Irrelevant perturbations of 2D integrable models

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Mainly based on work in collaboration with:

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JHEP 1307 (2013) 071, JHEP 1610 (2016) 112, JHEP11(2018)007, JHEP 1902 (2019), and arXiv:1904.09141

S. Dubovsky, R. Flauger and V. Gorbenko, Solving the simplest theory of quantum gravity, JHEP 1209 (2012) 133;

F. A. Smirnov and A. B. Zamolodchikov, On space of integrable quantum field theories, Nucl.Phys. B915 (2017);

L. McGough, M. Mezei, H. Verlinde, Moving the CFT into the bulk with TT, JHEP 1804 (2018) 010;

O. Aharony, S. Datta, A. Giveon, Y. Jiang, D. Kutasov, Modular invariance and uniqueness of TT deformed CFT, [arXiv:1808.02492];

A. Dei, A. Sfondrini, Integrable spin chain for stringy Wess-Zumino-Witten models, JHEP 1807 (2018) 109;

G. Bonelli, N. Doroud, M. Zhu, TT-deformations in closed form, JHEP 1807 (2018) 109;

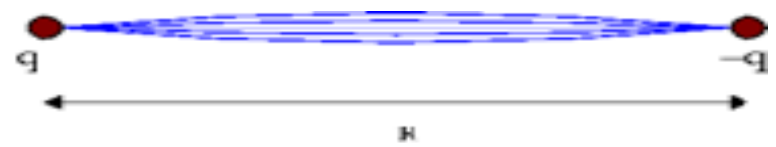
J. Cardy, The TT deformation of quantum field theory as random geometry, [arXiv:1801.06895]

L. Santilli, M. Tierz, Large N phase transition in TT-deformed 2d Yang-Mills theory on the sphere, JHEP 1901 (2019) 054, [arXiv:1810.05404]

M, Guica, An integrable Lorentz-breaking deformation of two-dimensional CFTs, JHEP 1901 (2019) 054, [arXiv:1810.05404]

Main motivations

The effective string theory for the quark-antiquark potential;



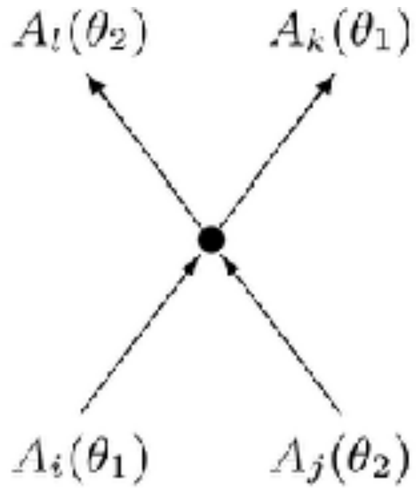
Emergence of singularities in RG/TBA flows with irrelevant perturbations;

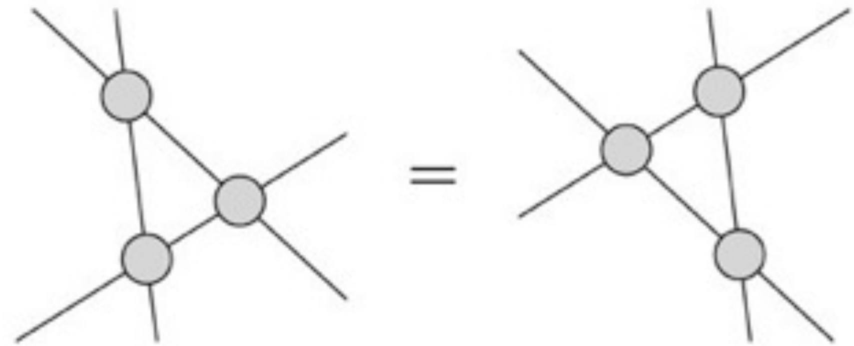
Relation between irrelevant perturbations and S-matrix CDD factor ambiguity;

Exact S-matrix and the CDD ambiguity

Consider a relativistic integrable field theory with factorized scattering:

$S_{ij}^{kl}(\theta_1 - \theta_2) \quad =$





Castillejo-**D**alitz-**D**yson ambiguity:

$$S_{ij}^{kl}(\theta) \to S_{ij}^{kl}(\theta)\, e^{i\delta_{ij}^{(\tau)}(\theta)}$$

The simplest possibility, consistent with the crossing and unitarity relations is:

$$\delta_{ij}^{(\tau)}(\theta) = \delta^{(\tau)}(m_i, m_j, \theta) = \tau\, m_i m_j \sinh(\theta)$$

The sine-Gordon NLIE

[1991: Klümper-Batchelor-Pearce, 1992: Destri-DeVega, 1996: Fioravanti-Ravanini- et al.]

$$
\begin{aligned}
f(\theta) &= -imR\sinh(\theta) + i\alpha \\
&\quad - \int_{\mathcal{C}_1} dy\, \mathcal{K}(\theta - y)\, \ln\left(1 + e^{-f(y)}\right) + \int_{\mathcal{C}_2} dy\, \mathcal{K}(\theta - y)\, \ln\left(1 + e^{f(y)}\right)
\end{aligned}
$$

For the ground state $\mathcal{C}_1 = \mathbb{R} + i0^+$ and $\mathcal{C}_2 = \mathbb{R} - i0^+$, but more more complicated contours appear for excited states.

$$
\mathcal{K}(\theta) = \frac{1}{2\pi i}\partial_\theta \ln S_{sG}(\theta)
$$

and

$$
\begin{aligned}
E(R) &= m\left[\int_{\mathcal{C}_1} \frac{dy}{2\pi i}\sinh(y)\ln\left(1 + e^{-f(y)}\right) - \int_{\mathcal{C}_2} \frac{dy}{2\pi i}\sinh(y)\ln\left(1 + e^{f(y)}\right)\right] \\
P(R) &= m\left[\int_{\mathcal{C}_1} \frac{dy}{2\pi i}\cosh(y)\ln\left(1 + e^{-f(y)}\right) - \int_{\mathcal{C}_2} \frac{dy}{2\pi i}\cosh(y)\ln\left(1 + e^{f(y)}\right)\right]
\end{aligned}
$$

replacing

$$\mathcal{K}(\theta) \to \mathcal{K}(\theta) - \frac{1}{2\pi}\partial_\theta \delta_{CDD}(\theta) = \mathcal{K}(\theta) - \tau \frac{m^2}{2\pi}\cosh(\theta)$$

we get

$$\begin{aligned} f(\theta) &= -i\, m\, \sinh(\theta)\, [R + \tau\, E(R,\tau)] + i\, m\, \cosh(\theta)\, \tau\, P(R,\tau) \\ &\quad - \int_{\mathcal{C}_1} dy\, \mathcal{K}(\theta - y)\, \ln\left(1 + e^{-f(y)}\right) + \int_{\mathcal{C}_2} dy\, \mathcal{K}(\theta - y)\, \ln\left(1 + e^{f(y)}\right) \end{aligned}$$

with

$$P(R,\tau) = P(R) = \frac{2\pi k}{R}, \quad k \in \mathbb{Z}$$

Therefore:

$$\begin{aligned} f(\theta) &= -i\, m\, \mathcal{R}_0\, \sinh\left(\theta - \theta_0\right) + i\alpha \\ &\quad - \int_{\mathcal{C}_1} dy\, \mathcal{K}(\theta - y)\, \ln\left(1 + e^{-f(y)}\right) + \int_{\mathcal{C}_2} dy\, \mathcal{K}(\theta - y)\, \ln\left(1 + e^{f(y)}\right) \end{aligned}$$

and

$$\sinh\theta_0 = \frac{\tau\, P(R)}{\mathcal{R}_0} = \frac{\tau\, P(\mathcal{R}_0)}{R}\,,\ \cosh\theta_0 = \frac{R + \tau\, E(R,\tau)}{\mathcal{R}_0} = \frac{\mathcal{R}_0 - \tau\, E(\mathcal{R}_0, 0)}{R}$$

Then

$$f(\theta | R, \tau) = f(\theta - \theta_0 | \mathcal{R}_0, 0)$$

which allows to compute the exact form of the τ-deformed energy level once its R-dependence is known at τ = 0. The result is:

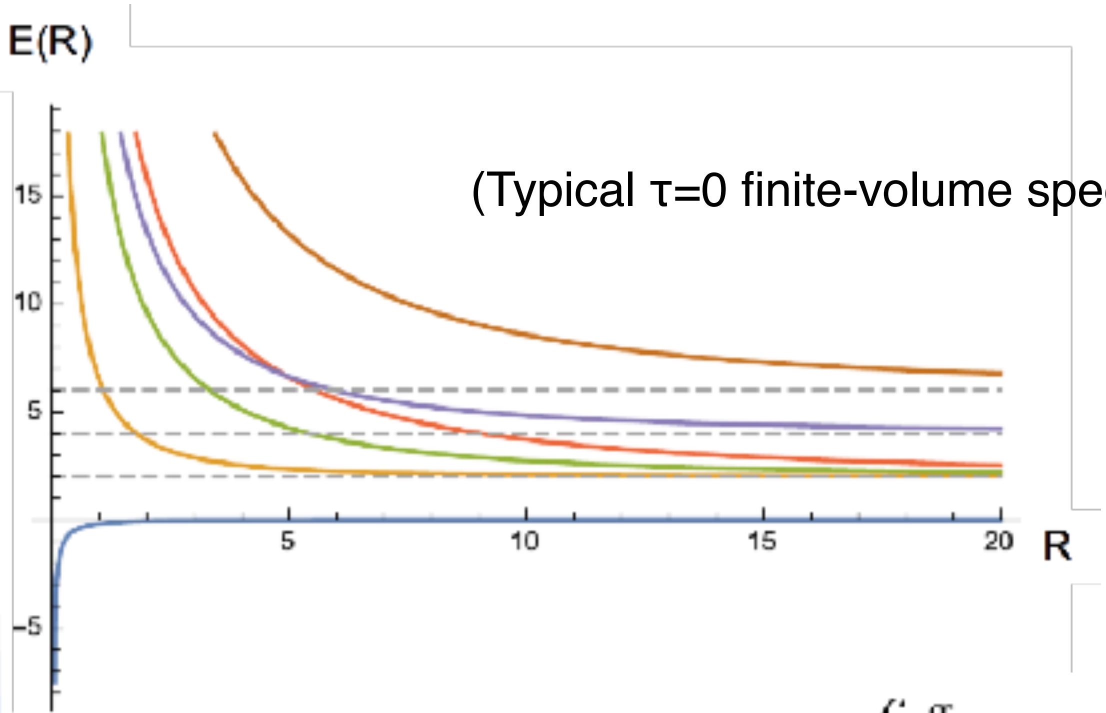
$$\begin{pmatrix} E(R,\tau) \\ P(R) \end{pmatrix} = \begin{pmatrix} \cosh(\theta_0) & \sinh(\theta_0) \\ \sinh(\theta_0) & \cosh(\theta_0) \end{pmatrix} \begin{pmatrix} E(\mathcal{R}_0, 0) \\ P(\mathcal{R}_0) \end{pmatrix}$$

therefore

$$E^2(R,\tau) - P^2(R) = E^2(\mathcal{R}_0, 0) - P^2(\mathcal{R}_0, 0)$$

We now have an implicit form of the solution of the inviscid Burgers equation with a source term:

$$\partial_\tau E_n(R,\tau) = \frac{1}{2}\partial_R\left(E_n^2(R,\tau) - P_n^2(R)\right) \qquad (\ \partial_\tau R = -E(R,\tau) \text{ at fixed } \mathcal{R}_0\,)$$

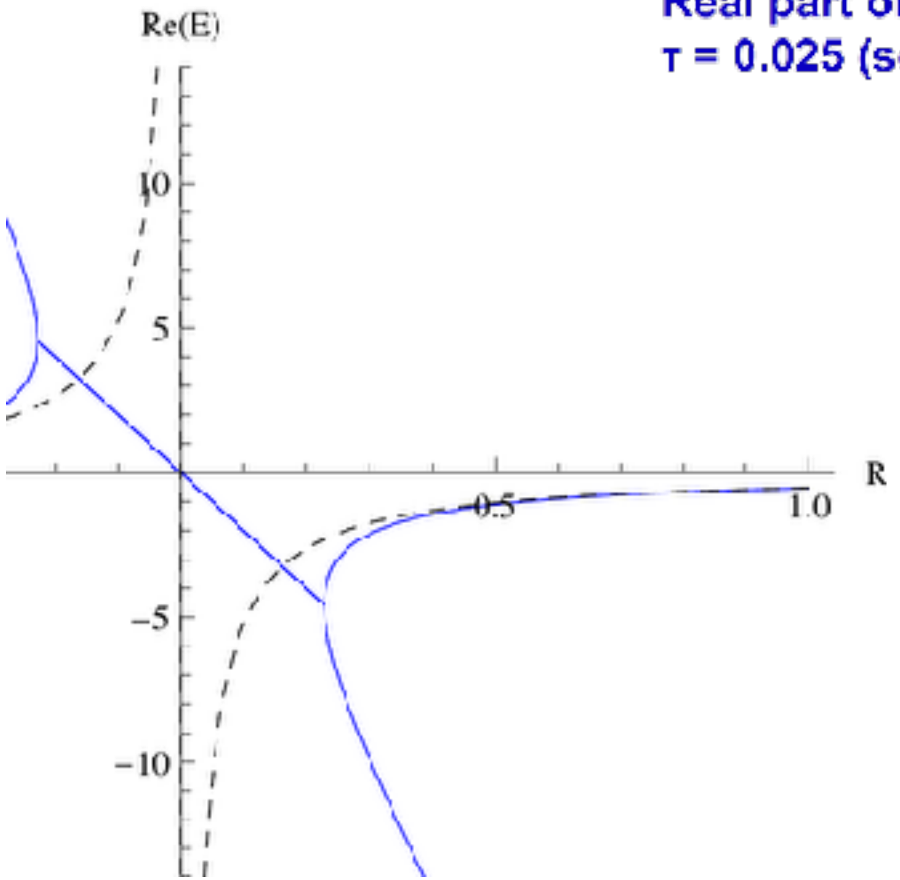


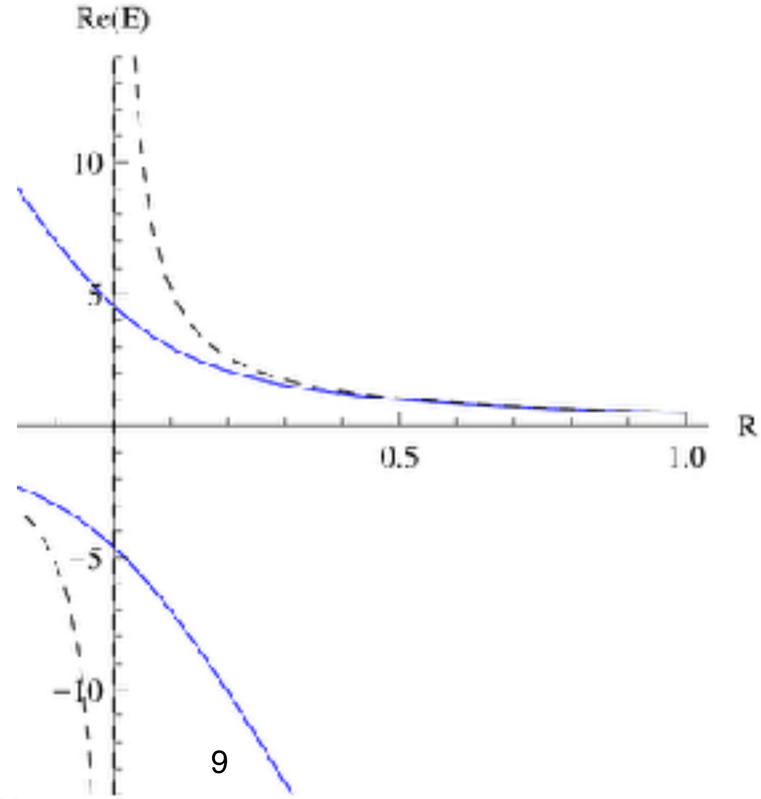
(Typical τ=0 finite-volume spectrum)

$$E(R,0) \sim -\pi\frac{c_{\text{eff}}}{6\,R}, \quad R \sim 0,$$

where, ceff= c – 24Δ is the “effective central charge” of the UV CFT state.

Real part of $E(R,\tau)$ for $\tau = 0$ (dashed line) and $\tau = 0.025$ (solid line), for $c_{eff} = 1$





Real part of $E(R,\tau)$ for $\tau = 0$ (dashed line) and $\tau = 0.025$ (solid line), for $c_{eff} = -1$

The CFT case

An extra CDD factor couples left (-) with right (+) movers and any NLIE or TBA equation leads to a pair of coupled algebraic equations:

$$E^{(+)}(R,\tau) = 2\pi \left(\frac{n_0 - c_{\rm eff}/24}{R + 2\tau E^{(-)}(R,\tau)} \right), \quad E^{(-)}(R,\tau) = 2\pi \left(\frac{\bar{n}_0 - c_{\rm eff}/24}{R + 2\tau E^{(+)}(R,\tau)} \right)$$

ceff= c – 24 Δ(primary), obtained by an energy-dependent shift:

$$R \to R + 2\tau E^{(\pm)}(R,\tau)$$

The total energy:

$$\begin{aligned} E(R,\tau) &= E^{(+)}(R,\tau) + E^{(-)}(R,\tau) \\ &= -\frac{R}{2\tau} + \sqrt{\frac{R^2}{4\tau^2} + \frac{2\pi}{\tau}\left(n_0 + \bar{n}_0 - \frac{c_{\rm eff}}{12}\right) + \left(\frac{2\pi(n_0 - \bar{n}_0)}{R}\right)^2} \end{aligned}$$

which matches the form of the (D=26, ceff=24) Nambu Goto spectrum, for a generic CFT, with τ=1/(2s), where s is the string tension.

Identification of the perturbing operator

Start from the equation:

$$\partial_\tau E_n(R,\tau) = \frac{1}{2}\partial_R \left(E_n^2(R,\tau) - P_n^2(R)\right)$$

and use the relations

$$E_n(R,\tau) = -R\langle n|\, T_{22}\,|n\rangle \ ,\ \partial_R E_n(R,\tau) = -\langle n|\, T_{11}\,|n\rangle \ ,\ P_n(R) = -\mathbb{i}R\langle n|\, T_{12}\,|n\rangle$$

since

$$T_{11} = -\frac{1}{2\pi}(\bar{T} + T - 2\Theta)\ ,\ T_{22} = \frac{1}{2\pi}(\bar{T} + T + 2\Theta)\ ,\ T_{12} = T_{21} = \frac{\mathbb{i}}{2\pi}(\bar{T} - T)$$

then

$$\partial_\tau E_n(R,\tau) = -\frac{R}{\pi^2}\langle n|\mathrm{T}\bar{\mathrm{T}}|n\rangle_R$$

with

$$\mathrm{T}\bar{\mathrm{T}}(z,\bar{z}) := \lim_{(z,\bar{z})\to(z',\bar{z}')} T(z,\bar{z})\bar{T}(z',\bar{z}') - \Theta(z,\bar{z})\Theta(z',\bar{z}') + \text{total derivatives}$$

Zamolodchikov's $\mathrm{T}\bar{\mathrm{T}}$ composite operator fulfills the following factorization property:

$$\langle \mathrm{T}\bar{\mathrm{T}} \rangle_n = \langle T \rangle_n \langle \bar{T} \rangle_n - \langle \Theta \rangle_n \langle \Theta \rangle_n$$

Putting all this information together:

$$\partial_\tau \ln Z(R, L, \tau) = \tfrac{1}{\pi^2} \langle \int_0^R dx \int_0^L dy \, \mathrm{T}\bar{\mathrm{T}}(z, \bar{z}) \rangle$$

Therefore, up to total derivatives:

$$\partial_\tau \mathcal{L}(\tau) = \det[T_{\mu\nu}(\tau)] \ , \ \mathrm{T}\bar{\mathrm{T}}(\tau) = -\pi^2 \det[T_{\mu\nu}(\tau)]$$

with $\mu, \nu \in \{1, 2\}$ and Euclidean coordinates (x_1, x_2).

Boson field theories with generic potential

$$\mathcal{L}^V(\vec\phi,0) = \partial\vec\phi\cdot\bar\partial\vec\phi + V(\vec\phi)$$

[Bonelli-Doroud-Zhu, Conti-Negro-Iannella-RT (2018)]

$$\mathcal{L}^V(\vec\phi,\tau) = \frac{V(\vec\phi)}{1-\tau V(\vec\phi)} + \frac{1}{2\bar\tau}\left(-1+\sqrt{1+4\bar\tau\mathcal{L}(\vec\phi,0)-4\bar\tau^2\mathcal{B}}\right)$$

$$\mathcal{B} = |\partial\vec\phi\times\bar\partial\vec\phi|^2$$

$$\mathcal{L}(\vec\phi,0) = \partial\vec\phi\cdot\bar\partial\vec\phi$$

with $\bar\tau = \tau(1-\tau V(\vec\phi))$.

Also:

$$\mathcal{H}^V(\vec\phi,\vec\pi,\tau) = \frac{V(\vec\phi)}{1-\tau V(\vec\phi)} + \frac{1}{2\bar\tau}\left(-1+\sqrt{1+4\bar\tau\mathcal{H}(\vec\phi,\vec\pi,0)+4\bar\tau^2\mathcal{P}^2(\vec\phi,\vec\pi)}\right)$$

$$\mathcal{H}(\vec\phi,\vec\pi,0) = \tfrac{1}{4}|\vec\phi'|^2 - |\vec\pi|^2 \qquad \mathcal{P}(\vec\phi,\vec\pi) = -\mathrm{i}\,\vec\pi\cdot\vec\phi' = -\mathrm{i}\,T_{12}(\tau)$$

The sine-Gordon model

$$\mathcal{L}_{\rm SG}(\phi,\tau) = \frac{V(\phi)}{1-\tau V(\phi)} + \frac{-1+S(\phi)}{2\tau\left(1-\tau V(\phi)\right)}\ ,\quad S(\phi) = \sqrt{1+4\tau\left(1-\tau V\right)\partial\phi\,\bar{\partial}\phi}$$

with

$$V = 2\frac{m^2}{\beta^2}\left(1-\cos\beta\phi\right)$$

and EoM

$$\partial\left(\frac{\bar{\partial}\phi}{S}\right) + \bar{\partial}\left(\frac{\partial\phi}{S}\right) = \frac{V'}{4S}\left(\frac{S+1}{1-\tau V}\right)^2\ , \qquad V' = 2\frac{m^2}{\beta}\sin\phi$$

$$\Downarrow$$

$\partial\bar{L} - \bar{\partial}L = \left[L,\bar{L}\right]$ (Lax consistency equation)

Deformed Conserved charges
(expansion in the spectral parameter λ)

A local change of coordinates

$$\mathcal{J}^{-1} = \begin{pmatrix} \partial_w z & \partial_w \bar{z} \\ \partial_{\bar{w}} z & \partial_{\bar{w}} \bar{z} \end{pmatrix} = \begin{pmatrix} 1 - \tau V & -\tau \left(\frac{\partial \phi}{\partial w} \right)^2 \\ -\tau \left(\frac{\partial \phi}{\partial \bar{w}} \right)^2 & 1 - \tau V \end{pmatrix}$$

$$\Downarrow$$

$$\phi^{(\tau)}(\mathbf{z}) = \phi^{(0)}\left(\mathbf{w}(\mathbf{z})\right) \ , \quad \mathbf{z} = (z, \bar{z}) \, , \quad \mathbf{w} = (w, \bar{w})$$

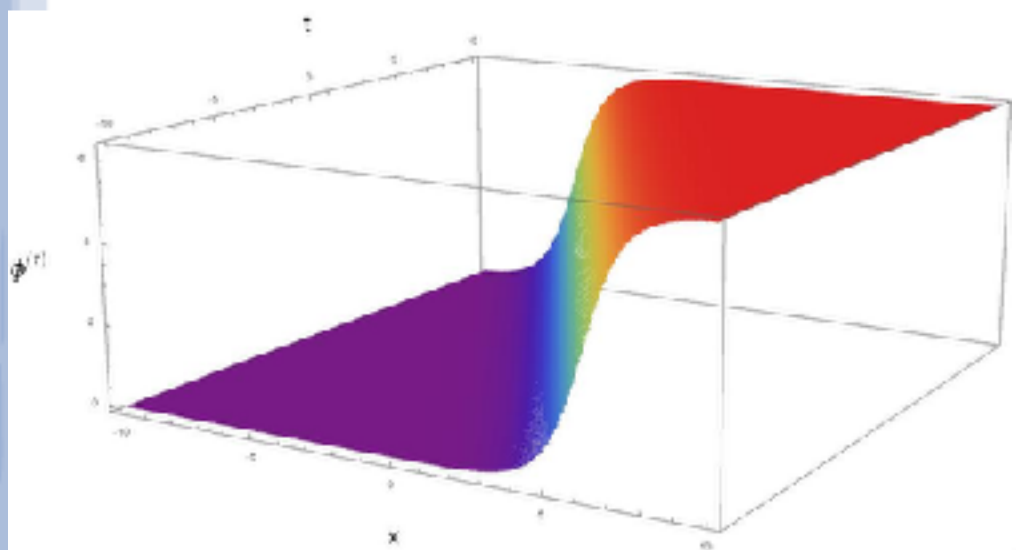
$$\partial \left(\frac{\bar{\partial} \phi}{S} \right) + \bar{\partial} \left(\frac{\partial \phi}{S} \right) = \frac{V'}{4S} \left(\frac{S+1}{1 - \tau V} \right)^2 \ , \qquad \Longrightarrow \qquad \frac{2 \frac{\partial}{\partial w} \frac{\partial}{\partial \bar{w}} \phi - V'}{\left(1 + \tau \left(\mathcal{K} - V\right)\right)^2} = 0$$

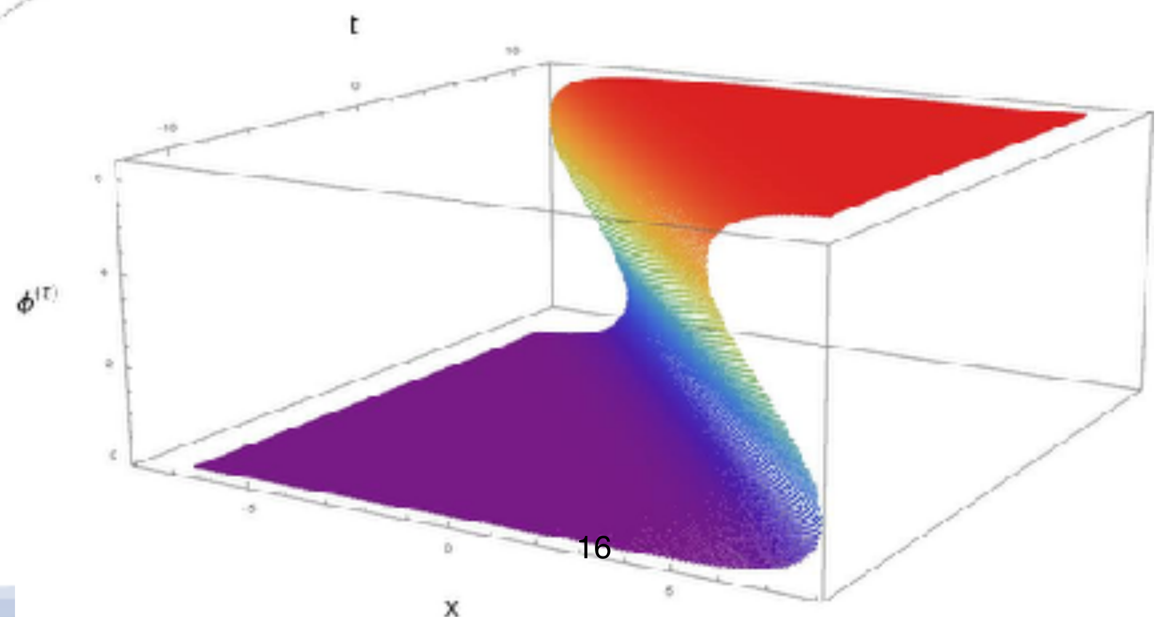
Where:

$$\mathcal{K} = \frac{\partial \phi(\mathbf{w})}{\partial w} \frac{\partial \phi(\mathbf{w})}{\partial \bar{w}}$$

The kink

$$\phi^{(0)}_{\text{1-kink}}(\mathbf{w}) = 4\arctan\left(e^{\frac{m}{\beta}\left(aw+\frac{1}{a}\bar{w}\right)}\right) \;,\; a = \sqrt{\frac{1-v}{1+v}}$$



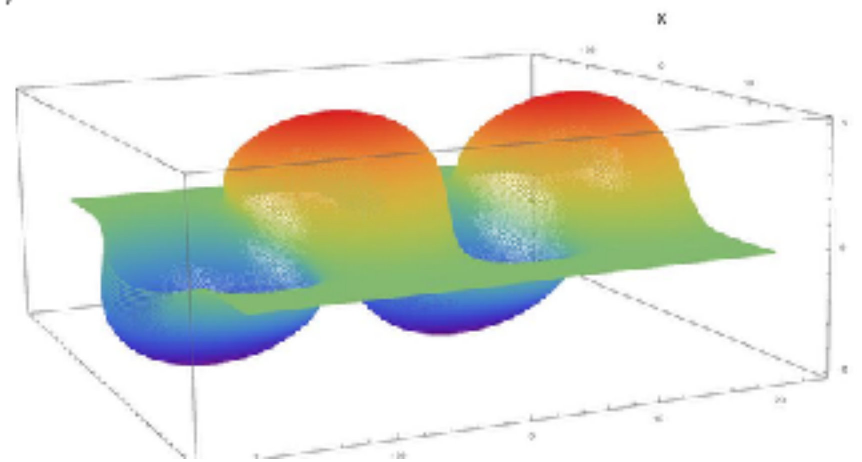


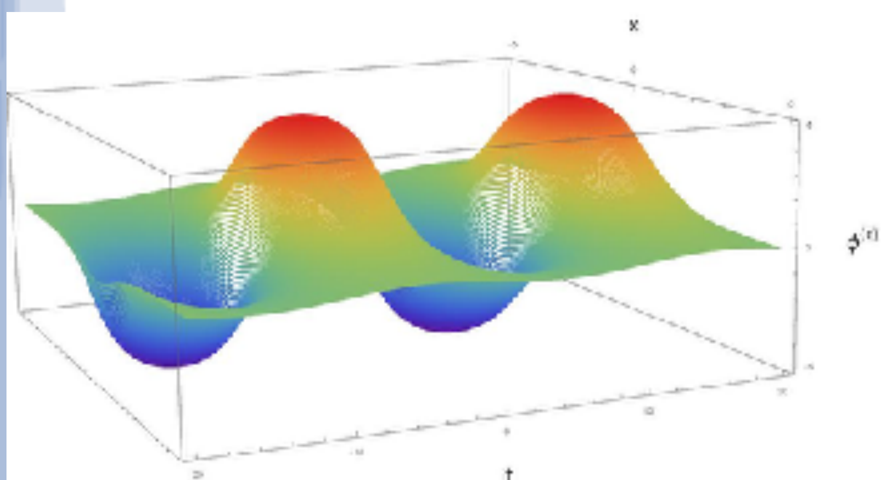
Deformed breather

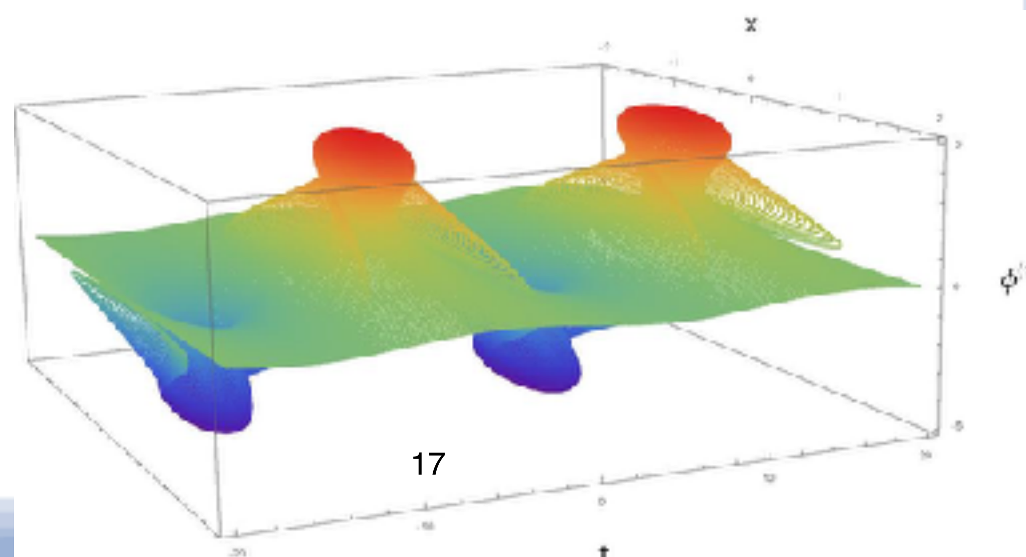
Start with the breather solution with envelope speed v = 0

$$\phi^{(0)}_{\text{breather}}(\mathbf{w}) = 4\arctan\left(\tan\psi\,\frac{\sin\left(-\frac{m}{\beta}(w-\bar{w})\cos\psi+\bar{k}\right)}{\cosh\left(\frac{m}{\beta}(w+\bar{w})\sin\psi+k\right)}\right)$$

$$T = \frac{2\pi}{\cos\psi}$$







Generic TTbar-deformed models

$$\mathcal{J}^{-1} = \begin{pmatrix} \partial_w z & \partial_w \bar{z} \\ \partial_{\bar{w}} z & \partial_{\bar{w}} \bar{z} \end{pmatrix} = \begin{pmatrix} 1 - \tau V & -\tau \left(\frac{\partial \phi}{\partial w}\right)^2 \\ -\tau \left(\frac{\partial \phi}{\partial \bar{w}}\right)^2 & 1 - \tau V \end{pmatrix} \Longleftrightarrow \begin{pmatrix} 1 - \tau\Theta(\mathbf{w}) & -\tau \bar{T}(\mathbf{w}) \\ -\tau T(\mathbf{w}) & 1 - \tau \Theta(\mathbf{w}) \end{pmatrix}$$

The solutions of the $T\bar{T}$-deformed EoMs are related to the $\tau = 0$ ones by

$$\boxed{dx^\mu = \left(\delta^\mu{}_\nu + \tau\, \widetilde{\mathbf{T}}^\mu{}_\nu(\mathbf{y}, 0)\right) dy^\nu \,, \quad \mathbf{y} = (y^1, y^2)}$$

with

$$\widetilde{\boldsymbol{T}}^\mu{}_\nu(\mathbf{y}) = -g^{\mu\delta} \epsilon_{\delta\rho}\, \boldsymbol{T}^{\rho\sigma}(\mathbf{y})\, \epsilon_{\sigma\nu} \,, \quad \epsilon_{\mu\nu} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{\mu\nu} \,, \quad \begin{cases} z = x^1 + i\, x^2 \\ \bar{z} = x^1 - i\, x^2 \end{cases} \,, \quad \begin{cases} w = y^1 + i\, y^2 \\ \bar{w} = y^1 - i\, y^2 \end{cases}$$

where $\boldsymbol{T}^{\mu\nu}(\mathbf{y})$ is the Hilbert stress-energy tensor associated to the undeformed theory.

$$\text{Notice that} \qquad \boxed{\frac{\partial^2 x^\mu}{\partial y^\rho \partial y^\tau} = \frac{\partial^2 x^\mu}{\partial y^\tau \partial y^\rho} \quad \Longleftrightarrow \quad \partial_\mu \mathbf{T}^\mu{}_\nu = 0}$$

Deformations induced by conserved currents with higher Lorentz spin

In complex coordinates $\mathbf{z}$, the continuity equations are

$$\bar{\partial} T_{\mathbf{s}+1} = \partial \Theta_{\mathbf{s}-1} , \quad \partial \bar{T}_{\mathbf{s}+1} = \bar{\partial} \bar{\Theta}_{\mathbf{s}-1} , \quad (\mathbf{s} \in \mathbb{N})$$

where the $\mathbf{s} = 1$ case of corresponds to the conservation of the energy and momentum.

We also have the spin-flip symmetry

$$\Theta_{s'-1} = \bar{T}_{s+1} , \quad T_{s'+1} = \bar{\Theta}_{s-1} ,$$

with $s' = \mathbf{s} < 0$, $s = |\mathbf{s}|$

More generally we can implement the change of variables with

$$\boxed{\mathbf{T} \quad \longrightarrow \quad \mathbf{T}_{\mathbf{s}}}$$

Therefore, the natural generalization which ensures the equality of mixed derivatives

$$\left(\mathcal{J}^{(s)}\right)^{-1} = \begin{pmatrix} \partial_w z & \partial_w \bar{z} \\ \partial_{\bar{w}} z & \partial_{\bar{w}} \bar{z} \end{pmatrix} = \begin{pmatrix} 1 + 2\tau\,\bar{\Theta}_{s-1}(\mathbf{w}) & 2\tau\,\mathrm{T}_{s+1}(\mathbf{w}) \\ 2\tau\,\bar{\mathrm{T}}_{s+1}(\mathbf{w}) & 1 + 2\tau\,\Theta_{s-1}(\mathbf{w}) \end{pmatrix}$$

The massless free boson

Consider the Lagrangian of a single massless boson field ϕ in the set of complex coordinates w

$$\mathcal{L}(\mathbf{w}) = \partial_w\phi\,\partial_{\bar{w}}\phi\ .$$

The EoMs are

$$\partial_w\partial_{\bar{w}}\phi = 0\ ,$$

therefore, some of the components of the stress-energy tensor are trivial.

There exists an infinite number of options for the choice of the basis of conserved currents. For example, both

$$\mathrm{T}^{(\mathrm{POW})}_{\mathbf{k}+1}(\mathbf{w}) = -\frac{1}{2}\left(\partial_w \phi\right)^{\mathbf{k}+1} \quad , \quad \Theta^{(\mathrm{POW})}_{\mathbf{k}-1}(\mathbf{w}) = 0 \; , \quad (\mathbf{k} \in \mathbb{N}) \; ,$$

and

$$\mathrm{T}^{(\mathrm{KG})}_{\mathbf{k}+1}(\mathbf{w}) = -\frac{1}{2}\left(\partial_w^{\frac{1+\mathbf{k}}{2}} \phi\right)^2 \quad , \quad \Theta^{(\mathrm{KG})}_{\mathbf{k}-1}(\mathbf{w}) = 0 \; , \quad (\mathbf{k} \in 2\mathbb{N}+1) \; ,$$

are possible sets of higher conserved currents. For simplicity, we will consider the negative spin power-type set of currents (s' =−s≤0, s=|**s**|≥0):

$$\left(\mathcal{J}^{(s')}\right)^{-1} = \begin{pmatrix} 1 - \tau \left(\partial_w \phi\right)^{s+1} & 0 \\ 0 & 1 - \tau \left(\partial_{\bar{w}} \phi\right)^{s+1} \end{pmatrix}$$

It follows immediately that the deformed EoMs are

$$\partial\bar{\partial}\phi = 0 \ ,$$

which reflects the fact that the **s** < 0 perturbations of CFT's do not mix the holomorphic and anti-holomorphic sectors. The level-k Hamiltonian and momentum deformed densities are:

$$\mathcal{H}_k^{(s')}(\mathbf{z},\tau) = \mathcal{I}_k^{(s')}(\mathbf{z},\tau) + \bar{\mathcal{I}}_k^{(s')}(\mathbf{z},\tau) \qquad \mathcal{P}_k^{(s')}(\mathbf{z},\tau) = \mathcal{I}_k^{(s')}(\mathbf{z},\tau) - \bar{\mathcal{I}}_k^{(s')}(\mathbf{z},\tau)$$

Where

$$\mathcal{I}_k^{(s')}(\mathbf{z},\tau) = -\big(\mathrm{T}_{k+1}^{(s')}(\mathbf{z},\tau) + \Theta_{k-1}^{(s')}(\mathbf{z},\tau)\big) \ , \quad \bar{\mathcal{I}}_k^{(s')}(\mathbf{z},\tau) = -\big(\bar{\mathrm{T}}_{k+1}^{(s')}(\mathbf{z},\tau) + \bar{\Theta}_{k-1}^{(s')}(\mathbf{z},\tau)\big)$$

with

$$\mathcal{I}_k^{(s')}(\mathbf{z},\tau) = \frac{\mathcal{I}_k(\mathbf{z})}{\Big(1 + 2\tau\,\mathcal{I}_s^{(s')}(\mathbf{z},\tau)\Big)^k} \ , \quad \bar{\mathcal{I}}_k^{(s')}(\mathbf{z},\tau) = \frac{\bar{\mathcal{I}}_k(\mathbf{z})}{\Big(1 + 2\tau\,\bar{\mathcal{I}}_s^{(s')}(\mathbf{z},\tau)\Big)^k}$$

The scattering phase

$$\mathcal{I}(\mathbf{z}) \longrightarrow \frac{I^{(+)}(R)}{R} \, , \quad \bar{\mathcal{I}}(\mathbf{z}) \longrightarrow \frac{I^{(-)}(R)}{R}$$

$$I_k^{(s',\pm)}(R,\tau) = \frac{R^k \, I_k^{(\pm)}(R)}{\left(R + 2\tau \, I_s^{(s',\pm))}(R,\tau)\right)^k}$$

The same relations can be obtained from the NLIE by introducing the phase factor:

$$\delta^{(s')}_{(\pm,\mp)}(\theta,\theta') = 0 \, , \quad \delta^{(s')}_{(\pm,\pm)}(\theta,\theta') = \pm\tau \, \frac{\hat{m}}{2} \hat{\gamma}_{s'} \, e^{\pm(\theta - s' \, \theta')} \, , \quad (s' = \mathbf{s} < 0) \, .$$

The natural generalisation to massive field theories is:

$$\delta^{(\mathbf{s})}(\theta,\theta') = \tau \, m \gamma_{\mathbf{s}} \sinh(\theta - \mathbf{s} \, \theta')$$

Explicit Lorentz breaking in a simple example

Consider the massless free-boson model:

$$\partial_w \partial_{\bar w} \phi = 0$$

and the vortex-type solution

$$\boxed{\phi(\mathbf{w},0) = d \ln\left(\frac{w+\xi}{\bar w+\bar\xi}\right) , \quad (\xi, \bar\xi \in \mathbb{C} , \quad d \in \mathbb{R})}$$

under the $\mathrm{T}\bar{\mathrm{T}}$ change of variables, involving

$$\mathrm{T}_2(\mathbf{w}) = -\frac{1}{2} \left(\partial_w \phi\right)^2 , \quad \Theta_0(\mathbf{w}) = 0$$

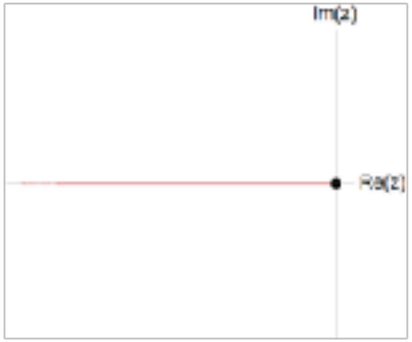
we get

$$z(\mathbf{w}) = w + \tau \frac{d^2}{(w+\xi)} , \quad \bar z(\mathbf{w}) = \bar w + \tau \frac{d^2}{(w+\xi)}$$

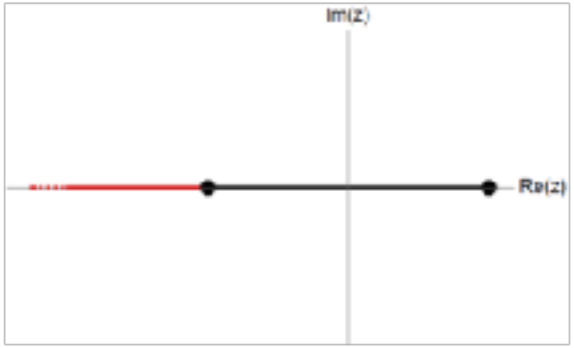
from which

$$\phi(\mathbf{z},\tau) = \phi(\mathbf{w},0) = d \ln\left(\frac{w+\xi}{w+\xi}\right) \equiv d \ln\left(\frac{z+\xi}{z+\xi}\right)$$

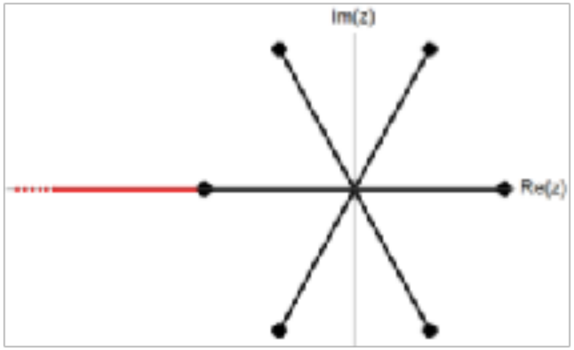
Therefore, the vortex solution is a fixed point of the $\mathrm{T}\bar{\mathrm{T}}$ flow.



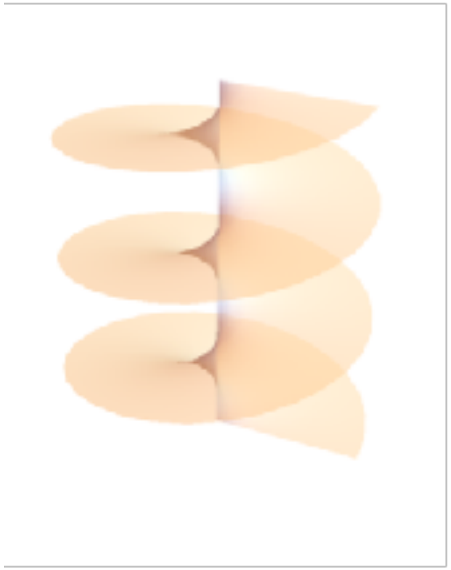
(a) s = 1



(b) s = −1



(c) s = −3



(a)

(b)

THANK YOU !