Gravitational form factors for finding mass and pressure distributions in hadrons

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https://sites.google.com/quark.kj.yamagata-u.ac.jp/pacspin2019

https://indico2.riken.jp/event/3039/

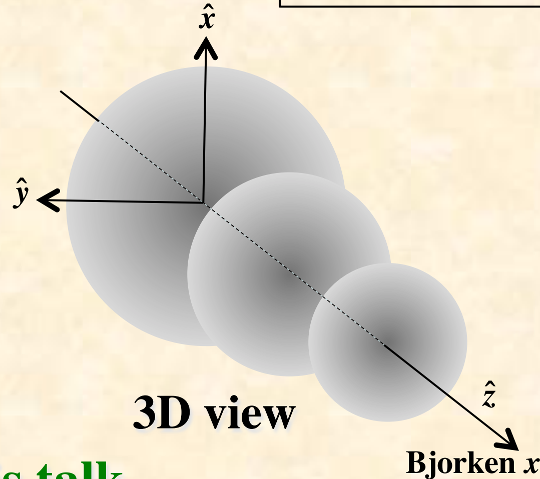
August 30, 2019

Proton (hadrons) puzzle studies by hadron tomography

Suda@PacSpin2019

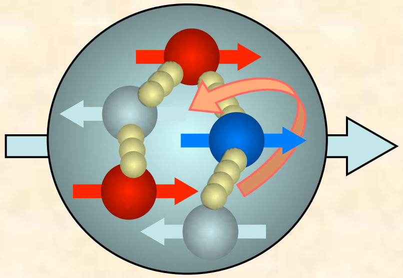
Hadron tomography

Proton radius puzzle

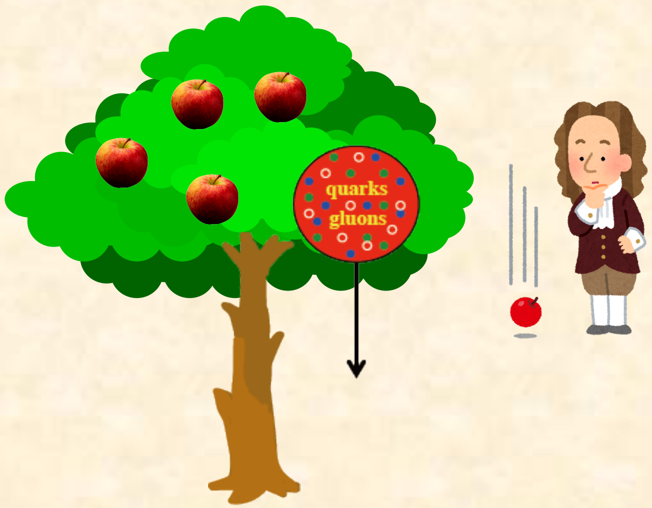


I discuss this topic in this talk.

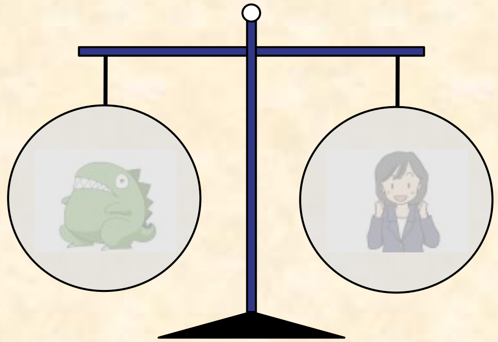
Origin of nucleon spin



Source of gravity (mass)



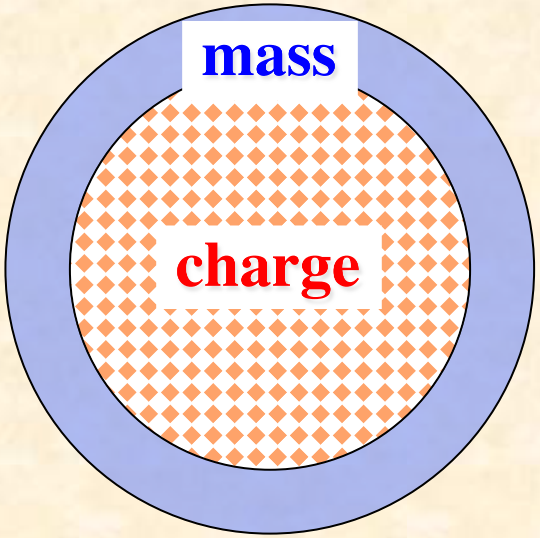
Exotic hadrons



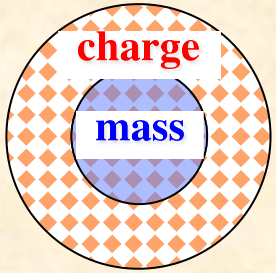
Relation to charge/matter distributions in unstable nuclei

One of major motivations for studying form factors of unstable nuclei is to find the difference between charge and matter distributions, namely the difference between proton and neutron distributions.

Unstable nuclei



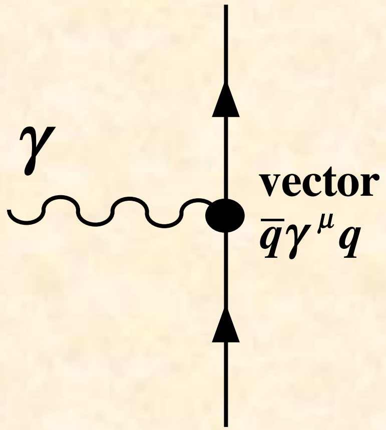
Pion in our work (2018)

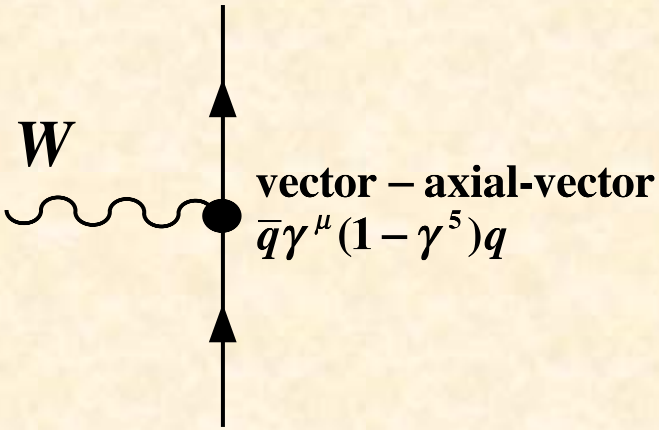


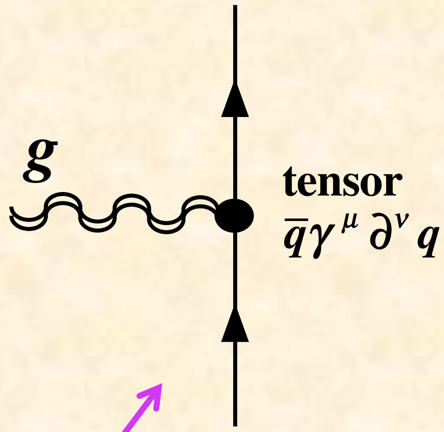
RIKEN-RIBF
(Radioactive Isotope Beam Factory)
US-FRIB
(Facility for Rare Isotope Beams)

Mass and pressure/shear force information for unstable nuclei can be obtained "in principle" by electron scattering through hadron tomography techniques.
→ Explained in my talk for the pion.

Why "gravitational" form factors of hadrons and nuclei ?







Electron-proton elastic scattering cross section:

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2 E_f\cos^2\frac{\theta}{2}}{4E_i^3\sin^4(\theta/2)}\left[\frac{G_E^2+\tau G_M^2}{1+\tau}+2\tau G_M^2\tan^2\frac{\theta}{2}\right],\quad \tau=-\frac{q^2}{4M^2}$$

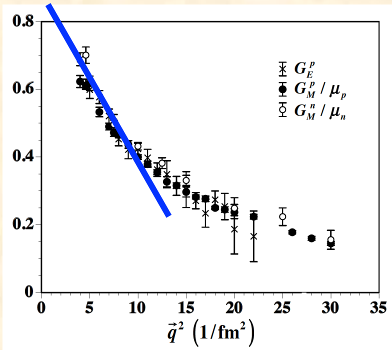
$$F(\vec{q})=\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\rho(\vec{x})=\int d^3x\left[1-\frac{1}{2}(\vec{q}\cdot\vec{x})^2+\cdots\right]\rho(\vec{x})$$

$$\langle r^2\rangle=\int d^3x\, r^2\rho(\vec{x}),\quad r=|\vec{x}|$$

$$\sqrt{\langle r^2\rangle}=\text{root-mean-square (rms) radius}$$

$$F(\vec{q})=1-\frac{1}{6}\vec{q}^{\,2}\langle r^2\rangle+\cdots,\qquad \langle r^2\rangle=-6\left.\frac{dF(\vec{q})}{d\vec{q}^{\,2}}\right|_{\vec{q}^2\to 0}$$

$$\rho(r)=\frac{\Lambda^3}{8\pi}e^{-\Lambda r}\ \Leftrightarrow\ \text{Dipole form: } F(q)=\frac{1}{\left(1+|\vec{q}|^2/\Lambda^2\right)^2},\quad \Lambda^2\simeq 0.71\ \text{GeV}^2$$



How about gravitational radius?

Proton-charge-radius puzzle:

$R_{\text{electron scattering}} = 0.8775$ fm

⇕

$R_{\text{muonic atom}} = 0.8418$ fm

Contents

- **Introduction**

  Origin of nucleon spin and mass

- **Hadron tomography and 3D structure functions**
- **Generalized distribution amplitudes**
- **Puzzle on hadron mass radius!?**

  Origin of nucleon mass and distributions:

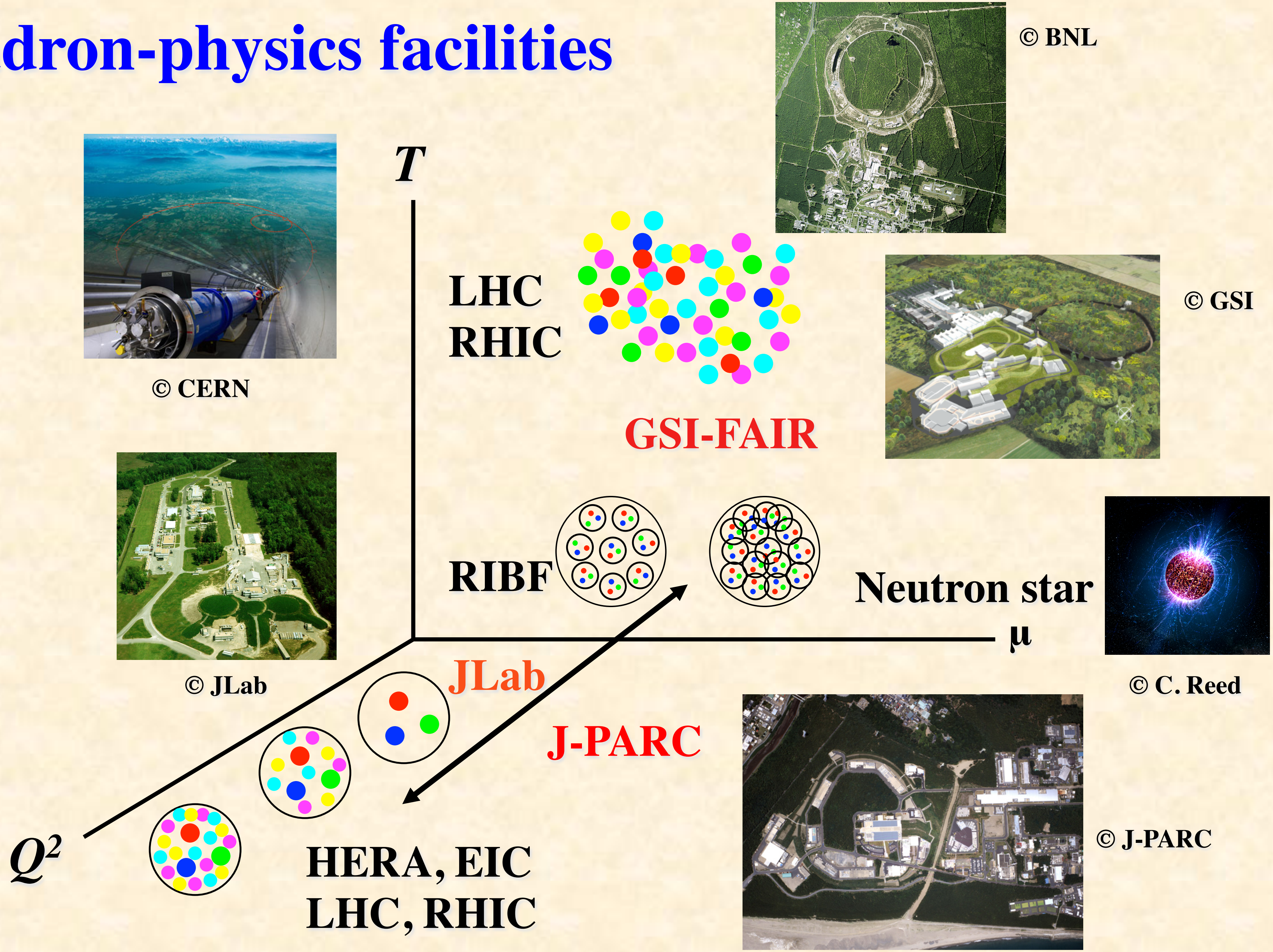
  Gravitational mass radius could be very difference from charge one.

- **Comments on hadron pressure and mass**
- **Summary**

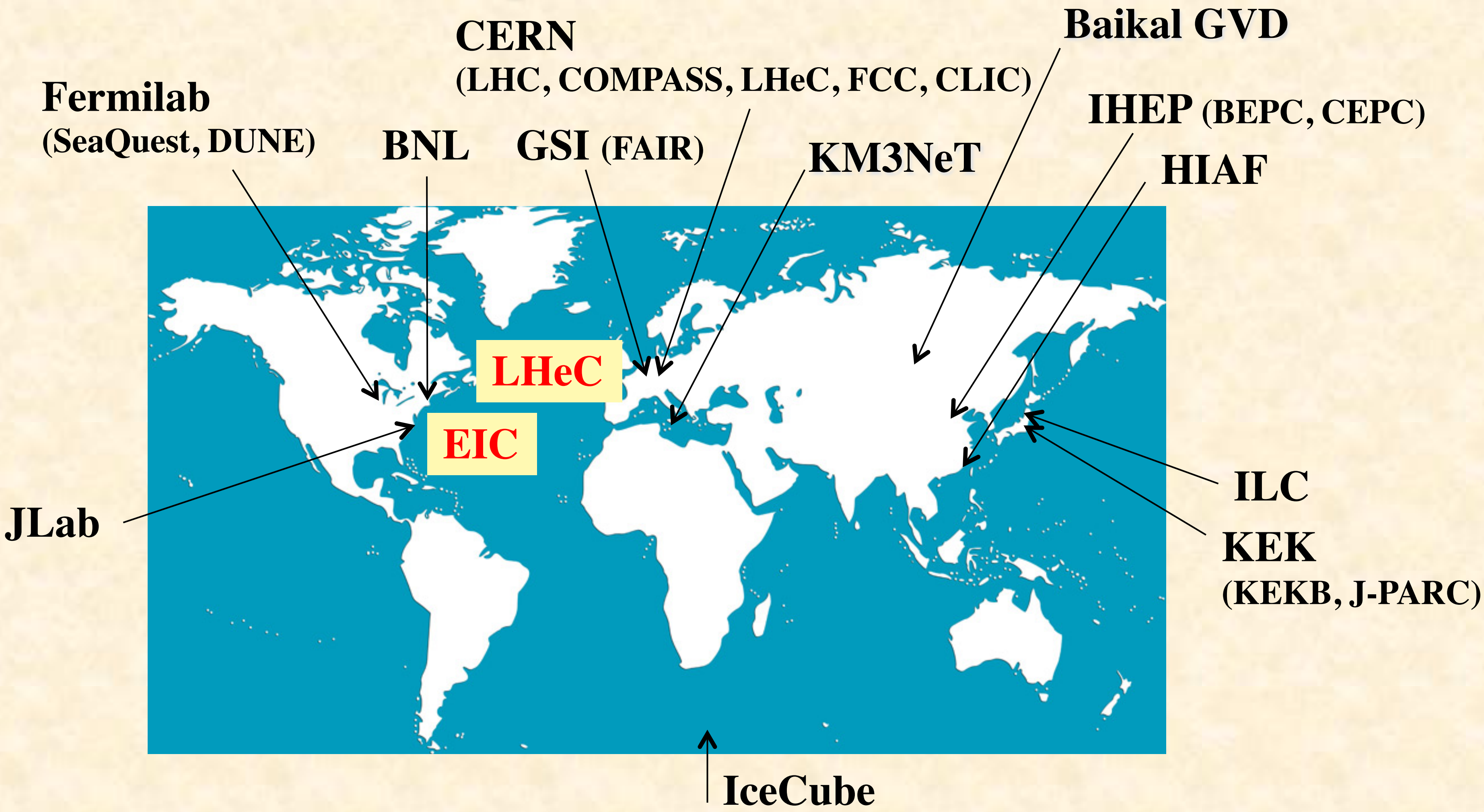
Introduction:

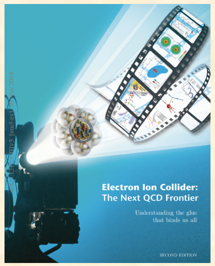
Origins of nucleon spin and mass

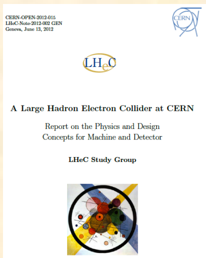
Hadron-physics facilities



Current and future experiments on structure functions



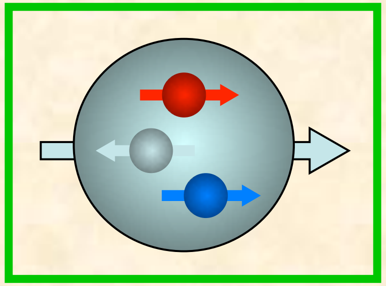


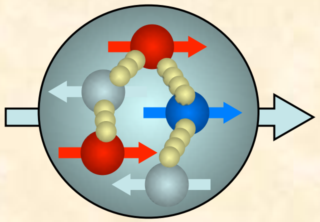


Among the future projects, electron-ion/hadron colliders are major new facilities in our DIS community.

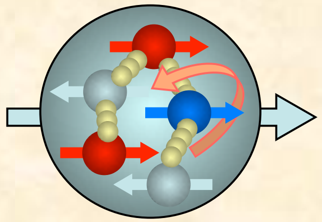
Recent progress on origin of nucleon spin

“old” standard model





gluon spin

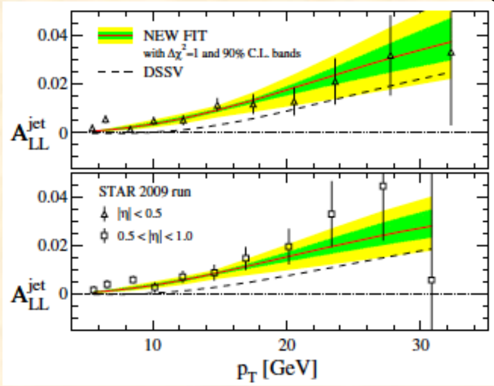


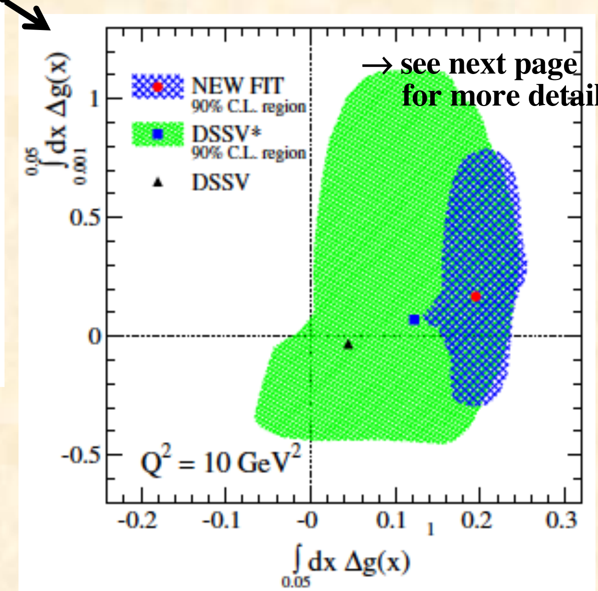
angular momentum

$$p_{\uparrow} = \frac{1}{3\sqrt{2}}\left(uud\left[2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \right] + \text{permutations} \right)$$

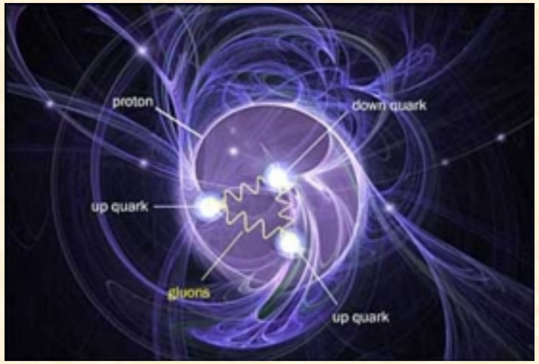
$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

$$\Delta\Sigma = \sum_i \int dx \left[\Delta q_i(x) + \Delta\bar{q}_i(x) \right] \rightarrow 1\ (100\%)$$





→ see next page for more details

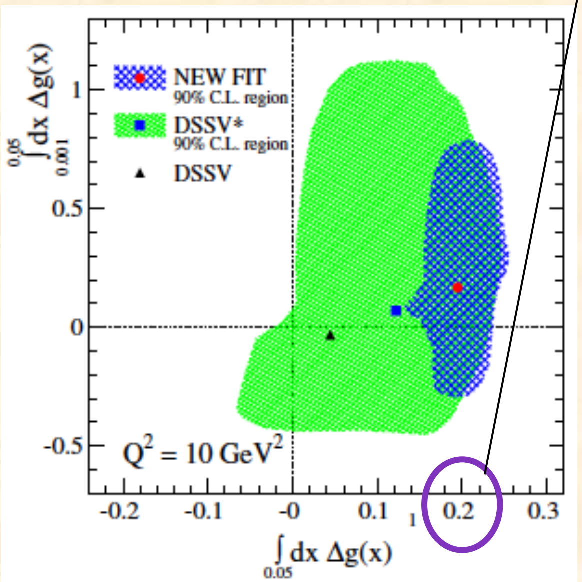


Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_{q,g}$$

Gluon polarization and nucleon spin

small-x contribution: $\int_{0.001}^{0.05} dx\, \Delta g(x)$



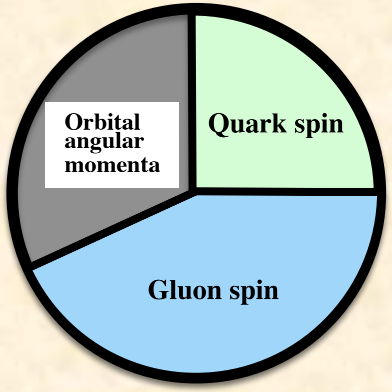
current experimental region: $\int_{0.05}^{1} dx\, \Delta g(x)$

In the current experimental region: $\int_{0.05}^{1} dx\, \Delta g(x) = 0.2$

$\frac{0.2}{1/2 \text{ spin}} = 40\%$: 40% of the nucleon spin is carried by the gluon spin!?

The major carrier of the nucleon spin could be gluons instead of quarks.
(CNN breaking news in 2014.)

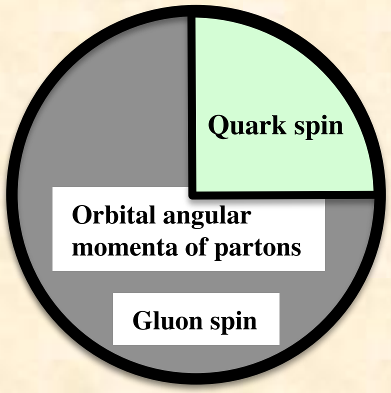
⇔ Completely different from the naive quark model.

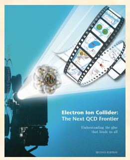


However, there are still large uncertainties from the small-x region.

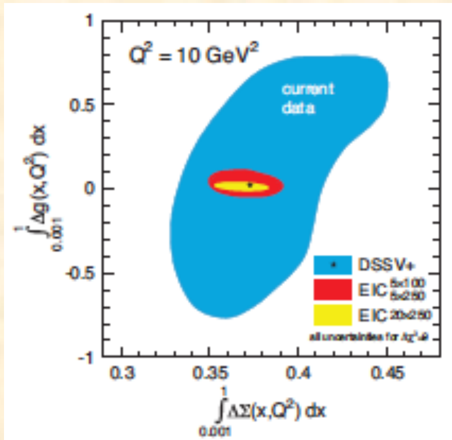
→ Importance of the EIC (electron-ion collider) project to measure small x.

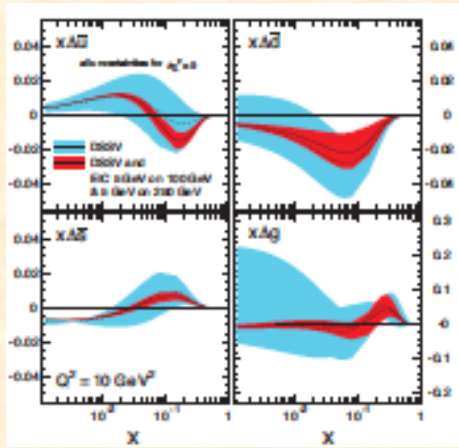
Real current experimental status →



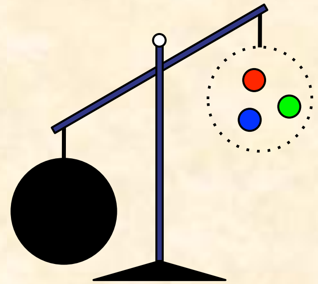


→ Electron-Ion Collider (~2025)





Origin of hadron masses

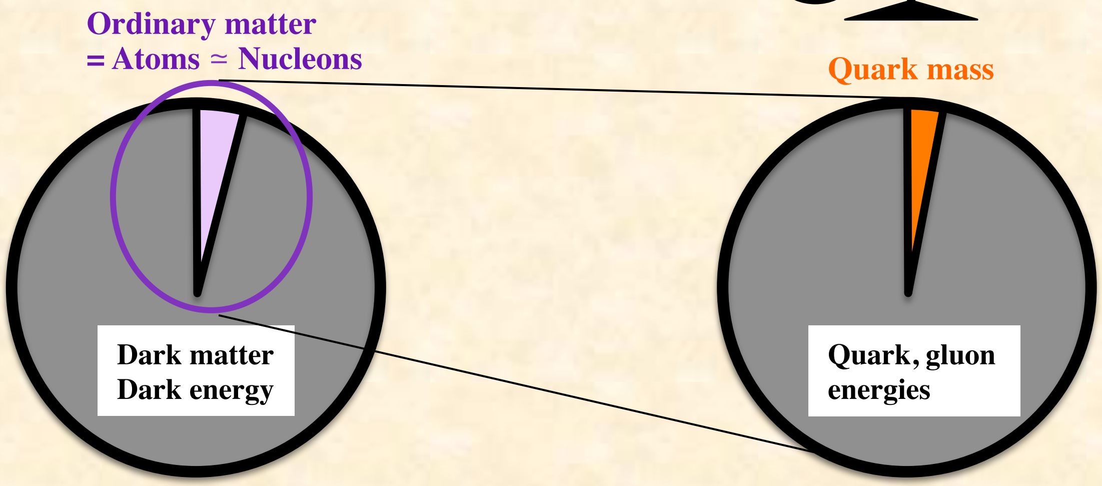


Mass and spin of the nucleon are two of fundamental quantities in physics.

Nucleon mass: $M = \left\langle p \middle| \int d^3x \, T^{00}(x) \middle| p \right\rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\ \alpha}(x)$$

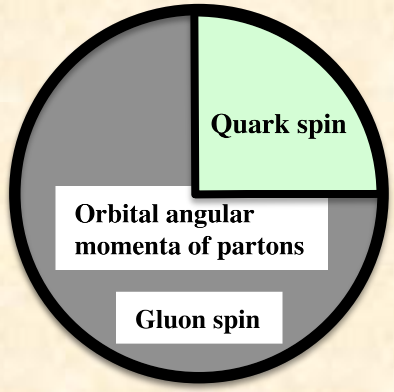


Dark matter **Origin of nucleon mass**

Nucleon spin: $\frac{1}{2} = \left\langle p \middle| J^3 \middle| p \right\rangle$

3rd component of total angular momentum: $J^3 = \frac{1}{2}\varepsilon^{3jk} \int d^3x \, M^{3jk}(x)$

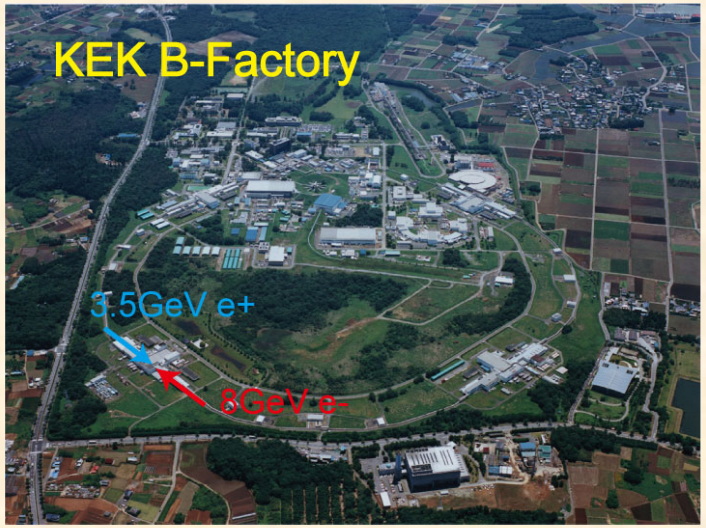
Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x) x^{\mu} - T^{\alpha\mu}(x) x^{\nu}$



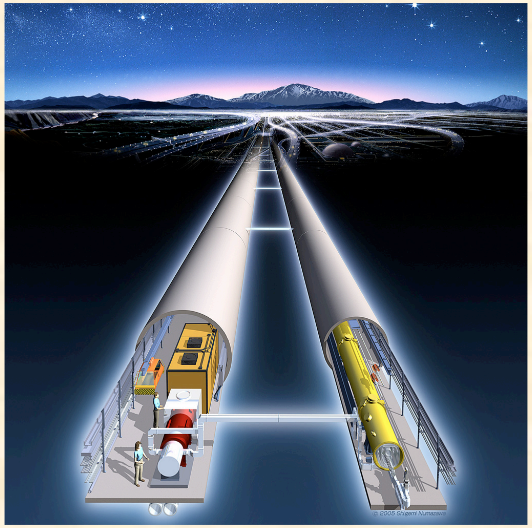
Origin of nucleon spin
(“Dark spin”)

Structure functions at Japanese facilities

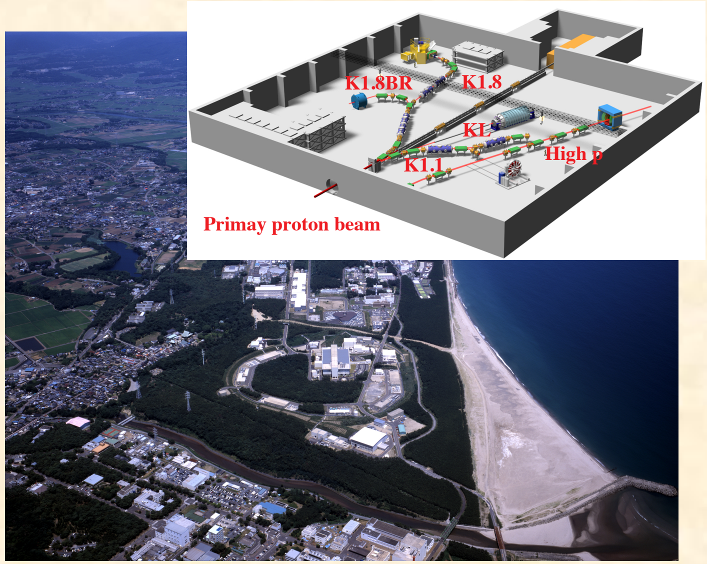
KEK B-factory



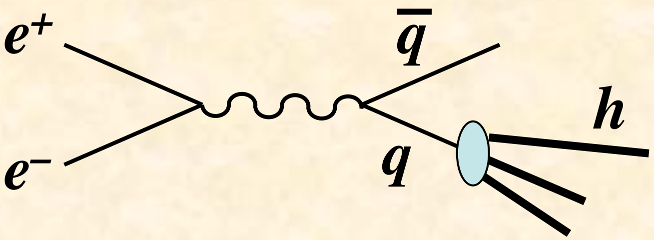
Linear Collider ?



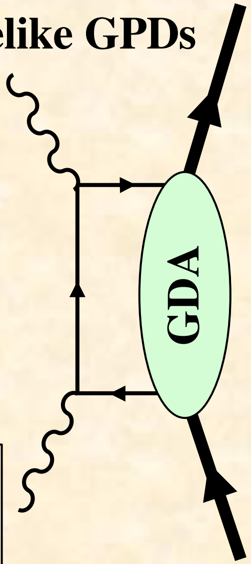
J-PARC



Fragmentation functions

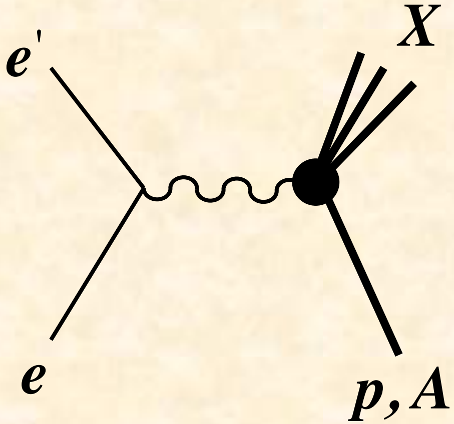


GDAs
= timelike GPDs

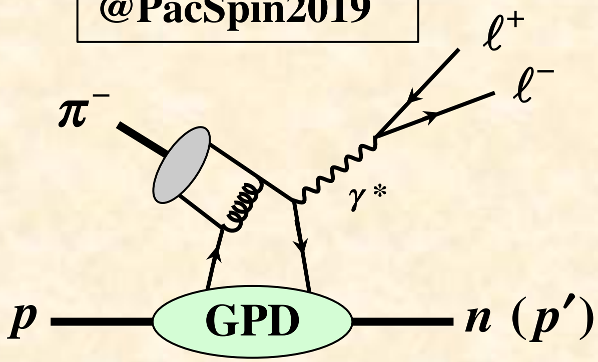


Uehara
@PacSpin2019

ILC-N
(fixed target)?



Sawada, Tanaka
@PacSpin2019

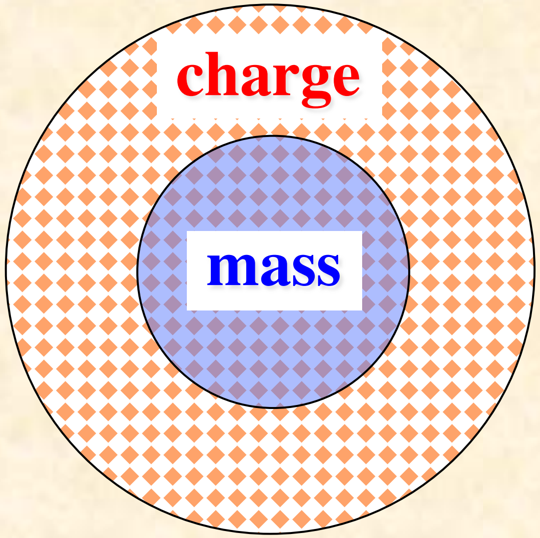


Hadron mass radius puzzle?

For pion

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm} \quad \Leftrightarrow \quad \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020;
(Erratum in v3 of arXiv:1711.08088).



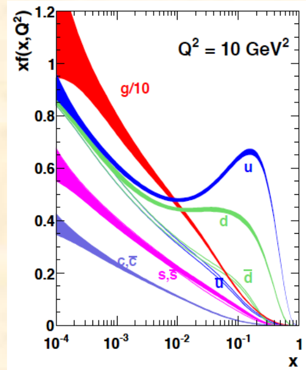
Mass radius seems to be much smaller than the charge radius for pion!?

Hadron tomography
(3D structure functions)

Wigner distribution and various structure functions

Form factor

PDF (Parton Distribution Function)



$\int dx\, d^2k_T$

$\int d^2k_T,\ \Delta \to 0$

GTMD $W(x, \vec{k}_T, \Delta) \xrightarrow[\Delta^+ \to 0]{}$ **Wigner** $W(x, \vec{k}_T, \vec{r}_T)$

$\Delta = p' - p$

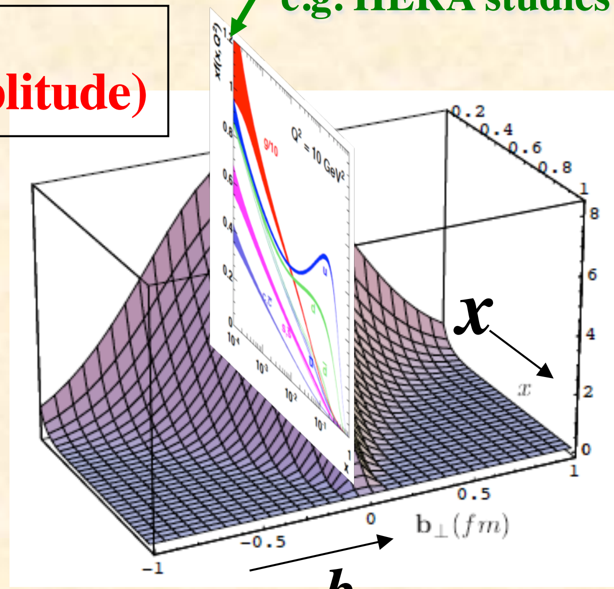
$\int d^2k_T$

e.g. HERA studies

3D world

GPD (Generalized Parton Distribution)

$\xrightarrow[s \leftrightarrow t]{}$ **GDA (Generalized Distribution Amplitude)**



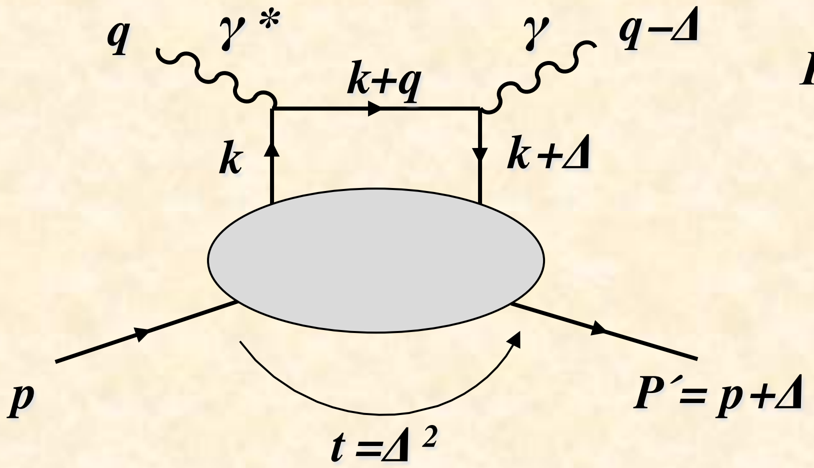
$b_\perp$

$\Delta \to 0$

TMD (Transverse Momentum Dependent) parton distribution

Our work by the two-photon process $\gamma\gamma \to h\bar{h}$.

Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2 p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewdness parameter $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2)\gamma^+\gamma_5\psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+}\left[\tilde{H}(x,\xi,t)\bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}(x,\xi,t)\bar{u}(p')\frac{\gamma_5\Delta^+}{2M}u(p) \right]$$

Forward limit: PDFs $H(x,\xi,t)\big|_{\xi=t=0} = f(x), \quad \tilde{H}(x,\xi,t)\big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t), \quad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$

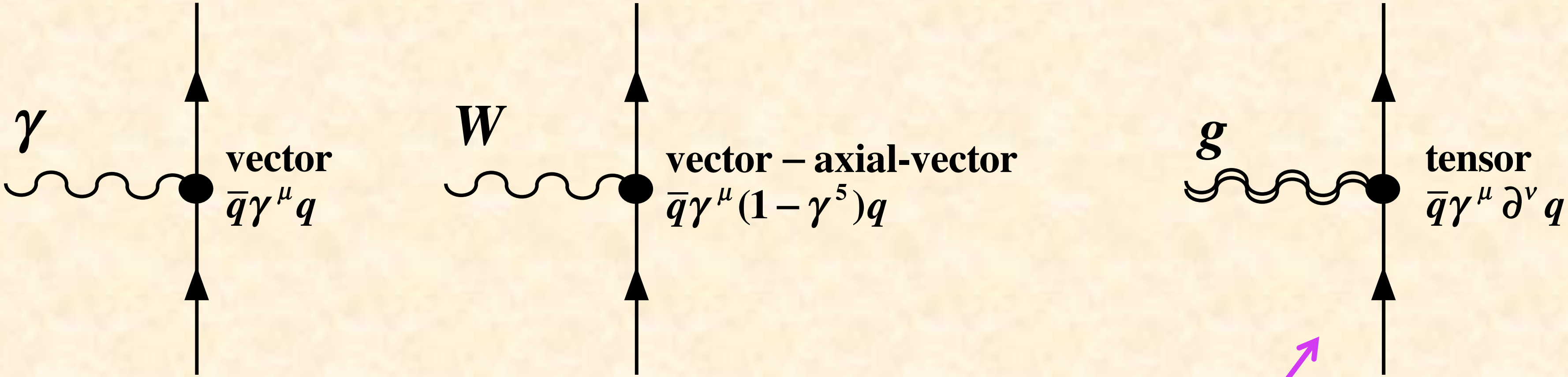
Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^{1} dx\, \tilde{H}(x,\xi,t) = g_A(t), \quad \int_{-1}^{1} dx\, \tilde{E}(x,\xi,t) = g_P(t)$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0) \right], \quad J_q = \frac{1}{2}\Delta q + L_q$

⇒ probe L_q, key quantity to solve the spin puzzle!

Why gravitational interactions with hadrons ?



Electron-proton elastic scattering cross section:

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2 E_f \cos^2\frac{\theta}{2}}{4E_i^3\sin^4(\theta/2)}\left[\frac{G_E^2+\tau G_M^2}{1+\tau}+2\tau G_M^2\tan^2\frac{\theta}{2}\right],\quad \tau=-\frac{q^2}{4M^2}$$

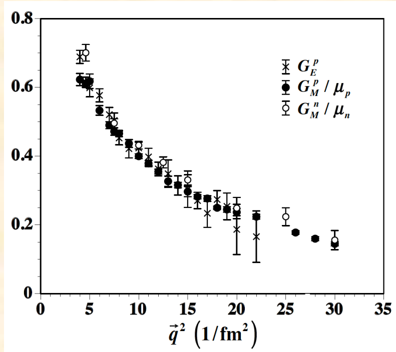
$$F(\vec{q})=\int d^3x\, e^{i\vec{q}\cdot\vec{x}}\rho(\vec{x})=\int d^3x\left[1-\frac{1}{2}(\vec{q}\cdot\vec{x})^2+\cdots\right]\rho(\vec{x})$$

$$\langle r^2\rangle=\int d^3x\, r^2\rho(\vec{x}),\quad r=|\vec{x}|$$

$$\sqrt{\langle r^2\rangle}=\text{root-mean-square (rms) radius}$$

$$F(\vec{q})=1-\frac{1}{6}\vec{q}^{\,2}\langle r^2\rangle+\cdots,\qquad \langle r^2\rangle=-6\left.\frac{dF(\vec{q})}{d\vec{q}^{\,2}}\right|_{\vec{q}^2\to 0}$$

$$\rho(r)=\frac{\Lambda^3}{8\pi}e^{-\Lambda r}\ \Leftrightarrow\ \text{Dipole form: } F(q)=\frac{1}{\left(1+|\vec{q}|^2/\Lambda^2\right)^2},\quad \Lambda^2\simeq 0.71\ \text{GeV}^2$$



How about gravitational radius?

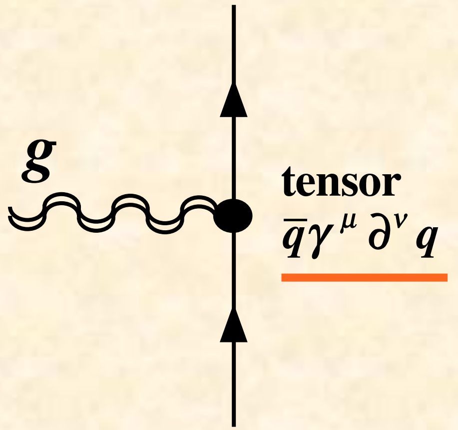
Proton-charge-radius puzzle:

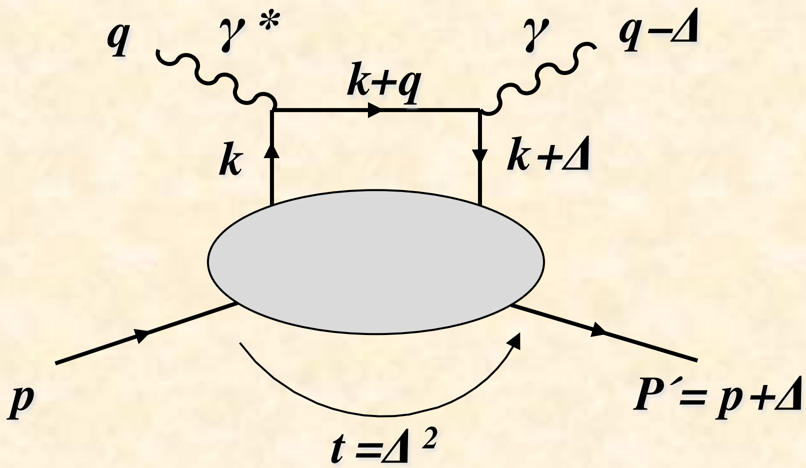
$R_{\text{electron scattering}}$ = 0.8775 fm

⇳

$R_{\text{muonic atom}}$ = 0.8418 fm

Gravitational sources and 3D structure functions





GPDs: $\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2)\gamma^+ q(z/2) | p \rangle \big|_{z^+=0,\vec{z}_\perp=0} = \frac{1}{2P^+}\left[H(x,\xi,t)\bar{u}(p')\gamma^+ u(p) + E(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p) \right]$

Angular momentum: $J_q = \frac{1}{2}\int_{-1}^{1} dx\, x\left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0) \right], \quad J_q = \frac{1}{2}\Delta q + L_q$

Non-local operator of GPDs/GDAs:

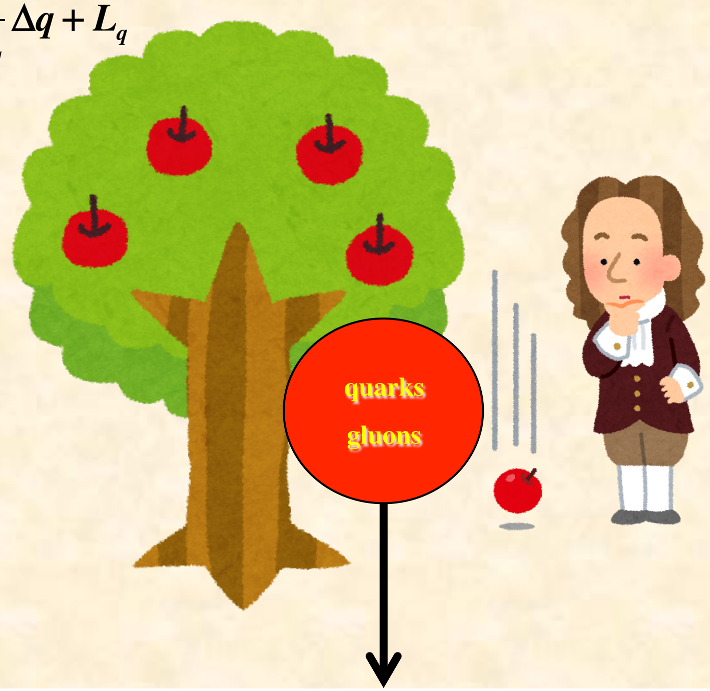
$$\left(P^+\right)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z^+=0,\vec{z}_\perp=0}$$

$$= \left(i\frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2)\gamma^+ q(z/2) \right]_{z=0}$$

$$= \bar{q}(0)\gamma^+ \left(i\overleftrightarrow{\partial}^+ \right)^{n-1} q(0)$$

= energy-momentum tensor of a quark for $n = 2$
(electromagnetic for $n = 1$)

= source of gravitiy



Generalized Distribution Amplitudes (GDAs)

and KEKB/ILC project

H. Kawamura and S. Kumano,
Phys. Rev. D 89 (2014) 054007.

S. Kumano, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020;
Erratum in v3 of arXiv:1711.08088.

GPD $H_q^h(x,\xi,t)$ and GDA $\Phi_q^{hh}(z,\zeta,W^2)$

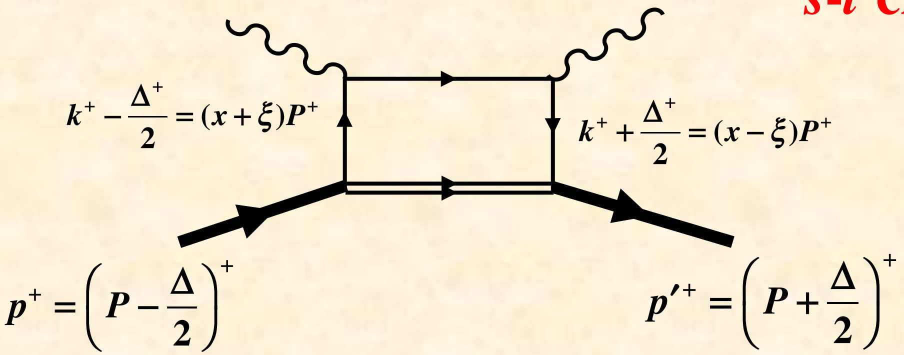
GPD: $H_q(x,\xi,t)=\int\frac{dy^-}{4\pi}e^{ixP^+y^-}\langle h(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|h(p)\rangle\Big|_{y^+=0,\vec y_\perp=0}$, $\quad P^+=\frac{(p+p')^+}{2}$

GDA: $\Phi_q(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle h(p)\bar h(p')|\bar\psi(-y/2)\gamma^+\psi(y/2)|0\rangle\Big|_{y^+=0,\vec y_\perp=0}$

DA: $\Phi_q^\pi(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle \pi(p)|\bar\psi(-y/2)\gamma^+\gamma_5\psi(y/2)|0\rangle\Big|_{y^+=0,\vec y_\perp=0}$

$H_q^h(x,\xi,t)$ $\longleftrightarrow$ $\Phi_q^{hh}(z,\zeta,W^2)$

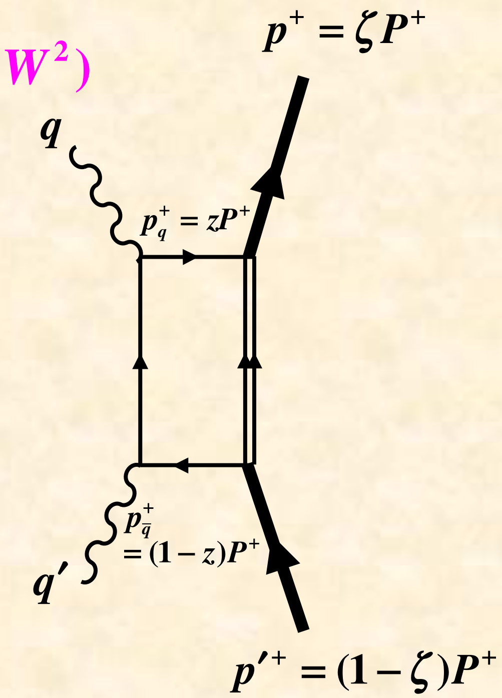
***s*-*t* crossing**



$z\Leftrightarrow\frac{1-x/\xi}{2}$

$\zeta\Leftrightarrow\frac{1-1/\xi}{2}$

$W^2\Leftrightarrow t$



JLab / COMPASS | **KEKB**

$P=\frac{p+p'}{2},\ \Delta=p'-p$

Bjorken variable: $x=\frac{Q^2}{2p\cdot q}$

Momentum transfer squared: $t=\Delta^2$

Skewdness parameter: $\xi=\frac{p^+-p'^+}{p^++p'^+}=-\frac{\Delta^+}{2P^+}$

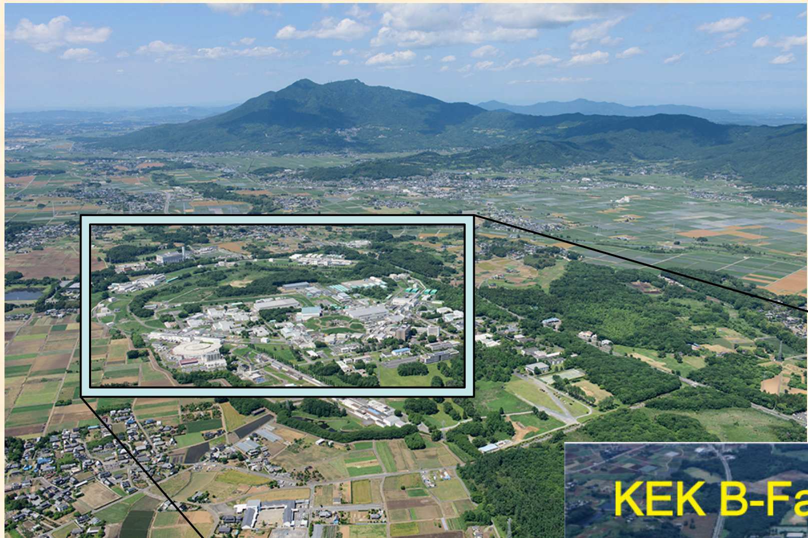
Bjorken variable for $\gamma\gamma^*$: $z=\frac{Q^2}{2q\cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar h$: $\zeta=\frac{p^+}{P^+}=\frac{1+\beta\cos\theta}{2}$

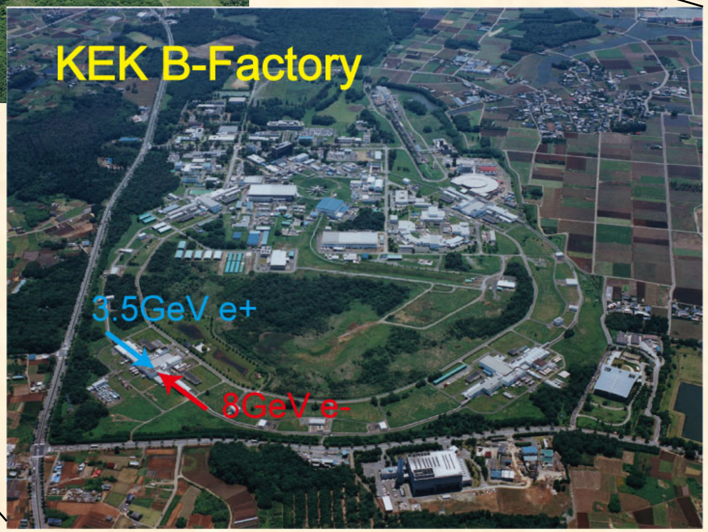
Invariant mass of $h\bar h$: $W^2=(p+p')^2$

Experimental studies of GDAs in future

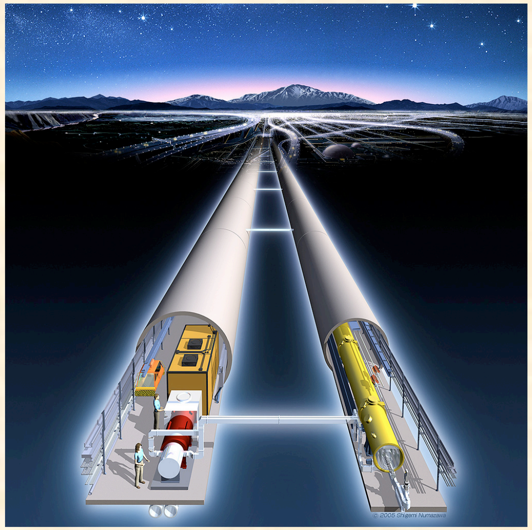
$\gamma\gamma \to h\bar{h}$ for internal structure of exotic hadron candidate h



KEK B-factory



Linear Collider ?



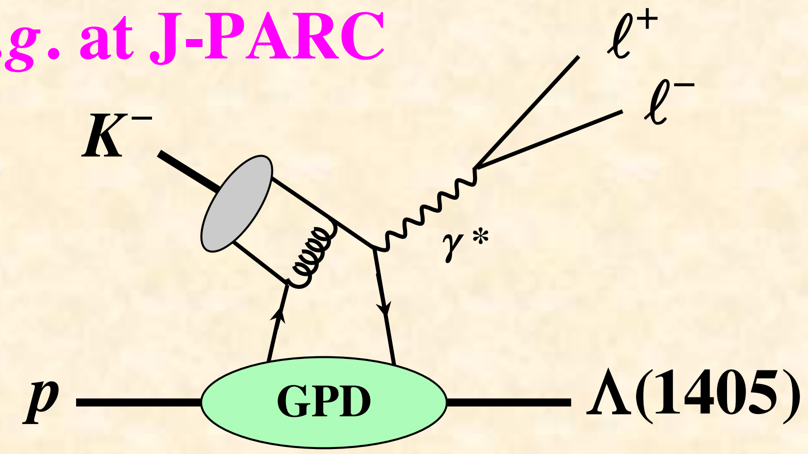
GPDs for exotic hadrons !?

Because stable targets do not exist for exotic hadrons, it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or → $s \leftrightarrow t$ crossed qunatity = GDAs at KEKB, Linear Collider

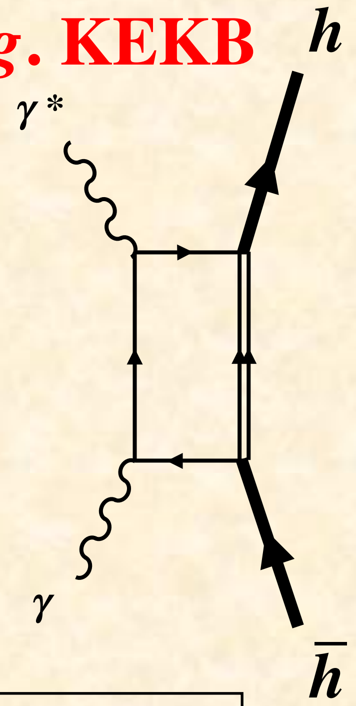
***e.g.* at J-PARC**



$K^-(\bar{u}s) + p(uud) \to \Lambda_{1405}(uud\bar{u}s) + \gamma^*$

Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate

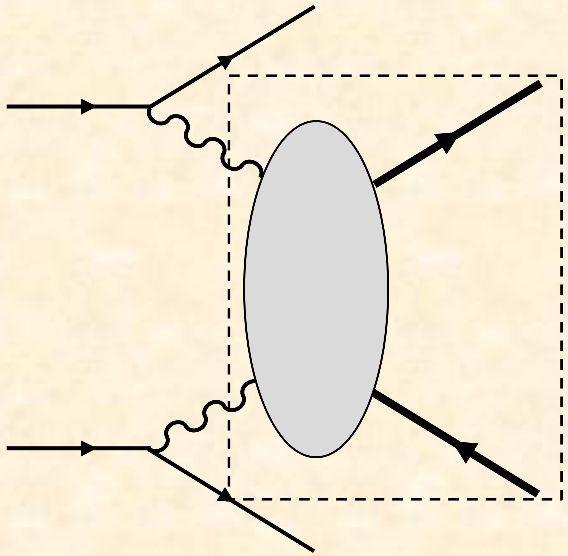
***e.g.* KEKB**



See H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006
for constituent-counting rule for exotic hadron candidates.

Generalized Distribution Amplitudes (GDAs) for pion

from KEKB measurements



$$\gamma\gamma \to h\bar{h}$$

Cross section for $\gamma^{*}\gamma \to \pi^{0}\pi^{0}$

$$d\sigma = \frac{1}{4\sqrt{(q\cdot q')^2 - q^2 q'^2}}(2\pi)^4\delta^4(q+q'-p-p')\overline{\sum_{\lambda,\lambda'}}|\mathcal{M}|^2 \frac{d^3p}{(2\pi)^3 2E}\frac{d^3p'}{(2\pi)^3 2E'}$$

$$q = (q^0,\ 0,\ 0,\ |\vec{q}|),\ \ q' = (|\vec{q}|,\ 0,\ 0,\ -|\vec{q}|),\ \ \ q'^2 = 0\ \text{(real photon)}$$

$$p = (p^0,\ |\vec{p}|\sin\theta,\ 0,\ |\vec{p}|\cos\theta),\ \ p = (p^0,\ -|\vec{p}|\sin\theta,\ 0,\ -|\vec{p}|\cos\theta)$$

$$\beta = \frac{|\vec{p}|}{p^0} = \sqrt{1-\frac{4m_\pi^2}{W^2}}$$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}\overline{\sum_{\lambda,\lambda'}}|\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu},\quad T^{\mu\nu} = i\int d^4\xi e^{-i\xi\cdot q}\langle\pi(p)\pi(p')|TJ_{em}^\mu(\xi)J_{em}^\nu(0)|0\rangle$$

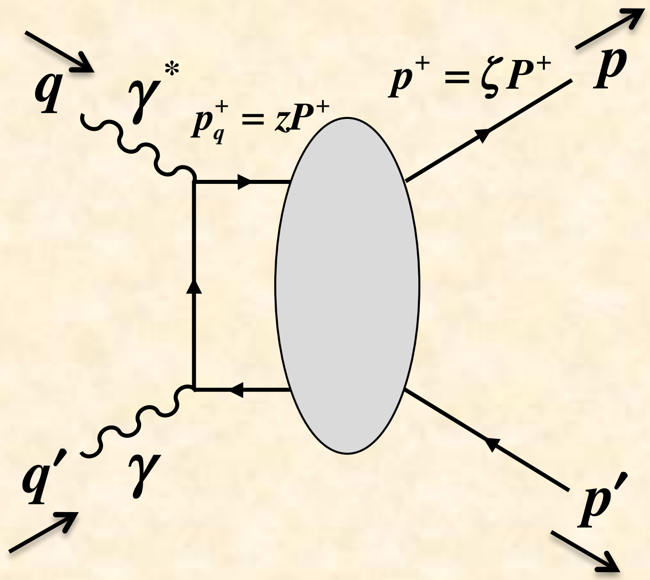
$$\mathcal{M} = e^2 A_{\lambda\lambda'} = 4\pi\alpha A_{\lambda\lambda'}$$

$$A_{\lambda\lambda'} = \frac{1}{e^2}\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')T^{\mu\nu} = -\varepsilon_\mu^\lambda(q)\varepsilon_\nu^{\lambda'}(q')g_T^{\mu\nu}\sum_q\frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

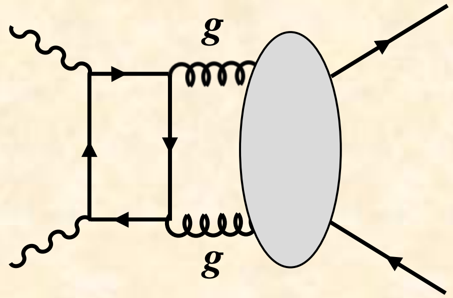
GDA: $$\Phi_q^{\pi\pi}(z,\zeta,s) = \int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle\pi(p)\pi(p')|\bar{\psi}(-y/2)\gamma^+\psi(y/2)|0\rangle\Big|_{y^+=0,\vec{y}_\perp=0}$$

$$A_{++} = \sum_q\frac{e_q^2}{2}\int_0^1 dz\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2),\quad \varepsilon_\mu^+(q)\varepsilon_\nu^+(q')g_T^{\mu\nu} = -1$$

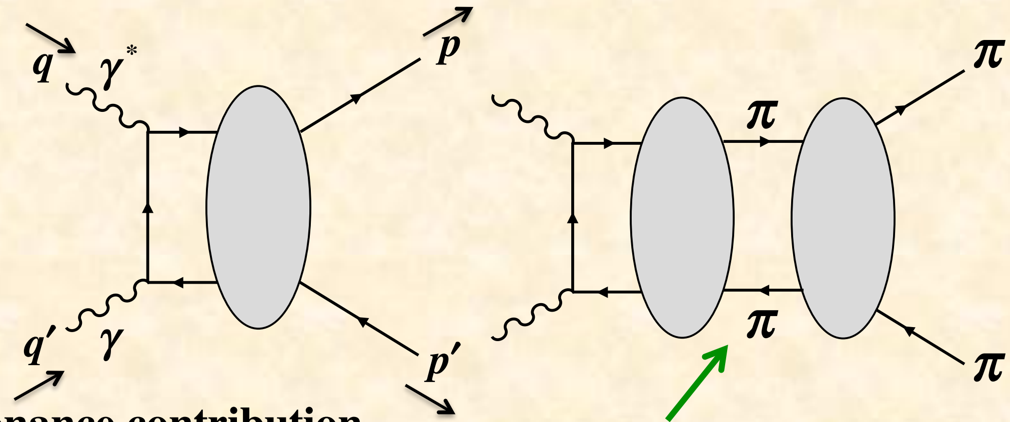
$$\frac{d\sigma}{d(\cos\theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m_\pi^2}{s}}|A_{++}|^2$$



Gluon GDA is higher-order term,
and it is not included in our analysis,



GDA parametrization for pion



Including intermediate resonance contributions

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{4(s+Q^2)}\sqrt{1-\frac{4m^2}{s}}\,|A_{++}|^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz\,\frac{2z-1}{z(1-z)}\Phi_q^{\pi\pi}(z,\zeta,W^2)$$

- **Continuum: GDAs without intermediate-resonance contribution**

$$\Phi_q^{\pi\pi}(z,\zeta,W^2) = N_\pi z^\alpha (1-z)^\alpha (2z-1)\zeta(1-\zeta)F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{\left[1+(s-4m_\pi^2)/\Lambda^2\right]^{n-1}},\quad n=2 \text{ according to constituent counting rule}$$

- **Resonances: Tthere exist resonance contributions to the cross section.**

$$\sum_q \Phi_q^{\pi\pi}(z,\zeta,W^2) = 18N_f z^\alpha (1-z)^\alpha (2z-1)\left[\tilde{B}_{10}(W) + \tilde{B}_{12}(W)P_2(\cos\theta)\right]$$

$$P_2(x) = \frac{1}{2}(3x^2-1)$$

$$\tilde{B}_{10}(W) = \text{resonance}\left[f_0(500)\equiv\sigma,\ f_0(980)\equiv f_0\right] + \text{continuum}$$

$$\tilde{B}_{12}(W) = \text{resonance}\left[f_2(1270)\right] + \text{continuum}$$

$f_0(500)$ or σ [g]
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400\text{–}550)$ MeV
Full width $\Gamma = (400\text{–}700)$ MeV

$f_0(980)$ [j]

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 10$ to 100 MeV

$f_2(1270)$

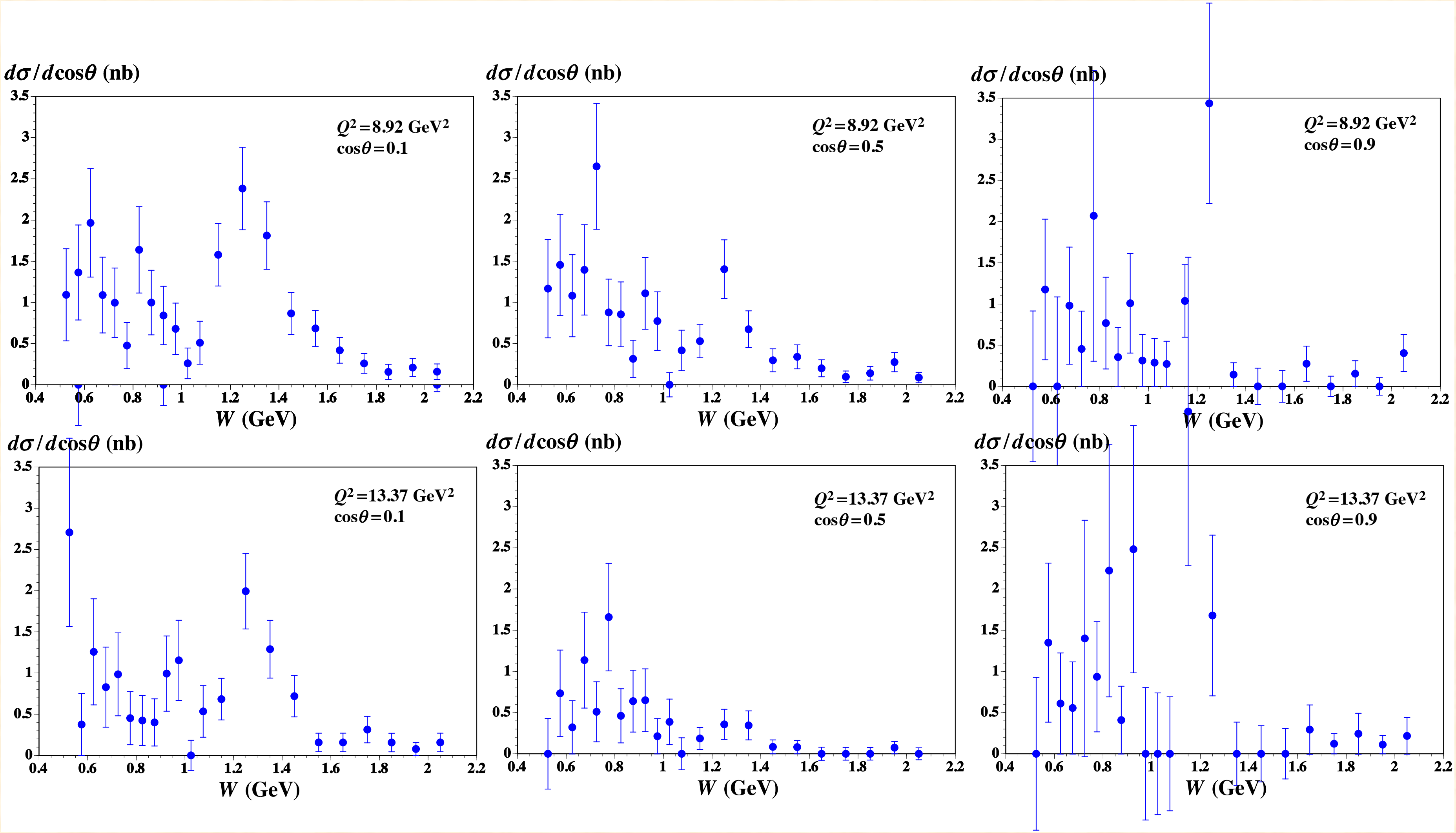
$I^G(J^{PC}) = 0^+(2^{++})$

Mass $m = 1275.5 \pm 0.8$ MeV
Full width $\Gamma = 186.7^{+2.2}_{-2.5}$ MeV (S = 1.4)

Analysis of Belle data on $\gamma\gamma^* \to \pi^0\pi^0$

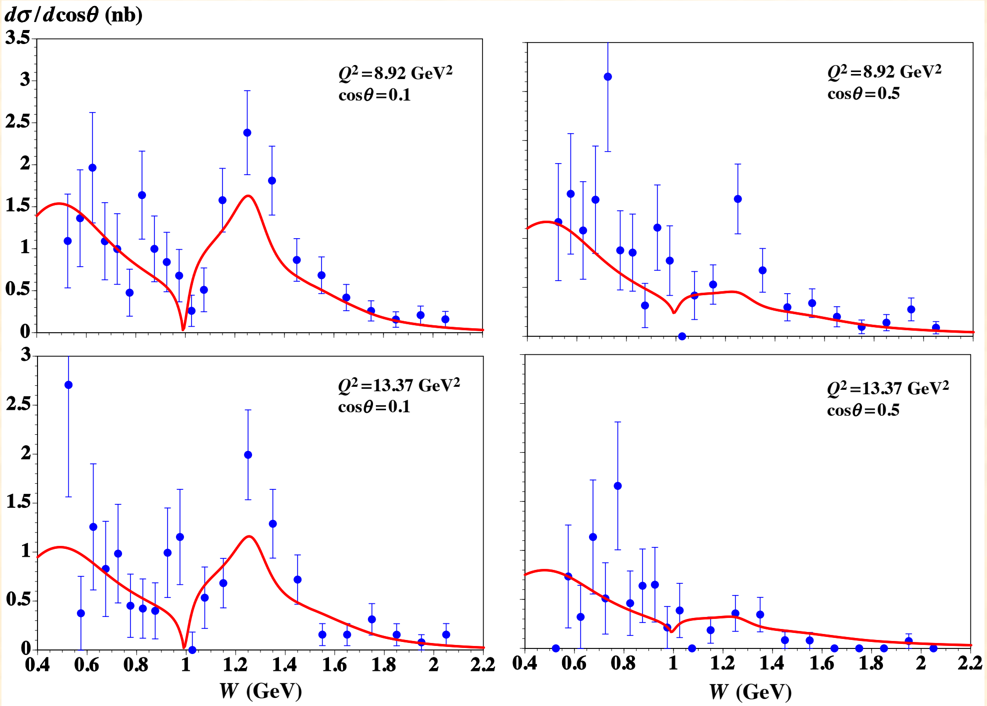
$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$

Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.

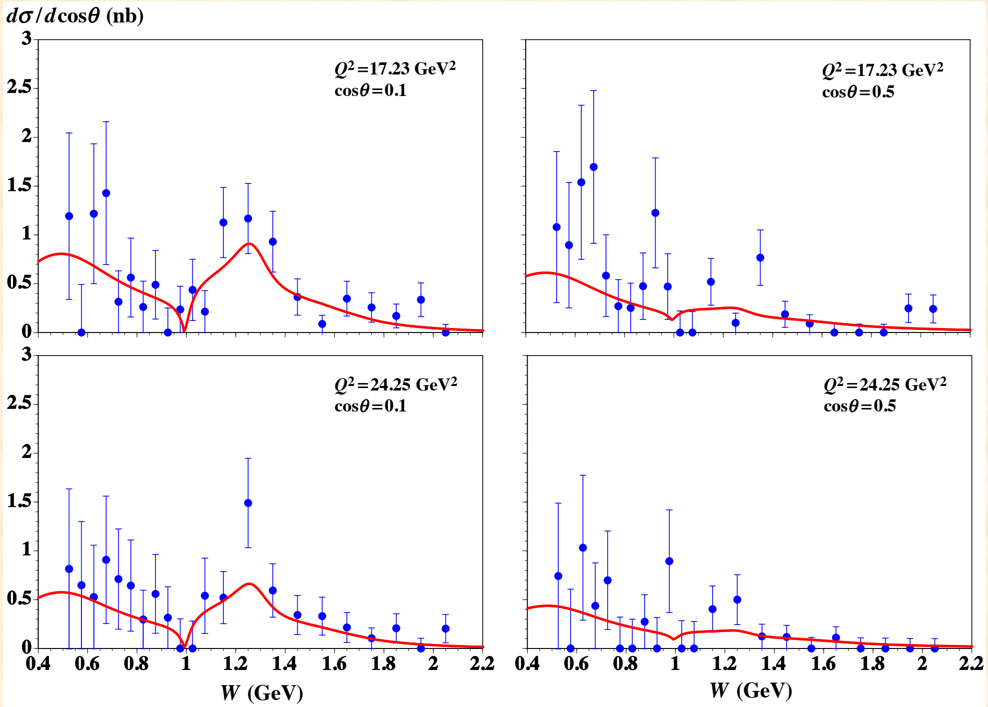


Analysis results for $\cos\theta = 0.1,\ 0.5$

$Q^2 = 8.92,\ 13.37\ \text{GeV}^2$



$Q^2 = 17.23,\ 24.25\ \text{GeV}^2$



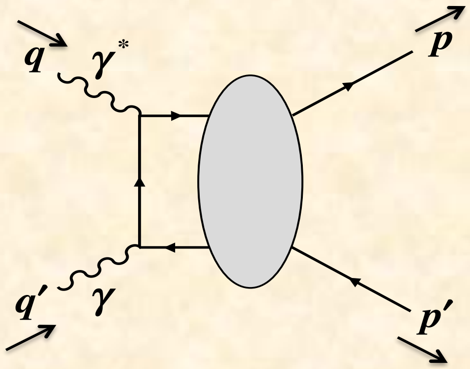
Gravitational form factors and radii for pion

$$\int_0^1 dz(2z-1)\Phi_q^{\pi^0\pi^0}(z,\zeta,s) = \frac{2}{(P^+)^2}\langle \pi^0(p)\pi^0(p')|T_q^{++}(0)|0\rangle$$

$$\langle \pi^0(p)\pi^0(p')|T_q^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[\left(sg^{\mu\nu} - P^\mu P^\nu\right)\Theta_{1,q}(s) + \Delta^\mu\Delta^\nu\Theta_{2,q}(s)\right]$$

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

See also Hyeon-Dong Son, Hyun-Chul Kim, PRD90 (2014) 111901.



$T_q^{\mu\nu}$: energy-momentum tensor for quark

$\Theta_{1,q}$, $\Theta_{2,q}$: gravitational form factos for pion

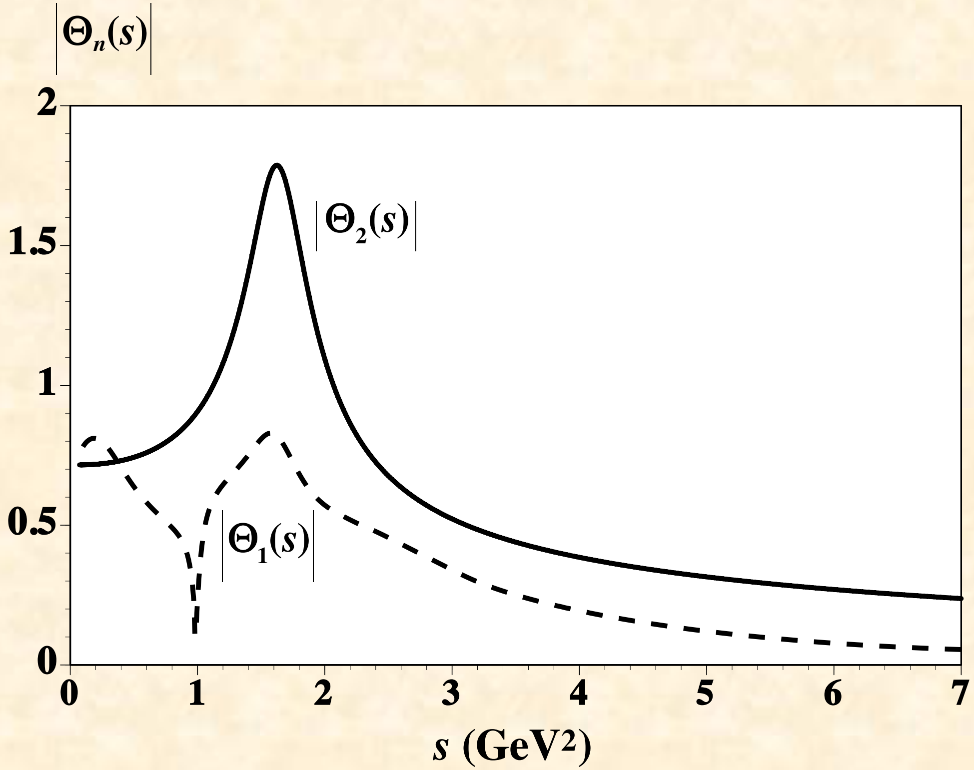
Analyiss of $\gamma^*\gamma \to \pi^0\pi^0$ cross section
$\Rightarrow$ Generalized distribution amplitudes $\Phi_q^{\pi^0\pi^0}(z,\zeta,s)$
$\Rightarrow$ Timelike gravitational form factors $\Theta_{1,q}(s)$, $\Theta_{2,q}(s)$
$\Rightarrow$ Spacelike gravitational form factors $\Theta_{1,q}(t)$, $\Theta_{2,q}(t)$
$\Rightarrow$ Gravitational radii of pion

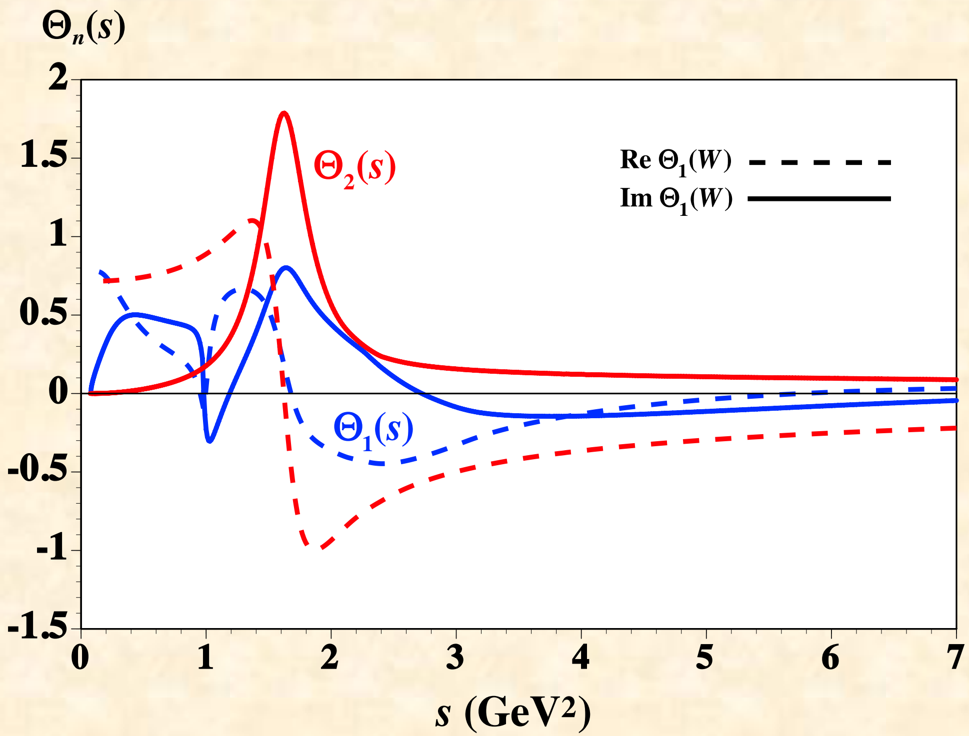
Gravitational form factors:
Original definition: H. Pagels, Phys. Rev. 144 (1966) 1250.
Operator relations: K. Tanaka, Phys. Rev. D 98 (2018) 034009.

Timelike gravitational form factors for pion

$$\langle \pi^a(p)\pi^b(p') | T_q^{\mu\nu}(0) | 0 \rangle = \frac{\delta^{ab}}{2}\left[(sg^{\mu\nu} - P^\mu P^\nu)\Theta_{1(q)}(s) + \Delta^\mu \Delta^\nu \Theta_{2(q)}(s) \right], \quad P = p + p', \quad \Delta = p' - p$$

- $\Theta_{1(q)}(s) = -\frac{3}{10}\tilde{B}_{10}(W^2) + \frac{3}{20}\tilde{B}_{12}(W^2) = -4B_{(q)}(s)$
- $\Theta_{2(q)}(s) = \frac{9}{20\beta^2}\tilde{B}_{12}(W^2) = A_{(q)}(s)$





Spacelike gravitational form factors and radii for pion

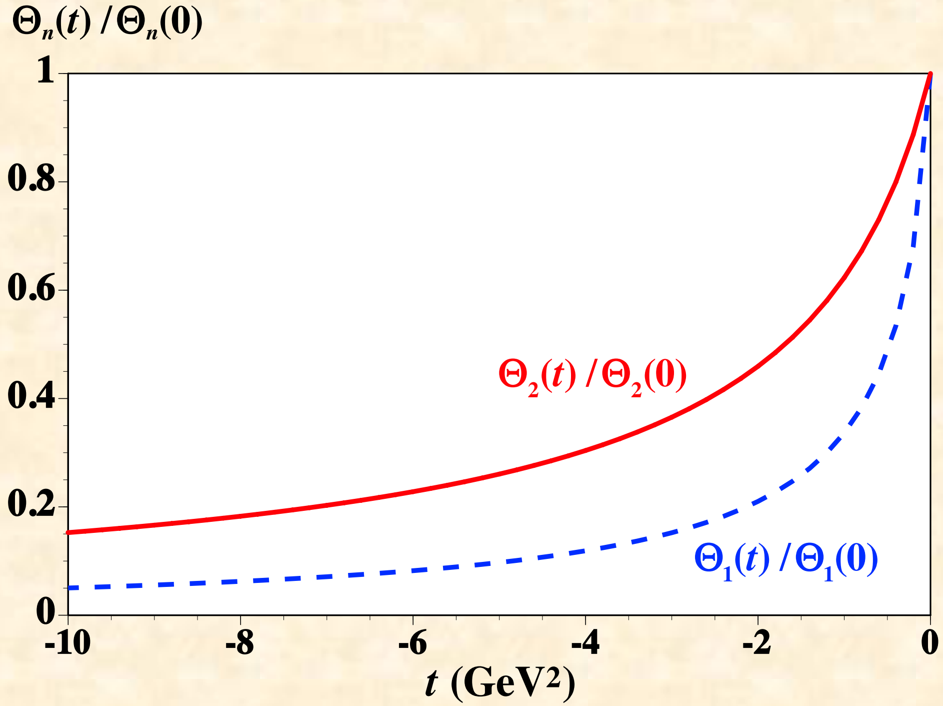
$$F(s) = \Theta_1(s),\ \Theta_1(s),\quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\,F(s)}{\pi(s-t-i\varepsilon)},\quad \rho(r) = \frac{1}{(2\pi)^3}\int d^3q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2}\frac{1}{r}\int_{4m_\pi^2}^{\infty} ds\, e^{-\sqrt{s}r}\,\mathrm{Im}\,F(s)$$

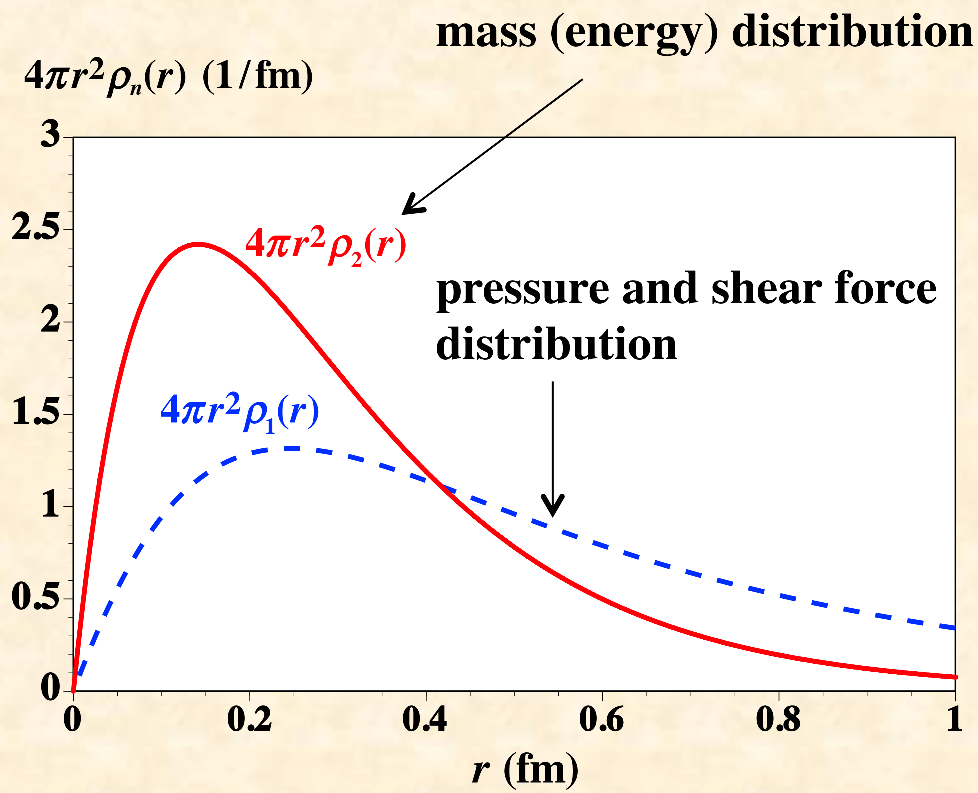
This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2\rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm},\quad \sqrt{\langle r^2\rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

← First finding on gravitational radius from actual experimental measurements

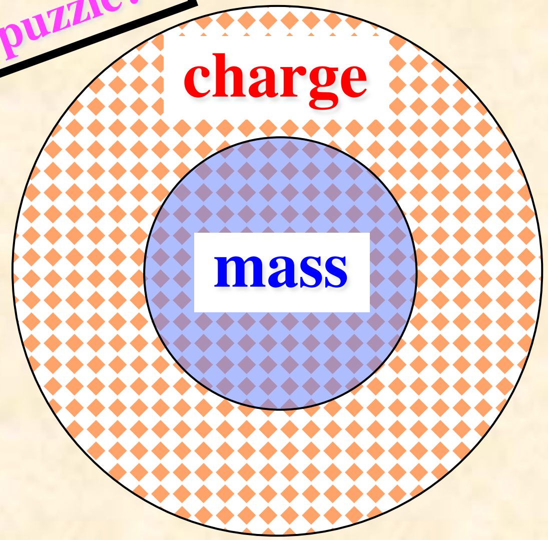
$$\Leftrightarrow \sqrt{\langle r^2\rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$





Hadron mass radius puzzle?

Hadron-mass radius puzzle?!



For pion

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm} \Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$

S. Kumano, Q.-T. Song, O. Teryaev, PRD 97 (2018) 014020;
Erratum in v3 of arXiv:1711.08088.

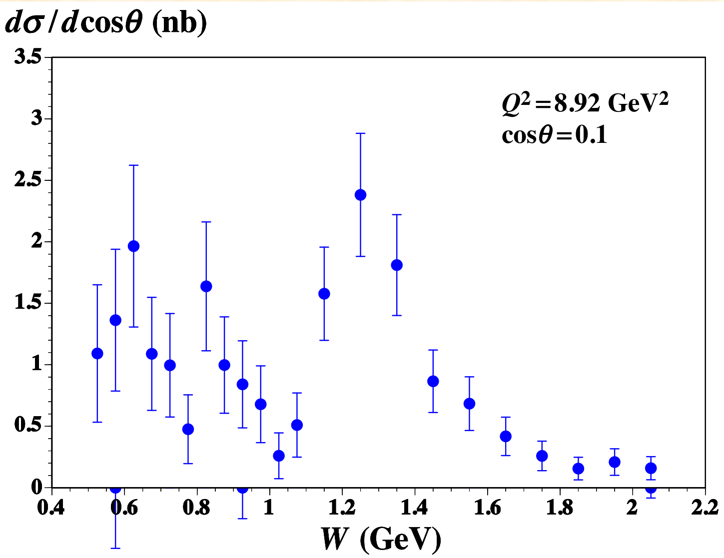
Mass radius seems to be much smaller than the charge radius for pion.

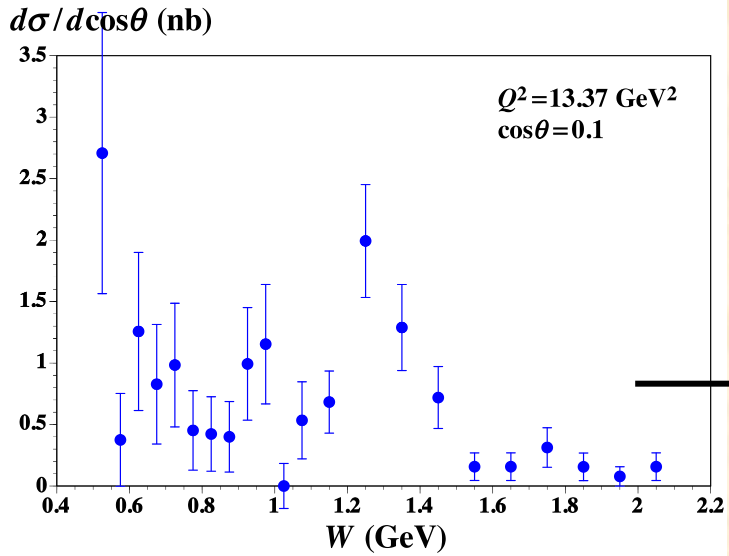
This is the first result on the mass radius from actual measurement,
so further studies are needed to find whether there is acutally a significant difference

Quarks contribute to both charge and mass distributions,
but gluons contribute to only the mass distribution.

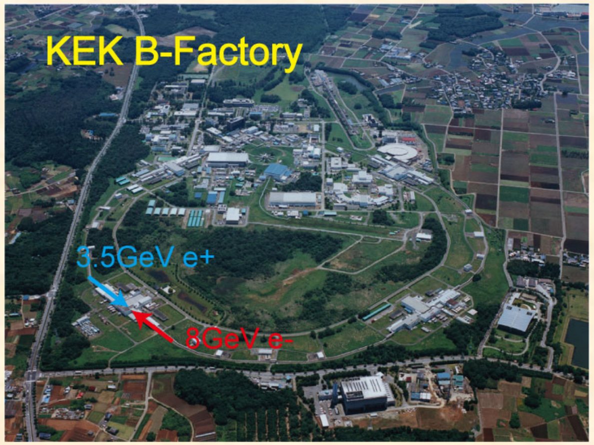
Electric interactions are repulsive (or could be attractive) and
gravitational interactions are always attractive,
so there would be some differences in both radii.
However, the difference of the factor of 2 may not be expected.

Super KEKB





The errors are dominated by statistical errors, and they will be significantly reduced by super-KEKB.

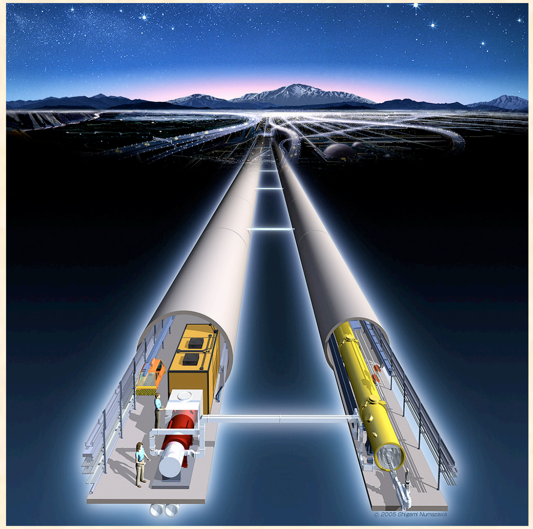


From KEKB to ILC

- **Very Large Q^2**
- **Large W^2**

for extracting GDAs

ILC



GSI-FAIR (PANDA)

arXiv:0903.3905 [hep-ex]

FAIR/PANDA/Physics Book i

Physics Performance Report for:

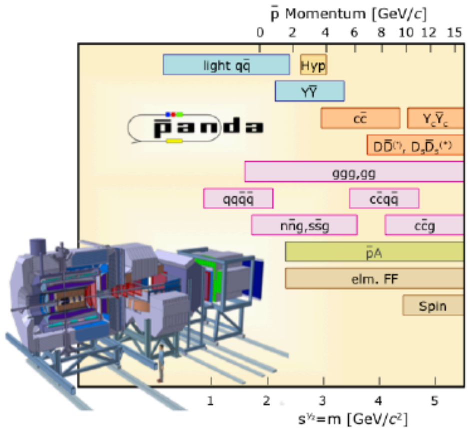
$\overline{\text{P}}$ANDA

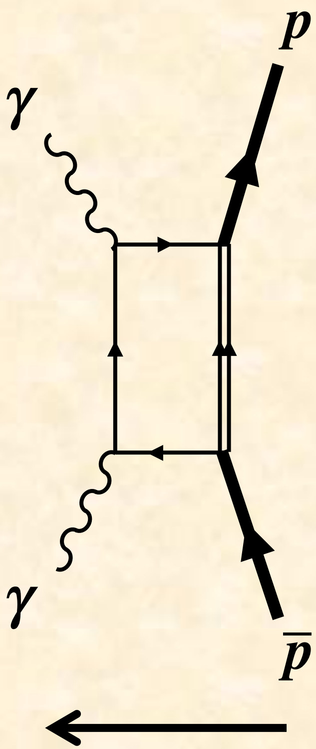
(AntiProton Annihilations at Darmstadt)

Strong Interaction Studies with Antiprotons

$\overline{\text{P}}$ANDA Collaboration

To study fundamental questions of hadron and nuclear physics in interactions of antiprotons with nucleons and nuclei, the universal $\overline{\text{P}}$ANDA detector will be build. Gluonic excitations, the physics of strange and charm quarks and nucleon structure studies will be performed with unprecedented accuracy thereby allowing high-precision tests of the strong interaction. The proposed $\overline{\text{P}}$ANDA detector is a state-of-the-art internal target detector at the HESR at FAIR allowing the detection and identification of neutral and charged particles generated within the relevant angular and energy range.
This report presents a summary of the physics accessible at $\overline{\text{P}}$ANDA and what performance can be expected.





GDAs for the proton!
(super-KEKB,
Uehara@PacSpin2019)

Comments on

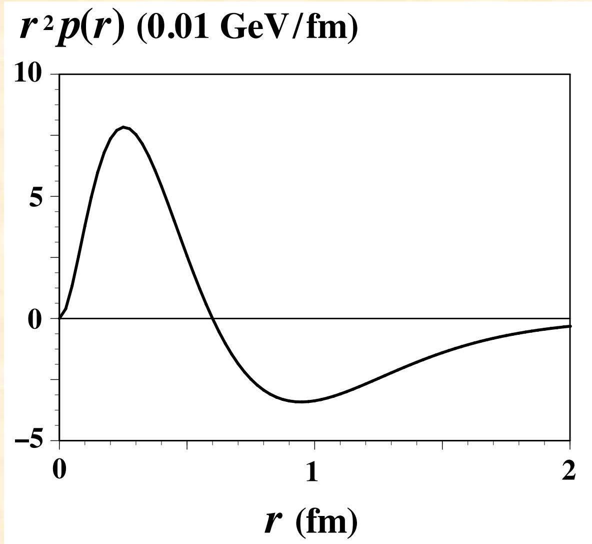
Hadron pressure and mass

Nucleon pressure

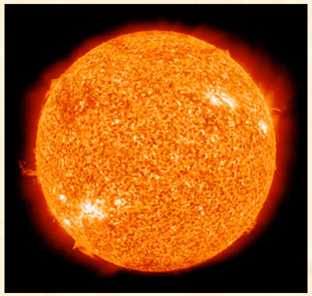
$$\langle N(p')|T_q^{\mu\nu}(0)|N(p)\rangle = \bar{u}(p')\left[A\gamma^{(\mu}\bar{P}^{\nu)} + B\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}Mg^{\mu\nu}\right]u(p)$$

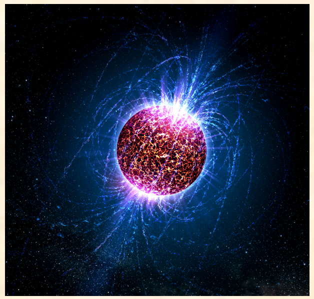
Recent progress

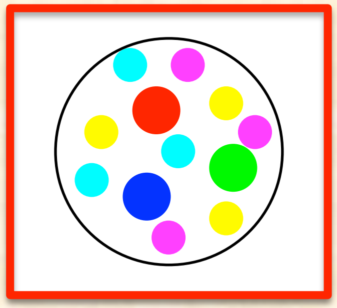
- V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557 (2018) 396;
- M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) 1830025;
- C. Lorce, H. Moutarde, and A. P. Tranwinski, Eur. Phys. J. C 79 (2019) 89.



Highest pressure in nature 1 Pa (Pascal) = 1 N/m^2







Earth atmosphere	Center of earth	Center of Sun	Neutron star	Hadron
10^5 Pa =1000 hPa	10^{11} Pa = 100GPa	10^{16} Pa = 10 PPa	10^{34} Pa	10^{35} Pa

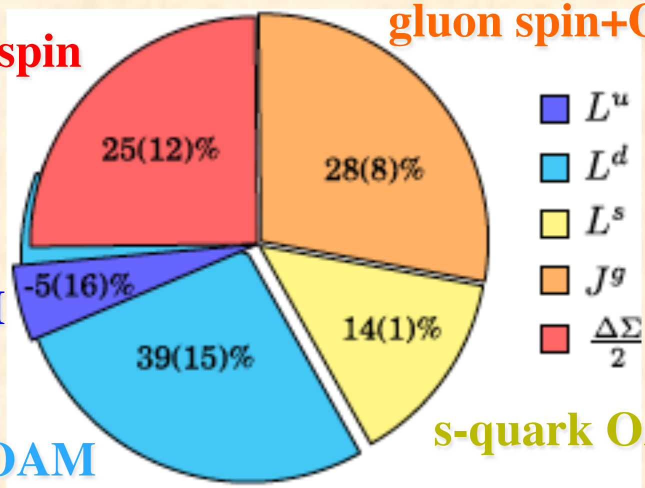
Origin of nucleon spin: decomposition

$$\frac{1}{2}=\langle p|J^3|p\rangle,\quad J^3=\frac{1}{2}\varepsilon^{3jk}\int d^3x\, M^{3jk}(x),\quad M^{\alpha\mu\nu}(x)=T^{\alpha\nu}(x)x^{\mu}-T^{\alpha\mu}(x)x^{\nu}$$

Gauge invariant decomposion: see review papers of M. Wakamatsu, Int. J. Mod. Phys. A29 (2014) 1430012;
E. Leader and C. Lorce, Phys. Rept. 541 (2014) 163;
and Y. Hatta (and S. Yoshida, K. Tanaka), Phys. Rev. D84 (2011) 041701;
Phys. Lett. B 708 (2012) 186; JHEP 1210 (2012) 080; 1302 (2013) 003.

$$\frac{1}{2}=\frac{1}{2}\Delta\Sigma+\Delta g+L_q+L_g,$$ $\Delta\Sigma$ = quark spin contribution, Δg = gluon spin contribution,
L_q = quark orbital-angular-momentum (OAM) contribution,
L_g = gluon orbital-angular-momentum (OAM) contribution

Lattice QCD estimate in M. Deke *et al.*, Phys. Rev. D 91 (2015) 0145505



Spin decomposion

- **quark spin 25%**
- **quark OAM 45%**
- **gluon spin + OAM 30%**

Hadjiyiannakou@DIS2018
Y.-B. Yang@PacSpin2019

Origin of nucleon mass

Nucleon mass: $M = \langle p | H | p \rangle, \quad H = \int d^3x\, T^{00}(x)$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2}\bar{q}(x) i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F_{\alpha}^{\ \nu}(x)$$

We need theoretical and experimental efforts to decompose nucleon mass for finding its origin.

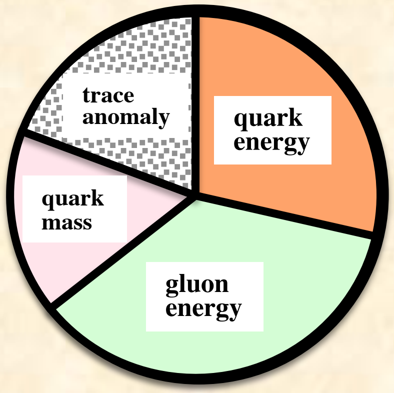
X. Ji, PRL 74 (1995) 1071.

$$T^{\mu\nu} = \hat{T}^{\mu\nu} + \bar{T}^{\mu\nu} = \left(T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \alpha} \right)_{\text{traceless}} + \left(-\frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \alpha} \right)_{\text{trace}}, \quad T^{\alpha}_{\ \alpha} = \bar{q}\, m\, q + \frac{\beta(g)}{2g} F^2$$

$$H = H_q(\text{quark energy}) + H_g(\text{gluon energy}) + H_m(\text{quark mass}) + H_a(\text{trace anomaly})$$

$$H_q = \int d^3x\, \bar{q}(x)\left(-i\vec{D}\cdot\vec{\alpha} \right) q(x), \quad H_g = \int d^3x\, \frac{1}{2}\left(\vec{E}^2 + \vec{B}^2 \right)$$

$$H_m = \int d^3x\, \bar{q}(x)\, m\, q(x), \quad H_s = \int d^3x\, \frac{9\alpha_s}{16\pi}\left(\vec{E}^2 + \vec{B}^2 \right)$$



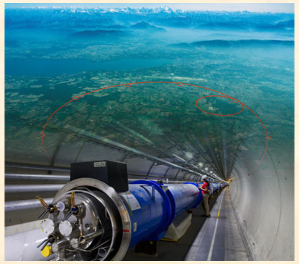
Recent progress on trace-anomaly, gravitational form factor, scale depdence in perturbative QCD:

Y. Hatta, A. Rajan, and K. Tanaka, JHEP 12 (2018) 008;
K.Tanaka, JHEP 01 (2019) 120.

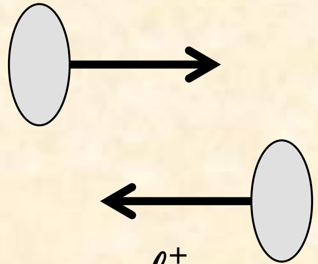
Prospects & Summary

Facilities to probe 3D structure functions (GPD, GDA)

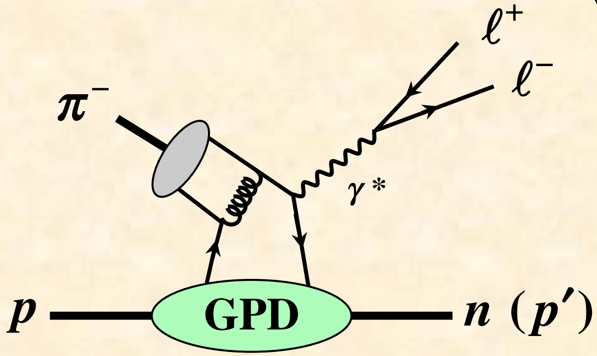
RHIC
LHC



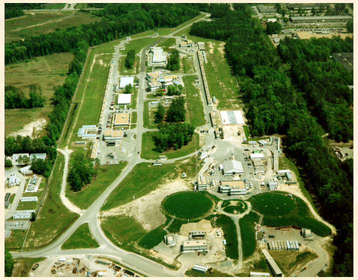
Ultra-peripheral collisions
for $\gamma^{*}\gamma \to h\bar{h}$??

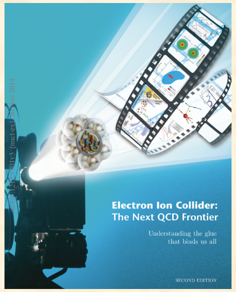


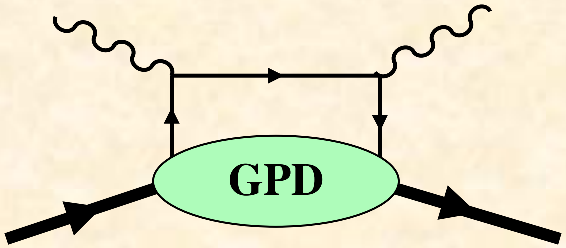
Fermilab
J-PARC
GSI-FAIR



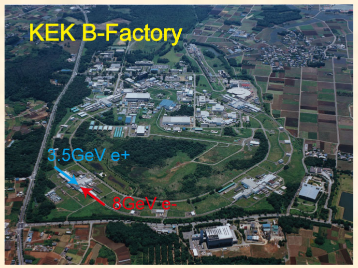
JLab
COMPASS
EIC

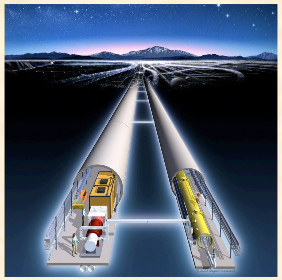




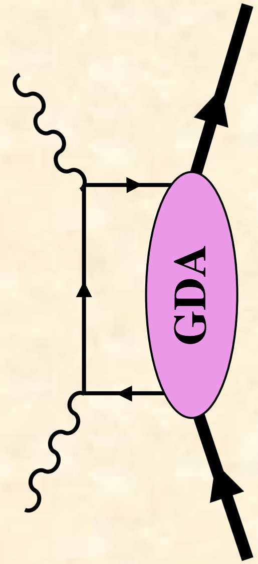


KEKB
ILC



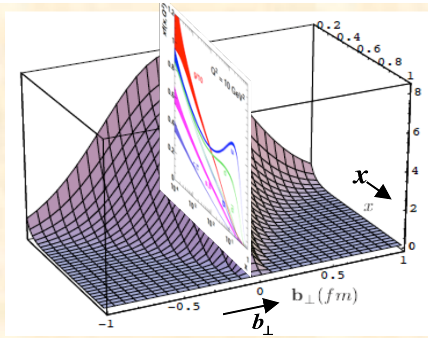


We studied this process.



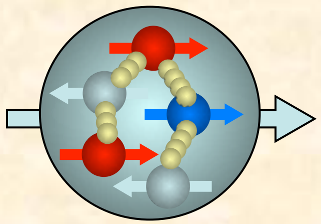
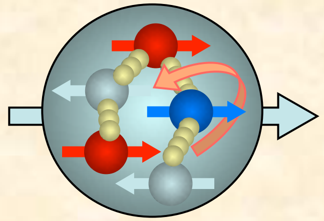
By hadron tomography

3D view of hadrons

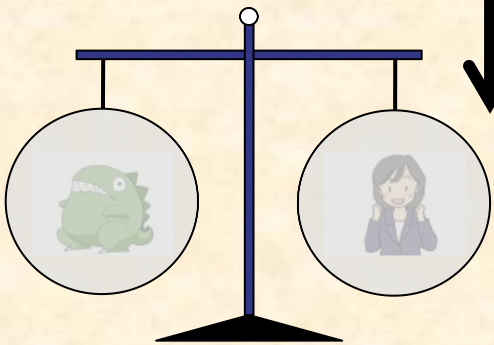


Origin of nucleon spin

By the tomography, we determine

 or  .

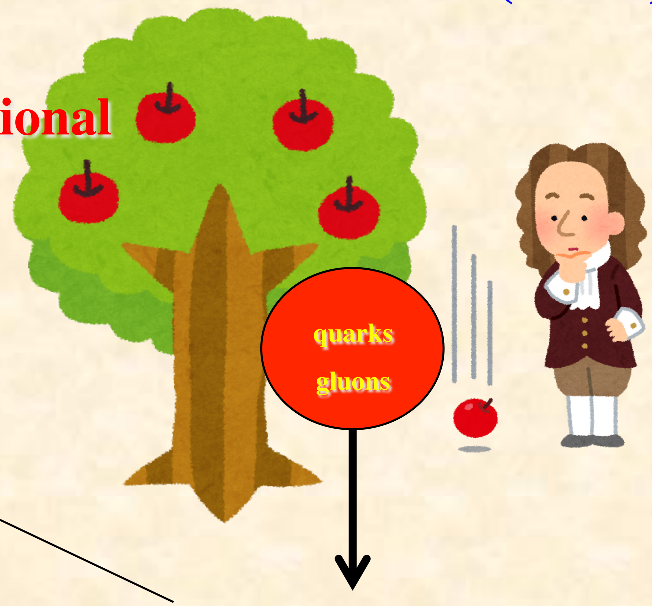
Exotic hadrons



By tomography, we determine  or  .

Origin of gravitational source (mass)

By tomography,
we determine gravitational sources in terms of quarks and gluons.



Summary

- Determination of GPAs (Generalized Distribution Amplitudes) for pion.
- Gravitational form factors Θ_1, Θ_2 were calculated from the GDAs.

Gravitational radii

mass radius: $\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39$ fm $\Leftrightarrow$ $\sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008$ fm

mechanical radius: $\sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88$ fm

→ Hadron-mass radius puzzle?

Hadron-mass radius is much smaller than charge radius?!

Theoretical efforts are needed, ⋯

Experimental GPDs, GDAs (COMPASS, JLab, super KEKB, ⋯, EIC)

→ Gravitational form factors (mass, pressure, shear force distributions)

Time has come to understand the gravitational sources and their interactions in microscopic (instead of usual macroscopic/cosmic) world in terms of quark and gluon degrees of freedom.

The End

The End