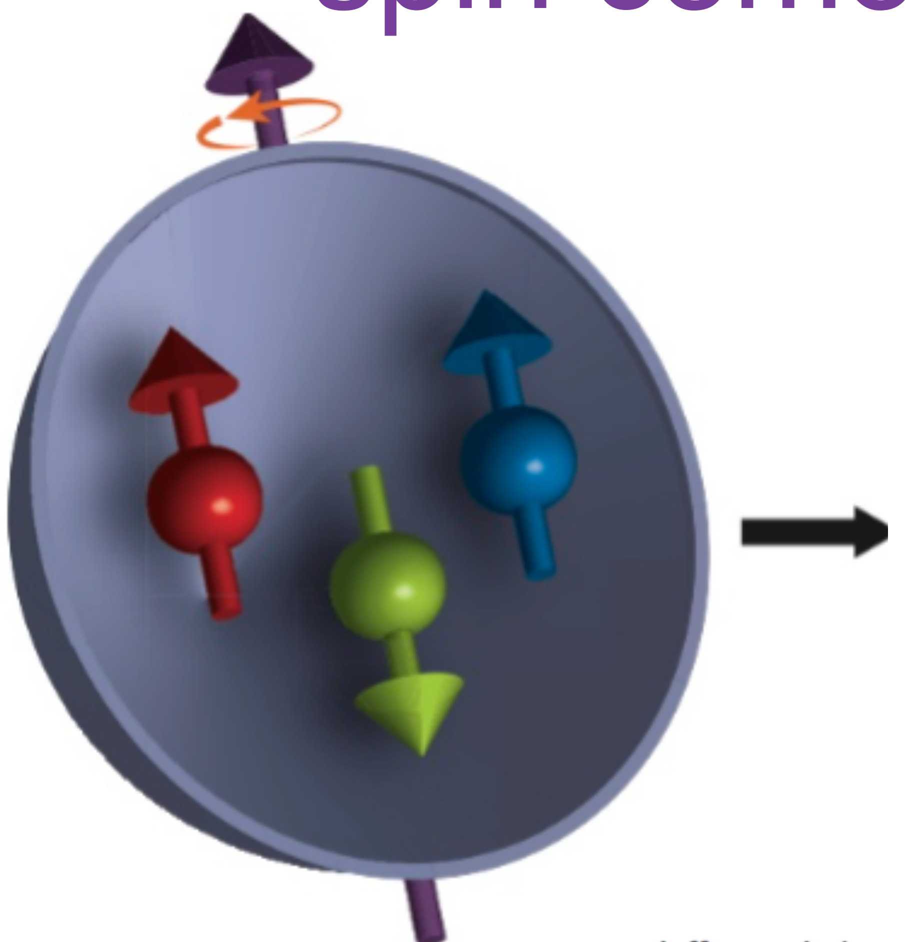
The 11th Circum-Pan-Pacific Symposium on High Energy Spin Physics

Proton Spin from quark and gluon

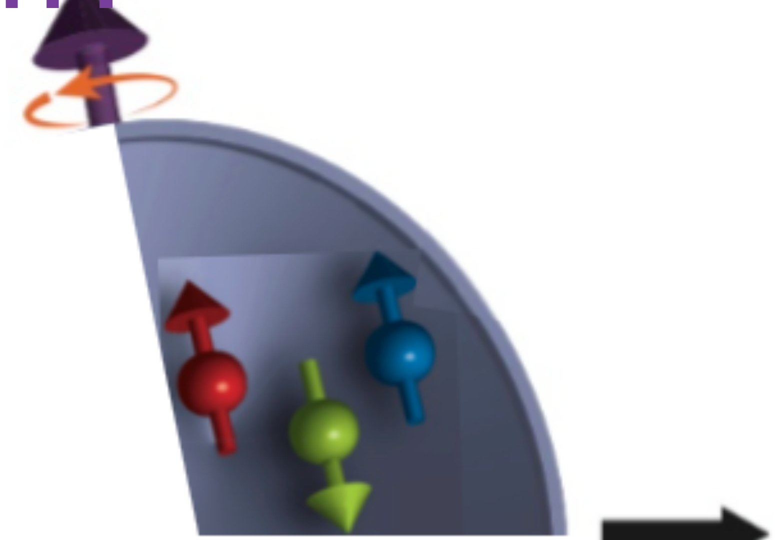
Yi-Bo Yang
ITP/CAS, China

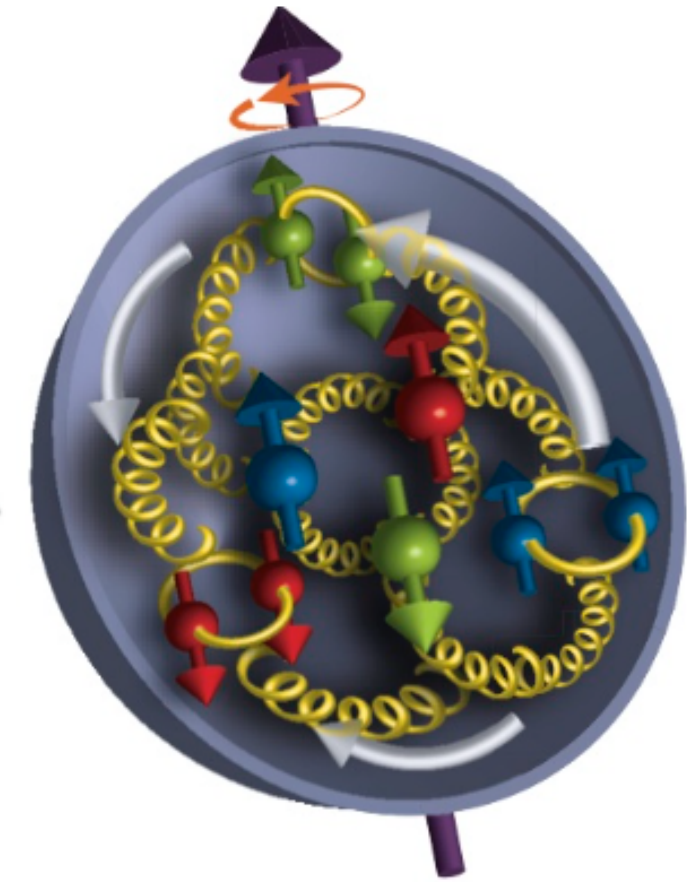
Aug. 27th. 2019

Where does the proton spin come from?



1980s





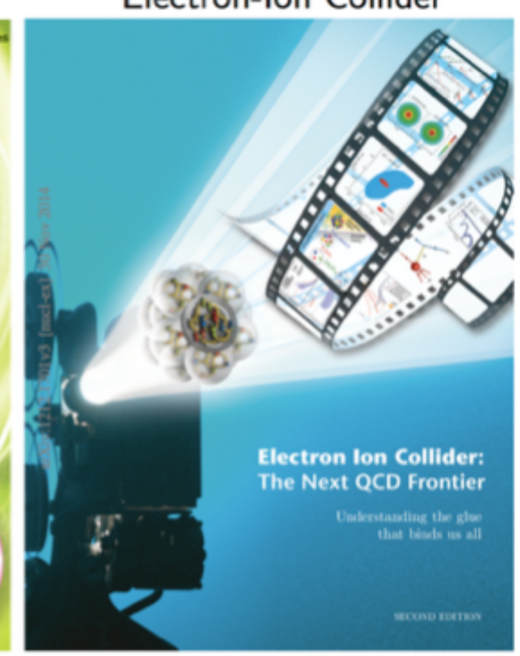
now

Only 30% of the proton spin comes from the quark spin, based on the experiments

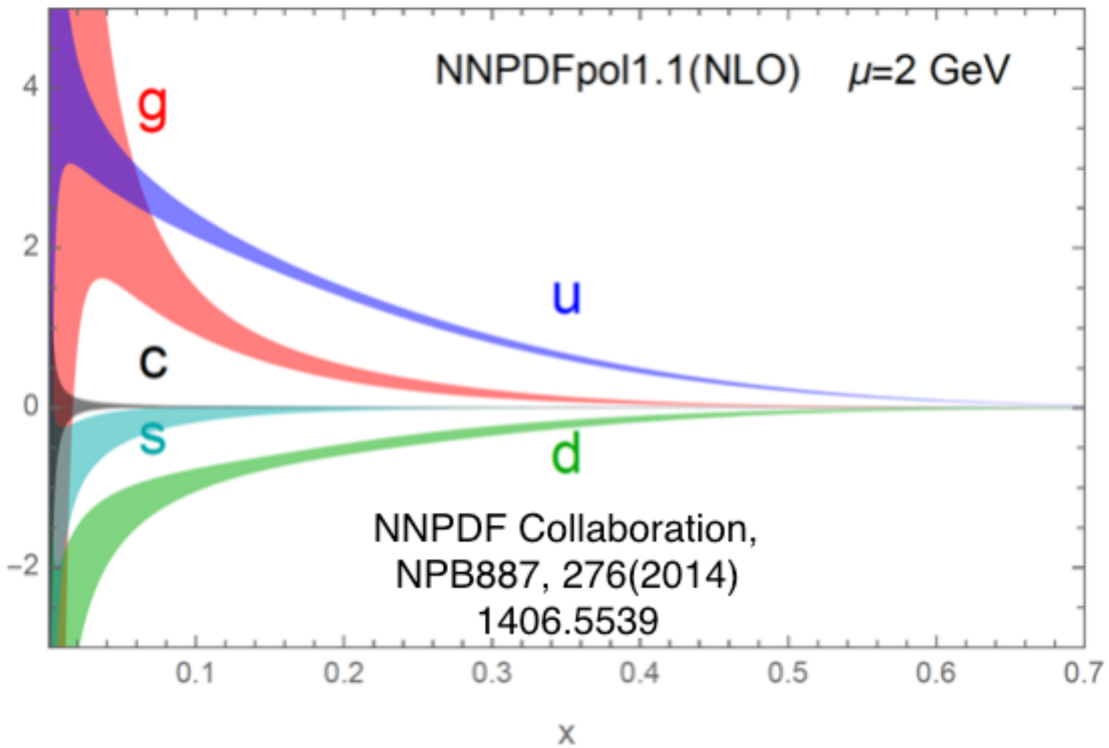
Jefferson Laboratory

Relativistic Heavy Ion Collider

A future Electron-Ion Collider



How does the spin of nucleon arise?



*Spin/helicity (**u**,**d**,**s**…,g): the **integration** of the polarized parton distribution function (PDF)*

$$\Delta q = \int_0^1 dx \Delta q(x)$$

$$\Delta G = \int_0^1 dx \Delta g(x)$$

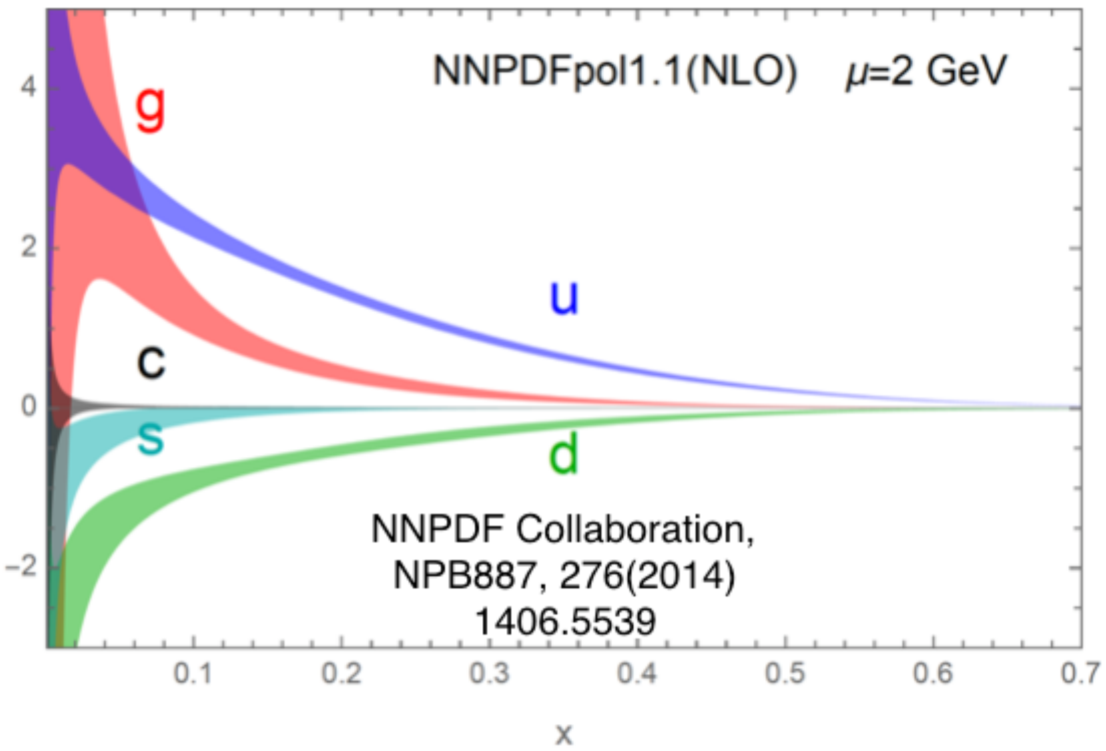
- The quark model (agrees with the lattice simulation at heavy quark limit):
  Δu→4/3, Δd→-1/3, Δs→0, Δg→0;

- **The polarized neutron decay:**
  Δu-Δd ≈ **1.2723(23);**
  PDG, CPC40, 100001 (2016)

- **The phenomenology fit of quark distribution based on Exp.:**
  Δu~0.8, Δd~-0.4, Δs~-0.1, Δg~0.4;
  EPJA52, 268 (2016), 1212.1701
  D. Florian, PRL 113, 012001 (2014), 1404.4293

- **The experiments** are quite **different** from the **naive theoretical understanding,** just becomes the **quark masses** in the real world are actually **light!**

Quark spin



*Quark spin/helicity (**u**,**d**,**s**…): the **integration** of the quark polarized parton distribution function (PDF)*

$$\Delta q = \int_0^1 dx \Delta q(x)$$

*Glue helicity (**g**): that of the gluon polarized PDF*

$$\Delta G = \int_0^1 dx \Delta g(x)$$

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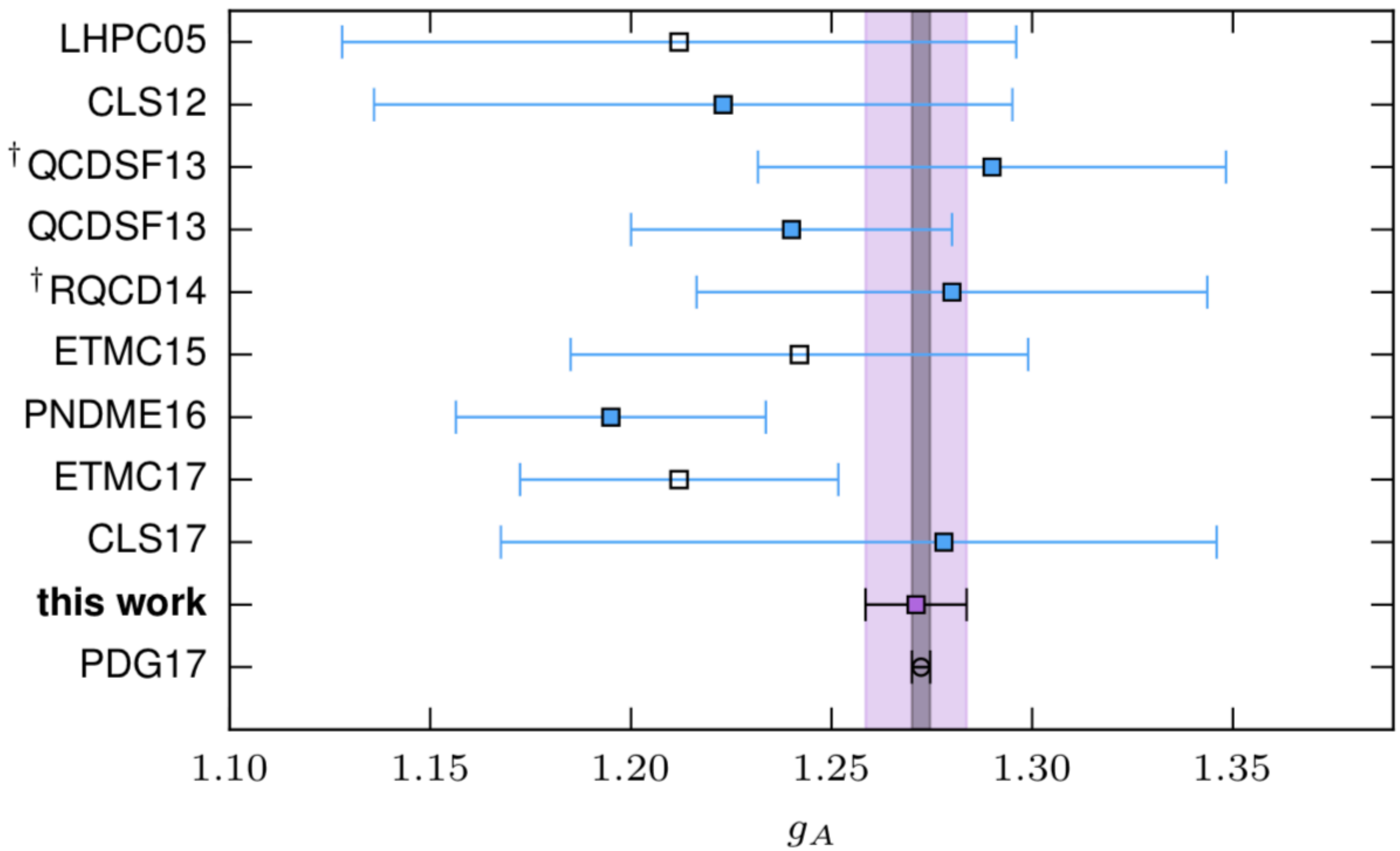
Quark spin

Iso-vector charge



C.C.Chang, et al., CalLat collaboration, Nature 558(2018)7708,91-94, 1806.08366

- Δu-Δd can be obtained from the neutron weak decay width, up to the iso-spin breaking and electric-weak effects;

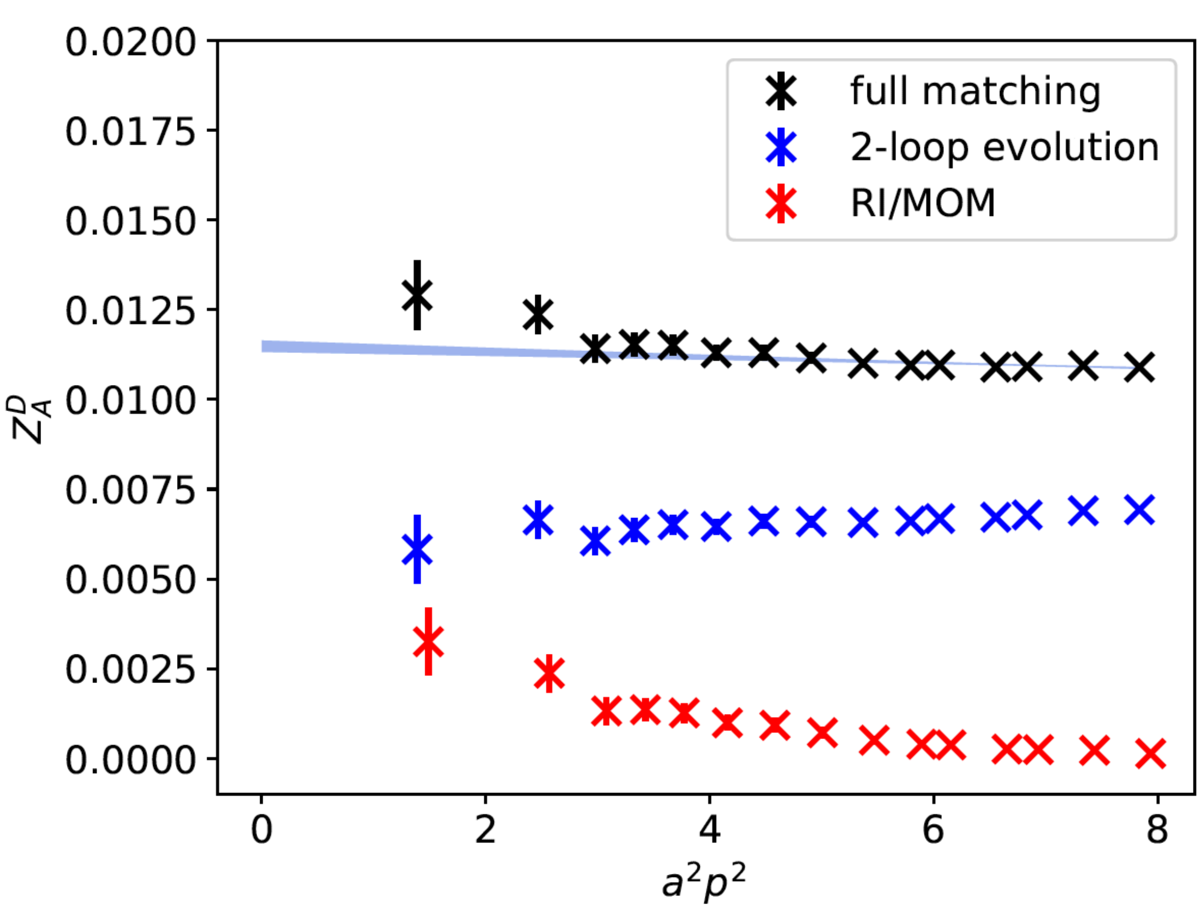
$$\langle N|(\bar{u}\Gamma u - \bar{d}\Gamma d)|N\rangle \simeq \langle P|\bar{d}\Gamma u|N\rangle$$

- It is scale independent and then don't need any renormalization;
- The uncertainty of the PDG value ~0.25%;
- The most precise lattice result so far has ~1% uncertainty.

Quark spin

Renormalization for the iso-scalar case

$$
\begin{pmatrix} \Delta u^{\overline{\mathrm{MS}}}(\mu) \\ \Delta d^{\overline{\mathrm{MS}}}(\mu) \\ \Delta s^{\overline{\mathrm{MS}}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_A + Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) & Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) & Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) \\ Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) & Z_A + Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) & Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) \\ Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) & Z_A^{\mathrm{D},\overline{\mathrm{MS}}\mathrm{I}}(\mu) & Z_A + Z_A^{\mathrm{D},\overline{\mathrm{MS}}}(\mu) \end{pmatrix} \begin{pmatrix} \Delta u \\ \Delta d \\ \Delta s \end{pmatrix}
$$

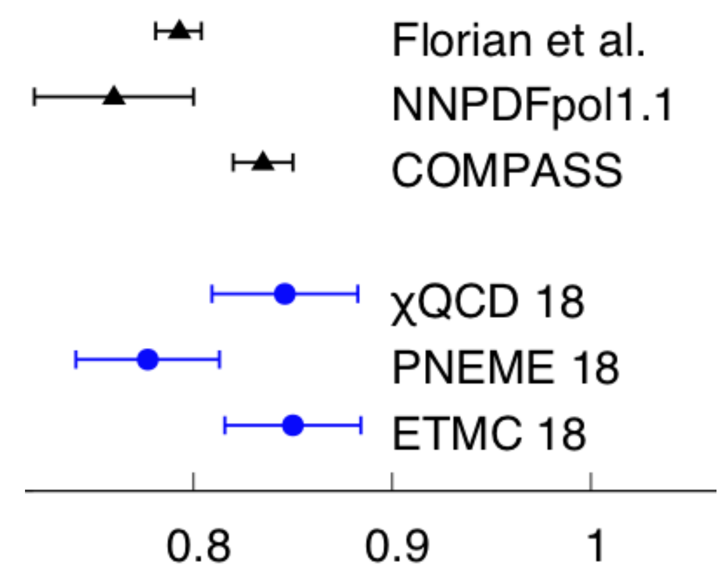


J. Liang, YBY, et al., χQCD collaboration, PRD98 (2018) 074505, 1806.08366

- *An accurate renormalization of the singlet axial vector current requires the **RI/MOM** calculation of the **quark loop** and also **the 2-loop matching.***
- *The only complete renormalization calculation with 2-loop finite piece so far.*

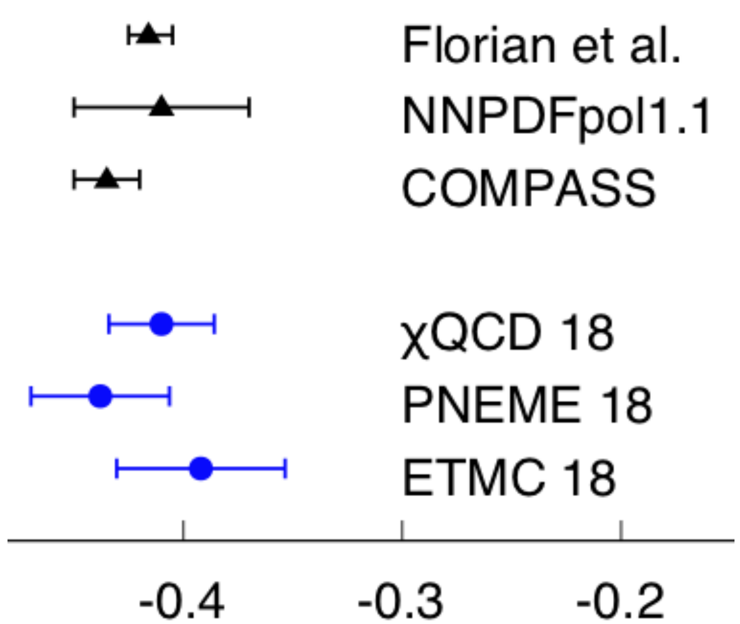
Quark spin: Present summary

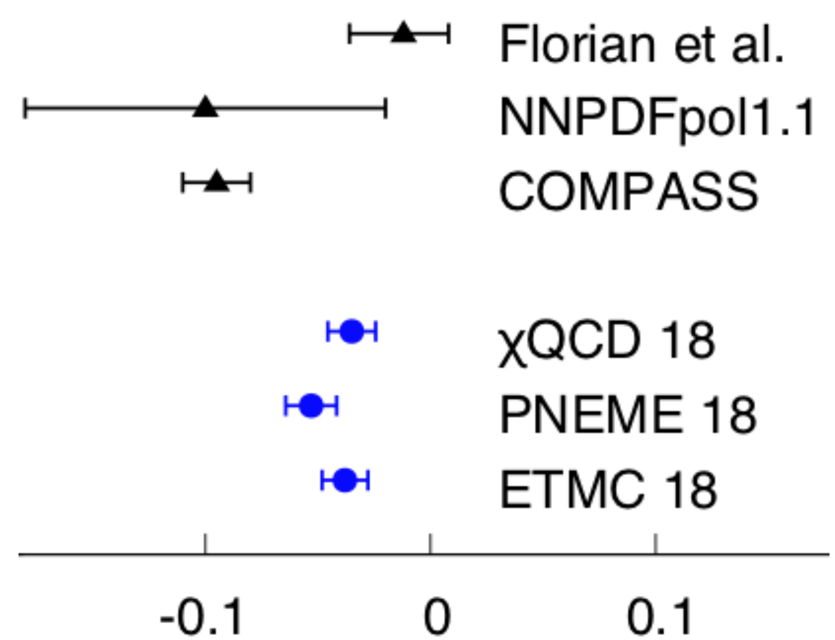
- Result from different groups are consistent.
- Uncertainty of Δu or Δd is comparable with the global fit
- That of Δs can be somehow better.



Δu

Δd





Δs

D. Florian, et.al, PRL113 (2014) 012001, 1404.4293

E. Nocera, et.al, NNPDF Collaboration, NPB887 (2014) 276, 1406.5539

C. Adoph, et.al, COMPASS Collaboration, PLB753 (2016) 18, 1503.08935

J Liang, **YBY**, et.al, χQCD Collaboration, PRD98 (2018) 074505, 1806.08366

H. Lin, et.al, PNEME Collaboration, PRD98 (2018) 094512, 1806.10604

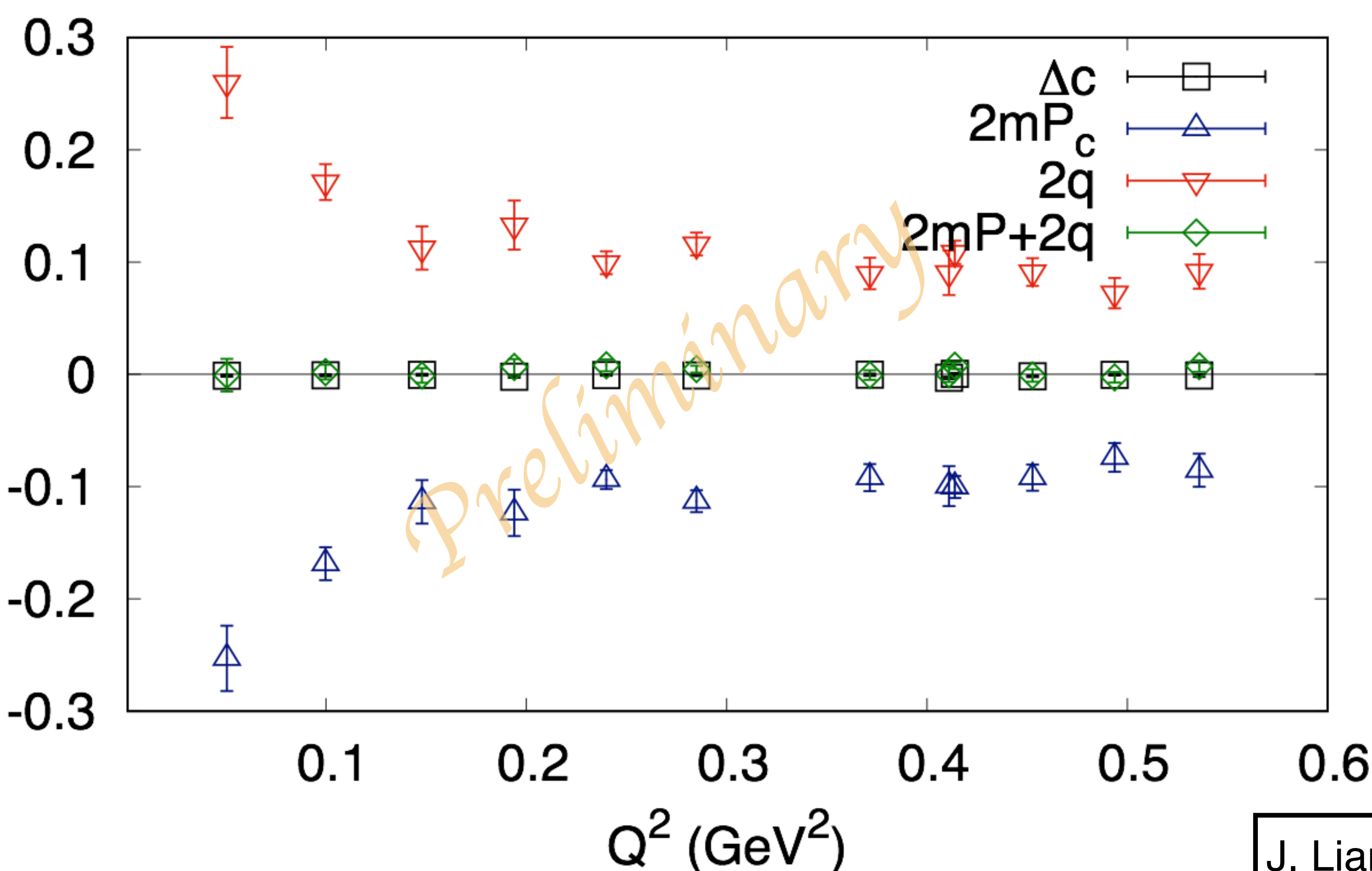
C. Alexandrou, et.al, ETMC collaboration, PRL119 (2017) 142002, 1706.02973, with 2018 updates

Quark spin

a story behind Δc

$$\langle ps \left| \vec{\mathcal{A}}_\mu \cdot \vec{s} \right| ps \rangle = \lim_{p'-p\to 0} \frac{i|\vec{s}|}{(\vec{p'} - \vec{p}) \cdot \vec{s}} \langle p', s| 2m_f \mathcal{P} - 2i \frac{\alpha_s}{4\pi} F\tilde{F} \, |p, s\rangle$$

$$= 2m_f \langle p, s| \int d^3x \; \vec{x} \cdot \vec{s} \; \mathcal{P}(x) |p, s\rangle - 2i \langle p, s| \int d^3x \; \vec{x} \cdot \vec{s} \; \frac{\alpha_s}{4\pi} F(x)\tilde{F}(x) \, |p, s\rangle$$



- The contribution from the glue anomaly turns out to be **very large** (~0.2-0.3 per flavor)!
- Accurate calculation of the light flavor cases is ongoing.

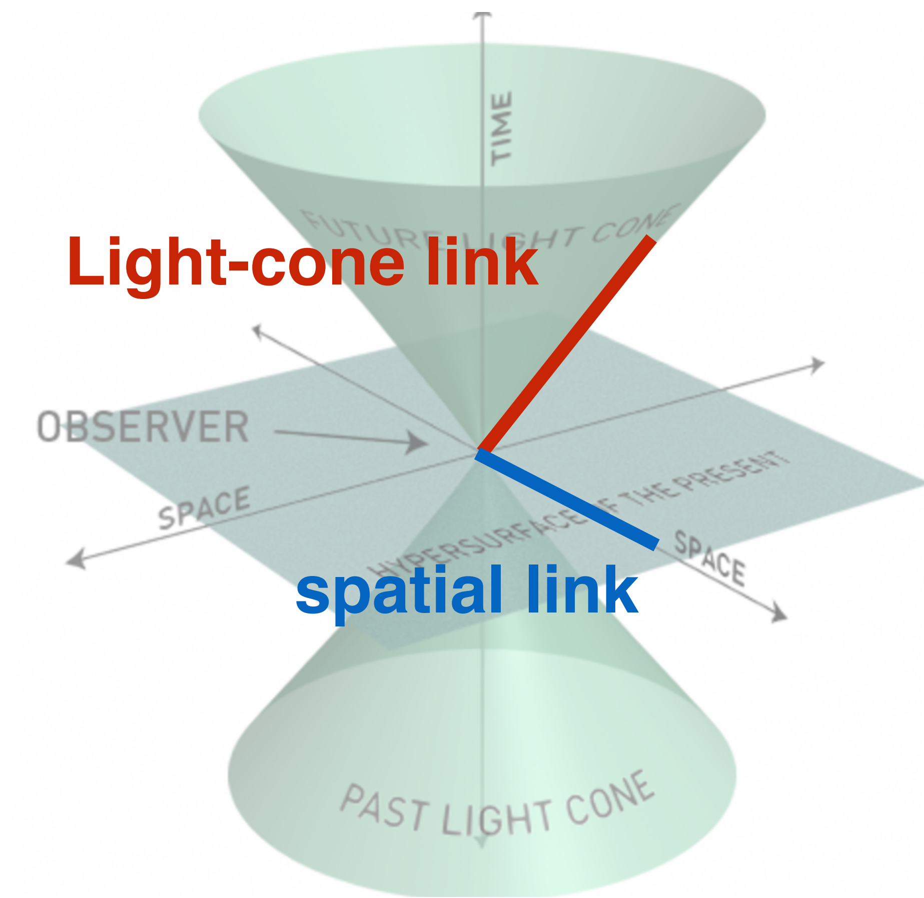
J. Liang, **YBY**, et. al., χQCD collaboration, in preparation

M. Gong, **YBY**, et. al., χQCD collaboration, PRD95, 114509 (2017), 1511.03671

Access the polarized PDF itself from lattice QCD

The original polarized quark PDF defined in the light front frame is,

$$\Delta q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+\gamma^5 \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$



The quasi-PDF is defined by

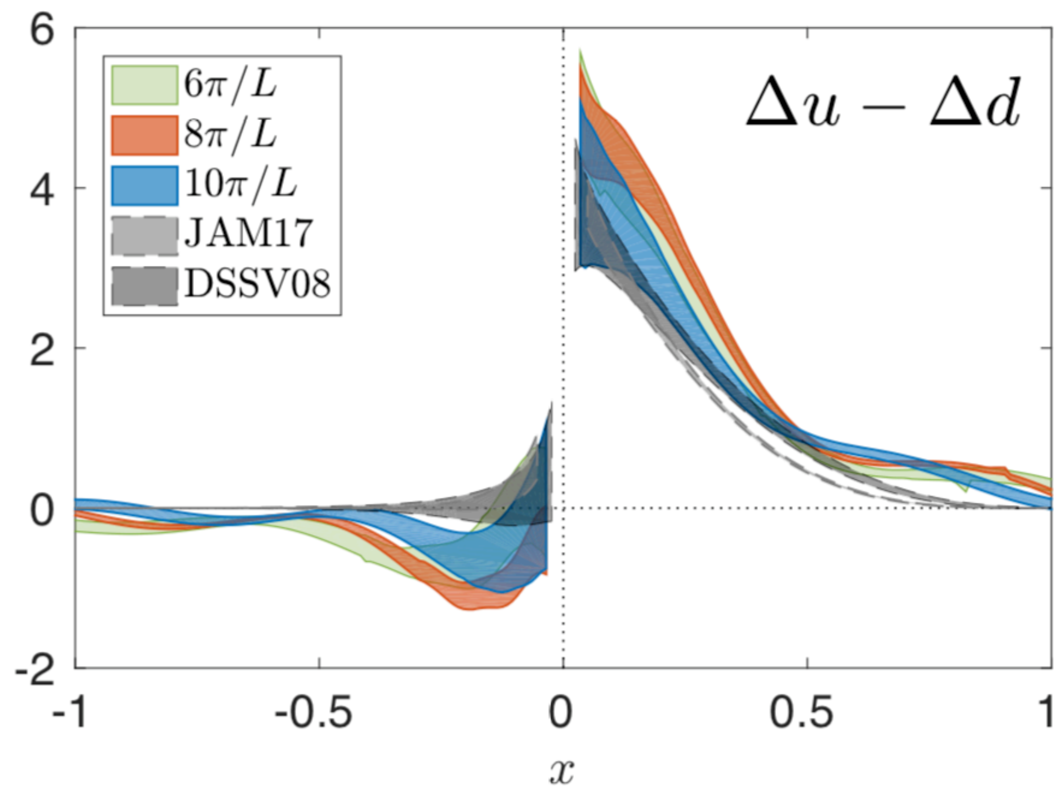
$$\Delta\tilde{q}(x,P_z,\tilde{\mu})_\Gamma = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{ixP_z z}\langle P|O_\Gamma(z)|P\rangle$$

With $O_\Gamma(z) = \bar{\psi}(z)\Gamma U(z,0)\psi(0)$,

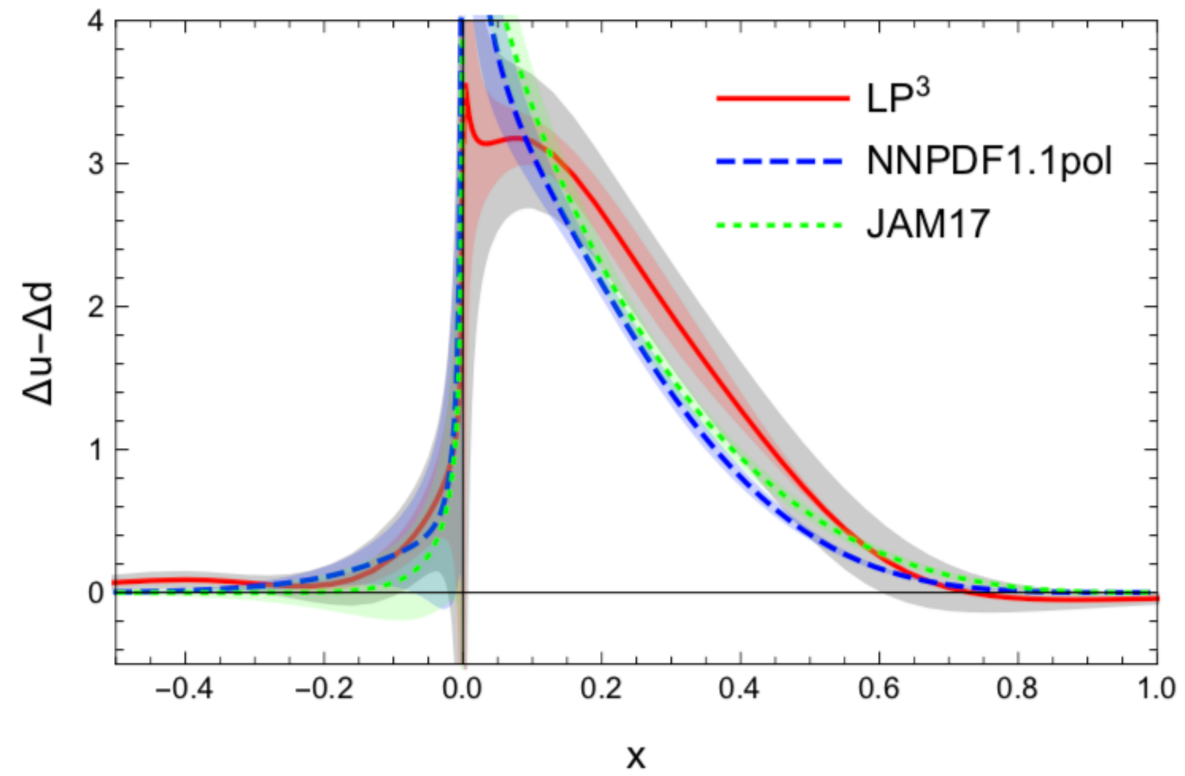
$\Gamma = \gamma_z\gamma^5$ *and* $U(z,0) = P\exp\left(-ig\int_0^z dz' A^z(z')\right)$

X.D. Ji, PRL110 (2013) 262002, 1305.1539
X. Xiong, et.al, PRD90 (2014) 014051, 1310.7471

Iso-vector quark polarized quasi-PDF



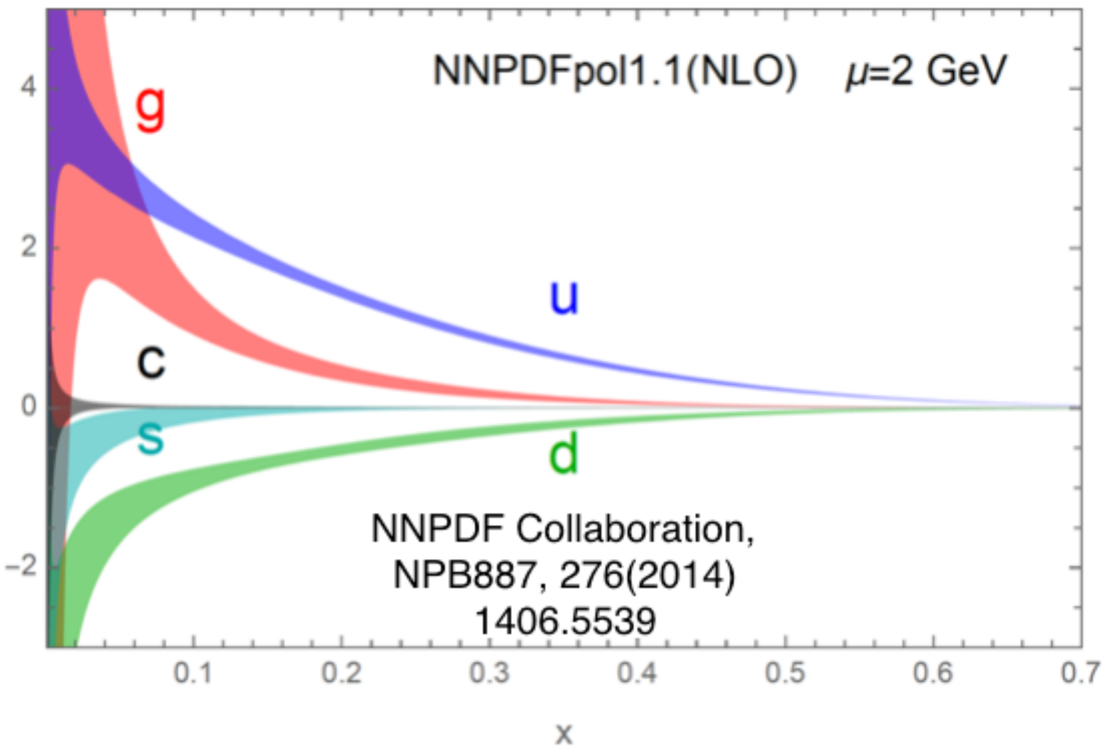
C. Alexandrou, et al, PRL 121 (2018) 112001, 1803.02685



H. Lin, et al, LP[3] Collaboration, PRL 121(2018) 242003, 1807.07431

- The shapes of from different simulation setup are still somehow different;
- The systematic uncertainties from finite lattice spacing, 2-loop matching, excited state contaminations and reconstruction scheme are under investigation.

glue spin



*Quark spin/helicity (**u**,**d**,**s**…): the **integration** of the quark polarized parton distribution function (PDF)*

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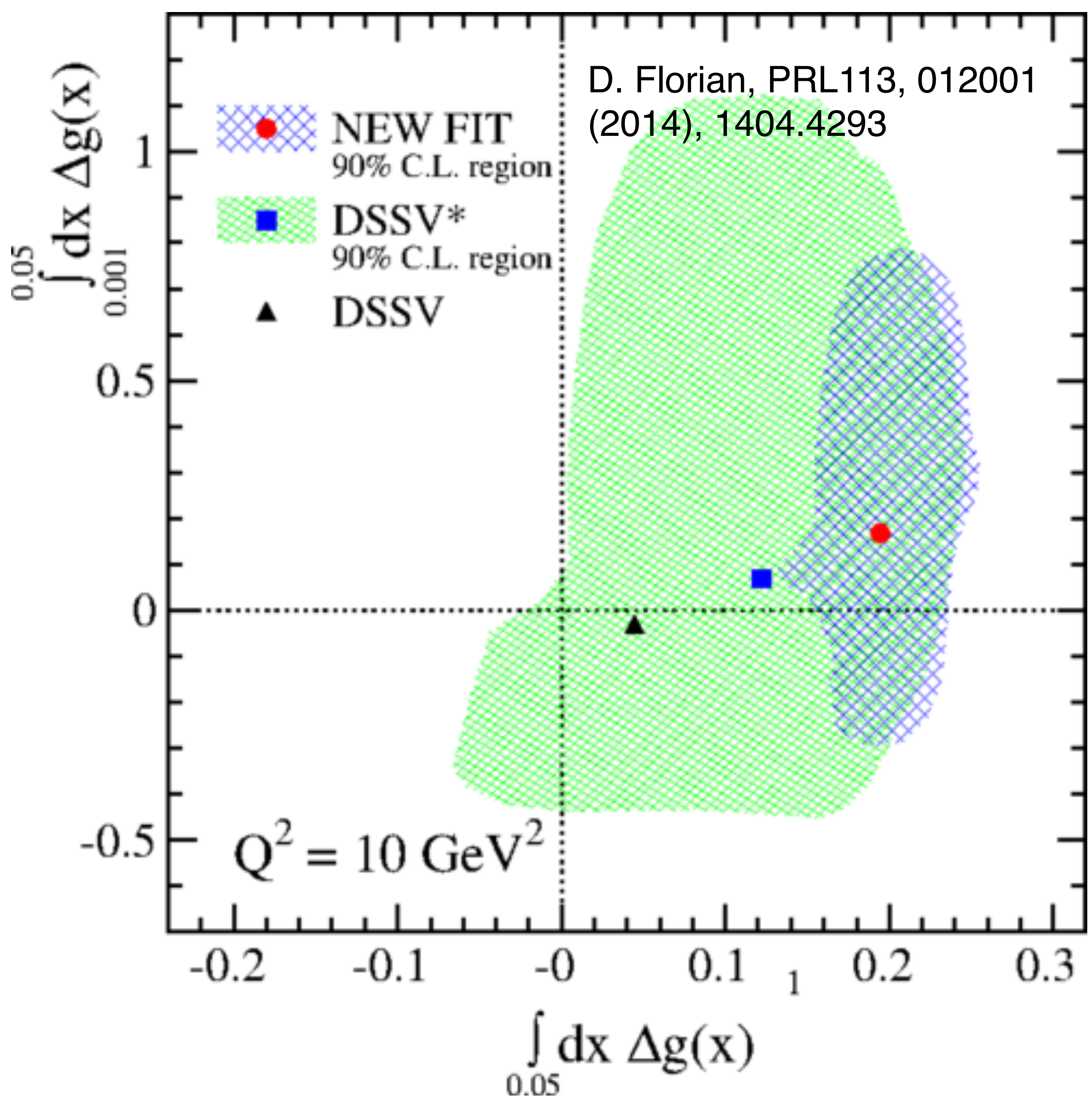
Proton spin

Glue helicity

Glue helicity $\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) | PS \rangle$



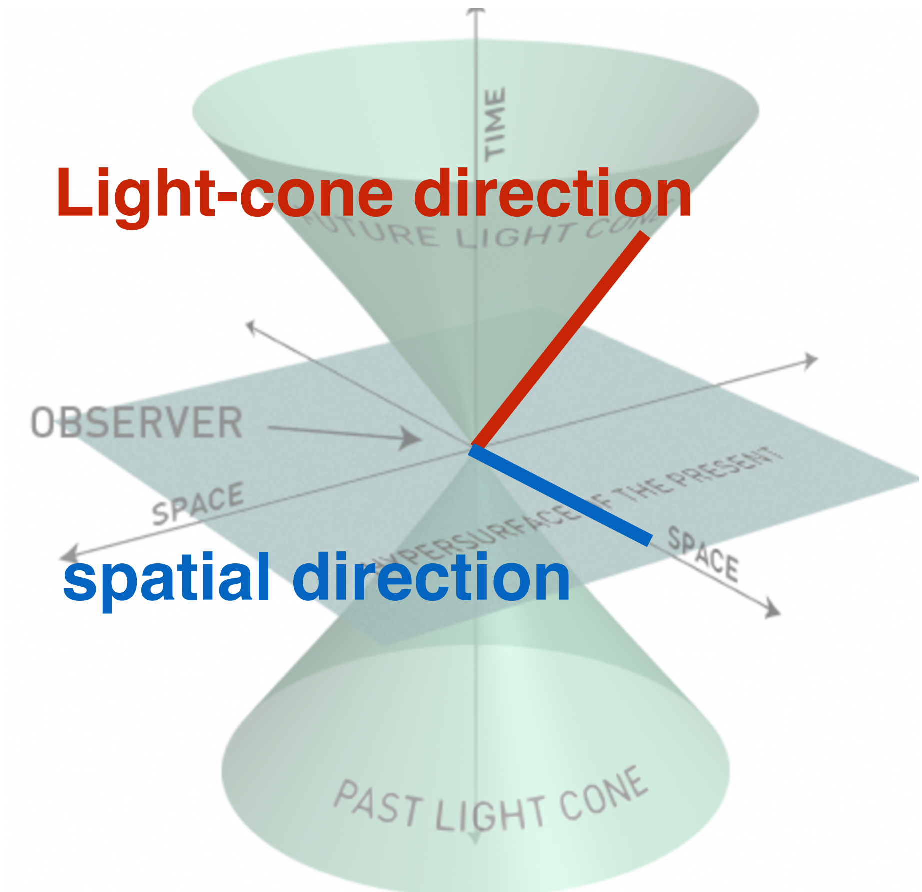
has been shown to be ~0.2 or 40% of the proton spin with large uncertainty, based the global analysis with experimental results from:

STAR NPA932, 500(2014),1404.5134
PHENIX PRD90, 012007(2014), 1402.6296
COMPASS PLB690, 466(2010), 1001.4654

Glue spin

Large momentum effective theory (LaMET)

$$O_{\Delta_G} = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-, 0))\right]^z = \vec{E}_{LC} \times \vec{A}_{LC},\ A^+_{LC} = 0$$



When nucleon is boosted:

- The Coulomb and Temporal gauge conditions become the light-cone one.
- **Glue spin** below becomes **glue helicity,** the integration of the glue polarized PDF, **at tree level**.

$$O_{S^c_G} = \vec{E}^c \times \vec{A}^c,\ \partial_i A^c_i = 0$$ ***Coulomb gauge***

or

$$O_{S^t_G} = \vec{E}^t \times \vec{A}^t,\ A^t_0 = 0$$ ***Temporal gauge***

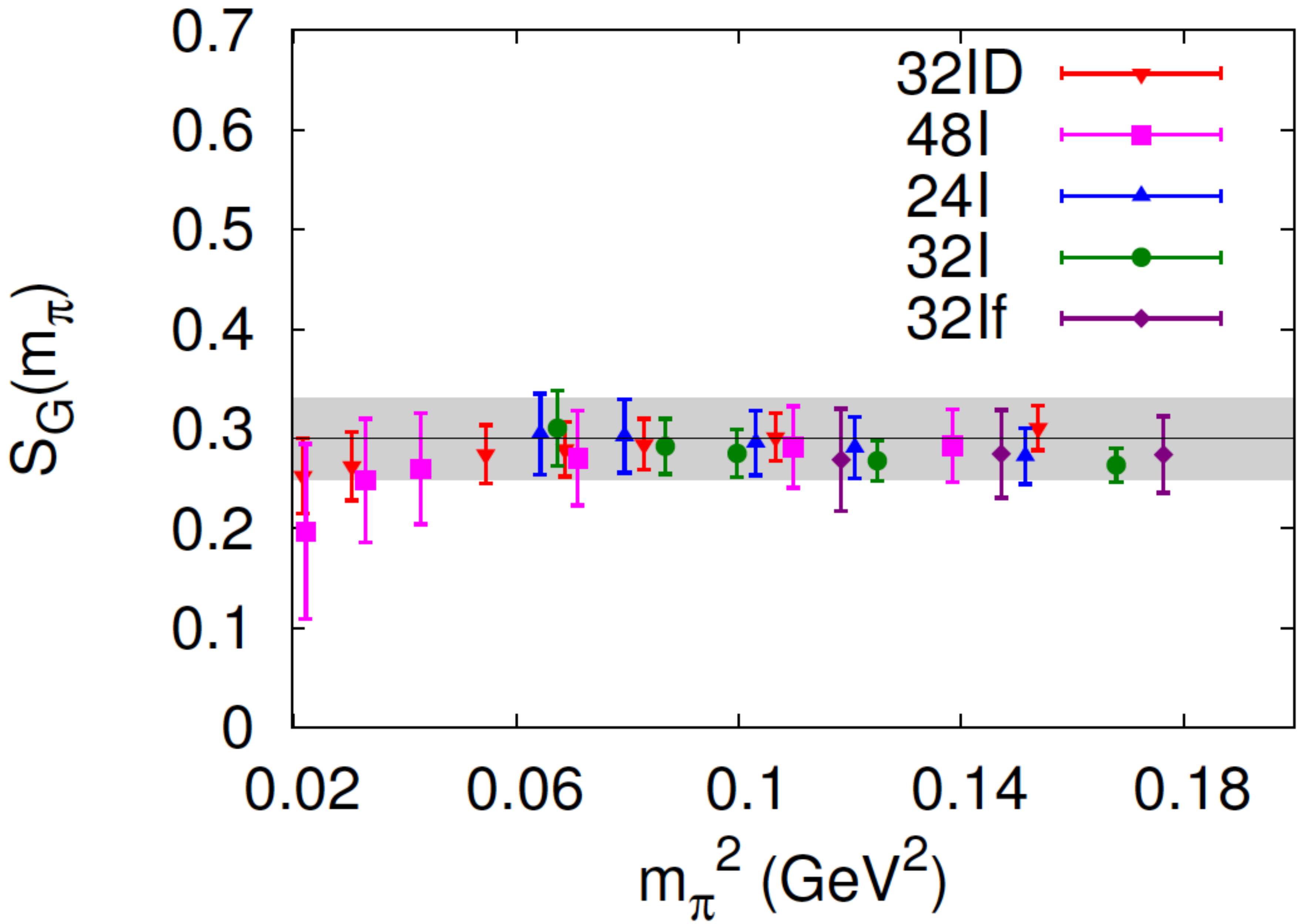
X. Ji, J. Zhang, and Y. Zhao, PRL111 112002 (2013), 1304.6708

The dependence

YBY, R. Sufian, et al., χQCD collaboration, PRL118, 042001(2017), 1609.05937
ViewPoint and Editor's suggestion

of m_π, a, and V

μ^2=10 GeV^2

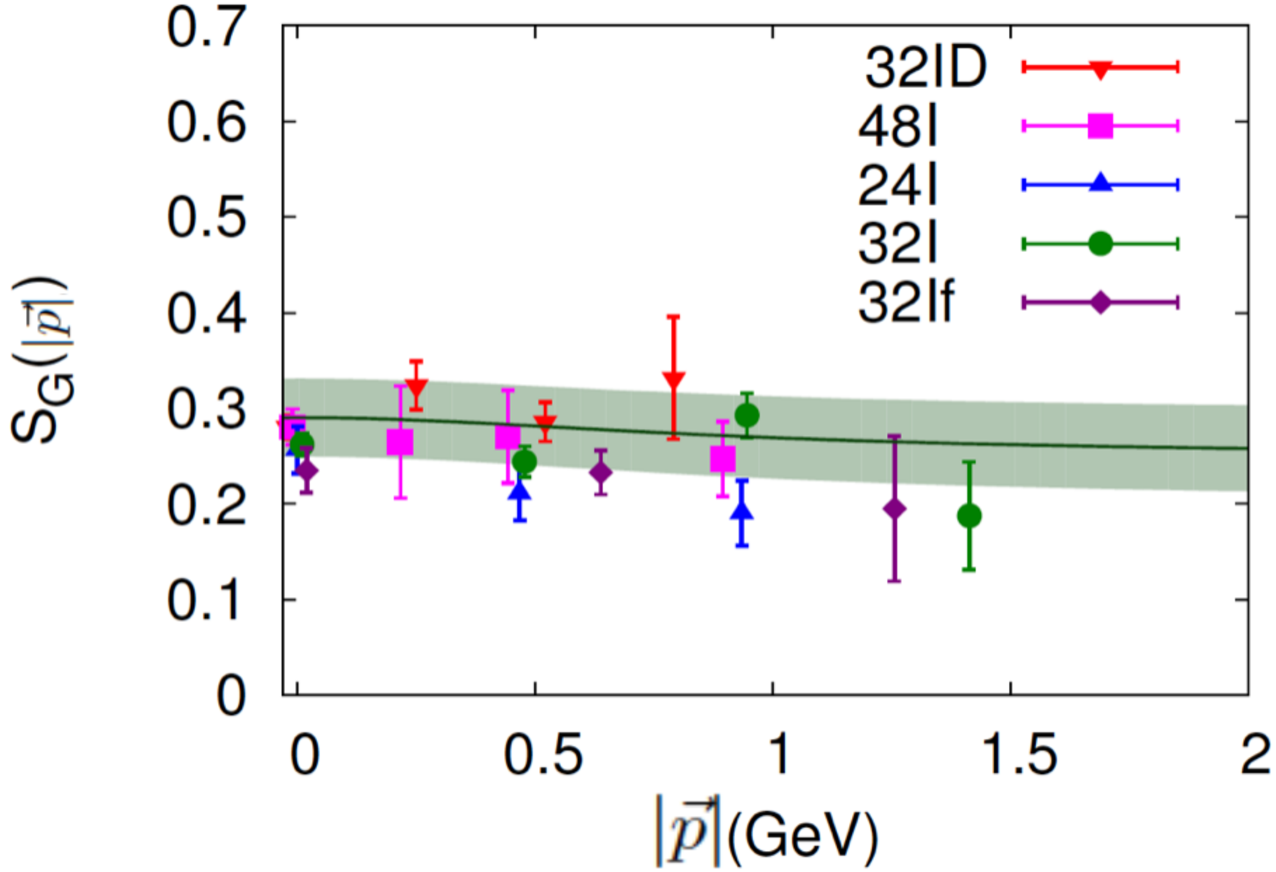


In the rest frame, the pion mass (both valence and sea), lattice spacing and volume dependences are mild.

Glue spin

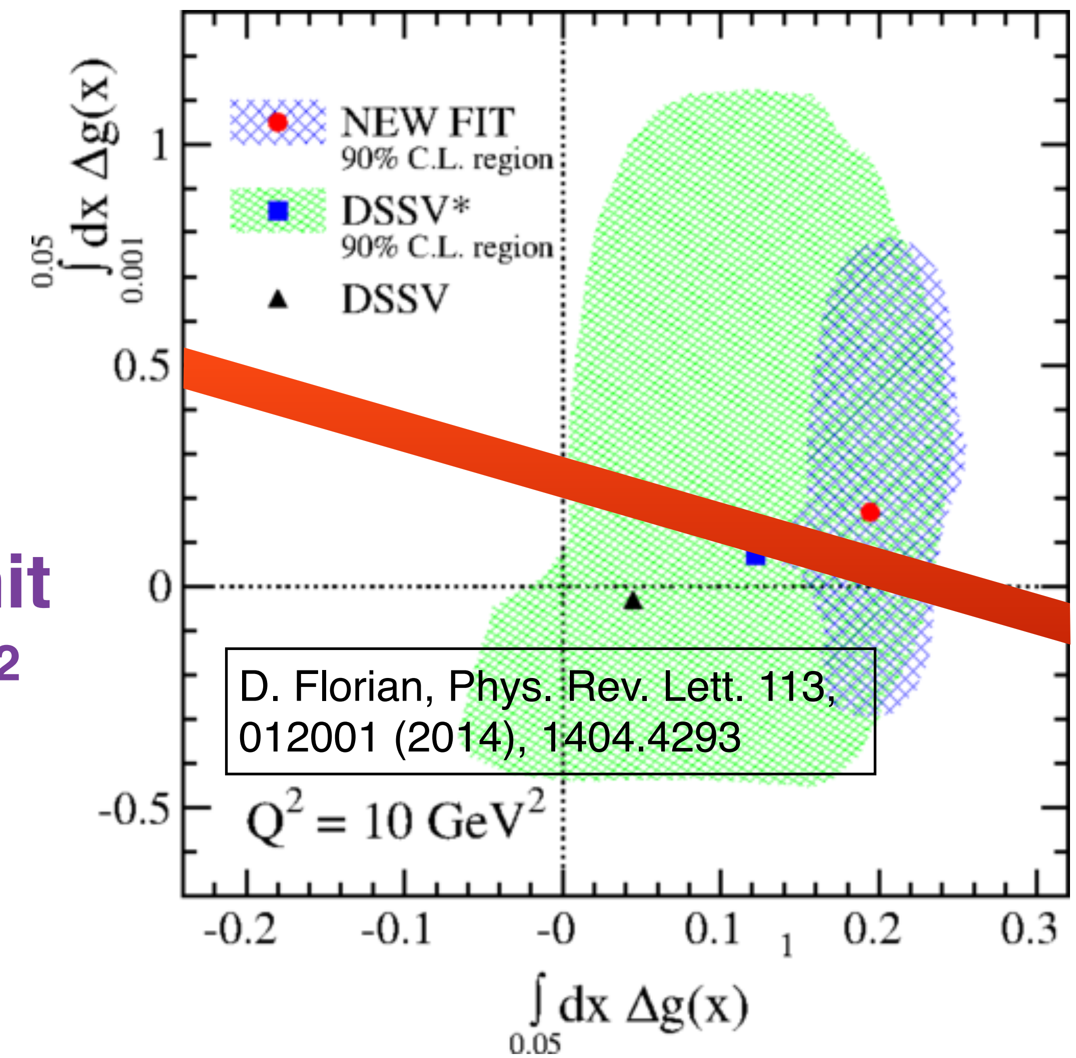
YBY, R. Sufian, et al., χQCD collaboration, PRL118, 042001(2017), 1609.05937
ViewPoint and Editor's suggestion

Results



Neglect the matching and apply an empirical form to fit the data,

$$\int_{0.001}^{0.05} \mathrm{d}x \Delta g(x) + \int_{0.05}^{1} \mathrm{d}x \Delta g(x) \simeq S_g$$

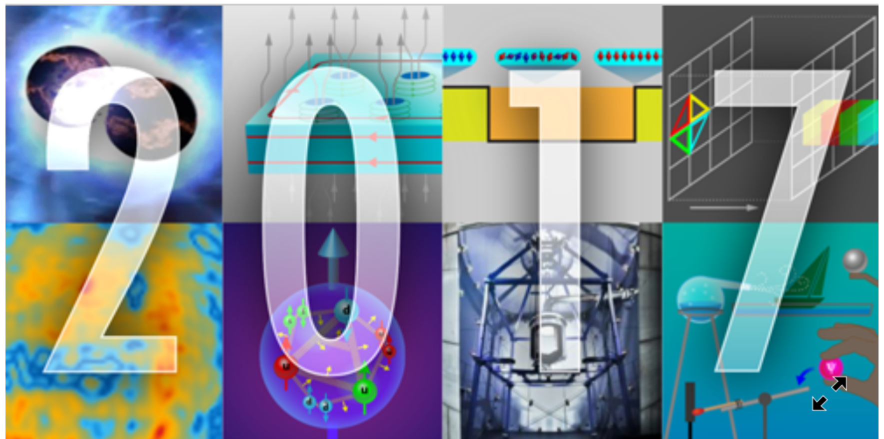


the glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2 will be

S_G=0.251(47)(16).

One of **eight**

YBY, R. Sufian, et. al., χQCD collaboration, PRL118, 042001(2017), 1609.05937
ViewPoint and Editor's suggestion



APS Highlights of 2017

https://physics.aps.org/articles/v10/137

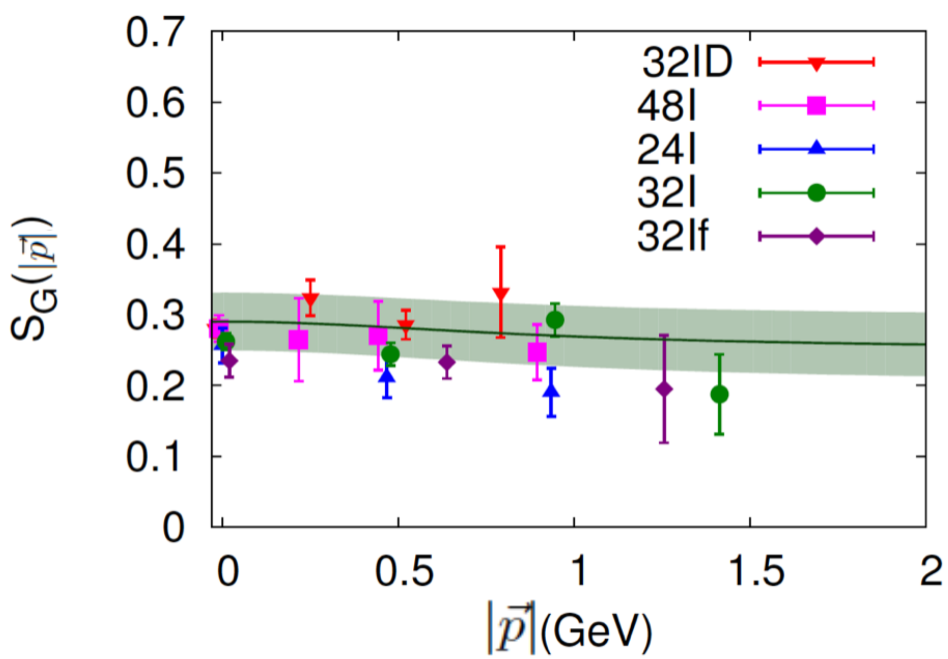
Gluons Provide Half of the Proton's Spin

The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from its gluons. The result is in agreement with recent experiments and shows how such lattice simulations can now accurately predict an increasing number of particle properties. The simulations also indicate that, despite being substantial, the gluon spin contribution is too small to play a major part in "screening" the quark spin contribution—which according to experiments is only 30%—through a quantum effect called the axial anomaly. The remaining 20% of the proton spin is thought to come from the orbital angular momentum of quarks and gluons.

Gluon unpolarized PDF from Lattice QCD

Revisit the glue spin

YBY, R. Sufian, et al., (χQCD), PRL118, 042001(2017), 1609.05937

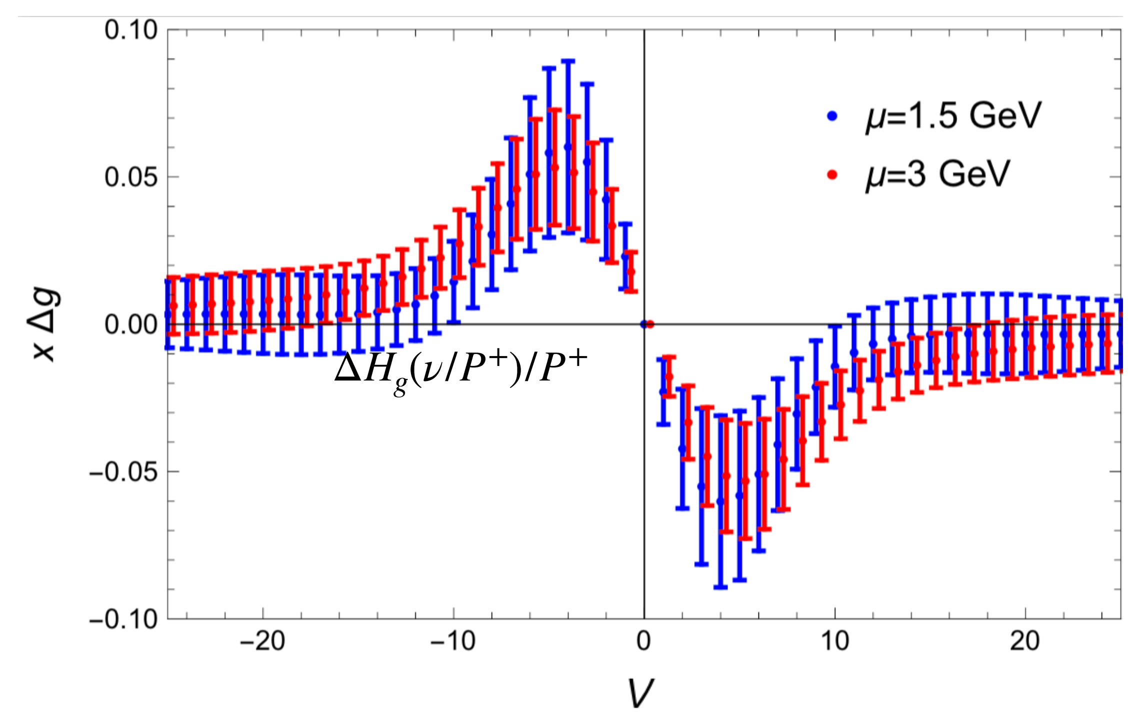


Glue spin under the Coulomb gauge

- *Sizable contribution to the proton spin;*
- *Convergence of the LaMET matching is poor at 1-loop level;*
- *Gauge dependence should be checked with the calculation under the other gauge conditions.*

Figure from Yu-Sheng Liu. Based on

E. R. Nocera, et.al. (NNPDF), NPB887, 276 (2014), 1406.5539



Glue spin can also be obtained through the following gauge invariant definition:

$$\Delta\tilde{g} = \int_0^\infty dz\, \Delta\tilde{H}_g(z)|_{P_z\to\infty} = \int_0^\infty dz\, \Delta H_g(z) + \mathcal{O}(\alpha_s) = \Delta g + \mathcal{O}(\alpha_s)\,.$$

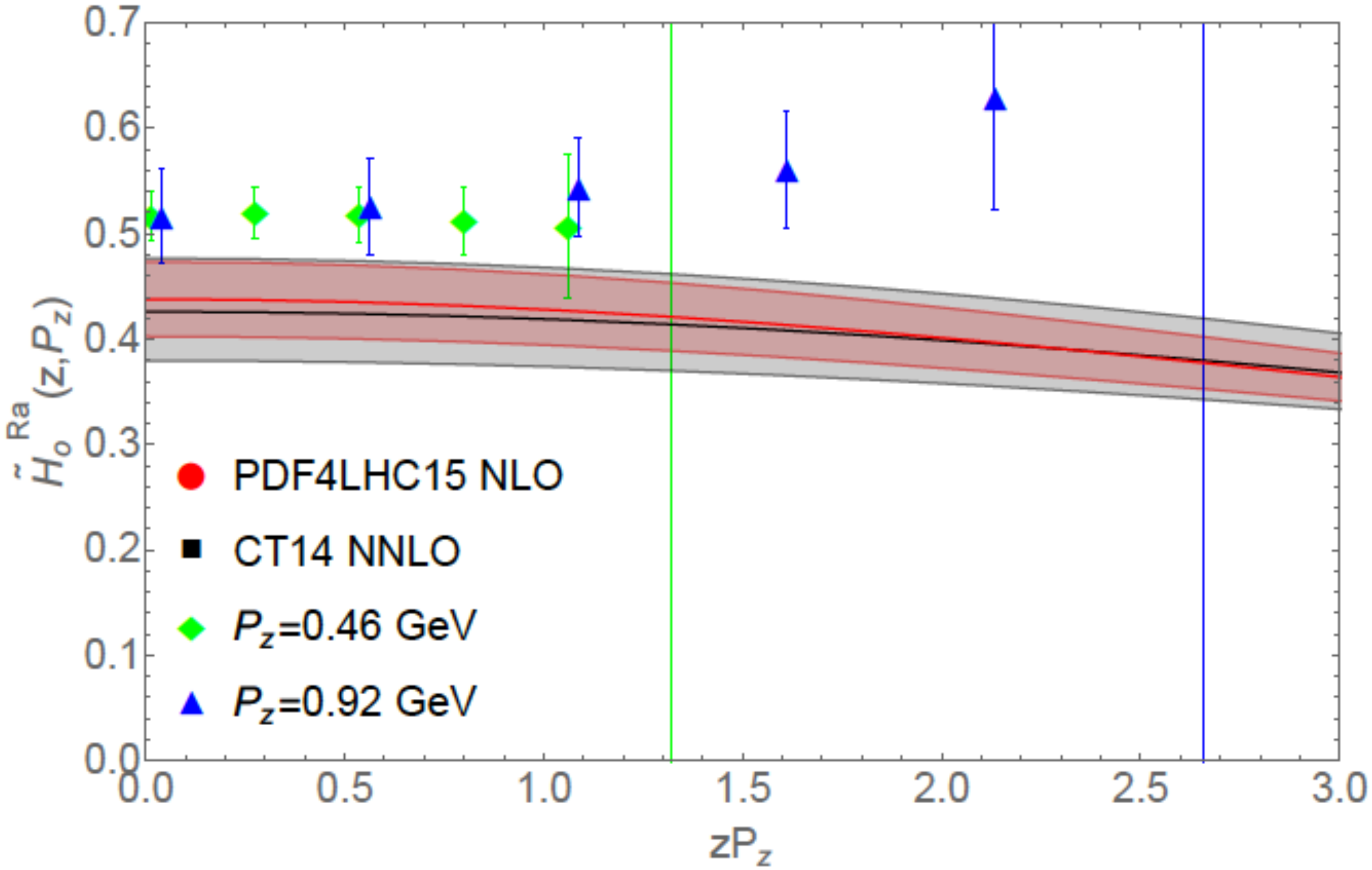
$$\Delta\tilde{H}_g(z) = \sum_{i=x,y} \langle PS| F_{iz,a}(z)(e^{\int_0^z igA_z(z')dz'})_{ab}\tilde{F}_{iz,b}(0)|PS\rangle$$

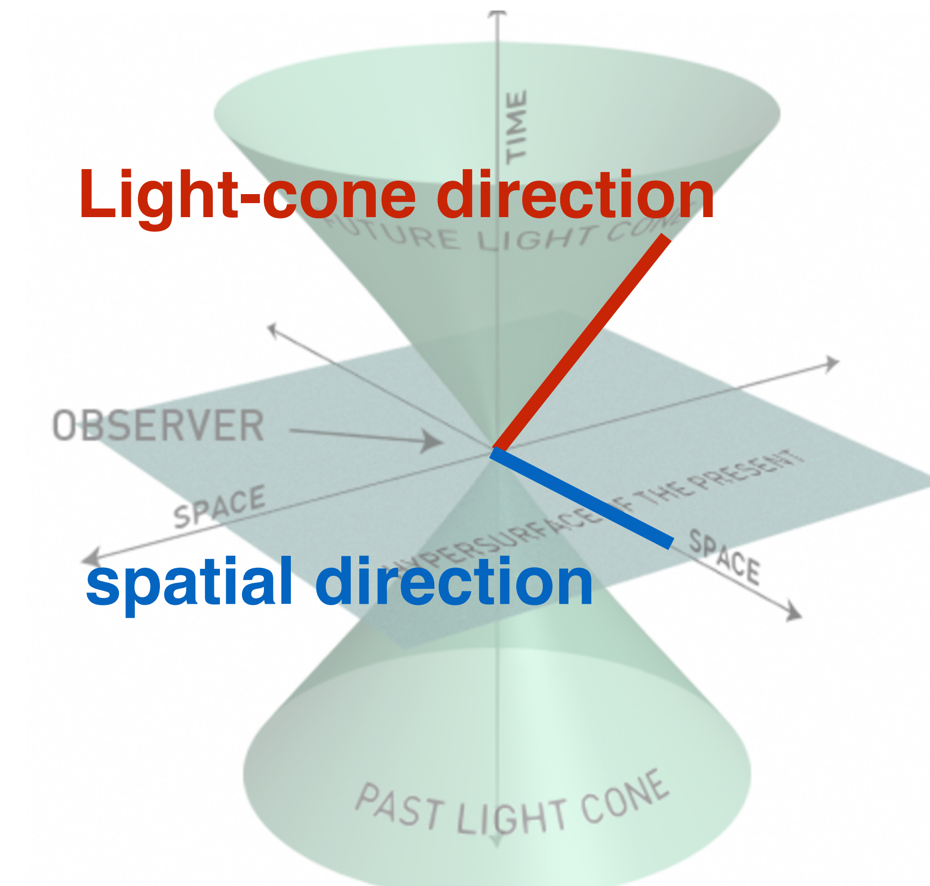
$$\Delta H_g(z) = \sum_{i=x,y}\langle PS|F_{+\mu,a}(\xi^-)(e^{\int_0^{\xi^-} igA^+(\eta^-)d\eta^-})_{ab}\tilde{F}^{+,b}_{\mu}(0)|PS\rangle = \int dx\, P^+ x e^{ix\xi^- P^+} g(x)$$

Investigation

on the unpolarized gluon PDF

Z. Fan, **YBY**, et.al, PRL121, 242001 (2018), 1808.02077

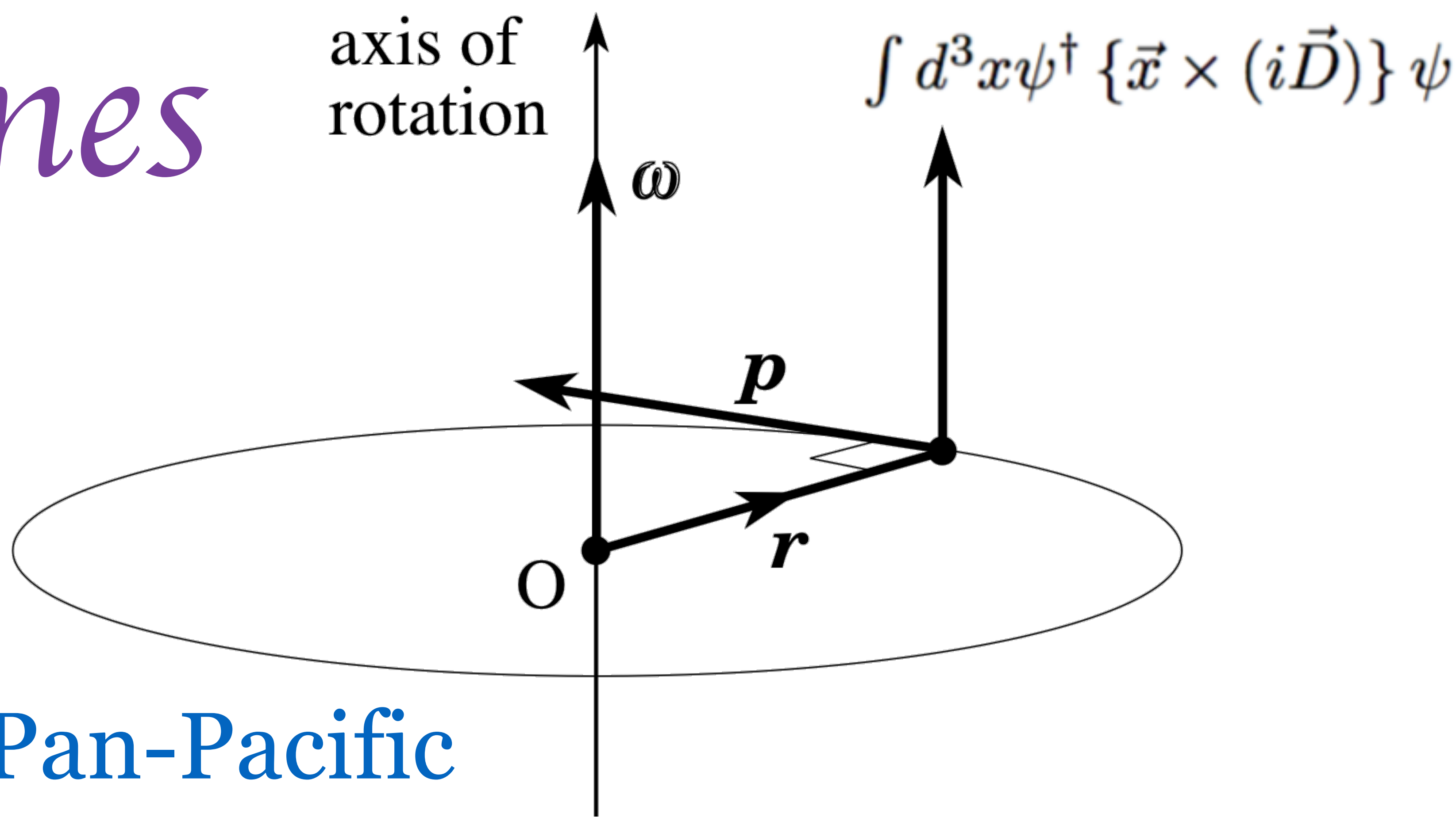




- **Lattice result v.s. FT of Pheno. gluon PDF**
- WITHOUT matching and mixing
- Much larger **momentum** and **statistics** are required.

$$H(\tau,\mu) = \frac{1}{2}\int_{-1}^{1} e^{i\tau x} x g(x,\mu)\mathrm{d}x$$

Angular momenta and Orbital ones



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Proton spin

Connections between decompositions

X. Ji, PRL78 (1997) 610, hep-ph/9603249

$$\vec{J} = \int d^3x \, \tfrac{1}{2} \, \overline{\psi} \, \vec{\gamma} \gamma^5 \, \psi + \int d^3x \psi^\dagger \{\vec{x} \times (i\vec{D})\} \psi + \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

$$\int d^3x \psi^\dagger \{\vec{x} \times \overrightarrow{A}\} \psi$$

quark spin

Different definitions of the quark OAM

Further decomposition of the glue AM

$$\vec{J} = \int d^3x \, \tfrac{1}{2} \, \overline{\psi} \, \vec{\gamma} \gamma^5 \, \psi + \int d^3x \psi^\dagger \{\vec{x} \times (i\vec{\nabla})\} \psi$$

$$\int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}] + \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]$$

glue spin *glue OAM*

R. L. Jaffe and A. V. Manohar, NPB337(1990)509

Proton spin

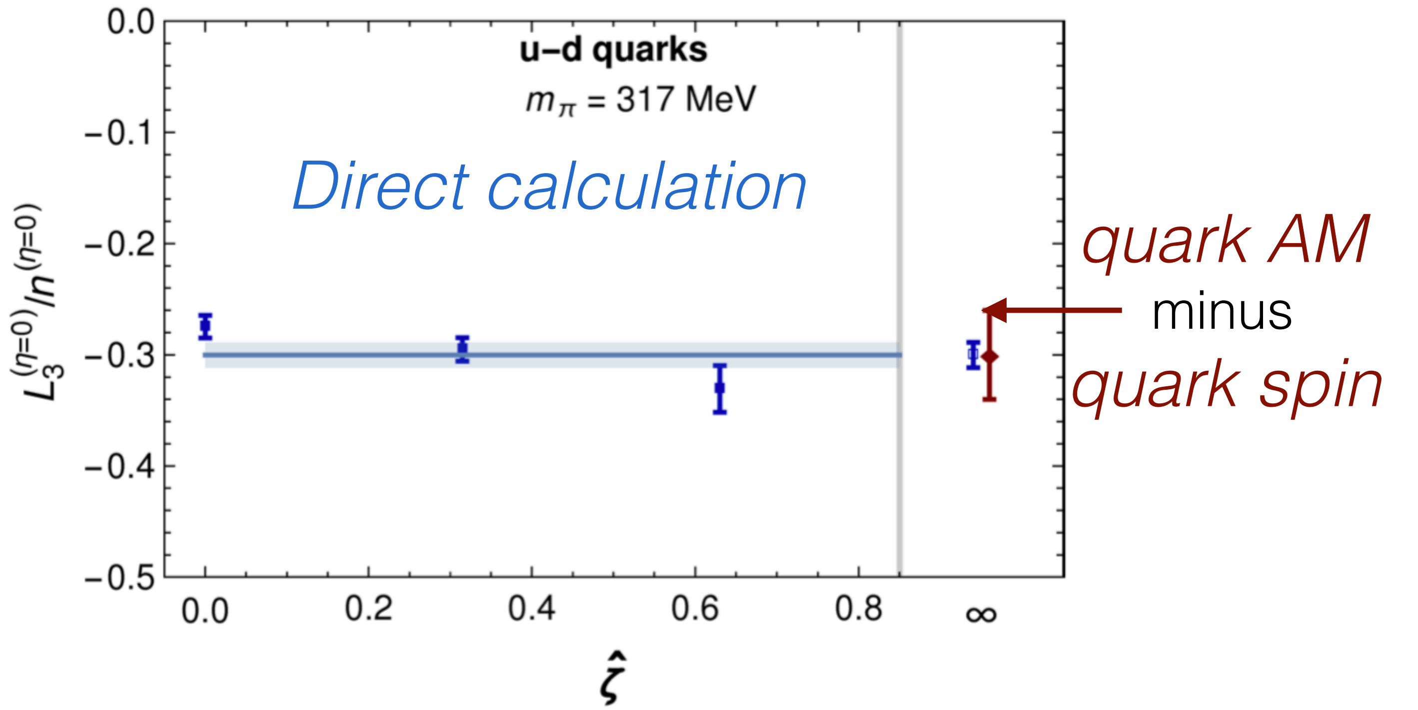
M. Engelhardt, et. al.,1901.00843

Direct calculation on OAM

The Ji quark orbital angular momentum,

$$\int d^3x \psi^\dagger \{\vec{x} \times (i\vec{D})\} \psi \ ,$$

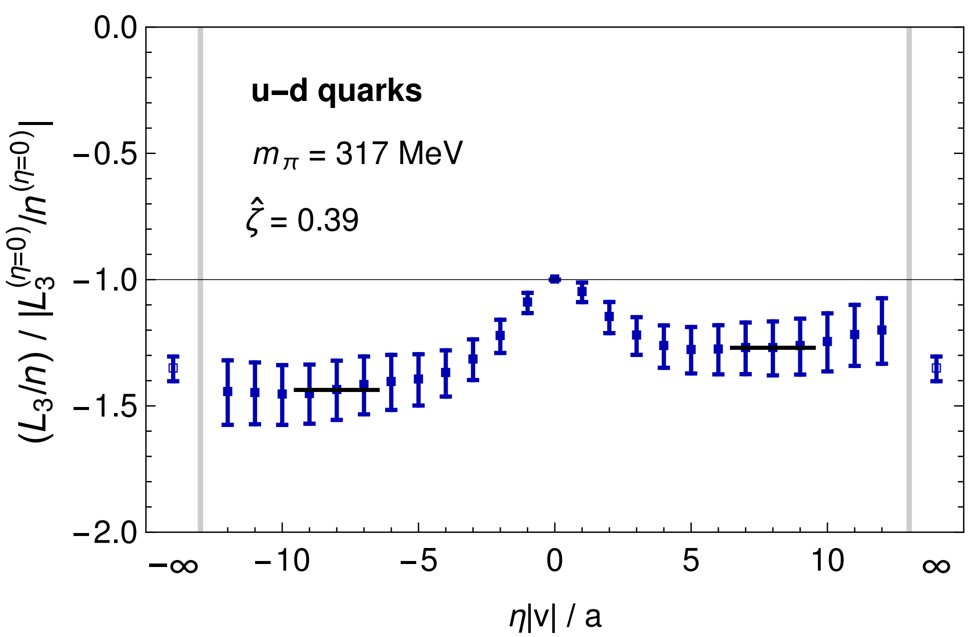
can be calculated with the quark bilinear operator with wilson links:



Similarly, Jaffe-Manohar quark OAM,

$$\int d^3x \psi^\dagger \{\vec{x} \times (i\vec{\nabla})\} \psi$$

can be calculated and ***the ratio over Ji OAM*** *seems to deviate from* ***one****.*



$$\frac{\text{Jaffe-Manohar OAM}}{\text{Ji OAM}}$$

Proton Spin decomposition

Calculation through the EMT form factors

X. Ji, PRL78 (1997) 610, hep-ph/9603249

Ji's angular momentum (AM) can be written in terms of the symmetrized energy momentum tensor (EMT) as,

$$J^{q,g} = \langle p,s| \int d^3x \, x \times \mathcal{T}^{\{0i\}q,g} |p,s\rangle, \qquad \mathcal{T}^{\{0i\}q} = \frac{1}{4}\bar{\psi}\gamma^{(0}\overleftrightarrow{D}^{i)}, \ \mathcal{T}^{\{0i\}g} = \vec{E}\times\vec{B}.$$

, with the form factors of the off-diagonal part of EMT defined by,

$$(p',s'|\mathcal{T}^{\{0i\}q,g}|p,s) = \left(\frac{1}{2}\right)\bar{u}(p',s')\left[T_1(q^2)(\gamma^0\bar{p}^i + \gamma^i\bar{p}^0) + \frac{1}{2m}T_2(q^2)\left(\bar{p}^0(i\sigma^{i\alpha}) + \bar{p}^i(i\sigma^{0\alpha})\right)q_\alpha + \frac{1}{m}T_3(q^2)q^0q^i\right]^{q,g} u(p,s),$$

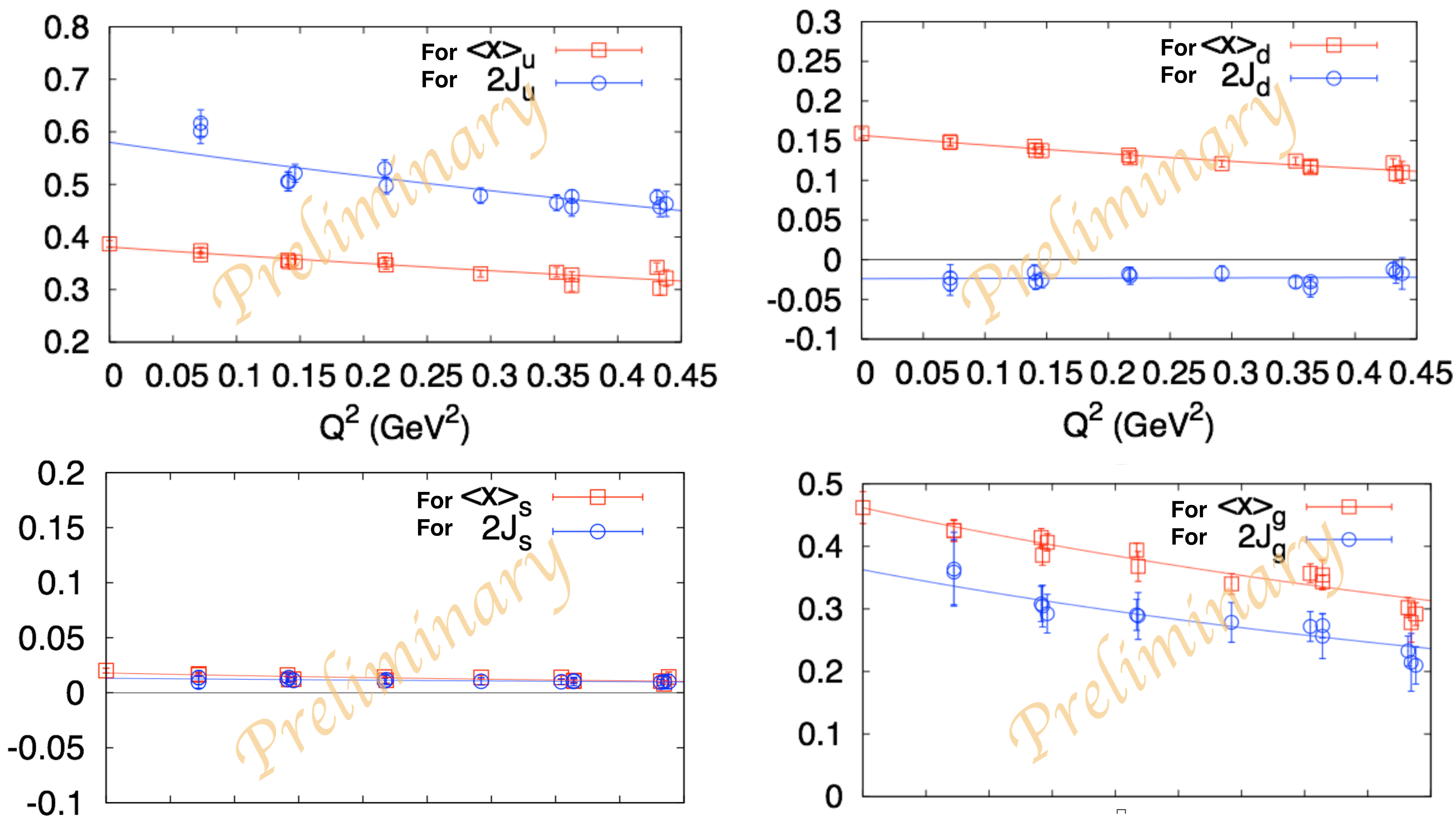
Ji's quark and glue AM correspond to the forward limit of the form factor combination,

$$J^{q,g} = \frac{1}{2}\left[T_1(0) + T_2(0)\right]^{q,g}$$

Angular momenta

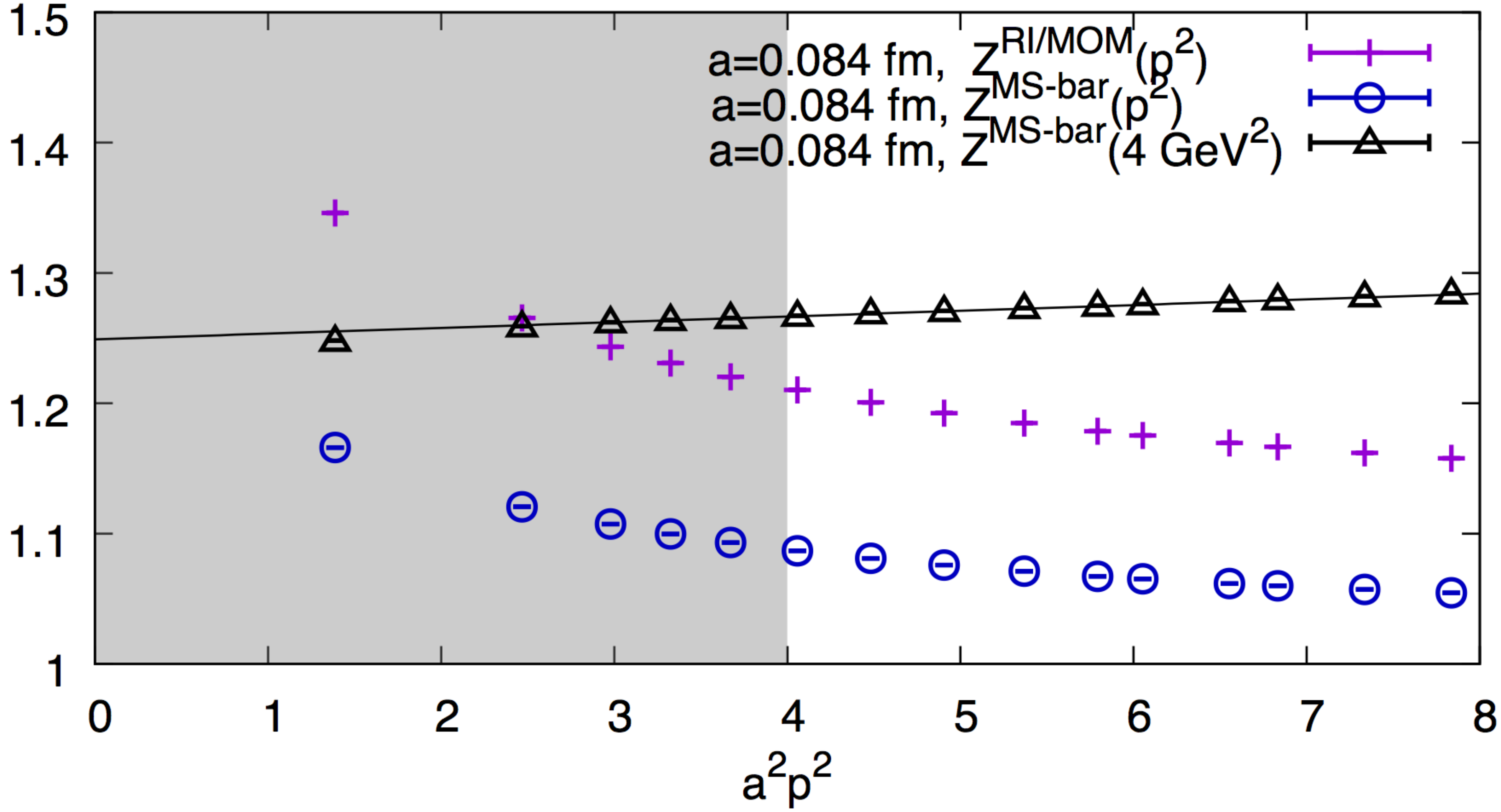
YBY, χQCD collaboration, 1904.04138

as the second moment of GPDs



Renormalization

of the **quark** momentum fractions



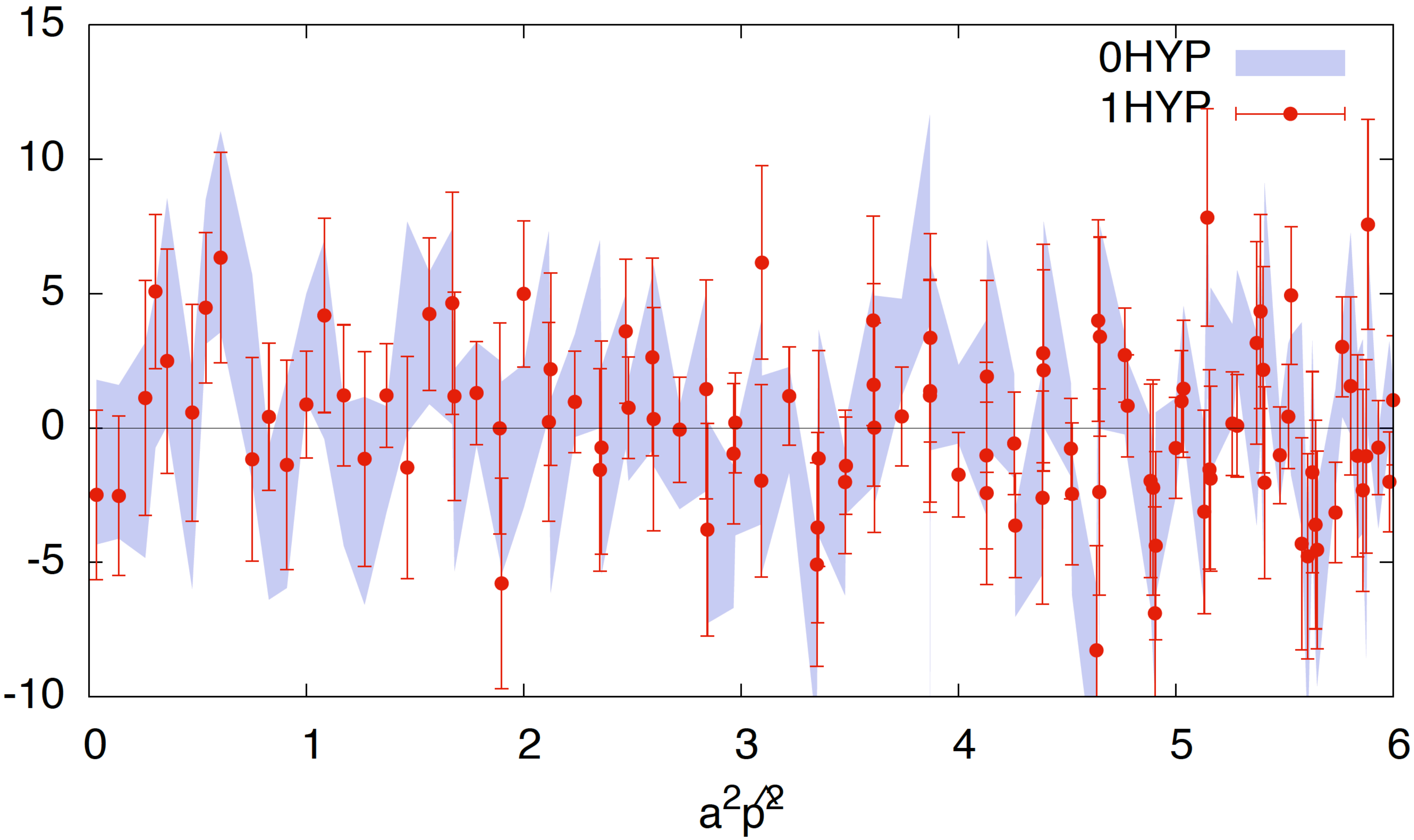
$$Z_{QQ}^{\overline{\rm MS}}(\mu) = \left[(Z_{QQ} R_{QQ}) \left(\mu_R, \frac{\mu}{\mu_R} \right) |_{a^2\mu_R^2 \to 0} \right]^{-1}$$

- Strong scale $\mu_R^2 = p^2$ dependence in the RI/MOM renormalization constant Z_{QQ} and converting ratio R_{QQ}
- But only the discretization error a^2p^2 left in final MS-bar renormalization constant at a fixed scale.

YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001, 1808.08677
ViewPoint and Editor's suggestion

Renormalization

of the **gluon** momentum fractions?



- Basically no signal with O(100) configurations with physical pion mass ensemble

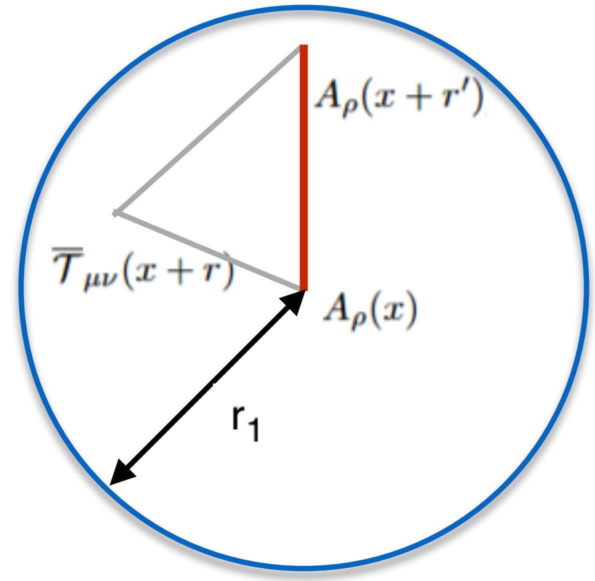
- Require significant improvement to get meaningful result

Gluon renormalization with CDER

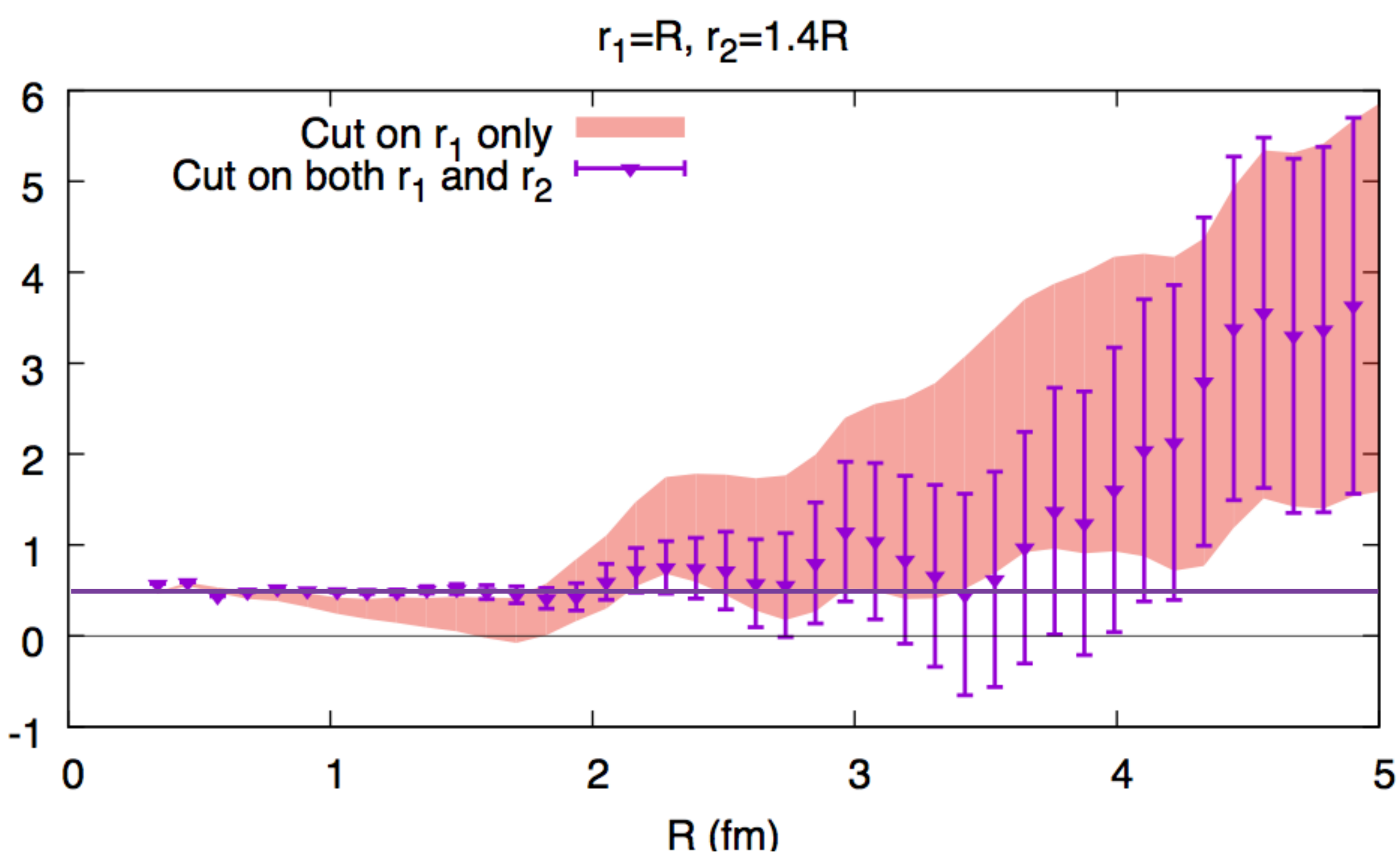
W. Sun, et.al, χQCD collaboration, CPC42, 063102(2018), 1507.02541
K. Liu, J. Liang, **YBY**, PRD96, 114504(2017), 1805.00531

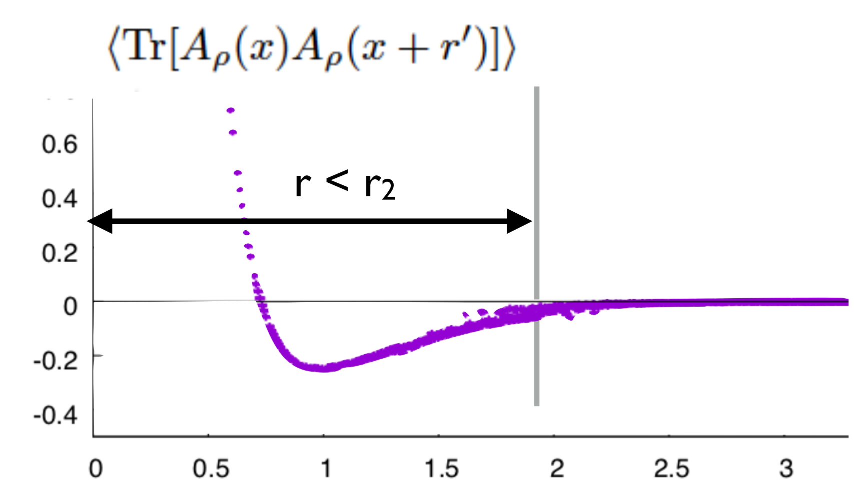
$$Z_T^{-1}(\mu_R^2) = \left. \frac{p^2 \langle (\overline{\mathcal{T}}_{\mu\mu} - \overline{\mathcal{T}}_{\nu\nu}) \, \mathrm{Tr}[A_\rho(p) A_\rho(-p)] \rangle}{2 p_\mu^2 \langle \mathrm{Tr}[A_\rho(p) A_\rho(-p)] \rangle} \right|_{\substack{p^2=\mu_R^2,\\ \rho\neq\mu\neq\nu,\\ p_\rho=0,\\ p_\nu=0}}$$

$$C_3(p) = \left\langle \int d^4x \, d^4y \, d^4z \, e^{ip\cdot(x-y)} \overline{\mathcal{T}}_{\mu\nu}(z) \, \mathrm{Tr}[A_\rho(x) A_\rho(y)] \right\rangle$$
$$\simeq \left\langle \int_{|r|<r_1} d^4r \int_{|r'|<r_2} d^4r' \int d^4x \, e^{ip\cdot r'} \overline{\mathcal{T}}_{\mu\nu}(x+r) \, \mathrm{Tr}[A_\rho(x) A_\rho(x+r')] \right\rangle$$



r_1: Cut on the $T_{\mu\nu}$ - A_ρ correlation





r_2: Cut on the A_ρ - A_ρ correlation

YBY, et. al., χQCD collaboration, PRD98(2018) 074506, 1805.00531

Gluon renormalization with CDER

W. Sun, et.al, χQCD collaboration, CPC42, 063102(2018), 1507.02541
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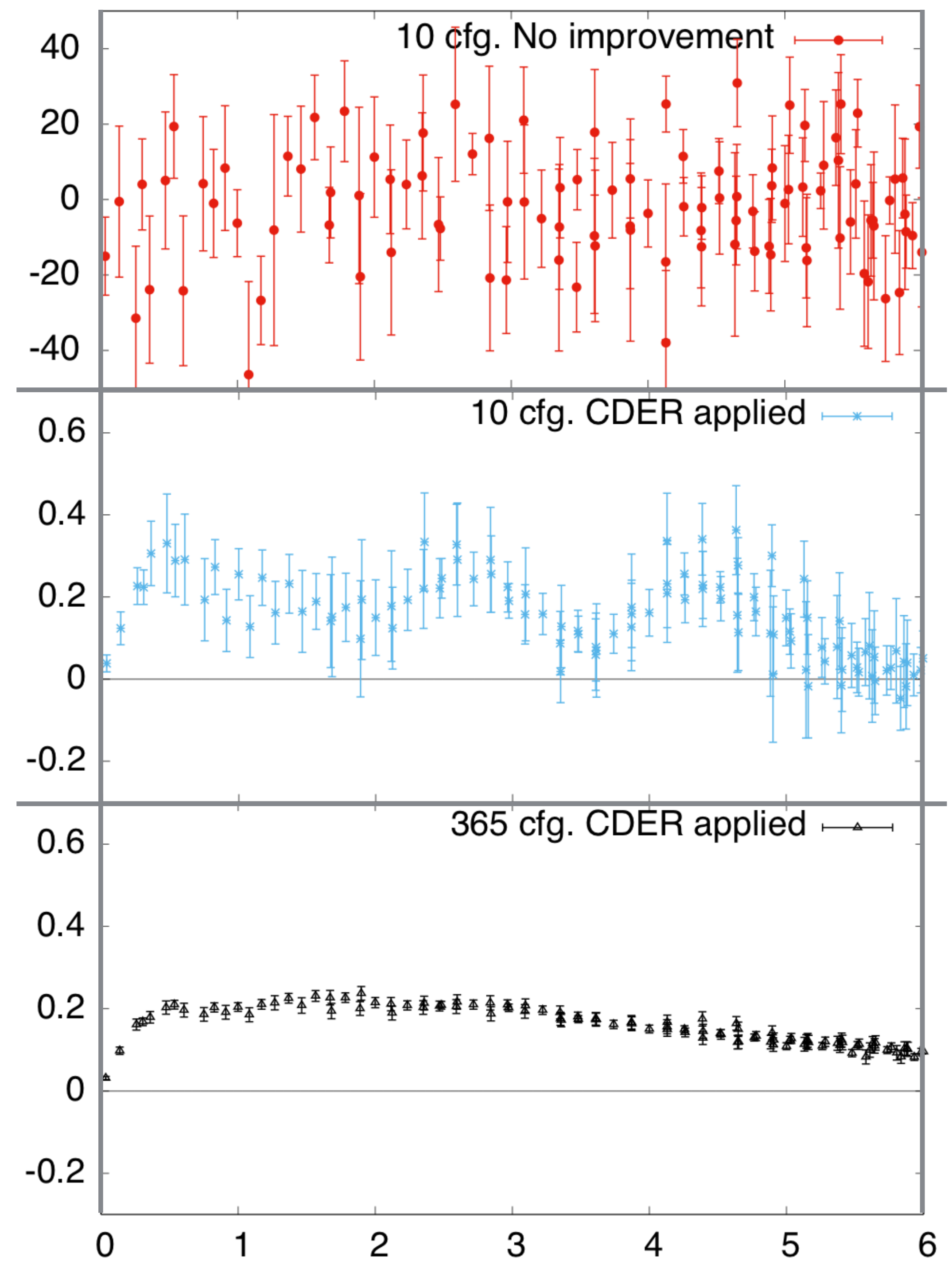
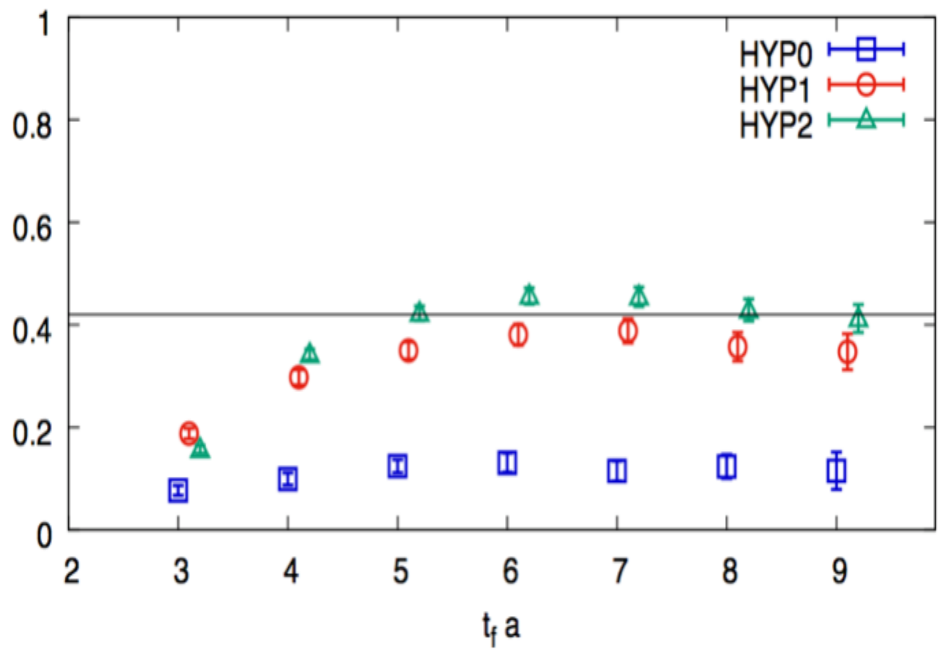
- Calculate the renormalization factor of the glue EMT non-perturbatively on a ~5 fm box will require ~30,000,000 configurations to make the uncertainty to be ~0.01;
- Taking the localization of the correlations between the glue fields/operators into account, the uncertainty can be reduced by a factor ~200;
- Use reasonable computer resource (~1M CPU hours) to increase the statistics, the ~0.01 uncertainty goal can be obtained with 365 configurations.

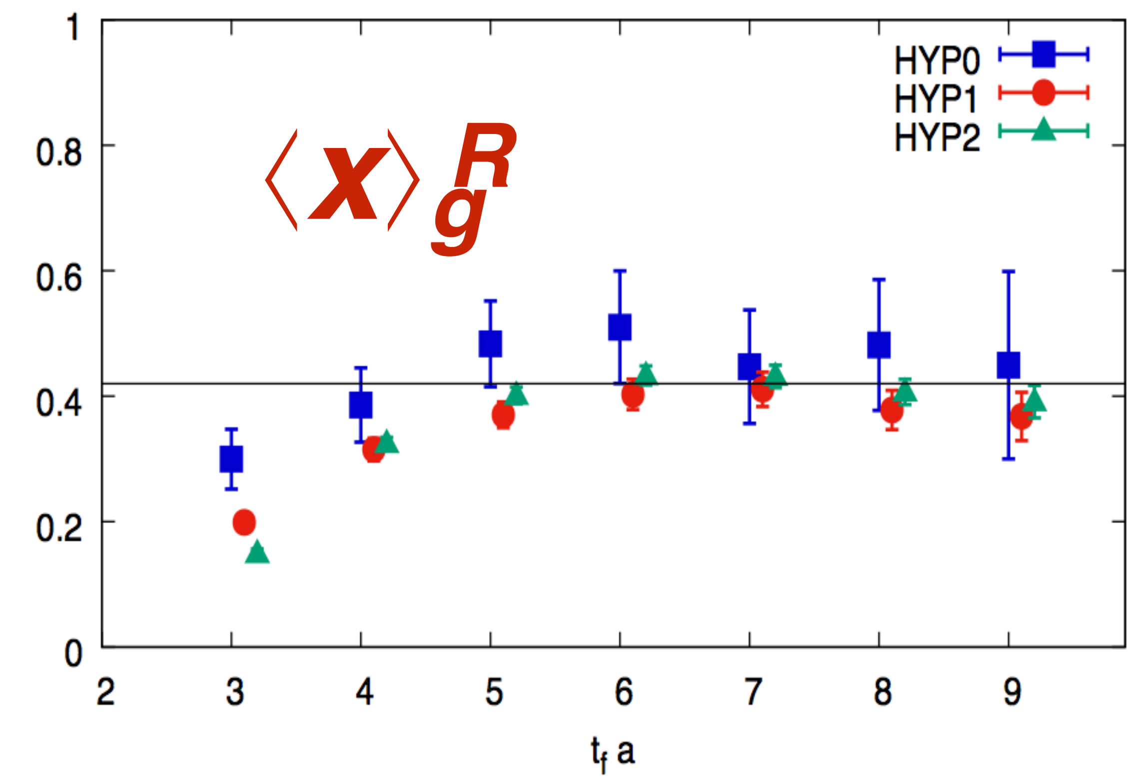


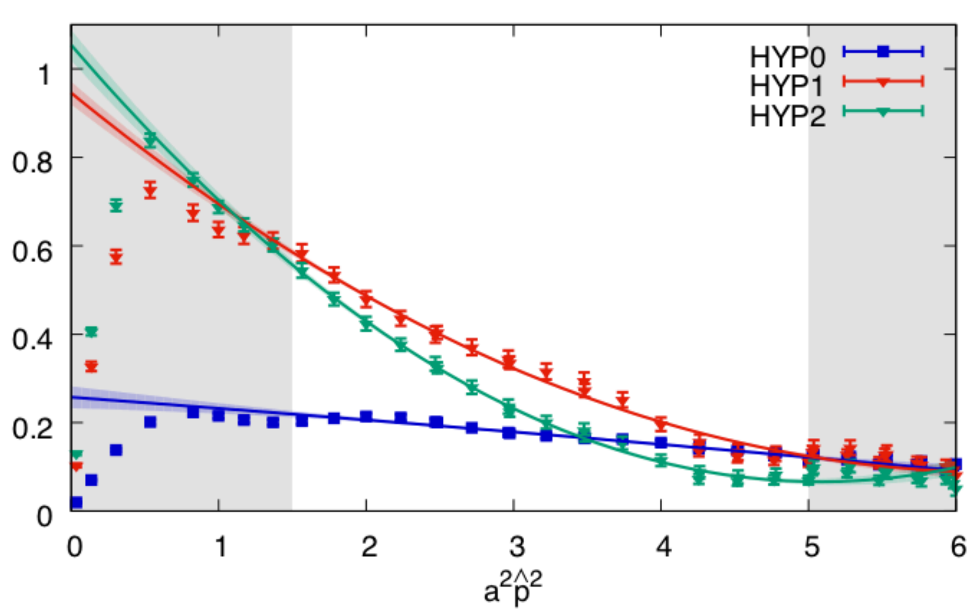
YBY, et. al., χQCD collaboration, PRD98(2018) 074506, 1805.00531

Non-perturbative renormalized glue momentum fraction



$$\frac{\langle x\rangle_g^{bare}}{Z^{-1}} = \langle x\rangle_g^{R}$$





- The lattice regularization effects are **fully cancelled** within the statistical uncertainties;
- The **non-perturbative renormalized quark/glue AM results** will come out soon.

YBY, et. al., χQCD collaboration, PRD98(2018) 074506, 1805.00531

Proton spin

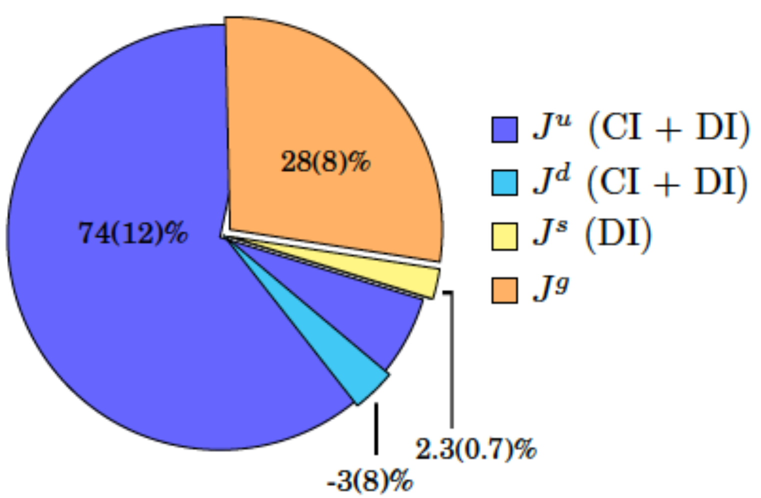
Lattice result of Ji AM

Quark ***AM***

Glue ***AM***

$$\vec{J} = \int d^3x\, \tfrac{1}{2}\, \overline{\psi}\, \vec{\gamma}\, \gamma^5\, \psi + \int d^3x\, \psi^\dagger \{\vec{x} \times (i\vec{D})\}\, \psi \; + \; \int d^3x\, 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

Quenched result

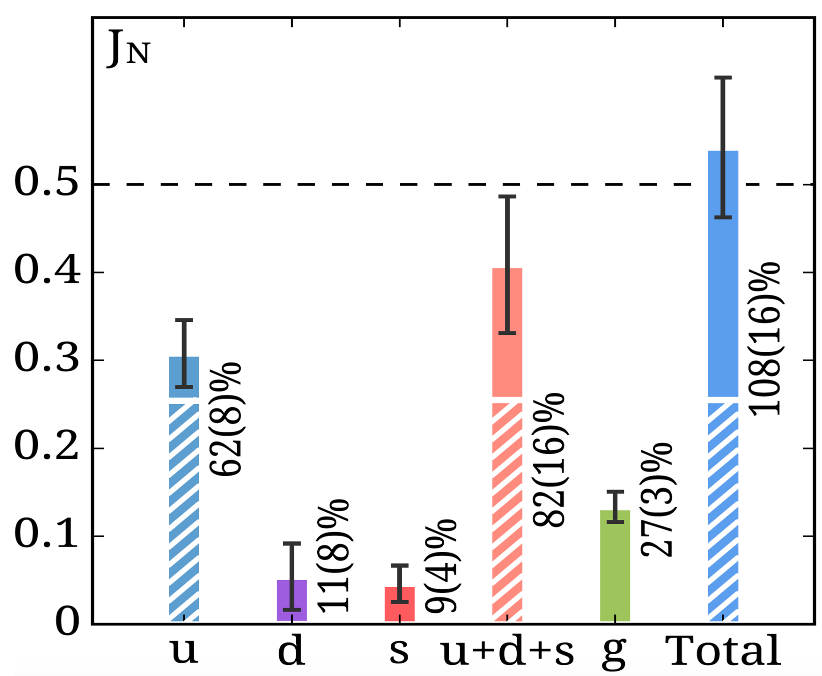


M. Deka, T. Doi, **YBY**, et. al., χQCD collaboration, PRD91, 014505 (2015), 1312.4816

2-flavor result

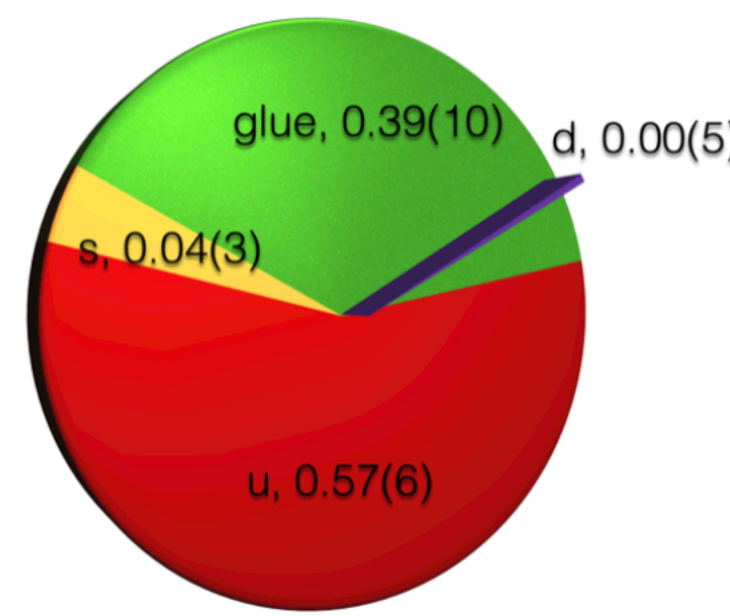
(neglecting the difference between the glue momentum and AM fractions)

EMTC 17: PRL119(2017), 1706.02973



1-loop perturbative renormalized

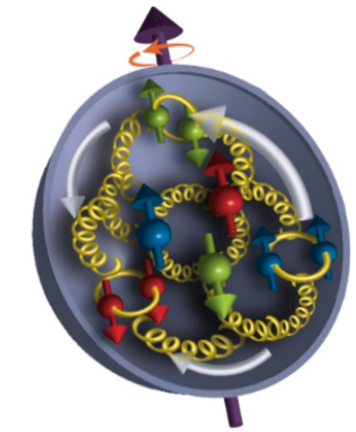
Preliminary *2+1 flavors result*



YBY, χQCD collaboration, 1904.04138

Non-perturbative renormalized

Summary



- We are very close to a state-of-the-art theoretical picture of the proton spin.
- **Increasing computation resources** and **technique breakthroughs** open the gates on the direct calculation of the glue spin and OAM.
- Fully non-perturbative renormalization becomes possible now.

At the right time, which will be soon, I myself will explain—and it will be an explanation that you'll think is reasonable—about everything that's occurred. Until then, be cheerful and think of each thing well.

Shakespeare: The Tempest: Act 5, Scene 1, 299-303