Polarized parton distribution functions from lattice QCD

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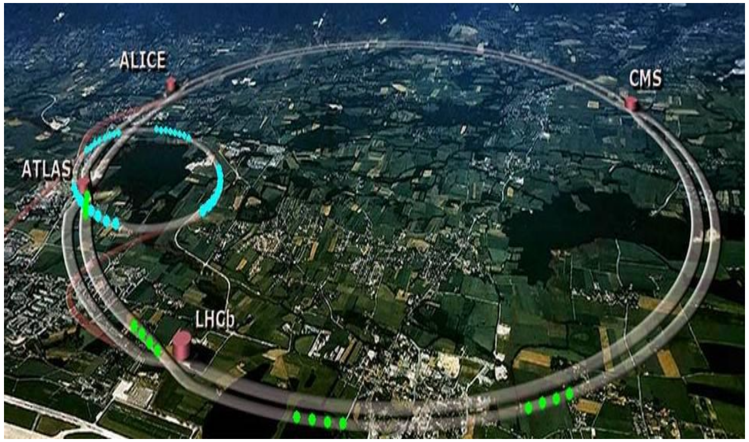
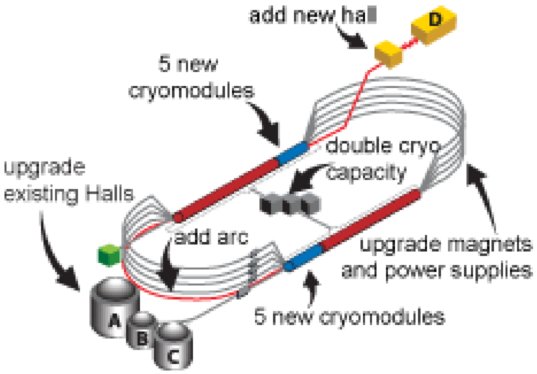
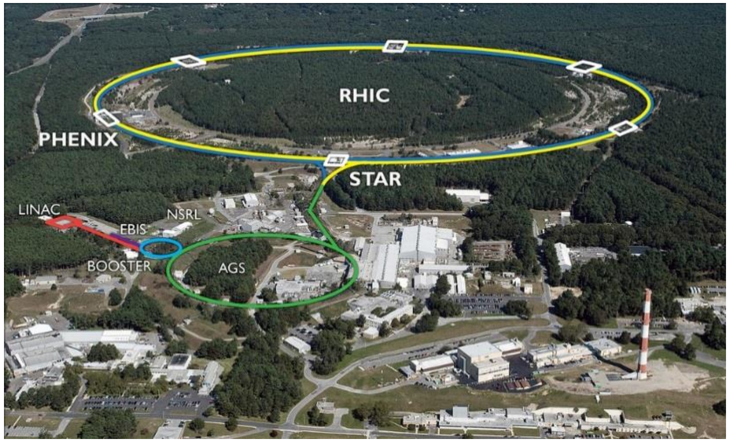
Introduction

- Exploring the partonic structure of hadrons is an important goal of high energy experiments

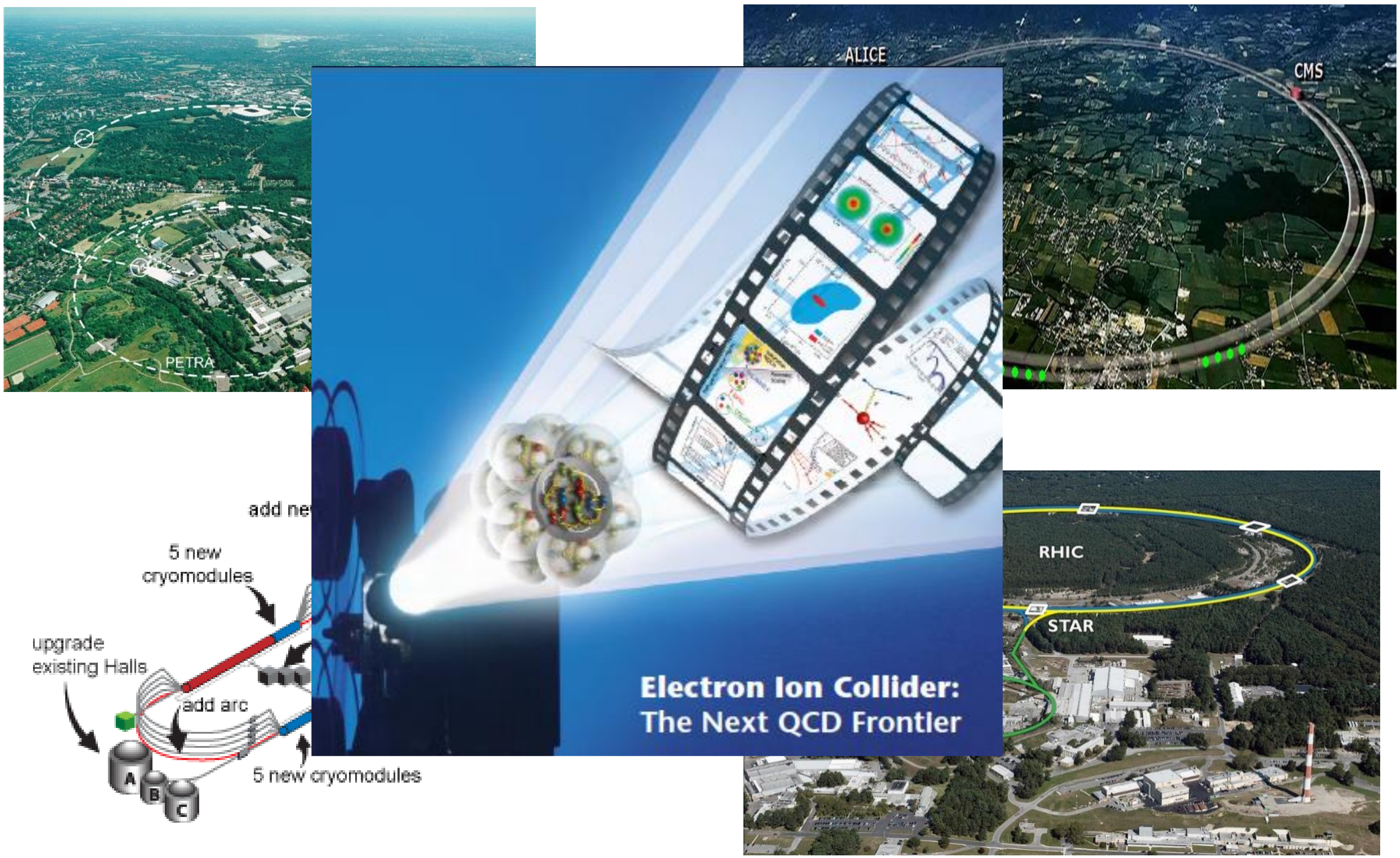






Introduction

- Exploring the partonic structure of hadrons is an important goal of high energy experiments



Introduction

- Extracting information on the partonic structure of hadrons from the experimental data relies on QCD factorization
  - e-H:

$$F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$$

Pert. calculable coefficient function

Universal nonperturbative PDF

  - H-H:

$$\frac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \frac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$$

  - DGLAP evolution

$$\frac{\partial f(x, \mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$

  - Global analysis

Introduction

- Extracting information on the partonic structure of hadrons from the experimental data relies on QCD factorization
  - In contrast to the unpol. PDFs, the pol. ones

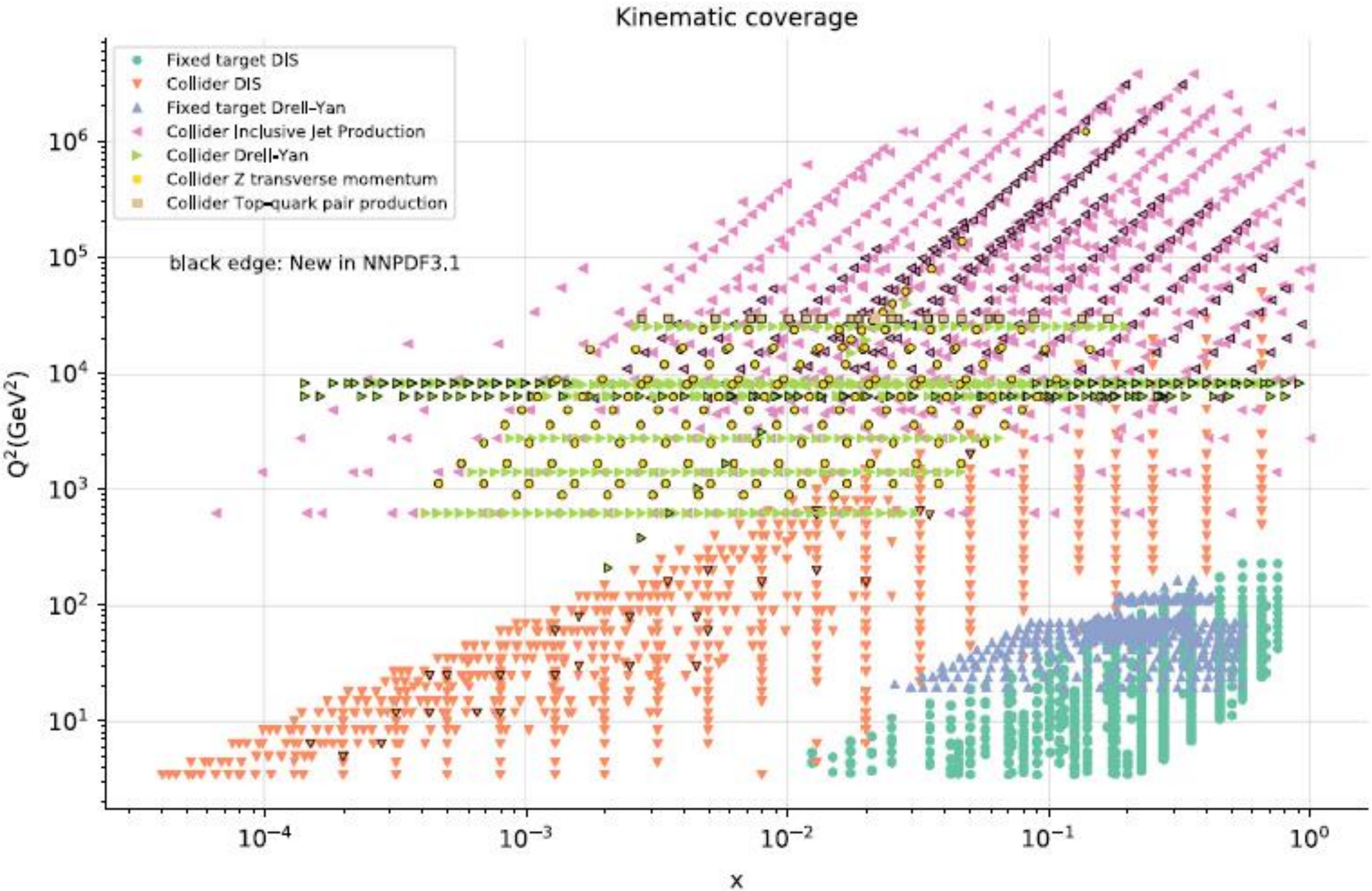
$$\Delta f(x, \mu^2) \equiv f^{\rightarrow}(x, \mu^2) - f^{\leftarrow}(x, \mu^2)$$

    play an important role in understanding the spin structure of the nucleon

  - But their determination is much less precise than the unpol. PDFs

    Limited
    - data points
    - kinematic coverage
    - available hard scattering processes

Introduction



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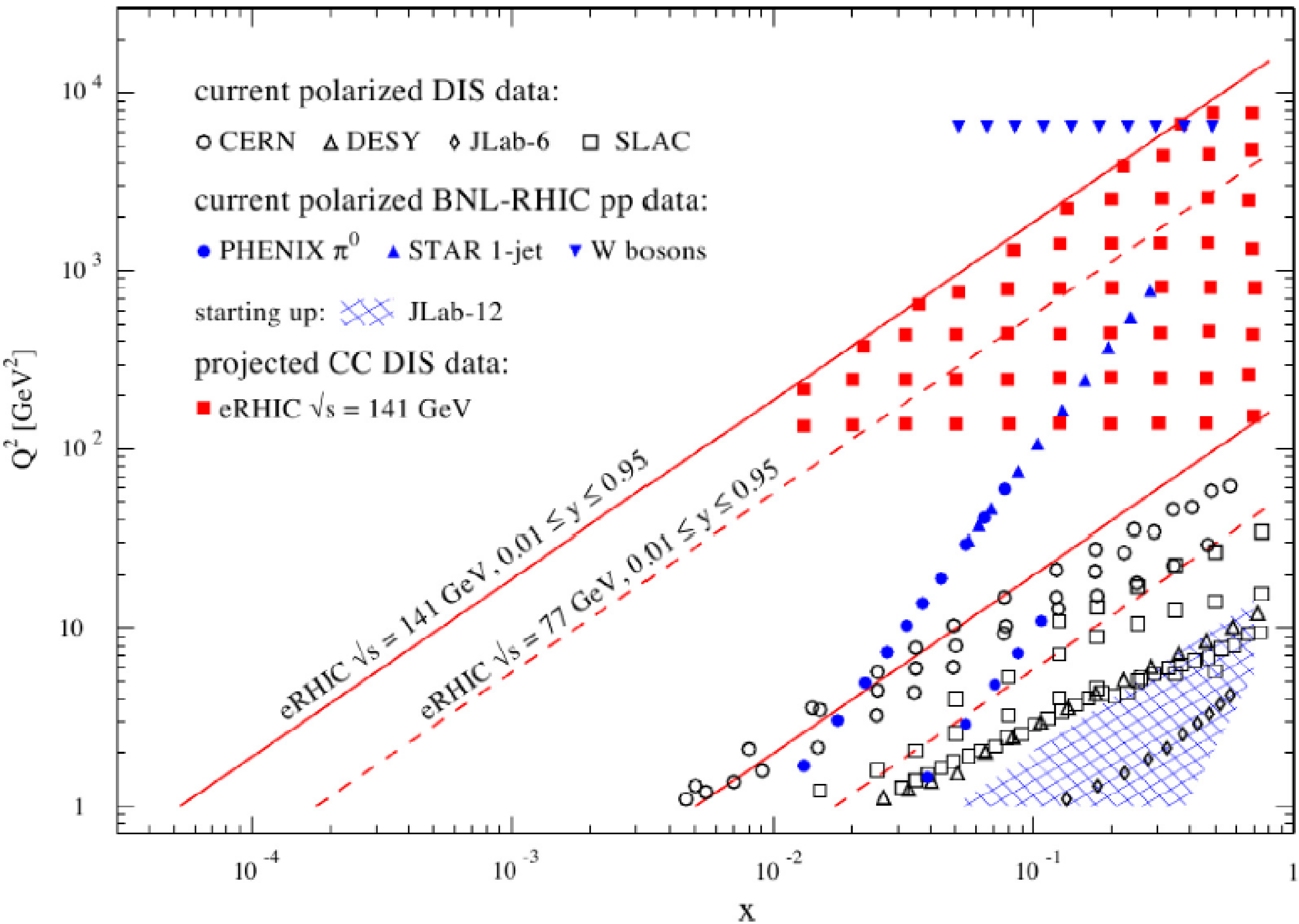

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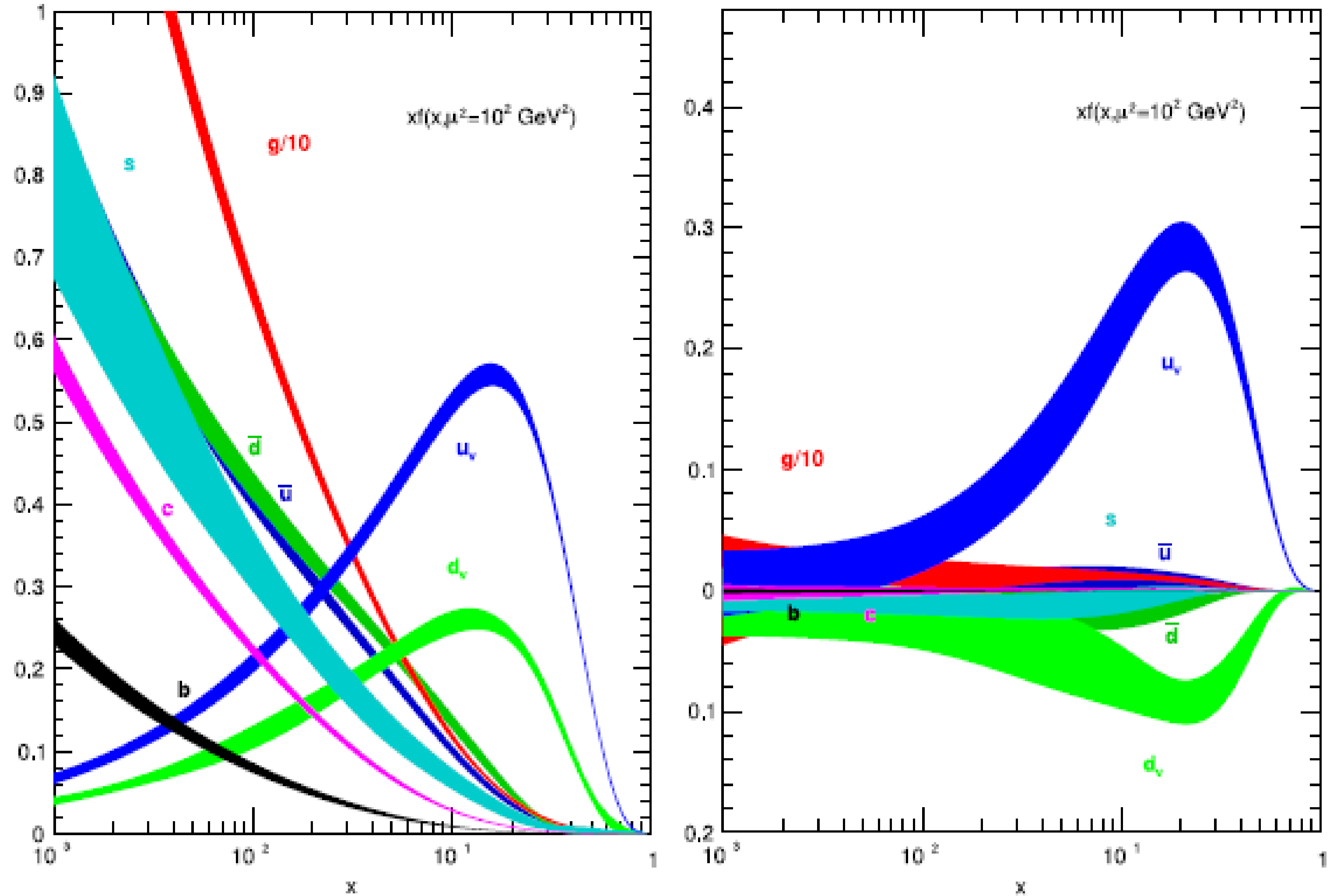
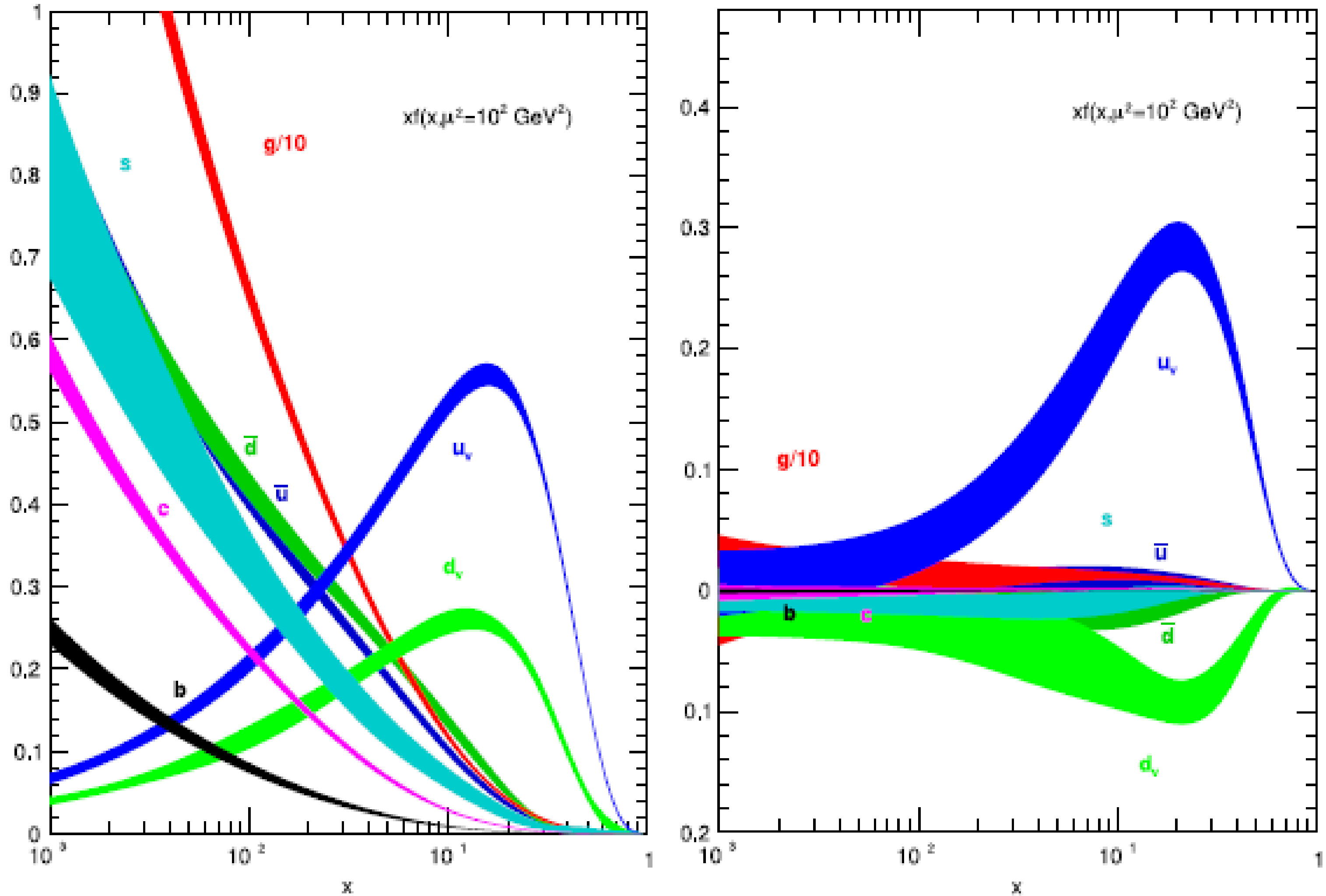


Global analysis (unpol. vs pol.) [Lin et al, Prog.Part.Nucl.Phys. 17′]

Introduction



Lattice QCD can be complementary!

Introduction

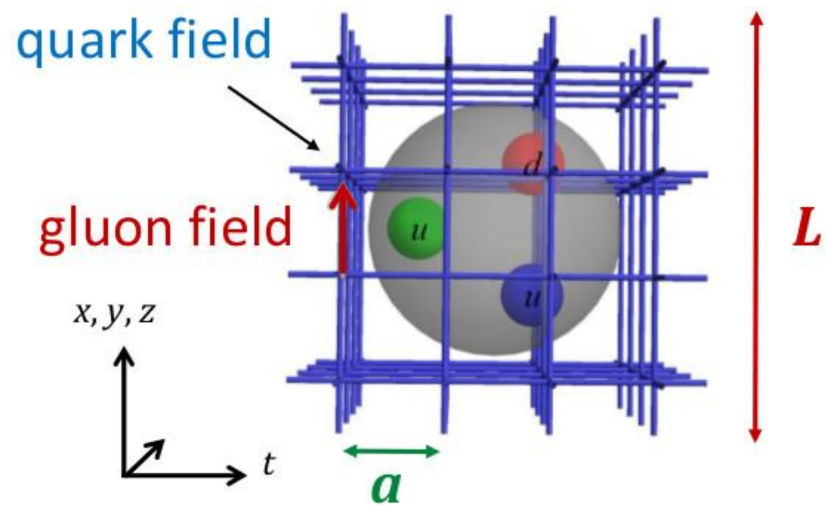
- Lattice QCD can help QCD global analysis
  - It is an ideal *ab initio* theoretical tool to investigate physics in strong-coupling regime
  - Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O\left(\bar{\psi},\psi,A\right)$$

in **Euclidean** space

$$t = -i\tau, \qquad e^{iS_M} \to e^{-S_E}$$

  - In analogy with statistical systems
  - Can be studied with Monte Carlo methods



Discrete spacetime

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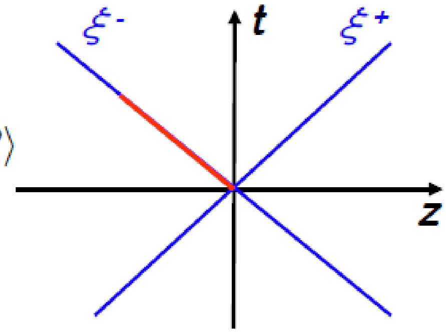
in **Euclidean** space

$$t = -i\tau, \qquad e^{iS_M} \to e^{-S_E}$$

  - Cannot be used to directly access quantities with real-time dependence, such as the PDFs

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

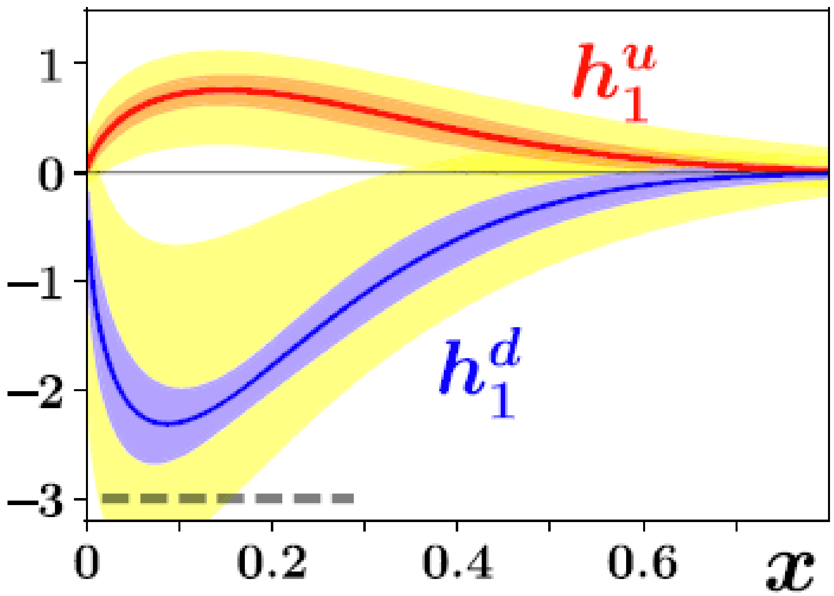


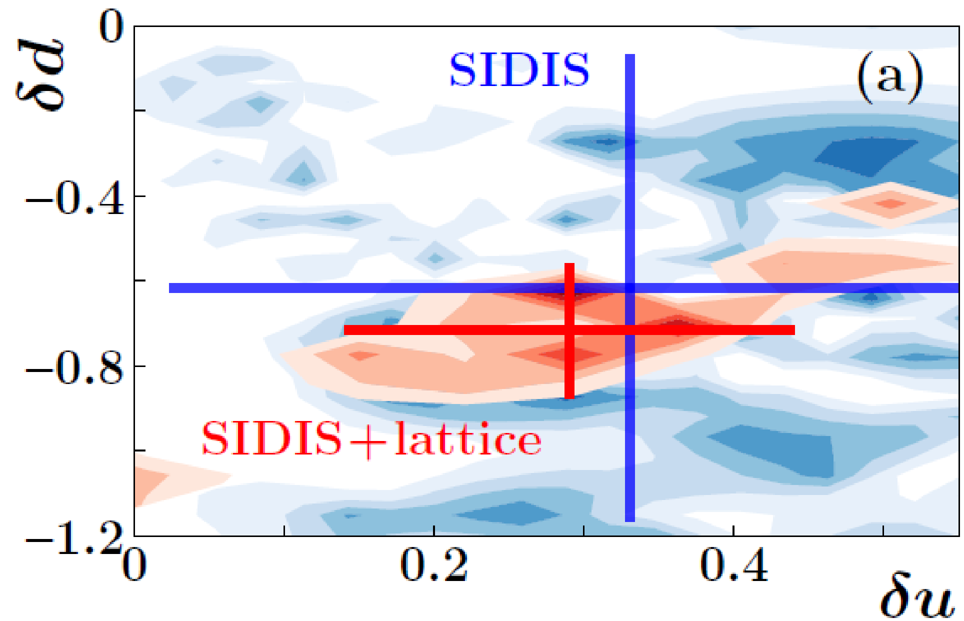
  - But their (first few) moments can be computed

$$\langle x^n(\mu^2)\rangle = \int_{-1}^{1} x^n q(x,\mu^2)$$

Introduction

- Lattice QCD can help QCD global analysis
  - Nucleon transversity distribution with lattice g_T [Lin et al, PRL 18′]





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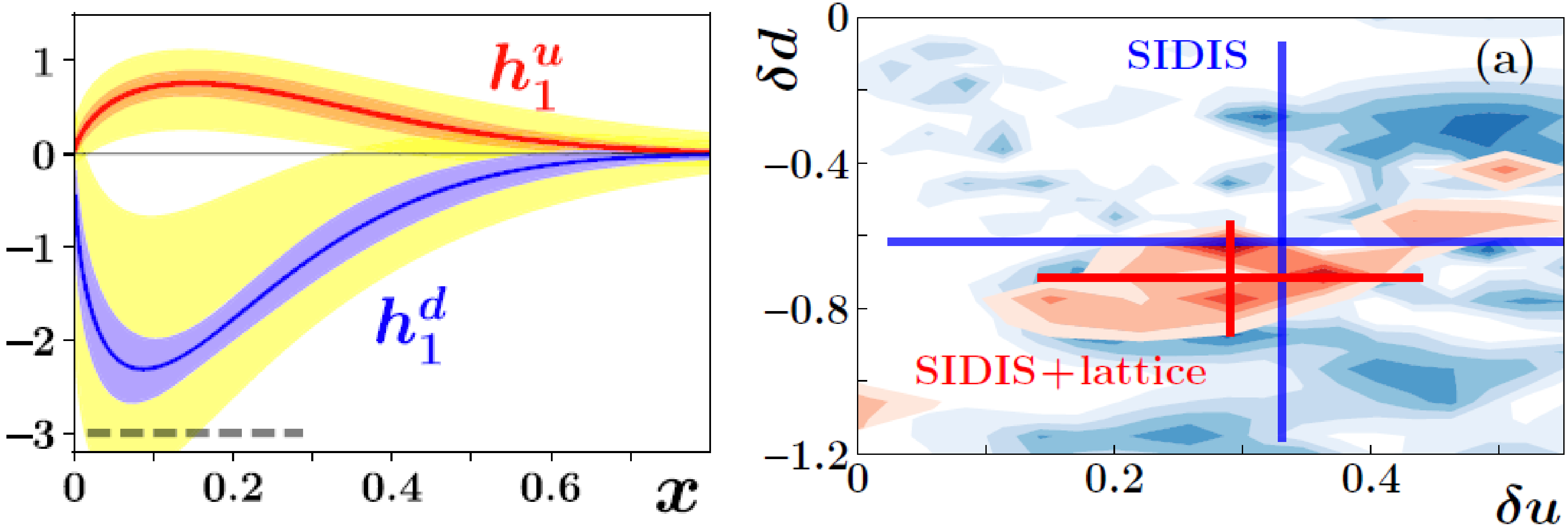


- Recently, also direct computation of PDFs from lattice QCD becomes possible (Large momentum effective theory [Ji, PRL 13′, Sci. China Phys. Mech. Astron. 14′])

PDFs from lattice

- PDFs are difficult to access on the lattice
  - Defined on the light-cone
  - Example [Collins and Soper, NPB 82′] ($\xi^{\pm} = (t \pm z)/\sqrt{2}$)

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

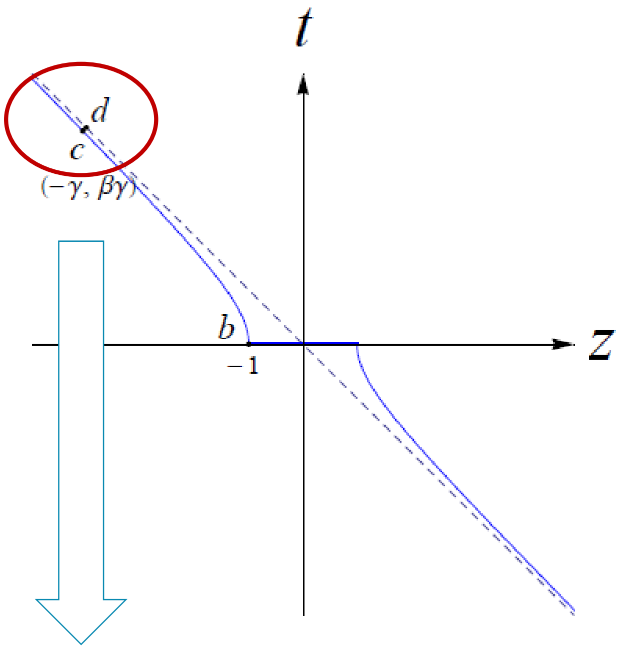
$$t^2 - x^2 = 0 \rightarrow -\tau^2 - x^2 = 0$$

- However, they were originally introduced by Feynman as the infinite momentum limit of frame-dependent quantities

$$q(x) = \lim_{P_z \to \infty} \tilde{q}(x, P_z)$$

- Boost to infinite momentum leads to light-cone correlations
- If we can construct a $\tilde{q}(x, P_z)$ such that it is calculable on the lattice, and all P_z-dependence can be systematically removed
- Then we can calculate $q(x)$!

PDFs from lattice



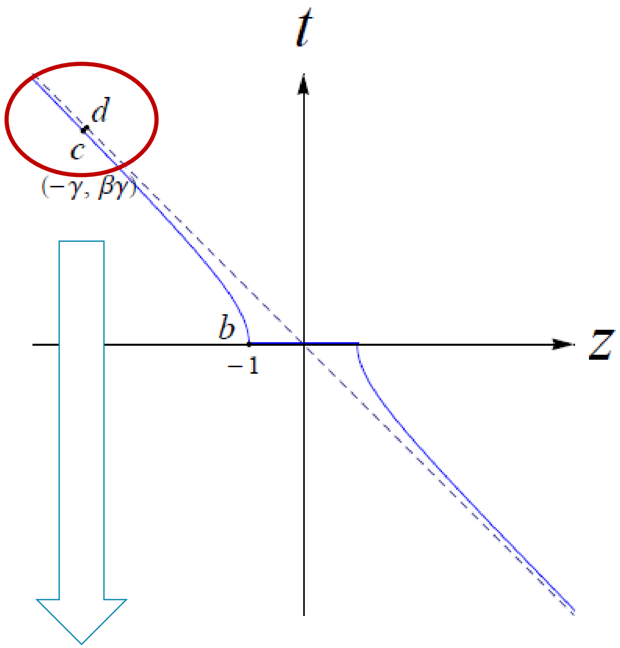
- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13′, Sci. China Phys. Mech. Astron. 14′]
  - Appropriately chosen $\tilde{q}(x, P_z)$ can be calculated on the Euclidean lattice, e.g.

$$\tilde{q}(x,\Lambda,P^z)=\int_{-\infty}^{\infty}\frac{dz}{4\pi}e^{izk^z}\langle P|\overline{\psi}(0,0_\perp,z)\gamma^z\exp\left(-ig\int_0^z dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

  - A finite but large P_z already offers a good approximation, where (leading) frame-dependence can be removed through a factorization formula

$$\tilde{q}(x,P_z,p_z^R,\mu_R)=\int_{-1}^{1}\frac{dy}{|y|}C\left(\frac{x}{y},r,\frac{yP_z}{\mu},\frac{yP_z}{p_z^R}\right)q(y,\mu)+\mathcal{O}\left(\frac{M^2}{P_z^2},\frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

PDFs from lattice



- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']

- Parton model is an effective theory for the nucleon moving at large momentum

Other proposals

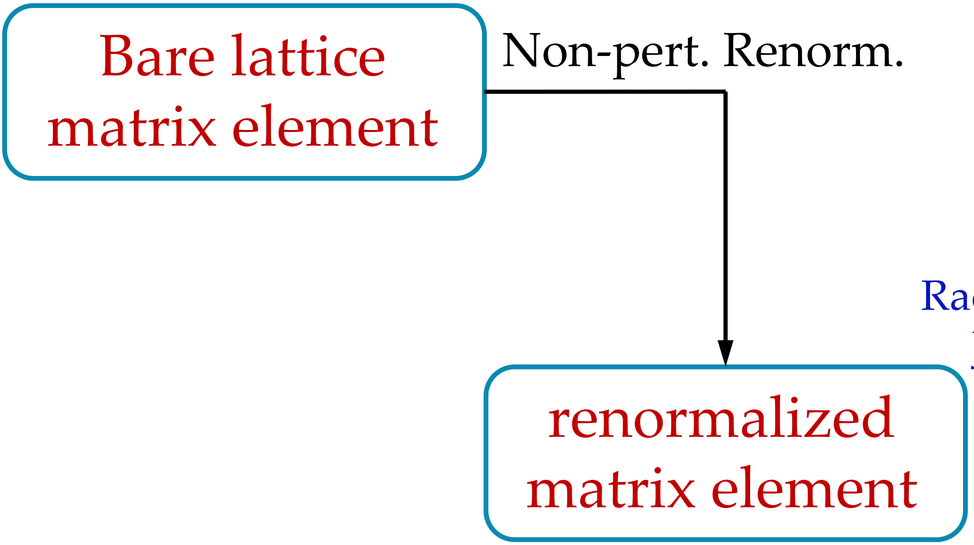
- Current-current correlation functions
- [Liu and Dong, PRL 94']
- [Detmold and Lin, PRD 06']
- [Braun and Müller, EPJC 08']
- [Davoudi and Savage, PRD 12']
- [Chambers et al., PRL 17']

- Lattice cross sections
- [Ma and Qiu, 14' & PRL 17']

- Ioffe-time /pseudo-distribution
- [Radyushkin, PRD 17']

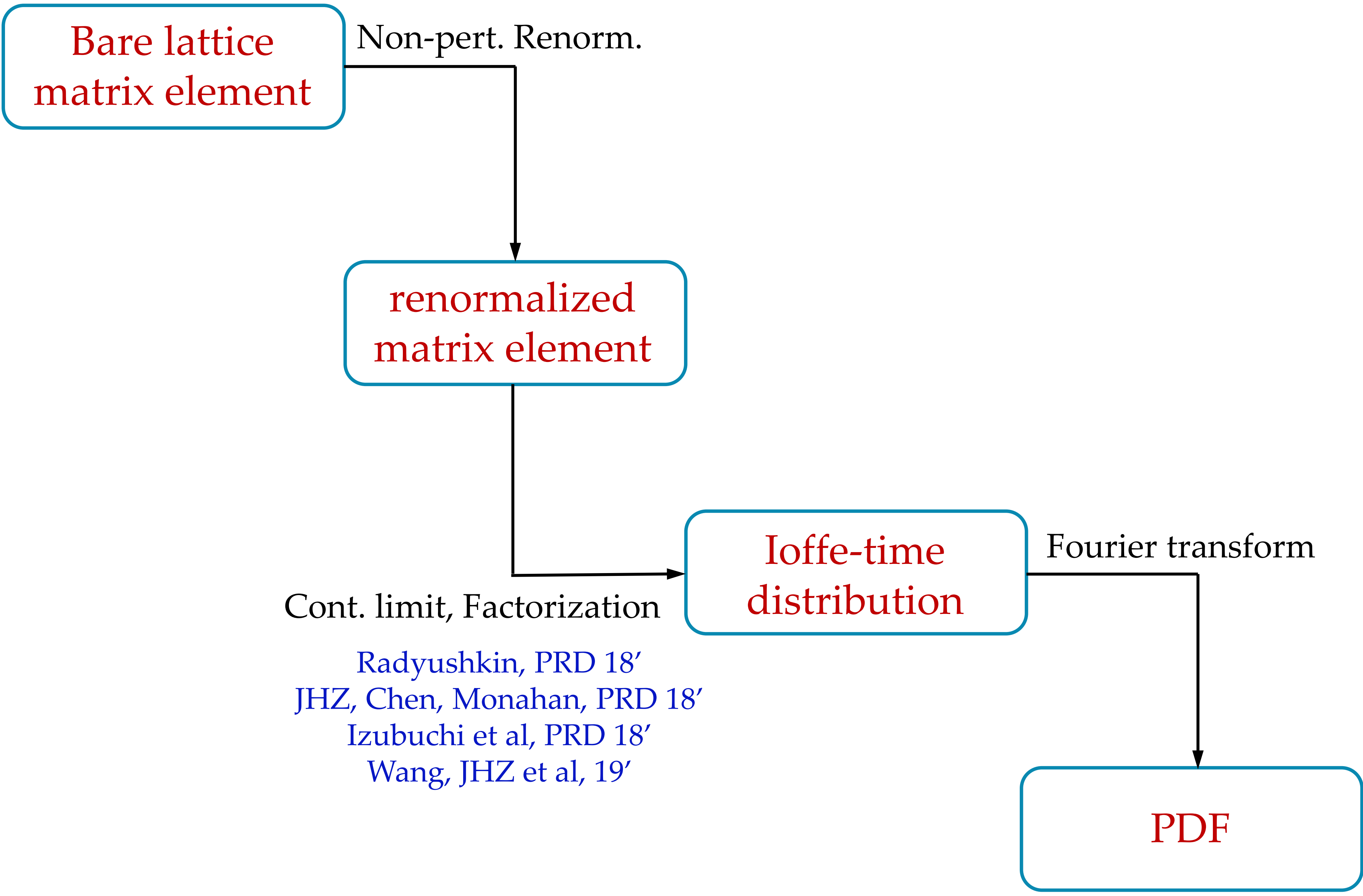
- **Mostly share similar spirit of computing correlations at spacelike separations**

PDFs from LaMET

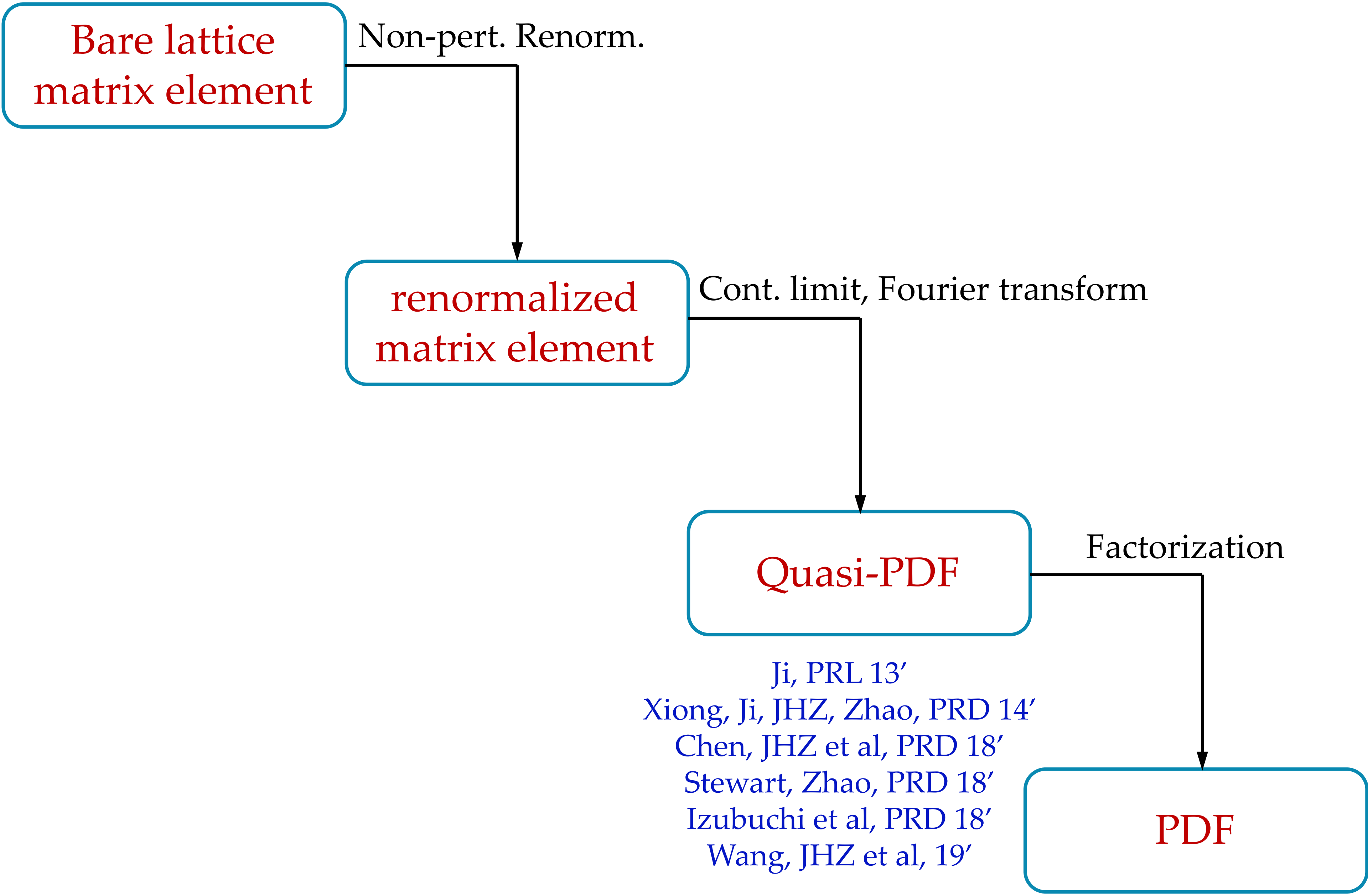


Ji, JHZ, Zhao, PRL 18'
Ishikawa et al, PRD 17'
Green et al, PRL 18'
Stewart, Zhao, PRD 18'
Chen, JHZ et al, PRD 18'
Alexandrou et al, NPB 17'
Monahan, Orginos, JHEP 17'
Radyushkin PRD 17' & Orginos et al, PRD 17'
JHZ et al, PRL 19' & Wang, JHZ et al, 19'
Li et al, PRL 19'

PDFs from LaMET



PDFs from LaMET



Polarized gluon quasi-PDFs

- Within LaMET, extensive studies have been carried out for unpolarized and polarized quark PDFs

- Let us focus on polarized gluon PDF [Collins, Soper, NPB 82']

$$\Delta f_{g/H}(x,\mu) = i\epsilon_{\perp ij}\int \frac{d\xi^-}{2\pi x P^+} e^{-i\xi^- x P^+}\langle P|F^{+i}(\xi^- n_+)\mathcal{W}(\xi^- n_+, 0; L_{n_+})F^{j+}(0)|P\rangle$$

- Naively expected gluon quasi-PDF operators
  - Replace + component of $\{F^{+i}, F^{j+}\}$ by $\{z, t\}$
  - In general, they might mix with other operators under renormalization

  - Auxiliary field approach [Dorn, Fortsch. Phys. 86', Ji, JHZ, Zhao, PRL 18']

$$\mathcal{L} = \mathcal{L}_{\rm QCD} + \overline{\mathcal{Q}}(x) i n\cdot D\mathcal{Q}(x)$$

  - For a space like n, no dynamical evolution for $\mathcal{Q}$
  - The two-point function of $\mathcal{Q}$ is

$$\int \mathcal{D}\overline{\mathcal{Q}}\mathcal{D}\mathcal{Q}\,\mathcal{Q}(x)\overline{\mathcal{Q}}(y)e^{i\int d^4x\mathcal{L}} = S_{\mathcal{Q}}(x,y)e^{i\int d^4x\mathcal{L}_{\rm QCD}} \qquad n\cdot D\, S_{\mathcal{Q}}(x,y) = \delta^{(4)}(x-y)$$

Polarized gluon quasi-PDFs

- Solution

$$S_Q(x,y) = \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x,y)$$
$$= \theta(x^z - y^z)\delta(x^0 - y^0)\delta^{(2)}(\vec{x}_\perp - \vec{y}_\perp)L(x^z, y^z)$$

  - δ-function ensures that the time and transverse components are equal, and therefore generates a spacelike Wilson line

  - The non-local gluon quasi-PDF operator can be replaced by a product of two local composite operators, e.g.

$$\Delta\mathcal{O}_g^{ij}(z_2, z_1) = J_1^{ti}(z_2)\bar{J}_1^{jz}(z_1)$$

$$J_1^{ti}(z_2) = F_a^{ti}(z_2)\mathcal{Q}_a(z_2), \qquad \bar{J}_1^{jz}(z_1) = \bar{\mathcal{Q}}_b(z_1)F_b^{jz}(z_1)$$

  - After integrating out $\mathcal{Q}$, the gluon quasi-PDF is recovered

Polarized gluon quasi-PDFs

- Local operator mixing [Joglekar, Lee, Annals Phys. 76′, Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

  - For $J_1^{\mu\nu}$, the operators allowed to mix are

$$J_2^{\mu\nu} = n_\rho (F_a^{\mu\rho} n^\nu - F_a^{\nu\rho} n^\mu) \mathcal{Q}_a / n^2,$$
$$J_3^{\mu\nu} = (-in^\mu A_a^\nu + in^\nu A_a^\mu)((in \cdot D - m)\mathcal{Q})_a / n^2,$$

  - The mass term is not forbidden in a cutoff regularization such as lattice regularization

  - General mixing pattern

$$\begin{pmatrix} J_{1,R}^{\mu\nu} \\ J_{2,R}^{\mu\nu} \\ J_{3,R}^{\mu\nu} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ 0 & 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{\mu\nu} \\ J_2^{\mu\nu} \\ J_3^{\mu\nu} \end{pmatrix},$$

Polarized gluon quasi-PDFs

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$$J_3^{\mu\nu} = (-i n^\mu A_a^\nu + i n^\nu A_a^\mu)((i n \cdot D - m)\mathcal{Q})_a / n^2,$$

  - The mass term is not forbidden in a cutoff regularization such as lattice regularization

  - Renormalization constants are not all independent

$$\begin{pmatrix} J_{1,R}^{z\mu} \\ J_{3,R}^{z\mu} \end{pmatrix} = \begin{pmatrix} Z_{22} & Z_{13} \\ 0 & Z_{33} \end{pmatrix} \begin{pmatrix} J_1^{z\mu} \\ J_3^{z\mu} \end{pmatrix}, \qquad J_{1,R}^{ti} = Z_{11} J_1^{ti}, \quad J_{1,R}^{ij} = Z_{11} J_1^{ij}$$

  - Different components have different renormalization due to Lorentz symmetry breaking

Polarized gluon quasi-PDFs

- Local operator mixing [Joglekar, Lee, Annals Phys. 76′, Collins, Renormalization]
  - Gauge-invariant operators
  - BRST exact operators
  - Operators that vanish by equation of motion

- We can identify appropriate building blocks and use them to construct multiplicatively renormalizable polarized gluon quasi-PDFs

$$\Delta O_g^1(z,0) = i\epsilon_{\perp,ij} F^{ti}(z_2) \mathcal{W}(z_2,z_1) F^{tj}(z_1),$$
$$\Delta O_g^2(z,0) = i\epsilon_{\perp,ij} F^{zi}(z_2) \mathcal{W}(z_2,z_1) F^{zj}(z_1),$$
$$\Delta O_g^3(z,0) = i\epsilon_{\perp,ij} F^{ti}(z_2) \mathcal{W}(z_2,z_1) F^{zj}(z_1),$$

- The renormalization comes from the endpoint renormalization and the Wilson line mass renormalization

RI/MOM scheme

- Nonlocal quasi-PDF operators at different *z* do not mix under renormalization. Two ways to perform renormalization:
  - Calculate the endpoint renormaliztion factors and the Wilson line mass counterterm nonperturbatively
  - Calculate the renormalization factors as a whole for each *z* (RI/MOM)

- Inserting gluon (quark) quasi-PDF operators into a quark (gluon) state yields finite mixing

- Taking it into account in RI/MOM renormalization helps improve convergence in the implementation of the matching

$$\begin{pmatrix} O_g^{(n)}(z,0) \\ O_q^s(z,0) \end{pmatrix} = \begin{pmatrix} Z_{11}(z) & Z_{12}(z)/z \\ zZ_{21}(z) & Z_{22}(z) \end{pmatrix} \begin{pmatrix} O_{g,R}^{(n)}(z,0) \\ O_{q,R}^s(z,0) \end{pmatrix},$$

with

$$O_q^s(z_1,z_2) = 1/2[\bar{q}_i(z_1)\Gamma W(z_1,z_2)q_i(z_2) - (z_1 \leftrightarrow z_2)]$$

RI/MOM scheme

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  - Calculate the endpoint renormaliztion factors and the Wilson line mass counterterm nonperturbatively
  - Calculate the renormalization factors as a whole for each *z* (RI/MOM)

- RI/MOM renormalization condition [Wang, JHZ et al, 19']

$$\left.\frac{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_R}{\mathrm{Tr}[\Lambda_{22}(p,z)\mathcal{P}]_{\mathrm{tree}}}\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=1,\qquad \left.\frac{[P_{ij}^{ab}\Lambda_{11}^{ab,ij}(p,z)]_R}{[P_{ij}^{ab}\Lambda_{11}^{ab,ij}(p,z)]_{\mathrm{tree}}}\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=1,$$

$$\left.\mathrm{Tr}[\Lambda_{12}(p,z)\mathcal{P}]_R\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=0,\qquad \left.[P_{ij}^{ab}\Lambda_{21}^{ab,ij}(p,z)]_R\right|_{\substack{p^2=-\mu_R^2\\ p_z=p_z^R}}=0,$$

$$h_{g,R}^{(n)}(z,P^z,\mu_R,p_z^R)=\bar{Z}_{11}(z,\mu_R,p_z^R,1/a)h_g^{(n)}(z,P^z,1/a)+\bar{Z}_{12}(z,\mu_R,p_z^R,1/a)/z\ h_q^s(z,P^z,1/a),$$
$$h_{q,R}^{s}(z,P^z,\mu_R,p_z^R)=\bar{Z}_{22}(z,\mu_R,p_z^R,1/a)h_q^s(z,P^z,1/a)+z\bar{Z}_{21}(z,\mu_R,p_z^R,1/a)\ h_g^{(n)}(z,P^z,1/a).$$

with

$$\bar{\mathcal{Z}}=\begin{pmatrix}\bar{Z}_{11}(z) & \bar{Z}_{12}(z)/z\\ z\bar{Z}_{21}(z) & \bar{Z}_{22}(z)\end{pmatrix}=\begin{pmatrix}Z_{11}(z) & Z_{12}(z)/z\\ zZ_{21}(z) & Z_{22}(z)\end{pmatrix}^{-1}$$

Factorization and matching

- Coordinate space

$$\tilde{h}_{q_i,R}(z,P^z,\mu) = \int_{-1}^{1} du\, \mathcal{C}_{q_iq_j}(u,\mu^2z^2) h_{q_j}(u\nu,\mu) + \int_{-1}^{1} du\, \mathcal{C}_{qg}(u,\mu^2z^2) h_g(u\nu,\mu).$$

$$\tilde{h}_{g,R}(z,P^z,\mu) = \int_{-1}^{1} du\, \frac{\mathcal{C}_{gg}(u,\mu^2z^2)}{\nu} h_g(u\nu,\mu) + \int_{-1}^{1} du \frac{\mathcal{C}_{gq}(u,\mu^2z^2)}{\nu} h_{q_i}(u\nu,\mu).$$

- Momentum space

$$\begin{aligned}
\tilde{f}^{(n)}_{g/H}(x,P^z,p_z^R,\mu_R) &= \int_{-1}^{1} \frac{dy}{|y|} \Big[C_{gg}\Big(\frac{x}{y},\frac{\mu_R}{p_z^R},\frac{yP^z}{\mu},\frac{yP^z}{p_z^R}\Big) f_{g/H}(y,\mu) + C_{gq}\Big(\frac{x}{y},\frac{\mu_R}{p_z^R},\frac{yP^z}{\mu},\frac{yP^z}{p_z^R}\Big) f_{q_j/H}(y,\mu)\Big] \\
&\quad + \mathcal{O}\Big(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big), \\
\tilde{f}_{q_i/H}(x,P^z,p_z^R,\mu_R) &= \int_{-1}^{1} \frac{dy}{|y|} \Big[C_{q_iq_j}\Big(\frac{x}{y},\frac{\mu_R}{p_z^R},\frac{yP^z}{\mu},\frac{yP^z}{p_z^R}\Big) f_{q_j/H}(y,\mu) + C_{qg}\Big(\frac{x}{y},\frac{\mu_R}{p_z^R},\frac{yP^z}{\mu},\frac{yP^z}{p_z^R}\Big) f_{g/H}(y,\mu)\Big] \\
&\quad + \mathcal{O}\Big(\frac{M^2}{(P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\Big),
\end{aligned} \tag{2.53}$$

- Perturbative matching coefficients have been available at one-loop

- Can be applied to computing the polarized gluon PDF, which in turn allows to determine the contribution of gluon helicity to proton spin

Summary and outlook

- Global analysis for polarized PDFs is much less precise than that for unpolarized ones

- Lattice QCD can be complementary, applying recent methodology to nucleon quark PDFs has yielded encouraging results

- Gluon PDF (pol. and unpol.)
  - Appropriate gluon quasi-PDF operators identified
  - Renormalization and factorization understood
  - Perturbative matching available at 1-loop

  - Awaiting systematic implementation on the lattice