Recent theory developments in QCD spin

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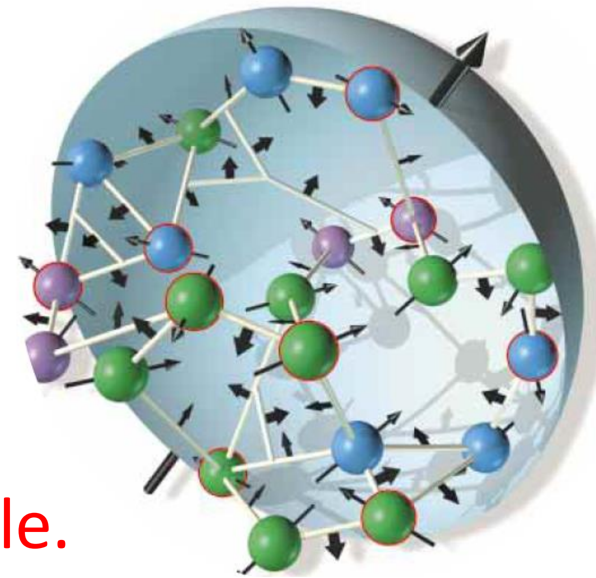
Longitudinal spin

The proton spin problem



The proton has spin ½.

The proton is not an elementary particle.

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity ($\frac{1}{2}\Delta\Sigma$)

Gluons' helicity (ΔG)

Orbital angular Momentum (OAM) (L^q, L^g)

Gauge invariant completion of JM decomposition

Chen, Lu, Sun, Wang, Goldman (2008)
YH (2011)

$$\langle PS|\epsilon^{ij}F^{i+}A^j_{phys}|PS\rangle = 2S^+\Delta G$$

$$\lim_{\Delta\to 0}\langle P'S|\bar{\psi}\gamma^+ i\overleftrightarrow{D}^i_{pure}\psi|PS\rangle = iS^+\epsilon^{ij}\Delta_{\perp j}L^q_{can}$$

$$\lim_{\Delta\to 0}\langle P'S|F^{+\alpha}\overleftrightarrow{D}^i_{pure}A^{phys}_\alpha|PS\rangle = -i\epsilon^{ij}\Delta_{\perp j}S^+L^g_{can}$$

where
(my choice)

$$A^\mu_{phys}(x) = \frac{1}{D^+}F^{+\mu} = \int_{x^-}^{\infty} dz^- W[x^-,z^-]F^{+\mu}(z^-,x_\perp)$$

$$D^\mu_{pure} = D^\mu - iA^\mu_{phys}$$

Polarized PDF global analysis

Recent determinations of polarised PDFs

	DSSV	NNPDF	JAM
DIS	☑	☑	☑
SIDIS	☑	☒	☑
pp	☑ (jets, π^0)	☑ (jets, $W^\pm$)	☒
statistical treatment	Lagr. mult. $\Delta\chi^2/\chi^2 = 2\%$ Monte Carlo	Monte Carlo	Monte Carlo
parametrization	polynomial (23 pars)	neural network (259 pars)	polynomial (10 pars)
features	global fit	minimally biased fit	large-x effects
latest updates	DSSV08 PRD 80 (2009) 034030 DSSV14 PRL 113 (2014) 012001	NNPDFpol1.0 NPB 874 (2013) 36 NNPDFpol1.1 NPB 887 (2014) 276	JAM15 PRD 93 (2016) 074005 JAM17 PRL 119 (2017) 132001

Talk by E. Nocera at Tianjin Workshop last week

Evidence of nonzero ΔG

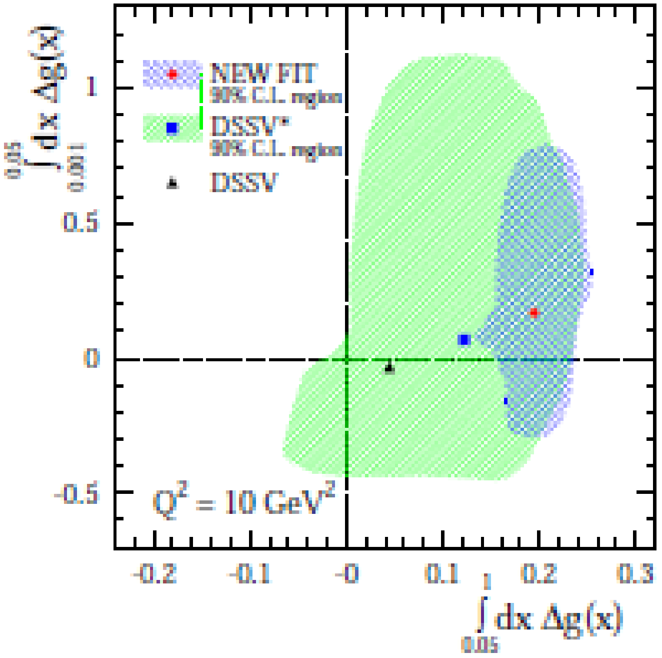
$$\int_{0.05}^{1} dx \Delta g(x, Q^2 = 10 GeV^2) = 0.20^{+.06}_{-.07} \quad \text{DSSV++}$$

$$\int_{0.05}^{0.2} dx \Delta g(x, Q^2 = 10 GeV^2) = 0.17 +- 0.06 \quad \text{NNPDFpol1.1}$$

$$\int_{0.001}^{0.8} dx \Delta g(x, Q^2 = 1\ GeV^2) = 0.5 +- 0.4 \quad \text{JAM15}$$

HUGE uncertainty from the small-x region

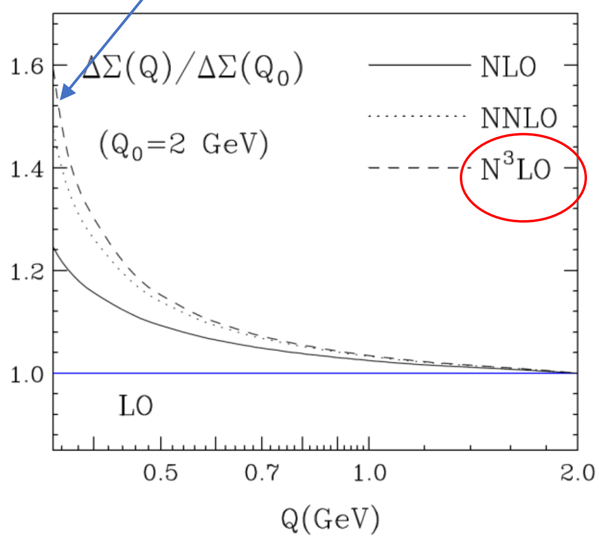
→ RHIC 510GeV,
Electron-Ion Collider



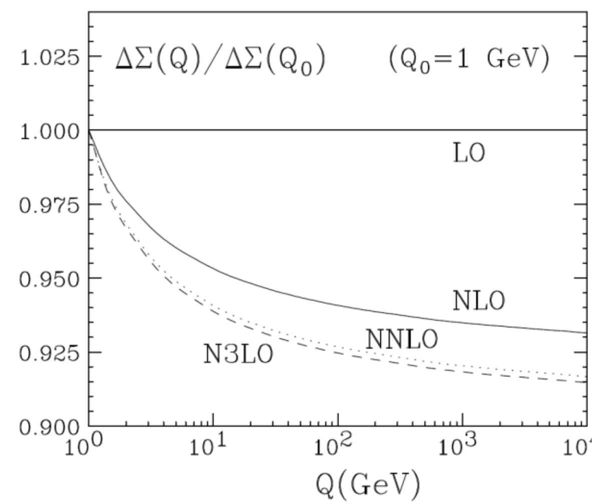
Helicity evolution to higher order

De Florian and Vogelsang (2019)

Approaching the quark model value?

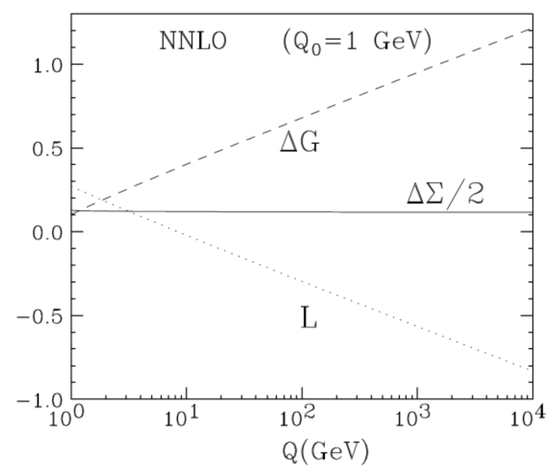


4 loops!



Evolution of OAM from

$$\frac{d\,\mathcal{L}(Q^2)}{d\ln Q^2} = -\frac{1}{2}\frac{d\,\Delta\Sigma(Q^2)}{d\ln Q^2} - \frac{d\,\Delta G(Q^2)}{d\ln Q^2}$$

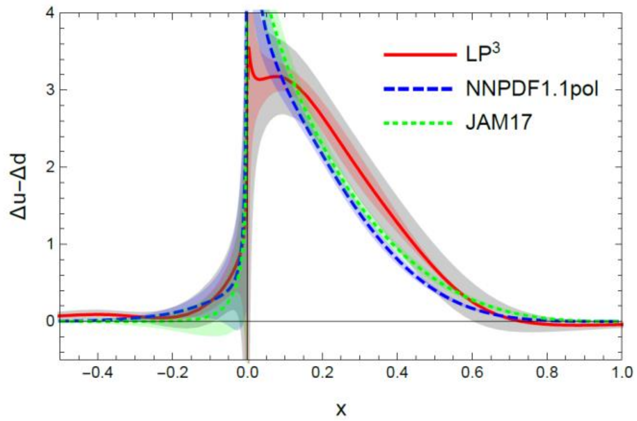


Helicity distributions from lattice QCD

Nowadays people compute PDFs on a lattice!

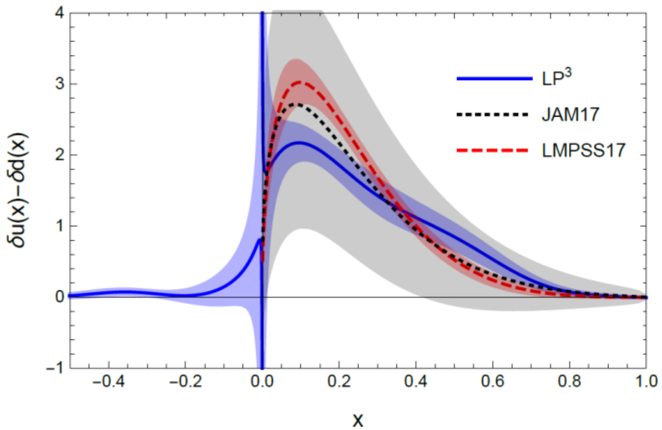
Polarized quark

Lin et al. 1807.07431



Transversity

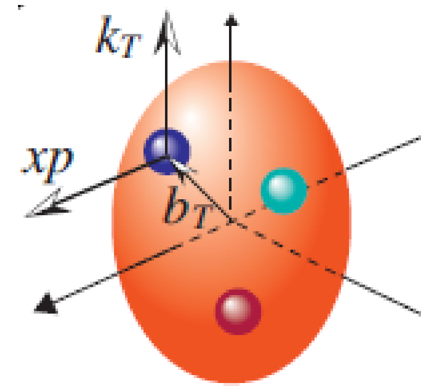
Liu, et al. 1810.05043



Polarized gluon (integrated)

Yang, et al. 1609.05937

The QCD Wigner distribution



Phase space distribution of partons in QCD—the `mother distribution'

Belitsky, Ji, Yuan (2004)

$$W(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2}\frac{dz^- d^2z_\perp}{16\pi^3} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle P-\frac{\Delta}{2}|\bar{q}(b-z/2)\gamma^+ q(b+z/2)|P+\frac{\Delta}{2}\rangle$$

$\int d\vec{b}_\perp$ ↓ $\qquad$ ↓ $\int d\vec{k}_\perp$

TMD $f(x,\vec{k}_\perp)$ $\qquad$ GPD $f(x,\vec{b}_\perp)$

$\int d\vec{k}_\perp$ ↘ $\qquad$ ↙ $\int d\vec{b}_\perp$ $\qquad$ ↘ $\int dx$

$f(x)$ PDF $\qquad$ $F(\vec{b}_\perp)$ Form factor

$\int dx$ ↘ $\qquad$ ↙ $\int d\vec{b}_\perp$

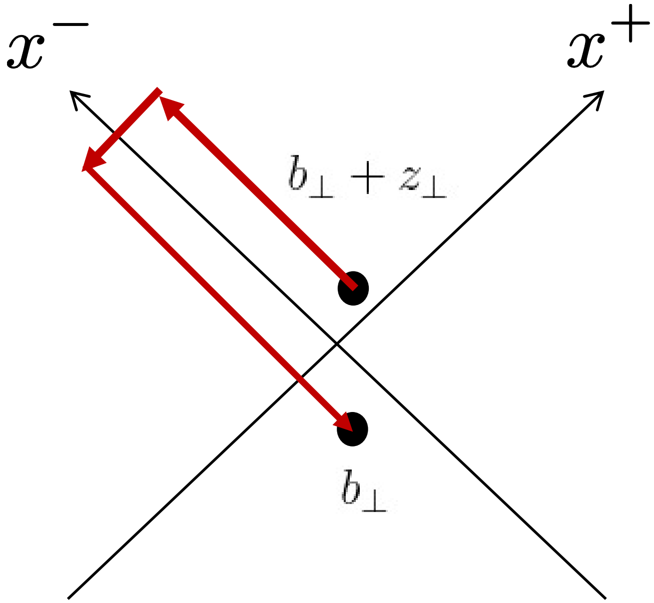
Q charge

OAM from the Wigner distribution

Lorce, Pasquini (2011);
YH (2011);
Lorce, Pasquini, Xiong, Yuan (2011)

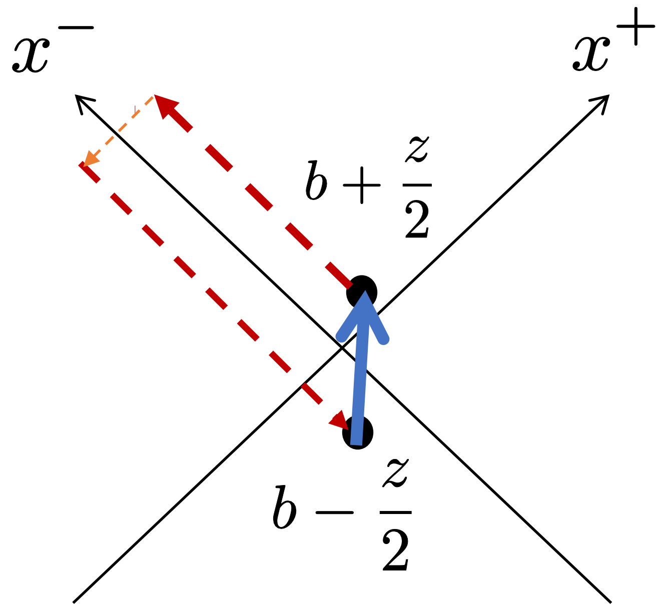
Define

$$L^q = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$



Jaffe-Manohar OAM from staple Wilson line

YH (2011)

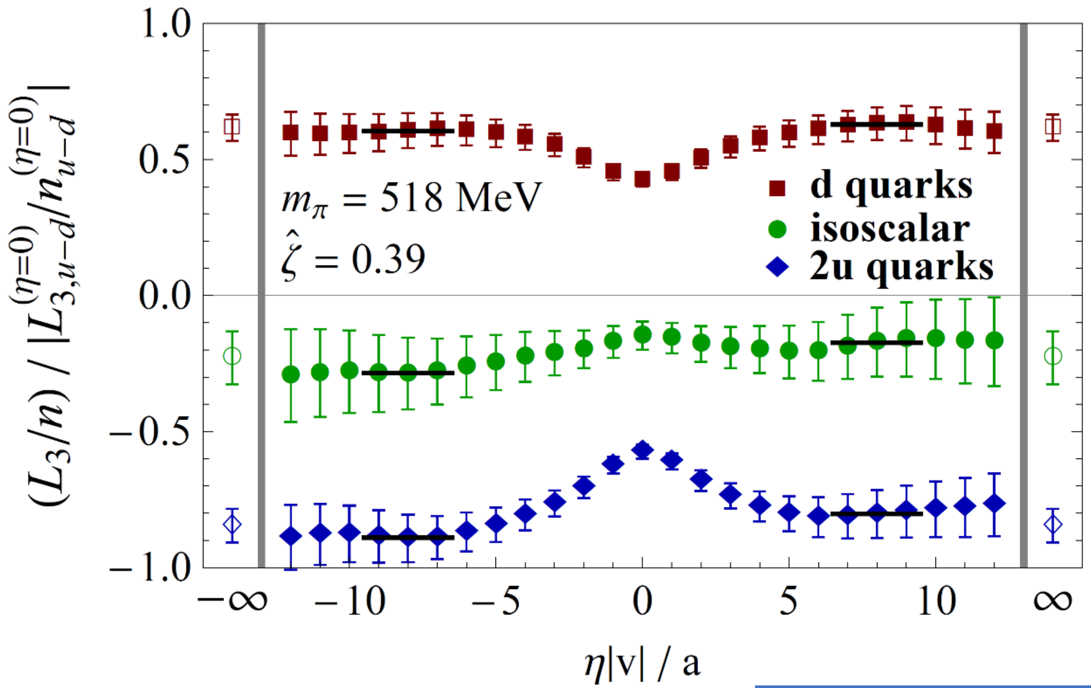


Ji OAM from straight Wilson line

Ji, Xiong, Yuan (2012)

Jaffe-Manohar vs. Ji: First lattice result

Engelhardt (2017)



Staple length

The difference: Potential OAM

$$L_{pot} \equiv L^q_{\rm Ji} - L^q_{can} = \int dx^- \langle \epsilon^{ij} b^i F^{+j} \rangle$$

torque acting on a quark

Burkardt (2012)

$$\sqrt{2} F^{+y} = -E^y + B^x = -(\vec{E} + \vec{v} \times \vec{B})^y$$

PDF of OAM

Define the x-distribution $L_{can} = \int dx L_{can}(x)$.

Hagler, Schafer (1998)
Harindranath, Kundu (1999)

$$L^q_{can} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

YH, Yoshida (2012)

$$\Rightarrow \quad L^q_{can}(x) = \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

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Wandzura-Wilczek part

$$= x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x') + E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1, x_2)\mathcal{P}\frac{3x_1 - x_2}{x_1^2(x_1 - x_2)^2}$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^2(x_1 - x_2)} .$$

genuine twist-three part

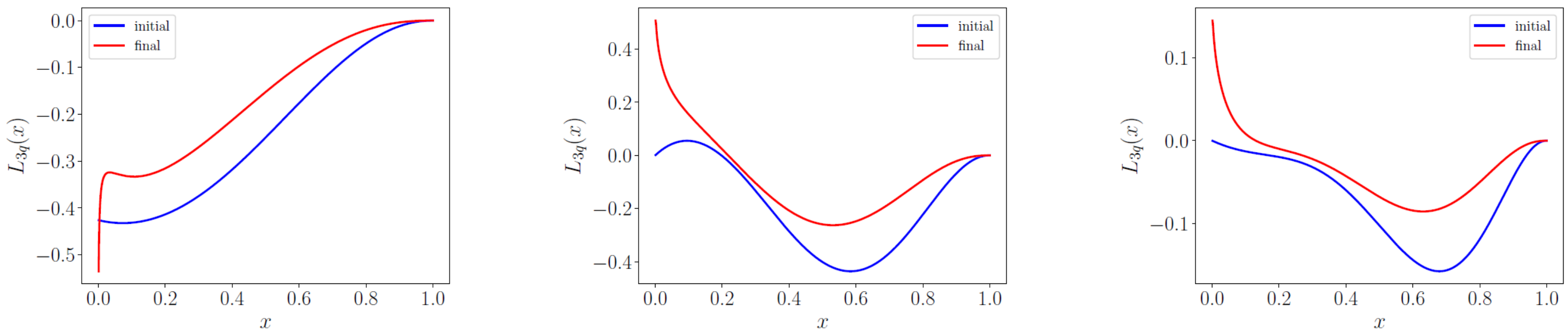
QCD evolution of OAM distributions $L_{q,g}(x)$

WW part → the same as twist-2 PDFs. Known to 3 loops.

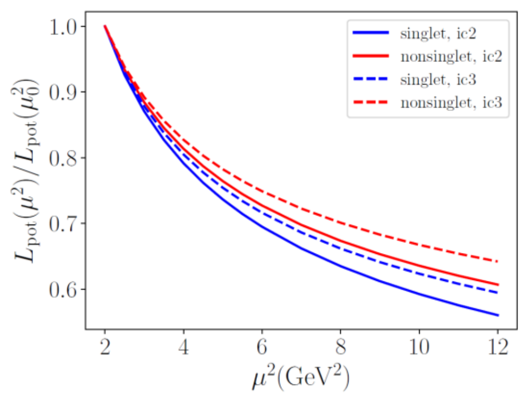
Genuine twist-3 part → Same as the evolution of Efremov-Teryaev-Qiu-Sterman function

Evolution of the genuine twist-3 part

YH, Yao (2019)



Scale dependence of the potential OAM



Spin at small-x?

All-order resummation of small-x double logarithms $(\alpha_s \ln^2 1/x)^n$
via InfraRed Evolution Equation   Kirshner, Lipatov (1983)

$$\Delta q(x) \sim \Delta g(x) \sim \left(\frac{1}{x}\right)^{3.45\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

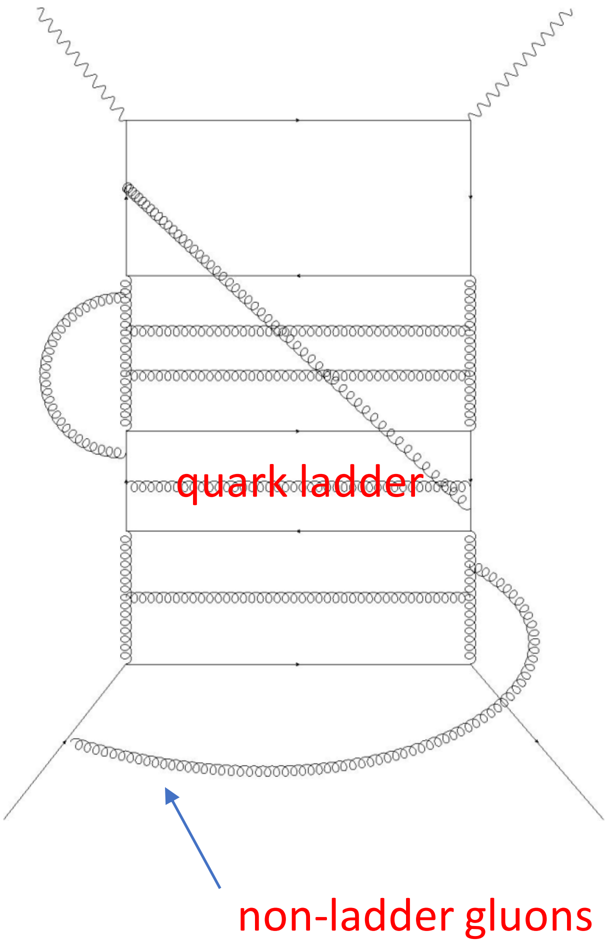
Bartels, Ermolaev, Ryskin (1996),

Disagree

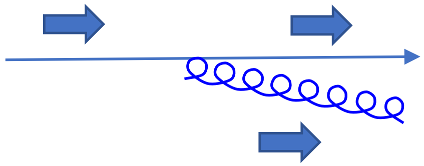
All-order resummation based on the polarized
dipole operator in the Wilson line approach to small-x

$$\Delta q(x) \sim \left(\frac{1}{x}\right)^{2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}} \qquad \Delta g(x) \sim \left(\frac{1}{x}\right)^{1.88\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Kovchegov, Pitonyak, Sievert (2015~)

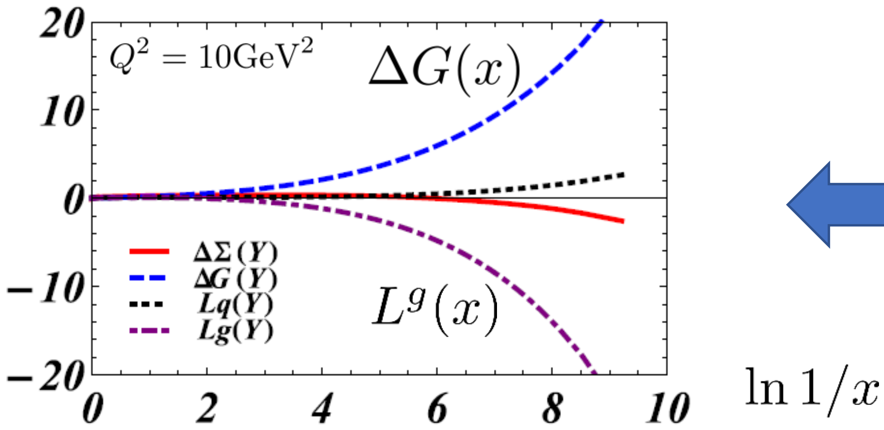


OAM at small-x



$x \ll 1$

Suppose a quark emits a very soft gluon.
Quark helicity unchanged.
From angular momentum conservation, gluon spin and OAM have to cancel.



Significant cancellation at small-x from one-loop DGLAP
YH, Yang (2018)

All-order result from the equation of motion and double log resummation

$$L_g(x) \approx -\frac{2}{1+\alpha}\Delta G(x) \sim \frac{1}{x^\alpha}$$

Boussarie, YH, Yuan (2019)

α from Bertels, Ermolaev, Ryskin

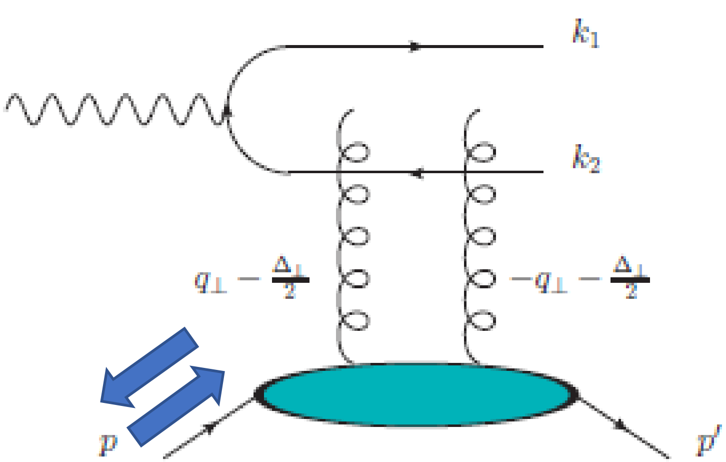
Measuring OAM at EIC

Ji, Yuan, Zhao (2016)
YH, Nakagawa, Xiao, Yuan, Zhao (2016)
Bhattacharya, Metz, Zhou (2017)

Exploit the connection between OAM and the Wigner distribution

$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

Longitudinal single spin asymmetry in diffractive dijet production



proton recoil momentum

$$\sigma^{\rightarrow} - \sigma^{\leftarrow} \propto \sin(\phi_P - \phi_\Delta)$$

dijet relative momentum

Need more work, more new ideas!

Transverse spin

Transversity global analysis

Radici, Bacchetta (2019)

First-ever extraction of transversity from data of SIDIS and pp collisions

SIDIS

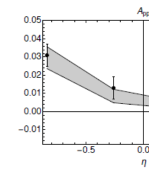
COMPASS: **18** data points

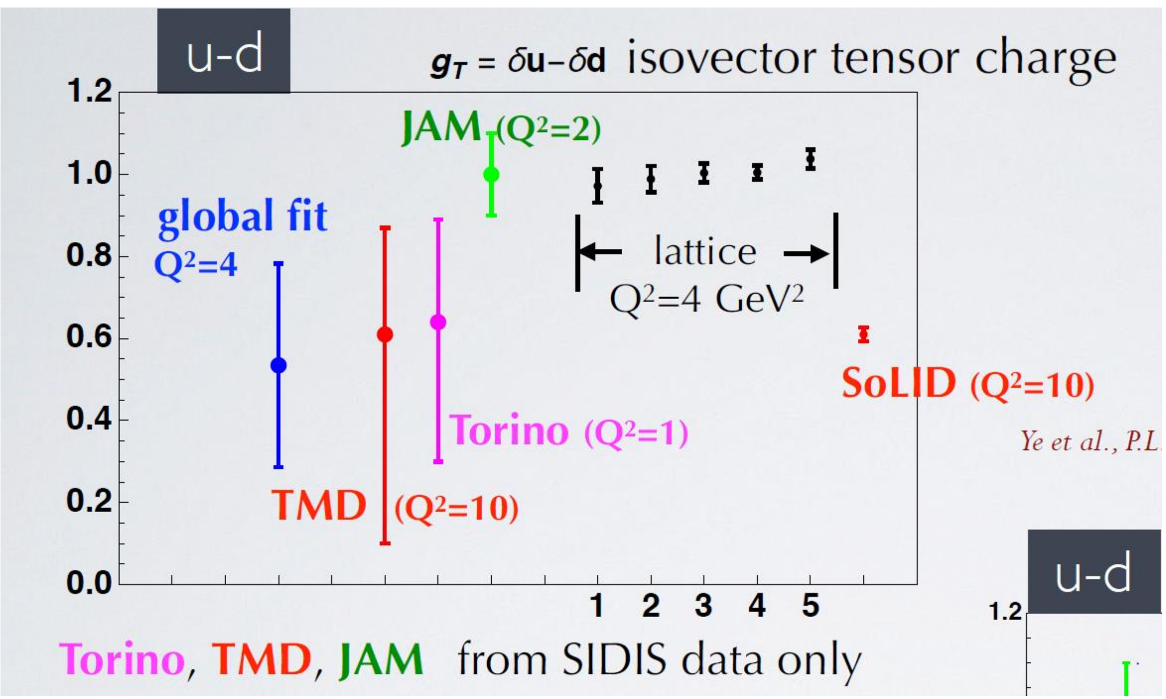
hermes: **4** data points

pp collisions

STAR: run 2006 (s=200 GeV^2)

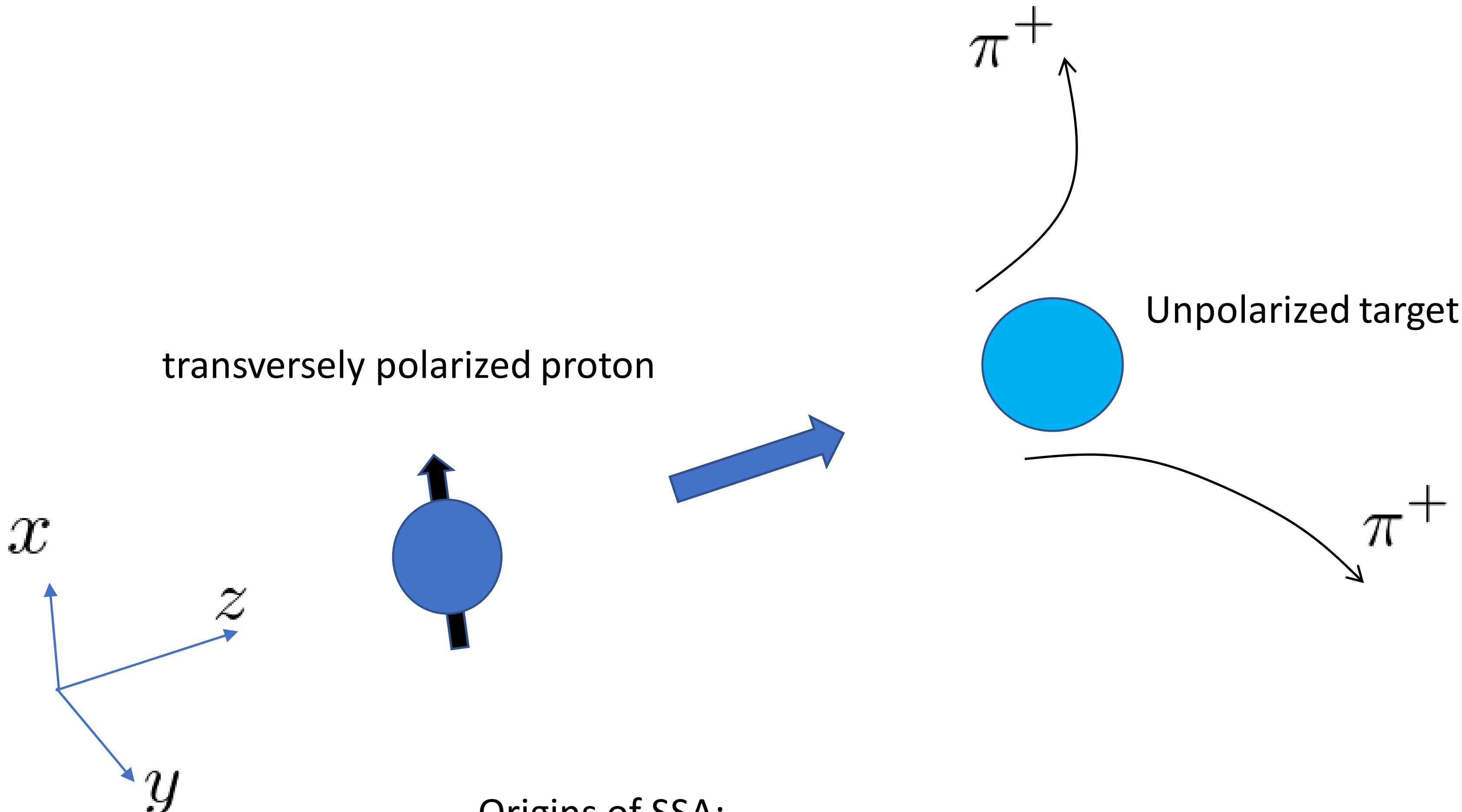
10 independent data points





Tension with the lattice data yet to be resolved.

Transverse Single Spin Asymmetry (SSA)



Origins of SSA:

kt factorization: Sivers, Collins

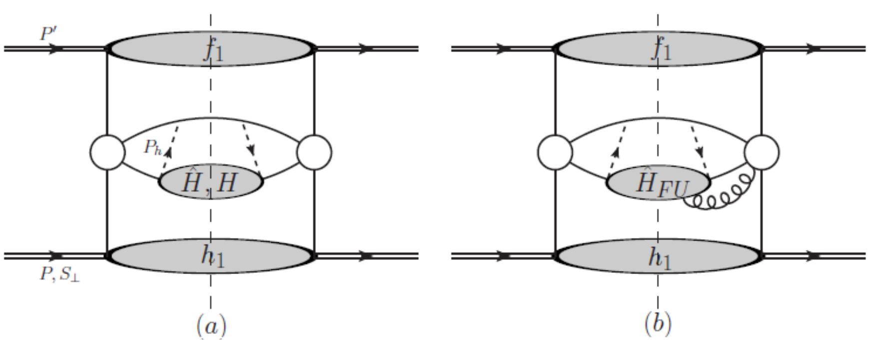
Collinear twist-3 factorization: ETQS, twist-3 fragmentation

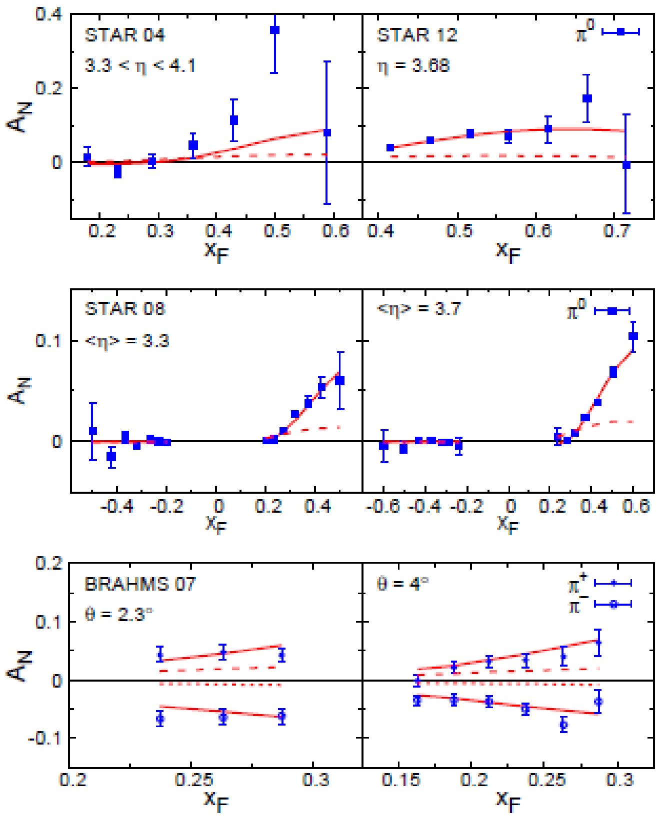
Fitting the pp data with twist-3 fragmentations

Kanazawa, Koike, Metz, Pitonyak (2014)
Gamberg, Kang, Pitonyak, Prokudin (2017)

RHIC pp data can be well described by assuming that SSA is dominated by genuine twist-3 fragmentations.

$$E_h \frac{d\sigma^{frag}}{d^3P_h} = 4M\alpha_s^2 \int \frac{dz}{z^3} h_1(x)\epsilon^{ij} S_{Ti} P_{hj} \frac{xz^6}{(P_{hT}^2)^3} \int dx' G(x') \times \left\{ -\mathrm{Im}\,\tilde{e}(z) + \int \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z}-\frac{1}{z_1}} \frac{\mathrm{Im}\hat{E}_F(z_1,z)}{N_c^2-1} \left(N_c^2 + \frac{1}{z_1\left(\frac{1}{z}-\frac{1}{z_1}\right)} \right) \right\}$$





Towards spin asymmetries at NLO

No all-order proof of factorization of SSAs in collinear twist-3 framework.

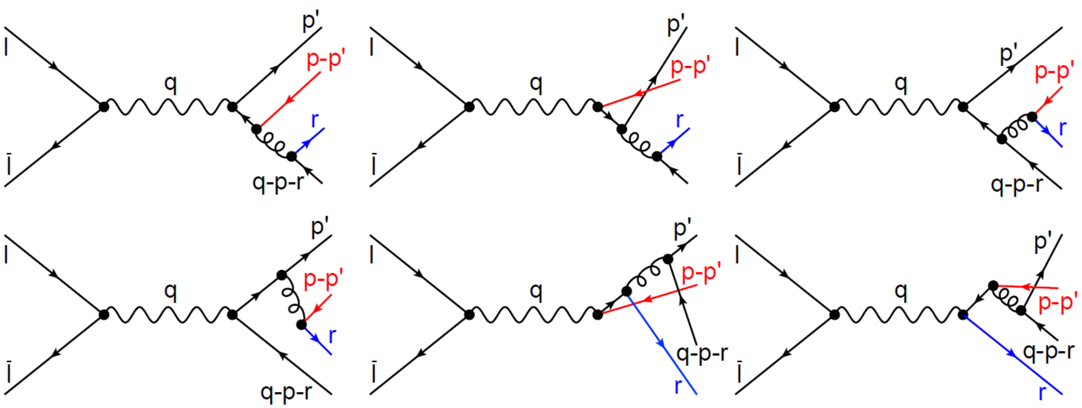
Check of factorization at NLO for a few observables.

- Pt weighted asymmetries in Drell-Yan and SIDIS

  Vogelsang, Yuan; Chen, Ma, Zhang; Kang, Vitev, Xing; Yoshida

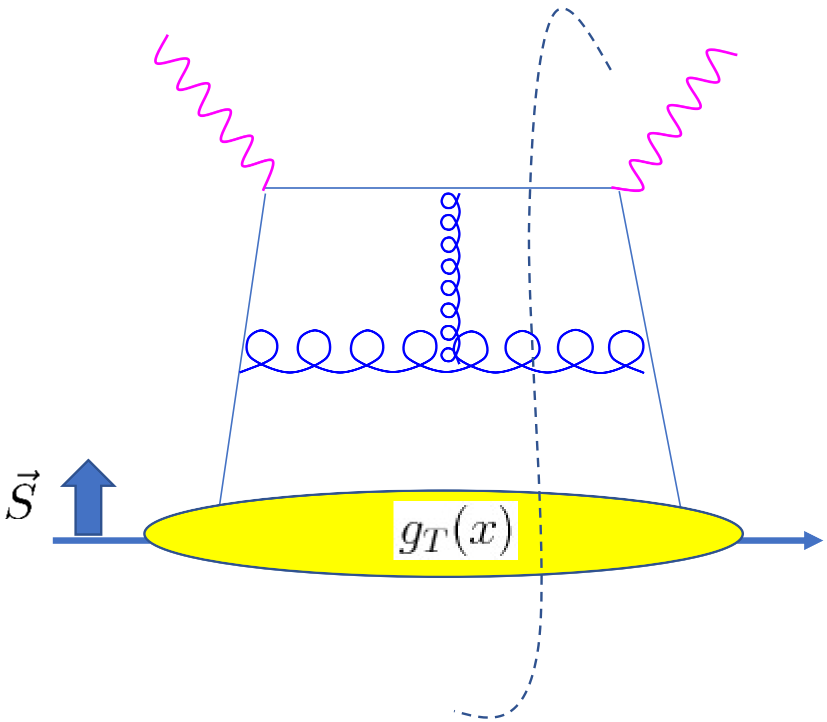
- Polarized hyperon production in e+e-

  Gamberg, Kang, Pitonyak, Schlegel, Yoshida



A yet another mechanism of SSA

Benic, YH, Li, Yang, to appear



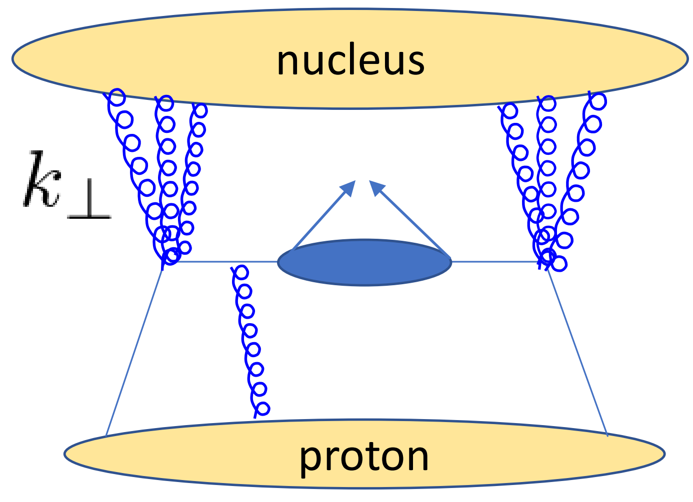
SSA of purely perturbative origin, first appears at two-loops
Suppressed by α_s compared to the known twist-three contributions
but $g_T(x)$ contains the Wandzura-Wilczek part.

SSA in pA

Boer, Dumitru, Hayashigaki
Kang, Yuan
Schaefer, Zhou
YH, Xiao, Yuan, Yoshida

In the forward region $x_F \sim 1$,
small-x effects of the nucleus are important

$$x = \frac{p_T^2}{szx_F} \ll 1$$



Study the A-dependence of A_N .
Different contributions have different A-dependences

$$A_N \sim A^{-1/3} \quad \text{or} \quad A_N \sim A^0$$

Data from RHIC available.
Tension between STAR and PHENIX data?

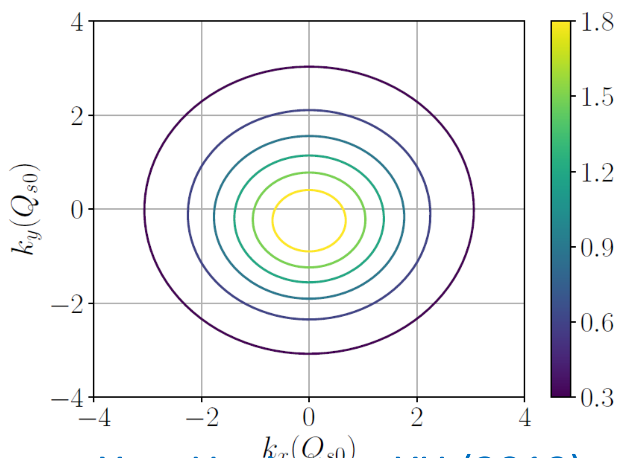
Gluon Sivers function

$$\frac{1}{xP^+}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{-ixP^+ z^- + ik_\perp \cdot z_\perp} \langle PS_\perp | 2\mathrm{Tr}[F^{+i}(z^-, z_\perp) U^{[\pm]} F^{+i}(0) U^{[\pm]}] | PS_\perp \rangle$$
$$= f_1^{[\pm\pm]}(x, k_\perp^2) - \frac{k_\perp \times S_\perp}{M} f_{1T}^{\perp[\pm\pm]}(x, k_\perp^2)\,,$$

Related to C-odd, three-gluon collinear correlation function
Beppu, Koike, Tanaka, Yoshida (2010)

Expected to be dominant in SSA of high-mass states, SSA in the backward region
Interesting physics cases at EIC Zheng, Aschenauer, Lee, Xiao, Yin (2018)

Related to the odderon exchange at small-x Zhou (2014)



Yao, Hagiwara, YH (2019)

Summary

- Longitudinal

  Uncertainties in gluon helicity distribution at small-x.
  → EIC
  → lattice, resummation

  OAM : Definition, evolution OK. Phenomenology missing!

- Transverse

  Transversity global analysis

  Many topics in SSA : Sign change, NLO, pA, gluon Sivers,