Cumulants of conserved charges and total transport currents in GHD: direct summation of matrix elements?

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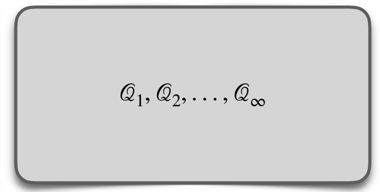
GHD and GGE

Isolated, out-of-equilibrium system



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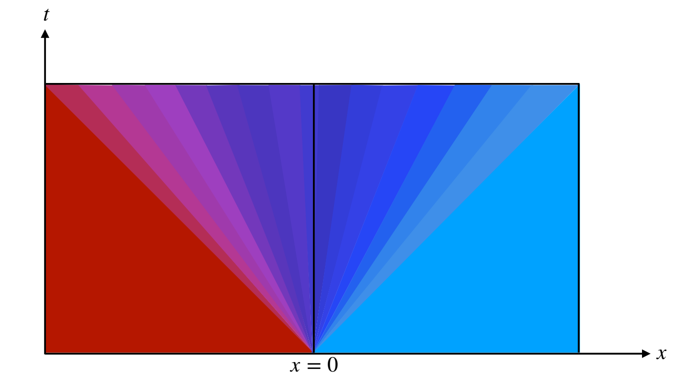

Hydrodynamic description

Local GGE at the fluid cell around x

$$\langle \mathcal{O}(x,t) \rangle = \frac{\mathrm{Tr}\left[e^{\sum_j -\beta_j(x,t)\mathcal{Q}_j}\mathcal{O}\right]}{\mathrm{Tr}\left[e^{\sum_j -\beta_j(x,t)\mathcal{Q}_j}\right]}$$

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The state at this cell

$$\{\beta_1, \beta_2, ...\beta_\infty\} \leftrightarrow \{\langle \mathcal{Q}_1 \rangle, \langle \mathcal{Q}_2 \rangle, ..., \langle \mathcal{Q}_\infty \rangle\}$$

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Equation of state that describes transport among neighboring cell

$$\partial_t \langle Q_j(x,t) \rangle + \partial_x \langle J_j(x,t) \rangle = 0.$$

Non-equilibrium steady state

The Fermi-Dirac factor of the state at the junction

$$f(\theta) = f^{\mathsf{L}}(\theta)\Theta(\theta - \theta^*) + f^{\mathsf{R}}(\theta)\Theta(\theta^* - \theta), \quad v^{\mathsf{eff}}(\theta^*) = 0$$

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Total transport crossing this point

$$J^{(t)} \equiv \int_0^t J(0,s)ds$$

Cumulant generating function (full counting statistics or Legendre transform of the large-deviation rate function)

$$F(\lambda) \equiv \lim_{t\to\infty} \frac{1}{t}\langle e^{\lambda J^{(t)}}\rangle = \sum_{n=1}^{\infty} \frac{\lambda^n}{n!} c_n$$

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satisfies a functional relation (see Doyon's talk or 1812.02082)

Individual cumulants

Steady current average

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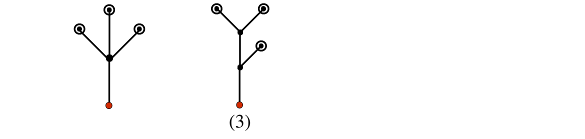
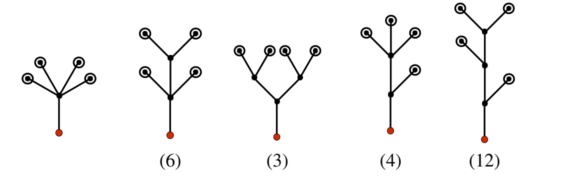
Then ...

$$c_3 = \int \frac{d\theta}{2\pi}(E')^{\text{dr}}(\theta)f(\theta)[1-f(\theta)]s(\theta)q^{\text{dr}}(\theta)\times \\ \times\left\{[1-2f(\theta)][q^{\text{dr}}(\theta)]^2 s(\theta) + 3\Big[(q^{\text{dr}})^2(1-f)s\Big]^{*\text{dr}}(\theta)\right\},$$

$$c_4 = \int \frac{d\theta}{2\pi}(E')^{\text{dr}}(\theta)f(\theta)[1-f(\theta)]\times\left\{\frac{Y(\theta)^2+6Y(\theta)+6}{[Y(\theta)+1]^2}s(\theta)[q^{\text{dr}}(\theta)]^4\right. \\ +3s(\theta)\{[(1-f)s(q^{\text{dr}})^2]^{\text{dr}}(\theta)\}^2 + 12s(\theta)q^{\text{dr}}(\theta)\{(1-f)sq^{\text{dr}}[(1-f)s(q^{\text{dr}})^2]^{\text{dr}}\}^{\text{dr}}(\theta) \\ \left.+6[f(\theta)-2)[q^{\text{dr}}(\theta)]^2[s(1-f)(q^{\text{dr}})^2]^{\text{dr}}(\theta) + 4s(\theta)q^{\text{dr}}(\theta)[(1-f)(f-2)(q^{\text{dr}})^3]^{\text{dr}}(\theta)\right\}$$

An unrelated problem, or is it?

$\langle \mathcal{Q} \rangle$	
$\langle \mathcal{Q}\mathcal{Q} \rangle^c$	
$\langle \mathcal{Q}\mathcal{Q}\mathcal{Q} \rangle^c$	
$\langle \mathcal{Q}\mathcal{Q}\mathcal{Q}\mathcal{Q} \rangle^c$	

Cumulants of conserved charges

Motivation

Traditional TBA

$$\mathrm{Tr}[e^{\sum_j -\beta_j \mathcal{Q}_j}]$$

This talk

$$\langle \mathcal{Q}_1 \mathcal{Q}_2 \dots \mathcal{Q}_n \rangle$$

$$\mathcal{Q}_j = \int dx Q_j(x,t)$$

Ultimate goal

$$\langle Q_1(x_1,t_1) Q_2(x_2,t_2) \dots Q_n(x_n,t_n) \rangle$$

Direct computation?

TBA free energy

$$F(\boldsymbol{\beta}) = \log \mathrm{Tr}\left[e^{\sum_j -\beta_j(x,t)\mathcal{Q}_j}\right] = L \int \frac{dp(\theta)}{2\pi} \log\left[1 + e^{-\epsilon(\theta)}\right]$$

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Cumulants of conserved charges are given by its derivatives

$$\langle \mathcal{Q}_j \mathcal{Q}_k \mathcal{Q}_l .... \rangle = (-1)^{\#}\frac{\partial}{\partial\beta_j}\frac{\partial}{\partial\beta_k}\frac{\partial}{\partial\beta_l}...F(\boldsymbol{\beta})$$

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$$\frac{1}{L}\langle\mathcal{Q}_j\rangle = \int\frac{dp}{2\pi}f(\theta)q_j^{\mathrm{dr}}(\theta)$$

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Higher cumulants: no rule to express in terms of particle distribution, Fermi-Dirac factor, simple dressing operations...

The free energy as a sum over trees [I.Kostov,D.Serban,D-L.V]

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- Bethe wavefunctions that diagonalize all the charges

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$$2\pi n_j = Lp(\theta_j) - i\sum_{k\neq j}^{N} \log S(\theta_j,\theta_k)$$

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$$Z(\beta) = \sum_{N\geq 0} \sum_{n_1<n_2<...<n_N} e^{-w(n_1,n_2,...,n_N)}$$

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- Removing the constraint by $1-\delta$ insertion

$$Z(\boldsymbol{\beta}) = \sum_{N\geq 0} \frac{(-1)^N}{N!} \sum_{n_1,...,n_N\in\mathbb{Z}^N} \sum_{r_1,..,r_N\in\mathbb{N}^N} \prod_{j=1}^{N} \frac{(-1)^{r_j}}{r_j} e^{-w(n_1^{r_1},...,n_N^{r_N})}.$$

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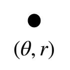
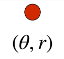
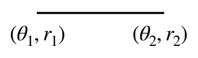
- Expand the Gaudin determinant to a sum over forests
- The free energy is a free sum over trees whose vertices (θ, r) live in $\mathbb{R} \times \mathbb{Z}^+$. Feynman rules



$$= \frac{(-1)^{r-1}}{r^2} e^{-rw(\theta)}$$



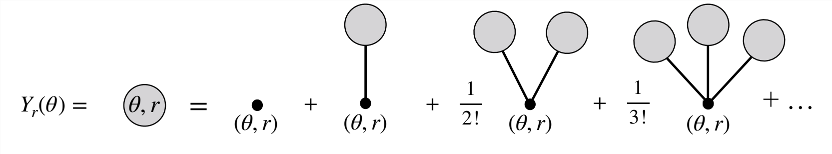
$$= Lrp'(\theta) \frac{(-1)^{r-1}}{r^2} e^{-rw(\theta)}$$



$$= r_1 r_2 K(\theta_2, \theta_1)$$

Summing over the trees

Let $Y_r(\theta)$ be the sum of all trees rooted at the point (θ, r)



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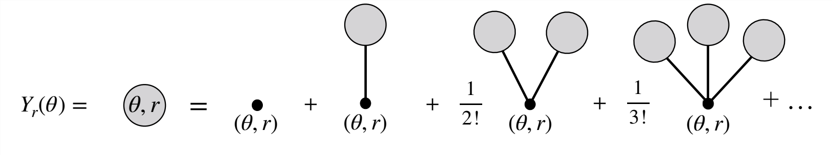

One can deduce from this structure that $Y_r(\theta) = (-1)^r Y_1^r(\theta)/r^2$ and

$$Y(\theta) = e^{-w(\theta)} \exp \int \frac{d\eta}{2\pi} K(\eta, \theta) \log[1 + Y(\eta)].$$

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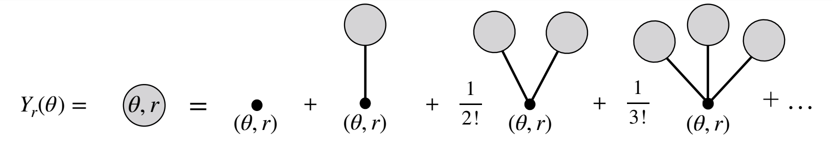

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The free energy is the sum over all trees

$$F(\beta) = L \int \frac{dp(\theta)}{2\pi} \sum_{r \geq 1} r Y_r(\theta) = L \int \frac{dp(\theta)}{2\pi} \log[1 + Y(\theta)].$$

Charge average

$$\langle \mathcal{Q}_j \rangle_{\boldsymbol{\beta}} = \mathrm{Tr}[e^{-\sum \beta_i \mathcal{Q}_i} \mathcal{Q}_j] / \mathrm{Tr}[e^{-\sum \beta_i \mathcal{Q}_i}]$$

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Insertion of identity

$$\langle \theta_N^{r_N}, \ldots, \theta_1^{r_1} | \mathcal{Q}_j | \theta_1^{r_1}, \ldots, \theta_N^{r_N} \rangle = \sum_{i=1}^{N} r_i q_j(\theta_i) \langle \theta_N^{r_N}, \ldots, \theta_1^{r_1} | \theta_1^{r_1}, \ldots, \theta_N^{r_N} \rangle$$

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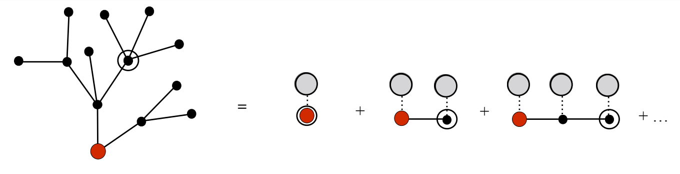
leads to a sum over trees with a charge insertion



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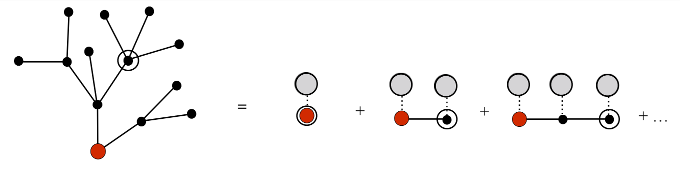

In terms of the dressing operation

$$\frac{1}{L} \langle \mathcal{Q}_j \rangle = \int \frac{dp}{2\pi} f(\theta) q_j^{\mathrm{dr}}(\theta)$$

Charge covariance

$$\langle \mathcal{Q}_j \mathcal{Q}_k \rangle = \mathrm{Tr}[e^{-\sum \beta_i \mathcal{Q}_i} \mathcal{Q}_j \mathcal{Q}_k] / \mathrm{Tr}[e^{-\sum \beta_i \mathcal{Q}_i}]$$

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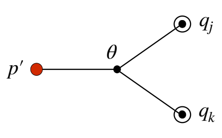
The covariance is given by a sum over trees with two leaves

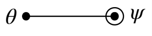
$$\frac{1}{L} \langle \mathcal{Q}_j \mathcal{Q}_k \rangle^{\mathrm{c}} =$$



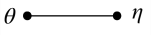
$$= \int \frac{d\theta}{2\pi} f(\theta)[1 - f(\theta)] (p')^{\mathrm{dr}}(\theta) q_j^{\mathrm{dr}}(\theta) q_k^{\mathrm{dr}}(\theta)$$

Higher cumulants

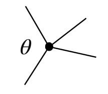
The n^{th} cumulants is given by the sum over all trees with a root that carries p' and n leaves carrying the n conserved charges



$$= \quad \psi^{\mathrm{dr}}(\theta)$$



$$= \quad K^{\mathrm{dr}}(\theta, \eta)$$



$$= \quad \sum_{r \geq 1} (-1)^{r-1} r^{d-1} e^{-r\epsilon(\theta)}$$

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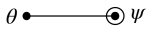

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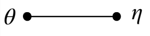

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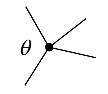


$$= \sum_{r\geq 1} (-1)^{r-1} r^{d-1} e^{-r\epsilon(\theta)}$$

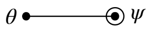
These trees are simple to generate

$$d_n = \sum_{\substack{p\in\mathcal{P}_n,\ |p|>1 \\ p=(a_1^{\alpha 1},\dots,a_j^{\alpha j})}} \prod_{i=1}^{j} \binom{d_{a_1} + \alpha_i - 1}{\alpha_i}.$$

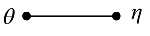
n	1	2	3	4	5	6	7	8	9	10
d_n	1	1	2	5	12	33	90	261	766	2312

Higher cumulants

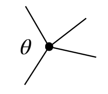
The n^{th} cumulants is given by the sum over all trees with a root that carries p' and n leaves carrying the n conserved charges



$$= \psi^{\text{dr}}(\theta)$$



$$= K^{\text{dr}}(\theta,\eta)$$



$$= \sum_{r\geq 1}(-1)^{r-1} r^{d-1} e^{-r\epsilon(\theta)}$$

Physical interpretation: virtual particles with anomalous correction to the bare charges

$$\psi^{*\text{dr}}(\theta) \equiv \psi^{\text{dr}}(\theta) - \psi(\theta) = \int \frac{d\eta}{2\pi} K^{\text{dr}}(\eta,\theta) f(\eta)\psi(\eta)$$

Cumulants of total transport currents

Comparison with diagrams

Simple modifications of the diagrams:

- Operator at the root: E' instead of p'
- Sign of the effective velocity at odd internal vertices

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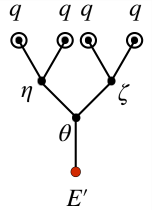
For instance



$$= 3 \int \frac{d\theta}{2\pi}\frac{d\eta}{2\pi}\frac{d\zeta}{2\pi}(E')^{\mathrm{dr}}(\theta)s(\theta)f(\theta)[1-f(\theta)]K^{\mathrm{dr}}(\theta,\eta)K^{\mathrm{dr}}(\theta,\zeta)$$

$$\times f(\eta)[1-f(\eta)]s(\eta)f(\zeta)[1-f(\zeta)]s(\zeta)[q^{\mathrm{dr}}(\eta)]^2[q^{\mathrm{dr}}(\zeta)]^2$$

Highly non-trivial matching with the fourth cumulant!

Conclusion

- We develop a new approach to express cumulants of conserved charges in GGE and total currents in GHD in an intuitive, effective way
- Does this combinatorial structure of individual cumulants translate into an analytic property of the full counting statistics?
- Matrix element of current cumulants? Drude weight?
- "Free energy" that generates current cumulants? Dual TBA?
- Large scale dynamical correlation functions?

Thank you for attention!