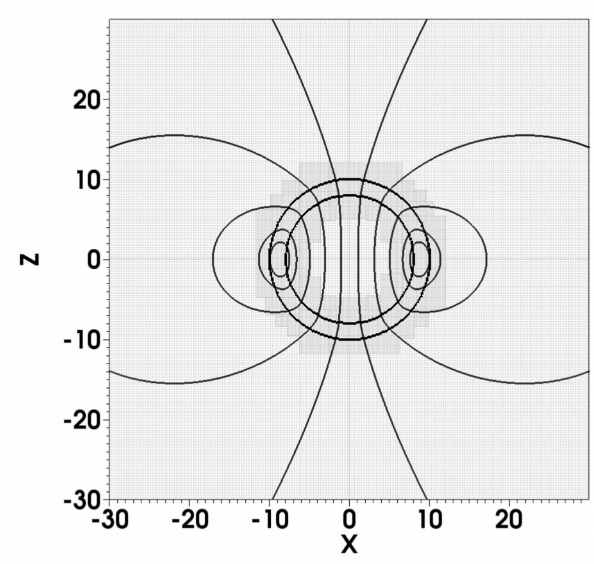


Equilibrium state

Vigano et al. (2018)

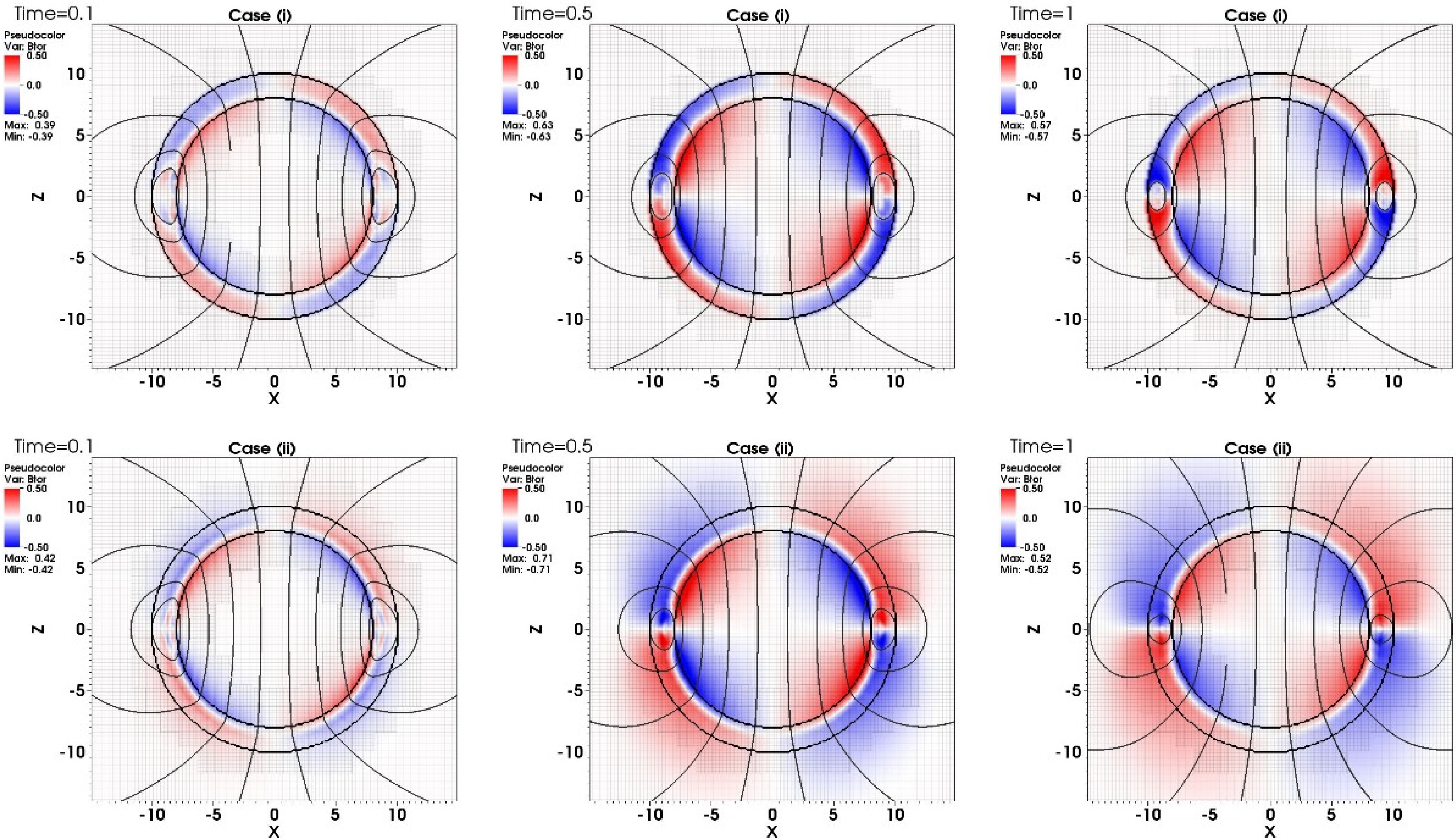
$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \left(f_d \vec{j} + f_h (\vec{j} \times \vec{B}) - f_a (\vec{j} \times \vec{B}) \times \vec{B} \right) = 0$$

- They treat Ohm dissipation, Hall drift and Ambipolar diffusion in both core and crust.
- FMR, higher order (5-th order) numerical scheme
  - Gougliatos's Hall MHD simulation is based on the spectral method.
- Numerical test only



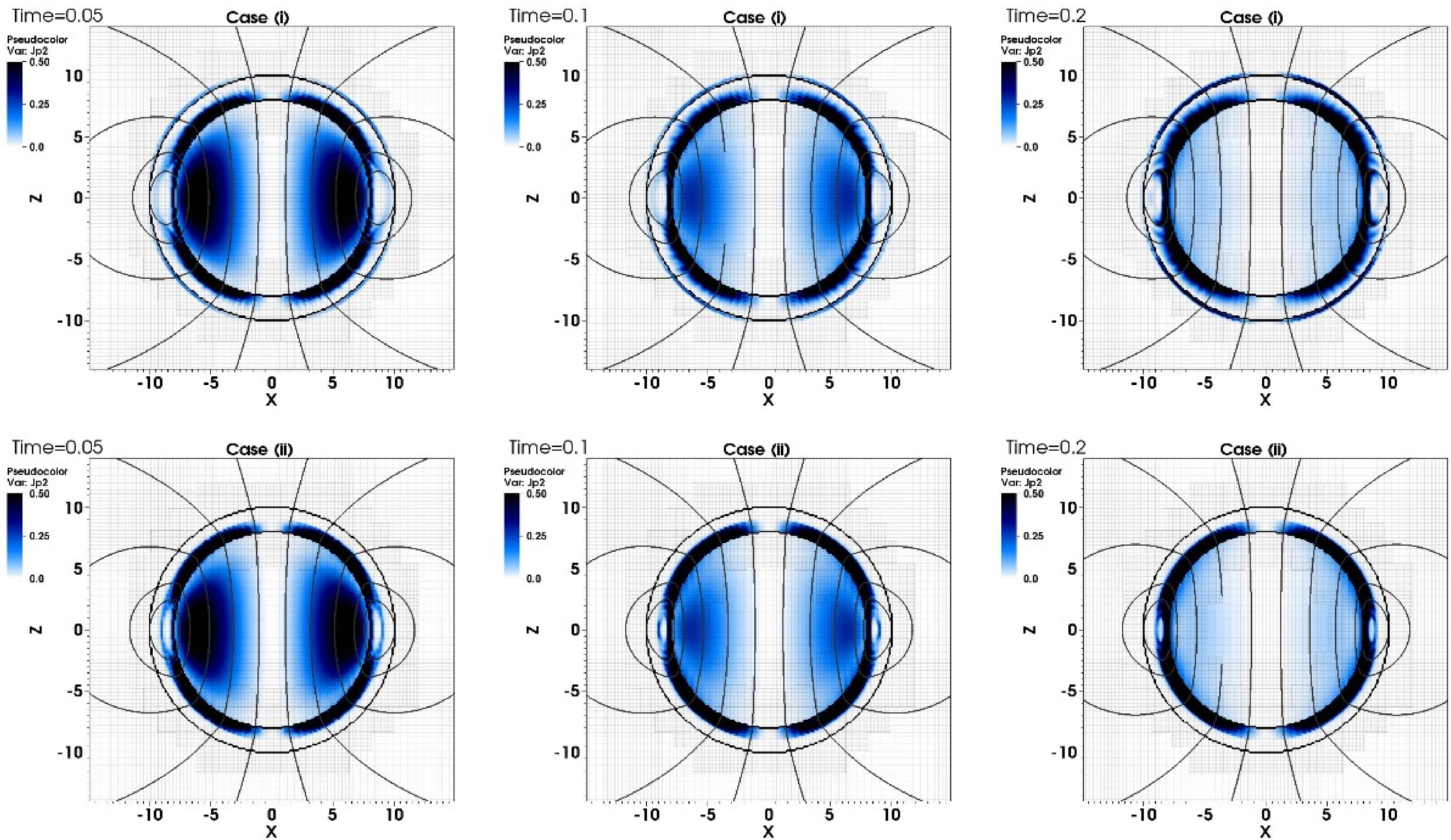
B

Vigano et al. (2018)



Vigano et al. (2018)

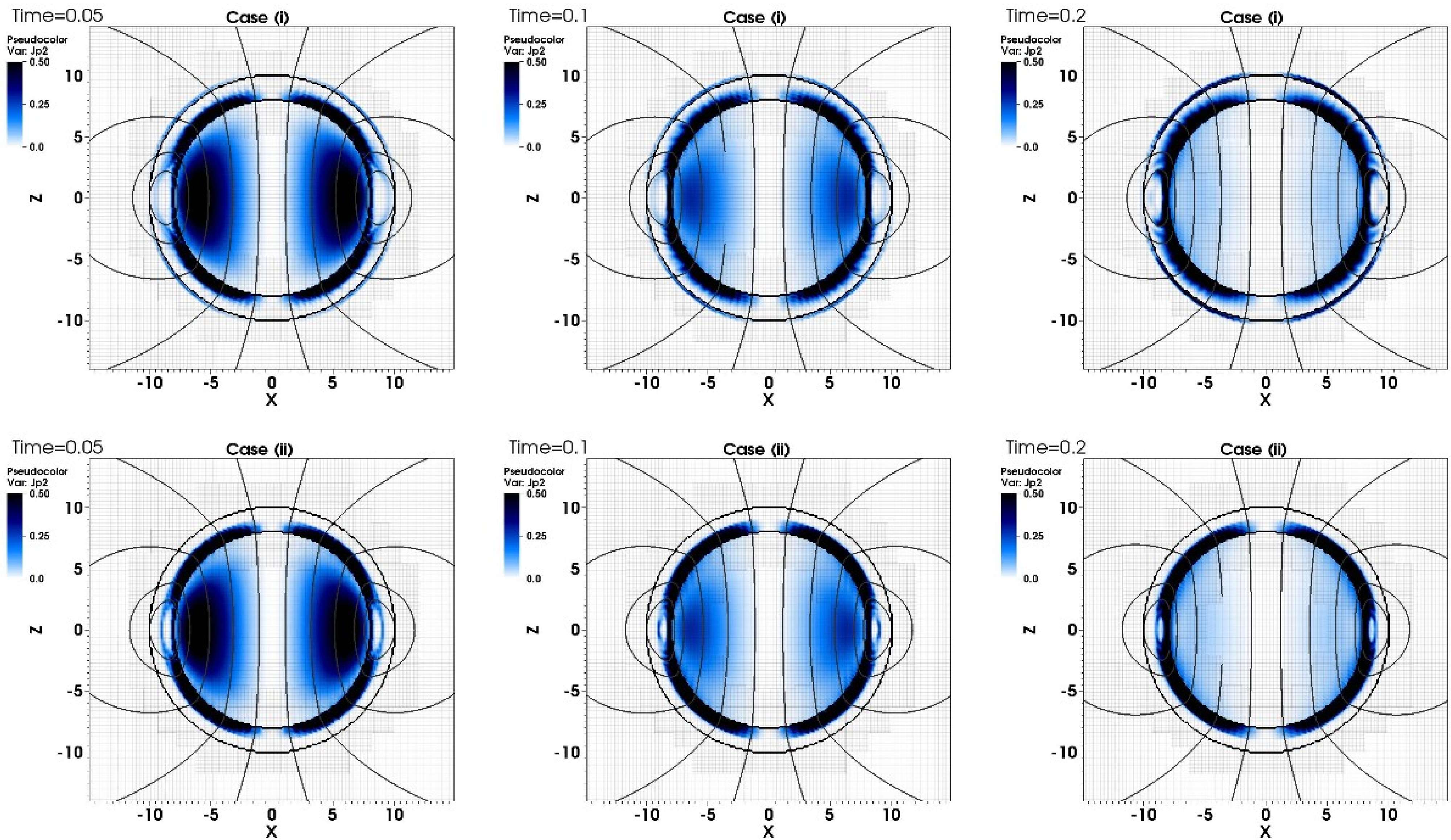
Current



Vigano et al. (2018)

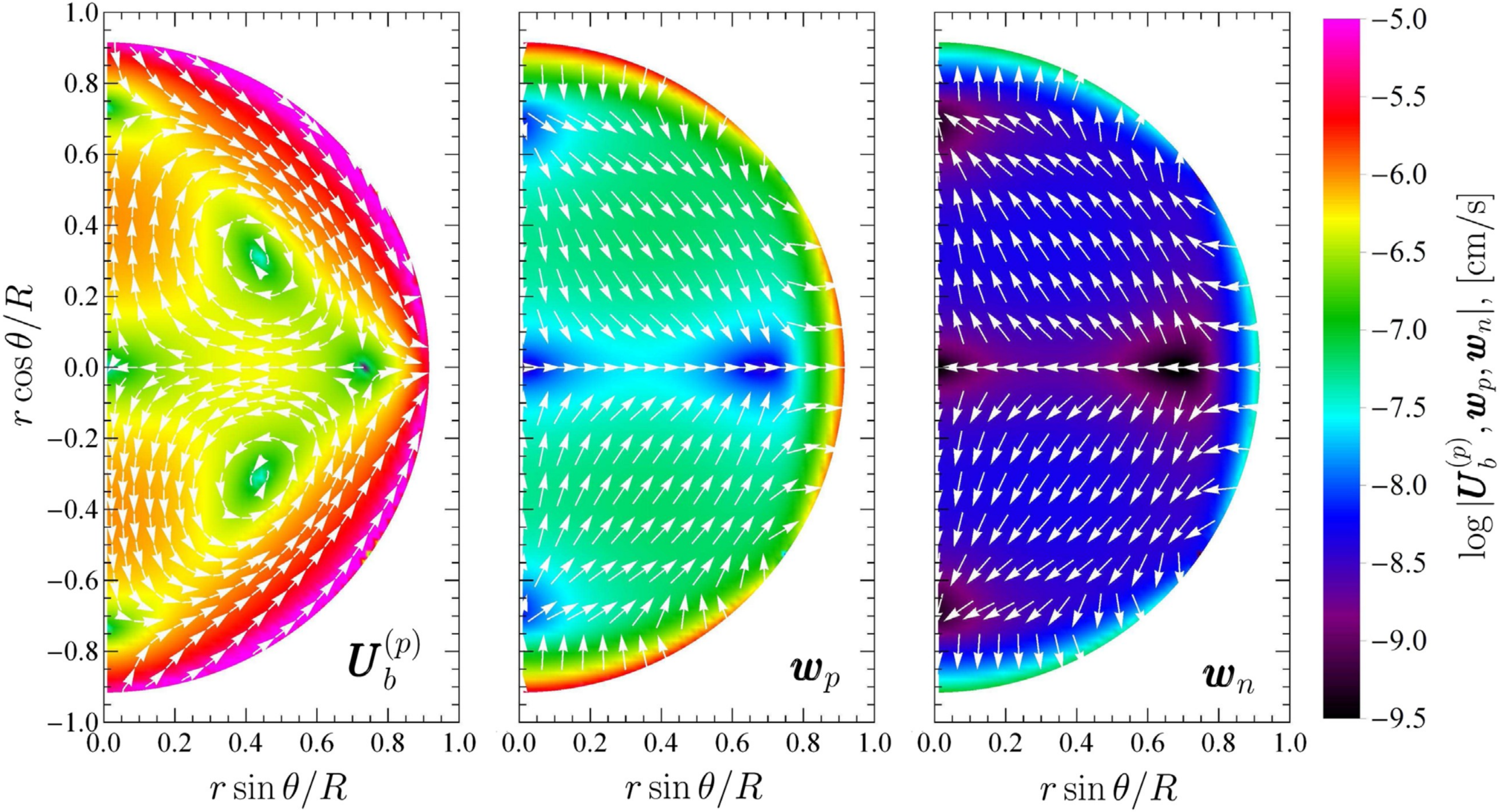
Current

Force-free field



Gusakov & Ofengeim (2018)

See also Passamonti et al. (2017b)



- Baryon (advection) velocity is faster than diffusion velocities.

→ Magnetic fields are “frozen in” rather than diffusion.

Frozen-in?

See also Passamonti et al. (2017b)

$$\frac{\partial(\rho \boldsymbol{v})}{\partial t} + \boldsymbol{\nabla} \cdot \left(\rho \boldsymbol{v}\boldsymbol{v} + \frac{\rho_{\rm c}\rho_{\rm n}}{\rho}\boldsymbol{w}\boldsymbol{w} + \frac{\rho_{\rm e}\rho_{\rm p}}{\rho_{\rm c}}\boldsymbol{u}\boldsymbol{u} \right) + \boldsymbol{\nabla} P + \rho \boldsymbol{\nabla}\Phi = \boldsymbol{F}_{\rm L} \,. \quad (20)$$

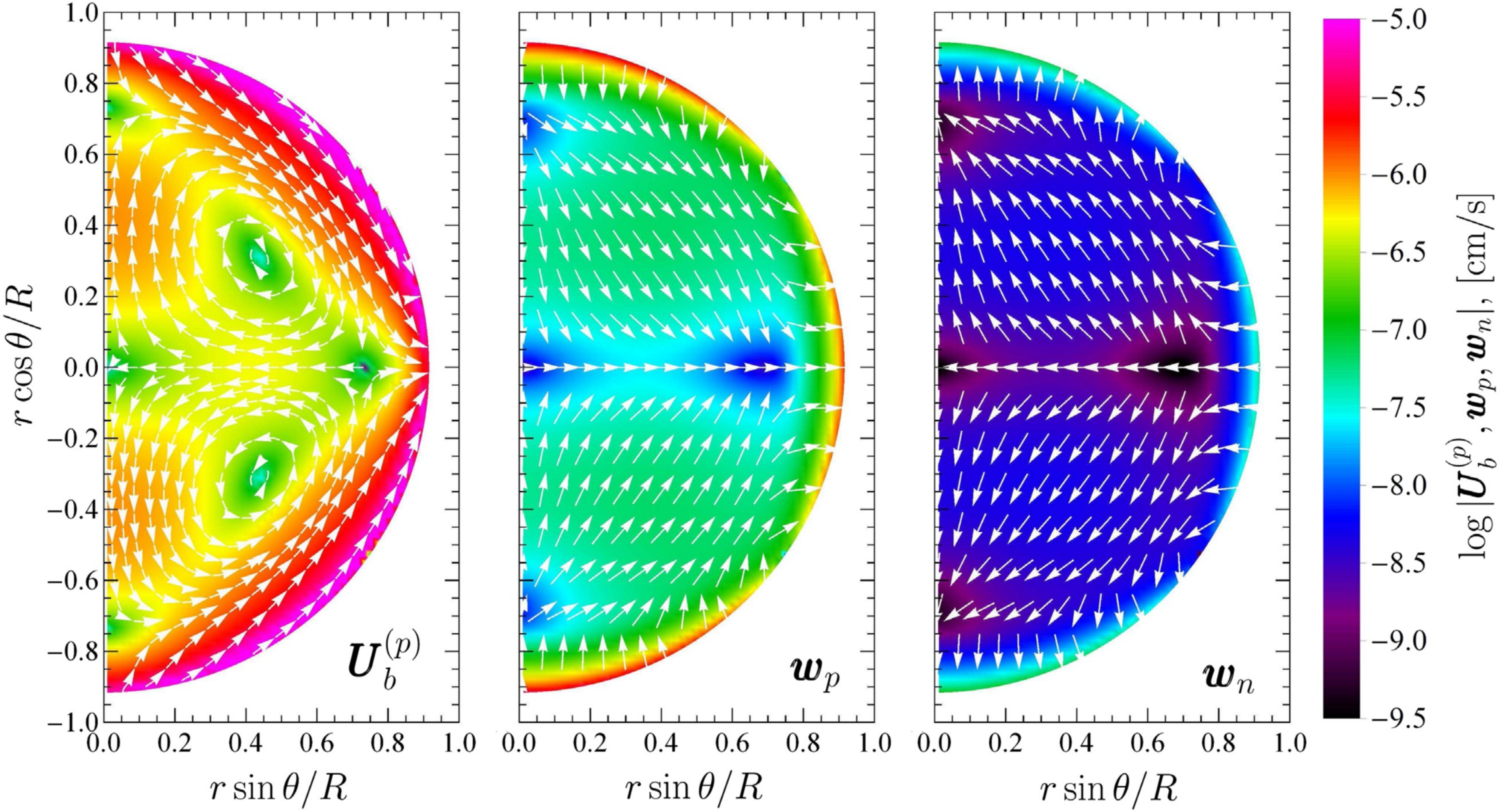
Although an evident improvement over the static assumption, the inclusion of velocities in the stationary regime must be accompanied by a new advective term in the induction equation, which may become dominant. Equation (20) is not easy to solve in the general case. As far as we know, in the literature there are not yet numerical solutions which describe the internal magnetic field of neutron stars with flow motion. Analytical solutions have been presented only for simplified cases (Chandrasekhar 1956; Tsinganos 1981, 1982).

Frozen-in equilibrium state with flows (Fujisawa et al. 2013)

$$\frac{1}{\rho}\nabla p = -\nabla \phi_{\rm g} - \nabla \phi_{\rm c} - \frac{1}{2}\nabla |\boldsymbol{v}|^2 + \boldsymbol{v} \times \boldsymbol{\omega} + \frac{1}{c\rho}\boldsymbol{j} \times \boldsymbol{B},$$

Speculation

See also Passamonti et al. (2017b)



Poloidal magnetic field becomes quadrupole?

- Dipole field is transformed into higher-order field.

Summary of ambipolar diffusion

- Ambipolar diffusion might be fast and efficient dissipation mechanism in a core.
  - Ambipolar equilibrium state?
  - Force balance in a core should be considered
- Treatment of a core-crust boundary is important.
- Baryon velocity could be faster than ambipolar diffusion (Ofsten & Gusakov 2018)
  - Core fields should be "frozen-in" the baryon flow?

Discussion and future works

- Magnetar's magnetic field might be dissipated (low field magnetar).
- Surface magnetic fields of old magnetars might be high order due to the Hall drift.
- Core magnetic fields of young magnetars might be dissipated by the ambipolar diffusion.
  - Ambipolar equilibrium state?. "Frozen-in"?