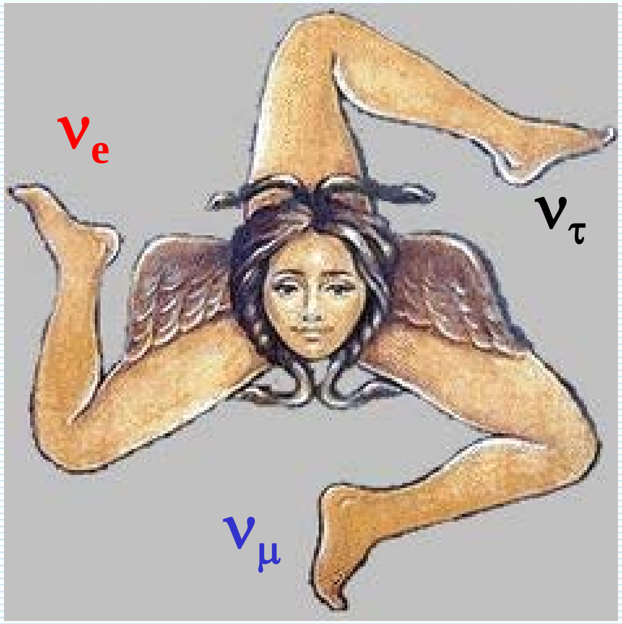
RIKEN, Japan, March 4, 2019



Massive neutrinos in nuclear processes

Fedor Šimkovic

Content

Acknowledgements: A. Faesler (Tuebingen), P. Vogel (Caltech), S. Kovalenko (Valparaiso U.), M. Krivoruchenko (ITEP Moscow), D. Štefánik, R. Dvornický (Comenius U.), A. Babič, A. Smetana, (IEAP CTU Prague), J.D. Vergados (Ioannina U.) …

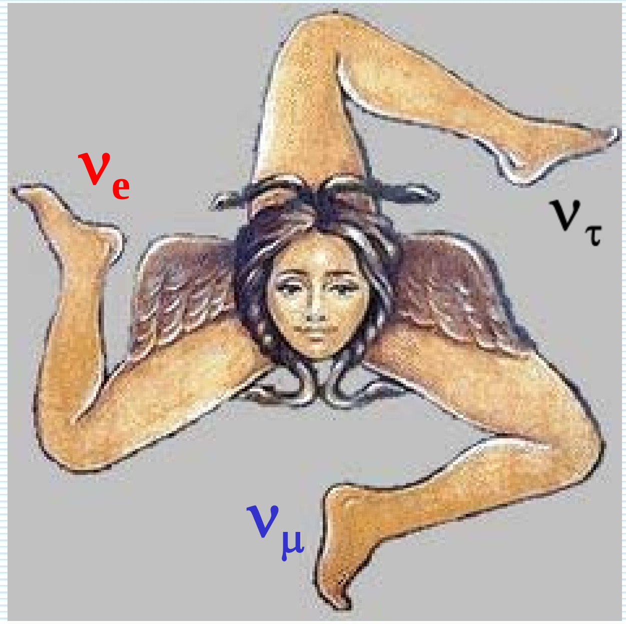
I. Introduction: Neutrino physics nowadays

After 63 years we know

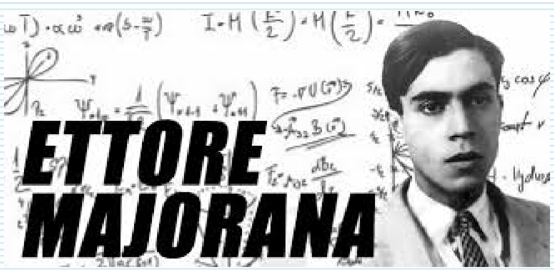
Fundamental ν properties

No answer yet

- 3 families of light (V-A) neutrinos: ν_e, ν_μ, ν_τ
- ν are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)

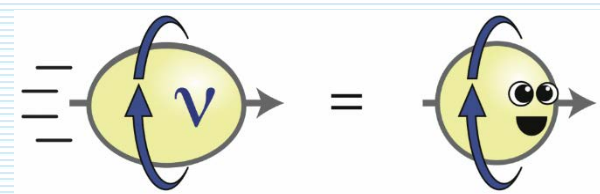


- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν?
- Sterile neutrinos?
- Statistical properties of ν? Fermionic or partly bosonic?

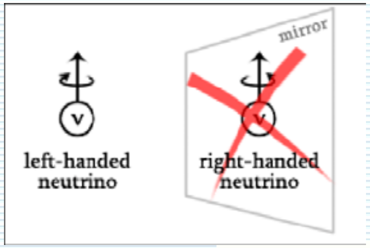


Currently main issue

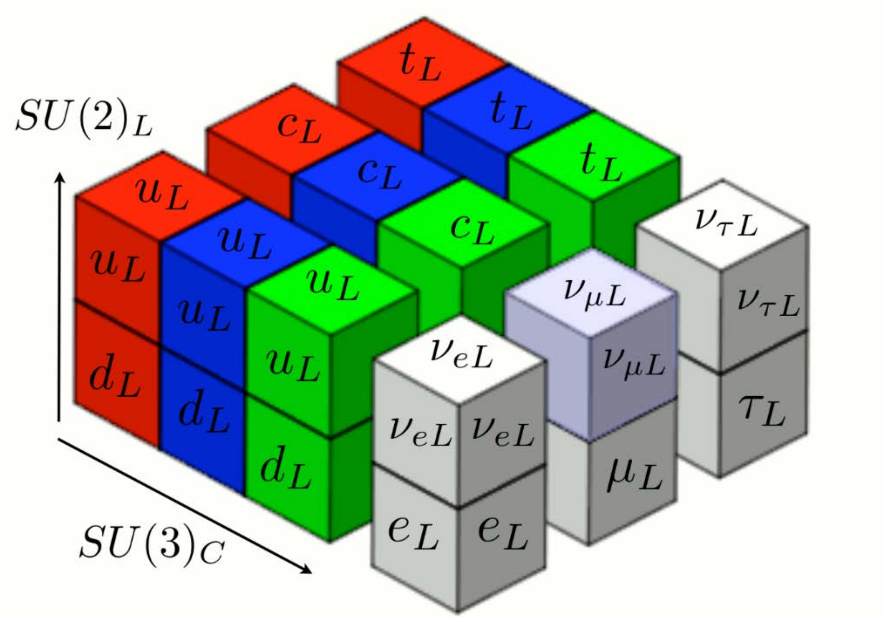
Nature, Mass hierarchy, CP-properties, sterile ν



The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties



Beyond the Standard model physics
(EFT scenario)



The absence of the right-handed neutrino fields in the SM is the simplest, most economical possibility. In such a scenario Majorana mass term is the only possibility for neutrinos to Be massive and mixed. This mass term is generated by the Lepton number violating Weinberg effective Lagrangian.

$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

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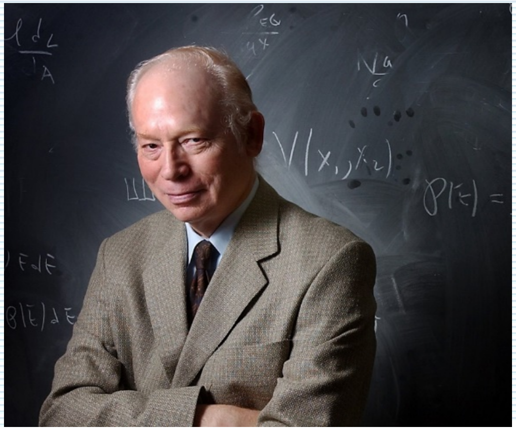
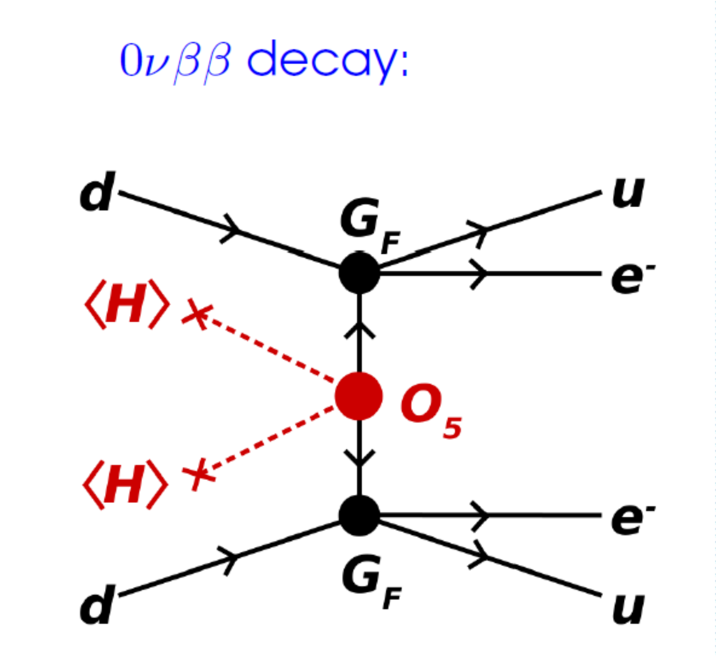


Weinberg, 1979: d=5



$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_i H)(L_j H)$$

. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. "We don't know anything about the details of those terms, but I'll swear they are there."

Majorana fermion

https://en.wikipedia.org/wiki/File:Ettore_Majorana.jpg

CNNP 2018, Catania, October 15-21, 2017

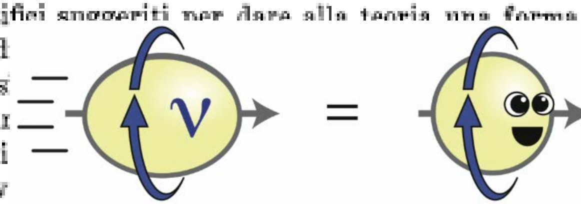
TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di Ettore Majorana

Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Sunto. - *Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di* Dirac *ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di « antiparticelle » corrispondenti ai « vuoti » di energia negativa.*

L'interpretazione dei cosidetti « stati di energia negativa » proposta da Dirac (¹) conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici suggeriti per dare alla teoria una forma simmetrica che si accord ... isfacenti; sia perchè s ... trica, sia perchè la sir ... iante tali procedimenti ... che possibilmente dov ... uova via che conduce più direttamente alla meta.



Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

(¹) P. A. M. Dirac, « Proc. Camb. Phil. Soc. », 30, 150, 1924. V. anche W. Heisenberg, « ZS. f. Phys. », 90, 209, 1934.

MESONIUM AND ANTIMESONIUM

B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor May 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 549-551 (August, 1957)

INVERSE BETA PROCESSES AND NONCONSERVATION OF LEPTON CHARGE

B. PONTECORVO

Joint Institute for Nuclear Research

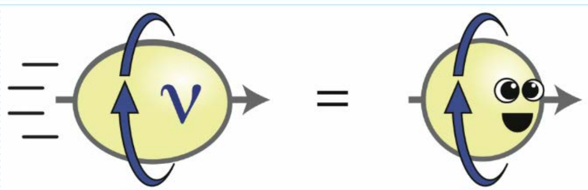
Submitted to JETP editor October 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 247-249 (January, 1958)

$\nu \leftrightarrow \bar{\nu}$ oscillation

(neutrinos are Majorana particles)

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.[5]



1968 Gribov, Pontecorvo [PLB 28(1969) 493]

oscillations of neutrinos - a solution of deficit of solar neutrinos in Homestake exp.

Observation of ν-oscillations = the first prove of the BSM physics

mass-squared differences: $\Delta m^2_{SUN} \cong 7.5\ 10^{-5}\ eV^2$, $\Delta m^2_{ATM} \cong 2.4\ 10^{-3}\ eV^2$

The observed small neutrino masses (limits from tritium β-decay, cosmology) have profound implications for our understanding of the Universe and are now a major focus in astro, particle and nuclear physics and in cosmology.

PMNS unitary mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

large off-diagonal values

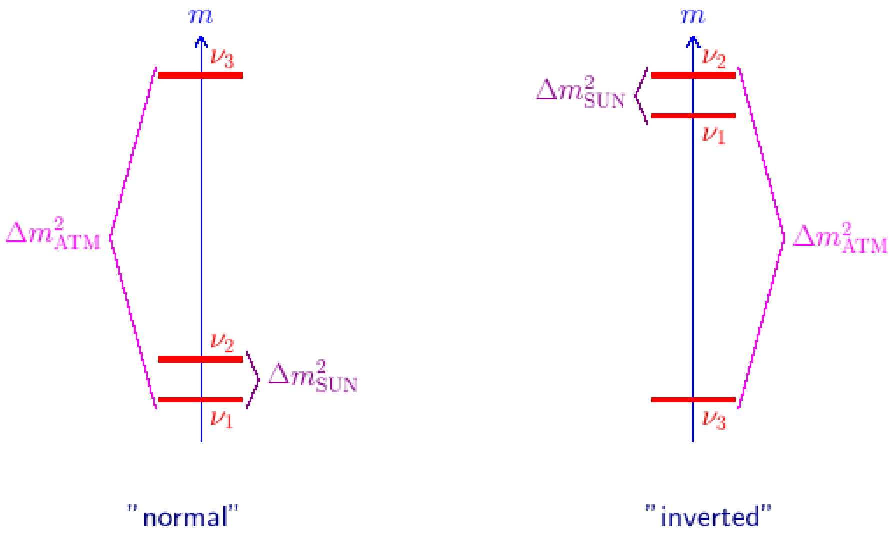
$$\begin{pmatrix} 0.82 & 0.54 & -0.15 \\ -0.35 & 0.70 & 0.62 \\ 0.44 & -0.45 & 0.77 \end{pmatrix}$$

3 angles: θ_{12}=33.36° (solar), θ_{13}=8.66° (reactor), θ_{23}=40.0° or 50.4° (atmospheric)

$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

unknown (CP violating) phases: δ, α_1, α_2

Neutrinos mass spectrum

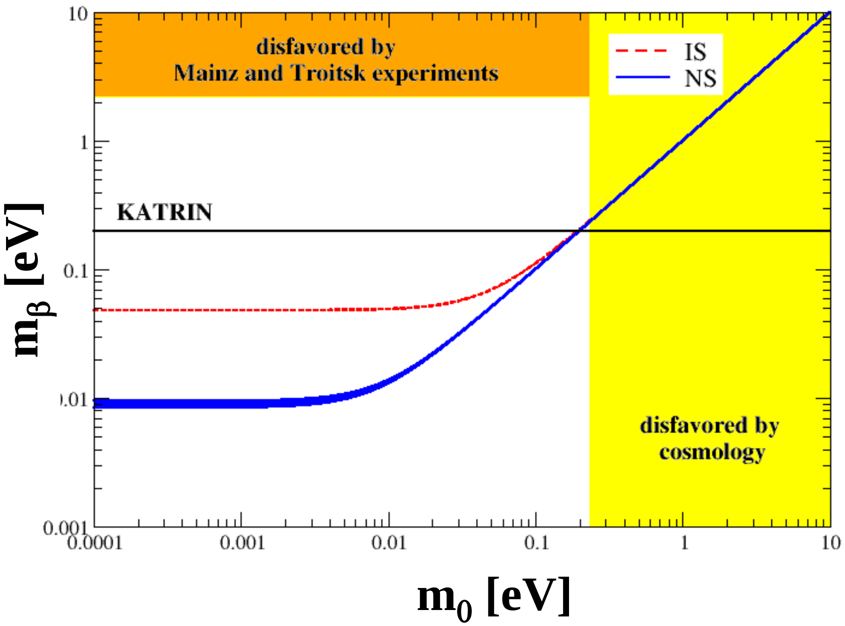


0νββ Measurements

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3\right|$$

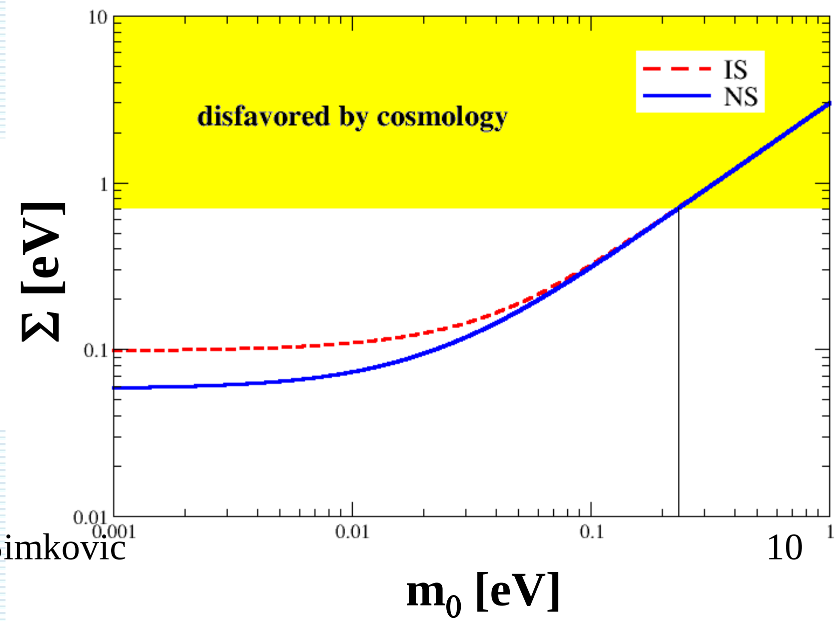
Beta Decay Measurements

$$m_\beta = \sqrt{c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2}$$



Cosmological Measurements

$$\Sigma = m_1 + m_2 + m_3$$

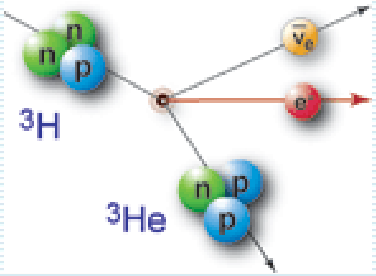


imkovic

II. Laboratory measurement of ν-mass

(tritium β–decay, forbidden β–decays, EC of ^{163}Ho)

Tritium beta decay: $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$

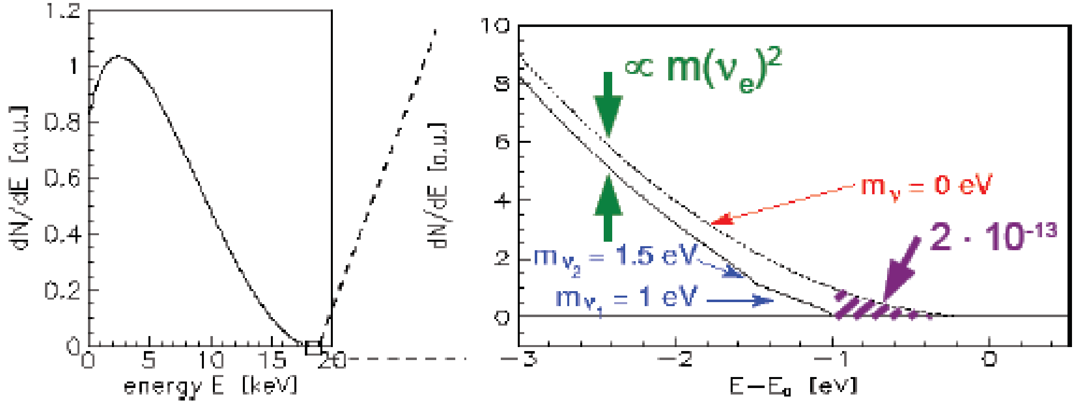


$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) pE (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

1934 – **Fermi** pointed out that shape of electron spectrum in β–decay near the endpoint is sensitive to **neutrino mass**

First measured by **Hanna** and **Pontecorvo** with estimation $m_\nu \sim 1$ keV [Phys. Rev. 75, 983 (1940)]

$Q = M_H - M_{He} - m_e$
$= 1858$ keV



Troitsk

$m_\nu^2 = -2.3 \pm 2.5 \pm 2.0$ eV^2

$m_\nu \leq$ 2.2 eV (95% CL.)

Mainz

$m_\nu^2 = -1.2 \pm 2.2 \pm 2.1$ eV^2

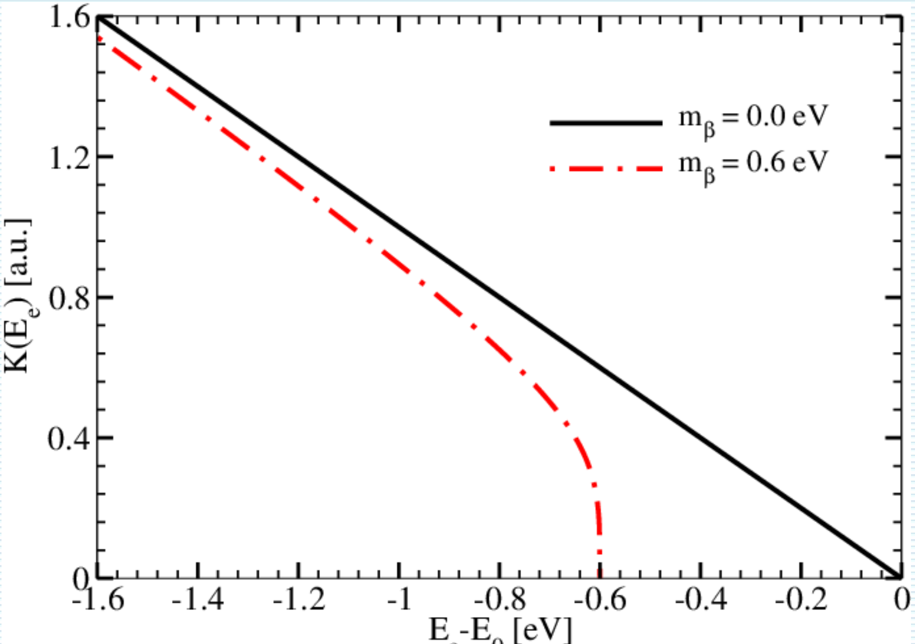
$m_\nu \leq$ 2.2 eV (95% CL.)

Kurie function

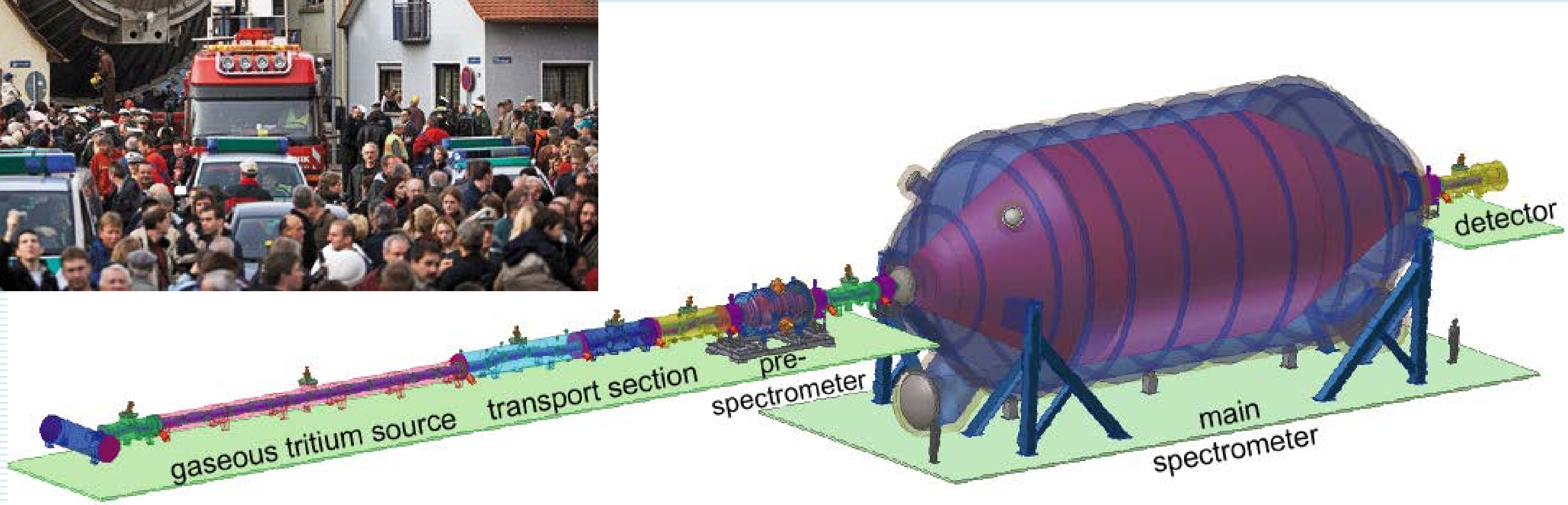
Franz Newell Devereux Kurie

$$K(E_e) = \sqrt{\frac{d\Gamma / dE_e}{p_e E_e F_0(Z_f, E_e)}} = \frac{G_\beta g_A |M|}{\sqrt{2\pi^3}} (E_0 - E_e) \sqrt[4]{1 - \frac{m_\beta^2}{(E_0 - E_e)^2}}$$

The advantage of Kurie plot is that non-linearity implies non-zero neutrino mass.



Karlsruhe TRItium Neutrino experiment (KATRIN)



$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 \; m_i^2}$$

Evidence for neutrino mass signal
KATRIN discovery potential:

m_β = 0.35 eV (5σ)
m_β = 0.30 eV (3σ)

No neutrino mass signal
KATRIN sensitivity

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2} < 0.2 \; eV \qquad m_\beta \approx m_1$$

Relativistic approach to ^3H decay
nuclear recoil (3.4 eV) taken into account

Standard approach

- non-relativistic nuclear w.f.
- nuclear recoil neglected
- phase space analysis

$$E_e^{max} = M_i - M_f - m_\nu$$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\,(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

Relativistic EPT approach (Primakoff)

- Analogy with n-decay
  (^3H,^3He) $\leftrightarrow$ (n,p)
- nuclear recoil of 3.4 eV by E_e^{max}
- relevant only phase space

$$E_e^{max} = \frac{1}{2M_f}\left[M_i^2 + m_e^2 - \left(M_f^2 - m_\nu^2\right)\right]$$

$$\begin{aligned}
\frac{d\Gamma}{dE_e} &= \frac{1}{(\pi)^3}(G_F \cos\theta_c)^2 F(Z,E_e) p_e \\
&\times \frac{M_i^2}{(m_{12})^2}\sqrt{y\left(y + 2m_\nu \frac{M_f}{M_i}\right)} \\
&\times \Bigg[(g_V+g_A)^2 y\left(y + m_\nu \frac{M_f}{M_i}\right)\frac{M_i^2(E_e^2 - m_e^2)}{3(m_{12})^4} \\
&\quad (g_V+g_A)^2 (y + m_\nu \frac{M_f + m_\nu}{M_i})\frac{(M_i E_e - m_e^2)}{m_{12}^2} \\
&\quad \times (y + M_f \frac{M_f + m_\nu}{M_i})\frac{(M_i^2 - M_i E_e)}{m_{12}^2} \\
&\quad - (g_V^2 - g_A^2) M_f \left(y + m_\nu \frac{(M_f + M_\nu)}{M_i}\right) \\
&\quad \times \frac{(M_i E_e - m_e^2)}{(m_{12})^2} \\
&\quad + (g_V - g_A)^2 E_e \left(y + m_\nu \frac{M_f}{M_i}\right)\Bigg]
\end{aligned}$$

$$\begin{aligned}
y &= E_e^{max} - E_e \\
(m_{12})^2 &= M_i^2 - 2M_i E_e + m_e^2
\end{aligned}$$

Numerics:
Practically the same dependence of Kurie function on m_ν for $E_e \approx E_e^{max}$

F.Š., R. Dvornický, A. Faessler, PRC 77 (2008) 055502

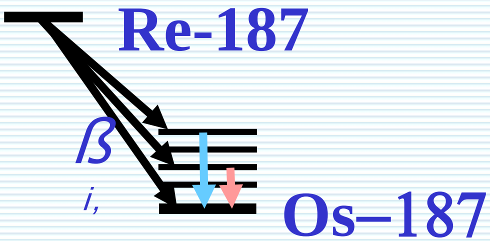
First unique forbidden beta-decays

$$\Delta J^{\pi} = 2^{-}$$

Nucleus:	^{79}Se	$^{93}Zr^{*}$	^{107}Pd	$^{135}Cs^{*}$	^{182}Hf	^{187}Re
Q[keV]:	151	60	34.1	0.5	104.6	2.469

*** - decay to the excited nuclear state**

Bolometer experiments for ^{187}Re



■ Rhenium experiments (MANU, MIBETA, MARE)

^{187}Re as ß-emitter: natural isotope content = 62.8 %

^{187}Re: unique 1st	
E_0	2.47 keV
$t_{1/2}$	$4.35\ 10^{10}$ y

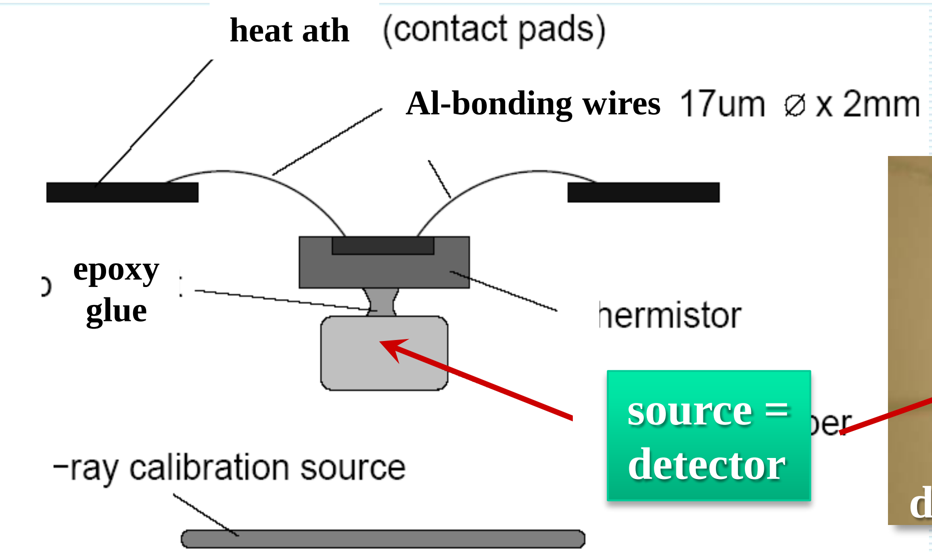
$$^{187}_{85}Re \rightarrow ^{187}_{86}Os + e^- + \bar{\nu}_e$$

$5/2^+ \rightarrow 1/2^-$ ´unique´ 1st forbidden transition (shape factor), BEFS

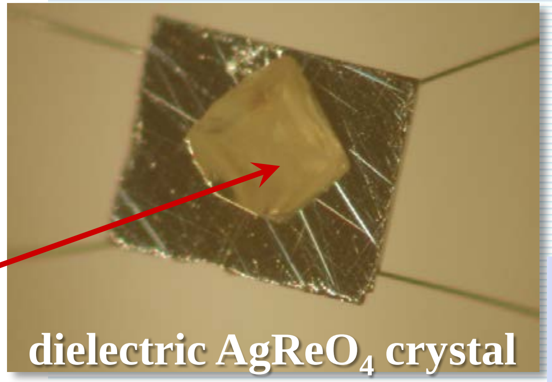
■ previous ^{187}Re-experiments MANU, MIBETA

MANU: metallic Rhenium — group in Genova

MIBETA: dielectric $AgReO_4$ crystals — group in Milano



measure entire ß-decay energy



The entire energy is measured in the detector, except the neutrino, including the molecular & atomic excitations

MIBETA: 10 crystals

Spectrum of emitted electrons in rhenium β-decay

Dvornický, F. Š., Muto, Faessler, PRC 83, 045502 (2011)

$$\frac{d\Gamma}{dE} = \frac{G_F^2 V_{ud}^2}{2\pi^3} |M|^2 pE(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}\, \frac{1}{3} R^2 \left(p^2 F_1(Z,E) + k^2 F_0(Z,E) \right)$$

$$k = \sqrt{(E_0 - E)^2 - m_\nu^2}$$

Electron $p_{3/2}$ decay channel clearly dominates

$$\Gamma_S / \Gamma_P = 1.011 \times 10^{-4}$$

In agreement with

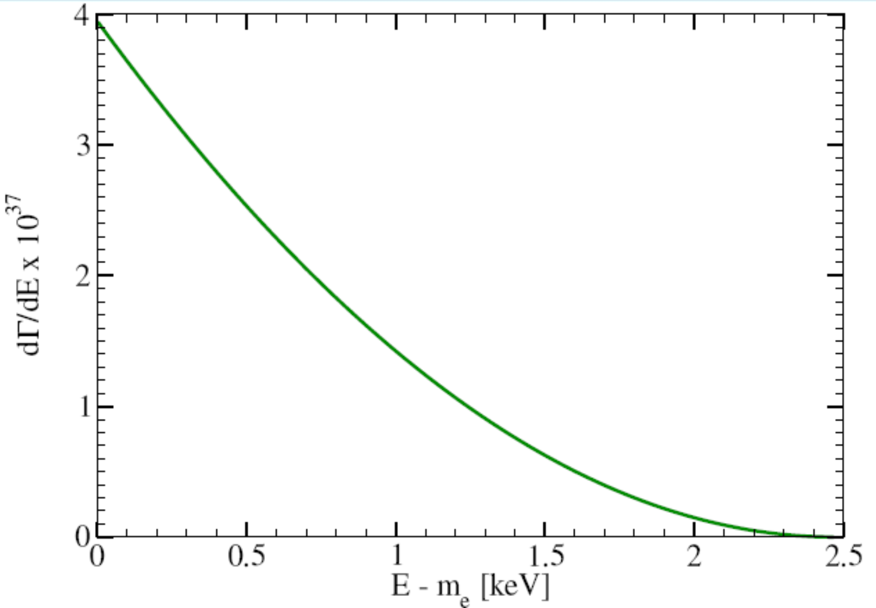
Arnaboldi et al.: PRL 96, 042503 (2006)

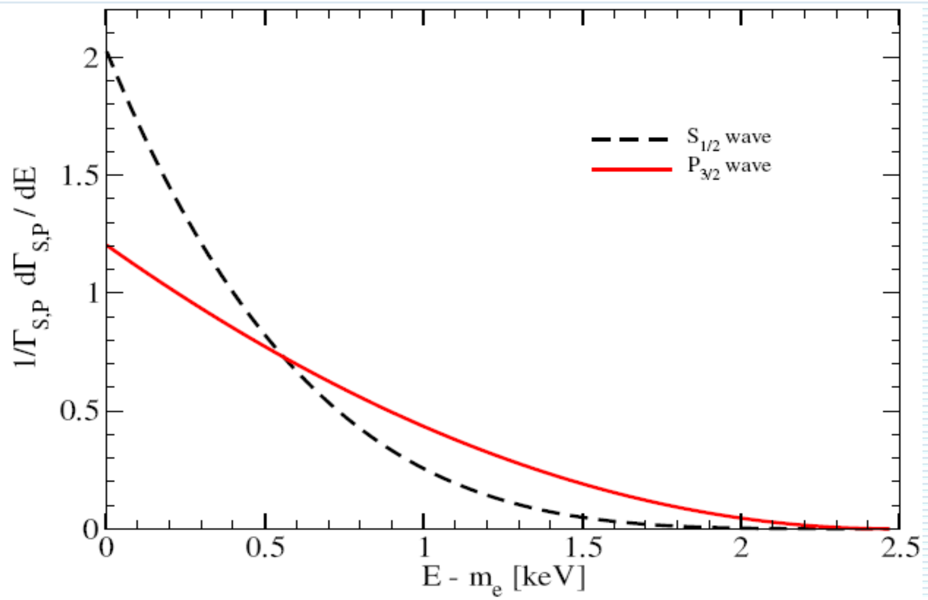
Electron in the $p_{3/2}$ state

$$p^{max} \cong 50 keV$$

Electron in the $s_{1/2}$ state

$$k^{max} = 2.47 keV$$



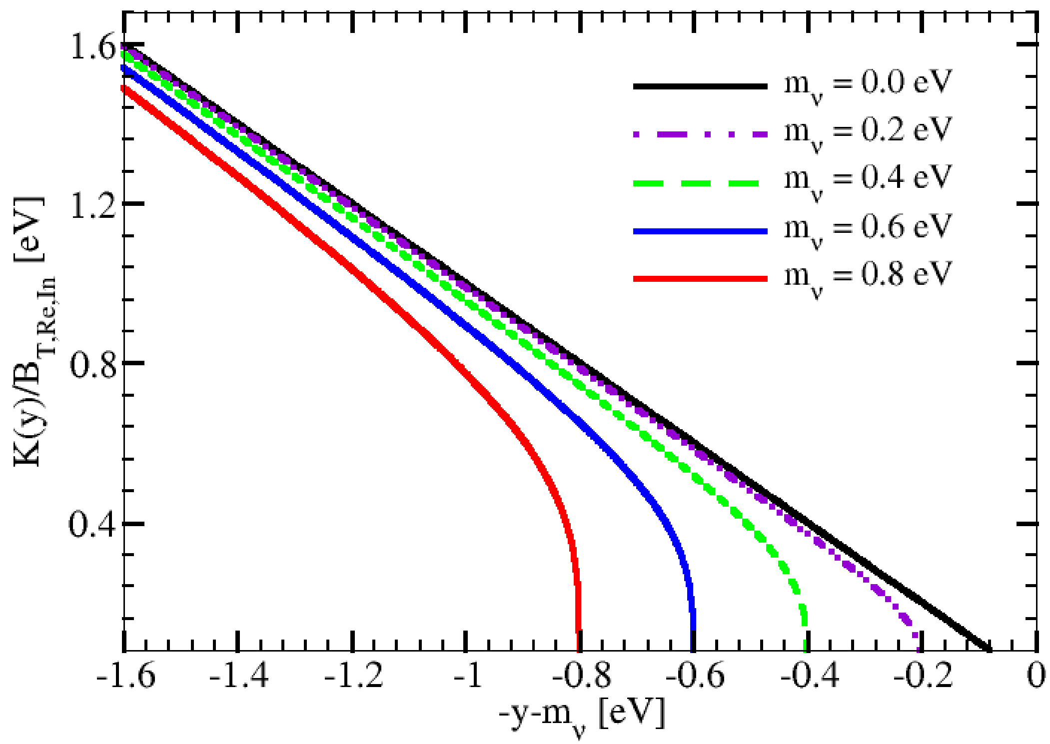


Kurie plots for tritium, rhenium and indium single β-decay

$$p^2 \frac{F_1(Z,E)}{F_0(Z,E)} \cong 1 + 2\frac{E - m_e}{m_e} \cong 1$$

$K(E)/B_{Re}, K(E)/B_{In} \cong K(y)/B_T$

Normalized Kurie functions become identical



Measuring ν-mass with electron-capture of ^{163}Ho - ECHO exp.

From ν phase space

$$\frac{d\Gamma}{dE_c} \propto (Q - E_c)\sqrt{(Q - E_c)^2 - m_\nu^2}$$

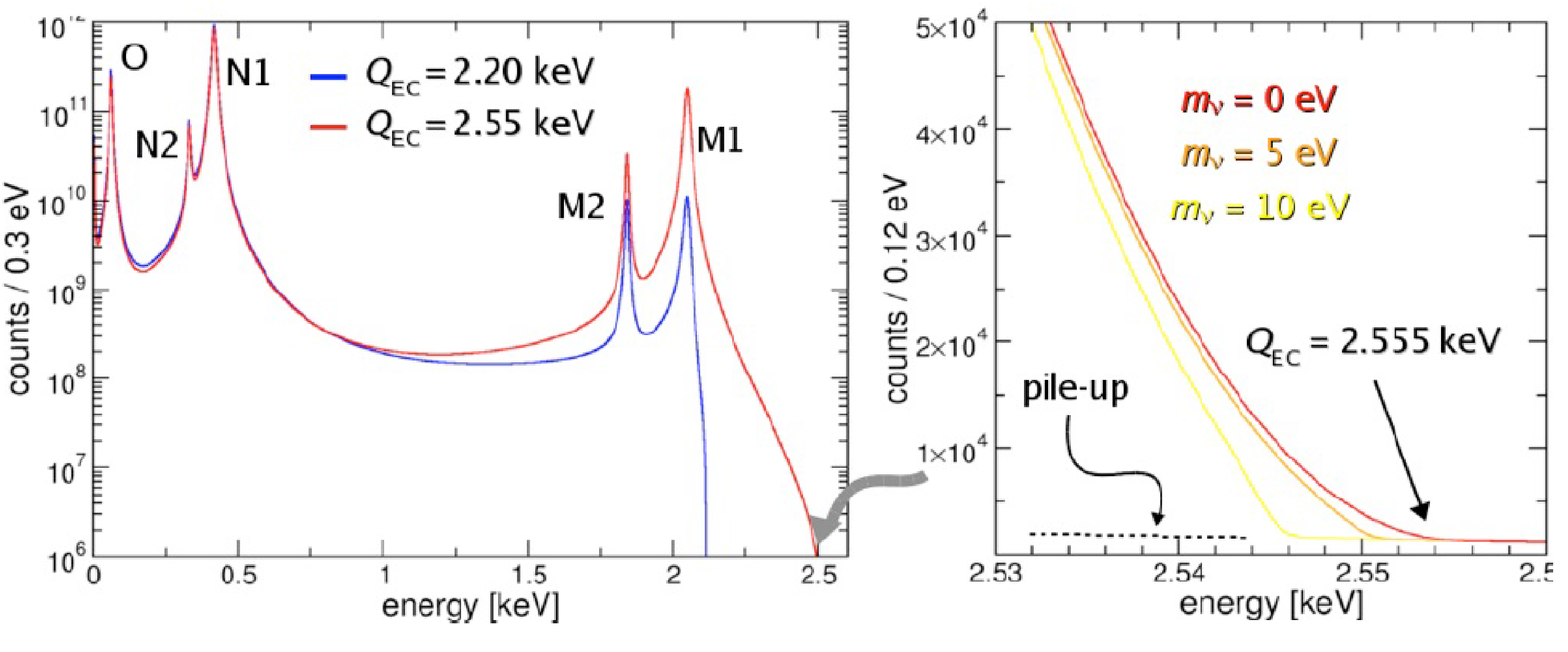
$$* \sum_{\mathrm{H}} \varphi_{\mathrm{H}}^2(0)\, B_{\mathrm{H}} \frac{\Gamma_{\mathrm{H}}}{2\pi} \frac{1}{(E_c - E_{\mathrm{H}})^2 + \Gamma_{\mathrm{H}}^2/4}$$

$$\Longrightarrow \mathcal{K}\,(Q - E_c)\sqrt{(Q - E_c)^2 - m_\nu^2}\,,$$

Not much progress in theory for a long period

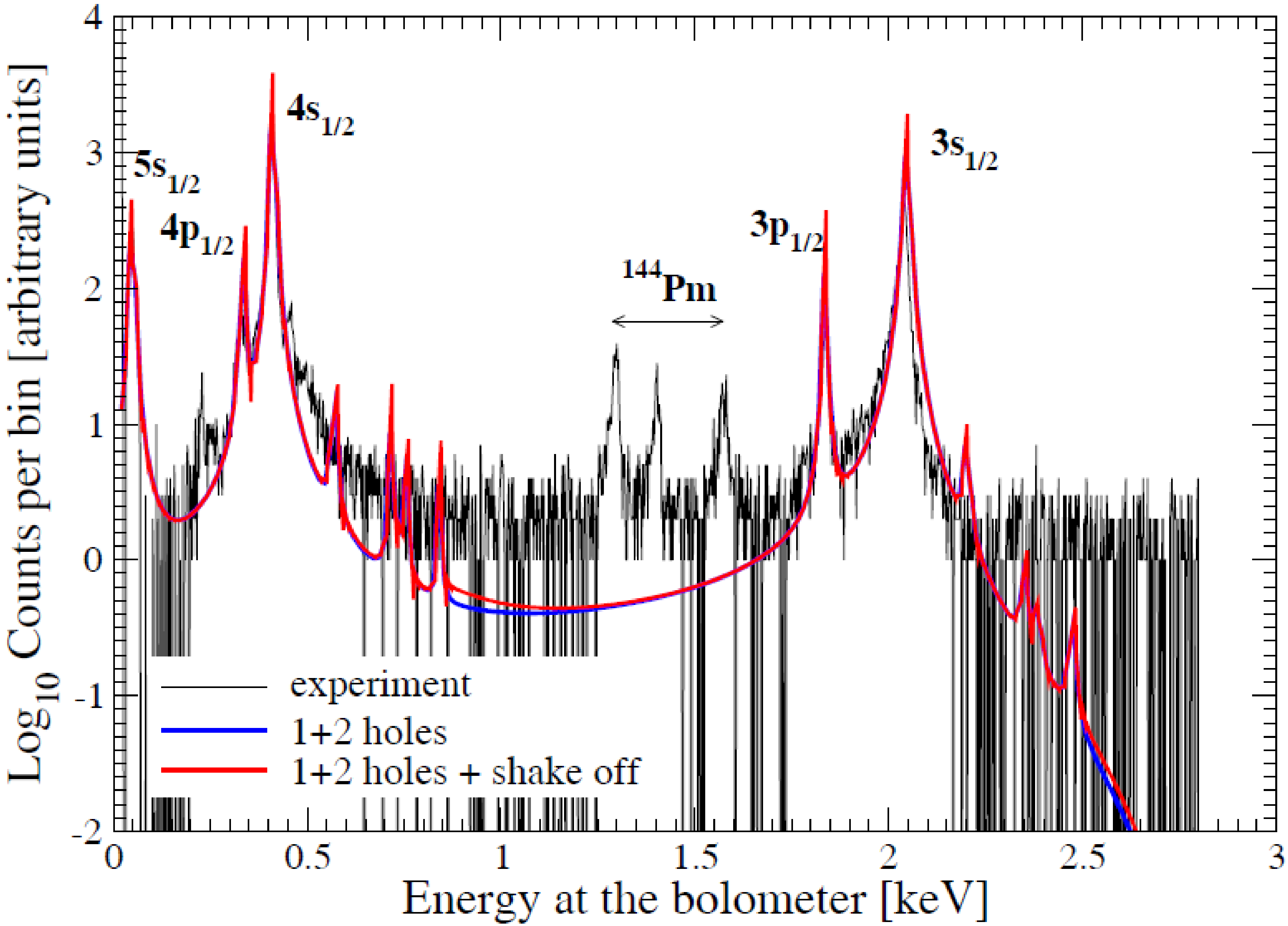
$$E_c = Q - m_\nu$$

2 keV/Γ_M= 2keV/13 eV≈ 100



1+2 holes and shake-off effect

A. Faessler, Ch. Enss, L. Gastaldo, F.Š., PRC 91, 064302 (2015) (two and 3 holes)
A. Faessler, L. Gastaldo, F.Š., PRC 95, 045502 (2017) (shake off)



III. Theory of 0νββ-decay

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

What is the nature of neutrinos?

ν $\Rightarrow$

GUT's

Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Only the 0νββ-decay can answer this fundamental question

Analogy with kaons: K_0 and $\bar{K}_0$

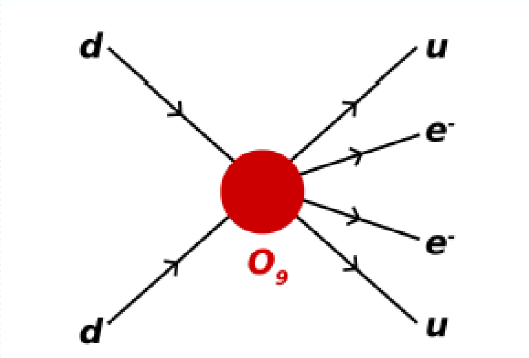
Fedor Simkovic

Analogy with π_0

Amplitude for (A,Z)→(A,Z+2)+2e⁻ can be divided into:

M. Hirsch, Pontecorvo school 2015

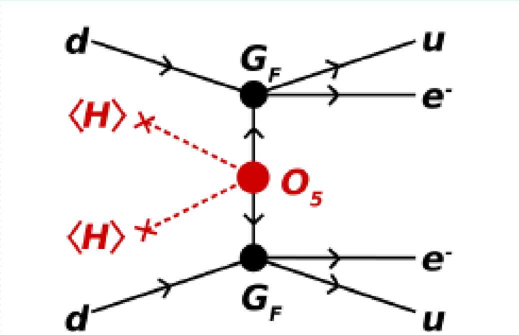
short range: d=9 (d=11)



mass mechanism: d=5

long range: d=7

+



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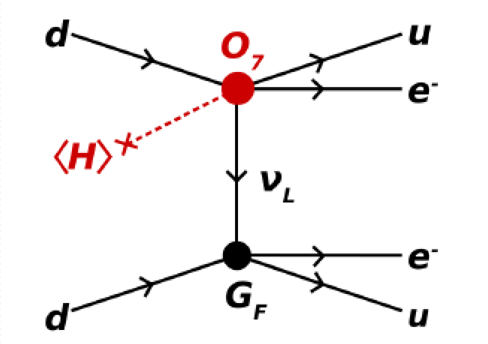


$$\mathcal{O}_5 \propto LLQd^cHHH^\dagger$$

$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^cHH^\dagger H$$

$$\mathcal{O}_7 \propto LQ\bar{e}^c\bar{Q}HHH^\dagger$$

$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_iH)(L_jH)$$

Weinberg, 1979

$$\mathcal{O}_2 \propto LLLe^cH$$

$$\mathcal{O}_3 \propto LLQd^cH$$

$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^cH$$

$$\mathcal{O}_8 \propto L\bar{e}^c\bar{u}^cd^cH$$

Babu, Leung: 2001
de Gouvea, Jenkins: 2007

$$\mathcal{O}_9 \propto LLLe^cLe^c$$

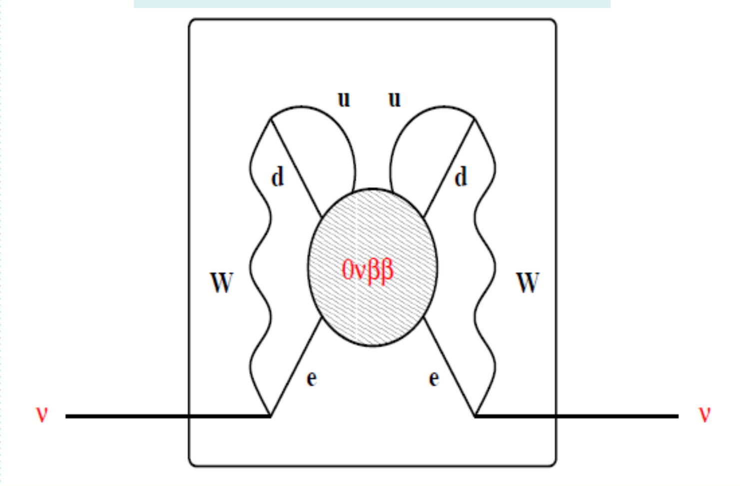
$$\mathcal{O}_{10} \propto LLLe^cQd^c$$

$$\mathcal{O}_{11} \propto LLQd^cQd^c$$

$$\cdots\cdots$$

Physics at LHC (Jose Valle talk)

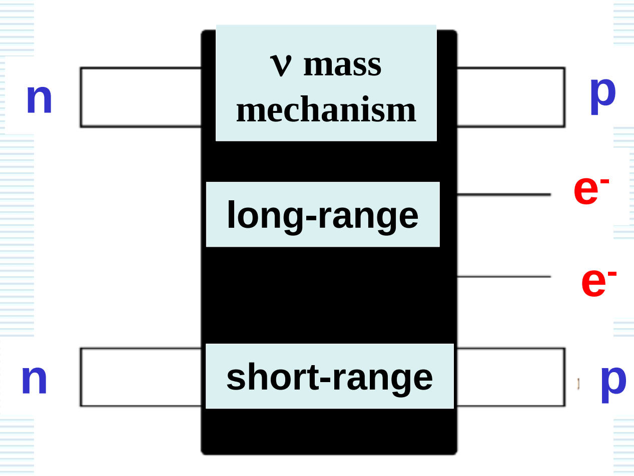
Schechter, Valle: PRD 1982



If 0νββ is observed the ν is a Majorana particle

II. Different 0νββ-decay scenarios

Can we say something about content of the black box?

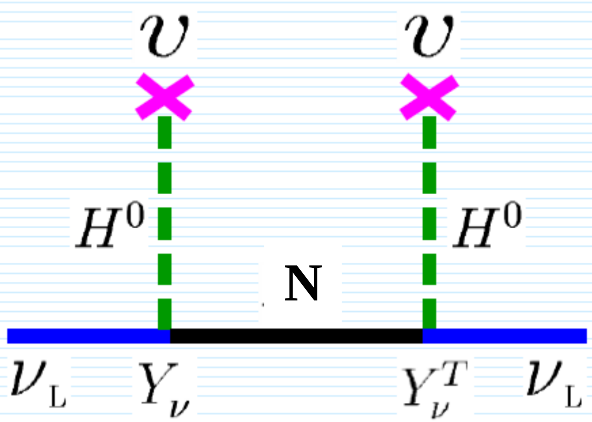


Considering

1. i. Sterile ν
2. ii. Different LNV scales
3. iii. Right-handed currents
4. iv. Non-standard ν-interactions

I.a. *The simplest $0\nu\beta\beta$-decay scenario: LHC & LNV scale Λ is too large*

$$\mathcal{L}_5^{eff} = -\frac{1}{\Lambda}\sum_{l_1 l_2}\left(\overline{\Psi}^{lep}_{l_1 L}\tilde{\Phi}\right)\acute{Y}_{l_1 l_2}\left(\tilde{\Phi}^T(\Psi^{lep}_{l_2 L})^c\right)$$



Heavy Majorana leptons N_i ($N_i = N^c_i$) singlet of $SU(2)_L x U(1)_Y$ group Yukawa lepton number violating int.

$$m_i = \frac{v}{\Lambda}(y_i v), \quad i = 1, 2, 3$$

$\Lambda \geq 10^{15}$ GeV

S.M. Bilenky, Phys.Part.Nucl.Lett. 12 (2015) 453-461

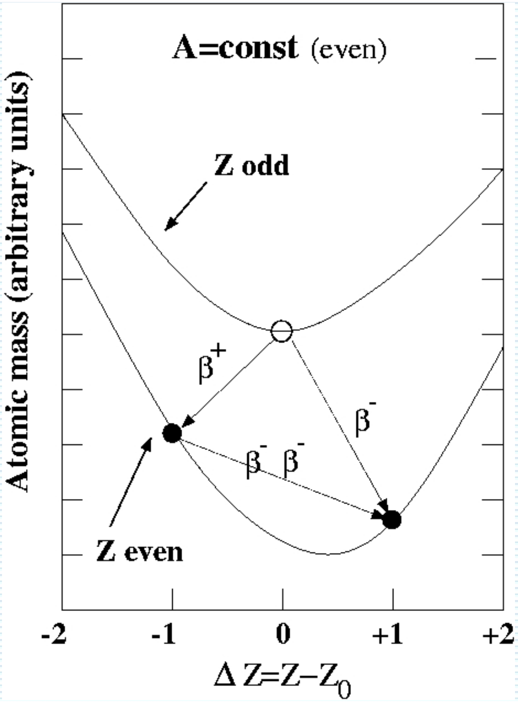
The three Majorana neutrino masses are suppressed by the ratio of the electroweak scale and a scale of a lepton-number violating physics.

The discovery of the ββ-decay and absence of transitions of flavor neutrinos into sterile states would be evidence in favor of this minimal scenario.

(A,Z) → (A,Z+2) + e⁻ + e⁻

$$\left(T^{0\nu}_{1/2}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M^{0\nu}_{\nu}\right|^2 G^{0\nu}$$

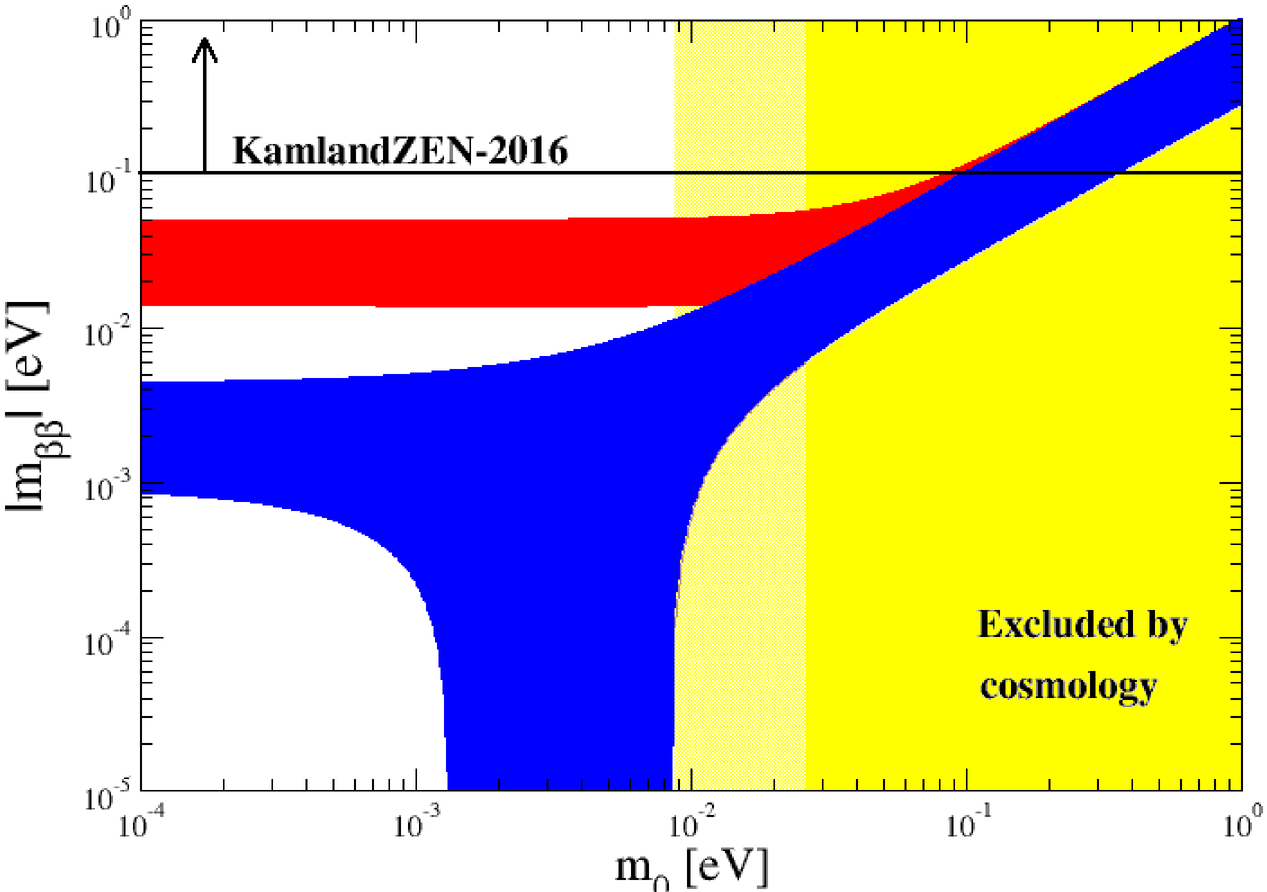
$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3\right|$$



transition	$G^{01}(E_0, Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\|M^{0\nu}\|^2$
$^{150}Nd \to {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \to {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \to {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \to {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \to {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \to {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \to {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \to {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \to {}^{76}Se$	0.79	2.040	8	?

The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

Effective mass of Majorana neutrinos



GUT's

m_1, m_2, m_3, θ_{12}, θ_{13}, α_1, α_2
(3 unknown parameters)

Complementarity of $0\nu\beta\beta$-decay, β-decay and cosmology

β-decay (Mainz, Troitsk)

$$m_\beta^2 = \sum_i |U_{ei}^L|^2 m_i^2 \leq (2.2 \text{ eV})^2$$

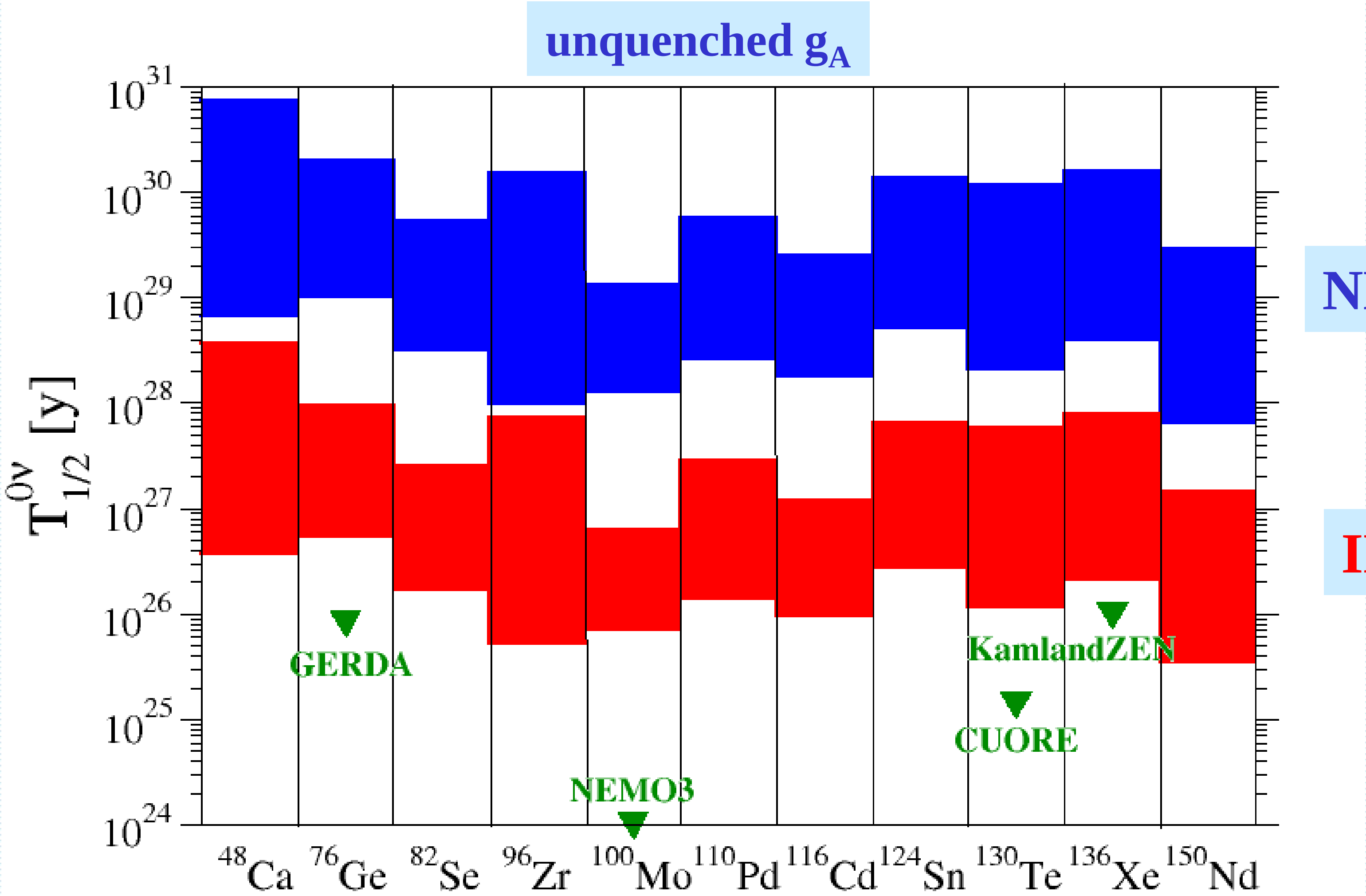
KATRIN: $(0.2 \text{ eV})^2$

Cosmology (Planck)

$$\Sigma < 110 \text{ meV}$$

$$m_0 > 26 \text{ meV (NS)}$$
$$87 \text{ meV (IS)}$$

0νββ –half lives for NH and IH with included undertainties in NMEe



NH: $m_1 \ll m_2 \ll m_3 \quad m_3 \simeq \sqrt{\Delta m^2}$

$m_1 \ll \sqrt{\delta m^2}. \quad m_2 \simeq \sqrt{\delta m^2}$

$1.4 \text{ meV} \le m_{\beta\beta} \le 3.6 \text{ meV}$

IH: $m_3 \ll m_1 < m_2 \quad m_1 \simeq m_2 \simeq \sqrt{\Delta m^2}$

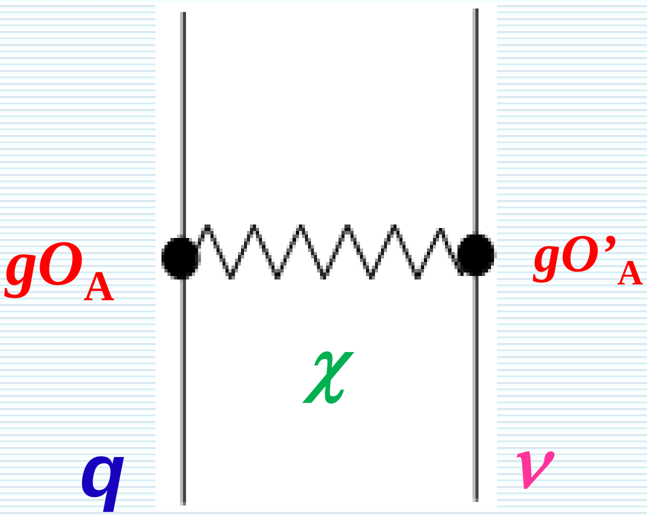
$m_3 \ll \sqrt{\Delta m^2}$

Lightest ν-mass equal to zero

$20 \text{ meV} \le m_{\beta\beta} \le 49 \text{ meV}$

Nuclear medium effect on the light neutrino mass exchange mechanism of the $0\nu\beta\beta$-decay

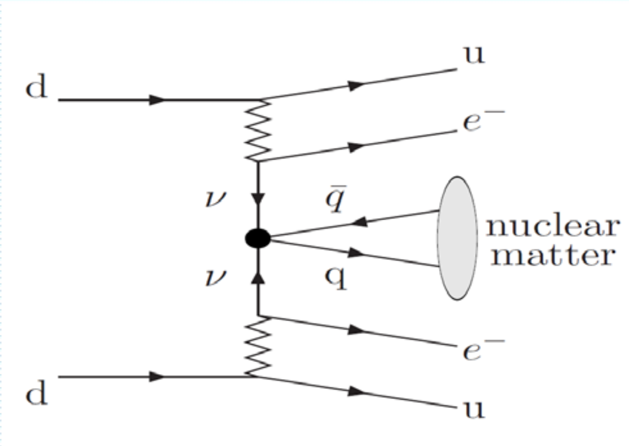
S.G. Kovalenko, M.I. Krivoruchenko, F. Š., Phys. Rev. Lett. 112 (2014) 142503



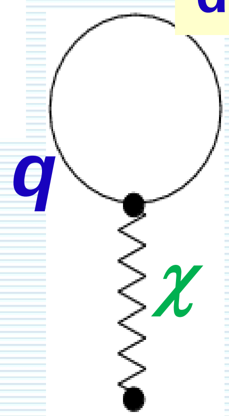
➡

Low energy 4-fermion
ΔL ≠ 0 Lagrangian

$$L_{\rm eff} = \frac{g^2}{m_\chi^2}\sum_A (\bar{q}O_A q)(\bar{\nu}O'_A \nu), \qquad m_\chi \gtrsim M_W .$$



density



oscillation experiments
tritium β-decay, cosmology

$$\Sigma_\nu^{\rm vac} = -\!\times\!-,$$

0νββ-decay

$$\Sigma_\nu^{\rm medium} = -\!\times\!- \; +$$

Mean field: $\bar{q}q \to \langle \bar{q}q \rangle$ **and** $\langle \bar{q}q \rangle \approx 0.5 \langle q^{\dagger} q \rangle \approx 0.25\,\mathrm{fm}^{-3}$

The effect depends on $\langle \chi \rangle = -\frac{g_\chi}{m_\chi^2} \langle \bar{q}q \rangle$

A comparison with G_F:

$$\frac{g_\chi g_{ij}^a}{m_\chi^2} = \frac{G_F}{\sqrt{2}} \varepsilon_{ij}^a$$

Typical scale: $\langle \chi \rangle g_{ij}^a = -\frac{G_F}{\sqrt{2}} \langle \bar{q}q \rangle \varepsilon_{ij}^a \approx -25\, \varepsilon_{ij}^a\ \mathrm{eV}$

We expect:

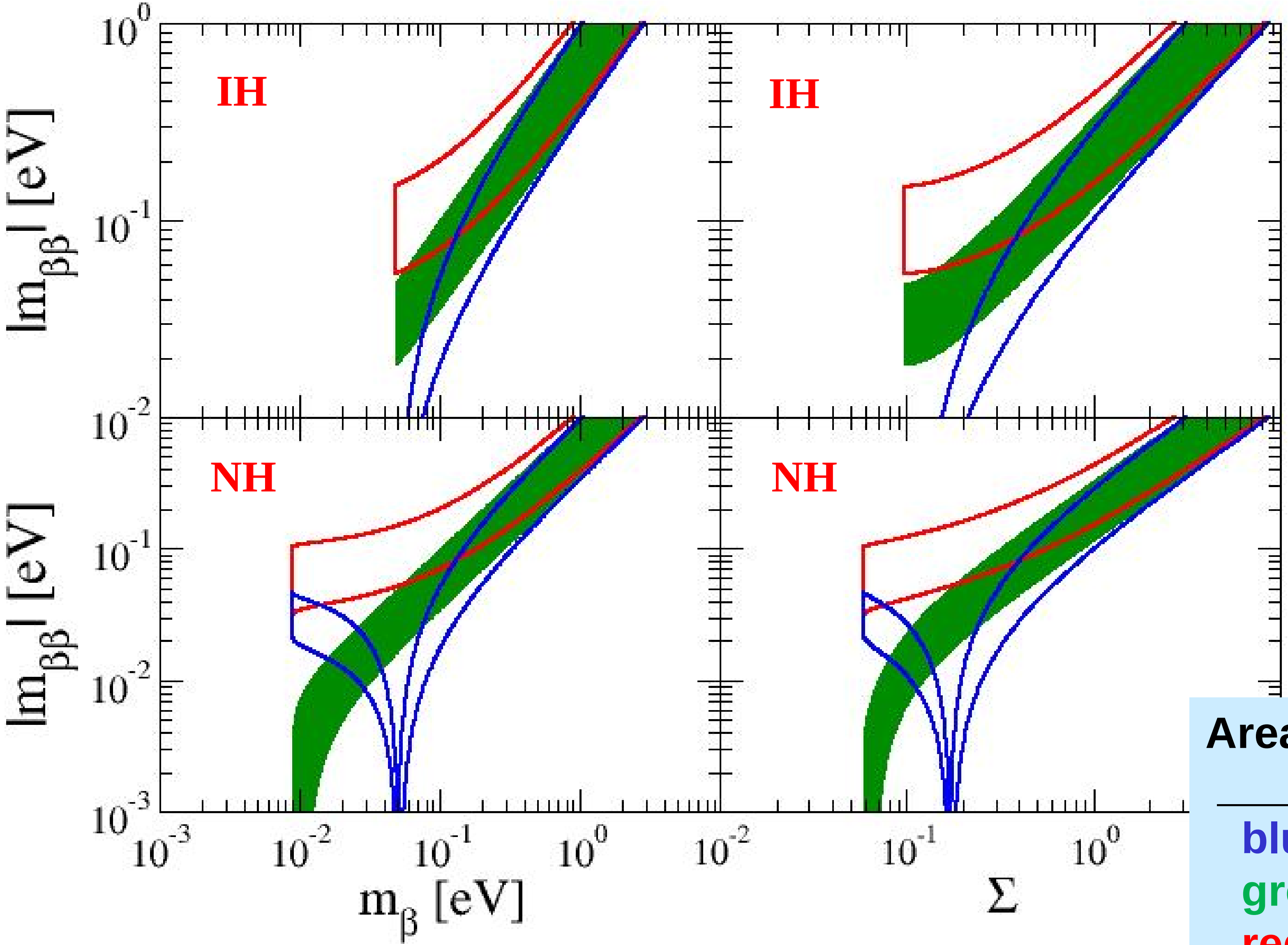
$$25\, \varepsilon_{ij}^a < 1 \to m_\chi^2 > 25 \frac{g_\chi g_{ij}^a \sqrt{2}}{G_F} \sim 1\,\mathrm{TeV}^2$$

Universal scalar interaction $g_{ij}^a = \delta_{ij} g_a \quad \varepsilon_{ij}^a = \delta_{ij} \varepsilon_a$

In medium effective Majorana ν mass

$$m_{\beta\beta} = \sum_{i=1}^{n} U_{ei}^2 \xi_i \frac{\sqrt{(m_i + \langle \chi \rangle g_1)^2 + (\langle \chi \rangle g_2)^2}}{(1 - \langle \chi \rangle g_4)^2}.$$

Complementarity between β-decay, 0νββ –decay and cosmological measurements might be spoiled



Area	$\langle\chi\rangle g_1$ [eV]
blue	-0.05
green	0
red	1

II.b. *The sterile ν mechanism of the 0νββ-decay*
(D-M mass term, V-A,SM int.)
Interpolating formula

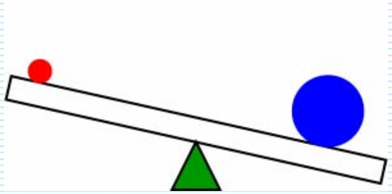
Dirac-Majorana mass term

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha}\, \nu_\alpha$$

Mixing of active-sterile neutrinos

small ν masses due to see-saw mechanism

$$\begin{pmatrix} 0 & m_D \\ m_D & m_{LNV} \end{pmatrix}$$



Light ν mass ≈ $(m_D/m_{LNV})\, m_D$
Heavy ν mass ≈ m_{LNV}

Neutrinos masses offer a great opportunity to jump beyond the EW framework via see-saw ...

Different motivations for the LNV scale Λ

Talk of Carlo Giunti

eV	keV	Fermi	TeV	GUT	Planck
light sterile ν	hot DM		LHC		
10^{-6} GeV	10^{-6} GeV	10^{-6} GeV	10^{3} GeV	10^{16} GeV	10^{19} GeV

r Si

Left-handed neutrinos: Majorana neutrino mass eigenstate N with arbitrary mass m_N

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g^4_{\rm A} \left| \sum_{\rm N} \left(U^2_{e\rm N} m_{\rm N} \right) m_{\rm p} \, M'^{\,0\nu}(m_{\rm N}, g^{\rm eff}_{\rm A}) \right|^2$$

General case

$$M'^{\,0\nu}(m_{\rm N}, g^{\rm eff}_{\rm A}) = \frac{1}{m_{\rm p} m_{\rm e}} \frac{R}{2\pi^2 g^2_A} \sum_n \int d^3x \, d^3y \, d^3p \times e^{i{\rm p}\cdot({\bf x}-{\bf y})} \frac{\langle 0^+_F | J^{\mu\dagger}({\bf x}) | n \rangle \langle n | J^\dagger_\mu({\bf y}) | 0^+_I \rangle}{\sqrt{p^2 + m_N{}^2}\left(\sqrt{p^2 + m_N{}^2} + E_n - \frac{E_I - E_F}{2}\right)}$$

light ν exchange

$$M'^{0\nu}(m_{\rm N} \to 0, g^{\rm eff}_{\rm A}) = \frac{1}{m_{\rm p} m_{\rm e}} M'^{0\nu}_\nu(g^{\rm eff}_{\rm A})$$

$$M'^{0\nu}(m_{\rm N} \to \infty, g^{\rm eff}_{\rm A}) = \frac{1}{m^2_{\rm N}} M'^{0\nu}_{\rm N}(g^{\rm eff}_{\rm A})$$

heavy ν exchange

Particular cases

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g^4_{\rm A} \times \times \begin{cases} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M'^{0\nu}_\nu(g^{\rm eff}_{\rm A}) \right|^2 & \text{for } m_{\rm N} \ll p_{\rm F} \\ \left| \langle \frac{1}{m_{\rm N}} \rangle m_{\rm p} \right|^2 \left| M'^{0\nu}_{\rm N}(g^{\rm eff}_{\rm A}) \right|^2 & \text{for } m_{\rm N} \gg p_{\rm F} \end{cases}$$

$$\langle m_\nu \rangle = \sum_{\rm N} U^2_{e\rm N} m_{\rm N}$$

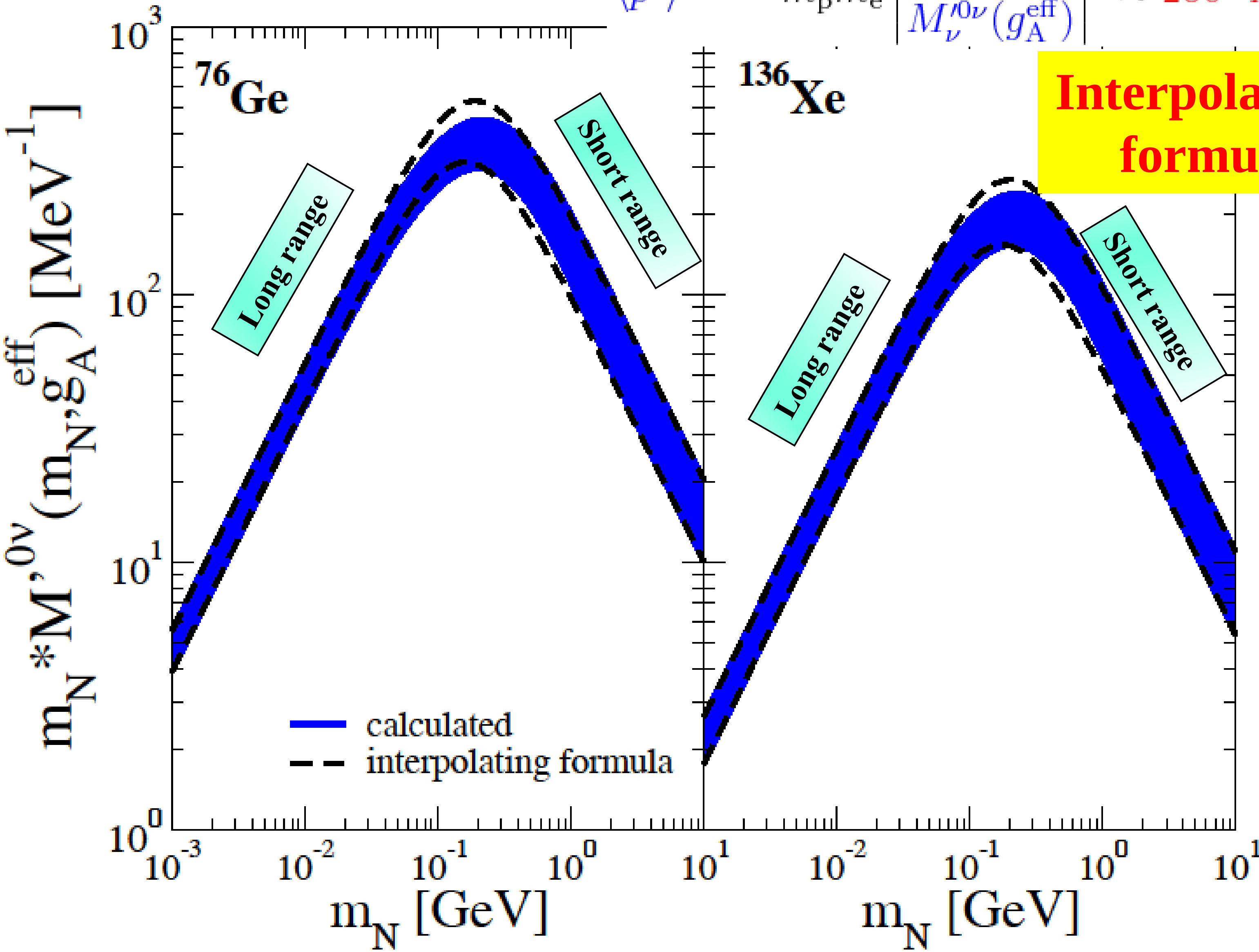
$$\left\langle \frac{1}{m_{\rm N}} \right\rangle = \sum_{\rm N} \frac{U^2_{e\rm N}}{m_{\rm N}}$$

$$[T^{0\nu}_{1/2}]^{-1} = \mathcal{A} \cdot \left| m_p \sum_N U^2_{eN} \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2,$$

$$\mathcal{A} = G^{0\nu} g_A^4 \left| M'^{0\nu}_N(g_A^{\text{eff}}) \right|^2,$$

$$\langle p^2 \rangle = m_p m_e \left| \frac{M'^{0\nu}_N(g_A^{\text{eff}})}{M'^{0\nu}_\nu(g_A^{\text{eff}})} \right| \approx 200\ MeV$$

Interpolating formula



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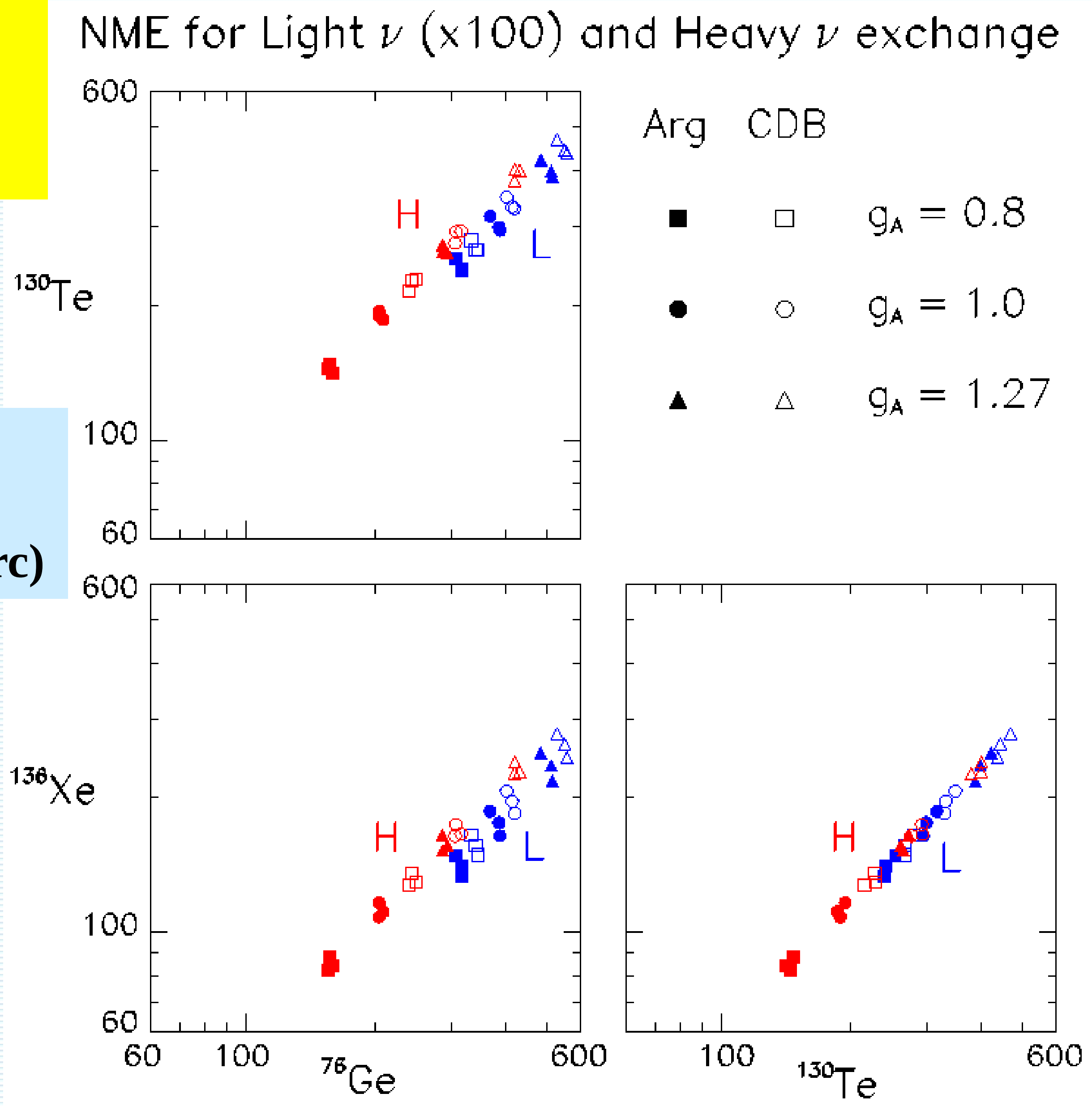
Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]

The light and heavy neutrino exchange are basically degenerate with the NME scaling factor
(^{76}Ge, ^{130}Te, ^{136}Xe)

Sqrt(<p^2>$_a$) =
175(11) MeV (Arg. src)
205(13) MeV (CDBonn src)

A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)



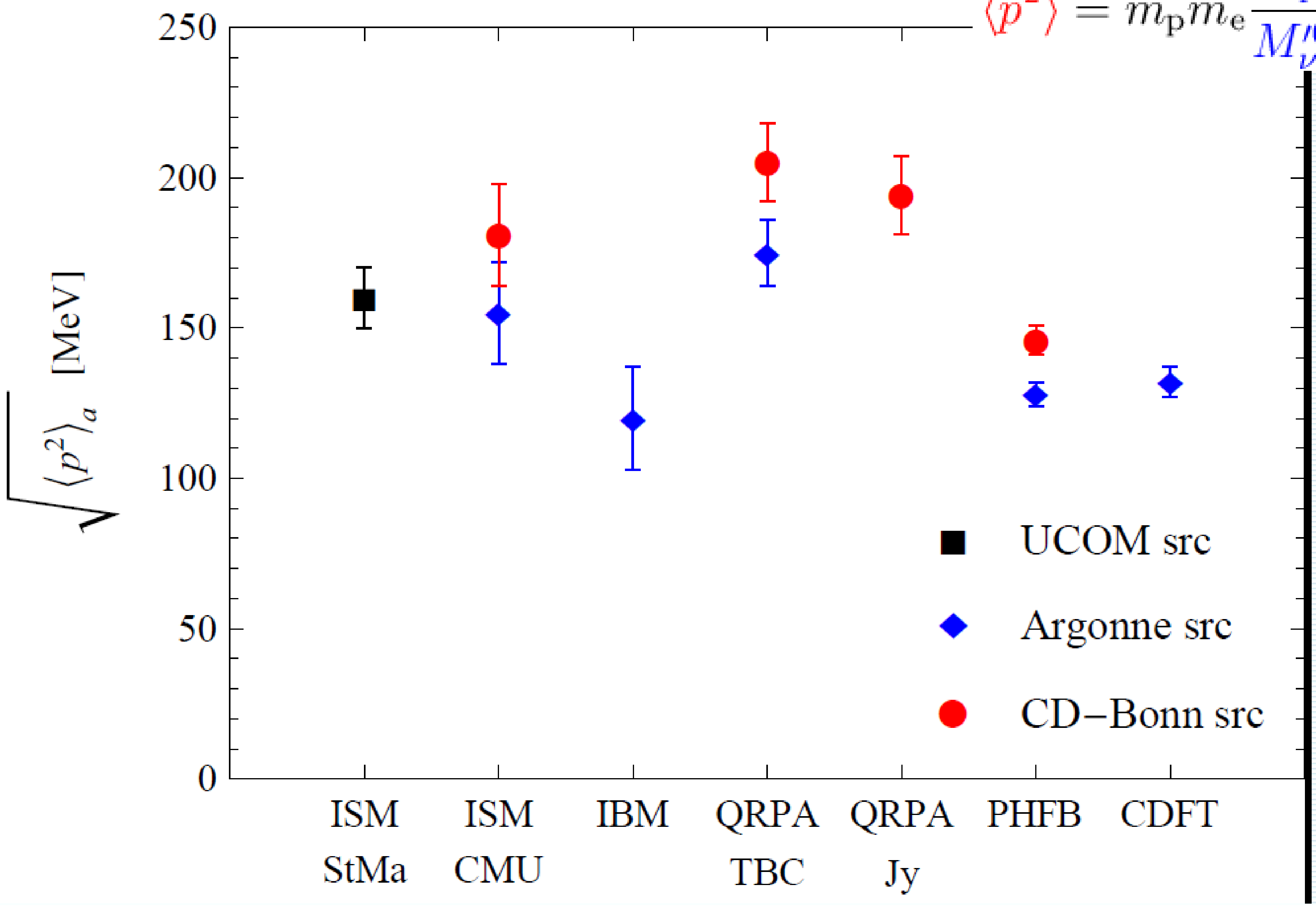
E. Lisi, A. Rotunno, F. Š., PRD 92 (2014) 093004

Interpolating formula is justified by practically no dependence <p^2> on A

$$[T^{0\nu}_{1/2}]^{-1} = \mathcal{A} \cdot \left| m_p \sum_N U^2_{eN} \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2,$$

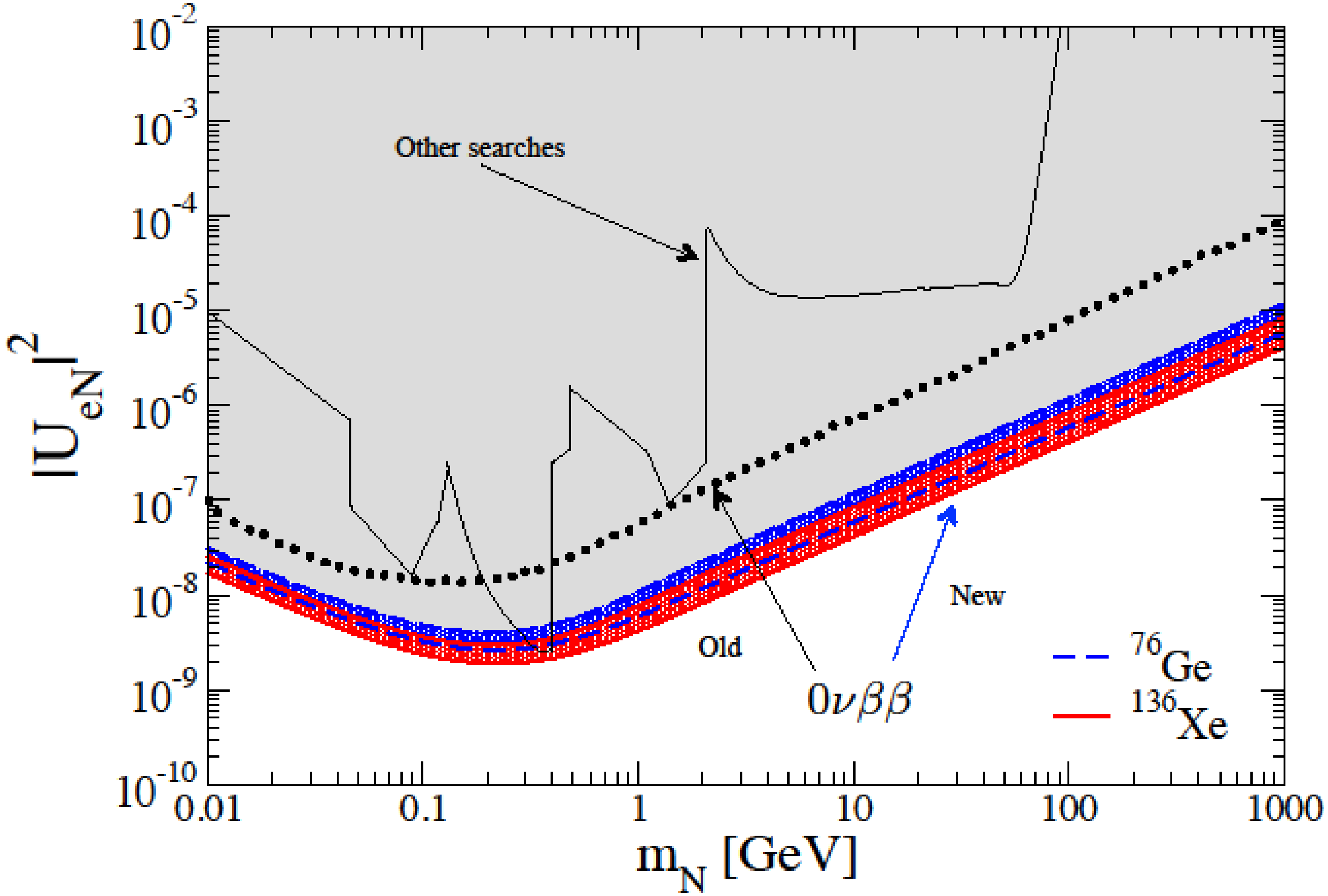
A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)

$$\langle p^2 \rangle = m_p m_e \frac{M'^{0\nu}_N}{M'^{0\nu}_\nu}$$



Exclusion plot in $|U_{eN}|^2 - m_N$ plane

$T^{0\nu}_{1/2}(^{76}Ge) \geq 3.0\ 10^{25}$ yr
$T^{0\nu}_{1/2}(^{136}Xe) \geq 3.4\ 10^{25}$ yr



Improvements: i) QRPA (constrained Hamiltonian by 2νββ half-life, self-consistent treatment of src, restoration of isospin symmetry …), ii) More stringent limits on the 0νββ half-life

II.c. The 0νββ-decay within L-R symmetric theories (interpolating formula)

(D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)

A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)

$$[T^{0\nu}_{1/2}]^{-1} = \eta^2_{\nu N}\ C_{\nu N}$$

$$C_{\nu N} = g_{\rm A}^4\ \left| M'^{0\nu}_{\nu} \right|^2\ G^{0\nu}$$

Mixing of light and heavy neutrinos

$$\nu_{eL} = \sum_{j=1}^{3} \left(U_{ej}\nu_{jL} + S_{ej}(N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^{3} \left(T^*_{ej}(\nu_{jL})^C + V^*_{ej}N_{jR} \right)$$

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

Effective LNV parameter within LRS model

$$\eta^2_{\nu N} = \left| \sum_{j=1}^{3} \left(U^2_{ej}\ \frac{m_j}{m_e}\ +\ S^2_{ej}\ \frac{\langle p^2\rangle_a}{\langle p^2\rangle_a + M^2_j}\ \frac{M_j}{m_e} \right) \right|^2 + \lambda^2 \left| \sum_{j=1}^{3} \left(T^2_{ej}\ \frac{m_j}{m_e}\ +\ V^2_{ej}\ \frac{\langle p^2\rangle_a}{\langle p^2\rangle_a + M^2_j}\ \frac{M_j}{m_e} \right) \right|^2$$

$$\langle p^2 \rangle = m_{\rm p} m_{\rm e} \frac{M'^{0\nu}_{\rm N}}{M'^{0\nu}_{\nu}}$$

6x6 PMNS see-saw ν-mixing matrix
(the most economical one)

6x6 neutrino mass matrix

Basis

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix} \qquad (\nu_L, (N_R)^c)^T \qquad \mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}$$

6x6 matrix: **15 angles, 10+5 CP phases**
3x3 matrix: 3 angles, 1+2 CP phases

3x3 block matrices U, S, T, V are generalization of PMNS matrix

Assumptions:
i) the see-saw structure
ii) mixing between different generations is neglected

$$\mathcal{U}_{\text{PMNS}} = \begin{pmatrix} U_{\text{PMNS}} & \zeta\,\mathbf{1} \\ -\zeta\,\mathbf{1} & U^{\dagger}_{\text{PMNS}} \end{pmatrix} \qquad \mathcal{U}_{\text{PMNS}}\,\mathcal{U}^{\dagger}_{\text{PMNS}} = \mathcal{U}^{\dagger}_{\text{PMNS}}\,\mathcal{U}_{\text{PMNS}} = \mathbf{1}$$

see-saw parameter

$$\zeta = \frac{m_{\text{D}}}{m_{\text{LNV}}}$$

6x6 matrix: **3 angles, 1+2 CP phases, 1 see-saw par.**

6x6 PMNS see-saw ν-mixing matrix
(the most economical one)

$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta\, 1 \\ -\zeta\, 1 & V_0 \end{pmatrix}$$

$$U_0 = U_{\rm PMNS}$$

A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)

$$V_0 = U^\dagger_{\rm PMNS} =$$
$$\begin{pmatrix} c_{12}\, c_{13}\, e^{-i\alpha_1} & \left(-s_{12}\, c_{23} - c_{12}\, s_{13}\, s_{23}\, e^{-i\delta}\right) e^{-i\alpha_1} & \left(s_{12}\, s_{23} - c_{12}\, s_{13}\, c_{23}\, e^{-i\delta}\right) e^{-i\alpha_1} \\ s_{12}\, c_{13}\, e^{-i\alpha_2} & \left(c_{12}\, c_{23} - s_{12}\, s_{13}\, s_{23}\, e^{-i\delta}\right) e^{-i\alpha_2} & \left(-c_{12}\, s_{23} - s_{12}\, s_{13}\, c_{23}\, e^{-i\delta}\right) e^{-i\alpha_2} \\ s_{13}\, e^{i\delta} & c_{13}\, s_{23} & c_{13}\, c_{23} \end{pmatrix}$$

Assumption about heavy neutrino masses M_i (by assuming see-saw)

Inverse proportional

$$m_i M_i \simeq m_D^2$$

$$M^{\rm R}_{\beta\beta} = \lambda \frac{\langle p^2\rangle_a}{m_D^2} \left|\sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\, m_j\right|$$

Proportional

$$m_i \simeq \zeta^2 M_i$$

$$M^{\rm R}_{\beta\beta} = \lambda\, \zeta^2 \left|\sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\, \frac{\langle p^2\rangle_a}{m_j}\right|$$

$M^{R}_{\beta\beta}$ depends on "Dirac" CP phase δ unlike "Majorana" CP phases α_1 and α_2

Heavy Majorana mass $M^{R}_{\beta\beta}$ depends on the "Dirac" CP violating phase δ

Contribution from exchange of heavy neutrino to 0νββ-decay rate might be large

Inverse proportional

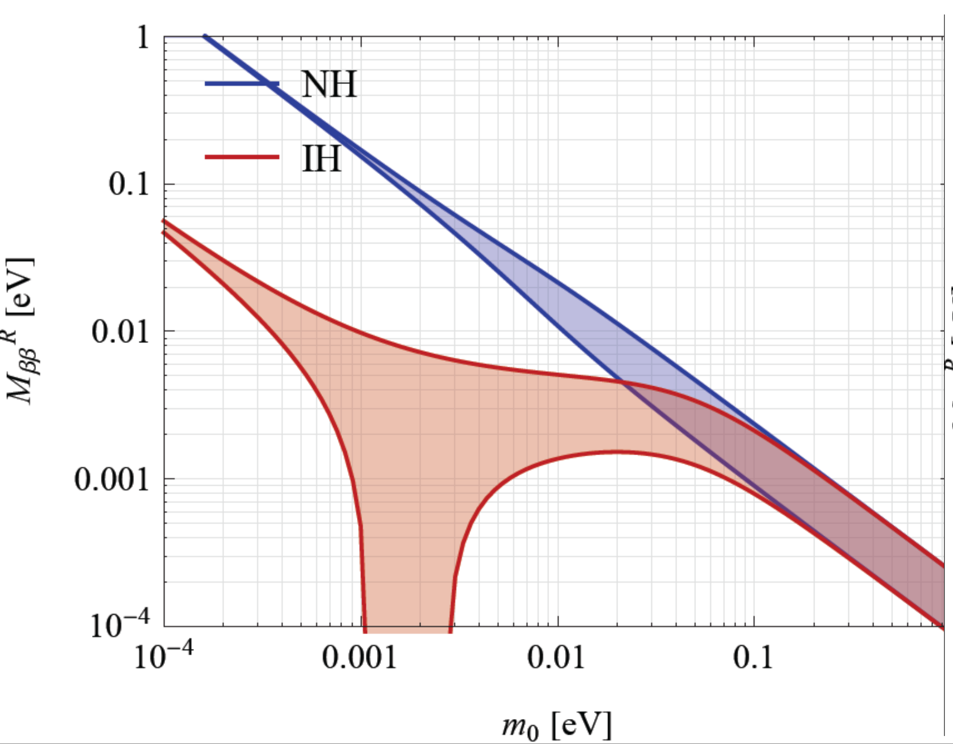
$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^{\mathrm{R}} = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 \, m_j \right|$$

$V_0 = U_{PMNS}^\dagger$

$M_i = m_D^2 / m_i \quad m_D \simeq 5$ MeV

$\lambda = 7.7 \times 10^{-4}$



Proportional

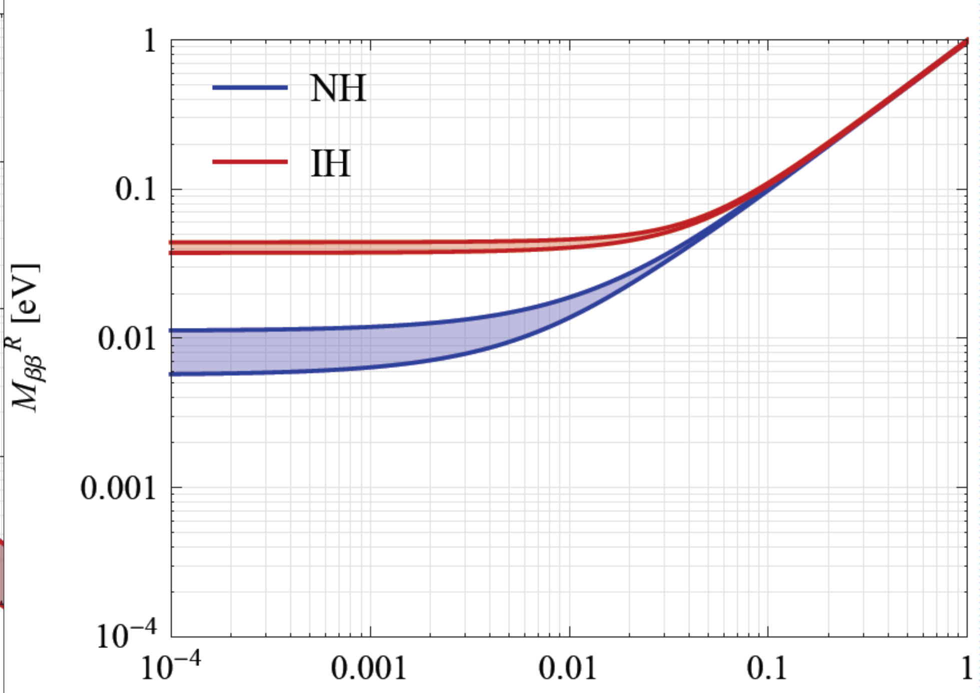
$$m_i \simeq \zeta^2 M_i$$

$$M_{\beta\beta}^{\mathrm{R}} = \lambda \, \zeta^2 \left| \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 \, \frac{\langle p^2 \rangle_a}{m_j} \right|$$

$V_0 = U_{PMNS}^\dagger$

$\zeta = m_i / M_i \quad \zeta^2 \simeq 5 \times \ 10^{-17}$

$\lambda = 7.7 \times 10^{-4}$



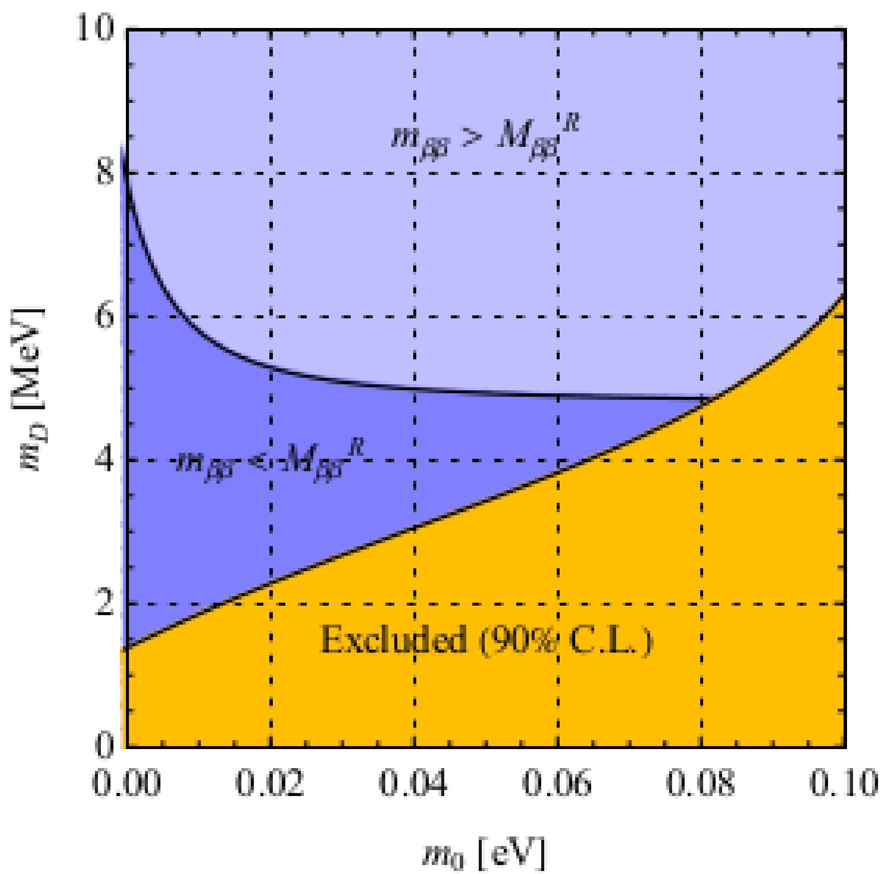
$$\eta^2_{\nu N} = \frac{1}{m_e^2}\left(m^2_{\beta\beta} + (M^R_{\beta\beta})^2\right)$$

See-saw scenario

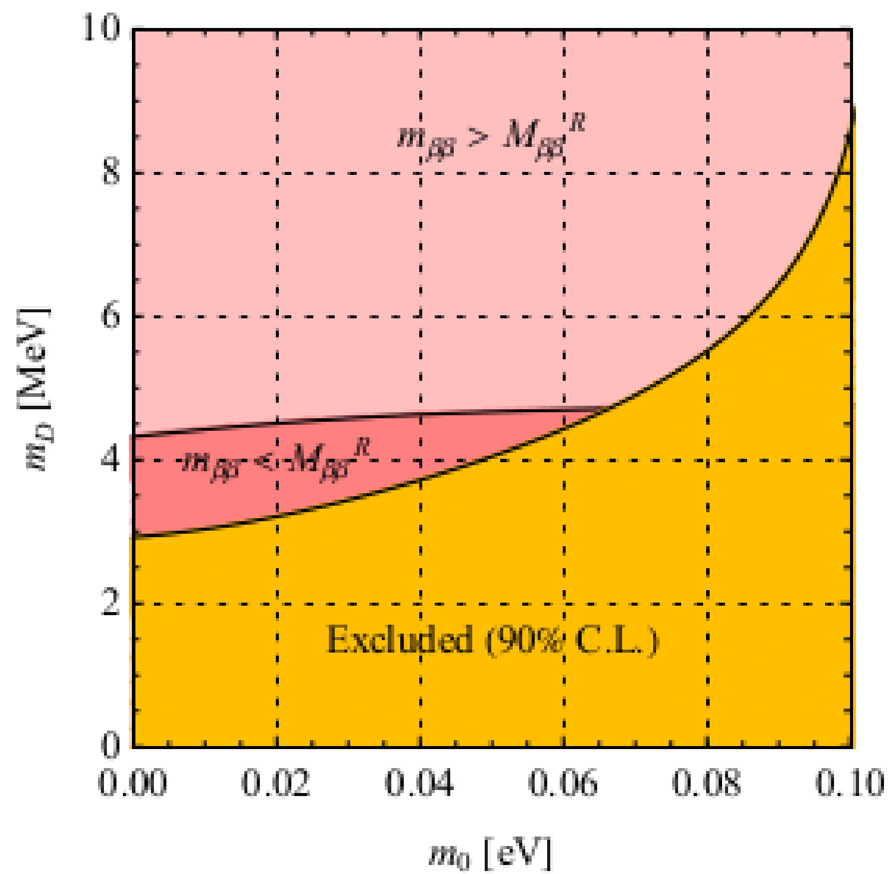
$$m_i M_i \simeq m_D^2$$

$$M^{\rm R}_{\beta\beta} = \lambda \frac{\langle p^2\rangle_a}{m_D^2} \left|\sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\ m_j\right|$$

Normal spectrum



Inverted spectrum



A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)

III. Resonant neutrinoless double electron capture

Neutrinoless double electron capture (resonance transitions)

$(A,Z)\rightarrow(A,Z-2)^{*HH'}$

0νECEC considered in 1955

R.G. Winter,
Phys. Rev. 1000, 142 (1955)

J. Bernabeu, A. DeRujula, C. Jarlskog,
Nucl. Phys. B 223, 15 (1983)

Atom mixing amplitude
ΔM

$$E \simeq E^* + E_{\rm H} + E_{\rm H'},$$

$$\Gamma \simeq \Gamma^* + \Gamma_{\rm H} + \Gamma_{\rm H'}.$$

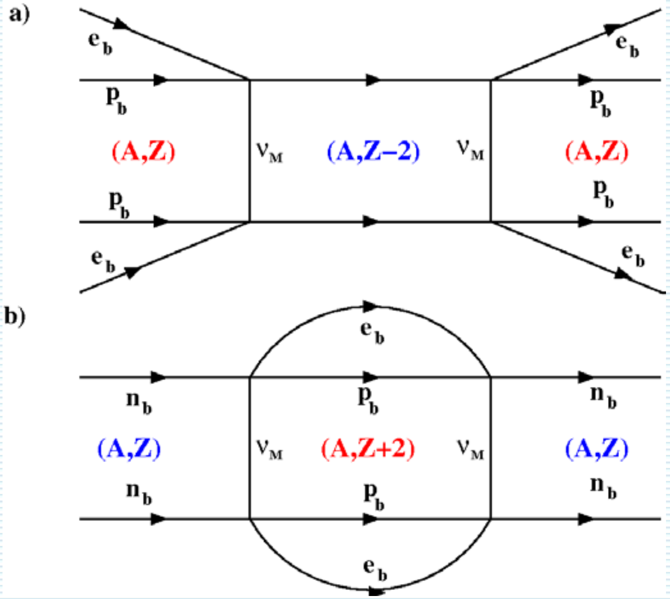
Decay rate

$$\frac{1}{\tau} \simeq \frac{(\Delta M)^2}{(Q-E)^2 + \frac{1}{4}\Gamma^2}\,\Gamma,$$

2νECEC-background depends strongly on Q-value

DEC transitions, abundance, daughter nuclear excitation, atomic vacancies and figure of merit of some isotopes [10]

Transition $Z \rightarrow Z-2$	Z-natural abundance in %	Nuclear excitation E^* (in MeV), J^P	Atomic vacancies H, H′	Figure of merit $Q-E$ (in keV)
$^{74}_{34}$Se → $^{74}_{32}$Ge	0.87	1.204 (2^+)	2S(P), 2S(P)	2 ± 3
$^{78}_{36}$Kr → $^{78}_{34}$Se	0.36	2.839 (2^+) 2.864 (?)	1S, 1S	$^{19}_{-6} \pm 10$
$^{102}_{46}$Pd → $^{102}_{44}$Ru	1	1.103 (2^+) 1.107 (4^+)	1S, 1S	$^{29}_{25} \pm 9$
$^{106}_{48}$Cd → $^{106}_{46}$Pd	1.25	2.741 (?)	1S, 1S	-8 ± 10
$^{112}_{50}$Sn → $^{112}_{48}$Cd	1.01	1.871 (0^+)	1S, 1S	-3 ± 10
$^{130}_{56}$Ba → $^{130}_{54}$Xe	0.11	2.502 (?) 2.544 (?)	1S, 1S 1S, 2S(P)	$^{8}_{-6} \pm 13$
$^{152}_{64}$Gd → $^{152}_{62}$Sm	0.20	0 (0^+)	1S, 2S	4 ± 4
$^{162}_{68}$Er → $^{162}_{66}$Dy	0.14	1.783 (2^+)	1S, 2S	1 ± 6
$^{164}_{68}$Er → $^{164}_{66}$Dy	1.56	0 (0^+)	2S, 2S	9 ± 5
$^{168}_{70}$Yb → $^{168}_{68}$Er	0.14	1.355 (1^-) 1.393 (?)	1S, 2S 2S, 2S	$^{-1}_{8} \pm 4$
$^{180}_{74}$W → $^{180}_{72}$Hf	0.13	0 (0^+) 0.093 (2^+)	1S, 1S 1S, 3S	$^{26}_{-4} \pm 17$
$^{196}_{80}$Hg → $^{186}_{78}$Pt	0.15	0.689 (2^+)	1S, 2S	26 ± 9



Oscillations of atoms

F.Š., M. Krivoruchenko,
Phys.Part.Nucl.Lett. 6 (2009) 485.

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PHYSICS OF ELEMENTARY PARTICLES
AND ATOMIC NUCLEI. THEORY

Mixing of Neutral Atoms and Lepton Number Oscillations*

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Received November 7, 2008

Abstract—We discuss oscillations of two neutral atoms which proceed with the violation of lepton number. One of the neutral atoms is stable, the other one represents a quasistationary state subjected to electromagnetic deexcitation. The system of neutral atoms exhibits oscillations similar to those of the system of neutral kaons and neutron–antineutron oscillations in the nuclear medium. The underlying mechanism is a transition of two protons and two bound electrons to two neutrons $p + p + e_b^- + e_b^- \longleftrightarrow n + n$. A signature of the oscillations might be an electromagnetic deexcitation of the involved unstable nucleus and atomic shell with the electron holes. A resonant enhancement of the neutrinoless double electron capture takes place when the atomic masses tend to be degenerate. Qualitative estimates show that in searches for lepton number violation oscillations of atoms might be a possible alternative to the conventional mechanism of the neutrinoless double β decay process with emission of two electrons.

Different types of Oscillations (Effective Hamiltonian)

$$H_{eff}^{K_0\overline{K_0}} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \Gamma_{12} \\ M_{12}^* - \Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of K_0-anti{K_0} (strangeness)

$$H_{\rm eff}^{\nu} = \begin{pmatrix} \cos\theta_{23} & -\sin\theta_{23} \\ \sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} E_3 - \frac{i}{2}\acute{\Gamma}_3 & 0 \\ 0 & E_2 - \frac{i}{2}\acute{\Gamma}_2 \end{pmatrix} \begin{pmatrix} \cos\theta_{23} & \sin\theta_{23} \\ -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

Oscillations of ν_l-$\nu_{l'}$ (lepton flavor)

$$H_{eff}^{n\overline{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of n-anti{n} (baryon number)

$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of Atoms (OoA) (total lepton number)

In analogy with oscillations of n-anti{n} (baryon number violation)

Full width of unstable atom/nucleus

Oscillations of atoms

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.

2x2 Hamiltonian matrix in the lepton in the lepton number basis

$$H_{\text{eff.}} = \begin{pmatrix} M_i & V \\ V & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

M_i, M_f ≈ tens of GeV
$V \leq 10^{-24}$ [$m_{\beta\beta}$/0.1 eV] eV
$\Gamma \leq$ tens of eV

Eigenvalues

$$\lambda_{\pm} = \frac{M_i + M_f}{2} - \frac{i}{4}\Gamma \pm \sqrt{V^2 + (\frac{M_i - M_f}{2} + \frac{i}{4}\Gamma)^2}$$

For $M_i = M_f$ the result od A. Gal hep-ph/9907334 Eqs. (11,12) is recovered

V is significantly smaller than Γ and M_i-M_f>0. In lowest order in V we get

$$\begin{aligned} \lambda_+ &= M_i + \Delta M - \frac{i}{2}\Gamma_{\rm LNV} \\ \lambda_- &= M_f - \frac{i}{2}\Gamma - \Delta M + \frac{i}{2}\Gamma_{\rm LNV} \end{aligned}$$

$$\begin{aligned} \Delta M &= \frac{V^2}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}(M_i - M_f) \\ \Gamma_{\rm LNV} &= \frac{V^2}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}\Gamma \end{aligned}$$

$\Gamma_{\rm LNV}$ – width for resonant 0νECEC

Resonant neutrinoless double electron capture

Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics A 859 (2011) 140–171
NUCLEAR PHYSICS A
www.elsevier.com/locate/nuclphysa

Resonance enhancement of neutrinoless double electron capture

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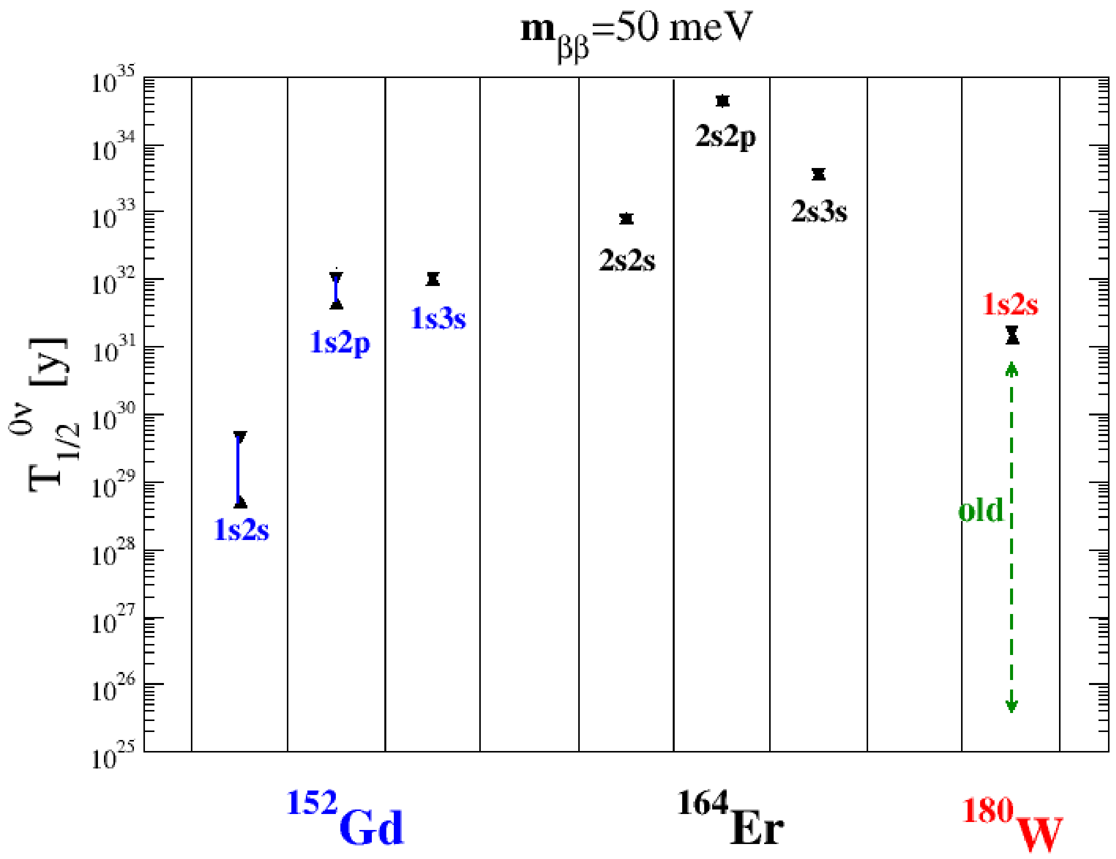
Available online 23 April 2011

0νεε **half-lives**

$m_{\beta\beta}$=50 meV



Nucleus	$(n2jl)_a$	$(n2jl)_b$	E_a	E_b	E_C	Γ_{ab} (keV)	Δ (keV)	$T_{1/2}^{min}$ (y)	$T_{1/2}^{max}$ (y)
^{152}Gd	110	210	46.83	7.74	0.34	2.3×10^{-2}	-0.83 ± 0.18	4.7×10^{28}	4.8×10^{29}
	110	211	46.83	7.31	0.32	2.3×10^{-2}	-1.27 ± 0.18	4.2×10^{31}	1.1×10^{32}
	110	310	46.83	1.72	0.11	3.2×10^{-2}	-7.07 ± 0.18	9.4×10^{31}	1.1×10^{32}
^{164}Er	210	210	9.05	9.05	0.22	8.6×10^{-3}	-6.82 ± 0.12	7.5×10^{32}	8.4×10^{32}
	210	211	9.05	8.58	0.23	8.3×10^{-3}	-7.28 ± 0.12	4.2×10^{34}	4.6×10^{34}
	210	310	9.05	2.05	0.11	1.8×10^{-2}	-13.92 ± 0.12	3.5×10^{33}	3.9×10^{33}
^{180}W	110	110	63.35	63.35	1.26	7.2×10^{-2}	-11.24 ± 0.27	1.3×10^{31}	1.8×10^{31}

A comparison

Resonance enhancement of neutrinoless double electron capture
M.I. Krivoruchenko, F. Š., D. Frekers, and A. Faessler,
Nucl. Phys. A 859, 140-171 (2011)

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$

$$e^- + e^- + (A,Z) \rightarrow (A,Z-2)^{**}$$

Perturbation theory

$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0,Z)\left|M^{0\nu}\right|^2$$

Breit-Wigner form

$$\Gamma^{0\nu ECEC}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma^2_{\alpha\beta}/4}\,\Gamma_{\alpha\beta}$$

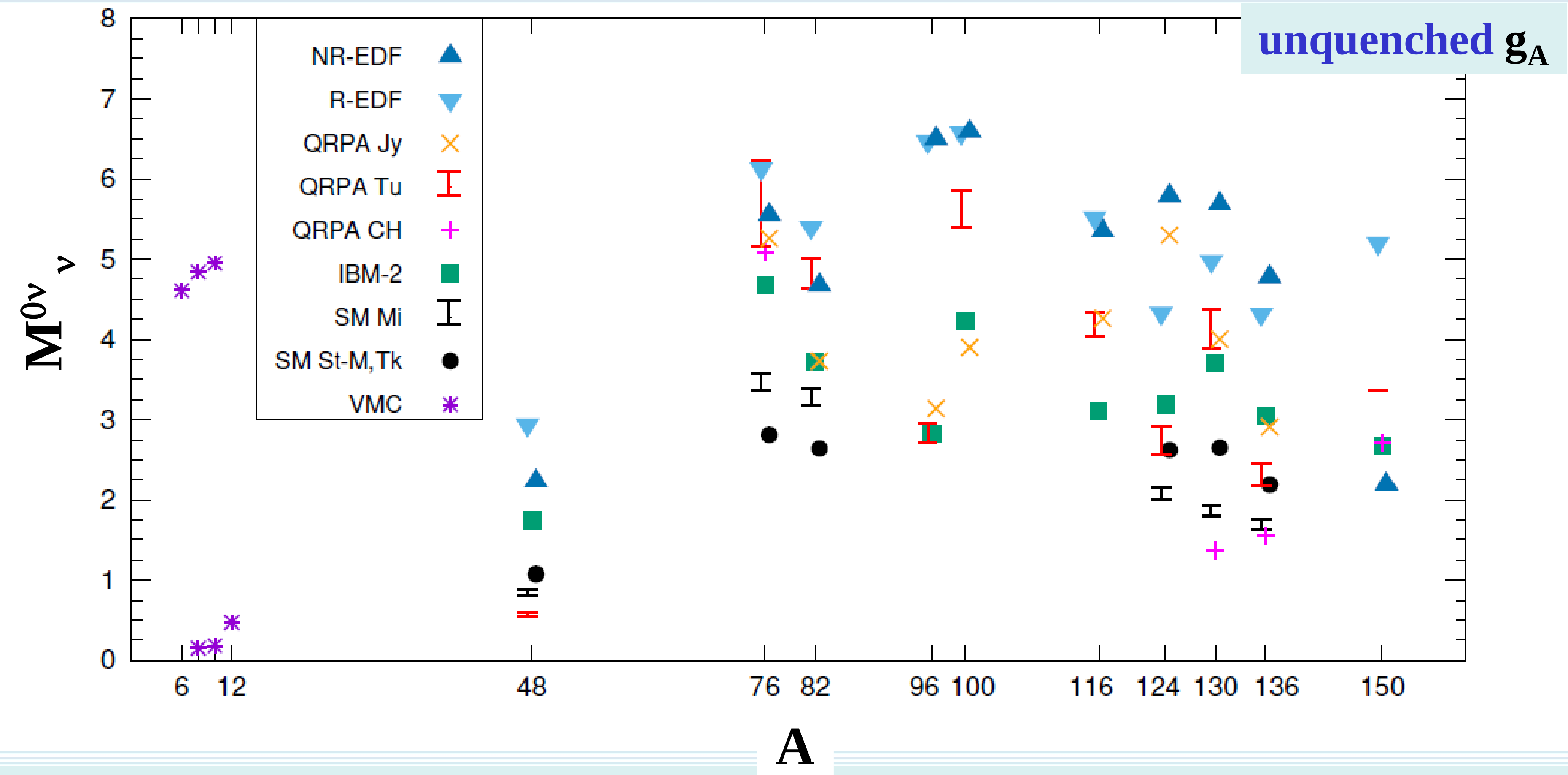
- **2νββ-decay background can be a problem**
- **Uncertainty in NMEs factor ~2, 3**
- **$0^+ \rightarrow 0^+, 2^+$ transitions**
- **Large Q-value**
- **^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, ^{136}Xe …**
- **Many exp. in construction, potential for observation in the case of inverted hierarchy (2020)**

- **2νεε-decay strongly suppressed**
- **NMEs need to be calculated**
- **$0^+ \rightarrow 0^+, 0^-, 1^+, 1^-$ transitions**
- **Small Q-value**
- **Q-value needs to be measured at least with 100 eV accuracy**
- **^{152}Gd, looking for additional**
- **small experiments yet**

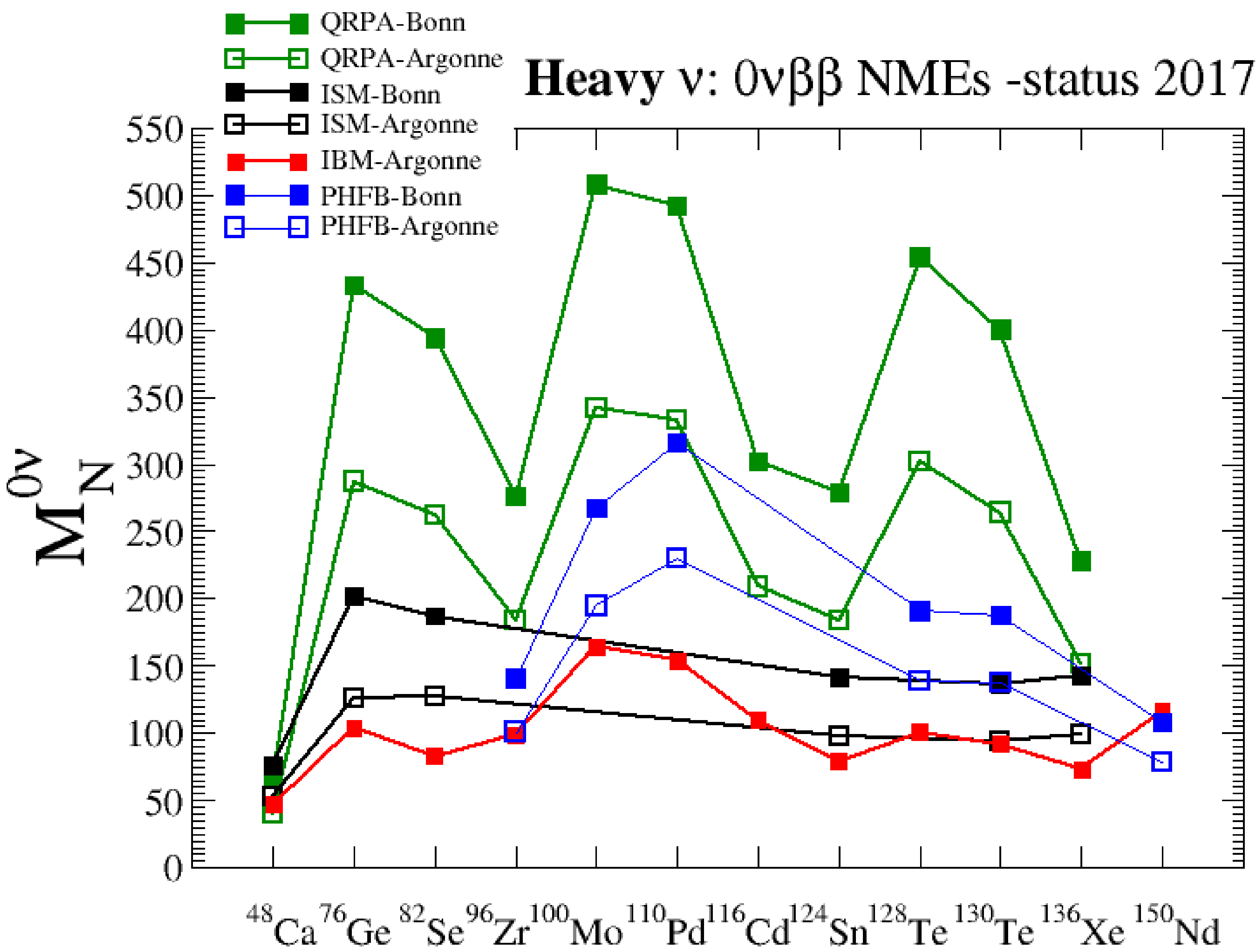
IV. Double beta decay NMEs
(there is a progress, but not easy task)

0νββ-decay NME (light ν mass) – status 2017

J. Engel, J. Menendez, Rept. Prog. Phys. 80, 046301 (2017)

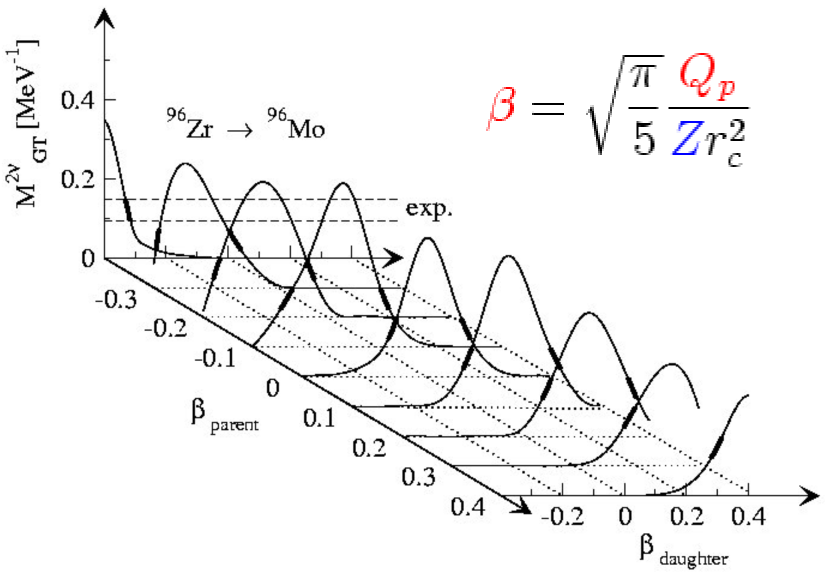


	mean field meth.	ISM	IBM	QRPA
Large model space	yes	no	yes	yes
Constr. Interm. States	no	yes	no	yes
Nucl. Correlations	limited	all	restricted	restricted



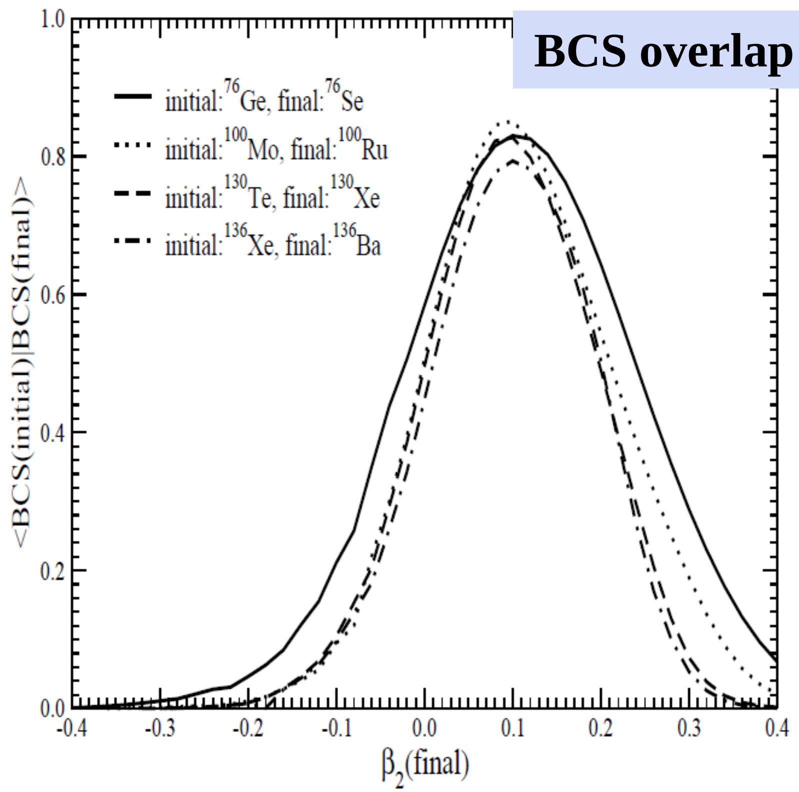
Heavy ν: 0νββ NMEs -status 2017
QRPA-Bonn
QRPA-Argonne
ISM-Bonn
ISM-Argonne
IBM-Argonne
PHFB-Bonn
PHFB-Argonne
$M_N^{0\nu}$
0
50
100
150
200
250
300
350
400
450
500
550
^{48}Ca ^{76}Ge ^{82}Se ^{96}Zr ^{100}Mo ^{110}Pd ^{116}Cd ^{124}Sn ^{128}Te ^{130}Te ^{136}Xe ^{150}Nd

Suppression of the 0νββ-decay NMEs due to different deformation of initial and final nuclei



The suppression of the NME depends on the relative deformation of initial and final nuclei

F.Š., Pacearescu, Faessler, NPA 733 (2004) 321

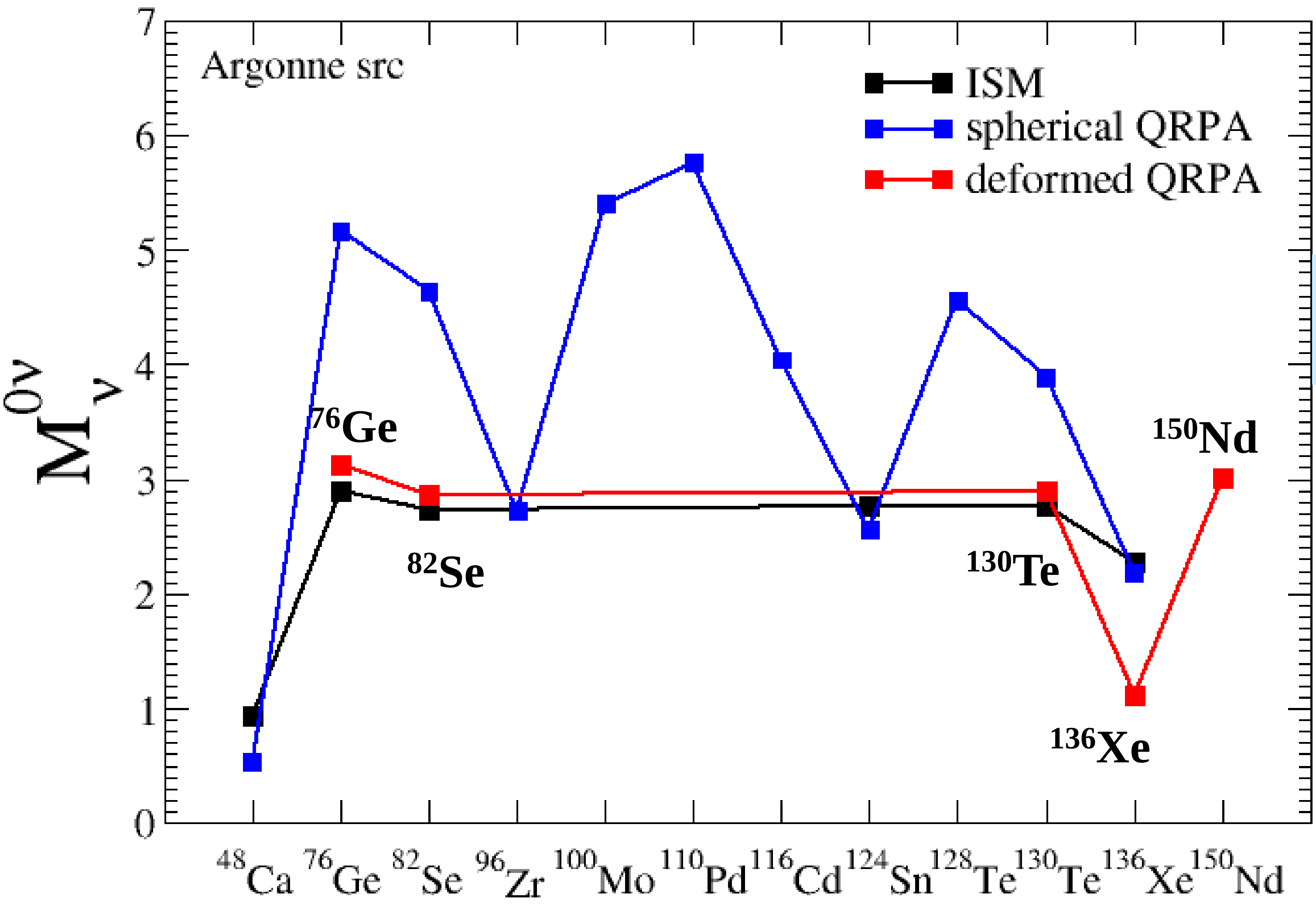


Systematic study of the deformation effect on the 2νββ-decay NME within deformed QRPA

Alvarez, Sarriguren, Moya, Pacearescu, Faessler, F.Š., Phys. Rev. C 70 (2004) 321

0νββ-decay NMEs within deformed QRPA with partial restoration of isospin symmetry (light neutrino exchange)

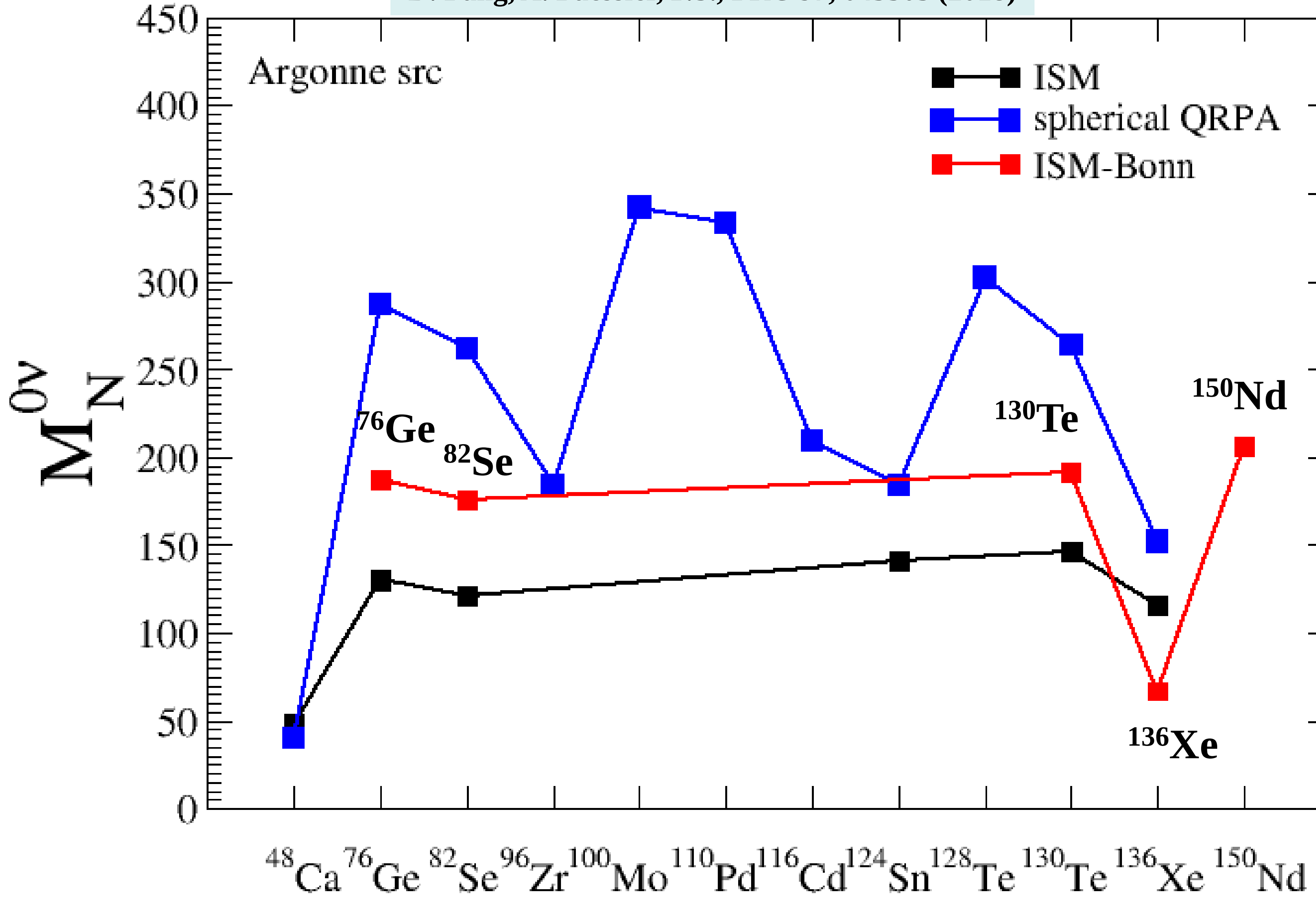
D. Fang, A. Faessler, F.Š., PRC 97, 045503 (2018)



Agreement by a chance?

0νββ-decay NMEs within deformed QRPA with partial restoration of isospin symmetry (heavy neutrino exchange, Argonne src)

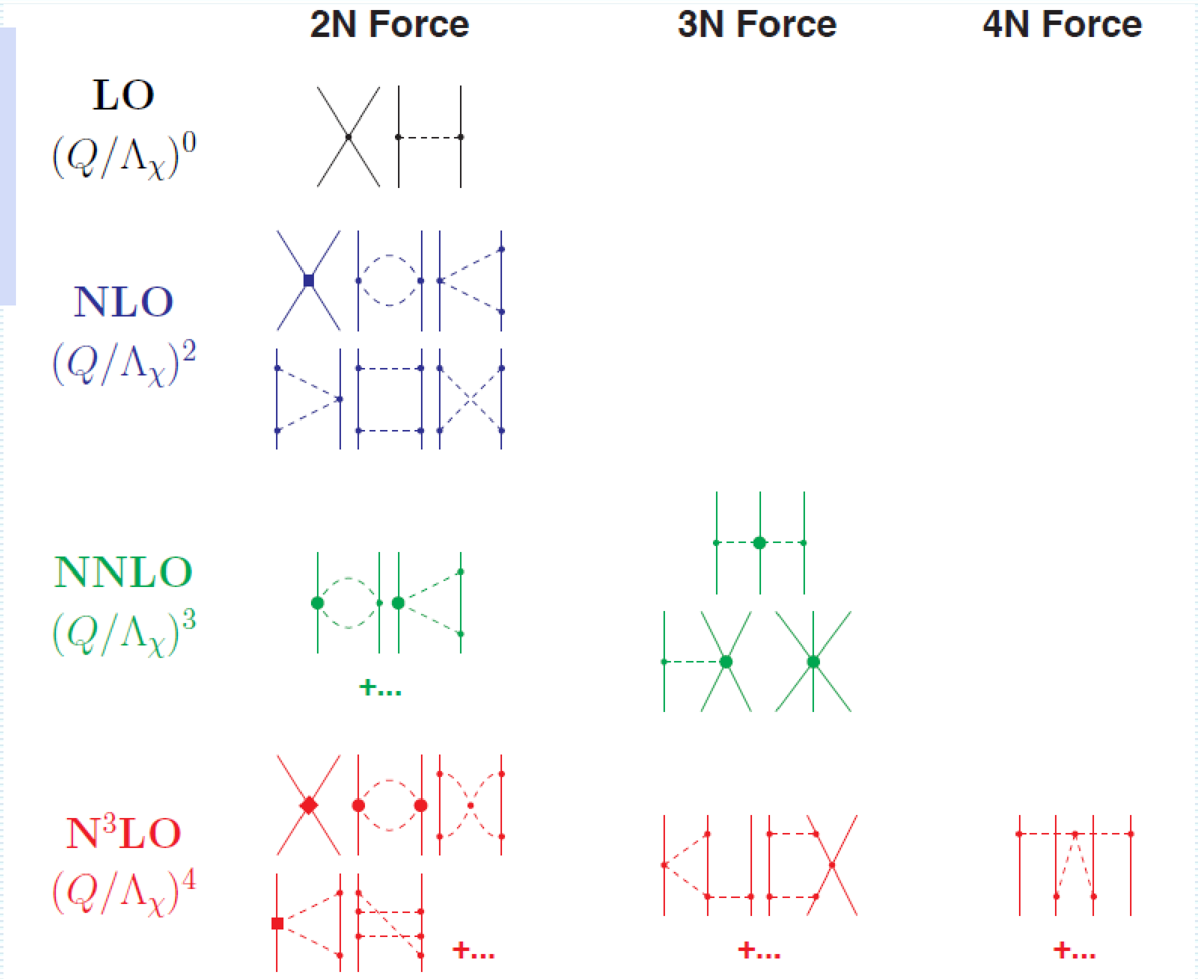
D. Fang, A. Faessler, F.Š., PRC 97, 045503 (2018)



Ab Initio Nuclear Structure
(Often starts with chiral effective-field theory)

Nucleons, pions sufficient below chiral symmetry breaking scale.
Expansion of operators in power of Q/Λ_χ. $Q=m_\pi$ or typical nucleon momentum.

A. Schwenk (Darmstadt U.)
P. Navratil (TRIUMPH)
J. Engel (North Caroline U.)
J. Menendez (Tokyo U.)

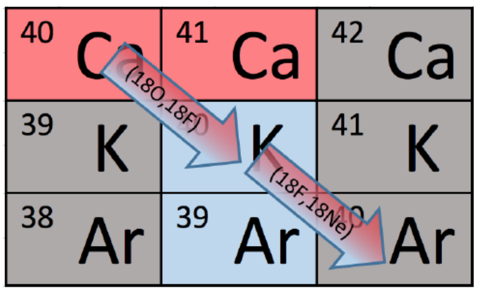


Calculation for the hypothetical 0νββ decay of ^{10}He:
$^{10}He \rightarrow {}^{10}Be + e^- + e$-

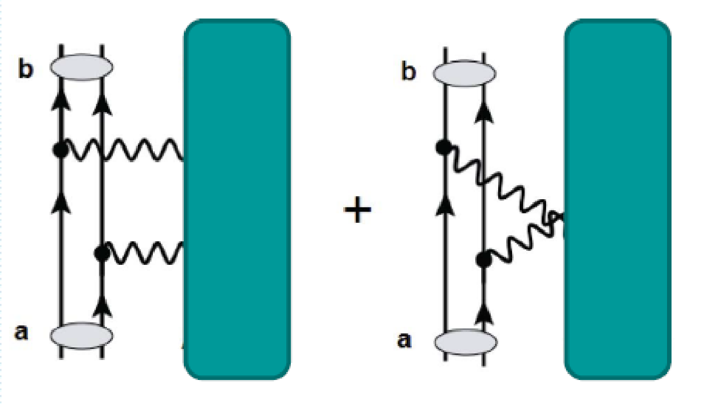
masses, spectra

Supporting nuclear physics experiments

(2νββ-decay ChER, pion and heavy ion DCX, nucleon transfer reactions etc)

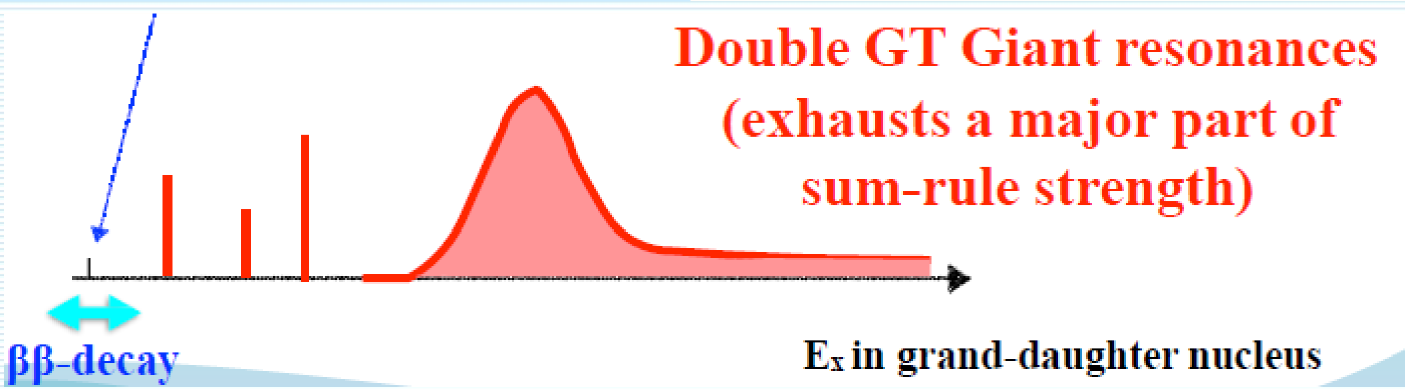


$$^{18}O + ^{40}Ca \rightarrow ^{18}F + ^{40}K \rightarrow ^{18}Ne + ^{40}Ar$$



H. Lenske group
Theory of heavy ion DCX and Connection to DBD NMEs

Heavy ion DCX:
NUMEN (LNC-INFN),
HIDCX (RCNP/RIKEN)



Understanding of the 2νββ-decay NMEs is of crucial importance for correct evaluation of the 0νββ-decay NMEs

$$(A,Z) \to (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

Both 2νββ and 0νββ operators connect the same states. Both change two neutrons into two protons.

Explaining 2νββ-decay is necessary but not sufficient

There is no reliable calculation of the *2νββ-decay NMEs*

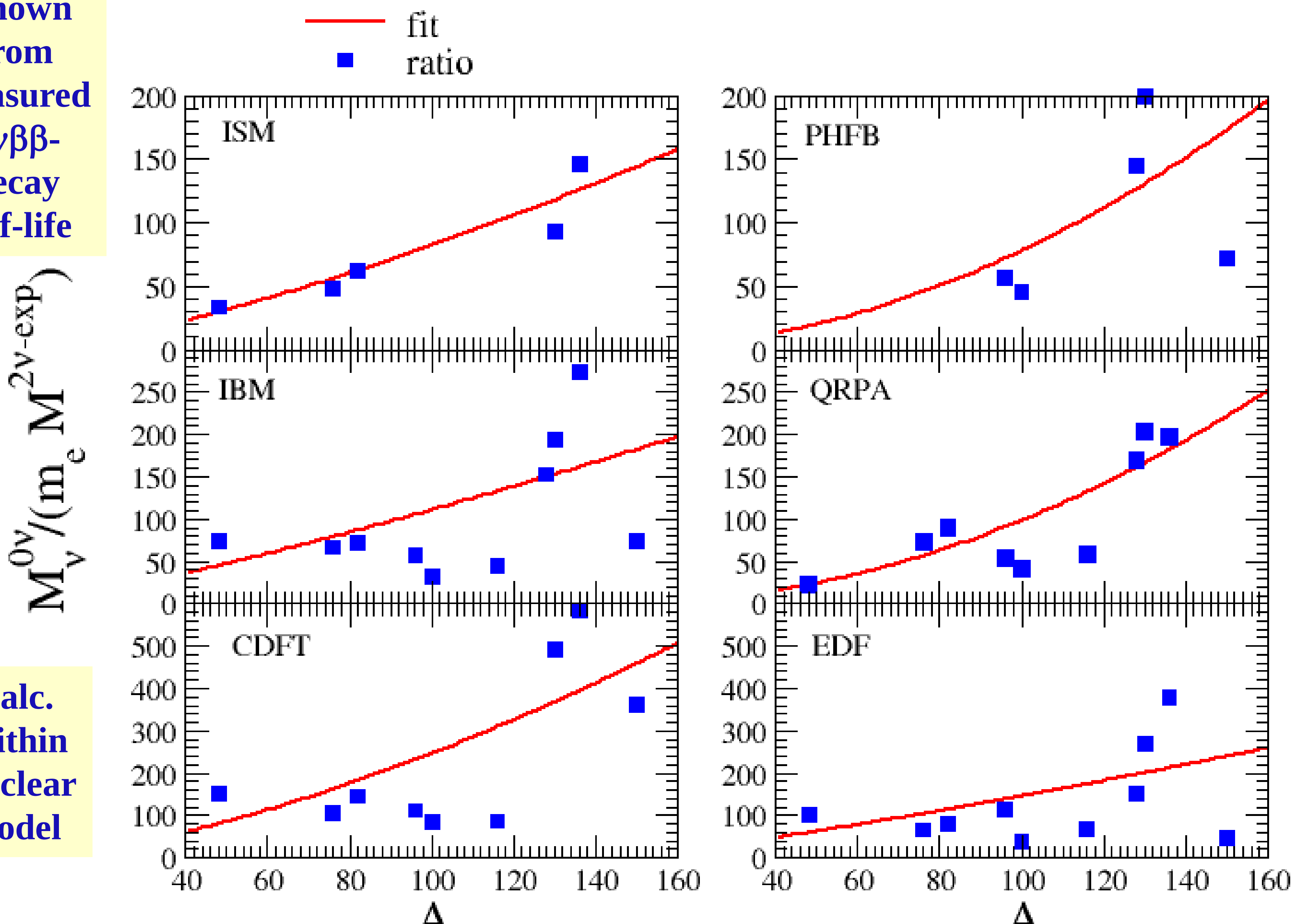
Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.) ISM (quenching, truncation of model space, spin-orbit partners)

Calculation via closure NME: IBM, PHFB

No calculation: EDF

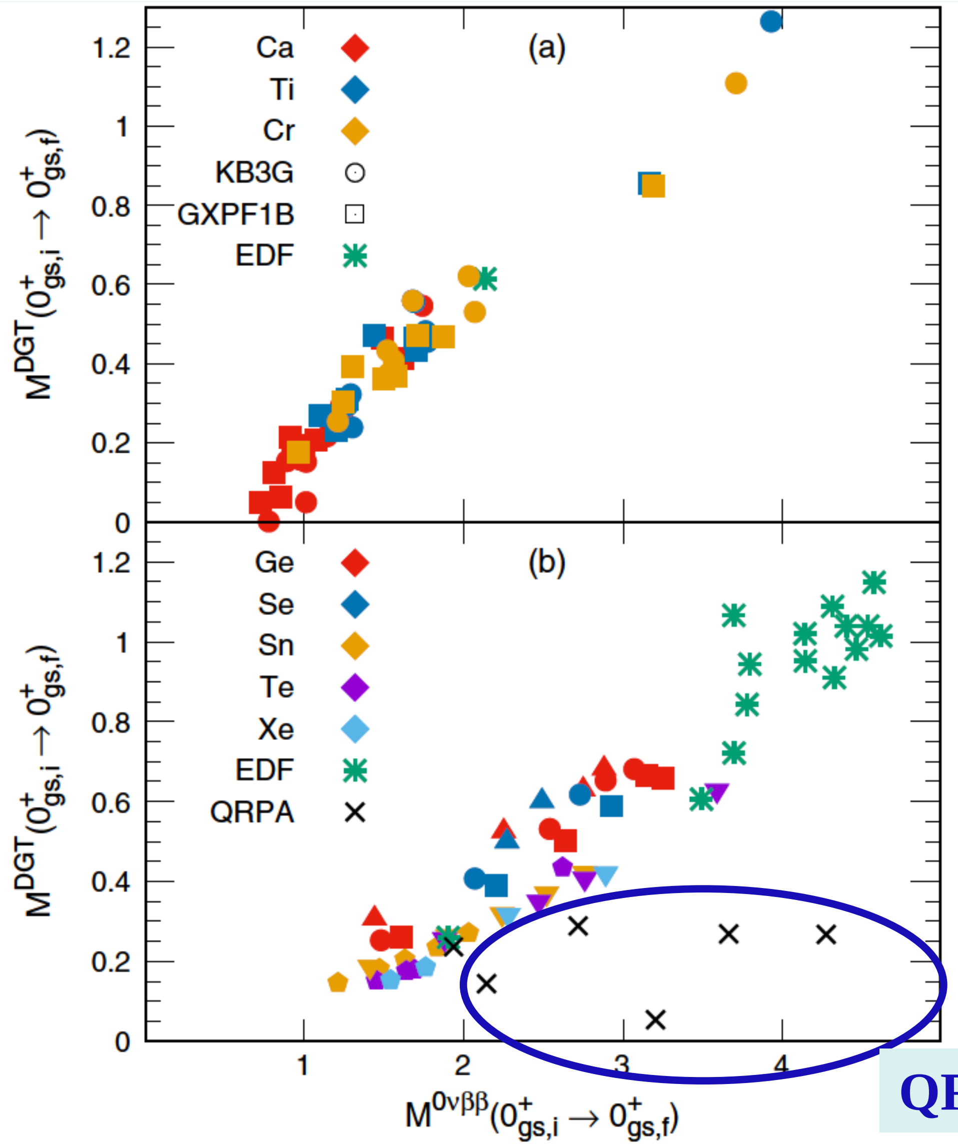
Is there a proportionality between 0νββ- and 2νββ-decay NMEs?

Known krom measured 2νββ-decay half-life



Calc. within nuclear model

$M^{0\nu} \propto M^{2\nu}_{GT\text{-}cl}$: ISM, EDF



QRPA?

ISM: N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)

Fedor Simkovic

$M^{DGT} = M^{2\nu}_{GT}$

	SSD	ChER
^{48}Ca		0.22
^{76}Ge		0.52
^{96}Zr		0.22
^{100}Mo	0.35	
^{116}Cd	0.35	0.30
^{128}Te	0.41	

EDF: 0.6 → 1.2
ISM: 0.1 → 0.7
IBM: 1.6 → 4.4
QRPA: |0.1| → |0.7|

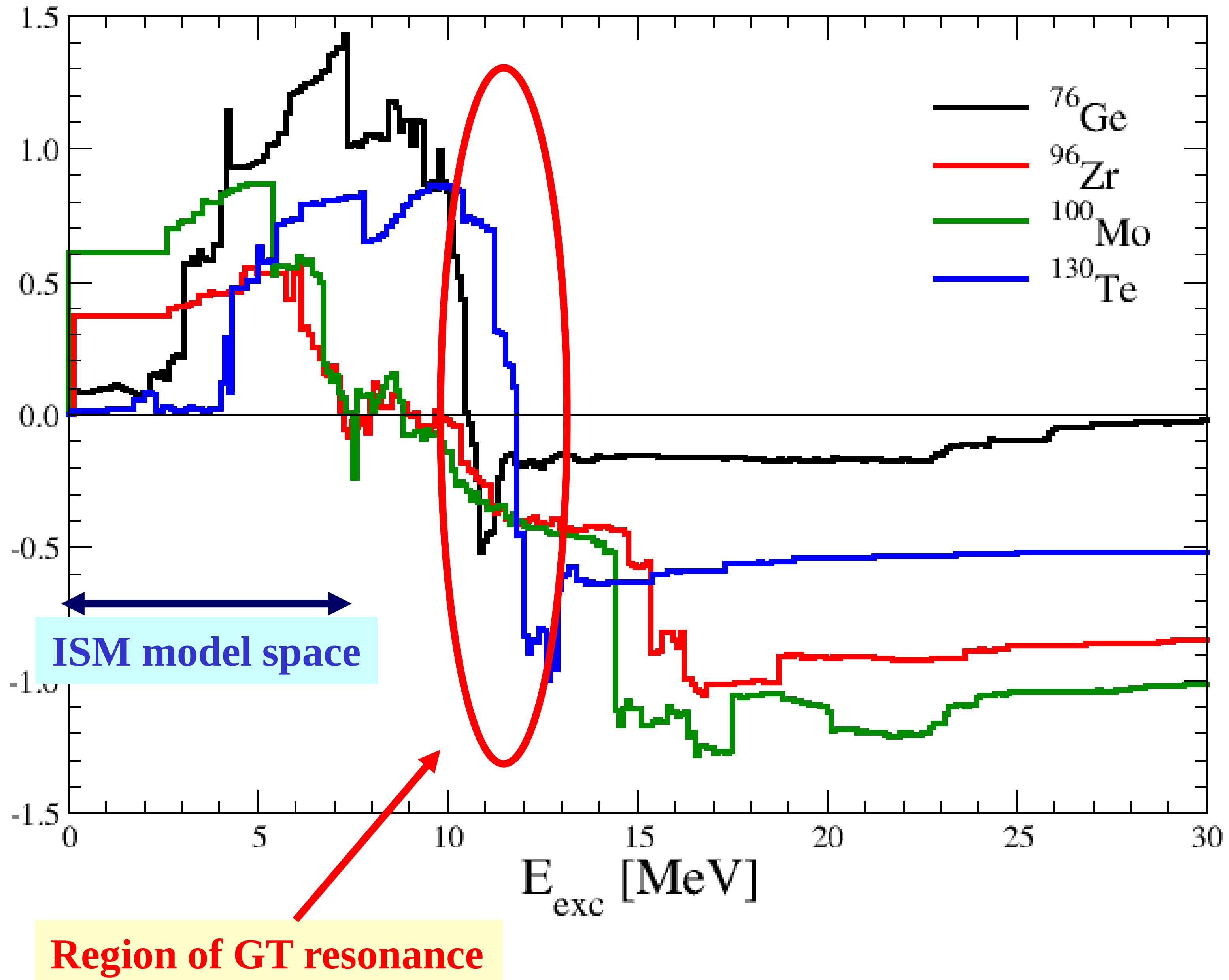
IBM: J. Barea, J. Kotila, F. Iachello, PRC 91, 034304 (2015)

QRPA: F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

M^{DGT} – only 1^+
$M^{0\nu}$ - contribution from many J^π (!)

QRPA: There is no proportionality between 0νββ-decay and 2νββ-decay NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)



Going to relative coordinates:

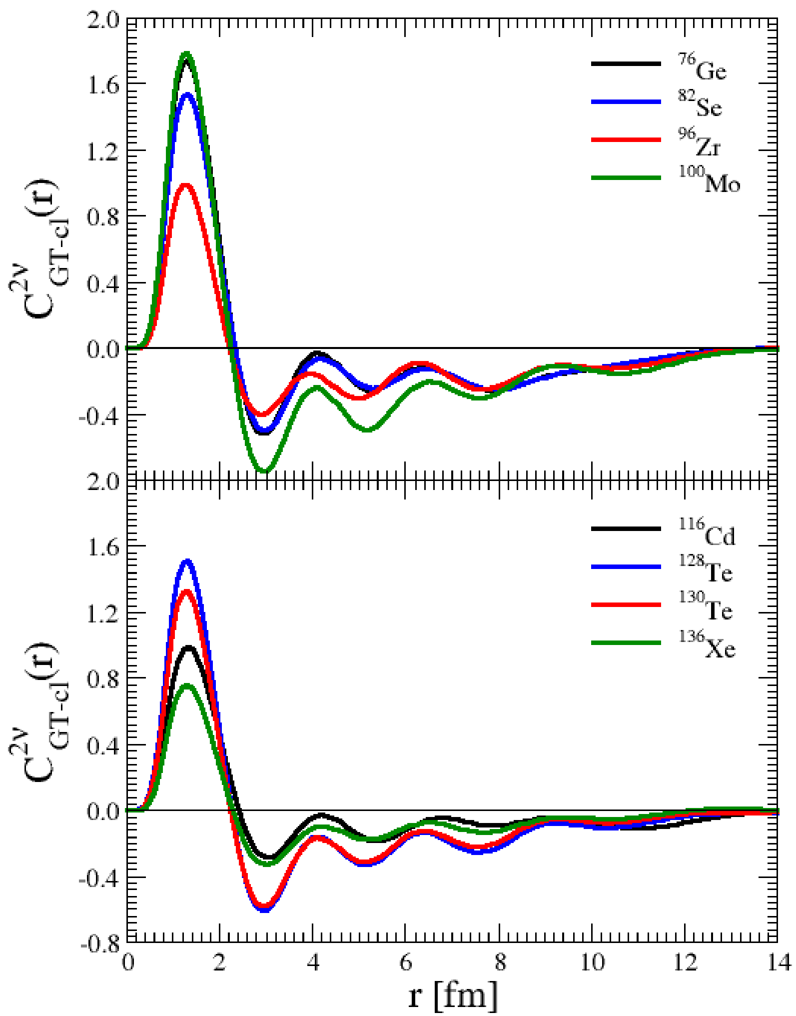
A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

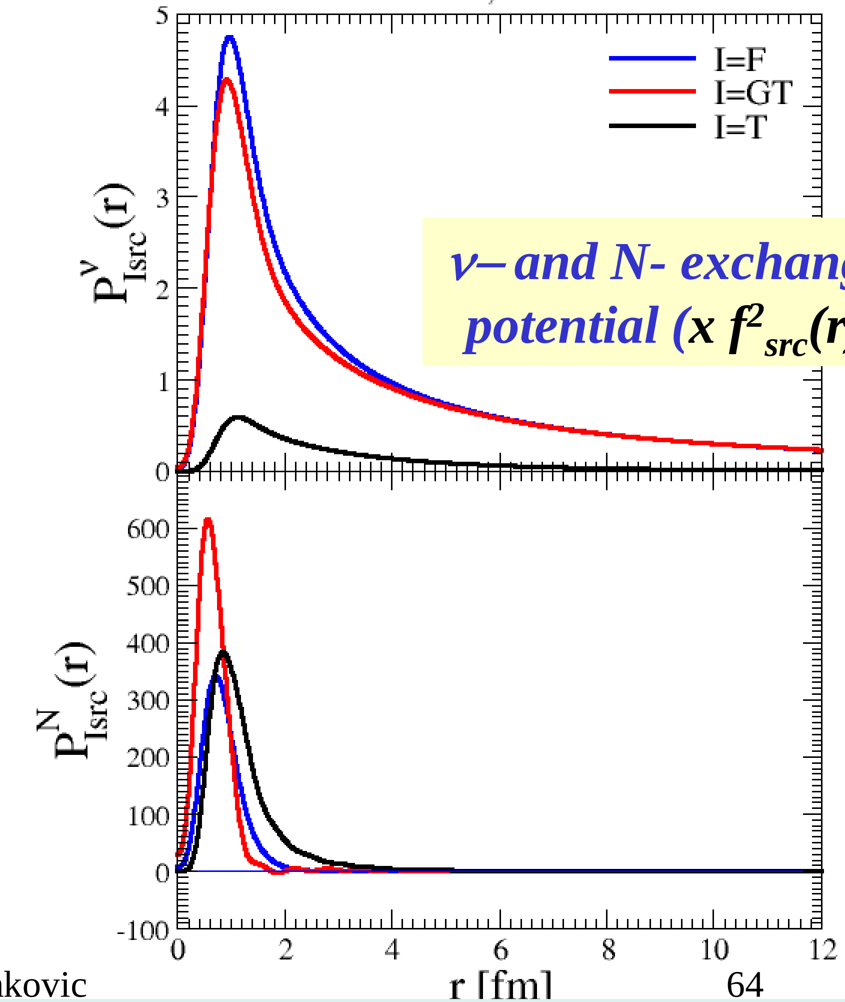
F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r) dr$$

$$M^{0\nu}_{\nu,N-I} = \int_0^\infty P^{\nu,N}_{I-src}(r)\ C^{2\nu}_{I-cl}(r)\ dr = \int_0^\infty f^2_{src}(r)\ P^{\nu,N}_{I}(r)\ C^{2\nu}_{I-cl}(r)\ dr$$

$$I = F,\ GT \text{ and } T$$





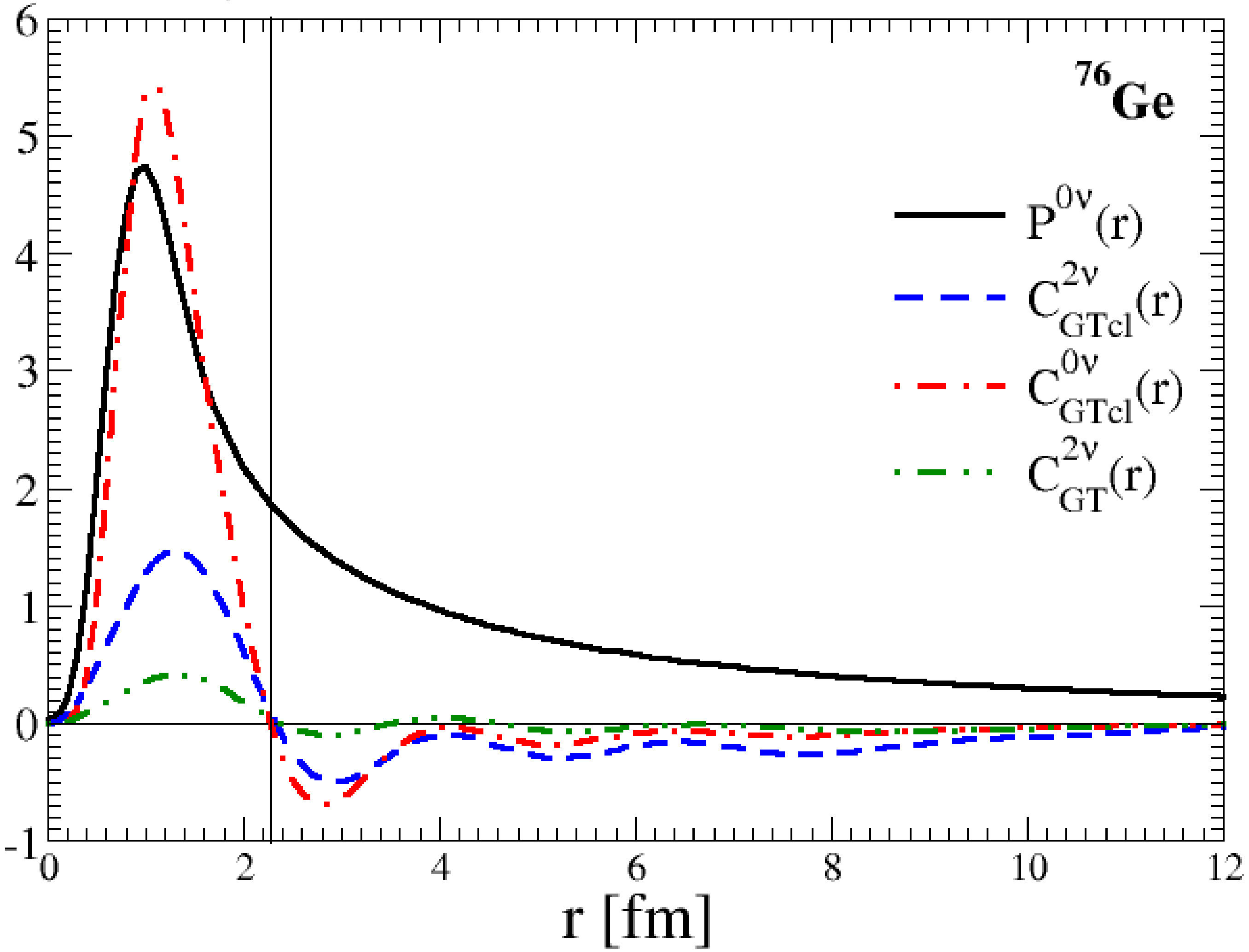
ν– and N- exchange potential (x $f^2_{src}(r)$)

r- relative distance of two decaying nucleons

Neutrino potential prefers short distances

$$M^{0\nu}_{\nu,N-I} = \int_0^\infty P^{\nu,N}_{I-src}(r)\, C^{2\nu}_{I-cl}(r)\, dr$$
$$= \int_0^\infty f^2_{src}(r)\, P^{\nu,N}_{I}(r)\, C^{2\nu}_{I-cl}(r)\, dr$$
$$I = F,\ GT \text{ and } T$$

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r) dr$$



Closure $2\nu\beta\beta$ GT NME

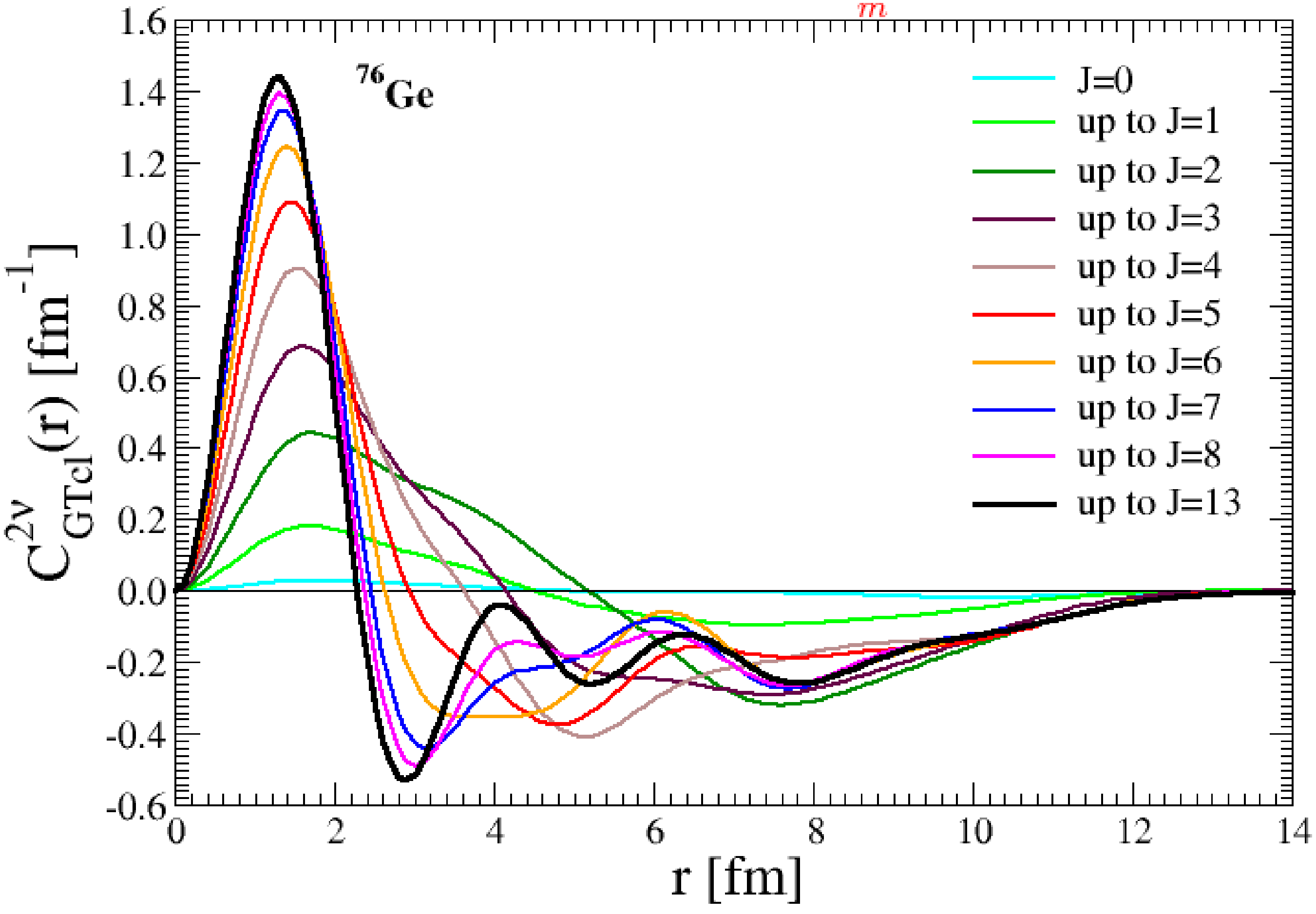
The only non-zero contribution from $J^\pi=1^+$

$$M^{2\nu}_{GT-cl} = \sum_{J^\pi} \int_0^\infty C^{2\nu}_{GT-J^\pi}(r)dr$$

$$M^{2\nu}_{GT-cl} = \sum_{J^\pi,m} < 0^+_f|\tau^+\vec{\sigma}|J^\pi, m > \cdot < J^\pi, m|\tau^+\vec{\sigma}|0^+_i >$$

$$\sum_m < 0^+_f|\tau^+\vec{\sigma}|1^+, m > \cdot < 1^+, m|\tau^+\vec{\sigma}|0^+_i >$$

=

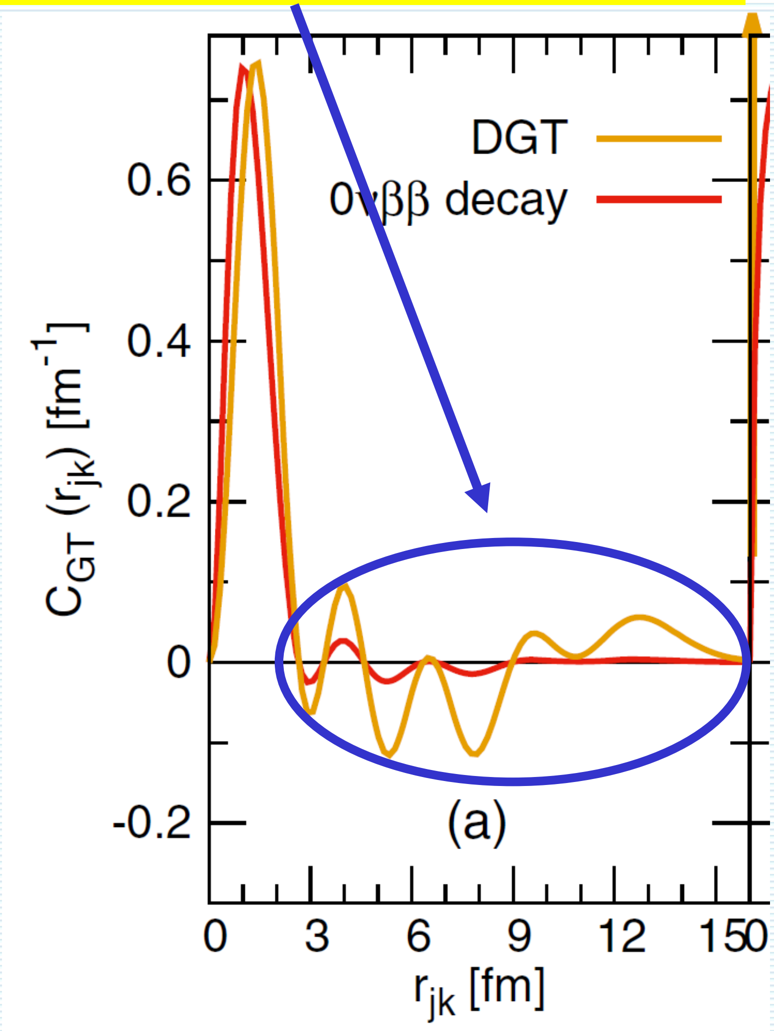


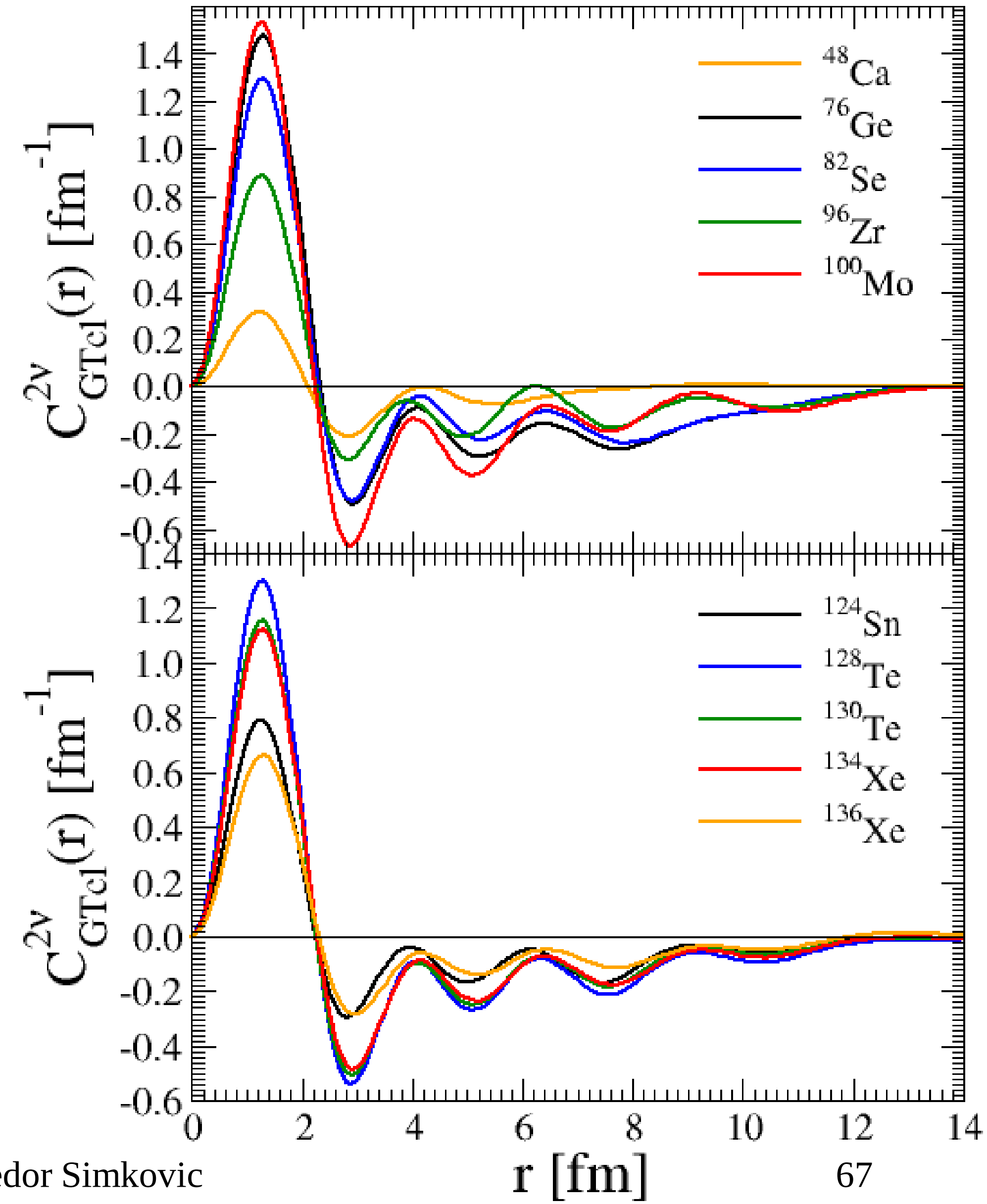
Many multipole contributions not included within the ISM due to truncation of the model space

QRPA: Bump ≈ - Tail => $M^{2\nu}_{cl} \approx 0$

Close to restoration of the SU(4) symmetry of residual Hamiltonian

ISM: Tail ≈ 0 (?!) => $M^{2\nu}_{cl}$ » 0

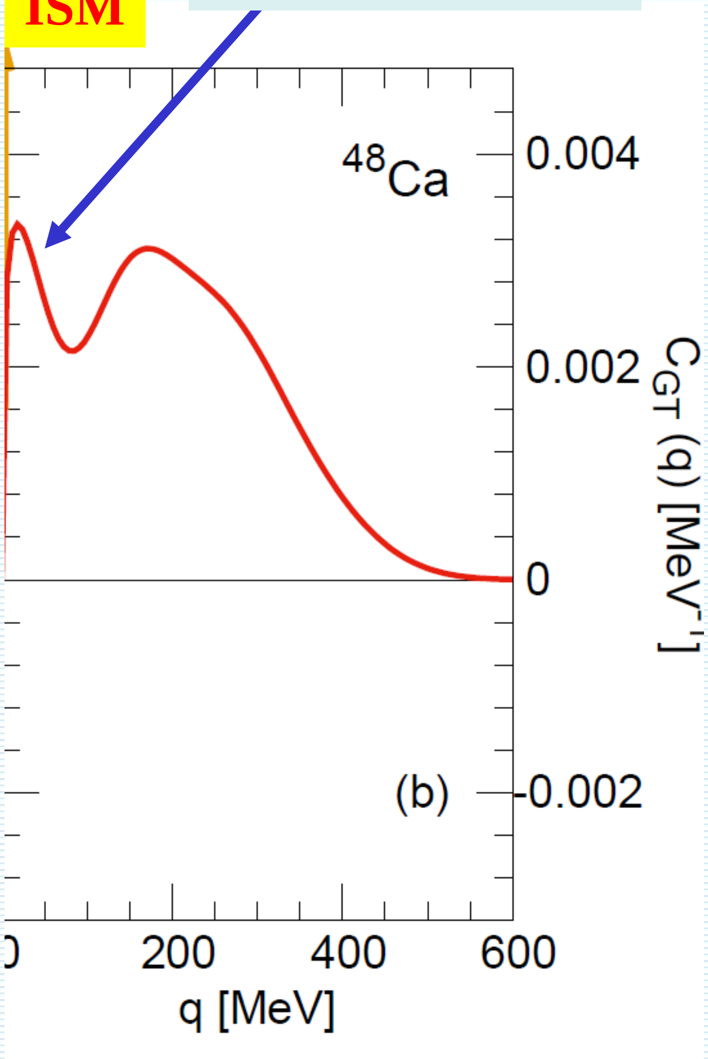




N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)

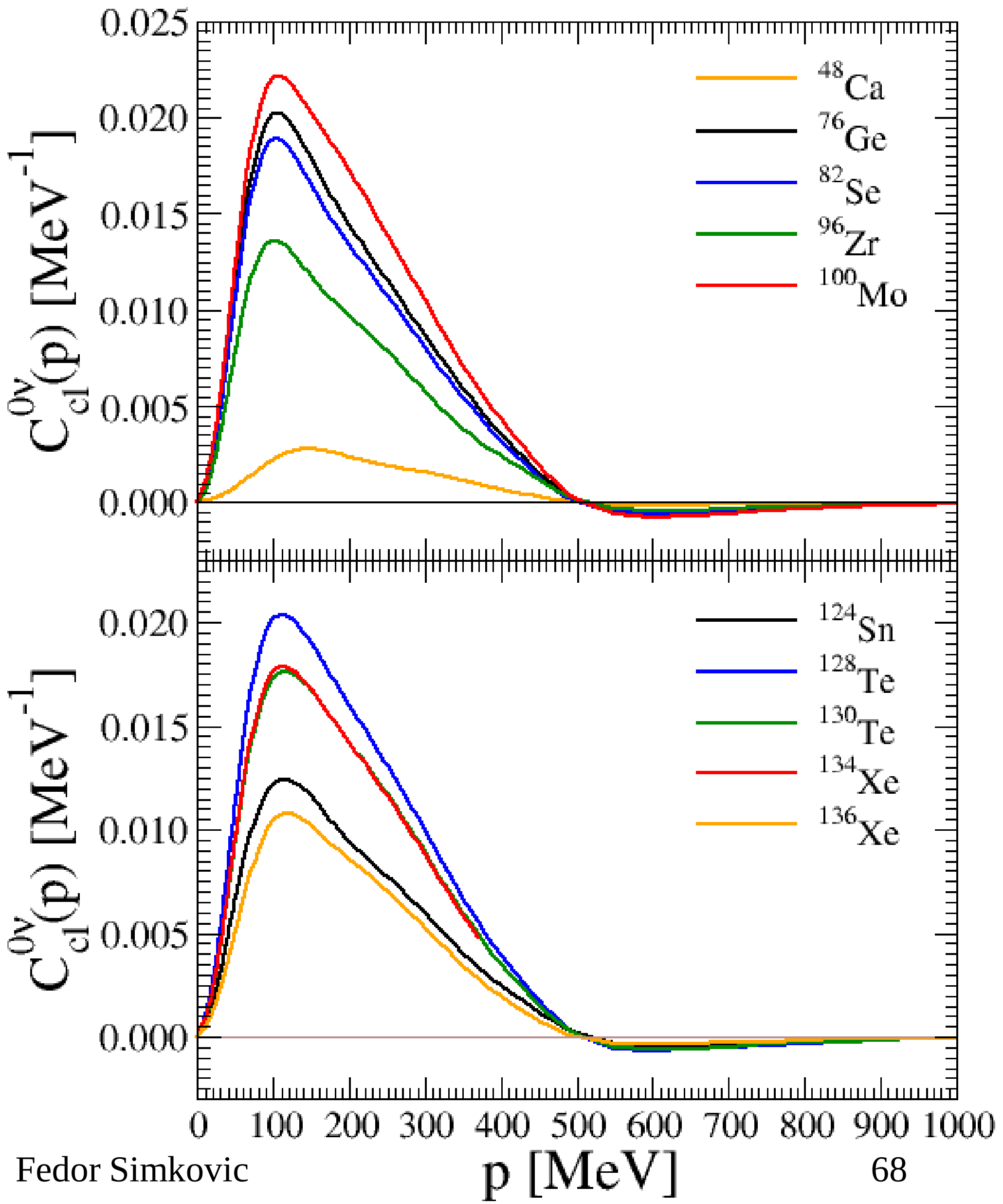
What is the origin of this peak?

ISM

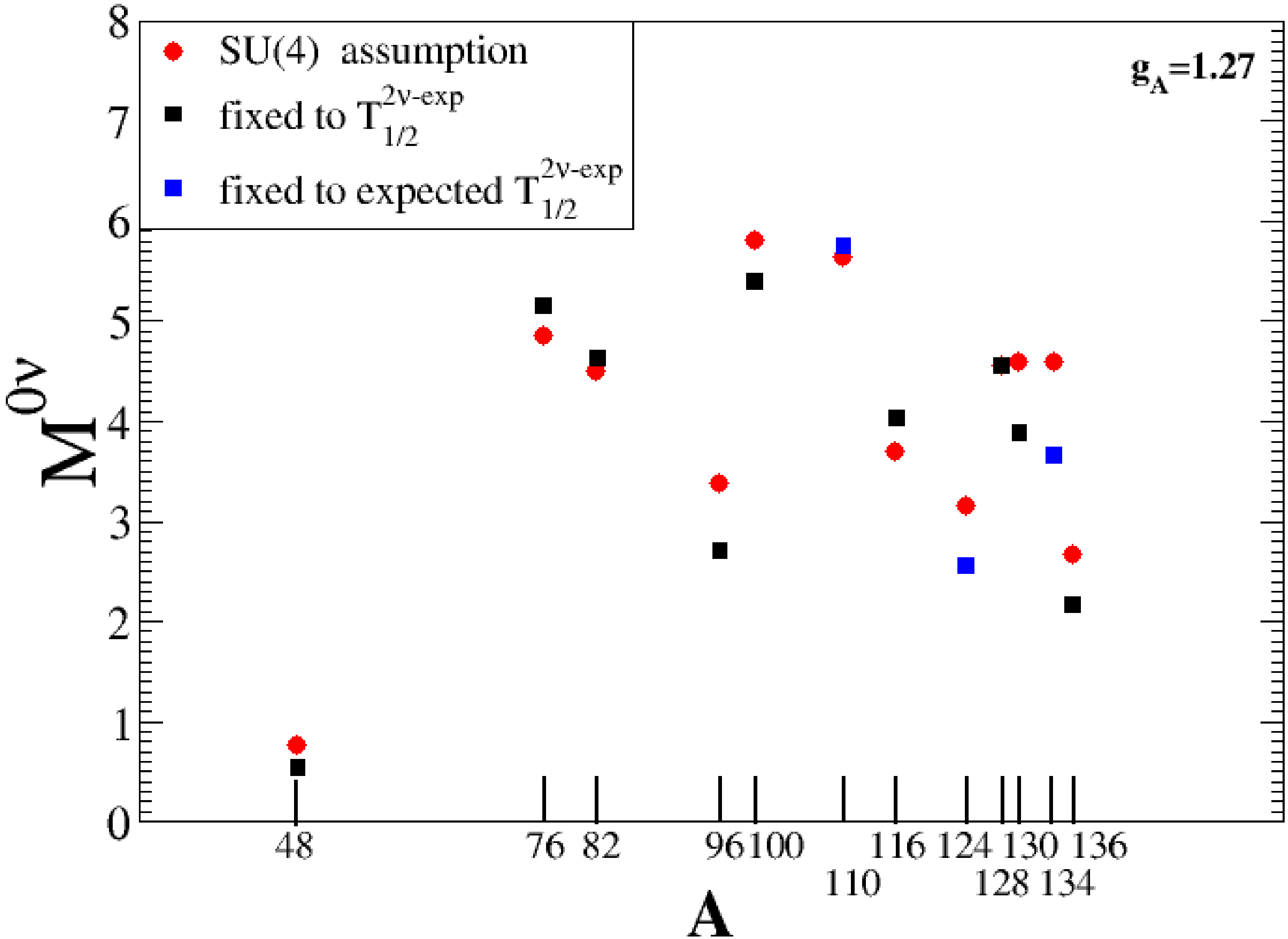


N. Shimizu, J. Menendez, K. Yako, Phys. Rev. Lett. 120, 14502 (2018)

QRPA



QRPA – SU(4) prametrization



2νββ–decay within the QRPA

(restoration of the SU(4) symmetry – $M^{2\nu}_{cl}=0$)

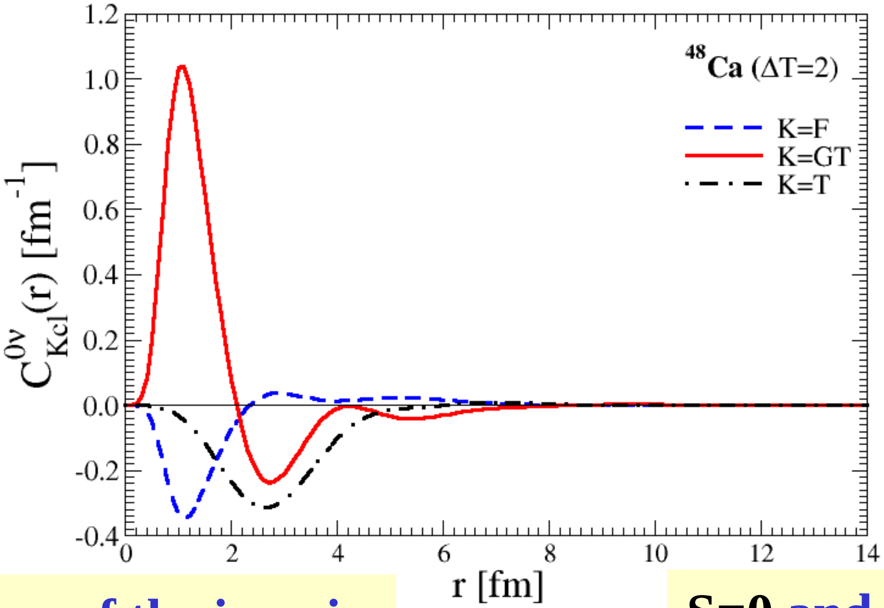
F.Š., A. Smetana, P. Vogel, PRC 98, 064325 (2018)

$$g_A^{\rm eff} = q \times g_A^{\rm free} = 0.901$$
$$g_A^{\rm free} = 1.269, \qquad q = 0.710$$

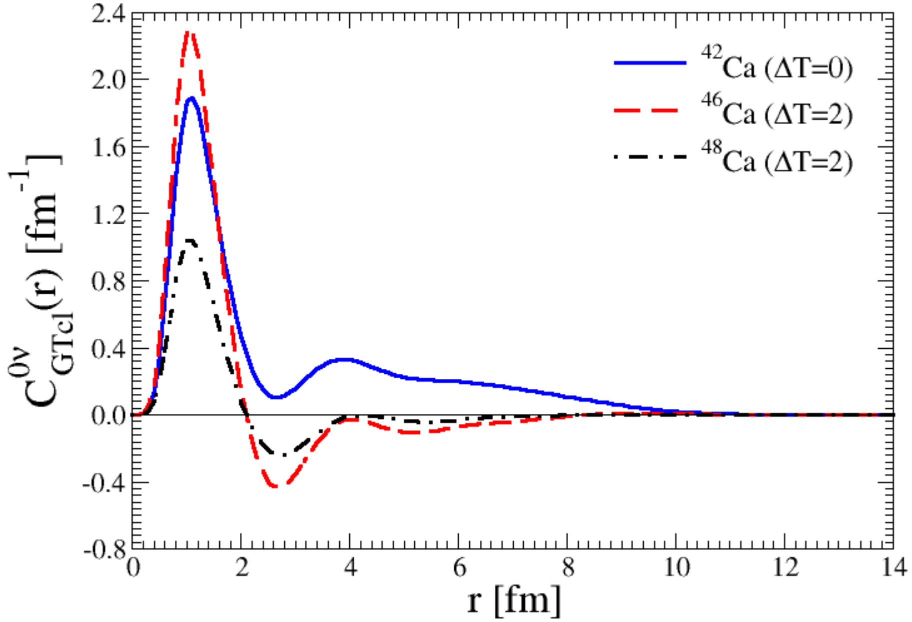
Nucleus	d^i_{pp}	d^f_{pp}	d^i_{nn}	d^f_{nn}	$g^{T=1}_{pp}$	$g^{T=0}_{pp}$	$M^{2\nu}_F$ [MeV^{-1}]	$M^{2\nu}_{GT} \times q^2$ [MeV^{-1}]	$M^{2\nu}_{exp}$ [MeV^{-1}]
^{48}Ca	-	1.069	-	0.982	1.028	0.745	-0.003	0.037	0.046
^{76}Ge	0.922	0.960	1.053	1.085	1.021	0.733	0.003	0.076	0.136
^{82}Se	0.861	0.921	1.063	1.108	1.016	0.737	0.001	0.070	0.100
^{96}Zr	0.910	0.984	0.752	0.938	0.961	0.739	0.001	0.161	0.097
^{100}Mo	1.000	1.021	0.926	0.953	0.985	0.799	-0.001	0.304	0.251
^{116}Cd	0.998	-	0.934	0.890	0.892	0.877	-0.000	0.059	0.136
^{128}Te	0.816	0.857	0.889	0.918	0.965	0.741	0.017	0.075	0.052
^{130}Te	0.847	0.922	0.971	1.011	0.963	0.737	0.016	0.064	0.037
^{136}Xe	0.782	0.885	-	0.926	0.910	0.685	0.014	0.039	0.022

QRPA

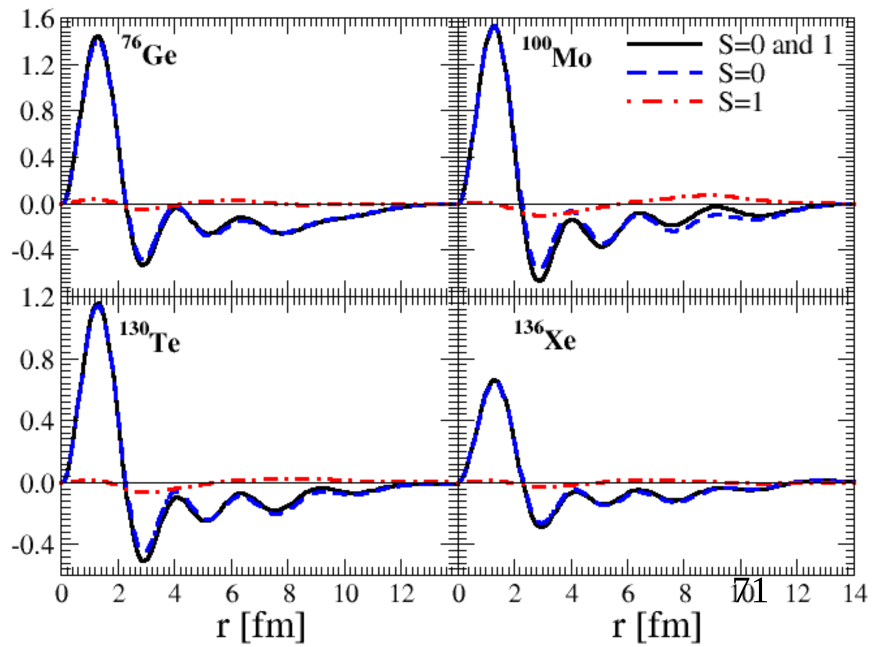
Fermi, Gamow-Teller and tensor



Role of the change of the isospin



S=0 and S=1 contributions



$M^{2\nu}_{F\text{-}cl} = 0$

The DBD Nuclear Matrix Elements and the SU(4) symmetry

D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

$M^{2\nu}_{GT\text{-}cl} = 0$

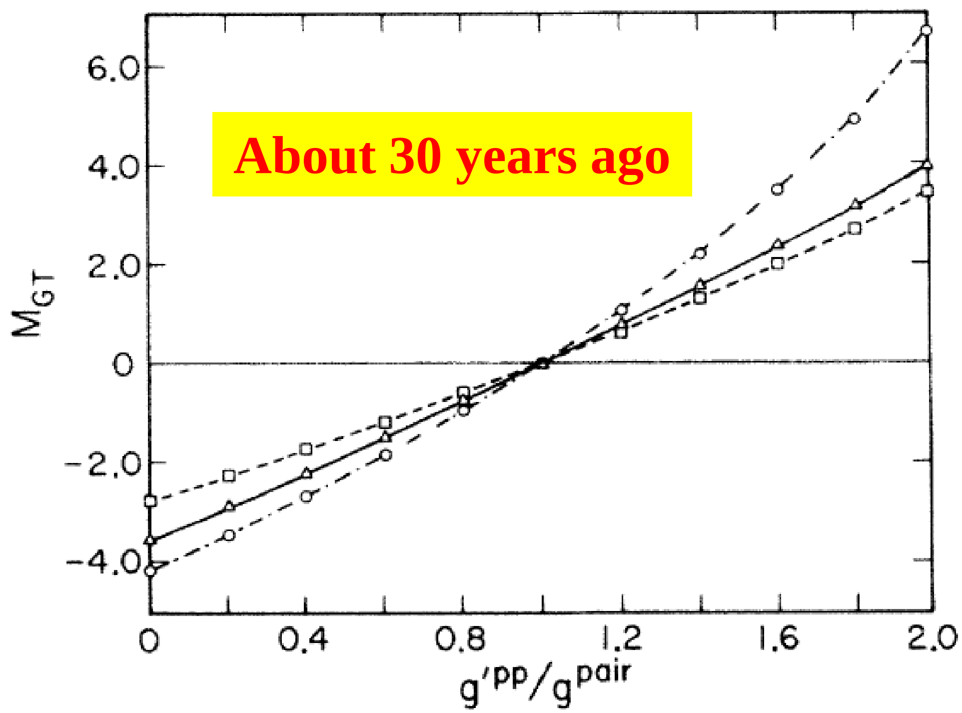
Suppression of the Two Neutrino Double Beta Decay by Nuclear Structure Effects

P. Vogel, M.R. Zirnbauer, PRL (1986) 3148

O. Civitarese, A. Faessler, T. Tomoda, PLB 194 (1987) 11

E. Bender, K. Muto, H.V. Klapdor, PLB 208 (1988) 53

…



The isospin is known to be a good approximation in nuclei

In heavy nuclei the SU(4) symmetry is strongly broken by the spin-orbit splitting.

What is beyond this behavior? Is it an artifact of the QRPA?

s.p. mean-field

Conserves SU(4) symmetry

$$H = \underbrace{e_n N_n + e_p N_p - g_{pair}\left(\sum_{M_T=-1,0,1} A^\dagger_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{a,b} E^\dagger_{a,b}E_{a,b}}_{H_0}$$

$$\underbrace{+ (g_{pair} - g_{pp}^{T=0}) \sum_{M_S=-1,0,1} A^\dagger_{1,0}(M_S,0)A_{1,0}(M_S,0) + (g_{pair} - g_{pp}^{T=1})A^\dagger_{0,1}(0,0)A_{0,1}(0,0)\,.}_{H_I}$$

H_I violates SU(4) symmetry

g_{pair}- strength of isovector like nucleon pairing (L=0, S=0, T=1, M_T=±1)

$g_{pp}^{T=1}$- strength of isovector spin-0 pairing (L=0, S=0, T=1, M_T=0

$g_{pp}^{T=0}$- strength of isoscalar spin-1 pairing (L=0, S=1, T=0)

g_{ph}- strength of particle-hole force

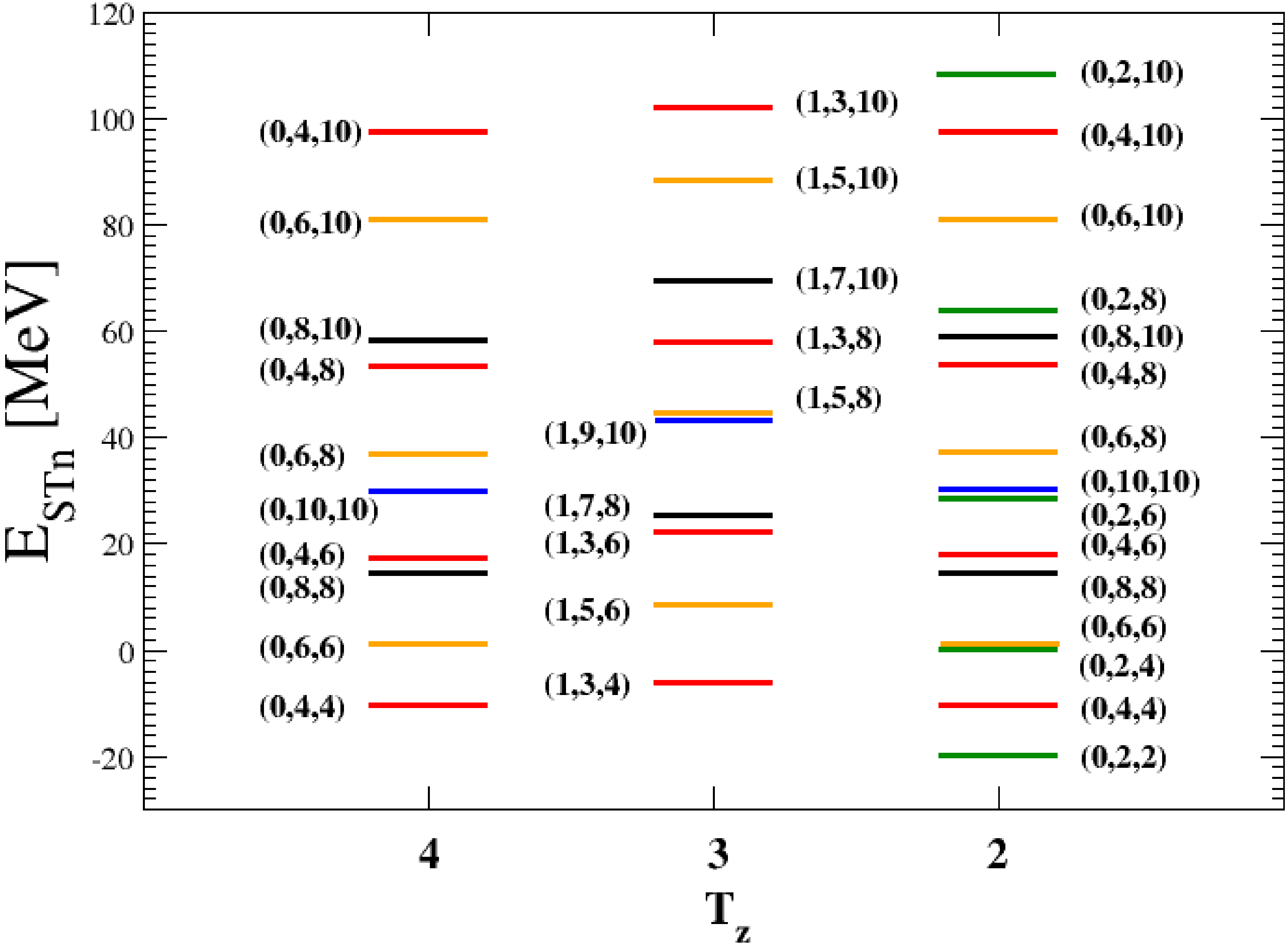
M_F and M_{GT} do not depend on the mean-field part of **H** and are governed by a weak violation of the **SU(4)** symmetry by the particle-particle interaction of **H**

$$M_F^{2\nu} = -\frac{48\sqrt{\frac{33}{5}}\left(g_{pair} - g_{pp}^{T=1}\right)}{(5g_{pair}+3g_{ph})(10g_{pair}+6g_{ph})}$$

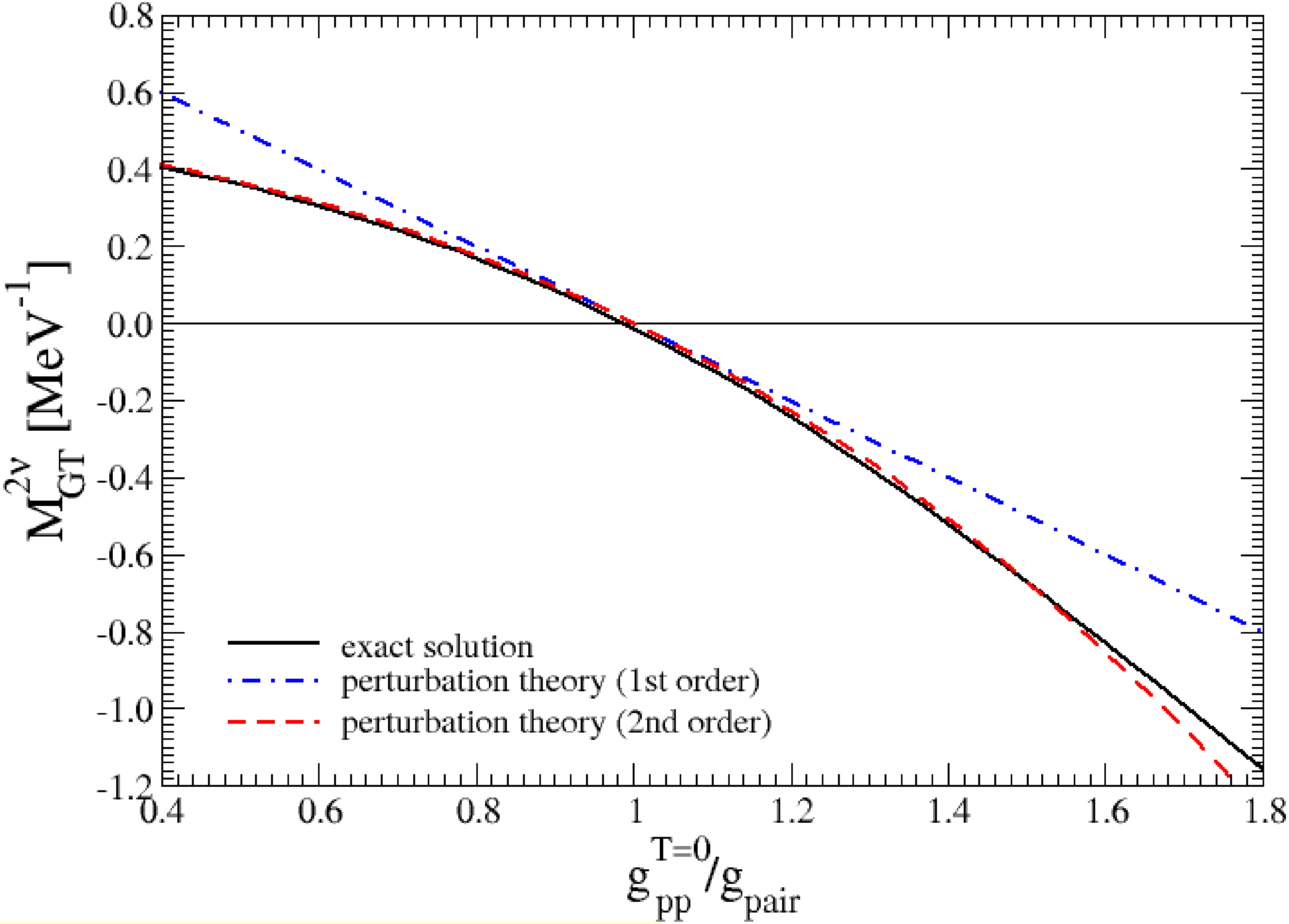
$$M_{GT}^{2\nu} = \frac{144\sqrt{\frac{33}{5}}}{5g_{pair}+9g_{ph}}\left\{\frac{(g_{pair}-g_{pp}^{T=0})}{(10g_{pair}+20g_{ph})} + \frac{2g_{ph}(g_{pair}-g_{pp}^{T=1})}{(10g_{pair}+20g_{ph})(10g_{pair}+6g_{ph})}\right\}$$

Energies of excited states for the case of conserved SU(4) symmetry

$M_F=0$, $M_{GT}=0$ (see SU(4) multiplets)



M_{GT} up to the second order of perturbation theory due to violation of the SU(4) symmetry by the particle-particle interaction of H

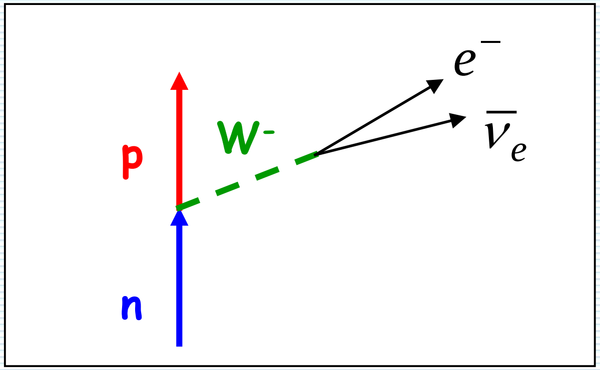


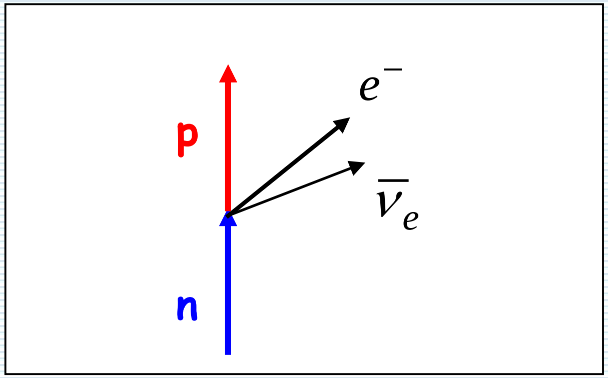
D. Štefánik, F.Š., A. Faessler, PRC 91, 064311 (2015)

V. Quenching of g_A

Quenching in nuclear matter: $g^{eff}_A = q\, g^{free}_A$

(from theory: $T_{1/2}^{0\nu}$ up 50 x larger)





$$\mathcal{L} = -\frac{G_\beta}{\sqrt{2}} \left[\overline{u}\gamma^\alpha(1-\gamma^5)d\right] \left[\overline{e}\gamma^\alpha(1-\gamma^5)\nu_e\right]$$

$$\mathcal{L} = -\frac{G_\beta}{\sqrt{2}} \left[\overline{p}\gamma^\alpha(g_V - g_A\gamma^5)n\right] \left[\overline{e}\gamma^\alpha(1-\gamma^5)\nu_e\right]$$

CVC hypothesis

g_V =1 at the quark level

g_V =1 at the nucleon level

g_V =1 inside nuclei

Quenching of g_A

g_A =1 at the quark level

g^{free}_A =1.27 at the nucleon level

g^{eff}_A = ? inside nuclei

ISM: $(g^{eff}_A)^4 \simeq 0.66$ (^{48}Ca), 0.66 (^{76}Ge), 0.30 (^{76}Se), 0.20 (^{130}Te) and 0.11 (^{136}Xe)

IBM: $(g^{eff}_A)^4 \simeq (1.269\, A^{-0.18})^4 = 0.063$

QRPA: $(g^{eff}_A)^4 = 0.30$ and 0.50 for ^{100}Mo and ^{116}Cd

$g_A^4 = (1.269)^4 = 2.6$

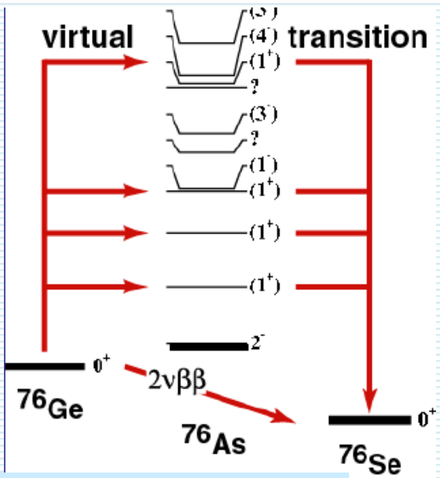
Quenching of g_A (from exp.: $T_{1/2}^{0\nu}$ up 2.5 x larger)

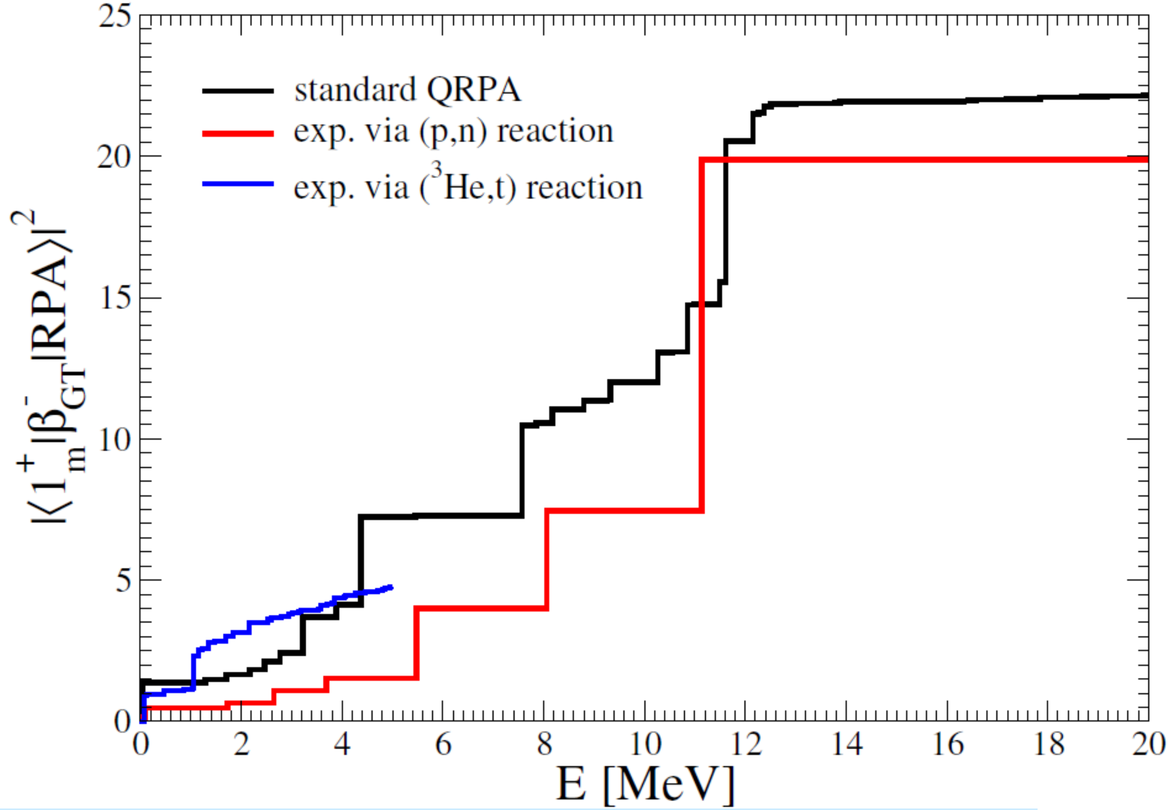
$(g^{eff}_A)^4 = 1.0$

Strength of GT trans. (approx. given by Ikeda sum rule =3(N-Z)) has to be quenched to reproduce experiment

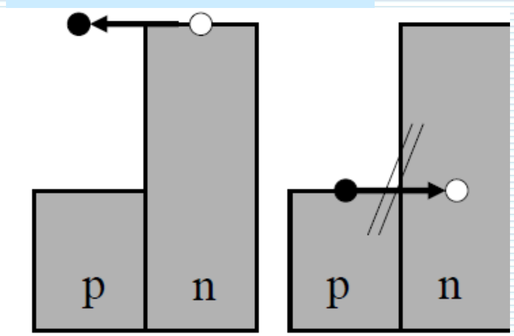
$^{76}_{32}Ge_{44} \Rightarrow$

$S_{\beta^-} - S_{\beta^+} = 3(N-Z) = 36$





Pauli blocking



Cross-section for charge exchange reaction:

$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{\mu}{\pi\hbar}\right]^2 \frac{k_f}{k_i} N_d \, |V_{\sigma\tau}|^2 \, |\langle f \,|\, \sigma\tau \,|\, i \rangle|^2$$

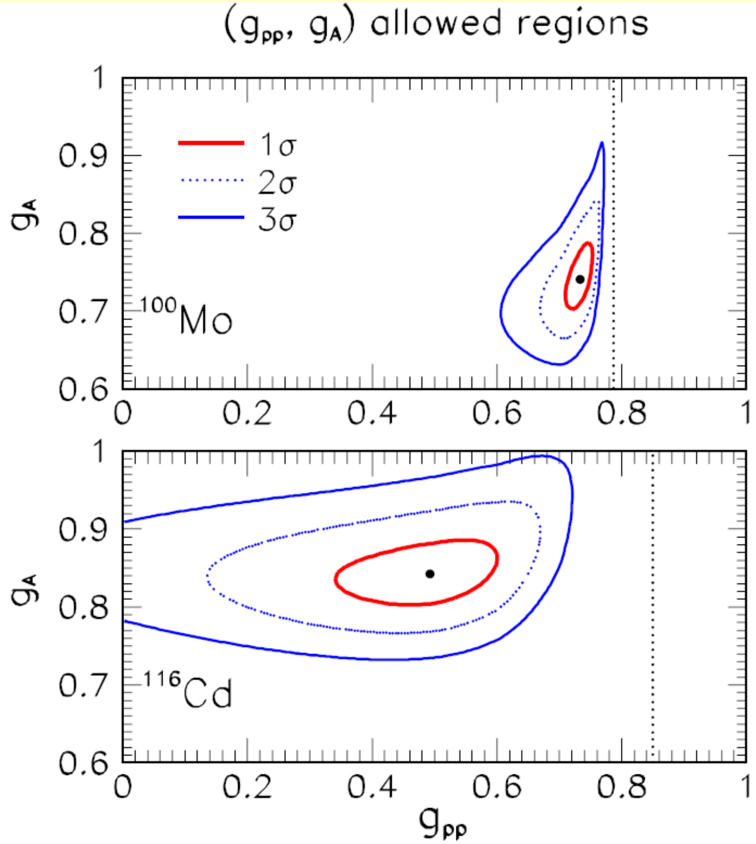
$q = 0!!$

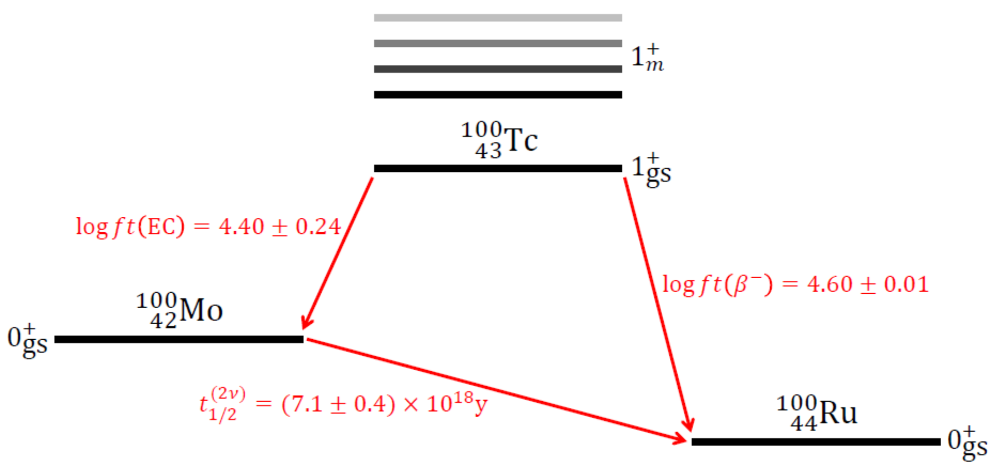
largest at 100 - 200 MeV/A

Faessler, Fogli, Lisi, Rodin, Rotunno, F. Š, J. Phys. G 35, 075104 (2008).

$(g^{eff}_A)^4 = 0.30$ and 0.50 for ^{100}Mo and ^{116}Cd, respectively (The QRPA prediction).

g^{eff}_A was treated as a completely free parameter alongside g_{pp} (used to renormalize particl-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of g^{eff}_A and g_{pp}, where possible, to the β-decay rate and β+/EC rate of the J = 1^+ ground state in the intermediate nuclei involved in double-beta decay in addition to the 2νββ rates of the initial nuclei, leads to an effective g^{eff}_A of about 0.7 or 0.8.





Extended calculation also for neighbor isotopes performed by

F.F. Depisch and J. Suhonen, PRC 94, 055501 (2016)

r Simkovic

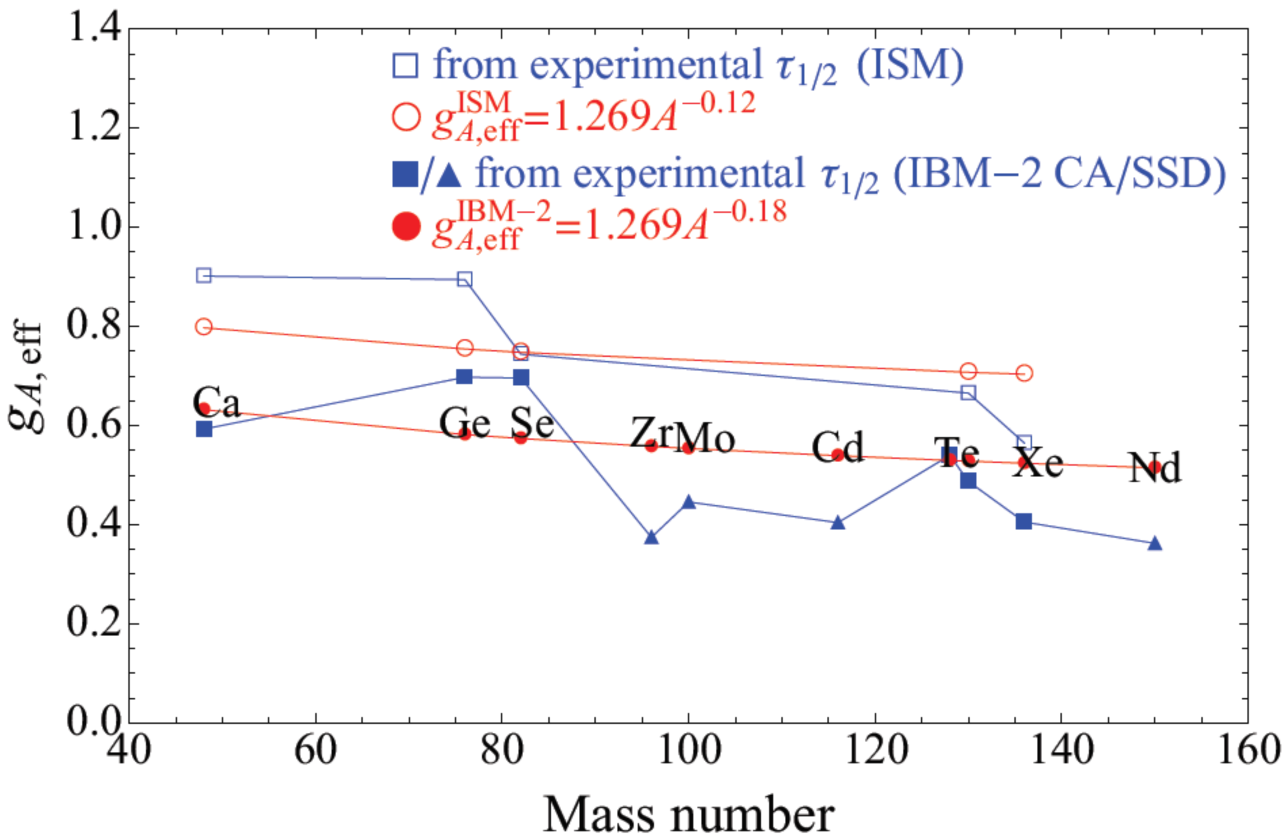
Dependence of g^{eff}_A on A was not established.

Quenching of g_A -IBM *($T_{1/2}^{0\nu}$ suppressed up to factor 50)*

$(g^{eff}_A)^4 \simeq (1.269\ A^{-0.18})^4 = 0.063$ (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.

It has been determined by theoretical prediction for the 2νββ-decay half-lives, which were based on within closure approximation calculated Corresponding NMEs, with the measured half-lives.

From F. Iachello



3/5/2019

J. Barea, J. Kotila, F. Iachello, PRC 87, 014315 (2013).

Improved description of the 0νββ–decay rate (and novel approach of fixing g_A^{eff})

Let perform Taylor expansion

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \varepsilon_{K,L}^2}$$

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2}$$

$$\epsilon_K = (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2$$
$$\epsilon_L = (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2$$

$$\epsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

We get

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\rm eff}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{\left|\xi_{13}^{2\nu}\right|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

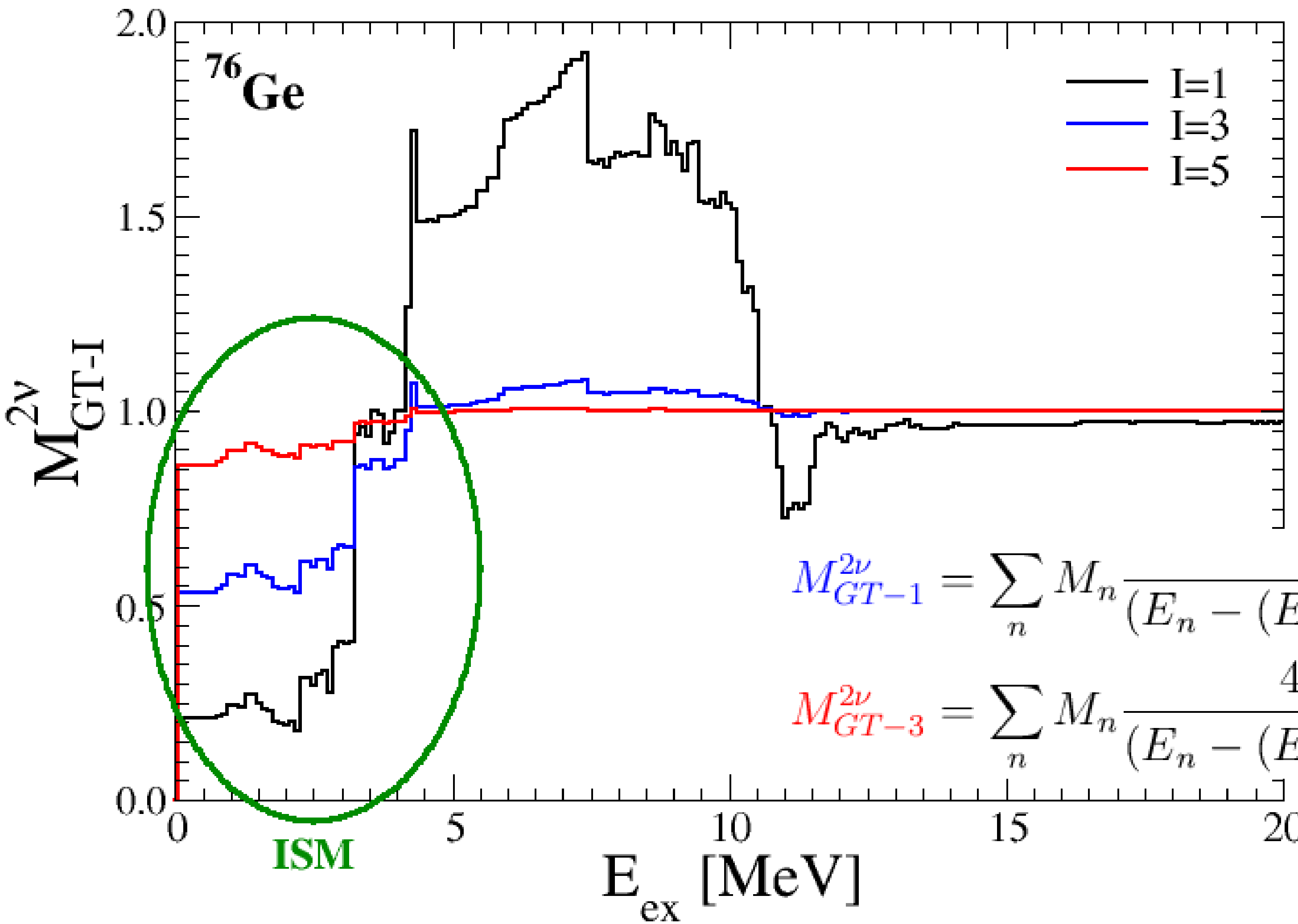
$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4\, m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

$$\xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

The g_A^{eff} can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)

The running sum of the 2νββ–decay NMEs (QRPA)

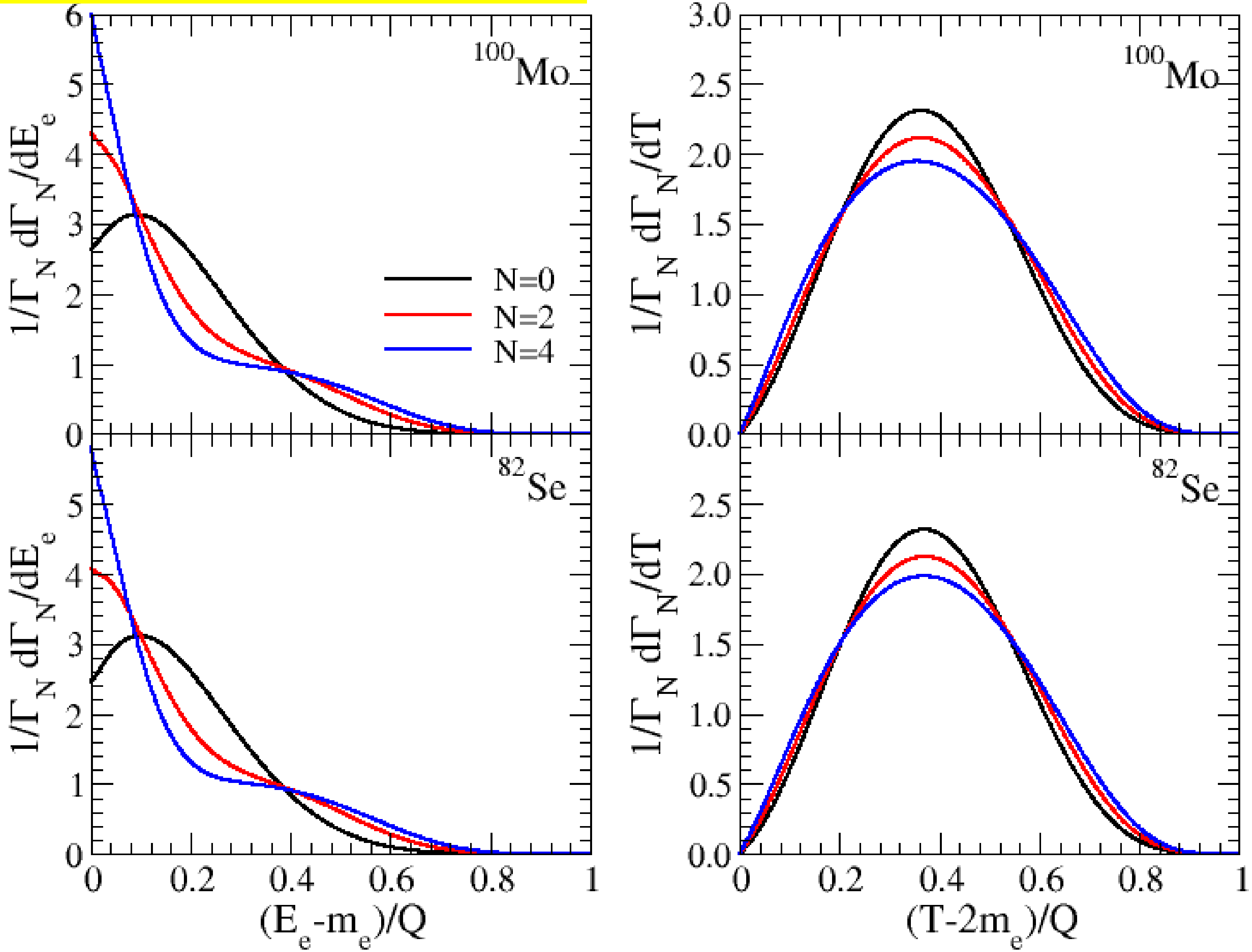


$$M^{2\nu}_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

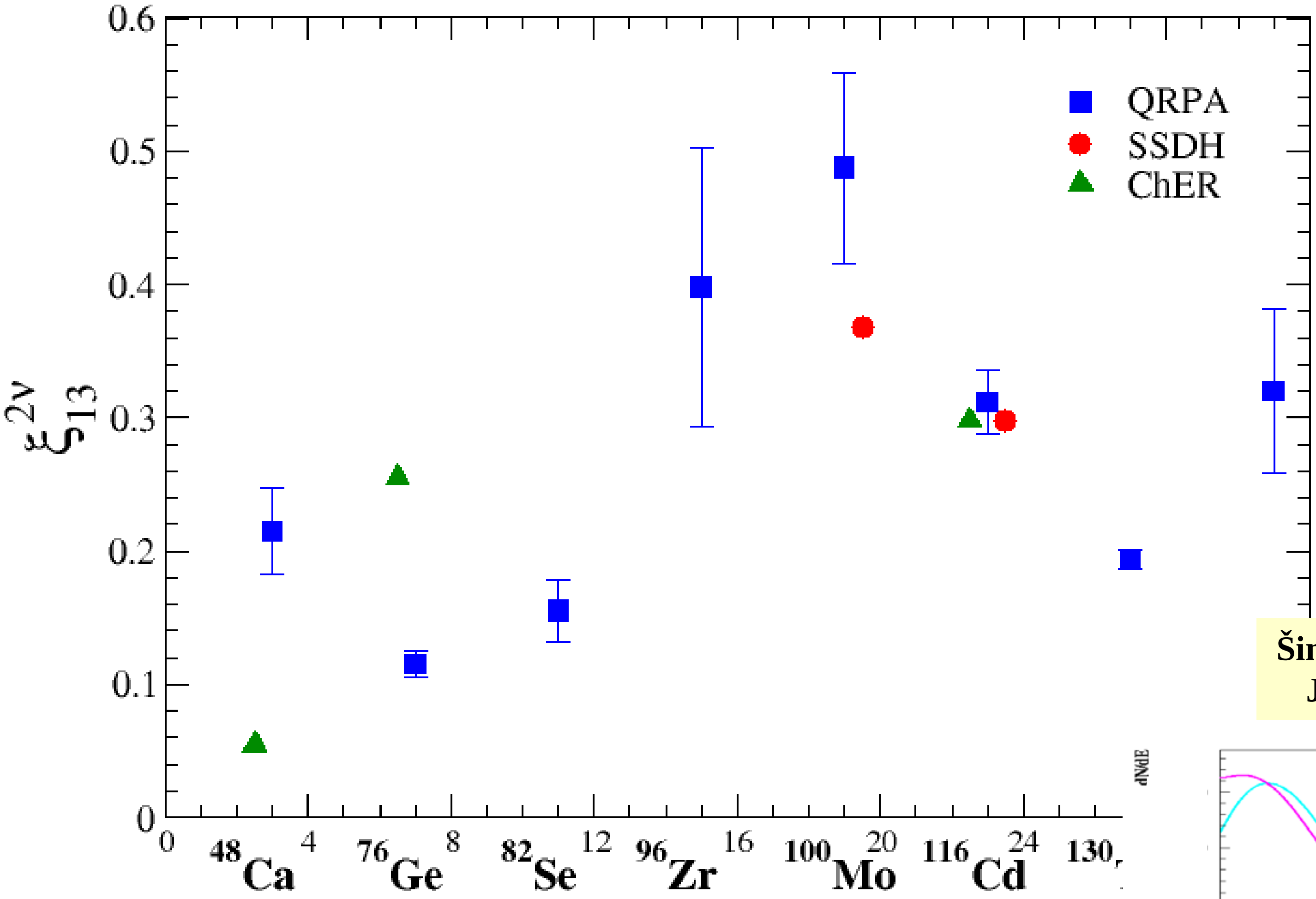
$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

Normalized to unity
different partial energy distributions

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \equiv \frac{\Gamma^{2\nu}}{\ln(2)} \simeq \frac{\Gamma_0^{2\nu} + \Gamma_2^{2\nu} + \Gamma_4^{2\nu}}{\ln(2)}$$

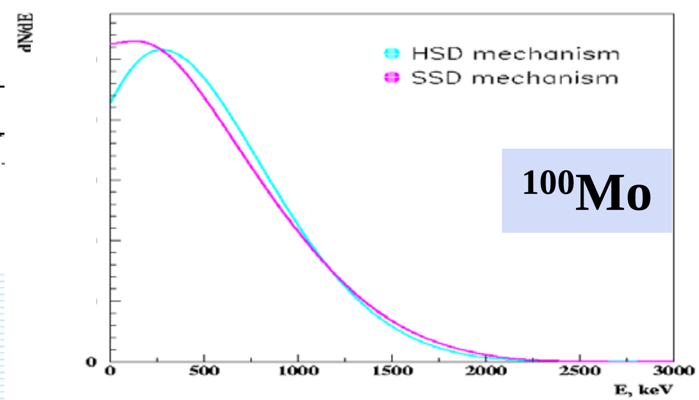


ξ_{13} tell us about importance of higher lying states of int. nucl.



HSD: ξ_{13}=0

Šimkovic, Šmotlák, Semenov
J. Phys. G, 27, 2233, 2001

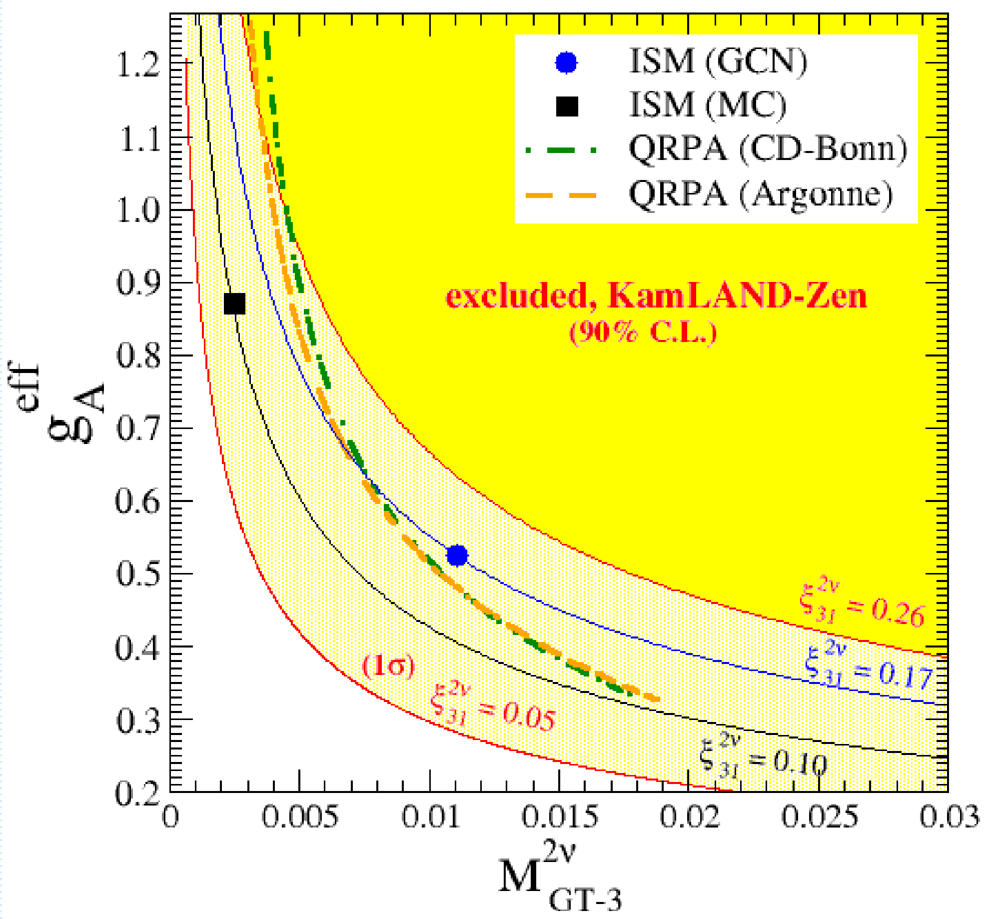


ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

Solution: measurement of ξ and calculation of $M_{GT\text{-}3}$

$M_{GT\text{-}3}$ have to be calculated by nuclear theory - ISM

$$\left(g_A^{\rm eff}\right)^2 = \frac{1}{\left|M^{2\nu}_{GT-3}\right|} \frac{\left|\xi^{2\nu}_{13}\right|}{\sqrt{T^{2\nu-exp}_{1/2}\left(G^{2\nu}_0 + \xi^{2\nu}_{13} G^{2\nu}_2\right)}}$$



KamLAND-Zen Coll. (+J. Menendez, F.Š.), arXiv: 1901.03871 [hep-ex]

VI. New modes of the double beta decay

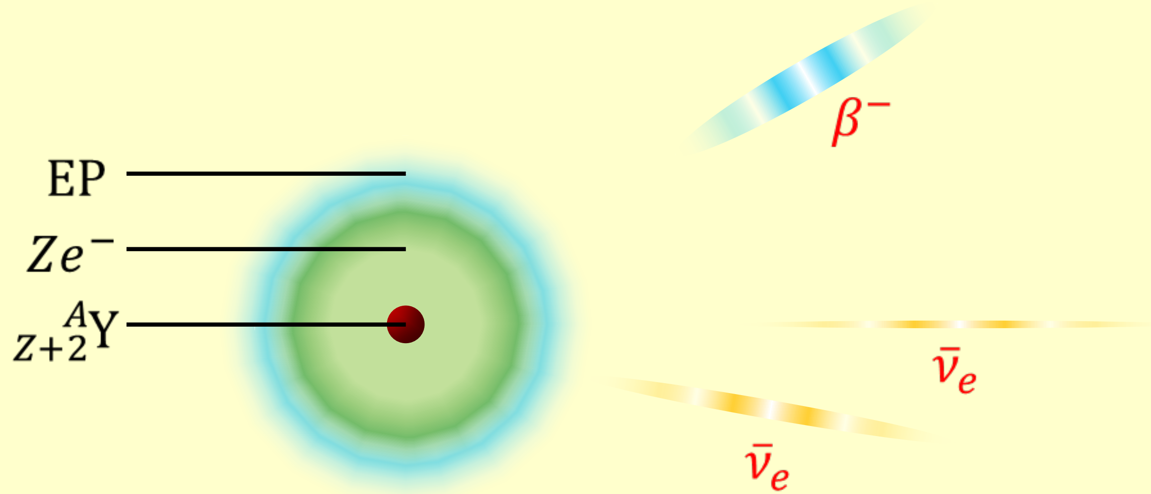
Double Beta Decay with emission of a single electron

A. Babič, M.I. Krivoruchenko, F.Š., PRC 98, 065501 (2018)

[Jung *et al.* (GSI), **1992**] **observed beta decay of** ${}^{163}_{66}\mathrm{Dy}^{66+}$ **ions with Electron Production** (EP) **in** K **or** L **shells:** $T_{1/2}^{\mathrm{EP}} = 47\ \mathrm{d}$

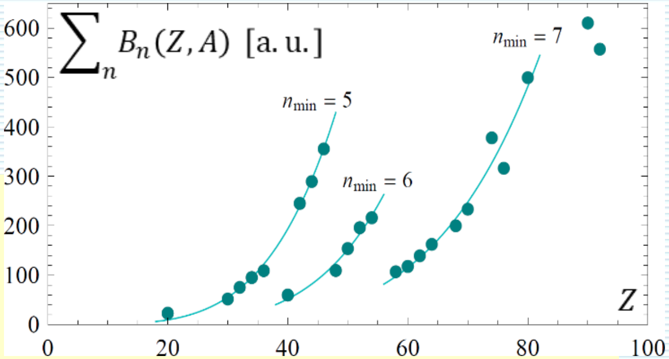
Bound-state double-beta decay $0\nu\mathrm{EP}\beta^-$ ($2\nu\mathrm{EP}\beta^-$) **with** EP **in available** $\mathrm{s}_{1/2}$ **or** $\mathrm{p}_{1/2}$ **subshell of daughter** 2+ **ion:**

$$ {}^{A}_{Z}\mathrm{X} \longrightarrow {}_{Z+2}^{\ \ A}\mathrm{Y} + e_{\mathrm{b}}^{-} + e^{-} + (\bar{\nu}_e + \bar{\nu}_e) $$



Search for possible manifestation in single-electron spectra…

Phase space factors



$0\nu EP\beta^-$ and $2\nu EP\beta^-$ phase-space factors:

$$G^{0\nu\mathrm{EP}\beta}(Z,Q) = \frac{G_\beta^4\, m_e^2}{32\pi^4 R^2 \ln 2} \sum_{n=n_{\min}}^{\infty} B_n(Z,A)\, F(Z+2,E)\, E\, p$$

$$G^{2\nu\mathrm{EP}\beta}(Z,Q) = \frac{G_\beta^4}{8\pi^6 m_e^2 \ln 2} \sum_{n=n_{\min}}^{\infty} B_n(Z,A) \int_{m_e}^{m_e+Q} \mathrm{d}E\, F(Z+2,E)\, E\, p \int_0^{m_e+Q-E} \mathrm{d}\omega_1\, \omega_1^2\, \omega_2^2$$

Single-electron spectra for ^{82}Se ($Q = 2.998\ MeV$):

Bound- and free-electron Fermi functions:

$$B_n(Z,A) = f_{n,-1}^2(R) + g_{n,+1}^2(R)$$
$$F(Z,E) = f_{-1}^2(R,E) + g_{+1}^2(R,E)$$

Relativistic electron wave functions in central field:

$$\psi_{\kappa\mu}(\vec{r}) = \begin{pmatrix} f_\kappa(r)\,\Omega_{\kappa\mu}(\hat{r}) \\ i g_\kappa(r)\,\Omega_{-\kappa\mu}(\hat{r}) \end{pmatrix}$$

$\kappa = (l-j)(2j+1) = \pm 1, \pm 2, \dots$

$j = |l \pm 1/2|$ $\quad \mu = -j, \dots, +j$

CALCULATION: GRASP2K

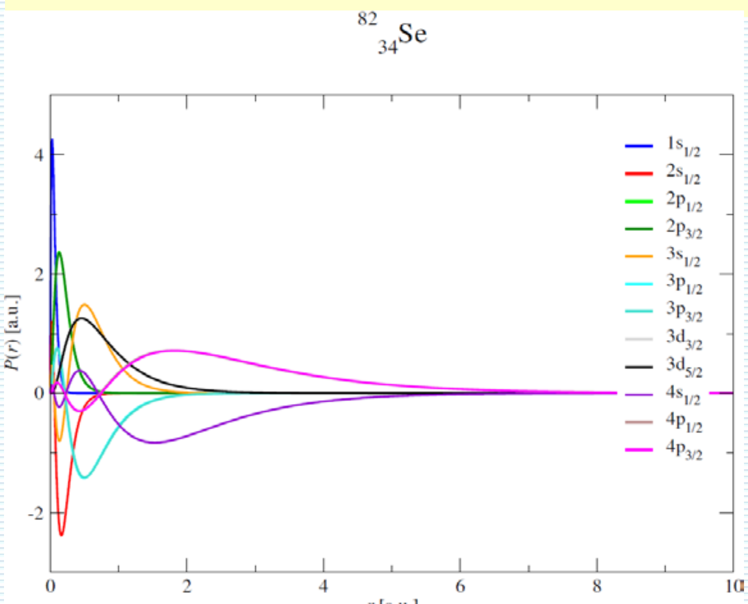
Stationary N-particle Dirac eq. with separable central atomic Hamiltonian $[a.u.]$:

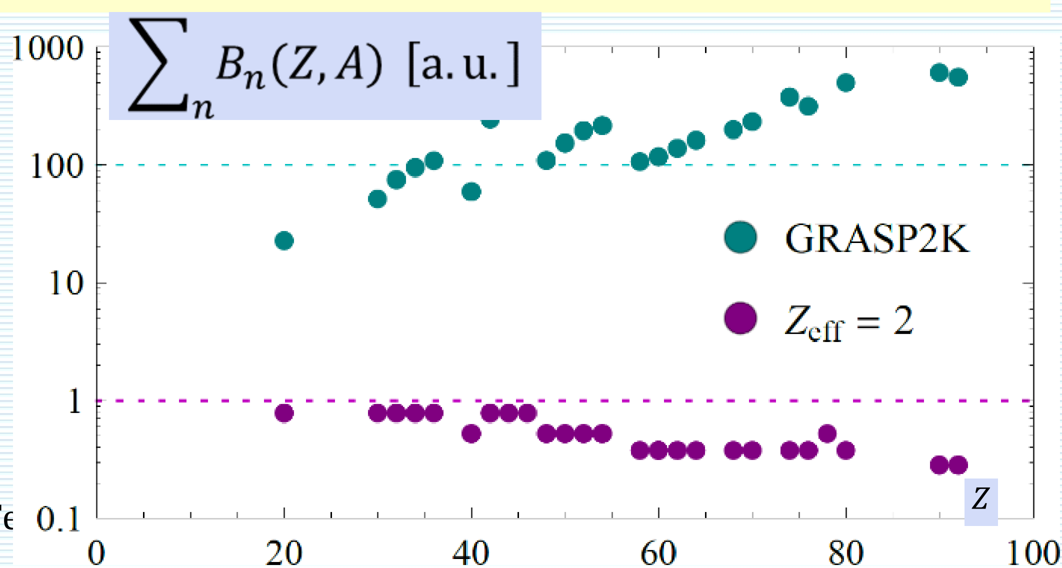
$$\left[\sum_{i=1}^{N} -i\nabla_i \cdot \vec{\alpha} c + \beta c^2 - \frac{Z}{r_i} + V(r_i)\right]\Psi = E\,\Psi$$

$$\Psi = \frac{1}{\sqrt{N!}}\begin{vmatrix} \psi_1(\vec{r}_1) & \cdots & \psi_1(\vec{r}_N) \\ \vdots & \ddots & \vdots \\ \psi_N(\vec{r}_1) & \cdots & \psi_N(\vec{r}_N) \end{vmatrix}$$

Multiconfiguration Dirac–Hartree–Fock package GRASP2K:

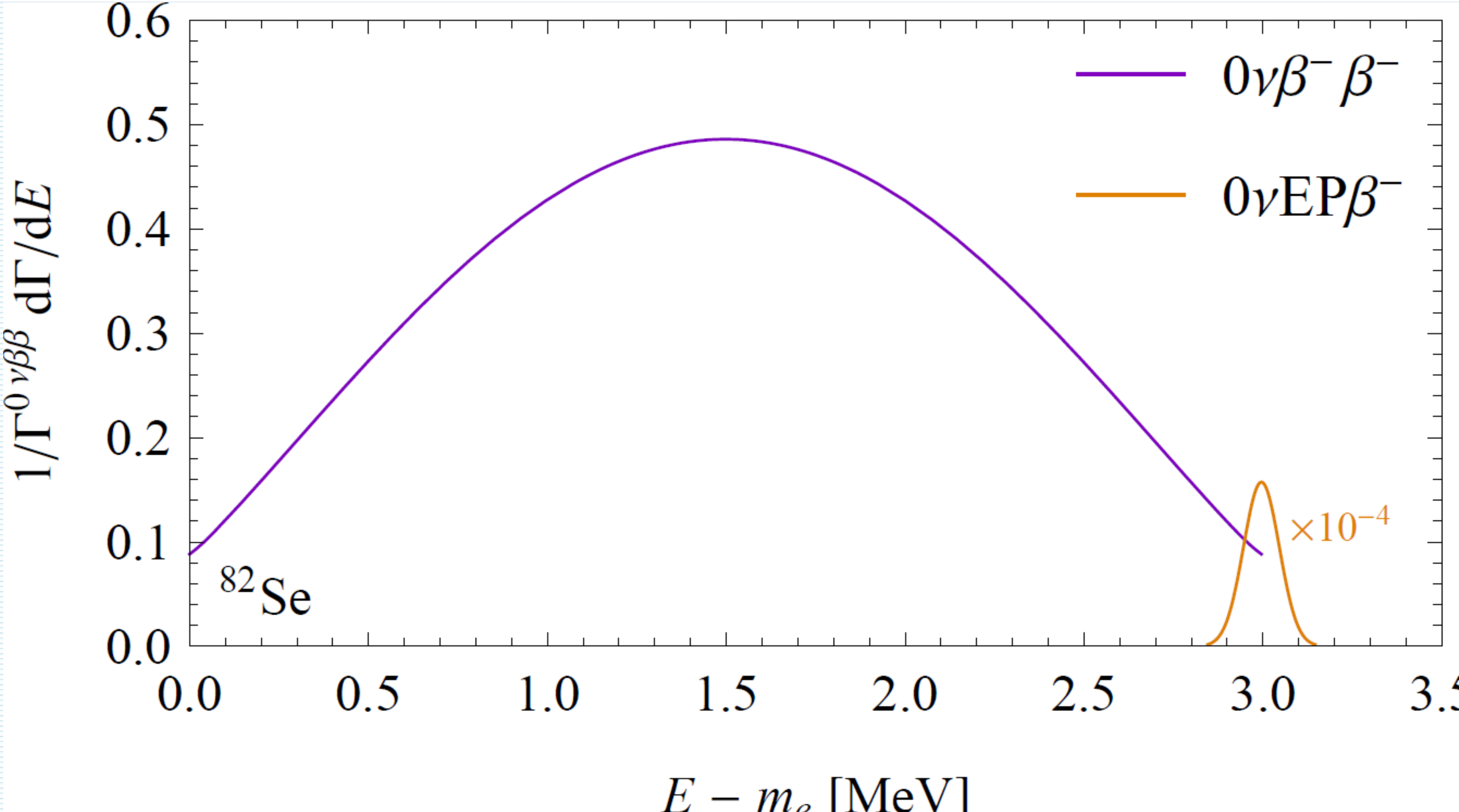
- **Fit of non-convergent orbitals:** $f_{n,-1}^2, g_{n,+1}^2(R) \approx aZ^b$
- **Fit of orbitals beyond $n = 9$:** $f_{n,-1}^2, g_{n,+1}^2(R) \approx cn^d$





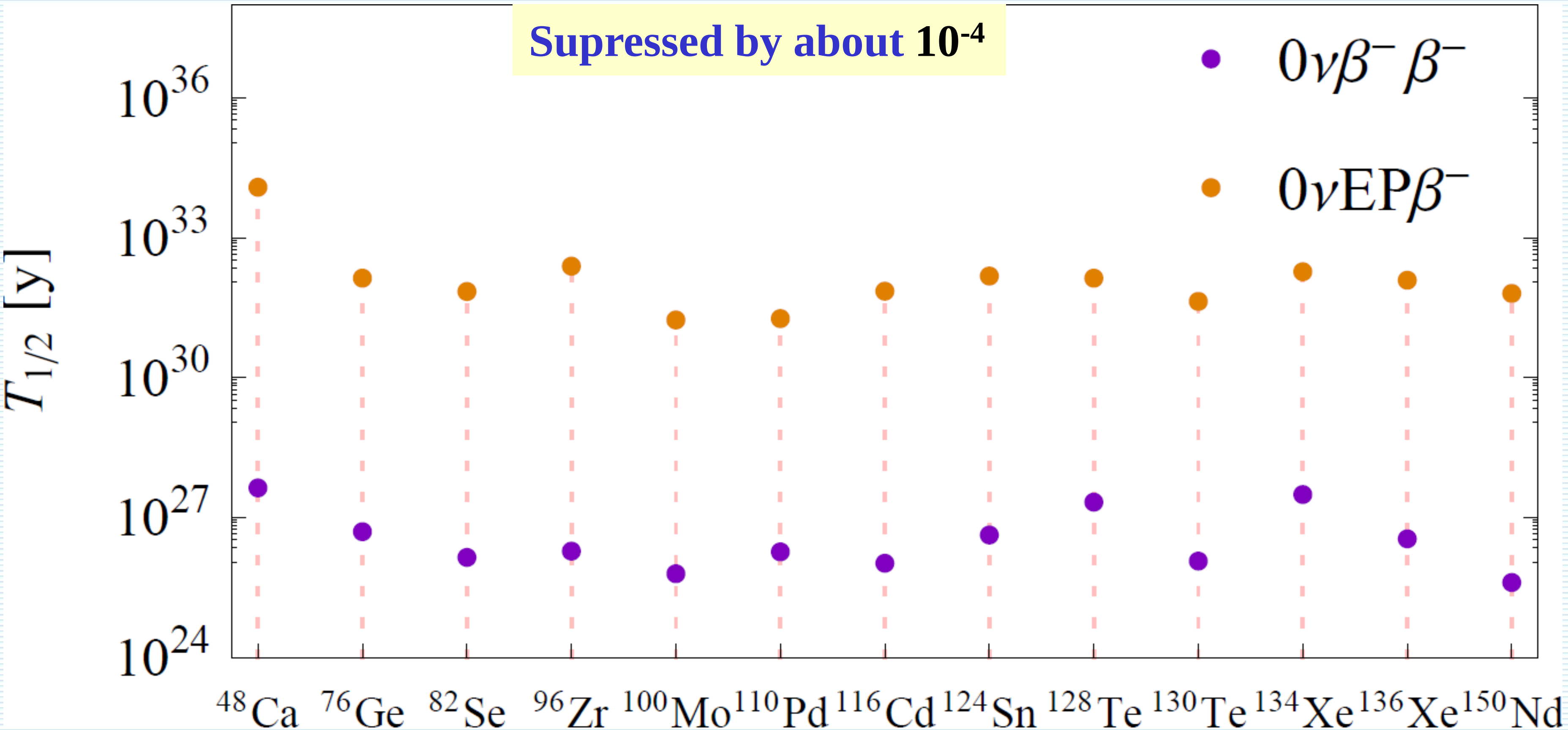
$0\nu EP\beta^-$ Single-Electron Spectrum (^{82}Se)

$0\nu\beta^-\beta^-$ and $0\nu EP\beta^-$ single-electron spectra $1/\Gamma^{0\nu\beta\beta}\, d\Gamma/dE$ *vs.* electron kinetic energy $E - m_e$ for ^{82}Se ($Q = 2.996$ MeV)



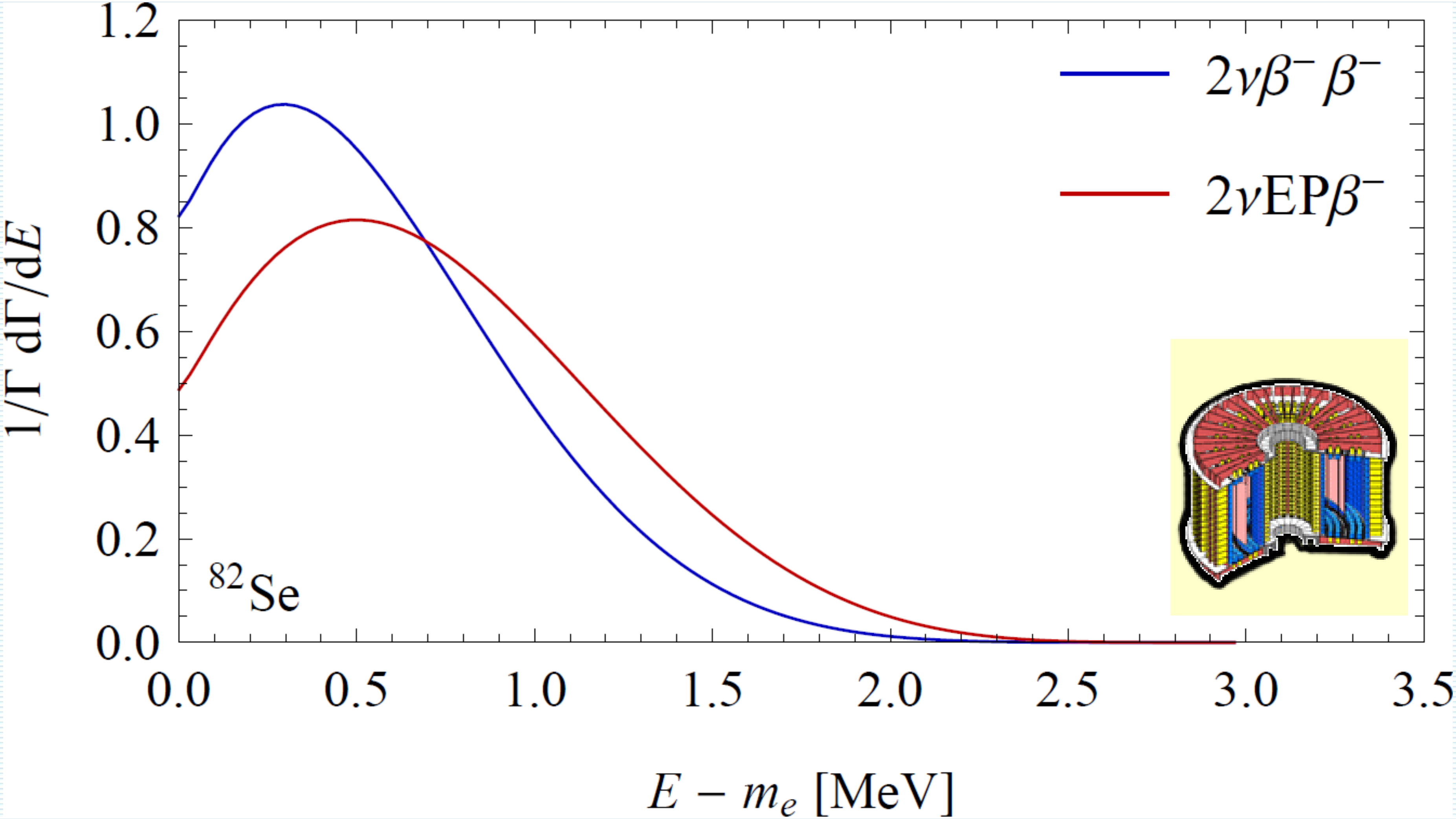
$0\nu\text{EP}\beta^-$ Half-Lives

$0\nu\beta^-\beta^-$ and $0\nu\text{EP}\beta^-$ half-lives $T_{1/2}^{0\nu\beta\beta}$ and $T_{1/2}^{0\nu\text{EP}\beta}$ estimated for $\beta^-\beta^-$ isotopes with known NME $|M^{0\nu\beta\beta}|$, assuming unquenched $g_A = 1.269$ and $|m_{\beta\beta}| = 50$ meV



$2\nu EP\beta^-$ Single-Electron Spectrum (^{82}Se)

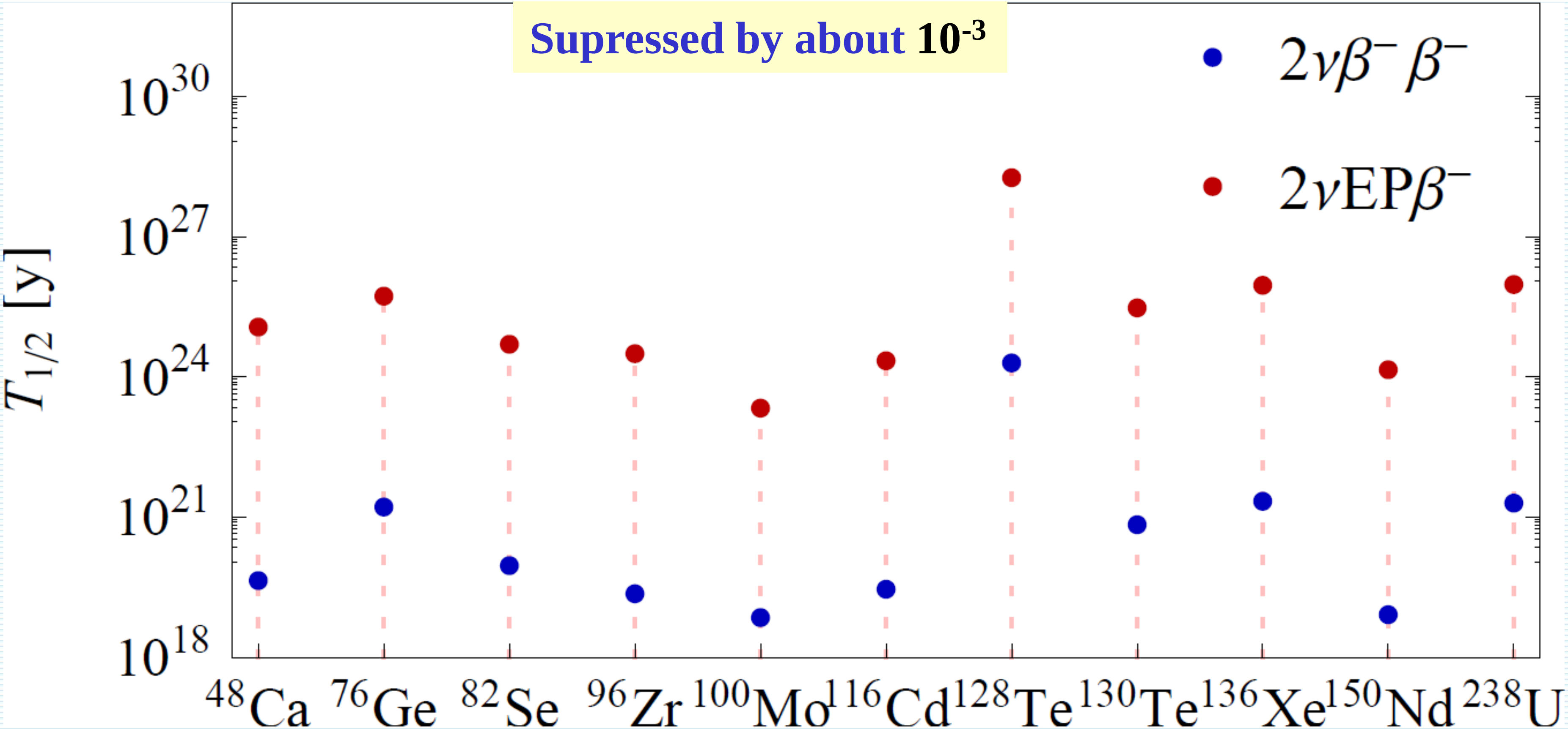
$2\nu\beta^-\beta^-$ **and** $2\nu EP\beta^-$ **single-electron spectra** $1/\Gamma\, d\Gamma/dE$ ***vs.*** **electron kinetic energy** $E - m_e$ **for** ^{82}**Se** ($Q = 2.996$ MeV)



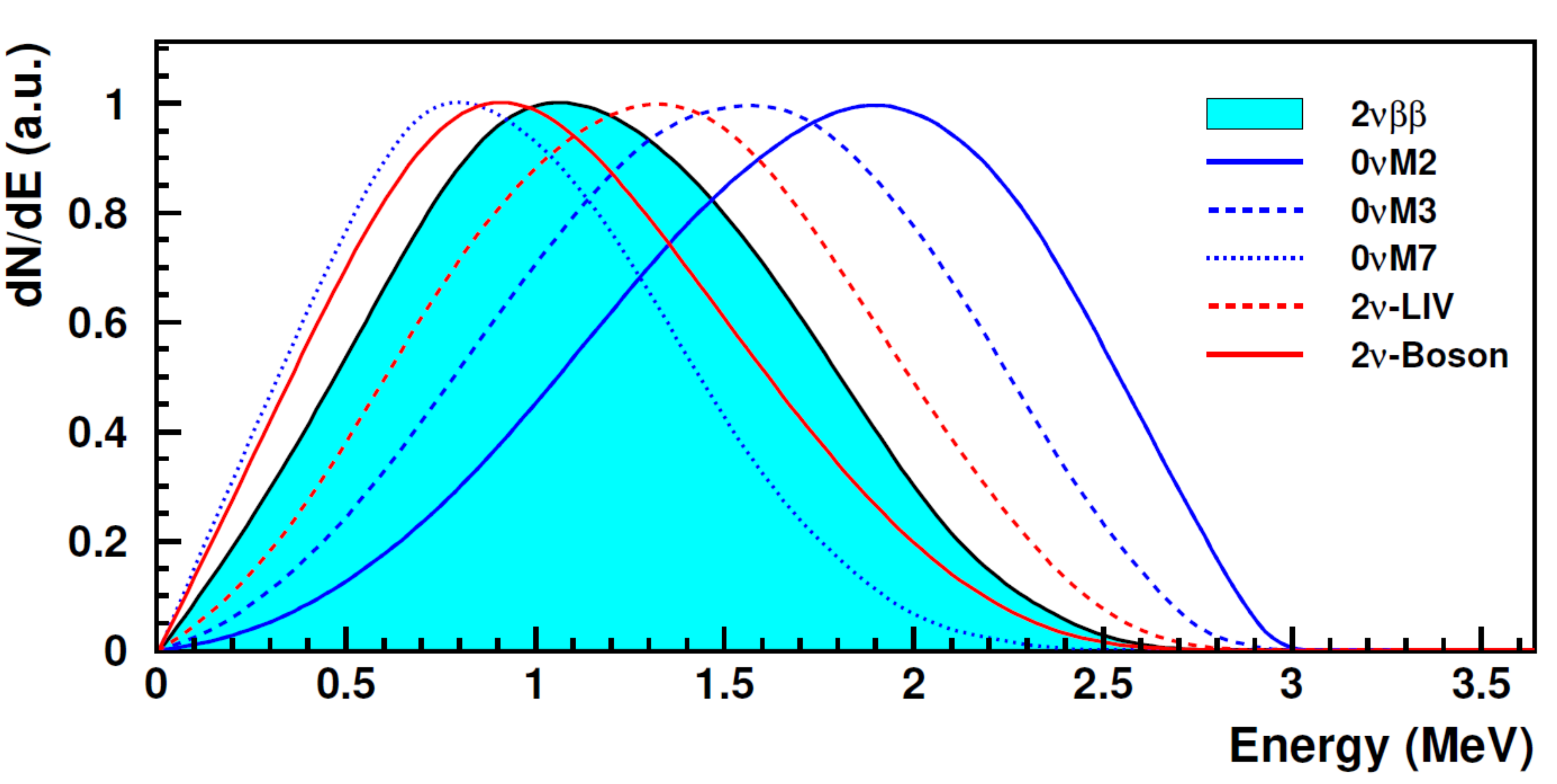
$2\nu\mathrm{EP}\beta^-$ Half-Lives predictions

(independent on g_A and value of NME)

$2\nu\beta^-\beta^-$ and $2\nu\mathrm{EP}\beta^-$ half-lives $T_{1/2}^{2\nu\beta\beta}$ and $T_{1/2}^{2\nu\mathrm{EP}\beta}$ calculated for $\beta^-\beta^-$ isotopes observed experimentally, assuming unquenched $g_A = 1.269$



Looking for a new physics with differential characteristics



Spectral index n

$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C(Q - \varepsilon_1 - \varepsilon_2)^n \; [p_1\varepsilon_1 F(\varepsilon_1)] \; [p_2\varepsilon_2 F(\varepsilon_2)]$$

VIII. Instead of Conclusion

LHC physics

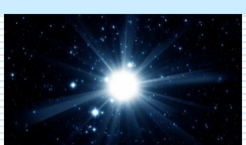
$$\frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

Neutrino physics

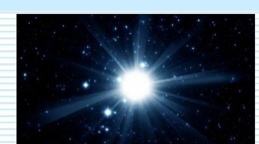
$$\frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)}$$

Progress in nuclear structure calculations is highly required

We are at the beginning of the Beyond Standard Model Road…



The future of neutrino physics is bright



VII Pontecorvo Neutrino Physics School

Sinaia, Romania, September 1-10, 2019

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Scientific program: **S.M. Bilenky** (JINR)
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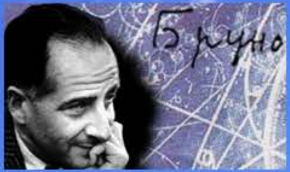
Secretaries:
- **A. Babic** (JINR)
- **O. Nitescu** (CIFRA, Bucharest)
- **J. Mansour** (CIFRA, Bucharest)

Lectures at the School

Introduction to ν-physics	Samoil Bilenky (JINR Dubna)
Theory of ν-masses and mixing	Alexei Smirnov (MPI Heidelberg)
ν-oscillation phenomenology	Boris Kayser (Fermilab)
ν-oscillation experiment:	
Solar ν-experiments	Oleg Smirnov (JINR Dubna)
Atmospheric ν-experiments	Juan Pablo Yanez (Univ. of Alberta)
Accelerator ν-experiments	Maury Goodman (Argonne National Laboratory)
Reactor ν-experiments	Yifang Wang (IHEP)
Spectra of ν's from reactor	Anna Hayes (Los Alamos National Lab)
Light sterile neutrinos:	
theory	Carlo Giunti (INFN Torino)
experiments	Vyacheslav Egorov (JINR Dubna)
Heavy sterile neutrinos	Dmitry Gorbunov (INR RAS Moscow)
Measurement of ν-mass	Kathrin Valerius (KIT in Karlsruhe)
0νββ-decay experiments	Andrea Giuliani (CSNSM in Paris)
0νββ-decay nuclear matrix elements	Javier Menendez (Univ. of Tokyo)
Coherent scattering of neutrinos	Henry Wong (Academia Sinica, Taipei)
ν-nucleus interactions	Jan Sobczyk (Wroclaw University)
Leptogenesis	Pascuale DiBari (Univ. of Southampton)
ν-telescopes	Nathan Whitehorn (Univ. of California)
ν-properties from cosmology	Richard Battye (University of Manchester)
Dark matter experiments	Nicolao Fornengo (INFN Torino)
Physics of gravitational waves	Imre Bartos (Univ.of Florida)
Everything about Higgs boson	Guenakh Mitselmakher (Univ. of Florida)
Statistics for ν-experiments	Thomas Schwetz (KIT in Karlsruhe)

VIII International Pontecorvo Neutrino Physics School



September 1 – September 10, 2019

Sinaia, Romania

ν_3

http://theor.jinr.ru/~neutrino19/

ν_2

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S. M. Bilenky - scientific program
A.G. Olshevskiy - vice-chairman
S. Stoica -vice-chairman
F. Šimkovic - vice-chairman
I. Štekl - vice-chairman

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C. Giunti
E.A. Kolganova
R. Leitner

ν_1

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A. Babič, J. Mansour, O. Nitescu

Introduction to ν-physics	Samoil Bilenky (JINR Dubna)
Theory of ν-masses and mixing	Alexei Smirnov (MPI Heidelberg)
ν-oscillation phenomenology	Boris Kayser (Fermilab)
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Solar ν–experiments	Oleg Smirnov (JINR Dubna)
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