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Energy-Momentum Tensor for massive hadrons

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QCD energy-momentum tensor

$$T^{\mu\nu} = \overline{\psi}\gamma^{\mu}\frac{i}{2}\overleftrightarrow{D}^{\nu}\psi - F^{a\mu\lambda}F^{a\nu}_{\lambda} + \frac{1}{4}g^{\mu\nu}F^{2}$$

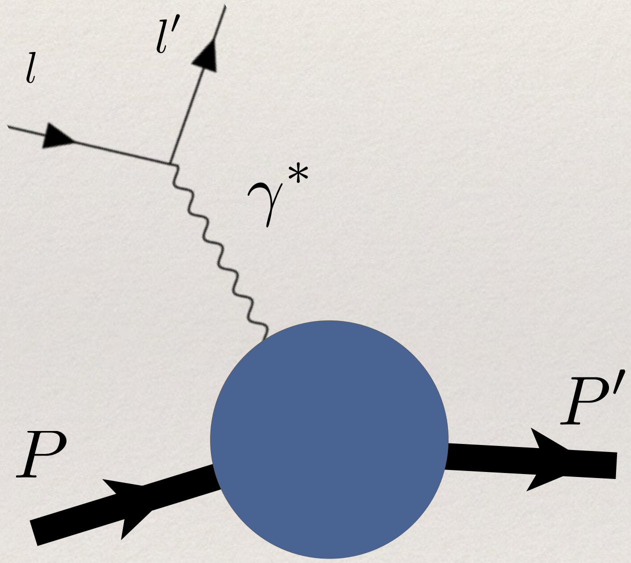
- Fundamental object in every theory (Noether's conserved current related to space-time translations)
- Contains info on mass and angular momentum distribution
- Contains info on the mechanical properties of hadrons (momentum distribution, pressure, etc..)
- Asymmetric object in theories with spin, symmetric otherwise.

EMT in scattering processes

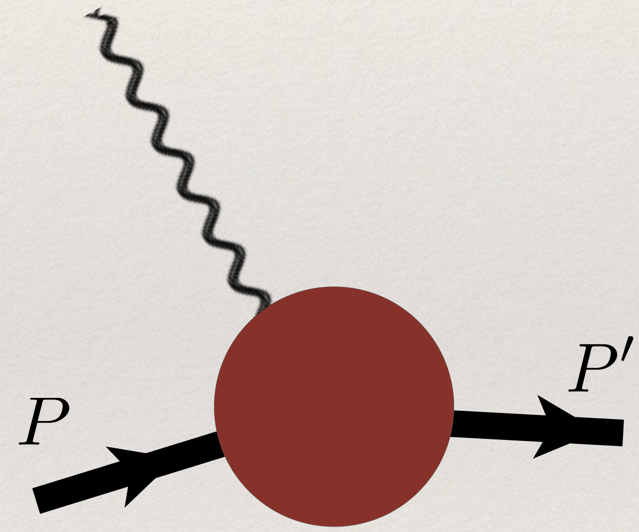
The EMT matrix element is written in terms of Form Factors, which in principle encode the nature of the interaction with gravity, just like the case of the electromagnetic current

BUT

The precise interaction with gravity is not known!



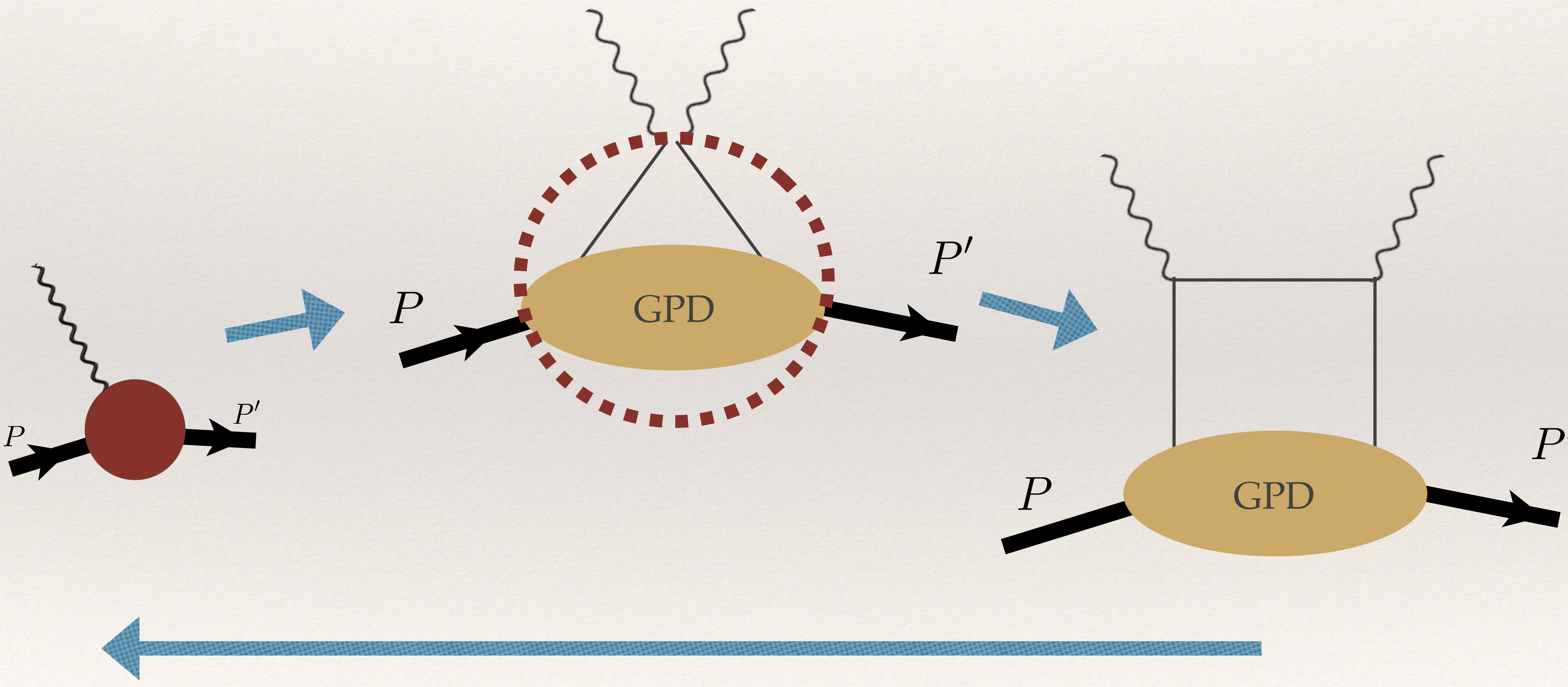
Electromagnetic probe, OK!



Impossible to probe directly through "graviton scatterings"

EMT in QCD processes

The only way to (indirectly) access the FFs is through the relation between the EMT matrix element and the Mellin moments of the GPDs



EMT Parametrizations and Form Factors

Redefine variables: $P = \frac{p' + p}{2}, \quad \Delta = p' - p, \quad t = \Delta^2$

$$\langle p', \lambda' \mid T_a^{\mu\nu}(0) \mid p, \lambda \rangle = \bar{\eta}_{\lambda'}(p') \Gamma_a^{\mu\nu}(P, \Delta)\, \eta_\lambda(p)$$

$a = q, g$

	Spin-0	Spin-1/2	Spin-1	Spin-3/2	Spin-2	etc.
$\eta_\lambda(p) \propto$	e^{ipx}	$u(p, \lambda)$	$\varepsilon_\alpha(p, \lambda)$	$u_\alpha(p, \lambda)$	$\varepsilon_{\alpha\beta}(p, \lambda)$	$\cdots$

Spin-0 and 1/2

Spin-0

$$\Gamma^{a\,\mu\nu}(P,\Delta) = 2M\left[\frac{P^\mu P^\nu}{M}\mathcal{A}^a(t) - \frac{1}{M}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\mathcal{C}^a(t) + g^{\mu\nu}M\bar{\mathcal{C}}^a(t)\right]$$

[Pagels (1966)]
[Donoghue, Leutwyler (1991)]
[Ji (1996)]

Spin-1/2

$$\Gamma^{a\,\mu\nu} = \frac{P^\mu P^\nu}{M}\mathcal{A}^a(t) + \frac{1}{M}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\mathcal{C}^a(t) + g^{\mu\nu}M\bar{\mathcal{C}}^a(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{4M}\mathcal{G}^a(t)$$

[Kobzarev, Okun (1962)][Pagels (1966)]
[Ji (1996)][Bakker, Leader, Trueman (2004)]
[Leader, Lorcé (2014)]

Non-conserved terms

Spin-1 (and higher…)

Spin-1 is important practically because of the applications on the deuteron

$$\Gamma^{a\,\mu\nu;\alpha\beta} = -\,2P^\mu P^\nu \left[g^{\alpha\beta}\mathcal{G}_1^a(t) - \frac{\Delta^\alpha\Delta^\beta}{2M^2}\mathcal{G}_2^a(t)\right] - \frac{1}{2}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\left[g^{\alpha\beta}\mathcal{G}_3^a(t) - \frac{\Delta^\alpha\Delta^\beta}{2M^2}\mathcal{G}_4^a(t)\right]$$
$$+ P^{\{\mu}\left(g^{\alpha\nu\}}\Delta^\beta - g^{\beta\nu\}}\Delta^\alpha\right)\mathcal{G}_5^a(t)$$
$$+ \frac{1}{2}\left[\Delta^{\{\mu}\left(g^{\alpha\nu\}}\Delta^\beta + g^{\beta\nu\}}\Delta^\alpha\right) - g^{\alpha\{\mu}g^{\nu\}\beta}\Delta^2 - g^{\mu\nu}\Delta^\alpha\Delta^\beta\right]\mathcal{G}_6^a(t)$$
$$+ g^{\alpha\{\mu}g^{\nu\}\beta}M^2\mathcal{G}_7^a(t) + g^{\mu\nu}g^{\alpha\beta}M^2\mathcal{G}_8^a(t) + \frac{1}{2}g^{\mu\nu}\Delta^\alpha\Delta^\beta\mathcal{G}_9^a(t)$$

[Holstein (2006)]
[Abidin, Carlson (2008)]
[Taneja, Kathuria, Liuti, Goldstein (2012)]
[Cosyn, SC, Freese, Lorcé (2019)]
[Polyakov, Sun, (2019)]

Non-conserved terms

Arbitrary spin

If we restrict ourselves to the terms O(Δ), the EMT for ANY spin reads:

[Boulware, Deser (1975)]
[SC, Lorcé, Lowdon, (2019)]

$$\langle p',\lambda' \mid T^{a\,\mu\nu}(0) \mid p,\lambda\rangle = \bar{\eta}_{\lambda'}(p')\left(P^{\{\mu}P^{\nu\}}\mathcal{A}^a(t) + P^{\{\mu}\Sigma^{\nu\}\rho}\Delta_\rho\mathcal{G}^a(t) + \cdots\right)\eta_\lambda(p)$$

$$\mathcal{G}(t) = A(t) + B(t)$$

Lorentz generator in the given spin representation

	Spin-1/2	Spin-1	Spin-3/2	Spin-2
$\Sigma^{\nu\rho}$	$\frac{i}{4}[\gamma^\nu,\gamma^\rho]$	$ig^{[\nu\alpha}g^{\rho]\beta}$	$ig^{[\nu\alpha}g^{\rho]\beta} + \sigma^{\nu\rho}g^{\alpha\beta}$	$ig^{[\nu\alpha_1}g^{\rho]\beta_2}g^{\alpha_2\beta_2} + (1\to 2)$

On-shell massive states

[Haag, (1996)]

- On-shell states with definite four momentum are defined as: [Lowdon, Chiu, Brodsky (2017)]

$$|p,\lambda;M\rangle = 2\pi\theta(p^0)\delta(p^2-M^2)|p,\lambda\rangle = \delta^{(+)}_M(p)|p,\lambda\rangle$$

- The definite-momentum states are distributional-valued, with normalization:

$$\langle p',\lambda';M|p,\lambda;M\rangle = (2\pi)^4\delta(p'-p)\delta^{(+)}_M(p)\delta_{\lambda'\lambda}$$

- Normalizable states result from integration with test functions:

$$|\text{state}\rangle = \int d^4p f(p)|p\rangle$$

- The EMT matrix elements has a distributional nature that needs to be taken into account:

[SC, Lorcé, Lowdon,(2019)]

$$\langle p',\lambda' \mid T^{a\,\mu\nu}(0) \mid p,\lambda\rangle =$$

$$\bar{\eta}_{\lambda'}(p')\left(P^{\{\mu}P^{\nu\}}\mathcal{A}^a(t) + P^{\{\mu}\Sigma^{\nu\}\rho}\Delta_\rho\mathcal{G}^a(t) + \cdots\right)\eta_\lambda(p)\delta^+_M(p')\delta^+_M(p)$$

Poincaré symmetry

[Lowdon, Chiu, Brodsky (2017)]

[SC, Lorcé, Lowdon,(2019)]

$$J^i \propto \epsilon^{ijk} \int d^4x\, \tilde{f}(x)\, \left[x^j T^{0k}(x) - x^k T^{0j}(x)\right]$$

Angular momentum

$$K^i \propto \int d^4x\, \tilde{f}(x)\, \left[x^0 T^{0i}(x) - x^i T^{00}(x)\right]$$

Boost

$$P^\mu \propto \int d^4x \tilde{f}(x)\, T^{0\mu}(x)$$

Momentum

Poincaré symmetry puts univocally constraints on: $\left(P^{\{\mu}P^{\nu\}}\mathcal{A}^a(t) + P^{\{\mu}\Sigma^{\nu\}\rho}\Delta_\rho \mathcal{G}^a(t) + \cdots\right)$

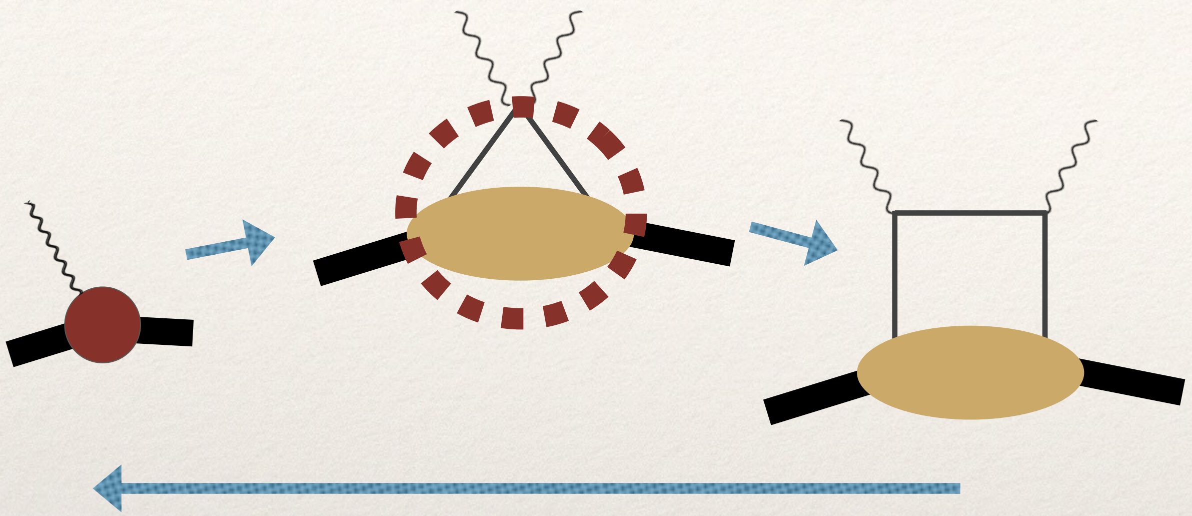
$\mathcal{G}(t) = A(t) + B(t)$

$$\mathcal{A}(0) = \mathcal{G}(0) = 1$$

Independent on the spin!

Massive on-shell states (**SC,Lorcé,Lowdon, arXiv:1905.11969**)
All on-shell states (**Lorcé,Lowdon arXiv:1908.02567**)

Gravitational Form Factors and GPDs



The gravitational Form Factors are related to the Mellin moments of the GPD matrix element!

Gravitational Form Factors and GPDs

The gravitational Form Factors are related to the Mellin moments of the GPD matrix element!

- GPD matrix element for quarks (for simplicity)

$$\langle p';\lambda'|\mathcal{O}^{\mu}_{qV}|p;\lambda\rangle = \frac{1}{2}\int_{-\infty}^{\infty}\frac{\mathrm{d}z}{2\pi}e^{i(Pn)zx}\left\langle p';\lambda'\middle|\overline{\psi}\left(-\frac{nz}{2}\right)\gamma^{\mu}\mathcal{W}_{[-\frac{nz}{2},\frac{nz}{2}]}\psi\left(\frac{nz}{2}\right)\middle|p;\lambda\right\rangle$$

- Second Mellin moments of GPDs

$$\int_{-1}^{1}\mathrm{d}x\,x\,\mathcal{O}^{\mu}_{qV} = \frac{1}{4(Pn)^2}\overline{\psi}(0)\gamma^{\mu}(i\overleftrightarrow{D}\,n)\psi(0)$$

- Relation with the EMT

$$\int_{-1}^{1}\mathrm{d}x\,x\,\mathcal{O}^{\mu}_{qV} = \frac{T^{\mu n}_{q}}{2(Pn)^2}$$

Ji's sum rule

Twist-2 GPD parametrization for any spin up to first order O(Δ)

$$\langle p', \lambda' | \mathcal{O}^n | p, \lambda \rangle = \bar{\eta}_{\lambda'}(p') \left[H_1(x,\xi,t) + i \frac{\Sigma^{n\rho}\Delta_\rho}{\bar{p}n} H_2(x,\xi,t) + \cdots \right] \eta_\lambda(p)\, \delta_M^{(+)}(p) \delta_M^{(+)}(p')$$

$$\int_{-1}^{1} dx\, x H_1(x,\xi,t) = A(t) + \cdots\,,$$

$$\int_{-1}^{1} dx\, x H_2(x,\xi,t) = G(t) + \cdots\,.$$

corrections from terms with a Δ-dependence

[SC, Lorcé, Lowdon,(2019)]

$$P^z = \sum_{a=q,g} \int_{-1}^{1} dx\, x H_1^a(x,0,0) = A(0) = 1,$$

$$J^z = \sum_{a=q,g} \int_{-1}^{1} dx\, x H_2^a(x,0,0) = G(0) = 1,$$

Comments

- Only from the knowledge of how the states transform under Poincaré transformation, we are able to derive momentum and angular momentum sum rules for any state (massive states in **SC,Lorcé,Lowdon, arXiv:1905.11969**, extension to the massless case in **Lorcé,Lowdon arXiv:1908.02567**)
- No need for explicitly treating the EMT spin by spin to obtain the Ji's sum rule for all spin
- The relation $\mathcal{A}(0) = \mathcal{G}(0) = 1$ automatically implies that, when defining $\mathcal{G}(t) = A(t) + B(t)$, the B(0) identically vanishes for all spins

Spin-multipoles expansion

The ultimate goal: finding an expression for the EMT for arbitrary spin.

How do we complete this expansion?

$$\left(P^{\{\mu}P^{\nu\}}\mathcal{A}^a(t) + P^{\{\mu}\Sigma^{\nu\}\rho}\Delta_\rho\mathcal{G}^a(t) + \cdots\right)$$

This can be seen as an expansion in spin-multipoles, built from the Lorentz generators.

$$\mathcal{I}^{\mu\nu} = g^{\mu\nu}$$ **"Monopole"**

$$\mathcal{D}^{\mu\nu} = \Sigma^{\mu\nu}$$ **Antisymm "Dipole"**

$$\mathcal{Q}^{\mu\nu,\rho\sigma} = \{\Sigma^{\mu\nu}, \Sigma^{\rho\sigma}\} - \frac{1}{6}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})\,\Sigma\cdot\Sigma$$ **STT "Quadrupole"**

$\cdots$

$$D^\mu = \mathcal{D}^{\mu\nu}\Delta_\nu \qquad Q^{\mu\nu} = g_{\lambda\tau}\mathcal{Q}^{\mu\lambda,\tau\nu} \qquad Q^{\mu\nu}_\Delta = \Delta_\lambda\Delta_\tau\mathcal{Q}^{\mu\lambda,\tau\nu}$$

Spin-1 revisited

$$I^{\mu\nu} = \frac{1}{4} g^{\mu\nu} g^{\alpha\beta}$$

$$D^{\mu} = \frac{1}{2}(g^{\mu\alpha}\Delta^{\beta} - \Delta^{\alpha} g^{\mu\beta})$$

$$Q^{\mu\nu} = \frac{1}{2}(g^{\mu\alpha} g^{\nu\beta} + g^{\nu\alpha} g^{\mu\beta}) - \frac{1}{4} g^{\mu\nu} g^{\alpha\beta}$$

The 9 FF of spin-1 corresponds to the following structures:

$$\left.\begin{array}{l} I^{\mu\nu},\ P^{\mu} I^{\nu P}, \Delta^{\mu} I^{\nu\Delta} \\ P^{\{\mu} D^{\nu\}} \\ Q^{\mu\nu},\ \Delta^{\{\mu} Q^{\nu\}\Delta},\ \eta^{\mu\nu} Q^{\Delta\Delta},\ P^{\mu} P^{\nu} Q^{\Delta\Delta},\ \Delta^{\mu}\Delta^{\nu} Q^{\Delta\Delta} \end{array}\right\}$$

linear combination of

$$\mathcal{G}_1(t) \quad \vdots \quad \mathcal{G}_9(t)$$

All the other possible multipoles are not independent on the previous ones

Ongoing…

[SC, Lorcé, Lowdon,in preparation]

- Spin-multipoles and regular counting provide the same number of FFs ✓
- There exist several non-trivial identities at the level of the Lorentz generators that relate the different structures ✓
- Find the complete parametrization for the EMT, where all the structures are linearly independent

Summary and Outlook

- The EMT matrix element is a crucial object to study hadrons.
- Exclusive processes give (indirect) access to gravitational FFs
- Characterizing the EMT for spin higher than 1/2 is useful to understand how the number of structures and whether the sum rules are spin-dependent
- There exist constraints on the FF that are universal for all spin-states
- A parametrization of the EMT for arbitrary spin will allow to derive relations and sum rules that are valid independently of the spin
- Such relations can be tested in experiments involving different hadrons